ADAM

- a model of the Danish economy

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Foreword

The first version of the macroeconomic ADAM model was solved in 1972. At that time, solving the model equations was a challenge. Today, ADAM is continuously used for a wide range of economic calculations by the economic ministries and a number of private users, including banks and interest groups.

ADAM's equations and variables are described on ADAM's website, and model users are offered courses in using ADAM. We have found that there is also a demand for a comprehensive book-form description of the model and its properties.

The present book describes specifically the model version that was solved for the first time in December 2009. With the december09 version, ADAM has been made smaller by reducing the number of industries from 19 to 12 and by splitting demand and import into fewer variables.

The production of the book represents a joint effort of all employees in Statistics Denmark's model group. At the time of writing, the group consisted of head of division Asger Olsen, consultants Tony M. Kristensen, Grane Høgh and Dan Knudsen, economists Kristian Søfeldt Engelund-Mikkelsen, Michael Osterwald-Lenum, Jacob N. Rasmussen and Dawit Sisay and research assistants Sofie Andersen, Marcus Mølbak Ingholt, Andreas Østergaard Iversen, Ralph Bøge Jensen and Mathias S. Matzen.

Dan Knudsen has edited the book with the assistance of Per Svensson.

Statistics Denmark, April 2013

Jan Plovsing / Asger Olsen

Contents

1. Introduction

ADAM (Annual Danish Aggregate Model) is a macroeconomic model of the Danish economy. ADAM represents the traditional synthesis between Keynesian and neo-classical theory. This means that in the short term production and employment are determined by the demand side, and in the long term they are determined by the supply side. ADAM is an empirical model, and most of the behavioural equations are estimated on national accounts data. Moreover, ADAM is characterized by being a large and fairly disaggregated model.

Use of ADAM

The purpose of ADAM is to have a tool for analysing the Danish economy. Main users of the model are the economic ministries, but ADAM is also used by some banks and interest organizations. The model is used mainly to assess the impact of economic policy interventions, but it is also used for economic projections, both short and long term.

For instance, the economic ministries use ADAM for short-term forecasting and short-term assessments but also for the long-term scenarios used in the annual convergence programs. It can also be mentioned that ADAM was used by the Danish Government's Commission on Climate Change in assessing the economic consequences of the Commission's energy policy scenarios until 2050. ADAM is formulated and maintained by the model group in Statistics Denmark.

Formulating ADAM

ADAM's behavioural equations are estimated separately before being inserted into ADAM, where they interact with a variety of definitions and non-estimated equations. This is a traditional approach to formulating macroeconomic models. The advantage should be that the behavioural equations are realistic and have good empirical properties. On the other hand, the overall model structure may be difficult to interpret and therefore difficult to use if the estimated equations do not have the expected theoretical properties, or if the equations do not work well together.

Emphasis on model interpretation

This interpretational challenge can concern both the long-run equilibrium and the dynamics of the model. In practice, most macroeconomic modelling comprise a phase of interpretation where the modeller goes back and try to re-estimate or re-formulate equations that makes it difficult to interpret the model. This means that the model equations are only accepted when it is possible to understand the whole model.

In the following presentation of ADAM, weight has been placed on interpreting the model properties, which are illuminated by a number of model calculations. The previous description of the ADAM model is found in Dam (ed.) (1996), which also mentions the development of ADAM since the beginning of the 1970s.

ADAM is a large model

With the present model version, a step has been taken towards making ADAM smaller and simpler by reducing the number of industries from

19 to 12 and by splitting demand and import into fewer variables. However, ADAM remains a large model. More specifically, the current ADAM version contains about 3500 variables of which 2500 variables are endogenous and 1000 are exogenous. Out of the 2500 equations explaining the endogenous variables, about 90 are estimated behave-oural equations, while the rest are identities or technical relations providing for example income tax and VAT revenues for the public sector or materials for industries in the input-output system of the model. The following presentation of ADAM describes model version Dec09, which was assembled for the first time in December 2009. The estimation periods of behavioural equations extend to 2007, which was the last year with final national accounts figures, when the model equations were estimated.

The number of variables and equations reflect that the input-output system of the 12 industries creates several additional variable and auxiliary equations to capture the many flows between industries and demand components. In addition, the model contains a large number of variables and equations to describe public finances, which are of special interest to model users in the economic ministries.

Focus on important equations and properties

In the following chapters, the presentation of ADAM is focused on explaining main relations and important model properties, including long-term properties. All model equations are not equally important, and many of them are similar, implying that the presentation will not cover all equations and all variables of the model.

Reading guide

The following chapters 2 to 11 are organized as follows: Chapter 2 gives an overview of the main features of ADAM. Chapters 3 to 7 present behavioural equations and the input-output system of ADAM. Chapters 8 and 9 explain the modelling of public finances and financial wealth, respectively. Chapter 10 describes a stylized ADAM baseline scenario, and chapter 11 illustrates model properties by means of calculations on the model.

If you are only interested in overall model properties, it suffices to read chapters 2 and 11. To some extent, chapters 10 and 11 refer to other chapters for description of particular model equations.

Use of variable names in the book

In all chapters, it is sought to limit the use of ADAM variable names, and all variables are explained when they occur for the first time. In sections on estimation of equations and in the discussion of model calculations, we make use of the actual ADAM names of the variables concerned, but especially in the presentation of individual equations and their properties, we have often chosen to shorten the ADAM name or to use, e.g. 'price' instead of the actual name of the price variable.

Notation of ADAM variables

Basic classification of a variable in ADAM is expressed by a single letter, which is written in capitals, when used in the name of a flow or a stock, and small letters in names of other variables, i.e. prices, interest rates and other rates etc. The classifying letters reflect the usual text-book standards. For example, *C* is consumption, *I* investment, *E* export, *M* import, *Y* domestic product or national income, *Q* employment, *S* tax, *T* transfer, *X* production, *K* capital stock, *p* is price, and *i* is interest rate.

An 'f' as the first letter of the name indicates that the variable is in fixed prices, chain volumes. The term "chain volumes" refers to the fixed price calculation in the national accounts, which is based on chain indices. In what follows, we shall often just refer to "fixed prices" or "quantities" and only mention that the variable is based on chain volumes when it is important. The chain index formula used is reported in box 3.8 in chapter 3. A 'p' as the first letter indicates that the variable is a price.

A classifying letter is usually followed by a suffix defining the variable. For instance, private consumption is called *Cp* in current prices, and public consumption is *Co*. The price of private consumption is called *pcp*, and private consumption in fixed prices is *fCp*.

In the equations presented, Dif and Dlog are operators indicating simple and logarithmic change, respectively. If a variable is lagged, the lag is placed in subscript, C_{-1} is the consumption in the previous year, and subscript -½ indicates a half-year lag, $C_{-½} = ½\cdot(C+C_{-1})$.

2. Presentation of ADAM

ADAM was originally conceived as a short-term macro model, where the economic development is driven by demand, while the supply side plays a minor role. Description of the business cycle and short-term fluctuations continues to play a significant role in ADAM. However, over the years there has been a focus on developing the long-term properties where the supply side is crucial, and today it is important that ADAM has a meaningful long-term solution.

The focus on long-term properties has been driven by the model users, who have applied ADAM for long-term projections and calculations. Besides, having a relevant long-term solution can improve the dynamic properties of the model.

The short term is still important in ADAM, which continues to be used as a short-term model and the need for analysing the business cycle has increased in recent years due to the financial crisis and the accompanying economic downturn. It is also an advantage that ADAM covers both the short and long term, because it means that the model describes the transition between the two. In practice, the economy is always in a transition period between the short and the long term.

Briefly put, ADAM can be seen as a compromise between on the one hand, the empirically oriented time series models and on the other hand, the theoretically oriented equilibrium models. As mentioned, the development has moved ADAM in the direction of the equilibrium models without transforming ADAM into a conventional equilibrium model.

The following section outlines the main properties of ADAM. It is followed by a section clarifying some key differences between ADAM and the typical framework of general equilibrium models, both long-term models and dynamic equilibrium models used to analyse the short term. The rest of the chapter presents the properties of ADAM by means of calculations on the model.

2.1 Main properties of ADAM

Demand-determined in the short term

It is important to differentiate between ADAM in the short term and ADAM in the long term. The immediate response of wages is so small that the wage variable can be seen as exogenous in the first year. This implies that ADAM at first works as a Keynesian short-term model where production passively adapts to changes in demand.

Supply-determined in the long term

After some years, the wage reaction and hence the effect on competetiveness becomes large enough, so that not only Danish production but also foreign demand, i.e. exports, respond significantly to a domestic demand shock. In the long term, only exports, not production, respond implying that total demand adapts to match total production, which in the long run is given from the supply side.

List of model properties

The central properties of ADAM are summarized in the following list and in an arrow diagram, figure 2.1, depicting the main structure of the model.

- ADAM describes a small open economy with exogenous exchange rate, so the Danish price increase is in the long term given from abroad.
- The price elasticity of external trade is not infinite. Thus, the price level is not given from abroad.
- The interest rate is exogenous and given from abroad, which is consistent with the exogenous exchange rate and restriction-free capital markets.
- Hourly wages are determined in an augmented Phillips curve. The Phillips curve is not vertical in the long term, but with inflation given from abroad, the model determines an equilibrium unemployment to which the economy returns in the long term. Thus, there is eventually full crowding-out in the labour market.
- The labour force is cyclical but not influenced by the tax level.
- Private consumption is a function of income and wealth. The wealth variable includes housing wealth, which is sensitive to the interest rate, and the income variable excludes savings in pension schemes.
- Production is described by a CES-function with labour, capital, energy and materials as inputs and with a moderate elasticity of substitution. Working hours and the underlying factor productivity are exogenous, while production per worker varies with the capital intensity and with the business cycle.
- The model contains a fully specified input-output system describing the values, quantities and prices of supply and use for:
  - 12 industries
  - 10 types of import
  - 8 types of consumption
  - 4 types of investment
  - 7 types of export.
- The model contains dynamic identities for the formation of wealth and for the relation between capital stock, investment and depreciation. These identities are often important for long-term model properties.
- There is no fiscal reaction function to ensure that public expenditures and revenues balance in the long term.
- Expectations are adaptive or constant.

Model properties can be changed ...

The properties listed above relate to the standard version of ADAM, which is made available to the users, and it is noted that at least some of the model properties are easy to change by introducing calibrated coefficients in the behavioural equations.

For example, calibration can be used in making labour force a decreasing function of the tax level, reflecting that households choose more leisure time if labour income is taxed more heavily. The necessary coefficient can, for instance, be determined on the basis of estimation results on micro data. This kind of calibrated supply effect entered the ADAM scenarios made in 2010 for the Danish Government´s Commission on Climate Change. Calibration is the normal practice in conventional equilibrium models, which often include a calibrated supply effect of taxation as the effect is central in economic theory, but difficult to estimate on macroeconomic time series.

ADAM as arrow diagram

The arrow diagram in figure 2.1 represents a simplified version of the interaction between equations and variables in the model. In particular, the diagram ignores the high level of details in ADAM and does not describe the dynamic adjustment in the model, so the impact represented by an arrow can appear both quickly and slowly.

Exogenous variables relate to: ...

In the diagram, the exogenous variables are put in boxes with bold rim. The diagram contains three major groups of exogenous variables.

... fiscal policy, ...

The first group comprises the fiscal instruments. There are several fiscal instruments in ADAM, which contains a lot of details on public finances, but the arrow diagram show only three fiscal instruments. Public consumption is located in the upper left part of figure 2.1 with margin texts “domestic demand” and “goods market”. The other two fiscal instruments in the diagram are the income tax rate and duty rate. Both are placed in the lower right part of the figure with the margin “public sector”.

… foreign sector, …

The second group of exogenous variables relate to the foreign sector. It consists of foreign demand, foreign prices and interest rates. All three are placed around the middle horizontal part of the diagram with the margin text “foreign”. The foreign prices are measured in Danish kroner, so the exogenous exchange rate is included in the prices. It is standard to assume that there is a relation between foreign demand, exchange rate, foreign price and interest rate. The model user can take this relation into account by applying results from international models.

… demography and productivity

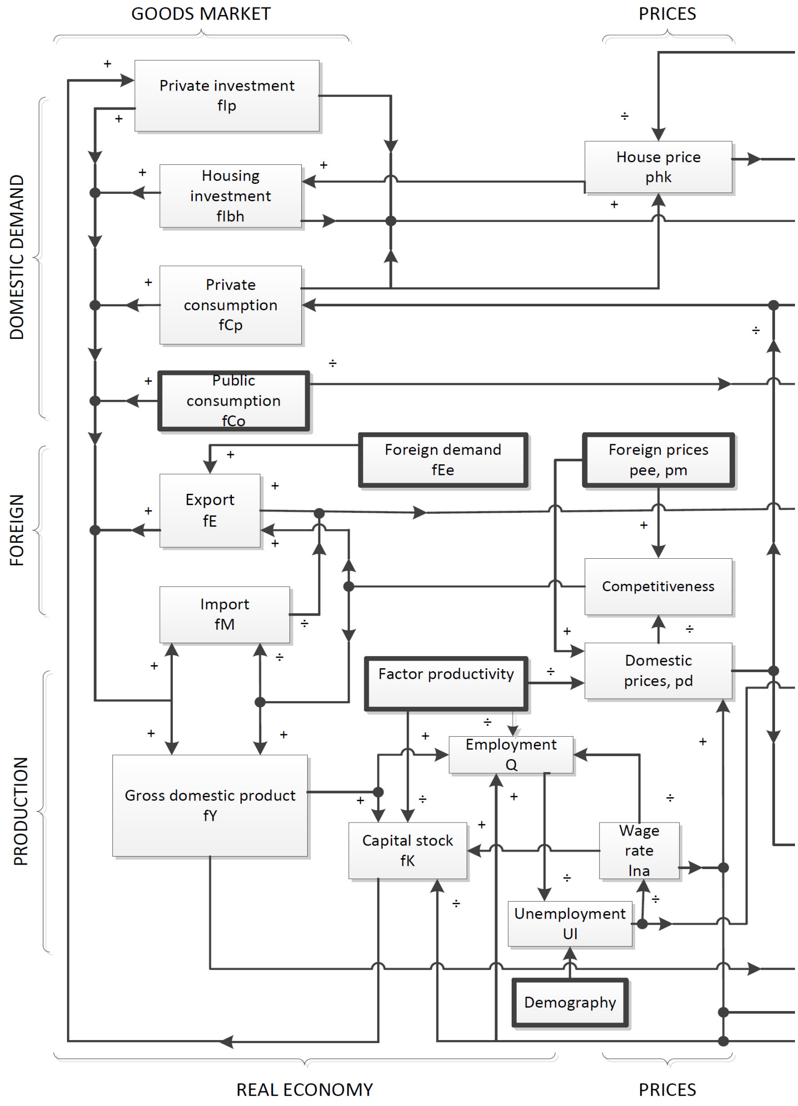
The third group of exogenous variables include demographics and factor productivity located in the lower left part of figure 2.1.

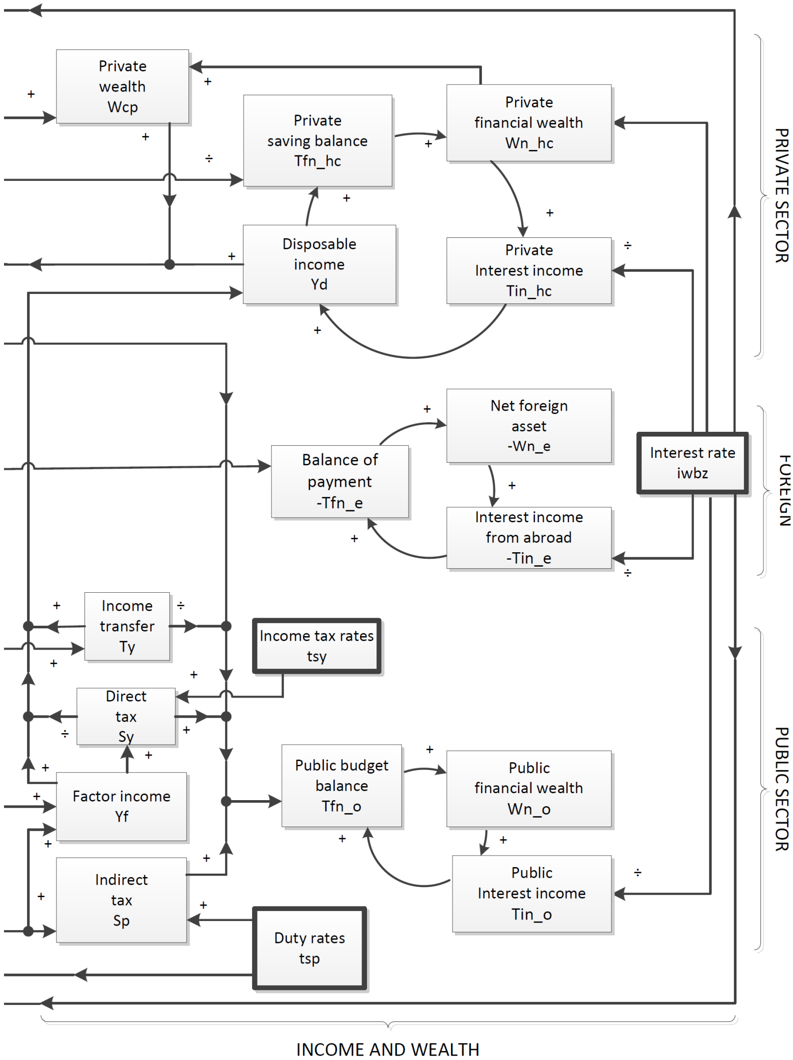
To explain the arrow diagram we first look at the reaction to an increase in public spending, thereafter we highlight the main effects of the rest of the exogenous variables.

When public consumption increases, GDP and imports increase

As mentioned, public consumption is located in the top-left part of the diagram. One of the two arrows from the public consumption box links up with arrows from the other demand variables and ends up indicating that the increase in public spending is distributed on imports and Danish GDP. If it is only the public employment that is increased, only GDP will be affected by the spending increase. If the public purchase from other sectors is increased, imports will react and GDP will consequently increase less.

Figure 2.1 **ADAM – arrow diagram**




Private wealth Wcp
Private saving balance Tfn_hc
Private financial wealth Wn_hc
Disposable income Yd
Private Interest income Tin_hc
Net foreign asset -Wn_e
Balance of payment -Tfn_e
Interest rate iwbz
Interest income from abroad -Tin_e
Income transfer Ty
Income tax rates tsy
Direct tax Sy
Factor income Yf
Public budget balance Tfn_o
Public financial wealth Wn_o
Indirect tax Sp
Public Interest income Tin_o
Duty rates tsp
PRIVATE SECTOR
FOREIGN
PUBLIC SECTOR
INCOME AND WEALTH

In the short term the cyclical effect is amplified ...

In the short term, the effect on GDP tends to stimulate demand and reinforce itself. First, the diagram shows that the capital stock increases when GDP increases, and this stimulates *business investment*. Second, the diagram shows that GDP generates income to the factors of production, which increases the disposable income, so that consumption and house price increase. The increase in house prices has an impact on housing investment and also stimulates consumption because housing wealth increases. The increased demand for private investment and private consumption is added to the initial increase in public spending, and thus the impact on GDP has reinforced itself.

... but the higher wage costs reduce exports,

However, there is also a tendency for the effect on GDP to dampen itself. To see this, we follow the arrow from GDP through employment to unemployment. The rate of unemployment falls, so wages rise, and through rising prices this deteriorates the competitiveness. The deteriorating competitiveness makes exports fall and import rise, and both effects displace Danish production. Thus, the GDP effect also dampens itself.

... and in the long term the effect on unemployment disappears

It may be added that in the long term, the self-damping relationship dominates over the self-reinforcing, so in the long term the expansionary effect of the demand shock disappears and that there is full crowding out in the model. It is not clear from the arrow diagram that the self-damping effect prevails in the long term, but it can be seen if the model is used to calculate the impact of a permanent shock to public spending.

The higher wage increases both labour productivity ...

In addition to the mentioned impact of wages on competitiveness the arrow diagram also shows that the wage costs affect both employment and capital stock directly with negative and positive sign, respectively. This reflects that capital to some extent replaces labour in the production function, when the wage rate increases. This moderate substitution effect implies that a little more can be produced by the same workforce.

... and real income

It is also noted that an arrow with a plus goes from domestic prices to factor income, so the higher the wage, the higher the nominal income. Meanwhile, the arrow with a plus going from nominal disposable income to consumption is linked with an arrow with a minus coming from the domestic prices. The latter arrow coming from domestic prices illustrates the negative effect on the purchasing power, when the price of consumer goods increases. If wages lifted income and consumer price proportionally, there would be no net effect on real income and on consumption in real terms. However, the income effect is larger than the price effect because the price of imported consumption is unaffected. Thus, private consumption is increased by the higher wage, while exports are reduced.

The increase in public consumption deteriorates the public budget…

There is not just an arrow with a plus going from public consumption to GDP. There is also an arrow with a minus going to the public budget balance, which is placed in the institutional half of the diagram. The arrow indicates that the immediate worsening of the budget balance amplifies itself by increasing public debt and hence public interest expenditures in accordance with the related arrows. The arrows form a circle, which seems, in the context, like a vicious circle for the public budget.

... while the private savings surplus is stabilized

The arrow diagram seems to indicate that also the private savings surplus or deficit can end up in a self-perpetuating vicious cycle where any worsening is reinforced by the interest on the growing debt. However, an arrow with a plus goes from private financial assets to private consumption. This connection implies that the private expenditures on consumption are reduced if the private savings surplus is reduced. Thus, the savings surplus of the private sector is stabilized by the model equations.

Tax funding of public consumption reduces income and private consumption

As mentioned, the public budget balance is not stabilized by any of the model equations, but the model user can choose, for example to raise income tax rates to finance an expansion of public consumption. Higher income tax rates increase tax revenues and reduce disposable income, cf. arrow diagram, which gives rise to a fall in private consumption and residential investment. This means that private domestic demand falls when the public demand rise is tax-financed, and on this basis the model arrives at a result quite different from the result of an unfinanced increase in public spending.

Duty funding reduces the purchasing power and private consumption

We can also improve public finances and keep the public budget unchanged by increasing the duty rates on consumption goods. This will raise public revenues, and private consumption will be influenced through higher prices, cf. the arrow from duty rates to domestic prices. The larger duty rates increase consumer price and the higher consumer price lead to a fall in real income and consumption. This effect is illustrated by the arrow with a minus going from domestic prices up to the horizontal arrow that links disposable income to consumption in real terms.

Higher foreign demand increases exports in the short term and the price of exports in the long term

Higher foreign demand will immediately increase exports, and there is an arrow going from exports to the balance of payments, which improves. In the real economy part of the arrow diagram, the increase in exports works much like the increase in public consumption. GDP rises and the increased employment will gradually increase wages and reduce competitiveness, implying that the immediate export increase is redressed gradually. Thus, in the long term the higher foreign demand is transformed into higher wages and higher export prices rather than into higher export volumes.

Higher foreign prices increase exports in the short term and the price of exports in the long term

Higher foreign prices will immediately improve the competitiveness as indicated in the arrow diagram, and this will increase exports and reduce imports. The increased demand for Danish production and employment will gradually increase wages and prices, including export prices, which will redress the initial improvement in competitiveness. In the long term, higher foreign prices are transformed into higher wages and higher domestic prices rather than into higher export volumes, see also the discussion in Section 4 of Chapter 11.

A change in interest rates affects both income ...

Changing the interest rates affect several model variables directly. We start by noting that the change in interest rates affects the interest flows between sectors and thereby also the private disposable income. This will yield an income effect on private consumption. If the private net interest income is positive and the interest rate is increased, the income effect will be positive.

... capital costs, house prices and bond prices

At the same time, an interest rate increase has a negative effect directly on the capital demand. Firstly, we note an arrow with a minus going from the interest rate down to the capital stock of the business sector and on to business investments. Thus, business investments fall when interest rates rise, making it more expensive to use capital. Secondly, an arrow with minus goes from the interest rate up to the house price, which falls when interest rates rise, and the fall in the house price makes housing investments fall. Moreover, the lower house price reduces the consumption-determining wealth variable, which is also reduced by the drop in the market value of the private net holding of bonds, cf. the arrow with a minus going from the interest rate to private financial assets.

Property price effect is important in the short term, income effect is important in the long term

The decline in wealth affects consumption negatively, and if, as stated, the higher interest rate raises income, there will be both a positive and a negative impact on consumption in the short term. A standard ADAM calculation on a rise in interest rates shows that the negative wealth-induced effect on consumption dominates in the short term. In the long term, the positive income effect becomes more important, cf. the discussion of a fall in interest rates in section 3 of chapter 11.

Higher productivity raises competitiveness and output

A rise in the productivity of production factors could, for instance, comprise a higher exogenous efficiency index of labour. This will reduce the immediate demand for labour-input, and the reduced employment will increase unemployment and reduce wages. Lower wages improve the competitiveness, so exports and GDP will rise and in the long term employment will remain unchanged, but there is a long-term increase in GDP due to the higher labour productivity.

Increasing the labour force raises unemployment in the short term ...

The term demographics refer to the size and age distribution of the population, both works as exogenous variables affecting the labour force. In ADAM, the labour force is determined by subtracting from the total population those groups that are outside the labour force. Many of the groups outside the labour force reflect the age distribution, e.g. the children, the normal pensioners, and the recipients of early retirement benefit. An increase in the labour force will increase the number of unemployed in the short term, see the arrow from demographics to unemployment.

... and raises competitiveness and employment in the long term

Higher unemployment will reduce wages, which improves the competitiveness. This will increase exports and cause GDP and employment to rise. The rising employment will redress the immediate increase in unemployment and in the long term, employment will expand proportionally to the labour force. Thus, demographics, which have an impact on the labour force, are the only exogenous factor in the arrow diagram that can change employment in the long term.

This concludes the outline of ADAM's main properties. In the following section 2.2, we compare ADAM with the more theoretically based equilibrium models.

2.2 ADAM and the equilibrium models

As mentioned in the introduction to the chapter, ADAM can both be compared with the long-term general equilibrium models of which DREAM is an example and with the dynamic equilibrium models, which also describe the short term.

ADAM and DREAM have similar long-term properties

ADAM's equations for consumption, investment, employment, foreign trade etc. differ in many respects from the corresponding equations in the Danish general equilibrium model DREAM. The specific differences are, however, not large enough to prevent the two models from having similar long-term properties, cf. Pedersen and Rasmussen (2001) who compare the long-term response of the two models to tax-funded changes in public consumption and in interest rates.

Consumption formation seems similar in the long term

As an example of this long-run similarity of the model properties, we can mention the determination of private consumption. The consumption functions of DREAM and ADAM look very different. ADAM's consumption function is a relatively simple equation where consumption depends on income and wealth. The DREAM consumption function explicitly considers the distribution of income on generations and distributes consumption over the life cycle of consumers, taking into account interest rates and expected income. Thus, in DREAM the formation of consumption explicitly involves the interest rate and is influenced by model-generated forward-looking expectations.

Notwithstanding the differences, both models end up producing a stylized long-term scenario where total consumption is proportional to total income and the consumption ratio is 1, provided that the economy is not growing.

Labour supply differs between ADAM and DREAM

Not all long-term properties are equally similar in the two models. As a major difference, Pedersen and Rasmussen highlight that the labour supply in ADAM is unaffected by income taxation, while DREAM's labour supply falls when the tax level rises. In a theoretical model, it is normal for the labour supply to respond to the wage income after tax, because households choose between work and leisure. This particular supply reaction is, however, difficult to estimate on macro time series. Consequently, the official version of ADAM does not have the effect, but as mentioned in the section on ADAM's main properties, it is relatively easy to supplement ADAM with a calibrated labour supply effect similar, for instance, to the labour supply effect in DREAM.

DREAM depends on public spending being funded

Another formal difference between the two models is that DREAM includes a public budget constraint to ensure that public spending is financed by revenues in the long term. Pedersen and Rasmussen impose an equivalent budget restriction on their ADAM calculations, whereby this difference disappears. In a simple application of ADAM, the long-term result will obviously reflect the lack of a fiscal function to ensure long-term balance in public finances.

ADAM compared to DSGE models

DREAM is not used for business cycle analysis, so there is no reason to compare the short-term properties of ADAM and DREAM. Instead, one can in general terms compare ADAM to the family of dynamic equilibrium models usually referred to as DSGE models (Dynamic Stochastic General Equilibrium), which are designed for analysis of the business cycle. There are many different DSGE models, see Chari (2010), who as a common feature emphasizes the clear theoretical assumptions of these models.

It is obvious that the short-term properties of ADAM are very different from what can be found in a neoclassical DSGE model where prices are flexible enough to prevent involuntary unemployment.

New-Keynesian DSGE models have sluggish prices like ADAM

It is more relevant to compare ADAM with the new-Keynesian model variant. In a new-Keynesian DSGE model, prices are less flexible, which should make the model easier to estimate and better at explaining the business cycle. Sluggish price formation is a traditional way to reconcile the Keynesian demand-driven short-term model with the neoclassical long-term equilibrium model. The price formation is also sluggish in ADAM, so on that point ADAM is similar to new-Keynesian DSGE model.

Differences between ADAM and DSGE models

There are also differences. Firstly, ADAM does not have rational expectations, secondly, ADAM has no political reaction function, and thirdly, ADAM is larger than a DSGE model.

DSGE models have rational expectations

The first difference mentioned concerns the formation of expectations. Dynamic equilibrium models have been motivated by an ambition to improve the model foundation by applying rational rather than adaptive expectations, cf. the so-called Lucas critique of economic models that are estimated with adaptive expectations. The critique says that models with backward-looking expectations break down if there is a fundamental change in economic policy.

In practice, rational expectations can be formulated as forward-looking model consistent expectations, where the model-calculated forecast of a variable is used to represent the expected value.

Effect of forward-looking expectations in ADAM

If ADAM were changed to applying model-consistent rather than adaptive expectations, the dynamic adjustment would probably be more focused and less volatile, without affecting the long-term equilibrium of the model. There have been attempts to insert forward-looking expectations in ADAM, see Veje (2001). Veje concludes that forward looking inflation expectations in the user cost variables of housing and factor demand will reduce the adjustment time and fluctuations of the model.

Forward-looking inflation expectations resemble exogenous inflation expectations

The rationale is that if expectations are backward looking, expected inflation will reflect the recent price increase, making the real interest rate countercyclical. In this case, the reaction in real interest rates will amplify the cyclical fluctuations. This can be avoided if expected inflation reflects a forecast on the Danish price increase based on fixed exchange rates, constant nominal interest rates and steady inflation of trading and currency partners. On this forward-looking basis, both expected inflation and real interest rates will be stable.

No empirical evidence of model-consistent expectations

You can achieve the same stability of the model-calculated real interest rate by exogenizing inflation expectations in ADAM. Throughout this book, ADAM is analysed with exogenous inflation expectations in the user cost variables. It is natural to apply exogenous inflation expectations in an economy with fixed exchange rate policy, but in general it has proved difficult to model expectations and there is no empirical evidence to suggest that model-consistent expectations are the right solution. For instance, the course of events leading up to the 2008 financial crisis illustrated that many economic agents are ready to expect that the recent trend continues and underestimate the risk of a turnaround.

DSGE models have political reaction functions

The second difference concerns the absence of policy reaction functions in ADAM. DSGE models traditionally emphasize the importance of having a monetary policy reaction function to describe how the central bank reacts to inflation and unemployment.

Monetary policy reaction functions are irrelevant in ADAM

A countercyclical response of the central bank may dampen the effect of demand shocks on output and employment. However, it seems irrelevant to incorporate a countercyclical monetary policy reaction in a Danish model, as the monetary policy of the central bank is dictated by the fixed exchange rate policy against the euro.

Fiscal reaction functions are more relevant ...

It would be more appropriate to include in ADAM, not a monetary but a fiscal policy reaction function. A fiscal reaction function could be designed to stabilize cyclical fluctuations through counter-cyclical fiscal policies. However, if the idea is to bring ADAM closer to the equilibrium models, there is more need for a fiscal reaction function to stabilize the public budget in the long term.

... but dispensable

In the absence of a long-term budget-stabilizing response function, we can make the ADAM-calculated effect of, e.g. higher public consumption comparable to a similar DSGE-calculated effect by financing the higher public consumption with a corresponding tax increase.

Without reaction function fiscal policy is controlled by the model user

When there is no fiscal reaction function in ADAM, it is partly because ADAM users normally prefer to specify the necessary fiscal policy without assistance from a reaction function that endogenizes one of the fiscal instruments. Moreover, it has proved difficult to make a robust fiscal reaction function, which can be used in all ADAM calculations.

ADAM stabilized by the response of competitiveness to unemployment

It should be added that without political reaction functions to stabilize the economy, it is the response of wages and thus of competitiveness to unemployment plus the response of foreign trade to competitiveness, which gives ADAM a long-term equilibrium. In models with political reaction functions, the reaction functions are often crucial for when and how the model reaches its equilibrium.

DSGE-models are smaller

The third and perhaps most striking difference between ADAM and a typical DSGE model is the size. ADAM is a fairly disaggregated model with several demand variables and 12 industries, implying that ADAM has many more variables than a DSGE model. DSGE models typically focus on a few macroeconomic issues and are therefore made more aggregate with normally one production function only.

ADAM can be used for multiple purposes

DSGE models are deliberately made small because it is easier to analyse a small model than a large. Conversely, ADAM can be used for several analytical purposes, and it may be added that in the U.S. where there are many suggestions for dynamic equilibrium models, traditional estimated macro models are, in practice, often used to quantify the impact of economic measures, cf. Mankiw (2006).

Summary on ADAM and equilibrium models

In summary, there is actually no big difference between ADAM and the equilibrium models concerning long-term properties, and it does not seem necessary to adopt the short-term properties of equilibrium models. As stated in the beginning of the chapter, ADAM can be seen as a workable compromise between equilibrium models and time series models and ADAM is used, in practice, for a variety of macroeconomic analyses.

In the remainder of this chapter, we illustrate, by means of model calculations, how ADAM responds to typical shocks to exogenous variables.

2.3 Calculations on ADAM

Exogenous ADAM variables changed ...

Model calculations typically consist of changing one or more of the exogenous variables in the model and calculate the consequences for endogenous variables. For example, public spending or foreign demand can be increased permanently. Such positive demand shocks will increase production and employment in the short term, but in the long term, demand shocks have no effect on employment in ADAM. To increase employment permanently requires a supply shock, for example, an expansion of the labour force or a reduction of the structural unemployment. The latter can be achieved by adjusting the wage equation of the model.

... relative to a baseline

As a starting point for calculating the effect of the exogenous shocks we need to make a baseline. The baseline represents a solution with respect to the 2500 endogenous model variables, given a stylised projection of the 1000 exogenous variables. It is necessary to have a baseline for making the exogenous shocks and calculating the resulting endogenous changes. Normally the impact of shocks depends, only to a small extent, on the baseline, but there are obvious exceptions. For instance, the effect of a change in interest rates will depend upon whether the private sector is a debtor or creditor in the baseline.

The following section 2.4 refers to a demand shock, which raises the public purchase of goods, while section 2.5 refers to a supply shock, which reduces the structural unemployment in the model.

2.4 Raising public purchase of goods by 1 per cent

An illustrative and often used shock to ADAM consists of changing the input of goods in the public production, i.e. we do not change the number of public employees but the public purchase of goods. In our specific calculation, the purchase of goods is raised by 1 per cent relative to the baseline for all years. This increase corresponds to about 1.5 billion in 2005-prices in 2010, which is the first year of the model calculation. This change in a single exogenous variable produces a new solution, and the difference between this new scenario of the economy and the baseline scenario illustrates the effects on the economy.

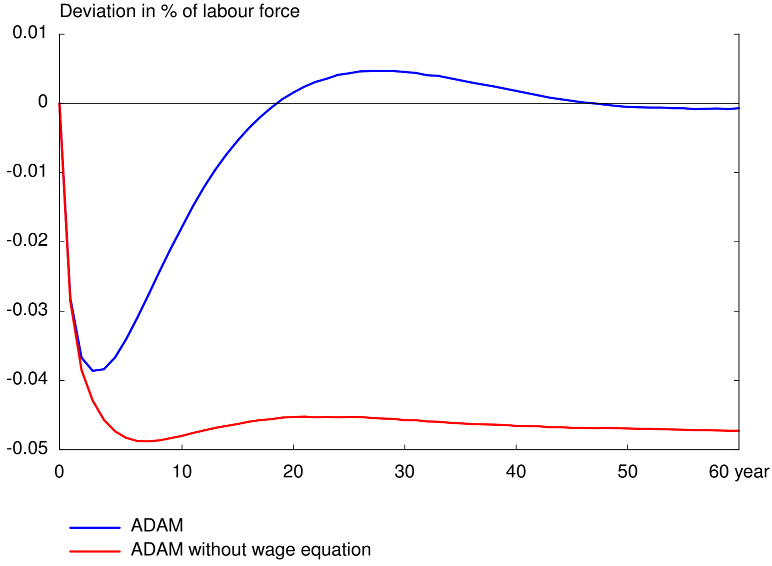
In the short term, activity rises and unemployment falls

In the short term, ADAM works as a Keynesian model in which demand determines production, so the immediate effect of increasing public consumption is that GDP rises. At the same time, employment increases and after a few years unemployment has fallen by 0.04 per cent of the labour force compared to baseline.

In the long term, competitiveness falls and unemployment is unchanged

This marks the peak of the impact on unemployment. The lower unemployment rate will increase wage growth and reduce competitiveness. Lower competitiveness makes the market share of exports fall and the market share of imports rise, which implies that Danish production is crowded out. Eventually, the impact on unemployment disappears and unemployment returns to and crosses the baseline 18 years after public consumption was increased. In figure 2.2, the blue curve depicts the effect on unemployment in per cent of the labour force.

Figure 2.2 **Effect on unemployment, public purchase of goods + 1%**



The long-term unemployment effect of zero reflects that ADAM in the long term works as an equilibrium model, where wages and prices adjust and the unemployment rate returns to its equilibrium, which is the unemployment rate in the baseline scenario. The crowding-out process is completed when the unemployment effect of the demand shock equals zero.

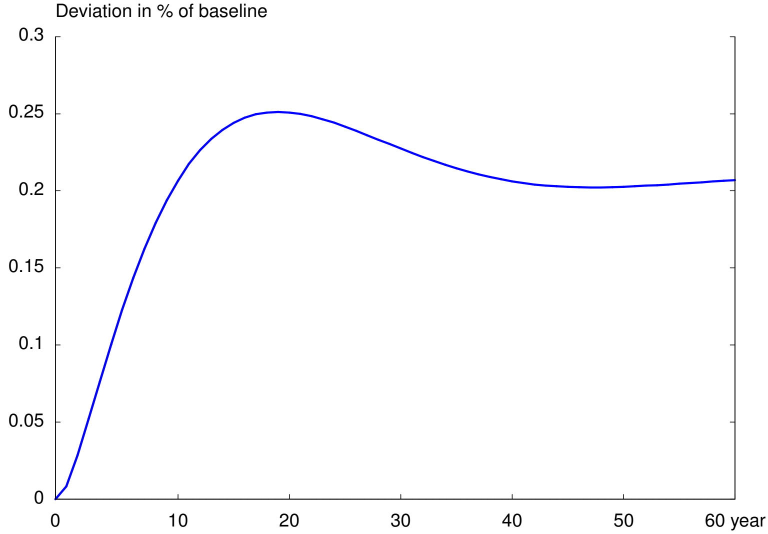
Wage equation crucial for long-term quilibrium in labour market

The difference between short- and long-term effects on unemployment hinges mainly on the wage formation in ADAM. If the wage equation is removed, the hourly wage will become exogenous and it will no longer affect competitiveness when unemployment falls. In this regime of rigid wages, unemployment can be reduced permanently by raising public purchases, cf. the red curve in figure 2.2.

The short-term increase in employment disappears if wages are endogenous, and during the crowding-out process unemployment is temporarily higher than in the baseline, cf. the blue curve in figure 2.2, which stays slightly above the zero line for a number of years.

This tendency for unemployment to overshoot its equilibrium for a period reflects a tendency for the wage rate to overshoot, which is illustrated in figure 2.3. As long as the wage level is above its long-term value, the production of exports is pushed down too far and unemployment is pushed above its baseline. The overshooting of wages and unemployment reflects that it takes time before a new wage level takes full effect on activity and unemployment.

Figure 2.3 **Effect on hourly wage, public purchase of goods + 1%**



Public finances are not in equilibrium

Although the wage reaction eventually gets unemployment back to the baseline, the overall economy is not back in equilibrium because the higher public purchases are deteriorating public finances. As already discussed, ADAM has no fiscal reaction function, so no fiscal instrument is automatically activated in order to finance the purchase of goods and keep public debt constant relative to GDP. Without revenue funding of the additional public consumption, the budget effect is negative and both public debt and interest outlays will be increasing relative to the baseline scenario, cf. figure 2.4, which shows the impact on public debt as share of GDP.

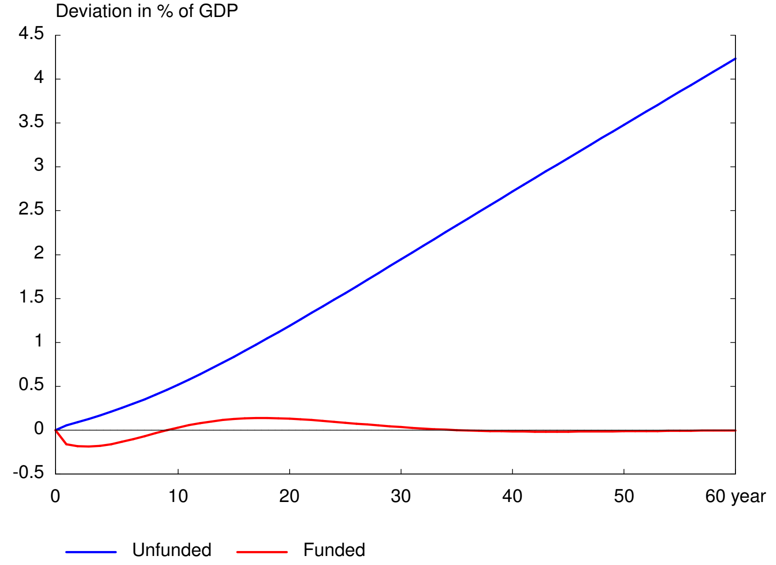
It is unsustainable if the public budget balance and, indeed, also the balance of payments deteriorates year after year. Sooner or later it will become necessary to finance the increase of public purchases by an increase of public revenues.

Funding the additional public purchase

It is here chosen to increase taxes to finance the additional public purchase of goods. More specifically, government income tax rates are increased by 1.5 per cent in all years to make the long-term effect on public debt zero, see figure 2.4.[1]

[1] The required tax funding is calculated using ADAM and further discussed in box 2 in chapter 11. The box explains that the mentioned permanent increase of income taxes must be supplemented by a modest temporary tax increase to secure that the long-term effect on public debt is exactly zero.

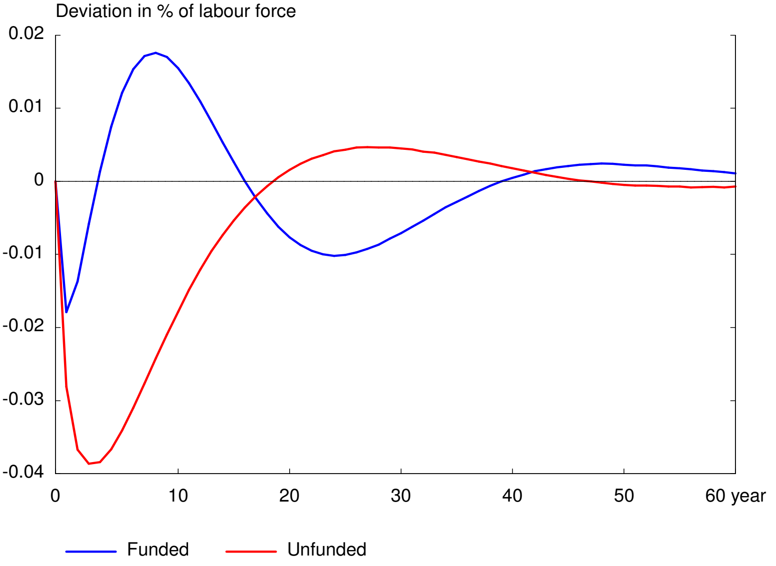
Figure 2.4 **Effect on public debt, public purchase of goods + 1%**



Tax financing changes the outcome

Without tax funding, it takes as stated 18 years before the unemployment effect crosses the zero line. With tax funding, it takes only 4 years, see figure 2.5 illustrating the impact on unemployment with and without tax funding. In return, unemployment fluctuates more when the purchase of goods is financed by higher taxes, and it takes just as long, with tax funding as well as without, for unemployment to be back in equilibrium at the baseline.

Figure 2.5 **Effect on unemployment, public purchase of goods + 1%**



Without tax-funding, the impact on employment is created by the wage and foreign trade equations in ADAM. With tax funding, the employment effect still reflects the reaction in competitiveness, but in addition it clearly reflects that the housing market fluctuates before reaching equilibrium.

Private domestic demand is adjusted instead of exports

We shall come back to the housing market. Generally, the tax funding implies that it is the private domestic demand, which falls to make room for the public purchase of goods. When the purchase of goods is unfunded it is exports, which fall and make room for the public purchase of goods.

Domestic demand is increased permanently when the expansion of public consumption is unfunded because there is no tax increase to reduce private demand. On the contrary, private consumption is increased in the long term as real income increases when domestic wages and prices increase, while import prices remain unchanged. The higher real income and improved terms of trade reflect that wages increase relative to the baseline as part of the crowding-out process, which reduces exports. The domestic demand and export effects of unfunded public purchases are illustrated by the two dashed curves in figure 2.6.

For a tax-funded expansion of public consumption, the initial increase in total domestic demand disappears when the tax increase reduces private demand. Consequently, there is less scope for long-term crowding-out through reduced competitiveness and also exports are only slightly affected in the long term, see the solid lines in figure 2.6.

Figure 2.6 **Effect on domestic demand and exports, public purchase of goods + 1%**

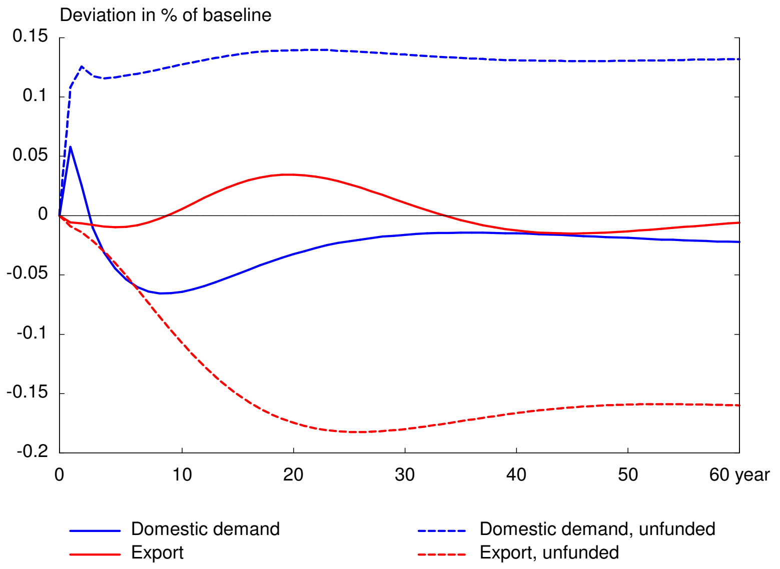
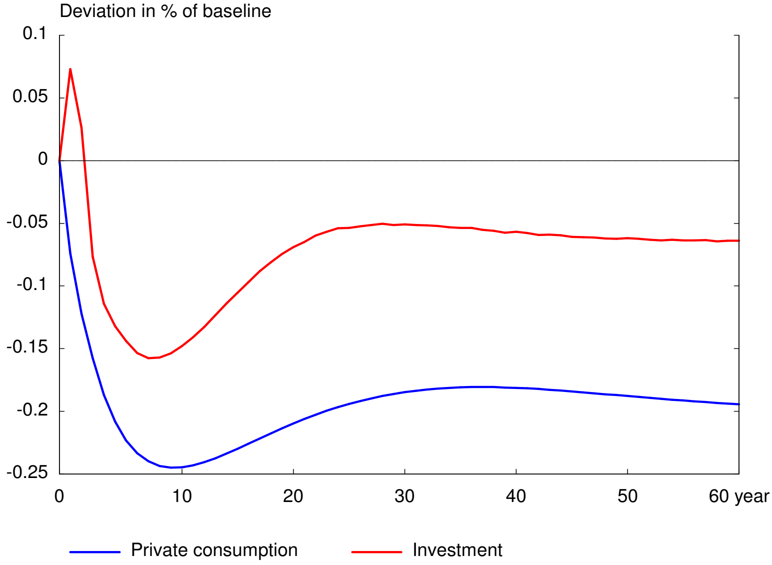


Figure 2.7 **Effects on demand, tax-funded public purchase of goods + 1%**



Tax funding reduces private consumption and investment

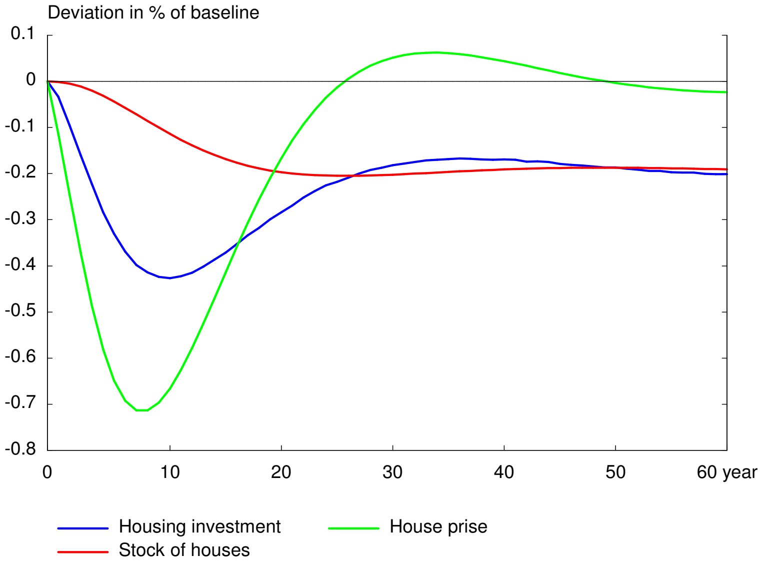
The decline in private demand relates to both consumption and investment, see figure 2.7. The increase of investments in the beginning of the calculation period concerns business investments, which increase enough to outweigh the fall in residential investments. After a few years, the fall in residential investments dominates and the strong investment reaction during the first decade primarily reflects the reaction of residential investments.

House price reaction to tax increase

When the tax increase reduces income and private consumption, it also reduces the demand for housing, and consequently both house prices, housing investments and the stock of houses fall relative to the baseline scenario. The price of houses is more flexible than the stock of houses, and the house price reacts significantly to the decline in housing demand. The fall in house prices peaks after eight years where the housing stock has fallen by so much that the supply of houses starts to approach its new and lower level, see figure 2.8.

In the long term, housing demand decreases and the stock of houses falls by 0.2 per cent relative to the baseline scenario. The fall corresponds to the decline in private consumption, excluding housing. The accompanying decline in house prices makes it less profitable to build new houses, and this reduces housing investment, which just like the house price fluctuates on its way to equilibrium.

Figure 2.8 **Effects on housing market, tax-funded public purchase of goods + 1%**



Also significant effect on housing investments

It is to be expected that housing investments for a period, and measured in relative terms, will react more than the stock of houses because 0.2 per cent of the housing stock equals 10 per cent of annual investments in the baseline. In the long term, housing investment will fall by 0.2 per cent similar to the housing stock, and only housing stock and housing investments will permanently change relative to the baseline in figure 2.8. The house price in the figure is measured relative to the price of housing investments, and this ratio will eventually return to the baseline.

It is primarily the fluctuations in housing investments and the impact of fluctuating house prices on private consumption, which create the fluctuations in domestic demand, in output, and in unemployment. Thus, it appears that the fluctuation of unemployment becomes smaller if the adjustment in the housing stock is slowed down by lowering the response of house prices and hence of housing investments to the decline in housing demand.

2.5 Permanent reduction of unemployment

Labour market policy can change structural unemployment

Just as ADAM can be used to assess the macroeconomic impact of fiscal policy, ADAM can also be used to assess the impact of labour market policy, for instance, of measures reducing the long-term, i.e. the structural, unemployment. In practice, the rate of unemployment cannot be frozen at a certain level. There will always be shocks and cyclical fluctuations at home and abroad to impact unemployment, but you can make unemployment fluctuate around a lower level in the long term.

The wage relation of the model makes structural unemployment a linear function of 1) the replacement ratio of unemployment benefits, i.e. the ratio between benefits and wages, and 2) a constant. In the model experiment considered here, the constant is adjusted downwards in order to make structural unemployment fall by 0.25 per cent of the labour force.

Wage growth is reduced in the short term

This downward adjustment of the wage relation immediately reduces the annual rate of wage increase by 0.14 per cent, e.g. from 3.5 to 3.36 per cent, reflecting that the structural unemployment rate enters with a coefficient of 0.55 in the wage equation that determines the rate of wage increase. The smaller wage growth improves competitiveness and increases Danish market shares. Subsequently, the larger market shares will increase production and reduce unemployment.

Long-term wage growth is unchanged, but long-term unemployment falls

Smaller unemployment stimulates wage growth, so the decline in wage growth is strongest in the first year of the model calculation. In the subsequent years, the initial impact on wage growth gradually disappears, and in the long term the rate of wage increase is as large as in the baseline scenario, and Danish prices are increasing in line with foreign prices. By contrast, unemployment will in the long run have decreased by 0.25 per cent of the labour force compared to the baseline, cf. figure 2.9, which shows the ADAM-calculated effect on wage growth and on unemployment.

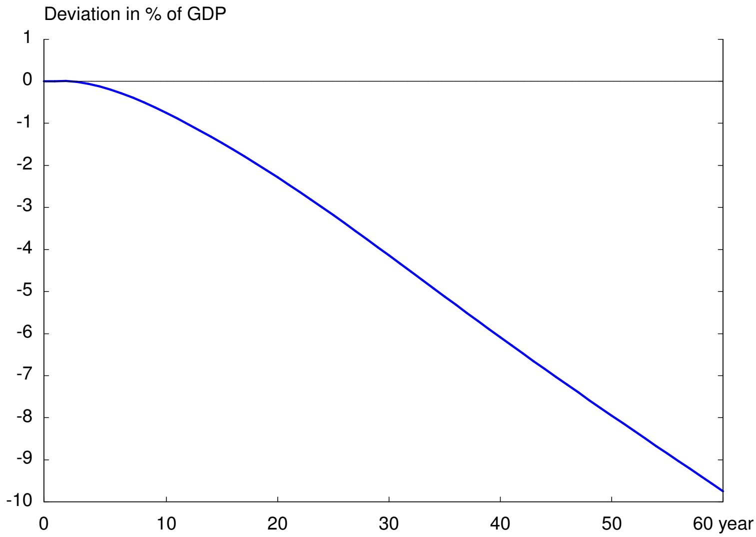
Figure 2.9 **Effect on wage growth and unemployment, structural unemploym. -¼%**

As mentioned, the structural unemployment of the model depends on the replacement ratio of unemployment benefits, but the structural unemployment can undoubtedly be affected by means of other instruments than the replacement ratio. The model experiment presented above does not say anything about the possible instruments, but the ADAM calculation illustrates the beneficial consequences of a permanent reduction in unemployment, not just for employment and output but also for public finances.

Public finances improved

Saved unemployment benefits, increased tax revenue and saved interest outlays mean that public debt will start falling relative to the baseline scenario, see figure 2.10.

Figure 2.10 **Effect on public debt, structural unemployment -¼%**



The permanent improvement of the public budge balance shows that an expansion in the supply of labour helps to finance public expenditures in the long term. Thus, increased labour supply can, for example, replace a tax increase.

3. Private consumption and housing market

Private consumption plus residential investments represent more than half of domestic demand and play a significant role for the cyclical movements in the overall economy. Private consumption alone takes up half of domestic demand and is by far the largest component in domestic demand. Housing investments are a significantly smaller component, which amounted to 7 per cent of domestic demand in 2006, when housing investments peaked. On the other hand, there are large cyclical movements in housing investments and in the price of houses. Moreover, housing and consumption interact, as house prices affect household wealth, which in turn affects consumption.

The interest in household consumption, including housing consumption, is not only related to the cyclical fluctuations of the economy. The ultimate purpose of production is consumption, so consumption constitutes a kind of proxy for the outcome of economic activity.

The main aggregate consumption function in ADAM determines total private consumption, excl. housing consumption. In the model, this aggregate consumption is input to a consumption system, which is a set of equations breaking down the aggregate into seven consumption components. The housing market is modelled with an equation for the price of houses and an equation for the stock of houses. Both housing investments and housing consumption can be derived from the latter.

Households are likely to determine their housing consumption and the rest of their consumption together, and the consumption and housing equations in ADAM should reflect this joint determination. The subsequent presentation of equations is divided into three parts. First, the equation for total consumption, excl. housing is presented, second, the consumption system, and finally the equations for the house price and stock of houses are presented. Each part consists of three sections, so that the chapter has nine sections.

3.1 Private consumption

Private consumption is basically explained by income and wealth. This is a standard approach, which has been used for years in ADAM's consumption formation, and it is also used in several similar models, see Muellbauer and Lattimore (1996). The long-term consumption equation is formulated in (3.1). Consumption is C, income Y, and wealth W. Greek letters are coefficients, β_1 is the consumption elasticity with respect to income, and β_0 is a scale factor.

(3.1) $$C = \beta_0 \cdot Y^{\beta_1} W^{1-\beta_1}$$

Desired consumption is a function of income and wealth

Equation (3.1) implies that income and wealth elasticities sum to one. Thus, consumption increases by 1 per cent in the long term if both income and wealth increase by 1 per cent. The equation also implies that long-term consumption ratio, C/Y, correlates positively with wealth relative to consumption, W/C. The interest rate is not included explicitly in (3.1), but the wealth variable includes housing wealth, implying that interest rates affects consumption via the impact on house prices.

Actual consumption and wealth ratios correlate to some extent

If the actual consumption is close to the desired consumption determined by the long-term consumption relation in (3.1), there will be a positive correlation between the actual consumption and wealth ratios. In figure 3.1, the two ratios are shown based on the ADAM-variables for equation (3.1). Consumption is measured excluding housing and also excluding car purchases, which are replaced by an imputed yield on cars, cf. the description of the variables in section 3.3 on the estimated consumption relation.

Figure 3.1 **Consumption/income and wealth/consumption ratios**

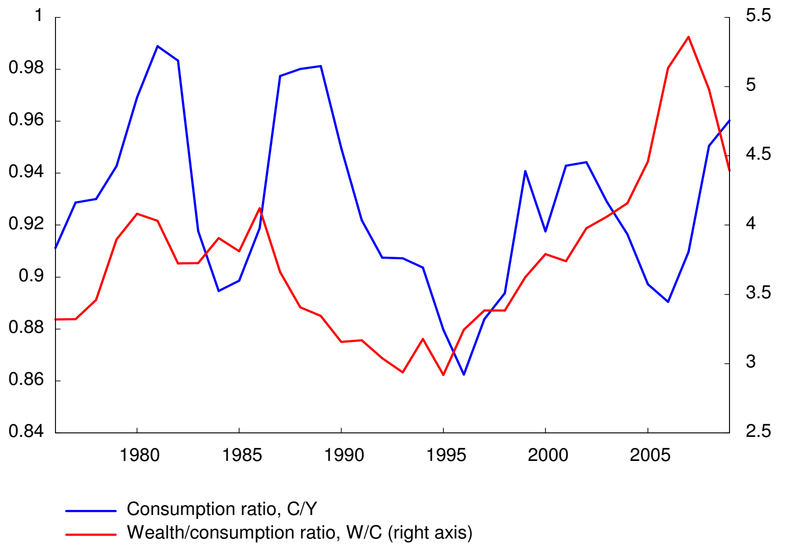


Figure 3.1 shows that there is a slight tendency for the consumption ratio to increase when wealth is high relative to consumption. This may reflect that high wealth moves consumption upward. Thus, figure 3.1 confirms to a certain degree that there is a correlation in data compatible with the long-term relation in (3.1).

As mentioned, it is only to a certain degree that figure 3.1 can confirm a positive correlation between the two ratios. It should be noted, e.g. that the ratio wealth/consumption clearly peaks in 2006, but the consumption ratio does not reach a similarly clear maximum in 2006 or 2007.

Income is more important than wealth

In summary, the figure suggests that income has been more important for consumption than wealth. Thus, it appears that the consumption ratio is considerably more stable than the wealth ratio, which is put on the right hand scale. Moreover, the consumption ratio seems to fluctuate around its own average during the period from 1975 to 2009, while the wealth ratio indicates a once and for all level shift during the period. In section 3.3, we estimate the consumption equation and it turns out to be easier to estimate the impact of income on consumption than the impact of wealth. The following section explains the properties of the estimated consumption equation.

3.2 Properties of the consumption equation

The resulting dynamic consumption equation has the logarithmic change in consumption on the left hand side, as detailed in section 3.3. On the right hand side, the contemporaneous income change enters with a coefficient of 0.4. The lagged log difference between desired and actual consumption enters with a coefficient of 0.407, and a comparison with the long-term relation in (3.1) shows that the long-term income elasticity, β_1, is 0.9, and (the log value) of the long-term equation scaling factor, β_0, is 0.200. The complete dynamic equation has a constant of 0. 011 and is shown in (3.2a).

(3.2a)

$$\text{dlog}(C) = 0.4 \cdot \text{dlog}(Y) + 0.407 \cdot \left(\log(Y_{-1}^{0.9} \cdot W_{-1}^{0.1})\text{-}0.200\text{-}\log(C_{-1})\right) + 0.011$$

Income is increased by 1 per cent ...

If income, Y, is increased by 1 per cent, consumption will grow 0.4 per cent in the same year. If wealth is exogenous, the long-term increase in consumption will be 0.9 per cent, corresponding to the long-term income elasticity. It is, however, artificial to assume exogenous wealth. When consumption increases less than income, savings will rise and total savings, Y-C, are accumulated in the wealth variable, W, which in principle can be endogenized by:

(3.2b)

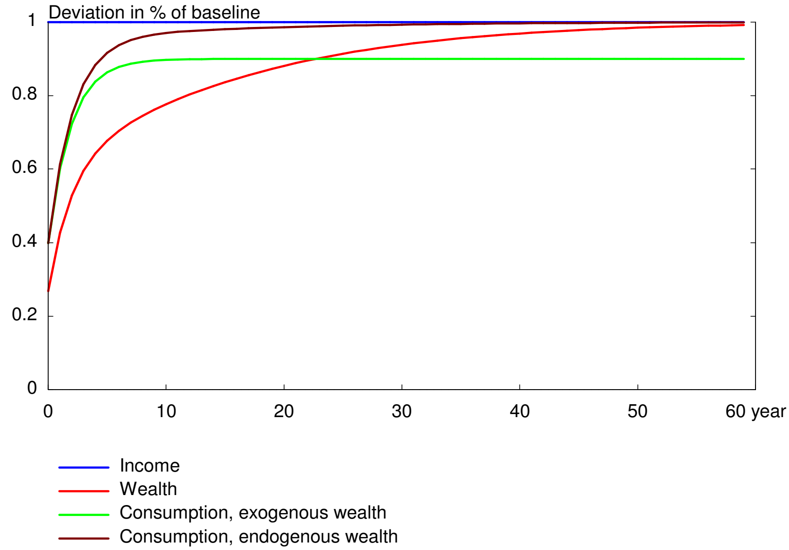
$$W = W_{-1} + Y \text{ - } C$$

... and in the long term both consumption and wealth will increase by 1 per cent

If wealth is endogenous, both wealth and consumption will eventually increase by 1 per cent as income. It does, however, take a long time before the accumulated savings make wealth rise by 1 per cent. After 40 years, wealth has increased by 0.9 per cent or 90 per cent of the long-term increase of 1 per cent, cf. figure 3.2. The illustrated adjustment speed partly reflects the relative size of income and wealth, which is one to three in the calculation. The higher the income is, the faster the wealth can be changed by traditional savings.

Wealth and hence consumption reacts more quickly if the change in wealth reflects not only savings, but also the increase in house prices, which usually accompanies an increase in income and consumption. This interaction between consumption and housing market is illustrated in section 3.8.2 describing the properties of the housing model.

Figure 3.2 **Effect on consumption, income + 1 per cent**



Propensity to consume is increased by 1 per cent

We will now illustrate what happens if adjustments are made to the consumption equation shown in (3.2a). The purpose of the calculation is to illustrate the effect of increasing the propensity to consume by 1 per cent, and the calculation is performed by increasing the scaling factor, β_0, of the long-term equation by 1 per cent. It corresponds to increasing the constant -0.200 in the parentheses of (3.2a) to -0.190. This 1 per cent upward revision of the long-term equation scaling factor corresponds also to lifting the income variable of the long-term consumption equation by 1.1 per cent without changing the income variable that determines savings and the wealth change in (3.2b).

The latter consideration refers to the use of a special income concept in the long-term consumption equation in ADAM, see the next section 3.3 describing the estimated equation. What is special is that pension savings are excluded from the income variable, which enters the long-term consumption equation. Thus, a decline in pension savings works like an increase in the propensity to consume.

In order to calculate the effect of a higher propensity to consume, the consumption equation is supplemented by the definitional equation for wealth in (3.2b) and with an equation defining income as the sum of income, excluding interest income plus the return on wealth:

(3.2c)
$$Y = Y_{exinterest} + interest \cdot W_{-1}$$

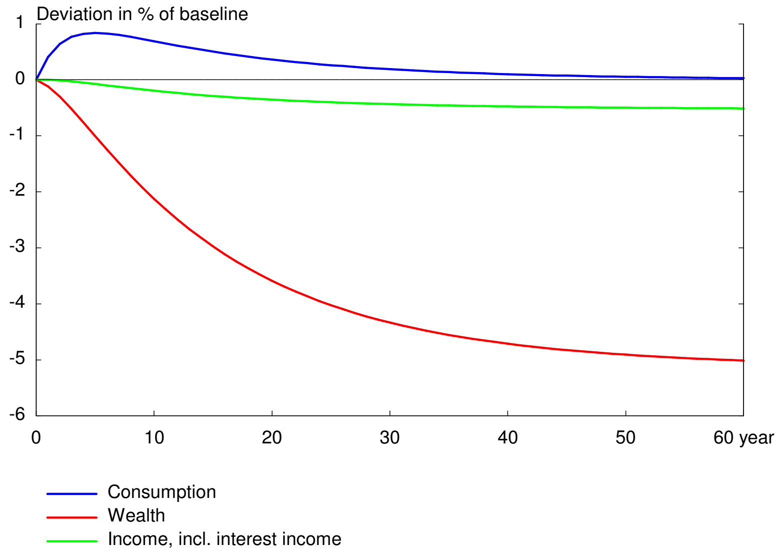
This equation implies that a decrease in assets will reduce total income, Y. Together, the two defining equations (3.2b) and (3.2c) represent a stylized version of the corresponding group of definitions in ADAM. When all three equations, (3.2a) - (3.2c) are used simultaneously, it is similar to applying a calculation on ADAM with all other behavioural equations than the consumption equation exogenized.

Consumption will rise by 1 per cent and in the long term wealth will fall

The short-term effect derived from increasing the propensity to consume by 1 per cent in the three-equation models (3.2a)-(3.2c) is an increase of 1 per cent in consumption, but since income has not increased, savings fall and wealth begins to decline. The adaptation does not stop until wealth, W, has dropped by 5.12 per cent and income, Y, including interest income has dropped by 0.52 per cent.

The weighted negative impact of the mentioned decrease in wealth and in total income equals 1 per cent (=.9·0.52+.1·5.12), which exactly cancels the increase in the propensity to consume.

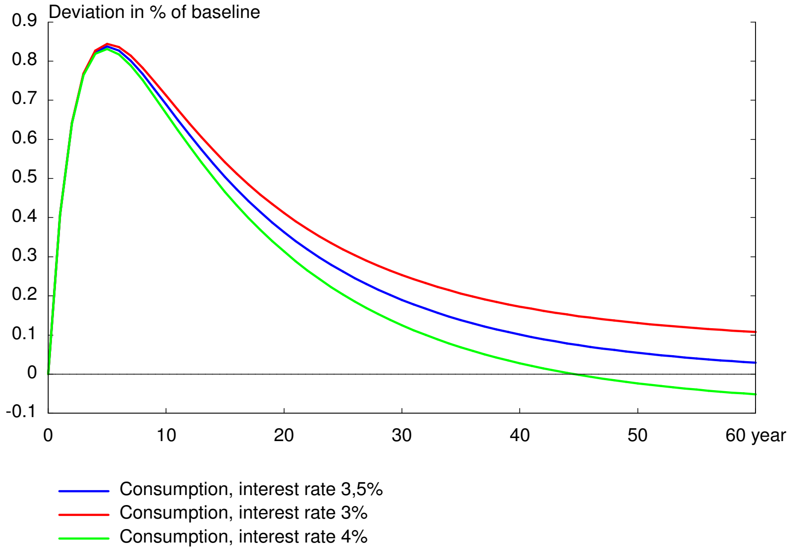
Figure 3.3a **Propensity to consume + 1%, growth and interest rate of 3.5%**



Consequently, the long-term effect on consumption is zero and consumption returns to the baseline after being above the baseline for a period, cf. the illustration in Figure 3.3a. The return of consumption to the baseline, despite of the permanent increase in the propensity to consume, reflects that the interest rate is assumed to equal growth rate. This is the normal assumption in a long-term ADAM projection.

When the interest rate equals the growth rates, for example, if both are 3.5 per cent, the income, Y, is reduced by 3.5 per cent of the decline in wealth, as the smaller wealth yields less interest income. At the same time, the decline in wealth reduces the growth-related need for savings, Y-C, by 3.5 per cent of the decline in wealth. Consequently, the consumption, C, is unchanged in the long term, as illustrated in figure 3.3.

Figure 3.3b **Propensity to consume + 1%, growth 3.5% interest rate 3, 3.5 and 4%**



If the interest rate is higher than growth rate, for example, 4 versus 3.5 per cent, the consumption will be reduced in the long term when the propensity to consume rises. The negative long-term consumption effect reflects that income, Y, is reduced by 4 per cent of the decline in wealth. Thus, the drop in income is larger than the drop in the need for savings, Y-C, which is reduced by only 3.5 per cent of the decline in wealth. If the interest rate is lower than the growth rate, income will fall less than savings, and consumption remains above baseline in the long term, see figure 3.3b.

Generally, the result reflects that consumption is equivalent to consumer income minus what it takes to keep wealth on the selected growth track. The argument about the significance of growth rate and interest rate in a steady state projection is elaborated in box 3.1.

We will refer to the three-equation model and to the calculation on the propensity to consume in the chapters 10 and 11, which concern the baseline scenario and the model analysis.

Box 3.1 **The significance of growth and interest rates in steady state**

If the exogenous income variable, $Y_{exinterest}$, grows at a constant rate, the long-term solution to the three-equation model (3.2a-3.2c) will get the same steady-state growth. This allows the change in wealth to be replaced by $growth \cdot W_{-1}$. If (3.2c) is also inserted into the wealth equation, (3.2b) we get the following equation for steady-state consumption.

(3.2b)*

$$C = Y_{exiterest} + (interest - growth) \cdot W_{-1}$$

The expression in brackets is the interest-rate growth differential. If this differential is zero, consumption will equal the exogenous income variable, $Y_{exinterest}$, irrespective of the size of wealth.

If the interest rate is higher than the growth rate, the wealth variable in equation (3.2b*) above will have a positive coefficient. In this case, the steady-state consumption depends on wealth, and consumption is reduced in the long run when wealth falls. If the interest rate is lower than the growth rate, the coefficient for wealth becomes negative and consumption will rise when wealth falls, as illustrated in figure 3.3b.

The equation in the box (3.2b*) for consumption in steady state is purely based on (3.2b) and (3.2c) and wealth appears in the equation as an undetermined variable. The wealth in steady-state depends on the consumption equation in (3.2a), implying that if the consumption equation is incorporated wealth as well as consumption can be determined. More specifically, the differenced variables in (3.2a) can be replaced by the steady-state growth rate, the lags can be eliminated, and the equation can be normalized on W

(3.2a)*

$$\log(W) = \left(\log(C / Y^{0.9}) + (1 - 0.4 \cdot growth - 0.011) / 0.407 + 0.200\right) / 0.1$$

For growth equal to the interest rate, the steady-state related three-equation model (3.2a*), (3.2b*) and (3.2c) is partly recursive as C can be determined from (3.2b*) alone. When growth differs from the interest rate, the equations are simultaneous, but also in this case it is easier to interpret the steady-state solution using the tailor-made system of equations: (3.2a*), (3.2b*) and (3.2c), instead of using the original equations (3.2a) (3.2b) and (3.2c).

3.3 The estimated consumption equation

The long-term equation

For the purpose of estimation, the long-term equation (3.1) is written with ADAM-notation for the variables and in log-linear form:

(3.1)*

$$\log(cpuxhw) = \beta_0^* + \beta_1 \log(ydl\_hc) + (1 - \beta_1) \log(wcp)$$

cpuxhw	desired consumption incl. yield on cars and excl. housing
ydl_hc	long-term disposable income in consumption equation
wcp	consumption-determining wealth

In the name, *cpuxhw*, the *w* at the end indicates that the variable does not represent actual, but wanted or desired consumption, while the *u* indicates that the car purchase found in the national accounts has been replaced by an imputed user yield on the stock of cars. The measuring of car consumption is discussed in section 3.4 describing the consumption components in ADAM.

Many macroeconomic models formulate a relation with consumption, income and wealth similar to the above equation (3.1*). However, this does not imply that the models are identical concerning their consumption formation, because the definitions of income and especially of wealth differ from model to model.

The consumption-determining wealth variable includes financial assets and housing wealth

In more theoretical models, wealth is often represented by the discounted value of expected future payments, including wages. In ADAM, the wealth variable comprises only financial assets and some real capital items. Specifically, the wealth variable in the consumption equation consists of total private sector's net financial assets plus the value of the stock of houses and the stock of cars. The net financial assets exclude shares and pension wealth enters with its value after taxes.

It could be argued that only the net financial assets should enter the wealth variable, because only the net financial assets represent a simple claim on the other sectors. Capital items such as machinery and buildings do not in the same way represent a claim on other sectors. Thus, the housing wealth of households is in principle based on the home owners paying rent to themselves, see Miles (1994).

Housing wealth makes it easier to borrow

On the other hand, there is empirical evidence that the market value of dwellings and the related possibilities for mortgage lending have an impact on private consumption. It is, therefore, as in many other models chosen to include housing wealth in the consumption-determining wealth variable. Business buildings and machinery are excluded. This reflects that business capital in ADAM is measured by replacement costs only and not by market prices. Thus, business capital constitutes a sluggish variable, which hardly affects the consumption decisions of households.

The income variable excludes pension savings

Although business capital is not included in the wealth variable, the economic situation for business does affect private consumption as the income variable of the consumption equation includes not only household income but also corporate income.

Specifically, the income, *ydl_hc*, of the long-term consumption equation is calculated as:

- The net disposable income of the private sector
- minus accrued interest on pension assets calculated net of taxation
- plus net withdrawals from pension funds to the private sector
- minus value added in the housing industry.

This income concept has been used in ADAM for some years. It differs from the normal national accounts compilation of private net disposable income because pension savings are removed. Thus, the accrued interest on pension assets is not included in income, and in return the net withdrawal from pension funds is included. Replacing the accrued interest income with net payments to pension fund members makes the income concept more liquid and hence more relevant to credit-rationed consumers. The value added of the housing industry is removed from the income variable, because the consumption is determined exclusive of housing.

The consumption equation explains the annual change in consumption

The difference between the desired consumption cf. (3.1*) and actual consumption is included as an explanatory variable in the estimated consumption equation. The equation has the real consumption change as explained variable on the left hand side, and on the right hand side enters the lagged residual from (3.1*) representing the difference between the desired and actual consumption. The residual is supplemented by short-term dynamics:

(3.3)

$$\mathrm{Dlog}\left(\frac{cpuxh}{pcpuxh}\right) = \alpha_0 + \alpha_1 \cdot \mathrm{Dlog}\left(\frac{ydk\_h}{pcpuxh}\right) + \gamma \cdot \log\left(\frac{cpuxhw_{-1}}{cpuxh_{-1}}\right)$$

Cpuxh	consumption incl. yield on cars and excl. housing
pcpuxh	price of consumption
Ydk_h	household disposable income adjusted for pension savings like *ydl_hc*

The short-term dynamics of (3.3) includes only the real income change and its coefficient, α_1, represents the marginal income elasticity. It is noted that the income variable, *ydk_h*, used in the short-term dynamics does not include the corporate income, which is assumed to affect consumption with a larger delay than household income.

It had been obvious to include the change in the house price to let wealth have a first-year effect on consumption, like income has a first-year effect. However, the change in the house price is not significant in the consumption equation and is therefore omitted.

The consumption equation (3.3) is an error correction equation. It is estimated using OLS and with (3.1*) to represent the desired consumption. The estimation result is shown in table 3.1.

Table 3.1 **Estimated consumption equation**

Variable	ADAM-name		Coefficient	Std. Err.
Private consumption	Dlog(*cpuxh/pcpuxh*)			
Constant		α_0	0.011	0.008
Disposable real income	Dlog(*ydk_h/pcpuxh*)	α_1	0.400	-
Error correction	log($cpuxhw_{-1}/cpuxh_{-1}$)	γ	0.407	0.078
Disposable income	log(ydl_hc_{-1})	β_1	0.900	-
Wealth	log(wcp_{-1})	$1-\beta_1$	0.100	-
Const. in long-term equation		β_0*	-0.200	-

Note: n=1975-2006 s=0,0218 R2=0,547 LM(AR1)=0,022 DF=-3,43

The first line of the table contains the explained variable. Under the table, the sample period, the standard deviation of the estimated residual, the R-squared, the LM-test statistic for first-order autocorrelation, and the Dickey-Fuller static for non-stationary residuals in the long-term relation are listed. The small LM statistic is chi-square distributed with 1 degree of freedom, so there is no evidence of autocorrelation. The numerically large Dickey-Fuller statistic seems to reject non-stationary residuals in (3.1*) indicating co-integration between consumption, income and wealth. It is noted that the long-term income and wealth elasticities are pre-set at 0.9 and 0.1, respectively. It cannot be rejected that the income elasticity is one, making the wealth elasticity zero. However, the wealth elasticity is set at 0.1, which is close to the free but insignificant estimate.

The short-term income elasticity, α_1, is set at 0.4. That is clearly less than one, making the short-term saving reaction relatively high. The 0.4 is slightly larger than the freely estimated value, but the difference is less than one standard deviation, and restricting α_1 to 0.4 cannot be rejected.

It is also noted that the long-term equation constant, β_0^*, has been set at -0.200. This provides the residuals of the long-term equation a mean of zero in the estimation period. The rest of the estimated constant is placed in α_0 in the short-term part of the consumption equation in (3.3). The α_0 of 0.011 provides the residuals of the consumption equation an average of zero in the estimation period.

Short-term dynamics affects the long-term consumption

The estimated error correction form is, as we have seen, divided into a short-term part, where the coefficients are called α and a long-term part, where coefficients are called β. This explicit division into short and long term does not mean that the long-term solution of the consumption equation in (3.3) is independent of its short-term part. 'Long-term solution' is here referring to a steady state path, where the rate of growth in income and consumption is constant.

To illustrate how the short-term dynamics affects the long-term solution we consider the error correction equation with the estimated short-term coefficients inserted and with a brief notation for the variables, *Cw* is the desired consumption, taken from the long-term equation. [1]

$$\text{dlog}(C) = 0.011 + 0.4 \cdot \text{dlog}(Y) + 0.407 \cdot \log(Cw_{-1}/C_{-1})$$

To clarify the impact of short-term dynamics on the level of consumption, the error correction equation is written with the contemporaneous consumption level on the left hand side.

$$\log(C) = \log(Cw) + \left[0.011 + 0.4 \cdot \text{dlog}(Y_{+1}) - \text{dlog}(C_{+1})\right]/0.407$$

It appears that actual consumption, C, is only equal to the desired consumption, Cw, in steady state, if the square brackets expression equals zero in steady state, and we cannot assume that this will be the case. In the estimation period, the short-term constant of 0.011 provides the square brackets expression a mean of zero, but growth in a steady-state scenario will generally differ from the average growth of the estimation period.

The problem occurs because the coefficient on income change is only 0.4 and not 1. In a simple steady-state projection, consumption and income will grow at the same rate, so if the coefficient on income change mentioned was 1 instead of 0.4, the expression in square brackets would automatically be 0.11 in any steady state with the same growth rate for income and consumption. In this case, it would be appropriate to set the short-term constant at zero and place the estimated constant exclusively in the long-run equation. In doing so, we would secure that the actual consumption, C, equals the desired consumption, Cw, at steady state, regardless of the growth rate.

The estimated coefficient on the income change is significantly lower than 1. Thus, it is not necessarily a good idea to restrict this coefficient to 1, which would prevent the short-term dynamics from affecting the equation's long-term solution. However, it can be attained that C is equal to Cw in steady-state by correcting the short-term constant to suit the trend, i.e. the growth rate, of the steady state. Such a 'trend correction' will neutralize the effect of the growth rate on the equation long-term solution, see box 3.2 on trend correction.

Box 3.2 **Trend correction**

As mentioned in the above text, the difference between the actual and desired consumption equals:

$$\left[0.4 \cdot \text{dlog}(Y_{+1}) + 0.0112 - \text{dlog}(C_{+1})\right]/0.407$$

This expression has an average of zero in the estimation sample, corresponding to the actual and desired consumption having the same average. The value of the expression in square brackets is obtained by inserting the sample average growth rates of 2.2 per cent for real

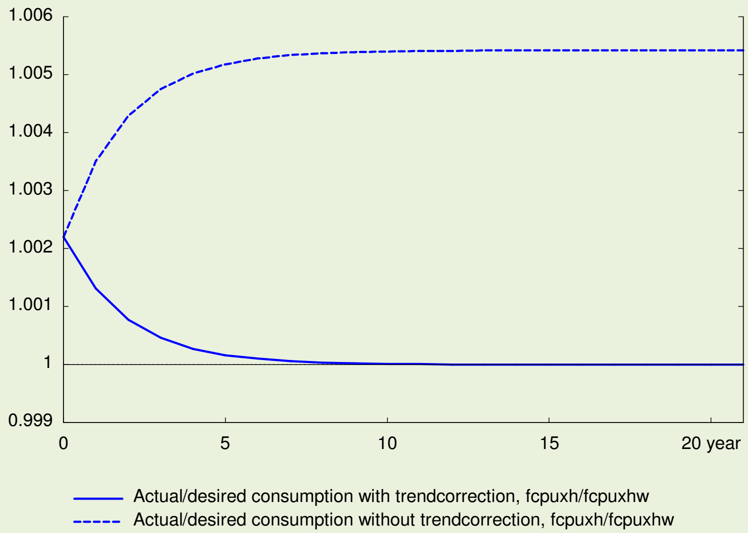
[1] The role of short-term dynamics for the long-term solution to an error correction equation is discussed by Currie (1981).

income and 2.0 per cent for real consumption, 0.4·0.022+0.0112-0.02=0. The 0.0112 represents the short-term constant, α_0, with 4 decimal places. The short-term constant is called the trend correction term, whose function is to balance the impact of the first order differences.

A long-term scenario does not normally establish the growth rates of the estimation sample. A long-term scenario is likely to be quite stylized with all volumes growing by say 1.5 per cent annually. In this case, the expression in the square brackets is no longer zero but 0.0022 (=0.4·0.015+0.0112-0.015), which implies that the actual consumption in the long term will be 0.5 per cent higher than the desired consumption (0.5=0.22/0.407).[2]

At the beginning of the stylized 1.5 per cent long-term scenario, it is natural if the actual consumption deviates from the desired consumption. If the trend correction term is reduced from 0.0112 to 0.009 a couple of years into the scenario, the difference between the actual and desired consumption will disappear gradually. Without this active trend correction the ratio between the actual and desired consumption will, as predicted, increase gradually to 1.005, see the illustration in the box figure using ADAM names for the actual and desired consumption.

Ratio between the actual and desired consumption



[2] If the consumption equation were re-estimated on a sample that reproduced the 1.5 per cent scenario, the long-term constant, β_0, would be estimated at -0.195, instead of the present -0.200 estimate. Thus, the question of trend correction resembles the issue in Hungnes (2010), who splits up structural breaks in VAR-models on growth-rate determining coefficients and long-term coefficients.

We shall return to the issue of trend correction in connection with the presentation of the wage relation in chapter 7 and the construction of the baseline scenario in Chapter 10.

3.4 Splitting up private consumption[3]

The seven components of the consumption system

ADAM's consumption system divides the total consumption, excl. housing consumption into seven components. The seven components of consumption, determined in the consumption system are:

- Driving, *fCgu*
- Car yield, *fCbu*
- Fuel, *fCe*
- Food, *fCf*
- Other goods, *fCv*
- Tourist expenditures *fCt*
- Services, *fCs*

The breakdown by consumption components contributes in ADAM to distribute private consumption to production industries and import categories and to determine the tax revenue. For example, the price of a passenger car is mainly taxes and imports, while households' purchase of services mainly affects the production in the private service industry. Thus, the consumption components differ in the content of domestic value added, imports from abroad and duties to the public sector.

Five of the seven consumption components listed above are directly taken from the national accounts, while the two transport-related expenditure components, driving and car yield are special ADAM-variables, and we return to those later. The corresponding national accounts components of private consumption are gasoline consumption and purchase of cars, and these two components can be derived from, respectively, driving and car yield. Thus, ADAM also determines consumption in the national accounts

On the chosen breakdown of consumption

The total private consumption in ADAM is divided into eight components: housing yield and the seven components determined in the consumption system. The share of the eight components in the 2006 consumption is shown in table 3.2.

[3] The consumption system is explained in detail in the electronic annex to chapter 3.

Table 3.2 **Consumption components in 2006**

	fCh Housing yield	*fCgu* Driving	*fCbu* Car yield	*fCe* Fuel	*fCf* Food	*fCv* Other goods	*fCt* Tourist expenses	*fCs* Services
Value	131.8	19.5	43.8	39.3	103.2	151.3	32.7	224.8
Share	0.192	0.027	0.055	0.058	0.136	0.191	0.042	0.299

Value in billions DKK chained values, and the share of the components' sum at current prices.

The component housing yield has been separated in order to determine housing yield together with housing stock in the equations for the housing market, cf. section 3.7. Services are the largest consumption component in table 3.2. The second largest is ‘other goods’, such as clothing and furnishings, while it is chosen to have food in a separate component. Services, other goods and foods, i.e. one service and two good components, are clearly the largest components in the consumption system, where they take up the lion's share, cf. table 3.2.

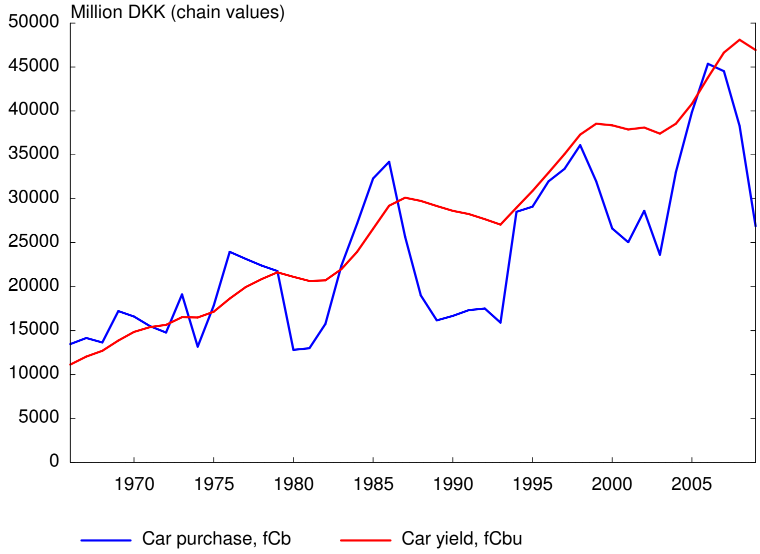
The four smaller consumption components in table 3.2 each play a special role in the model. The car yield component determines the cyclical purchase of cars. The driving component determines gasoline consumption, which together with fuel consumption describes household energy consumption. It is important that ADAM determines energy consumption when the model is used to asses energy and climate policy. Tourist expenses are a separate consumption component, because they also have a role as import component in ADAM.

Driving and car yield are variables in ADAM

Driving and car yield are, as mentioned, specific ADAM-variables. The car yield is determined as a constant factor multiplied by the stock of cars, and the consumption of driving is calculated as the national accounts gasoline consumption, *fCg*, multiplied by a fuel efficiency factor. The purpose of using car yield and driving is to describe consumers' car purchase as an investment decision and to be able to analyse changes in fuel efficiency.

When the consumption system provides the car yield variable it also provides the car stock, *fKncb*, which is a national accounts variable proportional to car yield. If the car stock is supplemented by a depreciation rate, the annual car purchase, *fCb*, can also be determined. The car stock is substantially larger than the annual car purchase, and a per cent increase in the stock of cars gives rise to an increase in the purchase of cars by several per cent. This stock-flow relation makes car purchases significantly more volatile than the car yield, cf. figure 3.4.

Figure 3.4 **Car purchase and car yield**



Nested structure of the consumption system

The consumption system has a so-called nested structure, which means that it lies in the system's equations that consumers choose between two consumption components at a time and not between all at once. The nested structure limits the number of parameters in the equations of the consumption system, thereby making it possible to estimate the system in steps and one or two nests at a time.

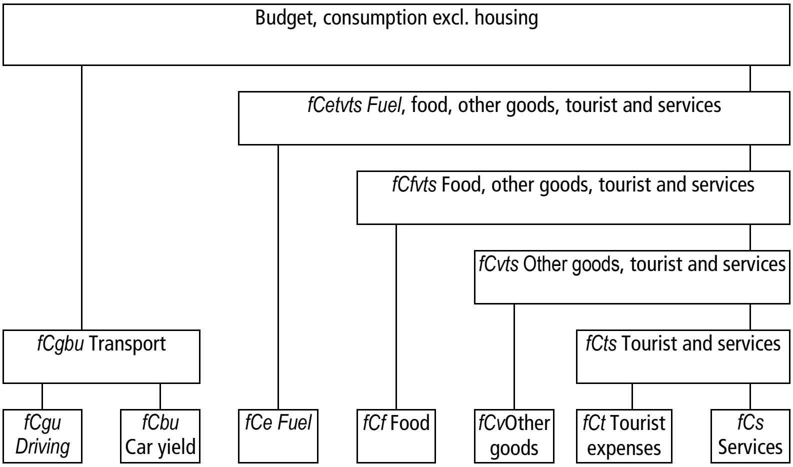
The nested structure is, as far as possible, constructed in such a way that consumption components are nested together when they naturally belong together. Components belonging together are either close substitutes or close complements. Moreover, the most flexible choices, such as tourist travel, lies deep in the nested structure, while less flexible choices like purchasing a car is among the first choices made.

The selected nesting structure is shown in figure 3.5, where it appears that the consumption system starts by distributing the budget on transport, *fCgbu*, and the rest, *fCefvts*. Transport is split into its two components, driving, *fCgu* and car yield, *fCbu*, while the rest is divided on fuel, *fCe*, and the rest, *fCfvts*. This process continues until the entire budget is allocated.

The budget of the consumption system equals consumption excl. housing

The total budget, which the consumption system allocates on seven categories, includes the household consumption without housing and with the consumption of cars transformed into a yield on the stock of cars. To household consumption is added the spending of foreign tourists in Denmark; and in ADAM notation the total budget is *fCpuetxh*.[4] The spending of foreign tourists is included in the budget, because the consumption components include the consumption of foreign tourists. Tourist consumption is small relative to the consumption of Danish households, and the consumption system does not distinguish between tourist and household demand. In Figure 3.5 and in the remaining chapter, we shall simply refer to the budget variable as consumption, excl. housing.

Figure 3.5 **Nested structure of the consumption system**



The long-term equations of the consumption system …

The consumption system behaves as if it is controlled by a representtative consumer that based on a utility function distributes the budget on consumption components in order to maximize utility. The representative consumer takes into account the relative prices and the substitution elasticity of the utility function, and it is chosen to consume less of consumption components that have become more expensive. The breakdown on consumption components is also affected by the size of the budget, since the components have different budget elasticity. Besides, fuel consumption is affected by the weather.

… reflect a flexible CES utility function

The utility function is a flexible CES utility function with a nested structure. The nested structure of the consumer choice has just been described, cf. figure 3.5. All CES functions are characterized by constant substitution elasticity, and the conventional CES functions also restrict the budget elasticity to 1. The adjective ‘flexible’ indicates two things: all budget elasticities can deviate from 1, and the consumption system contains other explanatory variables than just budget and relative prices.

In the long-term equilibrium of the consumption system, the breakdown of consumption reflects the specified utility function, and the derived optimal consumption shares are given by:

[4] Total budget equals the ADAM consumption variable excl. housing, *fCpuxh* cf. sections 3.1 to 3.3 plus the foreign tourist spending, *fEt*, mentioned in chapter 4, which lists the export components.

(3.4)
$$\frac{fC_i^*}{fC} = priceeffect_i \cdot DT_i$$

for $i = gu, bu, e, f, v, t, s$. The variable on the left hand side can be seen as a desired consumption share, defined as the desired consumption component in real terms, fC_i^*, divided by total consumption excl. housing, fC, also in real terms. The variable $priceeffect_i$ represents the total price-related effect on the share of the consumption component i, whereas DT_i represents the total non-price effect created by a budget elasticity different from 1, or by an additional explanatory variable.

Price effects reflect the chosen nesting

The determination of the price effect variable reflects the nesting of the applied CES utility function. For example, there are only two relative prices and two elasticities of substitution in the price effects for the components driving and car yield, located in the outer nest, whereas tourist expenditures and services, located in the innermost nest, are influenced by five relative prices and five elasticities. The seven price effects of the consumption system are shown in box 3.3.

Box 3.3 **Price effects in the consumption system**

The price effects of the 7 consumption components in (3.4) are:

(3.5)
$$BX_{gu} = \theta_{gu} \cdot \left(\frac{p_{gu}}{p_{gbu}}\right)^{-\sigma_{gu}} \cdot \left(\frac{p_{gbu}}{p}\right)^{-\sigma_{gbu}}$$

(3.6)
$$BX_{bu} = \theta_{bu} \cdot \left(\frac{p_{bu}}{p_{gbu}}\right)^{-\sigma_{gu}} \cdot \left(\frac{p_{gbu}}{p}\right)^{-\sigma_{gbu}}$$

(3.7)
$$BX_{e} = \theta_{e} \cdot \left(\frac{p_{e}}{p_{efvts}}\right)^{-\sigma_{e}} \cdot \left(\frac{p_{efvts}}{p}\right)^{-\sigma_{gbu}}$$

(3.8)
$$BX_{f} = \theta_{f} \cdot \left(\frac{p_{f}}{p_{fvts}}\right)^{-\sigma_{f}} \cdot \left(\frac{p_{fvts}}{p_{efvts}}\right)^{-\sigma_{e}} \cdot \left(\frac{p_{efvts}}{p}\right)^{-\sigma_{gbu}}$$

(3.9)
$$BX_{v} = \theta_{v} \cdot \left(\frac{p_{v}}{p_{vts}}\right)^{-\sigma_{v}} \cdot \left(\frac{p_{vts}}{p_{fvts}}\right)^{-\sigma_{f}} \cdot \left(\frac{p_{fvts}}{p_{efvts}}\right)^{-\sigma_{e}} \cdot \left(\frac{p_{efvts}}{p}\right)^{-\sigma_{gbu}}$$

(3.10)
$$BX_{t} = \theta_{t} \cdot \left(\frac{p_{t}}{p_{ts}}\right)^{-\sigma_{t}} \cdot \left(\frac{p_{ts}}{p_{vts}}\right)^{-\sigma_{v}} \cdot \left(\frac{p_{vts}}{p_{fvts}}\right)^{-\sigma_{f}} \cdot \left(\frac{p_{fvts}}{p_{efvts}}\right)^{-\sigma_{e}} \cdot \left(\frac{p_{efvts}}{p}\right)^{-\sigma_{gbu}}$$

(3.11)
$$BX_{s} = \theta_{s} \cdot \left(\frac{p_{s}}{p_{ts}}\right)^{-\sigma_{t}} \cdot \left(\frac{p_{ts}}{p_{vts}}\right)^{-\sigma_{v}} \cdot \left(\frac{p_{vts}}{p_{fvts}}\right)^{-\sigma_{f}} \cdot \left(\frac{p_{fvts}}{p_{efvts}}\right)^{-\sigma_{e}} \cdot \left(\frac{p_{efvts}}{p}\right)^{-\sigma_{gbu}}$$

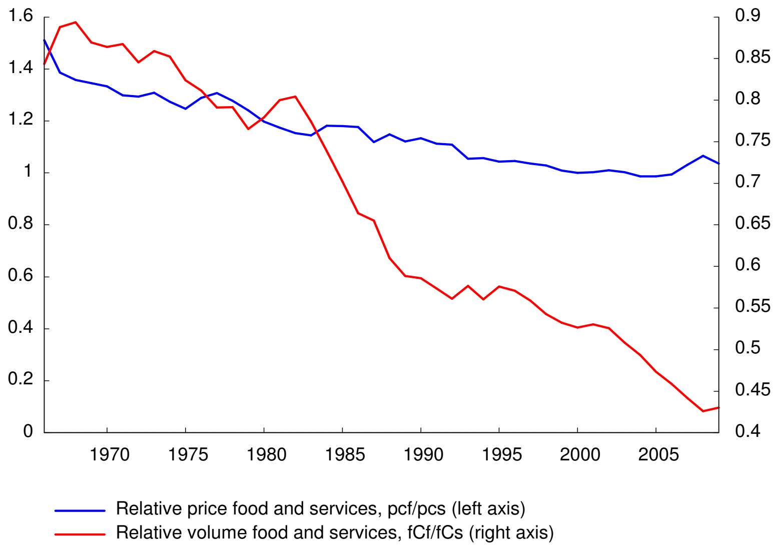
BX_i is the ADAM-term for price effect on component i, letter p with a subscript is the price of the component indicated in the subscript, p without subscript is the total price, the θs are estimated constants, while the σ's are the elasticities of substitution. The price effect on a consumption component reflects its position in the nested structure.

Driving and car yield in the outer nest can only substitute with each other and as an aggregate they substitute with the rest of consumption. Thus, there are only two relative prices and two elasticites of substitution in the top two box equations above. By contrast, there are 5 relative prices and 5 elasticites of substitution to determine the demand for services and tourist expenditures in the inner nest. Services and tourist expenditures are substitutes and as an aggregate they substitute with a third component 'other goods', the aggregate of those three components substitutes with food, the aggregate of the four substitutes with fuel, and finally the aggregate of the five substitutes with the transport aggregate of driving and car yield.

Budget elasticities are different from 1

With a conventional CES function, the consumption share in (3.4) would exclusively be determined by the price effects, but it seems inconsistent with data to explain the consumption share by price effects alone. For example, consumption of food has halved relative to the consumption of services since the 60s, while the relative price of food has declined by more than 30 per cent, cf. figure 3.6. Thus, the development in volumes cannot be explained by the development in prices. It is rather a low budget elasticity of foods, which reduces food consumption relative to service consumption.

Figure 3.6 **Relative price and volume, foods versus services**



Non-price variable allows the budget elasticity to differ from 1...

The non-price variable, DT_i, in long-term equation (3.4) makes the consumption share of component i a function of the total budget, which implies that the budget elasticity can be estimated. This will, e.g. give foods the role of necessity good with budget elasticity below 1, implying that the budget share decreases when the budget increases. At the same time, services have budget elasticity above 1 and appear as a luxury good, whose budget share increases when the budget increases. With these budget elasticities, the consumption system better explains the development in figure 3.6.

... introduces logistic trend and weather as explaining factors ...

Besides allowing the budget elasticity to differ from 1, the non-price variables introduce a logistic trend in the determination of car yield and a variable indicating cold weather (number of so-called heating degree days) in the determination of fuel consumption. The logistic trend for car yield captures the high growth rate in the car stock and car yield at the beginning of the estimation period.

... and helps determine services and tourist expenditures

For the two service components, services and tourist expenditures, the non-price variable is defined residually, so that the desired consumption complies with the budget restriction, which says that consumption components in current prices must sum to the budget in current prices, $fC{\cdot}p$:

(3.12)

$$fC \cdot p = \sum_i fC_i^* \cdot p_i$$

Subscript i in (3.12) represents the seven consumption categories *gu,bu,e,f,v,t,s*. The non-price effects of the consumption systems are outlined in box 3.4.

Box 3.4 **Non-price effects in the consumption system**

The non-price effects of the 7 consumption components in (3.4) are:

Driving: $DT_{gu} = constant1 \cdot (fC/U)^{\beta_{gu}}$

Car yield: $DT_{bu} = trend(fC/U)$

Fuel: $DT_e = constant2 \cdot coldweather^{\kappa}$

Food: $DT_f = constant3 \cdot (fC/U)^{\beta_f}$

Other goods: $DT_v = constant4 \cdot (fC/U)^{\beta_v}$

Services and tourist: $DT_s = DT_t = residual$

The three β-parameters are negative, which implies that the desired budget shares of driving, of food and of other goods fall when per capita consumption, *fC/U*, increases. For car yield, the *DT* variable is a logistic trend defined on per capita consumption. This logistic trend increases strongly in the beginning of the estimation period and reaches its horizontal part by the end of the estimation period. Such estimated logistic trends are found in other behavioural equations, and the functional form is illustrated in connection with the estimation of the house price equation in section 3.9.

Budget restriction in constant prices

Consumption components are national accounts variables, so the budget restriction in (3.12) must also hold in fixed prices. Since fixed-price magnitudes are calculated with the chain index formula, the fixed price budget restriction implies that the consumption components add up when multiplied by their one year lagged prices:

(3.12)*

$$fC = \sum_i fC_i^* \cdot \frac{p_{i,-1}}{p_{-1}}$$

The consumption system complies with this national accounting constraint, because the price, *p*, of total consumption is measured by the corresponding chain index formula for prices. Box 3.8, which is located at the end of the chapter, describes the applied chain index and chain index formula for both volume and price. All constant price items in ADAM are formulated using the chain index, implying that the box not only relates to the budget constraint of the consumption system but to all quantitative identities in ADAM.

It can be added that the price effects already described in box 3.3 formally implies that prices of consumption aggregates, incl. the overall aggregate *fC*, are measured with the CES-price formula following from the applied utility function. However, it is not advantageous to use the exact CES-price formula outside equilibrium, and the consumption system applies ADAM's national accounts price aggregates instead.

3.5 Properties of the consumption system

This section illustrates the properties of the consumption system by describing the effect on consumption components of shocks to the total budget and to the prices of consumption components. The purpose is to illustrate the response of the complete estimated consumption system, but we start by focusing on the long-term equation for driving.

Determination of driving as example

Consumption of driving, *fCgu*, is as mentioned placed in the outer nest of the consumption system, and its price effect includes two relative prices and two associated elasticities of substitution, cf. box 3.3 with price effects. The relative prices and the estimated elasticities of substitution, -0.344 and -0.560, enter equation (3.4*) below, which is one of seven equations that the general long-term equation (3.4) represents.

(3.4)*

$$\frac{fCgu^*}{fC} = 0.0519 \cdot \left(\frac{p_{gu}}{p_{gbu}}\right)^{-0.344} \cdot \left(\frac{p_{gbu}}{p}\right)^{-0.560} \cdot \left(\frac{fC}{U}\right)^{-0.101}$$

The driving-related non-price effect, DT_{gu}, is written in (3.4*) as budget per capita, *fC/U*, raised to the estimated elasticity -0.101, which corresponds to the budget elasticity of driving minus one. The multiplicative constant of 0.0519 comprises the estimated constant and an adjustment term applied to (3.4*).

Total budget raised by 1 per cent

It follows from (3.4*) that if total budget, *fC*, is raised by 1 per cent, the desired driving, *fCgu**, increases by 0.9 per cent, equivalent to 1 per cent minus 0.101 per cent drop in the budget share. We assume that the population, *U*, and relative prices are unchanged.

Cost of driving is raised by 1 per cent

To calculate the effect on driving from a one per cent rise in the price of gasoline and thus in the price of driving, p_{gu}, for a given budget, the effect on the two relative prices in (3.4*) must first be determined. The effect on relative prices depends on the share of driving in the transport aggregate and in total consumption excl. housing. According to box 3.5, the first relative price in (3.4*) increases by 0.67 per cent and the second increases by 0.3 per cent. Using the associated elasticities of substitution it can be calculated that driving falls by 0.4 per cent when the price of driving is raised by 1 per cent (-0,4=-0,334·0,67-0,560·0,3). This means that driving for constant real budget has an own-price elasticity of -0.4.

The fall in the consumption of driving implies that the rest of the consumption rises because the rest has become relatively cheaper. This argument excludes the consumption of car yield, *fCbu*, which falls slightly indicating that driving and car yield are complementary goods. Complementarity occurs because the substitution effect of the price rise on the transport aggregate (driving and car yield) is stronger than the substitution effect of car yield becoming cheaper compared to driving, see box 3.5.

Box 3.5 **Effect on relative prices and on the consumption of car yield**

If the share of driving is 33 per cent in the aggregate of driving and car yield, an increase of one per cent in the price of driving, p_{gu}, will raise the first relative price of (3.4*), p_{gu}/p_{gbu}, by 0.67 per cent, since the numerator rises 1 per cent, and the denominator rises 0.33 per cent. If driving and car yield together takes up 10 per cent of the budget, the second relative price of (3.4*), p_{gbu}/p, will rise by 0.3 per cent (0.3=0.33-0.1·0.33).

The applied weights of 33 and 10 per cent, respectively, reflect the related expenditure components at current prices. At current prices the component driving is equivalent to gasoline consumption, *Cg*, and *Cg*/(*Cg*+*Cbu*) equals about 33 per cent in ADAM databank, while (*Cg*+*Cbu*)/*Cpuetxh* equals about 10 per cent.

The long-term equation for the desired car yield, *fCbu**, resembles the long-term equation for the desired driving and contains, e.g. the same elasticities of substitution, cf. box 3.3. The multiplicative constant 0.0724 comprises the estimated constant and an adjustment term applied in the final year of the national accounts:

$$\frac{fCbu^*}{fC} = 0.0724 \cdot \left(\frac{p_{bu}}{p_{gbu}}\right)^{-0.344} \cdot \left(\frac{p_{gbu}}{p}\right)^{-0.560} \cdot trend\left(\frac{fC}{U}\right)$$

The car-yield-related non-price effect, DT_{bu}, consists of a logistic trend defined on the budget per capita.

The equation above illustrates how the consumption of car yield reacts when the price of driving increases by 1 per cent. For a given real budget per capita, it depends on the response of the two relative prices, p_{bu}/p_{gbu} and p_{gbu}/p. The former relative price falls by 0.33 per cent as the numerator is assumed unchanged, while the denominator, as already mentioned, increases by 0.33 per cent reflecting the share of driving in the transport aggregate. The other relative price, p_{gbu}/p, also enters equation (3.4*) for driving and, as already mentioned, it increases by 0.3 per cent

With relative price changes of minus 0.33 per cent and plus 0.3 per cent and with substitution elasticities of, respectively, -0.344 and -0.560, the consumption of car yield falls by 0.05 per cent ($-0.05=-0.334\cdot(-0.33)-0.560\cdot0.3$) when the price of driving rises by 1 per cent. Driving falls by 0.4 per cent, cf. the main text, so that the percentage fall in car yield is clearly less than the percentage fall in driving. However, car yield does fall, and the two transport-related goods are complementary in the estimated consumption system.

Illustrating the response of the entire consumption system

After illustrating how the long-term equation for driving reacts to budget and price changes, we now illustrate the response of the complete consumption system. The complete system consists not only of equilibrium equations for the seven consumption components, but also describes their dynamic adjustment to equilibrium.

Total budget raised by 1 per cent

The dynamic adjustment means that consumption components react differently in the short and long term, when the consumption system distributes a permanent increase of 1 per cent in the total consumption excl. housing on seven components. In the short term, the consumption share increases for both other goods and tourist expenditures, see the first column of table 3.3, which indicates that the first-year effect is larger than 1 per cent for both these consumption components. In the long term, however, the effect on other goods is slightly less than 1 per cent, cf. the last column in table 3.3, implying that in the long term consumption share for other goods falls. The long-term effect on tourist expenditures is similar to the short-term effect larger than 1.

Short-and long-term effect differ

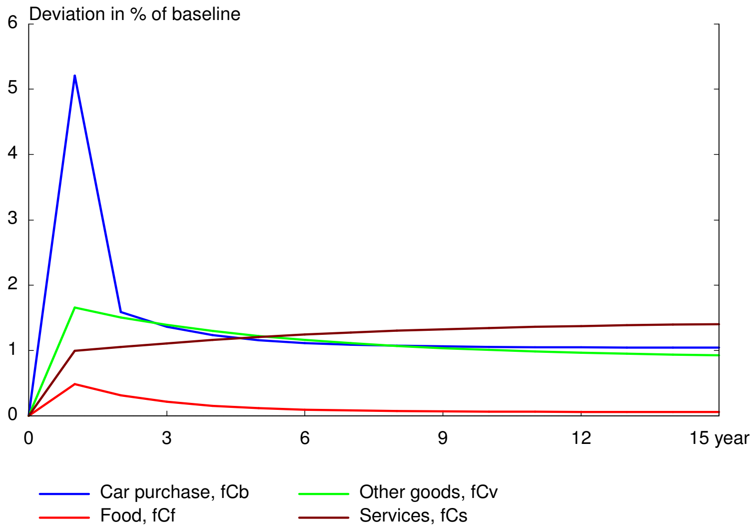
It is also apparent from the table that the consumption of food has the smallest long-term budget elasticity. In the long term, food consumption hardly increases, but in the first year food consumption increases by 0.5 per cent. Thus, similar to the component ‘other goods’, food consumption reacts stronger in the short than in the long term. This difference between the short- and long-term responses may reflect that it is relatively easy to increase, for instance, the consumption of food when the total budget is increased. It is noted that it is possible to increase a component in constant prices by purchasing more of the same but also by buying a better quality.

Table 3.3 **Effect of 1 per cent increase in consumption excl. housing**

	1. year	2. year	5. year	10. year	Long run
			per cent		
fCgu – driving	0.19	0.58	0.88	0.92	0.91
fCbu – car yield	0.76	0.88	1.02	1.05	1.04
fCe – fuel	0.27	0.59	0.93	1.01	1.00
fCf – food	0.49	0.32	0.12	0.07	0.05
fCv – other goods	1.66	1.51	1.23	1.01	0.87
fCs – services	1.00	1.06	1.21	1.35	1.45
fCt – tourist expenses	1.70	1.65	1.57	1.51	1.45
fCb – car purchase	5.21	1.59	1.16	1.06	1.04

Table 3.3 shows the effect on households' purchase of cars in the bottom line and it appears that although car yield and thus also car stock increases by less than 1 per cent in the first year, car purchases will increase by several per cent in the first year, see also figure 3.7, which shows the consumption reaction in the first 10 years of: car purchases, food, other goods and services.

Figure 3.7 **Effect of 1 per cent increase in consumption excl. housing**



Effect of price changes

In the following, the effect of price changes for a given nominal budget in DKK is presented. The budget is equivalent to consumption excl. housing, so the starting point is that the sum of consumption components is determined at current prices when price changes occur.

The nested CES utility function ensures that all own-price elasticities are negative, implying that when the price of a consumption component increases, the share of that component is reduced at constant prices. Moreover, the nested structure implies that a consumption component is substitutable with all components belonging to other nests.

In contrast, two consumption components belonging to the same nest may be complementary goods, if the substitution between them is small compared to their common substitution relative to the rest of the consumer components. Car yield and driving is, as already mentioned, an example of complementary goods in the consumption system.

Long-term price elasticities

The long-term price elasticities of the consumption system are given in table 3.4. With the budget pre-determined in nominal terms, the table shows the so-called uncompensated elasticities. Thus, the result of a 1 per cent price increase of a component will be affected by the implied reduction of the budget in real terms, and you cannot, for example, read from table 3.4 that the consumption system meets the Slutsky symmetry conditions, which relate to the compensated price elasticities. The implied impact on the budget in real terms, also means that driving and car yield fall slightly more in table 3.4 than in the calculations of box 3.5, where the real budget was assumed to remain unchanged when the price of driving increased.

Table 3.4 **Long-term price elasticities in the consumption system**

	pcg	*pcb*	*pce*	*pcf*	*pcv*	*pcs*	*pct*
fCgu – driving	**-0.437**	-0.146	-0.023	-0.054	-0.072	-0.121	-0.019
fCbu – car yield	-0.102	**-0.438**	-0.033	-0.077	-0.103	-0.172	-0.027
fCe – fuel	-0.014	-0.021	**-0.549**	-0.083	-0.109	-0.183	-0.029
fCf – food	0.018	0.027	0.033	**-0.430**	0.101	0.169	0.027
fCv – other goods	-0.010	-0.015	-0.026	-0.061	**-0.909**	0.138	0.022
fCs – services	-0.030	-0.044	-0.067	-0.167	-0.045	**-1.184**	0.108
fCt – tourist expense	-0.030	-0.044	-0.067	-0.167	-0.045	0.694	**-1.759**

The table shows that tourist expenditures and services are the most price-sensitive consumption components with own-price elasticities that are numerically larger than 1. The own-price elasticity of other goods is numerically close to 1, while the rest of the components have numerical own-price elasticities between 0.4 and 0.6.

The table also shows that the price of services has a clear negative effect on three of the other consumption components. This reflects that services are a large consumption component and an increase in the price of services has a relatively large positive impact on the overall consumer price, which reduces the real budget. Tourist expenditures and services appear to be close substitutes, which is reflected in the large positive price elasticity between these two components.

3.6 The estimated consumption system

Consumption shares determined in error correction equations

The consumption share of each consumption component is described by an error correction equation in which the logarithmic difference between the actual and desired budget share, $b\text{-}b^*$, is lagged and supplemented by the change in two variables. The first variable, *bx* is the logarithm of *BX* representing the effect from relative prices, see box 3.3. The second variable, *fc-u*, is the logarithm of per capita consumption where consumption excludes housing, but includes the consumption of foreign tourists.

(3.13)

$$\text{Dif}\, b_i = \alpha_{1,i}\, \text{Dif}\left(bx_i\right) + \alpha_{2,i}\, \text{Dif}\left(fc-u\right) + g_i - \gamma_i\left(b_{i,-1} - b^*_{i,-1}\right)$$

for i = gu, bu, e, f, v, t, s. Equation (3.13) represents the seven error correction equations of the consumption system. The Greek letters are coefficients and the g's are constants.

In principle, the seven error correction equations for budget shares should be estimated simultaneously. In practice, it is impossible, because there are more parameters than observations. However, the budget shares in outer nests are only influenced by the directly associated elasticities of substitution and a recursive procedure can be used that starts by estimating the two elasticities of substitution associated with the outer nest. Thereafter, the subsequent nests are estimated one by one to determine the rest of the substitution elasticities, while the elasticities estimated once are maintained. [5]

Estimated short- and long-term parameters

The most important estimated parameters are reported in table 3.5. The omitted parameters comprise constants and coefficients for specific variables such as the logistic trend in the demand for car yield.

Table 3.5 **Consumption system, estimation results**

	1st year price effect	1st year budget elasticity	Budget elasticity	Substi-tution elasticity	Error-correcting-parameter
	$\alpha_{1,i}$	$1+\alpha_{2,i}$	$1+\beta_i$	σ_i	γ_i
fCgu – driving	0.73(0.12)	0.19(0.13)	0.90(0.04)	0.34(0.04)	0.55(0.09)
fCbu – car yield	0.20(-)	0.77(0.12)	NA	0.56*(0.10)	0.45(0.10)
fCe – fuel	0.68(0.16)	0.28(0.33)	1.00(-)	0.52(0.05)	0.45(0.10)
fCf – food	0.71(0.29)	0.50(0.11)	0.05(0.07)	0.52(0.27)	0.40(0.12)
fCv – other goods	1.00(-)	1.68(0.13)	0.84(0.09)	1.00(-)	0.17(0.07)
fCt – tourist expense	0.52(0.15)	0.61(0.50)	NA	2.05(0.95)	0.13(0.07)
fCs – services	0.68(0.20)	1.00(-)	NA	NA	0.09(0.03)

* Substitution elasticity between the aggregate car yield + driving and the rest of consumption.
Standard errors in parenthesis.

[5] For further details, see the electronic appendix on consumption system estimation.

The first column indicates how much of the long-term price effect is reflected in the first year. The parameter $\alpha_{1,i}$ is between 0 and 1. The second column indicates how many per cent consumption increases in year one when the budget is increased by 1 per cent. The consumption increase corresponds to one plus the estimated 1st year budget share elasticity $\alpha_{2,i}$. The third column indicates how many per cent consumption increases in the long term, when the budget is increased by 1 per cent. Services and tourist expenditures do not increase by a given percentage when the budget is increased by 1 per cent, but increases to balance the decline of other consumer shares. The fourth column indicates the numerical value of the elasticity of substitution between the consumption component and the aggregate of other components entering the nest. Note the exception in the row with car yield. In this row, the elasticity concerns the substitution between car yield and the aggregate of car yield and driving. The fifth column contains the annual error correcting parameter, which determines the speed of adaptation to equilibrium.

Restrictions on parameters

Parameters without standard deviation are restricted to the value indicated. For example, the first year's price effect for car yield is restricted and larger than its unrestricted estimate. This is done to avoid an unrealistic jump in car purchases in the second year, while the first year's price effect is reduced for the component 'other goods' to avoid overshooting. Besides, the elasticity of substitution is restricted to 1 in the equation for other goods, because the free estimate is large and uncertain. None of the restrictions imposed can be rejected empirically.

3.7 Housing market

Dwellings are capital goods with a long life span, and the annual housing investments correspond to a small share of the stock of houses. Thus, in spite of large fluctuations in housing investments the stock of houses remains a sluggish variable. This means that when the demand for housing rises, e.g. due to a decline in interest rates, it is, at first, the house price, which rises in the short run, while the supply of dwellings is slow to react. Only in the longer term, the stock of dwellings will increase enough to meet the increased demand, and the house price will return to equilibrium.

Desired volume of housing

The housing market of ADAM is modelled to make housing prices serve as an endogenous variable making the desired volume of housing equal the actual volume of housing. This is a standard approach, which has been used in ADAM for several years and it implies that house prices respond faster than housing supply.

The basic equation for housing demand determines the desired volume of housing and hence the desired housing stock as a function of private consumption and of the relative price on housing consumption:

(3.14)

$$desiredhousingstock = \beta_0 \cdot consumption \cdot \left(\frac{usercostrate \cdot houseprice}{consumptionprice} \right)^{\beta_1}$$

It follows from (3.14) that housing demand rises by 1 per cent when private consumption (excl. housing consumption) rises by 1 per cent. In addition, there is a substitution effect of the ratio between the price of housing consumption and the price of the rest of consumption. The price of housing consumption is represented by the house price multiplied by a usercost rate that translates the house price into an annual rent paid by the home owner.

Housing volume determines housing consumption

In ADAM, the production and hence consumption of housing services is proportional to the volume of housing. Thus, equation (3.14) relates housing consumption to the rest of consumption with a volume elasticity of 1 and a negative price elasticity of β_1.

The definition of the ADAM user cost rate included in (3.14) implies that half of the households are assumed to be credit-rationed and focused on the nominal loan service, while the other half, which is not credit-rationed, focuses on the usual real economic user cost. The definition implies that the user cost rate is affected both by the use of short-term financing and the introduction of redemption-free loans. The user cost rate is discussed in more detail in section 3.9 describing the estimated housing model.

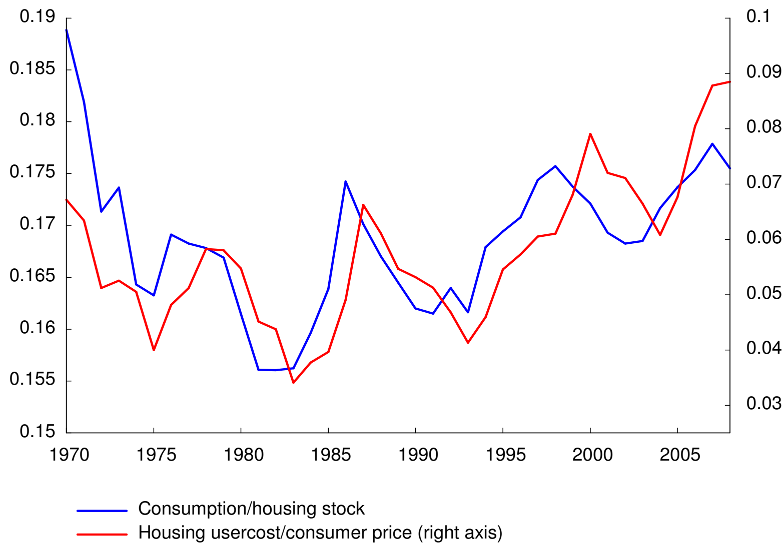
In the short term, housing volume is fixed and price is flexible

The model of housing markets implies that house prices are more responsive than volumes whenever the demanded housing volume differs from the actual housing volume. If the desired housing volume is larger than the actual volume, the house price will increase and make the desired housing volume decrease and approach the actual volume of housing.

If house prices are flexible, the desired housing stock will usually be close to the actual housing stock and following the housing equation in (3.14), there will be a positive correlation between, on the one hand, the ratio of consumption over actual housing stock and, on the other hand, the relative price of housing consumption. The relative price is shown in brackets in (3.14).

The data seems to confirm this correlation, cf. figure 3.8 comparing the two ratios.

Figure 3.8 **Consumption/housing stock and ADAM user cost/consumer price**



Data confirms equation for housing demand

From the mid-70s and until now, the ratio consumption/housing stock has four peaks, all of which are followed by a peak in user cost/consumer price. This correlation pattern can indicate that house prices are cyclical because the cyclical development in consumption drives the price of housing services.

The correlation between the two curves in figure 3.8 seems clear without being perfect. For example, it seems that the correlation breaks somewhat in the last years of the sample, where the relative price of housing is high in comparison to the ratio consumption/housing stock. The most outstanding break in the simple correlation is, however, that consumption has a clear maximum in relation to housing stock at the beginning of the period shown, but the relative price of housing was not particularly high at that time.

The discrepancy can reflect that house price formation was more restricted around 1970 because it was then more difficult to finance the sale of an existing dwelling than the sale of a newly built. The apparent deviation from the simple demand equation (3.14) at the beginning of the sample could justify a shortened estimation sample. However, it has been decided to introduce a logistic trend in the estimated equation, see section 3.9 describing the estimation of the house price equation.

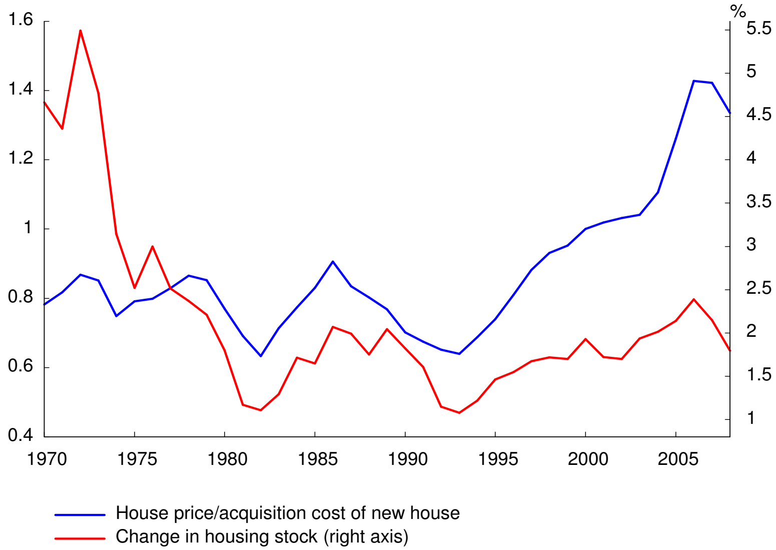
Tobin's q and the housing stock

In addition to determining the housing price, ADAM must also determine the housing supply. The point of departure is that the ratio between the market price of a house and the cost of constructing a new house determines the development of housing volume and thereby also the supply of housing. The ratio house price/acquisition cost is also called Tobin's q, and the higher this ratio is, the more advantageous it is to build houses. Thus, Tobin's q is assumed to explain the annual increase in the housing stock, see equation (3.15), where the parameter β_1 represents the short-term supply elasticity. The long-term supply elasticity is infinite, corresponding to a horizontal supply curve at the construction cost level.

(3.15)

$$\frac{housingstock}{housingstock_{-1}} = \beta_0 \cdot \left(\frac{houseprice}{constructioncost}\right)^{\beta_1}$$

Figure 3.9 **Tobin's q and the change in housing stock**



Data confirms only partially the equation for housing supply

Figure 3.9 confirms that there is a positive correlation in the data between the level of Tobin's q and the change in housing volume. However, it is also clear that the relationship has changed over time and that we may need an additional variable to explain that housing volume expanded rapidly in the early 70s, even though house prices and Tobin's q were not particularly high. The problem with explaining the housing volume in the first years of the estimation resembles the problem of explaining the house price, cf. the discussion of figure 3.8. To explain the pattern of the early 70s in figure 3.8, it was chosen to introduce a logistic trend to capture the restrictions on price formation at the beginning of the estimation period. It turns out that the same logistic trend plus a variable for subsidized housing can be used to explain the strong growth of housing supply in the early 70s.

Tobin's q is high at the end of the sample

Those additional variables do not solve every problem in the equation for the development of the housing volume. Specifically, there is a significant explanation problem in the later years of the estimation sample, where the growth in housing stock appears relatively small compared with the exceptionally high level of Tobin's q, see figure 3.9. The apparent shift in the relation between Tobin's q and housing growth corresponds to a jump in the equilibrium level of Tobin's q.

Regarding the level of Tobin's q in figure 3.9, it must be noted that the presentation of the equations for housing demand and supply have up till now disregarded that the house price data applied indicates the price of the entire property, incl. plot and not just building value, while the national accounts data for housing supply is a fixed-price inventory of the buildings without allowance for land value. We have taken account of this inconsistency between price and volume data in figure 3.9 and in the estimation of the supply equation (3.15) by supplementing construction costs with land prices in the denominator of Tobin's q. The estimation is discussed in section 3.9.

Difficult to adjust Tobin's q for land prices

In principle, the introduction of land prices in the denominator of Tobin's q will moderate Tobin's q, if the high house prices reflect a high price on the land and hence on the location of the house. In practice, however, it is difficult to identify the contribution of the land price to the total property price, and there is little difference between the official price index for land and the price index for property. Thus, despite of our formal adjustment for land prices the level of Tobin’s q looks unusually high in recent years, cf. figure 3.9.

3.8 Housing model properties

This section illustrates the properties of the house price and housing volume equations and how the two equations interact with each other. Moreover, it is illustrated how the housing market interacts with the equation for private consumption excl. housing. First, the properties of the housing equations are illustrated. Then, the interaction between the full housing market model and consumption is illustrated.

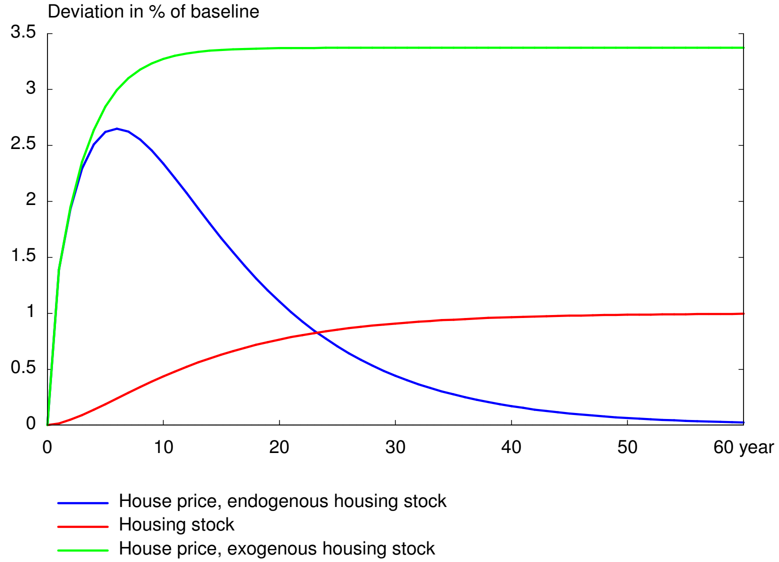
3.8.1 Properties of housing equations

The estimated equations for the house price and housing volume are based on, respectively (3.14) and (3.15), supplemented by short-term dynamics.

Higher consumption increases house prices in the short term ...

The properties of housing equations can be illustrated by increasing consumption and thereby the desired amount of housing by 1 per cent. We start by looking at the house price equation only for given housing volume. If the consumption variable of the house price equation increases by 1 per cent, it is followed by a first-year price increase of 1.4 per cent, as there is an estimated coefficient of 1.4 for the first-year consumption change in the house price equation. In the long run, house prices increase by 3.3 per cent with exogenous housing volume. The long-term result reflects that with a price elasticity of -0.3, it will take a house price increase of 3.3 (1/0.3) per cent to reduce demand by 1 per cent and thereby offset the effect of the 1 per cent increase in the consumption variable. The price reaction to a 1 per cent increase in demand with housing supply exogenous is illustrated in figure 3.10.

Figure 3.10 **Effect on house price and housing volume, consumption + 1%**



... and housing stock in the long term

If the equation for housing stock is incorporated, we have the full housing model, and the result will differ. The first-year effect on the price will be the same, but in the subsequent years the price effect will be smaller because the supply of housing will increase. The long-term house price is unchanged, and instead the housing volume has increased. The house price is unchanged in the long term, because housing supply is growing, as long as house prices exceed construction costs, which are exogenous in the calculation. Eventually, housing stock will have increased by 1 per cent similar to consumption, and consequently the relation between consumption and housing volume is unchanged, see figure 3.10.

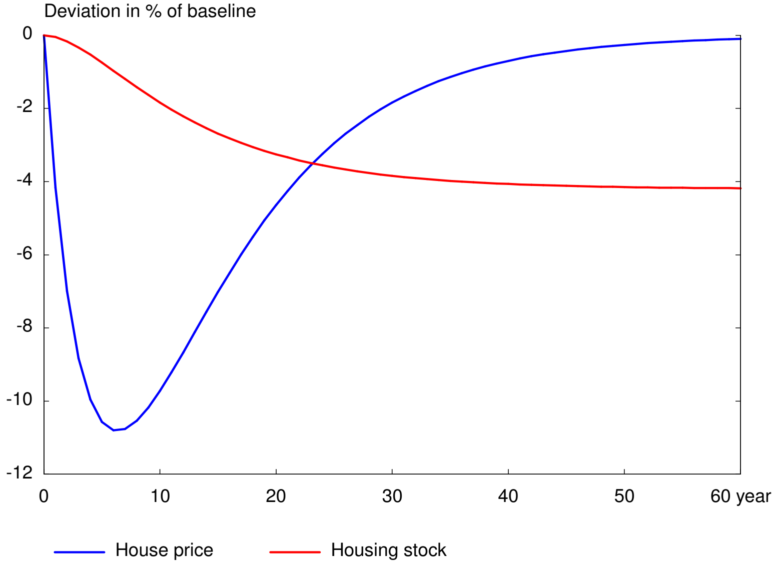
An increase in interest rates reduces house prices in the short term ...

When the interest rate rises by 1 per cent from, e.g. 3.5 to 4.5 per cent the user cost rate of the model will rise from 4.83 per cent in the baseline scenario to 5.58 per cent, corresponding to a relative increase of 15.3 per cent. In the first year, house prices fall by just over 4 per cent, reflecting that the change in user cost rate is included in the house price equation with a coefficient of –5.7 ($-5.7 \cdot (5.58-4.83) = -4.2$). In the long term and with housing stock held exogenous, house prices decrease by 13.3 per cent, which neutralizes the relative increase of 15.3 per cent in the user cost rate ($1.153 \cdot (1-0.133) = 1$). Thus, user cost rate multiplied by the house price remains unchanged in the long term. Consequently, the cost of housing is unchanged, and consumers will demand the same housing volume.

... and the housing volume falls in the long term

If the equation for housing stock is incorporated, house prices will remain unchanged in the long term, while the housing stock will decrease by 4.2 per cent. The volume decline reflects that with unchanged house prices, the cost of housing increases by 15.3 per cent similar to the user cost rate, and with a price elasticity of -0.3, this increase in the price of housing will reduce housing demand by 4.2 per cent ($1.153^{-0,3}$=1-0.042). The effect on house price and housing stock is illustrated in figure 3.11.

Figure 3.11 **Effect on house price and housing volume, interest rate + 1%**



3.8.2 Interaction of housing model and consumption

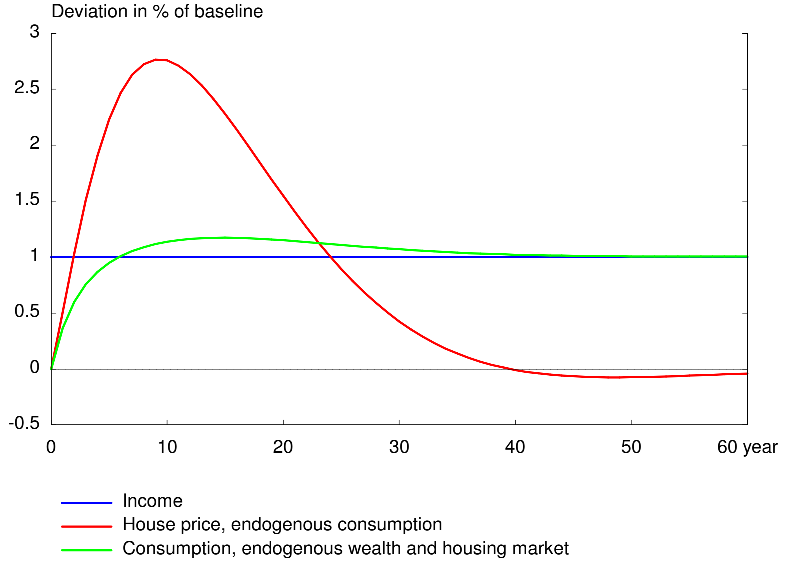
The housing market speeds up the reaction of consumption

If the housing equations are linked with the equation for consumption excl. housing and an equation describing the wealth variable, we will get a quicker adjustment in consumption, because housing wealth is part of the wealth used in the consumption equation. The interaction between housing and consumption does not alter the long-term reaction for either consumption or housing volume, but the dynamics will be affected.

Income Increase of 1 per cent increases consumption by more than 1 per cent in the medium term ...

Housing wealth is included with a 1-year lag in the consumption function, implying that in the first year consumption only reacts to the income change and increases by 0.4 per cent if income is increased by 1 per cent. Thus, the first-year effect on the house price is only 40 per cent of the 1.4 per cent that is the first-year house price increase when consumption increases by 1 per cent. However, the house price increase triggers a wealth effect on consumption in the following years, so that consumption and particularly house prices will have increased by more than 1 per cent after a few years.

Figure 3.12 **Effect on house price and consumption, income + 1%**



... and in the long term consumption and housing stock increase by 1 per cent

In the long term, the house price remains unchanged but the housing volume will have increased by 1 per cent in the long term similar to the rest of the wealth variable and consumption. The effect on consumption and on the house price is shown in figure 3.12. The effect of including the housing market model can be illustrated by comparing figure 3.12 to 3.2. In figure 3.2, which is placed in section 3.2 discussing the consumption equation properties, consumption is slower to adapt to an income increase of 1 per cent, because the accompanying reaction in wealth reflects only savings and not house prices.

Housing consumption also rises by 1 per cent

It can be added with respect to the calculation underlying figure 3.12 that housing supply and thereby housing consumption respond relatively slowly to volume increase, but the 1 per cent long-term increase in total consumption applies both to housing consumption and to the rest of total consumption. This long-term proportionality between housing consumption and other consumption reflects that the housing stock and thereby also housing consumption is linked to other consumption with a volume elasticity of one.

Increasing housing demand increases total consumption in the short term ...

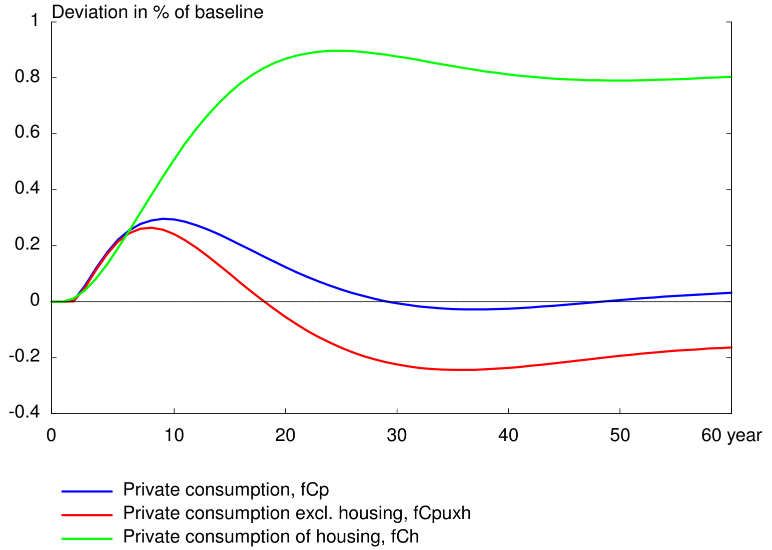
The calculation underlying figure 3.12 concerns the effect of a permanent increase in disposable income. If the upturn in housing demand is not created by a permanent income increase, but reflects a permanent increase in housing demand for a given income, the total consumption is still stimulated in the short term, because of the induced increase in house prices. In the long term, however, we will primarily see an impact on the composition and not on the level of consumption.

To illustrate this point, the long-term housing demand is lifted by 1 per cent, equivalent to a 1-per cent increase of the multiplicative constant, β_0, in equation (3.14). For a fixed relative housing price, this increases the housing consumption by 1 per cent relative to the rest of consumption. The consequence of this change in housing demand has been calculated on the ADAM model, and the result is shown in figure 3.13. It is not just housing consumption, but also the rest of consumption that increases in the short term. This is because higher housing demand immediately raises the house price, which triggers a positive direct wealth effect and increases consumption excl. housing.

... and changes the composition of consumption in the long term

In the long term, however, the positive effect on total consumption evaporates, because the extra consumption is financed through loans. There is no formal loan restriction on the private sector in ADAM, but in steady state private wealth grows in line with income, so that the private savings surplus, i.e. income minus consumption minus investment, eventually equals financial wealth multiplied by the growth rate. From this relation follows the long-term consumption effect of increasing housing demand and this effect comes close to zero, see box 3.6.

Figure 3.13 **Effect on private consumption, housing demand + 1%**



Box 3.6 **Long-term consumption effects of housing demand**

We seek an expression for consumption in steady state, as in box 3.2. The expression can be derived from the wealth equation with housing included. The wealth equation is written:

$$W = W_{-1} + (Y - C_h - C_{xh} - I_h) + \text{Dif}(W_h)$$

where the consumption-determining wealth, W, consists of financial wealth plus housing stock, W_h, so Dif(W) equals savings surplus plus Dif(W_h). Savings surplus equals income, Y, minus the sum of housing consumption, C_h, and consumption excl. housing, C_{xh}, and minus housing investment, I_h.

Income, Y, is split into interest income on financial assets, $interestrate \cdot (W_{-1} - W_{h,-1})$, and the rest of income, $Y_{exinterest}$. Besides, steady state change in wealth, W, is written as $growth \cdot W_{-1}$, and the same applies to housing wealth, W_h. With these rewrites inserted, we get the following equation for total consumption in steady state:

$$C_{xh} + C_h = Y_{exinterest} - I_h + (interestrate - growth) \cdot (W_{-1} - W_{h,-1})$$

If the interest rate equals the steady-state growth rate, the term with wealth variables disappears. If we could also assume that the return on housing capital corresponded to housing investment and that the rest of income was unaffected, total steady-state consumption would be unaffected, so that any increase in C_h would imply a similar decrease in C_{xh}. The baseline is, however, not calibrated to make the return on housing capital correspond to housing investment.[6]

Specifically, the return on housing capital is 15 per cent higher than residential investment in the baseline. The return is here measured as the difference between value added and payroll in the housing industry and the return is part of the income variable, $Y_{exinterest}$. The aforementioned difference between investment and capital returns takes up only 1.7 per cent of total consumption. Thus, an increase in housing stock by 1 per cent contributes to steady-state consumption by a modest 0.017 per cent.

The ratio between the two consumption components, C_{xh} and C_h, is affected by the relative housing price. In the long term, however, the relative price of housing remains largely unchanged, so that there is no significant long-term substitution effect on housing consumption. This means that if the long-term effect on consumption, excl. housing is x per cent, the long-term effect on housing consumption is 1 per cent higher, 1+x per cent, because the calculation lifts the housing demand relation by 1 per cent.

In the baseline, the two consumption components, consumption excl. housing and housing, takes up, respectively, 80 and 20 per cent of total private consumption. Thus, the total effect on consumption can be written as $0.8 \cdot x + 0.2 \cdot (1+x)$ per cent. If housing production increases by 1+x per cent, total consumption increases by $(1+x) \cdot 0.017$ per cent, cf. above, and x can now be determined from the equation, $0.8 \cdot x + 0.2 \cdot (1+x) = (1+x) \cdot 0.017$. The unknown x is calculated at -0.185 per cent and 1+x is 0.815 per cent. These two per cent figures are similar to the ADAM-calculated long-term effect on, respectively, consumption excl. housing and housing consumption, see figure 3.13.

The relative price of housing demand in equation (3.14) is not significantly affected in the long term, so housing consumption eventually increases 1 per cent more than consumption excl. housing. More specifically, the housing consumption increases around 0.8 per cent in the long term, while

[6] In ADAM, a permanent increase in housing wealth will increase the mortgage interest deduction, but also the property-related taxes. Thus, the net effect on disposable income is modest and we can defend the premise that income excluding housing yield is constant. In chapter 11 section 2, we calculate the effect of increasing the total propensity to consume, and in that calculation the increased interest deduction creates a significant impact on disposable income.

consumption excl. housing falls around 0.2 per cent, both compared to baseline, see figure 3.13. The effect on total consumption is very small in the long term, and the just mentioned effects on housing consumption and on the rest of consumption reflect that housing consumption and the rest of the consumption takes up, respectively, 20 and 80 per cent of total consumption, see box 3.6 for details.

3.9 Estimated housing model

The housing model has two estimated equations, one for the house price and one for housing stock.

House price equation

For the presentation of the estimated house price equation the ADAM-notation is applied. The desired housing stock is called *fKbhw*, consumption excl. housing *fCpuxh*, house price *phk,* the related user cost rate *buibhx,* and the price of consumption is *pcpuxh*. Furthermore, the equation is written with logarithms, and introduces a logistic trend, which is a fraction with an exponential function of time in the denominator. The new version of (3.14) is called (3.14*).

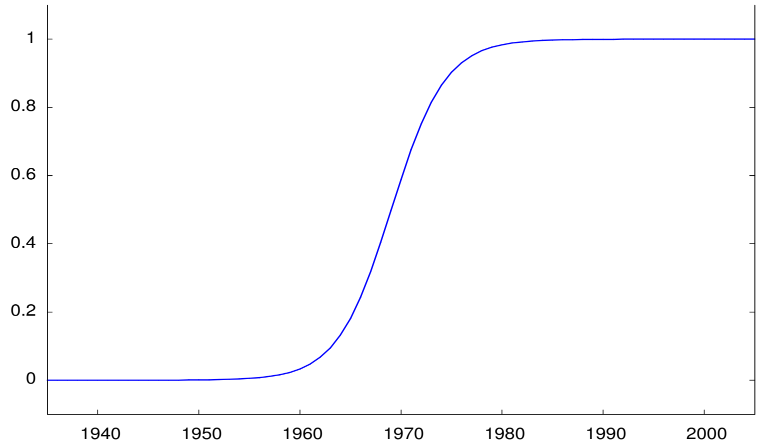
(3.14)*

$$\log(fKbhw) = \beta_0^* + \log(fCpuxh) + \beta_1 \cdot \log\left(\frac{buibhx\ phk}{pcpuxh}\right) + \frac{\beta_2}{1+e^{-0.3739\cdot(time-1969.04)}}$$

As already mentioned, the logistic trend captures that the housing stock was low at the beginning of the estimation period, but the low supply did not create a high house price.

The applied variable *time* indicates the year, the logistic trend lies between zero and one, and in 1969 the logistic trend attains both its median value ½ and its inflection point. The coefficient for *time* implies that the logistic trend is close to 1 from around 1980 onwards, see figure 3.14 where the logistic trend is drawn with 37 years on either side of 1969. The logistic trend resembles an S. The estimation period starts around 1970, implying that only the upper part of the S is being used.

Figure 3.14 **Logistic trend in the desired housing stock equation**



House prices are determined in an error correction equation, where the price change is determined by the lagged difference between desired and actual housing stock, supplemented by contemporaneous changes in private consumption and user cost rate. The estimated short-term dynamics includes the lagged residual as an explanatory variable. Thus, the equation residual called u follows a simple autoregressive process: $u = \rho \cdot u_{-1} + \varepsilon$, where ε is white noise. The total house price equation is shown in equation (3.16), and the estimated coefficients are shown in table 3.6.

(3.16)

$$\mathrm{Dlog}(phk) = \alpha_0 + \alpha_1 \cdot \mathrm{Dlog}(pcpuxh) + \alpha_2 \cdot \mathrm{Dlog}(fCpuxh)$$
$$+\alpha_3 \cdot \mathrm{Dif}(buibhx) + \alpha_4 \cdot d06 - \gamma \cdot \log\left(\frac{fKbh_{-1}}{fKbhw_{-1}}\right) + \rho \cdot u_{-1}$$

Table 3.6 **Estimated house price equation**

Variable	ADAM-name		Coefficient	Std. Err..
House prices	Dlog(*phk*)			
Constant		α_0	-0.009	2.964
Consumption price	Dlog(*pcpuxh*)	α_1	1.000	-
Consumption excl. housing	Dlog(*fCpuxh*)	α_2	1.388	0.254
User cost rate	Dif(*buibhx*)	α_3	-5.733	0.793
Dummy in 2006	*d06*	α_4	0.087	0.043
Error correction	log($fkbhw_{-1}/fkbhw_{-1}$)	γ	0.946	0.331
Relative price of housing	log(*buibhx phk/pcpuxh*)	β_1	0.300	-
Logistic trend	Cf. equation (3.9*)	β_2	1.430	2.966
Constant, long-term equat.		β_0	-0.501	-
Lagged residual		ρ	-0.689	0.163

Note: n=1973-2006 s=0,043 R2=0,781

The first variable on the right hand side is the consumer price increase with coefficient 1. Thus, the equation really explains the change in the real house price. The significant first-year effects from changes in both consumption and user cost help to make the house price a cyclical and interest-rate-sensitive variable in ADAM.

As explained in box 3.7, the applied user cost rate is an average of a conventional user cost rate, based on the real interest rate and a first-year debt service, which is based on the nominal interest rate. We note that the conventional user cost rate is negative in the mid-70s, when real interest rates after tax were negative. Thus, it is only possible to estimate the price elasticity, β_1 because the applied user cost rate incorporates the positive first-year debt service.

Box 3.7 **User cost rate in the house price equation**

Basically, a user cost rate summarizes four elements: Interest rate, inflation, tax and depreciation, see Poterba (1984). The conventional user cost rate can be written as:

$$usercost = i \cdot (1 - t) - dp + s + d$$

where i is the interest rate, t is the tax rate on negative capital income, dp is inflation, s is housing tax, and d is depreciation.

For use in ADAM's house price equation the conventional user cost rate is supplemented by an expression for the 1st year debt service:

$$1.yeardebtservice = i \cdot (1 - t) + s + d + a$$

where a represents repayment of the house loan and remaining variables are as in *usercost*. The applied ADAM user cost rate is an average of the user cost rate and the 1st-year debt service.

$$\begin{aligned} ADAMusercostrate &= 0.5 \cdot usercost + 0.5 \cdot 1.yeardebtservice \\ &= i \cdot (1 - t) - 0.5 \cdot dp + s + d + 0.5 \cdot a \end{aligned}$$

There are two differences between user cost and the 1st-year debt service. In user cost, inflation is deducted, and in the 1st-year debt service the repayment on the house loan is added. The deduction of inflation reflects that user cost in theory always reflects a real interest rate, since the price increase on a house makes it cheaper to own the house. In this perspective, the inflation component in the interest rate works as a debt repayment, increasing the net wealth of the house owner. This means that, contrary to the conventional user cost, the 1st-year debt service includes both a formal debt repayment called a and a real debt repayment of dp.

Although neither real nor formal repayment represents a cost, they may deter potential home buyers with financing problems. The problem with real repayments was evident, while inflation and interest rates were high in the 70s. At that time, the 1st-year debt service was rather high relative to the income of many home buyers.

At the same time, the high interest rate implied that the 1st-year formal debt repayment on a 30-year annuity was quite low. Today, inflation is lower and at the same time the lower level of interest rates means that formal debt repayment is higher. Thus, the issue of redemption-free loans is more important today than it would have been in the 70s.

Most of the historical variations in the ADAM user cost rate are created by changes in the interest rate. The applied interest rate is a weighted average of the 30 - and 1-year bond yield, and as the short interest rate has been the lowest in recent years, the use of short-term loans has reduced the average interest rate.

It was decided to insert a dummy that is one in 2006 and zero in other years. The dummy explains 8-9 per cent out of the roughly 20 per cent house price increase in 2006 when the housing market peaked. The need for a dummy in 2006 may reflect exaggerated house price expectations.

It is also noted that the long-term price elasticity of -0.3 for housing demand, cf. table 3.6, is numerically more than twice as large as the free estimate. The -0.3 is equivalent to the price elasticity in the earlier ADAM version. Keeping the old price elasticity makes the model more stable and the house price is less inclined to fluctuate when exogenous variables are shocked. On the other hand, dampening the price signal means that it takes longer to reach equilibrium in the housing market.

Housing stock equation

The acquisition cost of a new house used in the applied Tobin's q measure comprises not only the price of housing investment, *pibh*, to represent the construction cost, but also the price of land, *phgk*. The latter is included because the applied house price index, *phk*, includes the price of land. Housing stock is a national accounts item excl. land, and it would suffice to include the investment price in Tobin's q's if the house price, *phk*, was measured excl. price of land, but such data are not available. Thus, land price is included in the denominator of Tobin q, and taking logs and applying ADAM-notation turns equation (3.15) into (3.15*).

(3.15)*

$$\text{dlog}\,(fKbh) = \beta_0 + \beta_1 \log\left(\frac{phk}{0.8\,pibh + 0.2\,phgk}\right)$$

The estimated equation for housing stock is constructed from (3.15*) by lagging Tobin's q by one year and introducing some short-term dynamics. The short-term dynamics includes three variables: the contemporaneous change in Tobin's q, a variable with the number of subsidized dwellings under construction, *nbs*, relative to the housing stock, and the contemporaneous change in the logistic trend of the house price equation.

(3.17)

$$\text{Dlog}(fKbh) = \alpha_1 \cdot \text{Dlog}\left(\frac{phk}{.8\,pibh + .2\,phgk}\right) + \alpha_2 \cdot \frac{nbs}{fkbh_{-1}}$$
$$+\alpha_3 \cdot \text{Dif}\left(\frac{1}{1+e^{-0.3739\cdot(\text{tid}-1969.04)}}\right) + \beta_1^* \cdot \log\left(\frac{phk_{-1}}{.8\,pibh_{-1} + .2\,phgk_{-1}}\right) + \beta_0^*$$

The variable with subsidized housing captures that housing investments are affected immediately by subsidized housing under construction. The logistic trend is growing at the beginning of the estimation period, and the change in logistic trend can capture that housing stock grew faster at the beginning of the estimation period than suggested by the relatively low Tobin's q.

The estimated coefficients for (3.17) are shown in table 3.7.

Table 3.7 **Estimated housing stock equation**

Variable	ADAM-name		Coefficient	Std. Err.
Housing stock	Dlog(*fKbh*)			
Constant		β_0^*	0.018	0.001
Tobin's q change	Dlog(*phk*/(.8·*pibh*+.2·*phgk*))	α_1	0.015	0.007
Subsidized housing	*nbs*/*fKbh*$_{-1}$	α_2	0.862	0.288
Logistic trend	Dif(logistic trend)	α_3	0.234	0.040
Tobin's q	log(*phk*$_{-1}$/(.8·*pibh*$_{-1}$+.2·*phgk*$_{-1}$))	β_1^*	0.025	-

Note: n=1968-2006 s=0,0113 R2=0,936 LM(AR1)=3,816

To shorten an otherwise very long adjustment period, the coefficient for Tobin's q has been restricted to 0.025. The free estimate is 0.017, which is significantly smaller. The coefficient of 0.862 for the variable with subsidized housing can be interpreted as the price of a subsidized dwelling, 0.862 million DKK in 2000-prices.

The estimation sample goes back a few years further for the housing stock equation than for the house price equation. This means that there are more years with restrictions on the housing market when the housing stock equation is estimated, and the logistic trend does more for the estimation of the housing stock equation.

It may be added that housing investments are determined from the housing stock by adding the change in housing stock to the housing stock multiplied by the depreciation rate. Strictly speaking, we cannot use the simple sum of capital change and depreciation, because both housing stock and investment are national accounts items measured in constant prices by means of the chain index. Consequently, the chain index formula should be used, which is described in box 3.8.

Box 3.8 **Chain index in ADAM**

Motivating the chain index in national accounts
Before the chain index was introduced in the national accounts in 2005, volumes were weighted by their prices in a fixed base year. For periods far from the base year, this method can cause unrealistic results if there is significant and systematic variation in relative prices over time. The typical example is IT products versus conventional products. The price of IT products typically falls year by year, while the volume grows sharply, especially when, as in the national accounts price and volume are quality-adjusted.

For example, the U.S. price index of IT hardware and services has declined to a tenth since the index started in 1988, and the relative price of IT is reduced accordingly. If, today, IT constitutes 10 per cent of business investments in current prices, IT would constitute more than half if current investments were calculated in 1988 prices (10·10/(10·10+90)=0.53). It is, however, unrealistic and irrelevant to assume that IT has a share of 53 per cent in the investment volume.

This measurement problem can be avoided by using the chain index, where the 'base year' is always the previous year. Many countries have started in recent years to use the chain index in their national accounts, including Denmark. ADAM uses the official national accounts, and the quantitative identities in the model reflect the chain index formula of national accounts.

Chain index formula
In ADAM we use Laspeyres volumes similar to the national accounts. The Laspeyres chain index for volumes is:

(3.18)

$$fX = fX_{-1} \cdot \frac{fX_1 \cdot px_{1,-1} + fX_2 \cdot px_{2,-1}}{fX_{1,-1} \cdot px_{1,-1} + fX_{2,-1} \cdot px_{2,-1}}$$

fX: *Volume index for aggregate*
fX_i: *Volume of subcomponent i*
px_i: *Price of subcomponent i*

The formula says that the volume chain index equals the chain index in the previous year plus the volume growth in subcomponents weighted by previous year's prices.
The Laspeyres volume chain index corresponds to a chain Paasche price index *px*:

(3.19)

$$px = px_{-1} \cdot \frac{fX_1 \cdot px_1 + fX_2 \cdot px_2}{fX_1 \cdot px_{1,-1} + fX_2 \cdot px_{2,-1}}$$

If the Laspeyres volume is multiplied by the Paasche price, we get the value in current prices, and if we multiply the volume formula in (3.18) by the lagged price index, the right hand side can be reduced:

(3.20)

$$fX \cdot px_{-1} = fX_1 \cdot px_{1,-1} + fX_2 \cdot px_{2,-1}$$
$$\Leftrightarrow$$
$$fX = fX_1 \cdot \frac{px_{1,-1}}{px_{-1}} + fX_2 \cdot \frac{px_{2,-1}}{px_{-1}}$$

This formulation of the chain formula is used in ADAM. The volume aggregate, *fX*, is obtained by weighing the component volumes with lagged relative prices. The formulation illustrates that products with a weak price development will be getting less and less weight. With the old fixed-weight index, the volume aggregate was a simple sum of fixed-price variables, i.e. components multiplied by their base-year price. With the chain index, the price development plays a role whenever volumes are added. In equation 7 in chapter 6, the chain formula is applied to the relation between investment and capital stock, that is, to the relation between an annual stream variable and a stock variable, which relates to a point in time.

Note that the growth of a chain aggregate can be calculated by weighing the growth of subcomponents with their value share in the previous year. Thus, we have:

(3.21)

$$\frac{fX - fX_{-1}}{fX_{-1}} = \frac{X_{1,-1}}{X_{-1}} \frac{fX_1 - fX_{1,-1}}{fX_{1,-1}} + \frac{X_{2,-1}}{X_{-1}} \frac{fX_2 - fX_{2,-1}}{fX_{2,-1}}$$

where variables without an *f* represent values.

Calculating the chain aggregate of ADAM-variables
Finally, we will show how to construct a chain aggregate for total consumption, *fC*, from the two ADAM variables for private and public consumption, *fCp* and *fCo*.

With the old fixed-weight calculation, *fC* equaled *fCp* + *fCo* in all years. Compiled with the chain index formula, *fC* still equals *fCp* + *fCo* in the chosen base year, e.g. 2005, where fixed prices correspond to current prices. Thus, we immediately have *fC* in 2005 and can use (3.18) to calculate *fC* in all years after 2005. For the years before 2005, we normalize (3.18) on lagged *fC* and calculate *fC* using the re-normalized formula:

(3. 18*)

$$fC_{-1} = fC \cdot \frac{fCp_{-1} \cdot pCp_{-1} + fCo_{-1} \cdot pco_{-1}}{fCp \cdot pcp_{-1} + fCo \cdot pco_{-1}}$$

The corresponding price index, *pc*, can be calculated as *(Cp + Co)/fC*.

Summary of chapter 3

Chapter 3 has presented ADAM's formation of consumption and the equations for the house price and housing stock. The consumption depends on income and wealth, and the applied wealth variable includes housing wealth, which reflects the house price. The house price is determined by the user cost rate on housing capital and by consumption excl. housing, so that consumption affects the house price.

The interaction between the house price and consumption means that the housing market contributes to the modelled cyclical effect on demand and employment. Moreover, changes in interest rates will affect consumption indirectly by affecting the house price, which goes into the wealth variable used in the consumption function.

In the short term, it is primarily the house price that reacts to a demand change, but in the long-term, the house price is determined by construction costs, and the housing stock will adjust to the demand change.

ADAM divides total private consumption into eight components, one of which is housing consumption, which reflects the housing stock. The other seven components are determined by their relative prices and by total consumption excl. housing, which serves as budget variable in the consumption system determining the seven components. The system determines the stock of cars. The consumption of cars is the annual purchase of cars, which behaves as an investment variable sensitive to cyclical fluctuations.

4. Foreign trade

Exports are a key demand component equalling 50 per cent of GDP when sea transport and other exported services are included. Imports constitute a significant part of total resources, and the amount of imports impacts the size of domestic production.

Exports are basically determined by developments in the foreign export market, while imports depend on developments in the domestic market.

There is often a focus on the response of exports and imports to the ratio between Danish and foreign prices. The larger the price elasticity of foreign trade is, the faster is the crowding out of changes in demand, and the easier it will be for output and employment to reach a long-term equilibrium. The price elasticities of exports and imports in ADAM resemble what can be found in many foreign models, and they are slightly lower than in SMEC.

The price elasticity in foreign trade is important for the long-term properties of the model. In the short term, exports make the domestic production depend on foreign business cycles, while the cyclical reaction in imports dampens the short-term fluctuations in domestic production.

The starting point for modelling both exports and imports is Armington's market model, see Armington (1969). For imports, the model can be summarized as follows: A given Danish demand for a product may be covered by either domestic production or imports. These two types of resources basically adapt to cover total demand, but the division between imported and domestically produced resources depends on their relative price. Thus, imports are a function of demand and of the import price relative to the price of import-competing production. The same basic model is applied to exports, which can be analysed as imports from Denmark. Consequently, exports are modelled as a function of foreign demand and of the price of exports relative to the price of foreign import-competing production.

In the following, we explain the modelling and properties of the foreign trade equations in ADAM, first for exports and then for imports.

4.1 Exports

Market share depends on relative export price

The long-term elasticity of exports with respect to foreign demand is restricted to 1, as in other Armington-type models. Thus, it is a model of our market share, which is explained by the relative export price with estimated price elasticity. The price elasticity is represented by the coefficient β_1 in the long-term equation for the export market share:

(4.1)

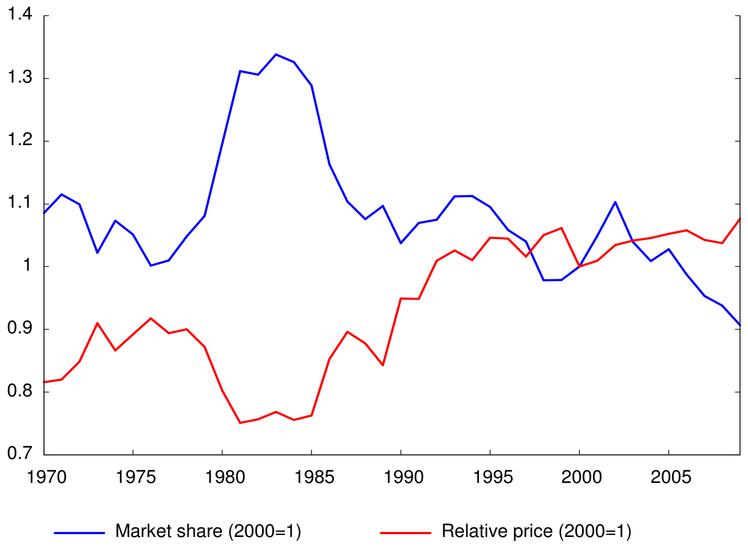
$$\frac{fE}{fEe} = \left(\frac{pe}{pee}\right)^{\beta_1}$$

fE	Exports in fixed prices
pe	Export price
fEe	Export market index in fixed prices
pee	Competitor price, weighted average of import prices for importing countries, converted to Danish kroner

If exports were infinitely price elastic, the Danish export price, *pe*, would be given from abroad and equal the foreign competitor price *pee*. The estimated price elasticity is, however, finite for all exported items and consequently the export price reflects the Danish costs of production. In ADAM, the export price is determined by weighing the prices of the related production and imports.

The price elasticity in (4.1) is negative because the market share decreases when the Danish export price increases relative to the foreign competitor price. The data also suggest that there is a conventional negative correlation between relative price and market share for Danish manufacturing exports, see figure 4.1 where manufacturing exports comprise SITC groups 5 to 9.

Figure 4.1 **Manufacturing exports, market share and relative price**



The negative correlation is seen especially in the period from the end of the 1970s and up until around 1990. The Danish krone was adjusted at the beginning of that period, and the clear downward fluctuation in Danish relative prices was accompanied by an upward fluctuation in the market share. Moreover, the percentage upturn in the market share variable was larger than the percentage downturn in the relative price variable, but after 1990, both volatility and correlation between the two variables has become less clear.

4.2 Export equation properties

Based on the data used for figure 4.1, the long-term price elasticity of manufacturing export is estimated at 2.00, first-year price elasticity is 0.75, while first-year demand elasticity is 0.62; see the related coefficients shown in table 4.1. Adjustment to the long-term export response comes gradually with an adjustment parameter of 0.15. This means that manufacturing exports every year adjusts by 15 per cent of the difference between wanted and actual export.

Tabel 4.1 **Summary of estimated export equations**

Export group	ADAM-name	Price elasticity Long term	Price elasticity 1st year	Adjustment	Demand elasticity 1st year
Food etc.	*fE01*	-1.65	-0.43	0.15	1.00
Materials	*fE2*	-1.78	-0.27	0.15	0.53
Manufacturing	*fE59*	-2.00	-0.75	0.15	0.62
Services ex sea transport	*fEsq*	-2.00	-0.72	0.15	1.00
Tourism revenues	*fEt*	-2.26	0.66	0.20	1.00
Estimated equations		-1.96	-0.60	0.15	0.77
Total exports		-1.58	-0.48	0.12	0.62

Export response with exogenous export price

The response of manufacturing exports to a 1-per cent increase in demand is shown in figure 4.2a, and the response to a 1-per cent increase in the competitor price is shown in figure 4.2b. The figures only report the reaction created by the export equation with the price of manufacturing exports as an exogenous variable. Thus, a 1-per cent increase in the competitor price reduces the relative price of exports by 1 per cent, and the long-term export response in figure 4.2b reflects the long-term price elasticity. It appears that it takes some years before the export adjusts by, e.g. 90 per cent of the long-term response.

Exports react less with endogenous export price

Figure 4.2 illustrates the response of manufacturing exports at given export prices. In ADAM, the export price is determined by domestic costs implying that an increase in foreign demand will not only increase the volume of exports but also the price of exports. This happens because the increased employment in export production makes wages and hence domestic costs rise. The induced increase in export price dampens the impact on export volume coming from the increase in export markets, and in this way ADAM converts the increase in export markets to a long-term improvement in the terms of trade.

An increase in competitor prices will also in the long term mainly increase the export price in ADAM and to a lesser extent increase the export volume, see also chapter 11, section 4, which analyses the model response to a 1-per cent increase of all externally determined prices in ADAM, including the competitor prices.

Figure 4.2a **Effect on exports, export market + 1%**

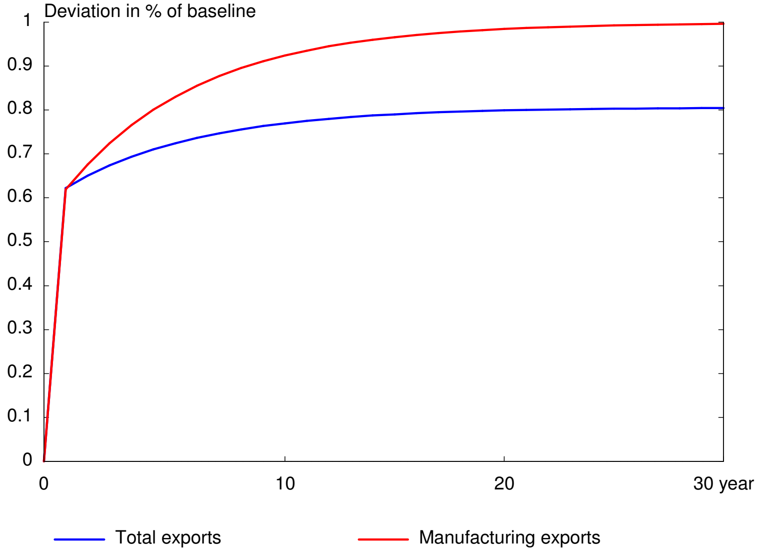
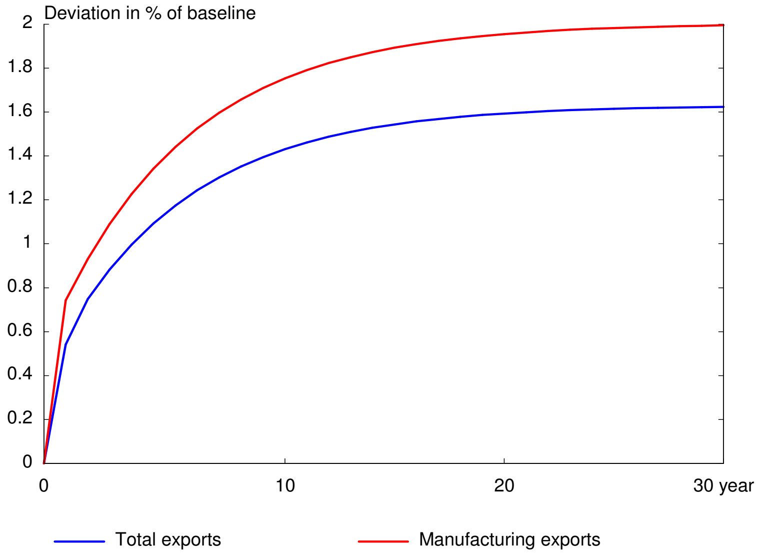


Figure 4.2b **Effect on exports, competitor price + 1%**



Export equations have been estimated for a total of five export groups, see table 4.1. For the export of ships etc., we use the results for manufacturing exports, while the remaining two export groups, energy and transport are exogenous.

Total exports react less than manufacturing exports

Figure 4.2 shows not only the response of manufacturing exports, but also that of total exports to a 1-per cent increase in demand and in competitor price. First-year response of total exports to a 1-per cent increase in demand is just over 0.6 per cent, which resembles the response of manufacturing exports, see also table 4.1. In the long term, total exports increase by less than 1 per cent because the two exogenous export components remain unchanged. Total exports are also less price sensitive than manufacturing exports, because the exogenous export components have, by definition, a price elasticity of zero.

Food, energy and sea transport depart from Armington model

It may be added that the estimated parameters for the export of foods, cf. the first line in table 4.1, are used in a special way. ADAM has an equation determining agricultural production so that the supply of agricultural production is price elastic and will respond if the world-market price of agricultural products changes relative to the Danish production costs. Consequently, the export of foods is determined residually as agricultural production minus domestic use of agricultural products, and the export price of foods follows the world market price of agricultural products.

Thus, the export of food differs from the standard Armington assumption that we can differentiate between the prices of Danish and foreign products. Moreover, the prices of the two exogenous export components are also determined outside the simultaneous model. Thus, the price of energy export, *pe3*, is tied to the exogenous import price of crude oil, while the export price of sea transport, *pess*, depends on the exogenous and world-market-determined output price of the sea transport industry, *pxqs*.

4.3 Export market and export components

Our export market is the import of our trading partners

Danish exports are imports of other countries making it natural to measure the market growth for exports as a weighted sum of the import growth of each of our trading partners. The importing countries do not have equal importance for Danish exports and the market index weight for a country reflects its share in Danish exports:

(4.2)

$$\frac{fEe}{fEe_{-1}} = \sum_j we_{j,-1} \cdot \frac{fEe_j}{fEe_{j,-1}}$$

fEe	Market index (2000=1)
we_j	Share in Danish exports of country *j*
fEe_j	Imports of country *j*

Denmark competes with other exporting countries

The market share of Danish exports is thus measured as imports from Denmark relative to total imports of the importing countries. The size of this market share is determined by the Danish exporters competing with other countries exporting to the importing country in question. The price of the competitors is measured as a weighted sum of the import prices in importing countries, where the weight for an importing country indicates the share of Danish exports going to the country.

The market index is measured based on data from OECD Economic Outlook, and the index includes importing countries with a share above 1 per cent in Danish exports of the product group in question. For manufacturing, almost all OECD countries and regions of Eastern Europe, the OPEC, and Southeast Asia enter the market index.

In total, there are 8 export components in ADAM, 5 goods and 3 services. The breakdown on components follows the foreign trade SITC classification, see table 4.2 showing the export share of each component.

Table 4.2 **Export components in ADAM**

Exports of	ADAM-name	SITC-group	Export share, 2000, per cent
Foods etc.	*E01*	0+1	13.0
Materials	*E2*	2+4	2.8
Energy	*E3*	3	5.9
Manufacturing	*E59*	5-9	44.5
Ships etc.	*E7y*	del af 79	1.2
Services excl. sea transport and tourists	*Esq*	-	13.3
Sea transport	*Ess*	-	14.3
Tourist revenues	*Et*	-	5.0
Total, DKK bn.	*E*		602.4

4.4 Estimation of export equations

For the purpose of estimation, the long-term equation (4.1) is written in logs and supplemented by short-term dynamics to make an error-correcting equation for, e.g. exports of manufacturing, SITC 5-9:

(4.3)

$$\mathrm{Dlog}(fE59) = \alpha_1 \cdot \mathrm{Dlog}(fEe59) + \alpha_2 \cdot \mathrm{Dlog}\left(\frac{pe59}{pee59}\right) + \alpha_3 \cdot \mathrm{Dif}(dum91) - \gamma \cdot \left[\log\left(\frac{fE59_{-1}}{fEe59_{-1}}\right) + \beta_1 \cdot \log\left(\frac{pe59_{-1}}{pee59_{-1}}\right) - \beta_2 \cdot dum91_{-1}\right]$$

If the elasticity, α_1, with respect to the first-year market change is less than the long-term elasticity of 1, Danish exports will lose some market share in the beginning of an economic upturn and gain market share in downturns. A sluggish export response to market changes may reflect that Danish export products are relatively insensitive to the business cycle. In addition, the estimated elasticity can be biased towards zero, because of measurement errors in the calculation of the foreign market.

We expect that the first-year price elasticity, α_2, is numerically smaller than the long-term price elasticity, β_1, because it takes time before a price effect attains its full effect on the volume demanded.

The German reunification increased the market close to Denmark and lifted exports to Germany. Consequently, the Danish market share increased without being driven by the relative price variable and to capture this, the export equation contains a dummy that is zero before 1991 and 1 thereafter.

The result of the estimation is shown in table 4.3. The modest Dickey-Fuller statistic confirms the impression from figure 4.1 that the long-run relation between market share and relative price is a weak relation. It is difficult to estimate the error correction parameter, γ, which in most export equations is set at 0.15. If we increase the 0.15 to get a faster adjustment to equilibrium, the numerical estimate of the long-term price elasticity will fall. With the chosen specification, there is no autocorrelation in the equation residual.

Table 4.3

Variable	ADAM-name		Coefficient	Std. Err.
Manufacturing export	Dlog(*fE59*)			
Market change	Dlog(*fEe59*)	α_1	0.62	0.085
Relative price change	Dlog(*pe59*/*pee59*)	α_2	-0.75	0.089
Lagged export	log($fE59_{-1}$)	γ	0.15	-
Relative price	log($pe59_{-1}/pee59_{-1}$)	β_1	-2.00	0.233
Reunification dummy	*dum91*	β_2	0.30	-
Dummy change	Dif(*dum91*)	α_3	0.05	-

Note: n=1971-2005 s=0,022 R2=0,768 LM(AR1)=0,756 DF= -2,530

Estimating an export equation involves the basic bias problem of simultaneity that an increase in the export price can reflect a pull from foreign demand or reflect a measurement error in the split into price and volume. In both cases, the error term in the estimated equation (error term not shown in equation 4.3) correlates with the contemporaneous export price change on the right hand side of the equation. This will bias the estimate of the coefficients in (4.3), and the potential bias problem is countered by including an equation for export prices. However, it has no significant effect on the outcome in table 4.3 that the export volume equation in (4.3) is estimated simultaneously with an export price equation. [1]

It may be added that the simultaneously estimated export price equations is not used in ADAM. In ADAM, the prices of the demand components, including Armington-modelled export components, are determined in input-output related equations as combinations of prices on production and imports, see the discussion in chapter 5 of ADAM's input-output system and related price determination.

[1] The estimation of the export equations is discussed further in the electronic annex.

4.5 Imports

Market share of imports reflects relative prices and international trend

For imports that compete with Danish production, the long-term import equation makes the import content in demand a function of the relative import price (import price over Danish production price) and of a trend. The latter can capture the development in the international division of labour.

(4.4)

$$\frac{fMz}{fAm} = \mu_0 \left(\frac{pm}{px} \right)^{\beta} \mathrm{f}(t)$$

fMz	Competing import, fixed prices
fAm	Market index for import, measured by weighing the growth in individual demand components in fixed prices cf. (4.5)
μ_o	Normal import market share in base year, when $pm = px$
pm	Price of imported good
px	Price of similar Danish produced good
β	Price elasticity
$f(t)$	Logistic trend for international division of labour

The import price, *pm*, is assumed to be exogenously determined and given from abroad, implying that in (4.4) the cost-determined Danish production price, *px*, is the only explanatory right-hand side variable, which is endogenous in ADAM.

The international division of labour has traditionally grown ...

The logistic trend variable is needed to explain the strong growth in import market share over the estimation period. This general growth in market share concerns most products and cannot be explained by relative prices. Instead, the trend expresses a growing international division of labour, for instance, due to better and cheaper transport options and to the development of the EU internal market. A logistic trend is also applied in the housing market equations to capture special market developments, see chapter 3.

... but the trend has declined in recent years

It should be added that the logistic trend plays a significant role in the beginning of the estimation period. In recent years, the estimated logistic trends have lost significance, which implies that import market shares are more consistent with relative prices.

It may also be noted that there is no logistic trend in ADAM's export equations because the international division of labour affects the export by affecting the export market.

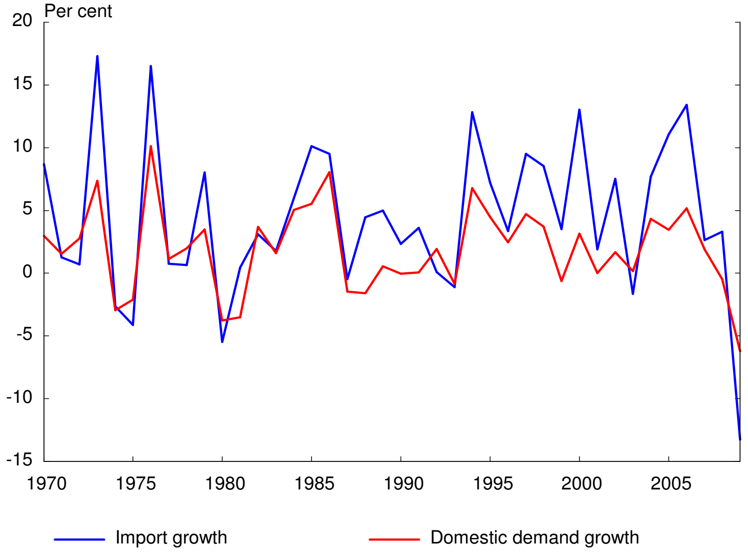
Short-term demand elasticity is larger than 1

The formulation of the import equation in (4.4) implies that the long-term elasticity of imports with respect to demand is restricted to 1, which is usual for Armington models. Short-term elasticity with respect to demand is estimated without restrictions and is in most import equations larger than 1. This implies that the import content in demand increases when demand is booming and falls when demand is stagnating.

The relatively strong and immediate import response to fluctuations in demand is illustrated in Figure 4.3, which compares growth in total imports to growth in domestic demand.

The more import responds to demand, the less Danish production needs to respond. Thus, the cyclicality of import dampens the fluctuations in Danish production.

Figure 4.3 **Import and domestic demand growth**



4.6 Import equation properties

The model has equations for three import groups and the main parameters are listed in table 4.3. The estimated long-term price elasticity of competitive imports is on average -0.9, but since a large part of the imports does not compete with Danish production and thus has a price elasticity of zero, the average import price elasticity is only -0.4. This means that imports account for a significantly smaller proportion of foreign trade price sensitivity than exports.

Table 4.3 **Estimated import equations**

Import group	ADAM-name	Price elasticity		Adjustment	Demand elasticity
		Long term	1. year		1. year
Food etc.	*fMz01*	-0.44	-0.27	0.57	1.23
Materials	*fMz2*	-0.93	-0.66	0.20	1.65
Manufacturing	*fMz59*	-0.96	-0.76	0.45	1.21
Estimated equations		-0.89	-0.69	0.45	1.24
Total imports	*fM*	-0.40	-0.31		1.11

As shown in the table, first-year demand elasticity is larger than 1, and it is also noted that all estimated adjustment parameters for imports are larger than 0.15, which is the standard adjustment parameter in most export equations.

Figure 4.4a **Effect on imports, demand + 1%**

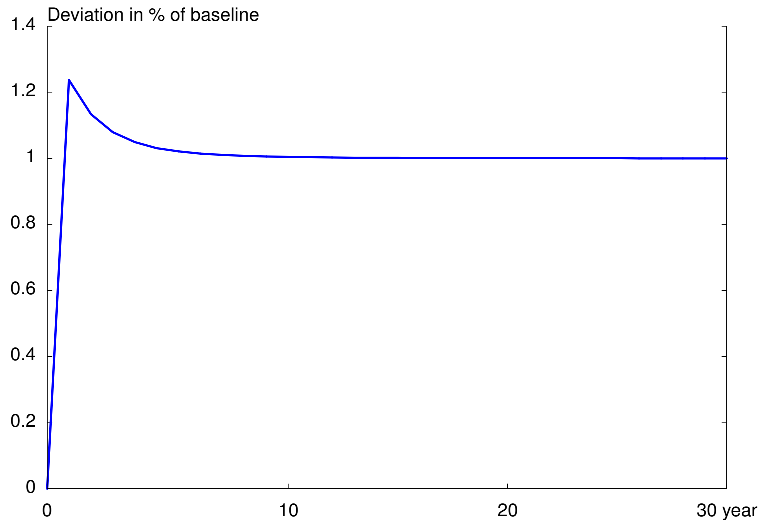
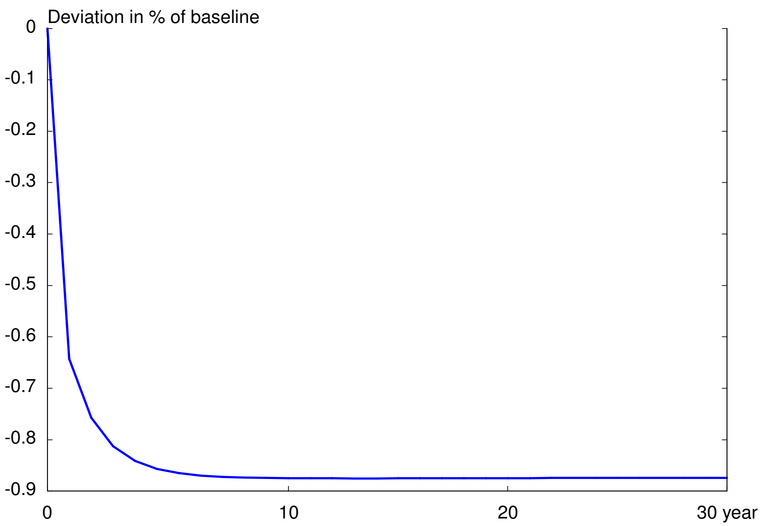


Figure 4.4b **Effect on imports, import price + 1%**



The reaction in the import equations of a 1-per cent increase in import demand, i.e. variable *fAm* in import equation (4.4), is illustrated in figure 4.4a. The reaction to a 1-per cent increase in the import price is illustrated in figure 4.4b.

Imports react faster than exports

Figure 4.4 can be compared with the corresponding figure 4.2 for exports, and it appears that imports react faster than exports, especially to demand shocks. As mentioned, the first year's import effect is significant and import rises by more than 1 per cent, when demand increases by 1 per cent. Imports are also relatively quick to reach their long-term response in the case of price shocks. On the other hand, imports react less to price changes than exports do, so the import reaction is not more important than the export reaction.

4.7 Import market and import components

The applied import market variable, *fAm*, weighs demand components with the size of their import content. Thus, it is taken into account that the import content differs between the demand components. Investment in machinery is an example of a demand component with a high content of imports, much higher than in, e.g. service consumption. The weighing is implemented by using the import content of the previous year.

(4.5)

$$\frac{fAm}{fAm_{-1}} = \sum_j \frac{M\_f_{j,-1}}{Mz_{-1}} \cdot \frac{f_j}{f_{j,-1}}$$

fAm	Import market
f_j	Demand component j
M_f_j	Import for demand component j
Mz	Total import

The formula in (4.5) implies that the rate of growth in the market variable equals a weighted average of growth rates in individual demand components with weights that reflect previous year's import allocation. This approach corresponds to the definition of the export market, shown in (4.2). The import market variable is calculated as an index with 2000 = 1, which means that the import market ratio *fMz/fAm* does not give the level of the import content.

Imports in ADAM are divided into 10 components, of which 8 are goods and 2 are services. ADAM's grouping of goods is based on the SITC classification, and the two service components are tourist spending and imports of other services. Below, table 4.4 lists the 10 import components.

Imports are divided into competing and non-competing

Several of the 10 import components are in the model divided further into two parts: a part competing with Danish goods and a non-competitive part, which passively follows demand. The market share of competing imports is sensitive to the relative price of imports and to the business cycle. Competing imports are marked by adding a *z* after *M* in the name of the import variable. For example, *fMz59* represents the competitive part of manufacturing imports. All estimated import equations concern competing imports.

Table 4.4 **Import components in ADAM**

Imports of	ADAM-name	SITC-group	Import share, 2000, per cent.
Food etc.	*M01*	0+1	7.3
Materials	*M2*	2+4	2.6
Coal and coke	*M3k*	32	0.3
Crude oil	*M3r*	333	1.4
Petroleum, electricity and gas	*M3q*	Rest af 3	3.5
Manufacturing	*M59*	5-9	49.3
Cars and trucks	*M7b*	del af 79	3.0
Ships etc..	*M7y*	del af 79	2.1
Services excl. tourists	*Ms*	-	25.0
Tourist spending	*Mt*	-	5.5
Total, DKK bn.	*M*		524.3

Non-competitive imports

Of the 10 import components, 6 are fully counted as non-competing imports. These are imports of:

- coal and petroleum etc., i.e. *fM3k* and *fM3r*, if a petroleum product is also produced in Denmark, the domestic price is assumed to equal the world market price
- cars, *fM7b*, as there is no competing Danish goods
- ships, aircraft and drilling platforms, *fM7y*, consisting of a few large units with a dubious price index
- other services, *fMs*, which primarily represent subcontracting for overseas transport, so there is no competing Danish product
- tourist spending, *fMt*, which is modelled as a component in private consumption, see section 3 of chapter 3.

For each of the remaining 4 import components, the non-competitive part comprises mainly imports for public consumption, for re-exports and for inventory investments. Imports for the public sector are supposed to be predominantly institutional determined and therefore counted as non-competing imports. Goods imported for re-exports without Danish value added, are probably not competing with Danish production and imports for inventories are modelled as inventory investments, see the section on inventory investments in Chapter 6

Apart from the non-competing import components that are modelled separately, e.g. inventory investments of imported goods, non-competing imports are made proportional to demand. This implies that the share of non-competing imports remains unchanged in each demand component. For example, imports of petroleum to oil refineries, *fM3q_Xng*, are set to follow the total energy input, *fVeng*, for the refineries, see the following equation:

(4.6)

$$\frac{fM3q\_Xng}{fM3q\_Xng_{-1}} = \frac{fVeng}{fVeng_{-1}}$$

4.8 Estimation of import equations[2]

The starting point for estimating the equations for competing imports is a logarithmic formulation of (4.4), which is used as a long-term equation for desired imports, *fMzw*.

(4.7)

$$\log(fMzw) = \log(fAm) + \beta \cdot \log\left(\frac{pm}{px}\right) + \mu_0 + \frac{\mu_1}{1+e^{-\theta\cdot(t-\tau)}}$$

The sum of constant and logistic trend in (4.7) can vary between μ_0 and $\mu_0+\mu_1$, and the coefficients θ and τ determine, respectively, the steepness of the logistic trend and the year with maximum trend, see the discussion of the logistic trend in housing market equations in chapter 3.

The lagged difference between desired and actual import is included in the error correction form of the import equation.

(4.8)

$$\text{Dlog}(fMz) = \alpha_1 \cdot \text{Dlog}(fAm) + \alpha_2 \cdot \text{Dlog}\left(\frac{pm}{px}\right) - \gamma \cdot \log\left(\frac{fMz_{-1}}{fMzw_{-1}}\right)$$

The indication of import group and industry is omitted in (4.7) and (4.8), but otherwise the names of the variables correspond to the ADAM-notation. The parameters of the logistic trend in (4.7) are not shown, but the other parameters in (4.7) and (4.8) were shown in table 4.3 for all three estimated import equations.

Equations (4.7) and (4.8) are generally estimated in one step and as a single equation that is non-linear in the trend parameters, θ and τ, but linear in the other parameters.

Summary of chapter 4

Foreign trade equations are important in a model of a small open economy. The focus is on foreign trade and especially on export price elasticity, which is important for how fast the economy returns to its equilibrium, and how much price adjustment it requires. ADAM's price elasticity of foreign trade is in line with many foreign models and slightly lower than in SMEC.

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[2] Due to problems involved in backcasting the import components, the import equations are not estimated in the presented model version. Instead, import equations continue unchanged from the previous model version.

5. Production and input-output

The chapter describes the determination of values and volumes for production, intermediate consumption and gross value added in the industries of ADAM. We also discuss the input-output-based price determination, which concerns the prices of demand components based on prices of imports and Danish production. Common to both value volume and price calculation is that they ensure the mutual consistency of the behavioural equations in ADAM by linking supply and demand.

5.1 Supply and demand

Equilibrium in the goods market

The starting point for ADAM's determination of production and gross domestic product is the simple equilibrium condition for goods markets found in textbooks:

(5.1)
$$Y + M = C + I + E$$

i.e. gross domestic product, Y, plus imports, M, equals the sum of consumption, C, investments, I, and exports, E, all at current prices.

However, for use in ADAM condition (5.1) is slightly reformulated. Firstly, it is important to divide the GDP into product taxes, Sv, and the rest, which is gross value added, Yf, so that $Y{=}Yf{+}Sv$. Secondly, in ADAM the equilibrium condition is formulated for production value, X, i.e. production calculated inclusive of the input of materials in production. The input of materials comprises goods and services used for intermediate consumption in industries. If V denotes total intermediate consumption, production can be written $X{=}Yf{+}V$.

Using these definitions, the equilibrium condition can be rewritten as

(5.2)
$$X + M + Sv = V + C + I + E$$

The left hand side of (5.2) is the total *resources* (supply) of goods and services at market prices, and it comes from either Danish production or imports incl. imposed taxes. The right hand side of (5.2) is total use (demand) of goods and services, and it consists of the total intermediate consumption, V, plus the three basic components of final demand: consumption, investment, and exports. Uses and resources must be equal to each other.

Not one, but many equilibria

ADAM contains not only one of these equilibrium conditions, but many. In principle, there is an equilibrium condition for each good. The introduced combination of three resources and four uses in (5.2) can be arranged as an input-output table:

(5.3)

$$X = X_V + X_C + X_I + X_E$$

$$M = M_V + M_C + M_I + M_E$$

$$Sv = Sv_V + Sv_C + Sv_I + Sv_E$$

where e.g. X_V represents Danish-produced input of materials, M_I is imports of capital goods, and Sv_C is taxes on consumption. The total equilibrium condition (5.2) is still valid and can be retrieved by summing the equations in (5.3).

Input-output table

The structure of the goods part in the input-output table is reflected by the right hand side of (5.3). There is a row for each resource, e.g. [X_V X_C X_I X_E] for Danish production, and a column for each use, e.g. [X_C M_C Sv_C] for consumption. The row for X shows the structure of demand for Danish production, i.e. the distribution on input of materials, consumption, investment, and exports. Column *C* shows the cost structure of consumption, i.e. the distribution on Danish production, imports and taxes.

To complete the picture, the goods part of the input-output table is often supplemented by gross value added at the bottom. Thus, the column for an industry will add up to its total gross production instead of its total inputs in production. In such a row of value added for industries, there are zeros in the cells of *C*, *I* and *E*.

Table 5.1 provides an overview of the 12 industries in ADAM. The entire input-output table for ADAM is shown in tables 5.A-5.C placed at the end of the chapter. Tables 5.A-5.C divide the gross value added of the 12 industries into gross profits, wages, and other taxes, as will be discussed in connection with equation (5.11).

Table 5.1 **Production value and employment in 12 industries, 2009**

	Variable	Production value X	Employment Q
		DKK mio.	1.000 persons
Agriculture	*a*	68 544	81
Extraction of hydrocarbons etc.	*e*	44 493	3
Oil refineries	*ng*	21 327	0,71
Electricity, gas and heating	*ne*	58 227	16
Food processing	*nf*	133 310	62
Manufacturing	*nz*	412 739	293
Construction	*b*	195 622	174
Housing	*h*	160 653	16
Sea transport	*qs*	137 608	19
Financial services	*qf*	160 226	85
Private services	*qz*	1 027 401	1 262
Public services	*o*	500 317	801

5.2 Model

Groupings in ADAM

ADAM distinguishes between 12 different industries of production, 10 types of imports, 3 types of product taxes, 9 types of consumption, 4 types of investments and 7 types of exports. Thus, in practice the input-output table is much larger (25 rows and 32 columns), but the principle is exactly the same as in the small input-output table in (5.3).

Names

The cells of the input-output table at current prices are included as variables in the model. Cell names follow a simple notation: for example, the variable *Xnf_Cf* denotes the cell with supplies of Danish manufactured food, *Xnf,* into food consumption, *Cf*, etc. Other examples: *M01_E01* is the re-exports of food products, i.e. of SITC groups 0 and 1, and *Spp_Cf* is excise duty on food consumption.

5 basic principles

The cells of the input-output table are endogenous variables in ADAM, and there are five basic principles for the determination of cells:

- Demanded volume is provided immediately, either as Danish production or as import.
- The split-up between Danish production and imports reflects the price-sensitive market shares determined in the import equations. In the short term, production is often more sluggish while import reacts more quickly to demand fluctuations cf. the estimated short-term import demand elasticity above 1.
- Price changes on resource components are passed on fully in the price of uses.
- As a starting point, the volume distribution in the column for a given use is assumed to remain unchanged from the previous year. Thus, for example, if car consumption increases by 1 per cent in volume, the car imports and the car-sale-related services will both grow by 1 per cent.
- As a starting point, prices in the row for a given resource are assumed to change proportionally from the previous year. Thus, for example, if the gasoline price increases by 1 per cent, the price of gasoline consumption will increase by 1 per cent for both households and industries.

Cells

These basic principles imply the following formula for determining the cells of the input-output table, here using imports for car consumption as example:

(5.4)

$$M7b\_Cb = M7b\_Cb_{-1} \cdot \frac{fCb}{fCb_{-1}} \cdot \frac{pm7b}{pm7b_{-1}}$$

The cell at current prices is determined in (5.4) as its value in the previous year inflated with the price of imported cars, *pm7b*, and increased with the volume growth in car consumption, *fCb*.

Volumes

Subsequently, the total volume of a resource is determined as the sum of cells in the row of the resource component divided by the price of the component. Again using car imports as example:

(5.5)

$$fM7b = \frac{M7b\_Xa + ... + M7b\_Cb + ... + M7b\_Im + ... + M7b\_E7b}{pm7b}$$

The summation in the numerator embraces all uses of car imports. The price of the resource is either exogenous, as in the case of imports, or determined in the price equations of the model.

Similar equations determine the volume of the other import components and of the production in industries, fX_i.

Prices

For all use components, i.e. demand components in ADAM, the net price (net of indirect taxes) is calculated as the sum of cells in the column of the demand component divided by the volume of the demand component. The volume of the demand component is either exogenous or determined in the demand equations of the model. For example, the net price of car consumption is given by:

(5.6)

$$pncb = \frac{Xa\_Cb + ... + M7b\_Cb + ... + Ms\_Cb}{fCb}$$

The summation in the numerator embraces all production industries and import components.

When the net price is determined, the market price of the demand component is found by adding taxes. For example, the price of food consumption is given by:

(5.7)

$$pcf = (pncf + tpcf) \cdot (1 + btgcf \cdot tg)$$

where *tpcf* is the rate of excise duty on food consumption, *tg* is the general VAT rate, and *btgcf* indicates the VAT load on food consumption. See chapter 8 on public finances for further discussion on the role of indirect taxes.

Similar equations determine the prices on the rest of the consumption components as well as on inputs of materials, pv_j, investments, pi_j, and exports, pe_j.

Special industries

It should be noted that some industries do not follow the normal principle that production adjusts to demand.

Production in agriculture, *Xa*, and the related food production, *Xnf*, are either exogenous or a function of the ratio between the exogenous food price and Danish costs. Thus, they do not respond to changes in food demand. Instead, it is food exports that adapt when there are changes in the food demand or agricultural production.

Extraction of hydrocarbons etc., *Xe*, is an exogenous production.

Production in housing, *Xh*, is proportional to the stock of houses, and is therefore slow to adjust to the housing demand. In the short term, it is rather the price of existing dwellings, which responds to demand. In the long term, housing stock and hence housing production are determined by housing demand, cf. the discussion of the housing model in chapter 3.

Production of public services, *Xo*, is determined by the exogenous employment of the public industry, and both public employment and production are seen as politically determined, see Chapter 8 on public finances.

For the special industries just listed, it is necessary to adjust the determination of input-output cells in various ways.

On import substitution

For products produced in both Denmark and abroad, it is necessary to modify the simple determination of input-output cells shown in (5.4). This is because changes in Danish competitiveness, i.e. changes in the relative price of Danish production and imports, affect the market share of imports determined in the import equations of the model. This mechanism must be reflected in the input-output cells.

As an example, we take imports of food to consumption of food, i.e. cell *M01_Cf*. The equation for this cell corresponds to the simple equation (5.4) with a correction for the change in the market share:

(5.8)

$$M01\_Cf = M01\_Cf_{-1} \cdot \frac{fCf}{fCf_{-1}} \cdot \frac{pm01}{pm01_{-1}} \cdot kfmz01$$

The correction factor, *kfmz01*, expresses the growth in the market share of food imports, *M01*, measured in volumes. This market share is determined by the import equation for food, and if the market share increases by 1 per cent, *kfmz01* equals 1.01. Note that the change in the market share of *M01* is assumed to have the same proportional effect on all cells in the *M01* row.

When the import share is changed, the share of domestic production must be changed in the opposite direction. Thus, the total market is not affected by a shift in import share; the gain of one market share is the loss of the other.

Food imports are competing with the production of the Danish food industry, *Xnf*. Consequently, when food imports rise because of a higher market share, the additional imports are deducted in the food production. This means that the cell equation for food production delivered to food consumption is modified to:

(5.9)

$$Xnf\_Cf = \left(Xnf\_Cf_{-1} - M01\_Cf_{-1} \cdot (kfmz01 - 1)\right) \cdot \frac{fCf}{fCf_{-1}} \cdot \frac{pm01}{pm01_{-1}}$$

If the import market share is unchanged, i.e. *kfmz01*=1, the equation works as the simple cell equation in (5.4). But if the import market share increases by, e.g. 1 per cent, the import cell *M01_Cf* will increase by 1 per cent, cf. (5.8), while the Danish production cell in (5.9) will decrease by the same amount in kroner. Thus, the sum of the two cells will remain unchanged.

Gross value added

When the production and the input of intermediate consumption are determined, gross value added, GVA, can be determined as production minus intermediate consumption:

(5.10)

$$Yf_i = X_i - V_i$$

"Other taxes on production"

A part of gross value added goes to the public sector in the form of other net taxes on production, *Spz,* (net means net of subsidies). These production taxes are distributed across industries. For example, agriculture pays property tax and receives EU subsidies, while the housing industry pays property tax and receives housing subsidies, see also the chapter on public finances.

Gross surplus

Finally, the "gross operating surplus and mixed income" for short the residual income, can be determined by:

(5.11)

$$Yr_i = Yf_i - Spz_i - Yw_i$$

That is to say: Gross value added minus other net taxes on production and minus wages. Residual income covers capital costs and profit.

Totals for *Yf*, *Spz* and *Yw* are made by a summation taking place over the 12 industries.

5.3 Properties

In the case of a positive demand shock, ADAM's total output and employment will increase in the short term and thereafter decrease in the long term, reverting to the baseline. This is the general effect on the total economy. In contrast, production and employment in individual industries will not necessarily return to the starting point.

If the public purchase of goods and services increase by 1 per cent, the accompanying wage increase will be instrumental in crowding out exports, but the higher wage increases the purchasing power of households, so both public and private consumption will increase in the long term. It is primarily manufacturing that produces the exported goods, so in the long term value added and employment will be smaller in manufacturing. On the other hand, the private service industries mainly produce for the domestic market, so these industries will be larger in the long term. Figures 5.1 and 5.2 illustrate the effects on gross value added and employment in the manufacturing industries, specifically the *n*-industries *nz*, *nf*, *ng* and *ne*, as well as in the service industries *qz*, *qs* and *qf*.

Figure 5.1 **Public purchase + 1%, effect on gross value added**

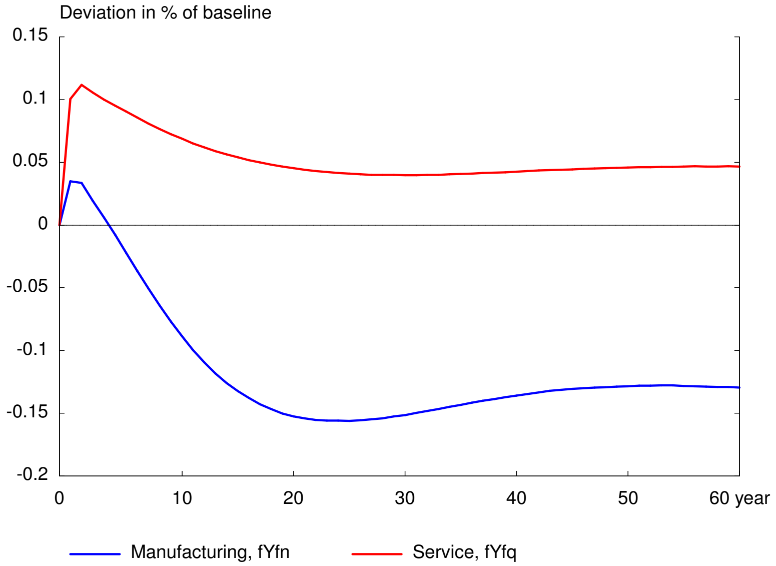
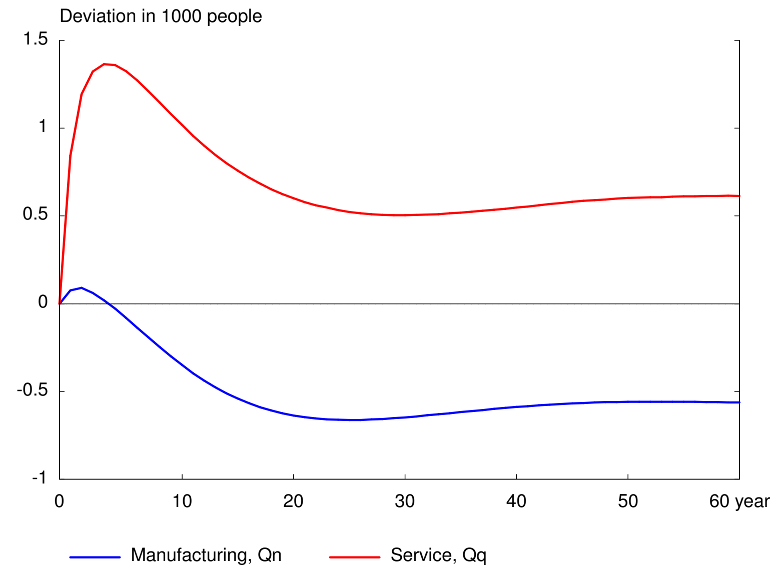


Figure 5.2 **Public purchase + 1%, effect on employment**



Box 5.1 **Adjusting in the input-output system**

It is not easy to make adjustments in the equations of the input-output system without running the risk of difficulties with the many definitional relations that must apply.

In general, adjustments to the input-output cells must be made so that either the row or column sum remains unchanged: If one cell is adjusted upwards, another cell must be adjusted downwards.

If the column sum remains unchanged as in the example of import substitution, the adjustment will affect the distribution of a given demand component on resource components, such as imports and Danish production.

If instead the row sum remains unchanged, the adjustment will affect the distribution of a given resource price on demand components. The problem may be that a price increase of Danish production is expected to have an impact on domestic prices, but not on the export price. This can be obtained by entering the row of the production industry in question and in this row we adjust upwards a domestic cell and downwards an export-cell by the same amount.

This kind of adjustment in the pass-through of resource prices is, in practice, the most frequent intervention in the input-output system, and a mechanism for price adjustment has been specified in the input-output equations. More specifically, special adjustment terms have been specified for the net price of every demand component, e.g. *Jdpncf* for the net price on food consumption. These adjustment terms are located in cells with input from the service industry (*qz*), for example supply to food consumption, and a change in the adjustment term can be interpreted as a change in trading profit (wholesale and retail trade). The sum of the special adjustment terms is automatically offset in the price of inventory investment, *pnil*, to ensure that the row identity for the *qz* industry holds, also when the problem is ignored by the model user. Obviously, the drawback of the automatic adjustment is that the price of inventory investment becomes difficult to interpret.

ADAM's input-output system is further discussed in a number of model group papers on *dst.dk/adam*.

Table 5.A **Input-output table, input to industries. 2007**

	Xa	Xe	Xng	Xne	Xnf	Xnz	Xb	Xqz	Xqs	Xqf	Xh	Xo	**Total**
	DKK bn..												
Xa	8	0	0	1	38	0	2	1	0	0	0	2	**51**
Xe	0	1	20	11	0	1	1	0	0	0	0	0	**34**
Xng	2	0	1	0	0	1	1	4	0	0	0	0	**9**
Xne	1	0	0	4	2	5	0	8	0	0	0	3	**24**
Xnf	9	0	0	0	13	1	0	8	0	0	0	1	**33**
Xnz	3	1	0	1	5	85	45	33	2	2	0	6	**184**
Xb	1	1	0	4	1	2	2	21	0	2	19	5	**58**
Xqz	10	2	1	3	16	73	56	294	3	15	1	65	**538**
Xqs	0	0	0	0	0	0	0	7	0	0	0	1	**7**
Xqf	5	0	0	1	1	3	2	16	2	23	13	2	**69**
Xh	-	-	-	-	-	-	-	-	-	-	-	-	**-**
Xo	0	0	0	0	0	1	0	6	0	1	0	11	**20**
M01	4	0	0	0	15	1	0	6	0	0	0	1	**27**
M2	1	0	0	0	4	6	2	0	0	0	0	0	**14**
M3k	0	0	-	3	0	0	-	0	-	-	0	0	**3**
M3r	-	-	6	-	-	-	-	-	-	-	-	-	**6**
M3q	1	0	0	1	0	1	1	6	27	0	0	0	**39**
M59	3	0	1	0	6	109	25	31	2	1	0	13	**190**
M7b	-	-	-	-	-	0	-	-	-	-	-	-	**0**
M7y	-	-	-	-	-	0	0	0	-	-	-	0	**1**
Ms	1	2	0	1	5	17	3	81	117	6	1	8	**241**
Mt	-	-	-	-	-	-	-	-	-	-	-	-	**-**
Spm	0	0	0	0	0	0	0	0	0	0	0	0	**1**
Spp+Spr	1	2	0	0	1	2	2	9	0	0	4	2	**23**
Spg	0	0	0	0	0	1	0	14	0	4	5	21	**45**
Spz	- 6	- 0	- 0	0	- 0	- 1	0	1	- 0	3	4	- 3	**- 1**
Yw	9	2	0	6	22	122	64	370	6	49	4	275	**929**
Yr	14	55	1	22	9	50	17	168	13	30	99	31	**507**
Total	**70**	**66**	**31**	**57**	**139**	**481**	**223**	**1 084**	**172**	**135**	**150**	**446**	**3 054**

Note: The last three rows, *Spz*, *Yw* and *Yr* distribute value added in the industry on production taxes, wages, and gross operating surplus.

Table 5.B Input-output table, input to consumption and investments. 2007

	Cf	Cv	Ce	Cg	Cb	Ch	Cs	Ct	-Et	Co	Im	Ib	It	Il	**Total**
	DKK bn.														
Xa	2	1	0	-	-	-	0	-	-	1	0	-	- 0	1	**5**
Xe	0	0	-	-	-	0	0	-	-	-	0	-	-	2	**3**
Xng	0	0	1	4	-	-	0	-	-	-	0	-	-	1	**7**
Xne	-	0	19	-	-	2	0	-	-	-	0	-	-	0	**22**
Xnf	28	1	-	-	-	-	0	-	-	0	0	-	-	1	**30**
Xnz	0	14	0	- 0	0	1	2	-	-	1	36	-	-	7	**61**
Xb	-	0	-	-	-	4	0	-	-	7	0	153	-	-	**164**
Xqz	34	74	2	2	9	7	142	-	-	31	49	13	-	2	**365**
Xqs	-	0	-	-	-	-	1	-	-	-	0	-	-	-	**1**
Xqf	-	0	-	-	-	-	56	-	-	0	3	-	-	-	**60**
Xh	-	0	-	-	-	150	0	-	-	-	0	-	-	-	**150**
Xo	-	0	-	-	-	-	30	-	-	393	1	0	-	-	**424**
M01	19	1	-	-	-	-	-	-	-	0	-	-	- 0	0	**20**
M2	0	1	1	-	-	0	-	-	-	-	0	-	-	1	**3**
M3k	-	0	0	-	-	-	-	-	-	-	-	-	-	0	**0**
M3r	-	-	-	-	-	-	-	-	-	-	-	-	-	1	**1**
M3q	-	0	1	3	-	-	-	-	-	-	-	-	-	- 0	**4**
M59	0	38	-	0	1	1	2	-	-	5	50	-	-	8	**104**
M7b	-	-	-	-	17	-	-	-	-	0	8	-	-	0	**26**
M7y	-	0	-	-	-	-	-	-	-	-	5	-	-	1	**7**
Ms	-	1	-	-	-	0	4	-	- 36	0	5	0	-	0	**- 26**
Mt	-	-	-	-	-	-	-	38	-	-	-	-	-	-	**38**
Spm	0	1	0	0	0	0	0	-	-	0	0	-	- 0	0	**1**
Spp+Spr	11	0	12	10	15	1	- 3	-	-	0	10	2	-	0	**58**
Spg	22	31	9	5	6	2	20	-	-	2	9	26	-	-	**131**
Spz	-	-	-	-	-	-	-	-	-	-	-	-	-	-	**-**
Yw	-	-	-	-	-	-	-	-	-	-	-	-	-	-	**-**
Yr	-	-	-	-	-	-	-	-	-	-	-	-	-	-	**-**
Total	**116**	**163**	**46**	**24**	**47**	**168**	**255**	**38**	**- 36**	**440**	**177**	**194**	**- 0**	**25**	**745**

Table 5.C Input-output table, input to exports. 2007

	E01	E2	E3	E59	E7y	Es	Et	Total	Total uses
	DKK bn.								
Xa	7	6	-	-	-	0	-	**11**	**70**
Xe	-	1	28	0	-	1	-	**29**	**66**
Xng	-	0	15	0	-	-	-	**15**	**31**
Xne	-	-	11	-	-	-	-	**11**	**57**
Xnf	68	4	-	2	-	2	-	**75**	**139**
Xnz	0	3	0	224	2	7	-	**237**	**481**
Xb	-	-	-	-	-	1	-	**1**	**223**
Xqz	10	7	0	46	0	117	-	**181**	**1 084**
Xqs	-	-	-	-	-	163	-	**163**	**172**
Xqf	-	-	-	0	-	6	-	**6**	**135**
Xh	-	-	-	-	-	-	-	**-**	**150**
Xo	-	-	-	-	-	2	-	**2**	**446**
M01	9	-	-	-	-	-	-	**9**	**56**
M2	-	2	-	-	-	-	-	**2**	**19**
M3k	-	-	0	-	-	-	-	**0**	**4**
M3r	-	-	0	-	-	-	-	**0**	**7**
M3q	-	-	7	-	-	-	-	**7**	**49**
M59	-	-	-	88	-	-	-	**88**	**381**
M7b	-	-	-	4	-	-	-	**4**	**30**
M7y	-	-	-	-	3	-	-	**3**	**11**
Ms	-	-	-	-	-	3	36	**38**	**253**
Mt	-	-	-	-	-	-	-	**-**	**38**
Spm	0	0	0	1	0	-	-	**1**	**3**
Spp+Spr	- 1	0	- 0	- 0	-	-	-	**- 1**	**80**
Spg	-	-	-	-	-	-	-	**-**	**176**
Spz	-	-	-	-	-	-	-	**-**	**-1**
Yw	-	-	-	-	-	-	-	**-**	**929**
Yr	-	-	-	-	-	-	-	**-**	**507**
Total	**94**	**23**	**61**	**365**	**5**	**302**	**36**	**885**	**5 595**

6. Production factors

Labour, capital equipment and materials are used by businesses to produce. Capital equipment includes machinery and buildings. Machines are usually more involved in the production process than buildings, and in ADAM it is chosen to divide the capital stock into machinery and building capital. Materials include a wide range of goods and services provided as input to businesses, either output from other Danish businesses or import from abroad. Model users often focus on the input of energy, which is part of materials used in businesses. Therefore, the input of materials has been divided into energy and other materials, including services. With both capital equipment and input of materials divided into two, there are five inputs for the production functions in ADAM.

Labour is the most important factor of production. The labour input in terms of hours determines the number of employed, which together with labour supply, determines unemployment. Thus, production is linked to the labour market and the wage formation

Changes in capital equipment are transformed into changes in investment, which generates a feedback from the factor demand to the demand for goods. This starts a cyclical accelerator process where increased production creates a need for even more production to meet investment requirements. The effect on investment can be significant, as small relative fluctuations in capital stock can create large relative fluctuations in annual investment.

As mentioned, model users often focus on the input of energy, which e.g. affects CO2 emissions. Businesses' total consumption of materials is important for the import of foreign goods.

Demand for the five factors of production is typically modelled by assuming that the whole industry is represented by a company that minimizes costs for a given production and given factor prices. The ability to replace an expensive production factor with a cheap factor depends on the production function of the industry. The larger the elasticity of substitution, the more price-sensitive is the demand for production factors. The price-sensitivity implies that factor demand helps to determine the response of the economy to changes in, e.g. energy prices or in the interest rate, which enters the price of using capital equipment.

It can be added that the resulting factor demand equations will be used in chapter 7 to determine production prices, which in the long term are proportional to the cost of factor input per unit produced.

The following section provides an overview of ADAM's factor block, i.e. the system of factor demand equations. This is followed by a section on factor block properties and a section on estimated equations.[1] After this review of the factor demand follows a section on businesses' inventory investments and a section describing how the labour force is determined.

6.1 Factor demand in ADAM

There are 12 industries in ADAM. In principle, every industry has its own production function and thus its own factor demand. All production functions include 5 factors of production: machinery, *K*, labour, *L*, energy, *E*, buildings, *B*, and non-energy materials *M*. However, the function form differs and the twelve industries can be divided into three groups according to the structure of their production function.

Industries sorted by production structure

The first group consists of six so-called *KLEBM*-industries, where *K* and *L* are put in the inner structure of the production function, while materials, *M*, is put in the outer.

- *a*: agriculture etc.
- *b*: construction
- *nf*: food processing
- *nz*: manufacturing, excl. *ne*, *nf* and *ng*
- *qf*: financial services
- *qz*: private services, excl. *qf* and *qs*

The second group is the so-called KLBME-industries, where energy is put in the outer production structure:

- *ne*: energy supply (electricity, heating and gas)
- *ng*: oil refineries
- *qs*: sea transport

The third and last group comprises three industries with a simple non-estimated factor demand:

- *e*: extraction of hydrocarbons
- *h*: housing
- *o*: public services

Private services is the largest industry

The 12 industries differ in terms of size and composition of factor inputs, cf. Table 6.1, which shows the industry shares of production and factor inputs in current prices. The industries with the largest production are manufacturing (*nz*), private services (*qz*), and public services (*o*). Together, these three ADAM industries account for almost

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[1] For further discussion, see the electronic annex.

2/3 of total production and private services alone accounts for more than 1/3.

Table 6.1 **Industry share of production and factor input, 2006**

	a	b	ne	Nf	ng	nz	qf	qs	qz	e	h	o	Total
	Per cent												
X	2.3	7.5	2.1	4.7	1.0	15.5	4.4	5.1	35.3	2.3	4.9	15.0	100.0
K	6.4	3.5	2.1	4.2	0.7	15.6	2.4	11.5	45.4	2.7	-	5.5	100.0
L	3.1	7.2	0.5	2.3	0.0	11.6	2.8	0.9	42.9	0.1	0.6	28.0	100.0
E	4.8	3.4	16.1	2.5	19.1	7.3	0.4	18.8	20.2	0.8	0.3	6.2	100.0
B	4.1	0.5	4.9	1.1	0.1	3.1	0.9	0.4	17.9	1.3	52.6	13.2	100.0
M	3.0	9.7	0.7	7.5	0.1	20.0	3.8	7.6	35.0	0.5	2.9	9.2	100.0

Industry consumption of factor input

The industry with the highest production, private services, is also the largest consumer of four of the five factor inputs. The exception is building input of which more than half is used for housing. Public service industry also accounts for a relatively high proportion of the building input, which includes non-building constructions, e.g. roads and bridges. It should particularly be noted that public services accounts for almost 30 per cent of employment.

In general, the three major industries, private and public services and manufacturing, consume a large share of the inputs, but it is noted that more than half of the energy input goes to sea transport (*qs*) and to two minor energy manufacturing industries: Electricity, heating and gas (*ne*) and oil refineries (*ng*). Sea transport is also the third largest in terms of machinery, which includes means of transport, while construction (*b*) is a clear number four with regard to the use of labour. The selected breakdown by industry is justified in box 6.1.

Box 6.1 **Selecting the industry breakdown in ADAM**

As starting point for a rough breakdown of private industries, it can be argued that there are four types of industries: 1) the so-called primary industries (*a*+*e*), which utilizes natural resources, 2) manufacturing (*n*), which exports a lot, 3) construction (*b*), which is particularly sensitive to the business cycle, and 4) private services (*q*). However, we have ended up with 11 and not 4 private industries in ADAM due to special considerations.

Extraction of hydrocarbons (*e*) has been singled out in order to model an important tax base and to help determine the demand for energy imports. Also, energy supply (*ne*) and oil refineries (*ng*) have been singled out to model the energy flow in the economy. For example, crude oil is extracted by the *e*-industry, processed into fuel oil by the *ng*-industry, and transformed into heating by the *ne*-industry.

The food-processing industry (*nf*) is singled out to be modelled as the agricultural processing industry. Sea transport (*qs*) is taken out of private services because, similar to energy extraction and agriculture, it is an industry with exogenously determined world prices. Financial services (*qf*) has been singled out because its production is calculated in a special way, and last but not least, a special industry has been made for housing (*h*) where production follows the stock of housing capital.

The following describes how the desired factor demand is determined, starting with the nine industries where the production functions are estimated.

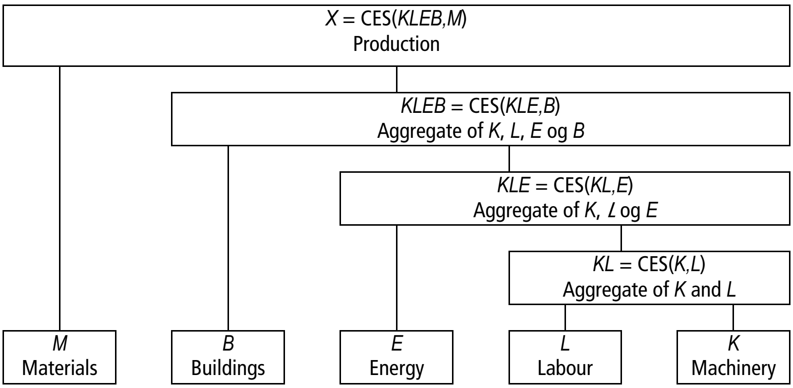
Factor block structure is nested

The system of ADAM equations determining the demand for production factors is often referred to as the factor block, and like the consumption system in chapter 3, the factor block is structured as a nested system. The advantage is that factor inputs are grouped to reduce the number of cross-price elasticities, which would have been necessary to estimate.

Typical structure is KLEBM

The typical nesting is, as already mentioned, *KLEBM* with machines, *K*, and labour, *L*, in the inner nest, see figure 6.1 with a *KL* aggregate in the lower right hand side of the table. This means that capital and labour are substitutable and for each industry we estimate an elasticity of substitution between *K* and *L*. These two factor inputs combine to create a *KL* aggregate, which in the next-inner nest substitutes with energy input E. We can imagine that these three factors, *K*, *L* and *E*, are used at a production line to process raw materials or semi-manufactured products. If the input of machinery is increased, the input of labour may be reduced, and by using more energy both machinery and labour may, to a minor extent, become redundant.

Figure 6.1 **Factor block nesting structure KLEBM-industries**



The *KLE* aggregate of capital, labour and energy is allowed to substitute with building capital, *B*, and form a *KLEB* aggregate, which in the outer nest is supplemented with non-energy materials, *M*. The total *KLEBM* aggregate represents the result of all five factors of production and corresponds to total production of the industry.

In the three *KLBME*-sectors, it is not materials but energy input that are placed in the outer nest where energy can substitute with the four-input aggregates, *KLBM*, and thus reflect the relative price of energy. The *KLBME*-sectors are characterized by a large energy input, and two of the three industries produce energy products.

Desired factor input determined by cost minimization

The desired factor demand of an industry is determined by minimizing costs at given production and given prices of production factors. For each industry, the desired demand for the 5 production factors reflects production and relative factor prices in basically the same way as the demand for 7 private consumption components reflects the budget and relative consumption prices.

With machinery, K, as example, the desired volume of machinery, K^*, in industry i is written as:

(6.1)

$$K_i^* = e_{i,K}^{-1} \cdot priceeffekt_{i,K} \cdot X_i$$

The first variable on the right hand side is the reciprocal value of the capital efficiency index, $e_{i,K}$. The variable called *priceeffect* reflects the effect of relative factor prices, and X_i is the production of the industry.

Factor demand is proportional to production

Factor demand, see (6.1), is proportional to production, because all production functions are homogeneous of the first degree. Production functions are also nested CES functions with an elasticity of substitution for each nest.

Factor demand depends on relative factor prices ...

Since K is located in the innermost nest together with labour L, the price effect in (6.1) reflects the relative price of K and L and three more relative prices. All four relative prices enter the price effect raised to their corresponding substitution elasticity, see equation (6.6) in box 6.2, which presents the desired long-term demand for all five factors of production. The input of materials in the outer nest depends on only one relative price and the price dependence disappears when the corresponding elasticity of substitution is zero.

... and on factor efficiency

There are industry-specific and multiplicative efficiency indices for all five factors of production, as illustrated in (6.1). Thus, the production functions are not formulated directly in the production factors, such as K and L, but in the factors of production multiplied by their efficiency index , e_K and e_L, that is in $e_K{\cdot}K$ and $e_L{\cdot}L$. This approach of formulating the production factors in efficiency units is often used in growth theory.[2]

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[2] The classic example is a Cobb-Douglas production function with capital and labour and an efficiency index, A, formally associated with labour:

$$F(A,K,L) = F(K, A\cdot L) = K^{\alpha}(A\cdot L)^{1-\alpha}$$

In ADAM, this equation is generalized by first replacing A with two efficiency indices, e_K for K and e_L for L. Afterwards, the Cobb-Douglas function is replaced by a CES function:

$$F(e_K\cdot K, e_L\cdot L) = \left(\theta^{1/\sigma}\cdot(e_K\cdot K)^{(\sigma-1)/\sigma} + (1-\theta)^{1/\sigma}(e_L\cdot L)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

This is the KL-aggregate in ADAM. If both efficiency indices rise by 1 per cent, the aggregate rises by 1 per cent. An industry has 5 inputs with their own efficiency index, and if all 5 efficiency indices rise by 1 per cent, the industry production will also rise by 1 per cent.

The corresponding efficiency-adjusted factor prices are user cost and hourly wage divided by the efficiency indices, i.e. respectively u/e_K and w/e_L. This implies that the efficiency correction reduces the price of factor input with the same multiplicative factor that increases factor input. The role of the efficiency indices in factor demand equations is shown in box 6.2.

Efficiency indices reflect the technical progress

ADAM's efficiency indices depend on a polynomial in time, and the indices are determined as part of the factor block estimation, discussed in section 6.3. The indices represent technical progress, education and other efficiency-enhancing factors, and all efficiency indices are normalized to 1 in 2000. For example, the efficiency index for labor in manufacturing is 1.13 in 2006, which implies that a worker in 2006 could do the same in 1 hour as a worker in 2000 could do in 1.13 hours. The hourly wage increased from DKK 171 in 2000 to DKK 228 in 2006, but the efficiency-adjusted hourly wage only increased from DKK 171in 2000 to DKK 202 in 2006, 202=228/1.13.

Box 6.2 **Long-term factor demand equations**

For *KLEBM* industries the desired input of the 5 factors is given by:

(6.2)
$$M^* = \theta_M \cdot e_M^{-1} \left(\frac{p_M / e_M}{p_{KLEBM}} \right)^{-\sigma_M} \cdot X$$

(6.3)
$$B^* = \theta_B \cdot e_B^{-1} \left(\frac{u_B / e_B}{p_{KLEB}} \right)^{-\sigma_B} \cdot \left(\frac{p_{KLEB}}{p_{KLEBM}} \right)^{-\sigma_M} \cdot X$$

(6.4)
$$E^* = \theta_E \cdot e_E^{-1} \cdot \left(\frac{p_E / e_E}{p_{KLE}} \right)^{-\sigma_E} \cdot \left(\frac{p_{KLE}}{p_{KLEB}} \right)^{-\sigma_E} \cdot \left(\frac{p_{KLEB}}{p_{KLEBM}} \right)^{-\sigma_M} \cdot X$$

(6.5)
$$L^* = \theta_L \cdot e_L^{-1} \cdot \left(\frac{p_L / e_L}{p_{KL}} \right)^{-\sigma_K} \cdot \left(\frac{p_{KL}}{p_{KLE}} \right)^{-\sigma_E} \cdot \left(\frac{p_{KLE}}{p_{KLEB}} \right)^{-\sigma_B} \cdot \left(\frac{p_{KLEB}}{p_{KLEBM}} \right)^{-\sigma_M} \cdot X$$

(6.6)
$$K^* = \theta_K \cdot e_K^{-1} \cdot \left(\frac{u_K / e_K}{p_{KL}} \right)^{-\sigma_K} \cdot \left(\frac{p_{KL}}{p_{KLE}} \right)^{-\sigma_E} \cdot \left(\frac{p_{KLE}}{p_{KLEB}} \right)^{-\sigma_B} \cdot \left(\frac{p_{KLEB}}{p_{KLEBM}} \right)^{-\sigma_M} \cdot X$$

X is industry production, e is a factor-specific efficiency index, p is price, either on a single factor of production or on an aggregate of factors, cf. the subscript on p. The price of capital is called u as in user cost, θ is a factor-specific constant, and the σ's are substitution elasticities of the CES production function. There is an equivalent set of equations for each of the 6 *KLEBM* industries. The 3 *KLBME* industries have a similar structure in their factor demand, but with energy in the outer nest, instead of materials.

We see that the number of relative prices increases, the further into the nesting structure we get, and to determine the K and L in the inner nest 4 relative prices are required. Prices of factor aggregates are chained Paasche price indices based on the efficiency-adjusted factor prices. The chained Paasche price approximates the formally correct

CES-price, as the latter can cause problems outside of equilibrium.

In each demand equation, the associated efficiency index appears twice. First, the reciprocal efficiency index appears as a general scaling factor, and second, the price of the factor input is divided by the efficiency index. We note that the efficiency index is eliminated from the equation if the substitution elasticity is 1, and our use of factor-specific efficiency indices generally presumes that the substitution elasticity differs from 1. The use of efficiency indices in ADAM is discussed in Thomsen (2000).[3]

The equations for the desired factor input function as long-term equations in the factor block. In the short term, factors of production adapt to the desired level via error correcting equations. The adaptation must be gradual, because it is associated with costs

Cost of factor input affects production price

Besides determining their factor input, most businesses also determine the price of their production. This price setting is modelled so that it is linked to the cost minimization, which ensures that the industry also maximizes its profit. Price equations are described in Chapter 7.

Determining investment and employment

The factor block determines the level of machinery, buildings, and working hours, and from these factor inputs, the level of investments and employment in terms of persons can be determined.

Investments in machinery by a given industry in a given year reflect the investments needed by the industry to achieve the calculated level of machinery at year-end. More specifically, the level of investments is calculated by the following identity[4]:

(6.7)

$$I_K = \left(K - (1-\delta_K)\cdot K_{-1}\right)\frac{p_{K,-1}}{\frac{1}{2}\cdot p_{I,-1} + \frac{1}{2}\cdot p_I}$$

I is investment, δ is the depreciation rate of capital, and p is the price of either capital K or investment I, cf. subscript on p. The fraction indicates the ratio between capital and investment price at the beginning of the year.

Equation (6.7) says that gross investment in a given year equals capital stock at year-end minus the capital stock, which has survived from the previous year. This gross change of the capital stock is multiplied by the price ratio between capital and investment in order to convert from capital to investment prices. A similar equation describes the transition from building capital to construction investments.

Employment, Q, in a given industry is determined as the number of hours worked, L, divided by average working hours:

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[3] For further details about efficiency and price indices, see the electronic annex on formulating the factor block.

[4] The dynamic identity in (6.7) is discussed in the electronic annex.

(6.8)
$$Q = \frac{L}{a_S h_S + a_W h_W}$$

where a is the share of, respectively, self-employed, subscript S, and employees, W, while h is the corresponding average number of working hours. The breakdown on self-employed and employees is made to split factor income into wages for the employees and gross operating profits for the self-employed.

For three industries: extraction of hydrocarbons (e), housing (h) and public services (o), factor demand is not estimated. Most factor input is either exogenous or follows production in a simple manner.

In the extraction of hydrocarbons, energy input is exogenous, while materials and labour follow production. Building capital is exogenous, and machinery investments follow a three-year-average of changes in production.

In the housing industry, building capital is given by the stock of dwellings, which is not determined in the factor block but in the housing model that also determines the house price, cf. chapter 3. There is no machinery in housing and the input of energy, materials and labour follow production. The latter three inputs concern the caretaker service included in housing industry.

In public services, energy input follows total production, while input of materials follows value added. Machinery and building capital follow from investments in machinery and construction investments, which are both exogenous. Input of hours worked is determined from employment, which is also exogenous.

The amount of, e.g. machinery in the total economy is the sum of machinery in the 12 industries.

6.2 Properties of factor demand

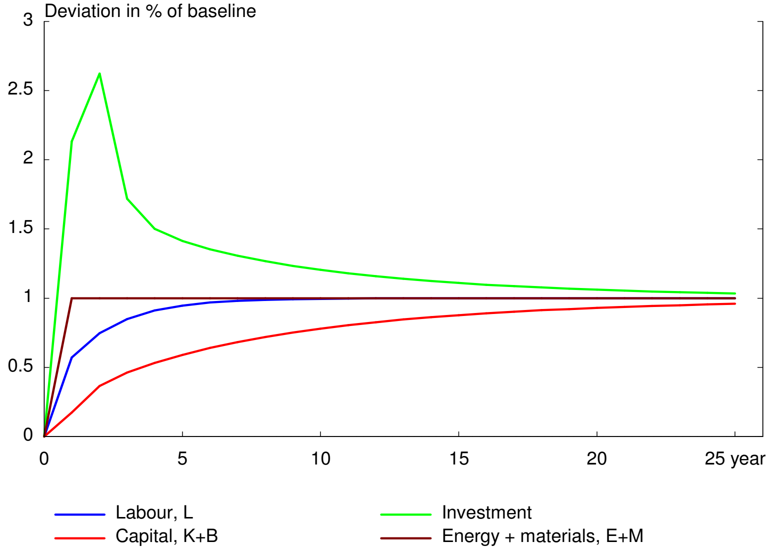
The following illustrates the response of factor demand to exogenous changes in production, factor prices, and productivity in the 9 estimated industries, which we here treat as one single industry. Thus, for example, a wage increase of 1 per cent is a wage increase of 1 per cent in all 9 industries, and the reported effect on, e.g. labour input is the sum of labour changes in the 9 industries.

The illustration concerns only the factor demand equations, implying that the effect on factor input reflects only those equations. When the entire ADAM model is involved in the calculation, a wage change, for example, would come with a number of secondary effects on prices and production etc., and an ADAM-calculated effect on factor inputs would reflect these accompanying effects. It should be noted that we are here illustrating the estimated error-correcting equations, so that the factors of production respond differently in the short and long term.

Effect of increasing production by 1 per cent

A production increase of 1 per cent will in the long term increase all factor inputs and thus also total production costs by 1 per cent. The input of both material and energy increases by 1 per cent in the first year, while labour and capital increase by less than 1 per cent in year 1, see figure 6.2 showing the average factor response for all 9 estimated industries to a 1-per cent production increase. Since production increases by more than factor inputs in year 1, costs per unit produced will fall in year 1, and this mechanism makes the gross profit pro-cyclical at given prices.

Figure 6.2 **Effect of increasing production by 1 per cent**



The sluggish response of labour reflects that labour's short-term elasticity with respect to production is not restricted to one, and the estimated inertia of labour makes labour productivity pro-cyclical. This inertia may be due to labour hoarding. There are costs of firing and hiring people, and businesses often have a labour reserve.

Both machinery and building capital respond even more slowly than labour to production growth. Especially building capital is slow to react, and even after 10 years building capital has far from increased by 1 per cent. If capital stock were to adjust fully in year 1 and increase by 1 per cent, investments would increase by several per cent in year 1 and thereafter be 1 per cent higher than the baseline. The actual sluggish adjustment of capital lifts investments by more than 1 per cent above the baseline for a number of years, until capital has reached the 1 per cent higher level.

Although building capital is particularly slow to react to an increase in production, the first-year percentage response of building investments is larger than the response of machinery investments, see table 6.2. This reflects that building capital is very large relative to investments, since buildings depreciate slowly.

Table 6.2 **Effect of 1 per cent increase of production**

	1. year	2. year	5. year	10. year	Long-term
	Percentage change				
Machinery – *K*	0.29	0.63	0.84	0.95	1.00
Labour – *L*	0.57	0.75	0.95	1.00	1.00
Energy – *E*	1.00	1.00	1.00	1.00	1.00
Buildings – *B*	0.10	0.20	0.43	0.67	1.00
Materials – *M*	1.00	1.00	1.00	1.00	1.00
Total costs	0.78	0.86	0.95	0.98	1.00
Labour productivity	0.43	0.25	0.05	0.00	0.00
Machinery investments	1.95	2.56	1.11	1.03	1.00
Building investments	2.60	2.77	2.20	1.65	1.00

The labour productivity in table 6.2 is measured as value added per hours worked. Value added corresponds to production less cost of materials, energy and non-energy. The table shows that the productivity of labour is unaffected in the long term, but in the short term labour productivity rises because of the aforementioned sluggishness in labour input.

Effect of changing factor prices

The long-term effect of changing factor prices for a given output can be summarized by the long-term price elasticities of factor inputs, which here concern the 9 estimated industries combined.

The long-term price elasticities are shown in table 6.3. As expected, there are negative numbers for own-price elasticities in the table diagonal. Most cross-price elasticities are positive or close to zero. In the six *KLEBM* industries, the substitution elasticity is zero in the two outer nests, so these industries have straight zeros in the two bottom rows that describe the price effects on buildings and materials. In the *KLBME* industry for energy supply, *ne*, there is a minor substitution between the *KLBM*-aggregate and energy input, and, consequently, for this industry there are not straight zeros in the two bottom rows.

In the other two *KLBME*-industries and in the six *KLEBM*-industries, materials, M, and building capital, B, are both proportional to production in the long term. Apart from the small effects in the industry for energy supply, an increase in the price of either buildings or materials will not create substitution. The only effect is that production becomes more expensive.

A price increase of energy will trigger substitution between energy and a KL-aggregate of machinery and labour. The energy price increase does not affect the relative use of machinery and labour, both of which will rise slightly.

Rising the cost of labour, i.e. the hourly wage, by 1 per cent will in the long term decrease the use of labour by 0.06 per cent and increase the use of machinery by 0.20 per cent, see the long-term elasticities in table 6.3. The percentage decrease in labour is smaller than the increase in machinery, because labour has the highest output elasticity.

Table 6.3 **Long-term price elasticities**

	K-user cost	Wage	*E*-price	*B*-user cost	*M*-price
Machinery – *K*	**-0.21**	0.20	0.01	-0.00	-0.00
Labour – *L*	0.05	**-0.06**	0.01	-0.00	-0.00
Energy – *E*	0.02	0.06	**-0.10**	0.01	0.01
Buildings – *B*	-0.00	-0.00	0.01	**-0.01**	-0.00
Materials – *M*	-0.00	-0.00	0.00	-0.00	**-0.00**

The impact of wage increases is smaller in the short term, cf. the illustration in table 6.4, which shows that the first year impact on labour, i.e. the own-price elasticity, is only half of the long-term elasticity. For machinery, the first-year price elasticity with respect to wages is only a quarter of the long-term elasticity. However, annual investment is only a fraction of capital stock, so that the percentage effect on machinery investments is relatively high in the first years after the wage increase.

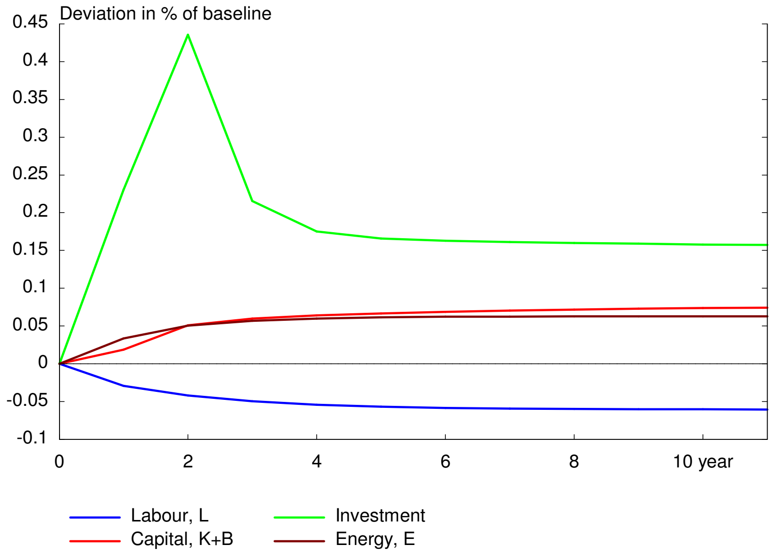
Table 6.4 **Effect of 1 per cent increase in hourly wage**

	1. year	2. year	5. year	10. year	Long-term
	Percentage change				
Machinery – *K*	0.05	0.13	0.17	0.19	0.20
Labour – *L*	-0.03	-0.04	-0.06	-0.06	-0.06
Energy – *E*	0.03	0.05	0.06	0.06	0.06
Buildings – *B*	-0.00	-0.00	-0.00	-0.00	-0.00
Materials – *M*	-0.00	-0.00	-0.00	-0.00	-0.00
Total costs	-0.00	0.00	0.00	0.00	0.00
Labour productivity	0.03	0.05	0.06	0.07	0.07
Machinery investments	0.32	0.60	0.23	0.22	0.20
Buildings investments	0.00	-0.01	-0.01	-0.00	-0.00

The impact of wage increases on factor demand is also illustrated in figure 6.3, which shows the effect of 1 per cent increase in hourly wage. In practical terms, both labour and energy input seem to adapt to the wage increase after 5 years. For capital inputs it takes longer, but most of the adjustment in capital has also happened after 5 years.

It may be noted that labour productivity increases when wage increases, because the input of labour falls at a given production. However, there is no real efficiency gain associated with the higher labour productivity, because the cost of additional capital outweighs the drop in labour costs.

Figure 6.3 **Effect of 1 per cent wage increase**



The cost of using capital is measured as a user cost that represents the cost of renting the capital stock for a year, see box 6.3 on user cost. The expression for user cost is mainly formulated with a view to the long term, where the economy is in equilibrium.

Box 6.3 **User cost**[5]

The applied prices of materials and energy are price indices from the national accounts, and the wage cost per hour worked can, in principle, also be derived from the national accounts. However, there are no statistics for user cost, i.e. the cost of using capital stock for a year.

User cost, u, is made with a formula of Jorgenson type, see Jorgenson (1963), which assumes full debt financing:

$$u = \frac{1 - t \cdot \delta^T}{1 - t}[(1-t) \cdot i + \delta - (1-\delta) \cdot \pi] \cdot p \qquad (6.9)$$

t is the marginal corporate tax rate, δ is depreciation rate, i is interest rate, π is price increase, and p is the investment price. Superscript T indicates that δ is a tax-related depreciation rate.

The intuition behind (6.9) is that the user cost after tax, i.e. $u\cdot(1\text{-}t)$, for 1 unit of machine corresponds to the investment price adjusted for the present value of tax depreciations, $p\cdot(1\text{-}t\cdot\delta^T)$, and spread out on the

[5] For more details on user cost, see the electronic annex on new capital equations.

life span of the machine by multiplying with the square-bracket-expression in (6.9). The square brackets contain: the interest rate after tax plus annual depreciation rate minus price increase on the machine. If, for instance, we expect the price of machinery to fall, as the computer price often does, it will increase user cost and tend to postpone investment.

The equation for user cost on buildings is similar to (6.9), but the investment price and depreciation rate are different and a bond yield enters instead of the bank lending rate.

Effect of increased efficiency

A 1-per cent increase in an efficiency index immediately reduces the related factor demand by 1 per cent, but as the price of efficiency units falls by 1 per cent, the decline in factor input is moderated by a substitution effect. The substitution effect implies that the percentage decrease in the use of a factor is smaller than the increase in its efficiency. However, with ADAM's estimated CES functions the substitution effect is far from large enough to prevent a fall in factor demand.

For example, if the efficiency index for labour increases by 1 per cent, the input of labour falls by 0.93 per cent, see table 6.5, which shows the long-term effect of 1 per cent increase in five efficiency indices, cf. the five columns of the table. More specifically, each efficiency index in the table represents efficiency indices for the nine estimated industries.

Table 6.5 **Long-term efficiency elasticities**

	e_K	e_L	e_E	e_B	e_M
K	-0.78	-0.20	-0.01	0.00	0.00
L	-0.05	-0.93	-0.01	0.00	0.00
E	-0.02	-0.06	-0.89	-0.01	-0.01
B	0.00	0.00	-0.01	-0.98	0.00
M	0.00	0.00	-0.00	0.00	-0.99

Increases in efficiency affect other factors via substitution

When the efficiency index of a production factor changes, it will influence the demand for all factors. If the efficiency index of labour increases by 1 per cent and makes labour 1 per cent cheaper per efficiency unit, the demand for machinery and energy will fall by, respectively, 0.20 and 0.06 per cent, see table 6.5. This secondary effect of labour efficiency on machinery and energy corresponds to the effect of 1 per cent fall in hourly wage, cf. price elasticities in table 6.3.

6.3 Estimated factor demand equations

For the purpose of estimation, the lagged difference between the actual and desired factor demand is inserted in an error correction formula:

$$\text{Dif}\left(k\right) = \alpha_1 \text{ Dif}\left(kx^*\right) + \alpha_2 \text{ Dif}\left(x\right) + g - \gamma\left(k_{-1} - k_{-1}^*\right) \quad (6.10)$$

where machinery is used as example. The letter k indicates the logarithm of the actual machinery, and k^* is the logarithm of the desired machinery. The equation for the desired machinery was shown in a simplified version in (6.1) and more detailed as equation (6.6) in box 6.2. In the short-term dynamics of the error correction equation, the desired machinery k^* is split into x, which is production, and kx^*, summarizing the effect of efficiency indices and relative prices in (6.1). Greek letters are estimated coefficients, and g is a constant. There are similar equations for labour, L, energy, E, buildings, B, and materials M. Thus, for each of the nine estimated industries we have five error correction equations.

Stepwise estimation

For each industry, the five error correction equations are estimated in four steps. For the six *KLEBM*-industries, we first estimate the equation for materials placed in the outer nest. Then follows the equation for building capital, the equation for energy, and finally we estimate the two error correction equations for machinery and labour placed in the inner nest with common parameters in their long-term equations. In the three *KLBME*-industries, the sequence of estimations starts with the energy equation in the outer nest. This recursive approach implies that we estimate an elasticity of substitution in the most outer nest, where the elasticity is found, and use the estimate in the inner nests.[6]

Table 6.6

Substitution elasticities in factor block, σ's

	a	*nf*	*nz*	*b*	*qz*	*ne*	*ng*	*qs*	*Qf*
	KLEBM-industries					*KLBME*	Leontief-industries		
K vs. *L*	**0.41**	**0.41**	**0.32**	**0.20**	**0.31**	**0.00**	0.00	0.00	0.00
	(0.21)	**(0.23)**	**(0.13)**	**(0.24)**	**(0.04)**	**(-)**	(-)	(-)	(-)
E vs. *KL*	**0.35**	**0.00**	**0.31**	**0.10**	**0.12**	**0.30**	0.00	0.00	0.00
	(0.07)	**(-)**	**(0.12)**	**(-)**	**(0.09)**	**(0.08)**	(-)	(-)	(-)
B vs. *KLE*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
M vs. *KLEB*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Figures in brackets are standard errors of estimated coefficients.

Many substitution elasticities are restricted to zero

Table 6.6 illustrates the elasticity of substitution in the 9 factor demand equations based on CES functions. It has not been possible to estimate positive and significant substitution elasticities for oil refineries, sea transport and financial services, cf. the three columns of zeros to the right in Table 6.6, where these three industries are referred to as Leontief industries. The 9 industries of table 6.6 are included in the following three tables showing the rest of the equation coefficients.

It also appears from table 6.6 that the substitution elasticity between materials and *KLEB*-aggregate and the one between buildings and *KLE*-aggregate are insignificant and set to zero in all industries. Moreover, it is noted that the estimated substitution elasticity of 0.3 between E and

[6] For further details, we refer to the electronic annex on factor block estimation.

the KLBM-aggregate in energy supply, *ne*, is placed in the second row of the table.

Estimated substitution elasticities are small

The substitution elasticities in table 6.6 are all relatively small and far from the elasticity of 1 found in Cobb-Douglas production functions. Thus, most factor price fluctuations will only create modest changes in the use of production factors. The estimated substitution elasticities help to determine the factor price elasticities shown in table 6.3. The more inside the nesting structure a factor of production is placed, the more relative prices and substitution elasticities will affect the factor demand; see the long-term demand equations in box 6.2.

Table 6.7

Error correction parameters in factor block, γ's

	a	*Nf*	*nz*	*b*	*qz*	*ne*	*ng*	*qs*	*qf*
	KLEBM-industries					*KLBME*	Leontief-industries		
K	0.20	0.20	0.21	0.20	0.83	0.20	0.20	0.20	0.20
	(-)	(-)	(0.04)	(-)	(0.08)	(-)	(-)	(-)	(-)
L	0.40	0.40	0.46	0.40	0.40	0.40	0.40	0.40	0.40
	(-)	(-)	(0.10)	(-)	(-)	(-)	(-)	(-)	(-)
E	0.82	0.50	0.42	0.42	0.57	0.41	0.50	0.35	0.53
	(0.19)	(0.14)	(0.16)	(0.15)	(0.15)	(0.15)	(0.15)	(0.13)	(0.15)
B	0.10	0.10	0.21	0.10	0.10	0.10	0.10	0.10	0.10
	(-)	(-)	(0.03)	(-)	(0.04)	(-)	(-)	(-)	(-)
M	0.62	0.27	0.64	0.84	0.14	0.68	0.86	0.62	0.53
	(0.18)	(0.14)	(0.16)	(0.16)	(0.11)	(0.19)	(0.17)	(0.13)	(0.15)

Figures in brackets are standard errors of estimated coefficients.

Error-correcting parameters differ

Error-correcting parameters corresponding to the parameter γ in equation (6.10) are given in table 6.7. The larger the parameter, the more rapidly the actual factor use adjusts to the desired factor use. In most industries, error-correcting parameters for buildings, machinery, and labour are restricted in order to shorten the adjustment time. Most parameters are significant in free estimation, but without restrictions the resulting adjustment time is typically very large. For most industries, the restricted error-correcting parameters are within a 5 per cent interval of confidence, but in some cases it has been chosen to restrict individual estimates beyond the confidence interval.

In most industries, labour input reacts faster than machinery to deviations from long-term equilibrium, and machinery reacts faster than building capital.

First-year's response to factor prices

First-year's effects of factor prices on factor demand are illustrated in table 6.8, where the numbers indicate the size of first-year's price effect relative to the long-term price. There is no price effect on materials and buildings when they are in the outer nests, because the elasticity of substitution is zero, and in general, there is no price effect in the refining industry, in sea transport, and in financial services, as all substitution elasticities are set to zero in those three industries, cf. table 6.6. The figures for first-year's price effect relative to long-term price effect are all between 0 and 1, implying that the adjustment to the long-term effect of relative prices is gradual. Most parameters in table 6.8 are restricted to values within a 5 per cent confidence interval.

Table 6.8

First-year's impact of price effects in factor block, α_1's

	a	*Nf*	*nz*	*b*	*qz*	*ne*	*ng*	*qs*	*qf*
	KLEBM-industries					*KLBME*	Leontief-industries		
K	**0.10**	**0.10**	**0.10**	**0.30**	**0.32**	**0.20**	0.00	0.00	0.00
	(-)	**(-)**	**(-)**	**(-)**	**(0.12)**	**(-)**	(-)	(-)	(-)
L	**0.20**	**0.46**	**0.60**	**0.50**	**0.50**	**0.30**	0.00	0.00	0.00
	(-)	**(0.60)**	**(-)**	**(-)**	**(-)**	**(-)**	(-)	(-)	(-)
E	**0.50**	**0.00**	**0.60**	**0.50**	**0.50**	**0.50**	0.00	0.00	0.00
	(-)	**(-)**	**(-)**	**(-)**	**(-)**	**(-)**	(-)	(-)	(-)
B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
M	0.00	0.00	0.00	0.00	0.00	**0.50**	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Figures in brackets are standard errors of estimated coefficients.

First-year's response to production

Coefficients for the contemporaneous change in production are shown in table 6.9. For energy input and other materials, the first-year coefficient for production is set to 1 in all industries. This first-year elasticity of 1 corresponds to the long-term impact from production, and since the input of materials is not affected by relative prices, the input is always proportional to production. In most cases, the restriction to 1 cannot be rejected at a 5 per cent level. For the inputs of labour and of capital, i.e. machinery and buildings, it has been decided to restrict the first-year elasticity to, respectively, 0.4 and 0.1, if the restriction cannot be rejected at a 5 per cent confidence interval. The restriction prevents that employment and investments jump two years after a production change.

Table 6.9

First-year's production elasticities in factor block, α_2's

	a	nf	nz	b	qz	ne	ng	qs	qf
	KLEBM-industries					*KLBME*	Leontief-industries		
K	0.30	0.19	0.10	0.36	0.36	0.20	0.20	0.30	0.43
	(0.12)	(0.10)	(-)	(0.07)	(0.07)	(-)	(-)	(0.06)	(0.07)
L	0.40	0.36	0.67	0.72	0.57	0.30	0.40	0.35	0.40
	(-)	(0.17)	(0.07)	(0.08)	(0.08)	(-)	(-)	(-)	(-)
E	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
B	0.10	0.12	0.08	0.10	0.10	0.10	0.09	0.10	0.04
	(-)	(0.07)	(-)	(-)	(-)	(-)	(0.04)	(-)	(-)
M	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Figures in brackets are standard errors of estimated coefficients.

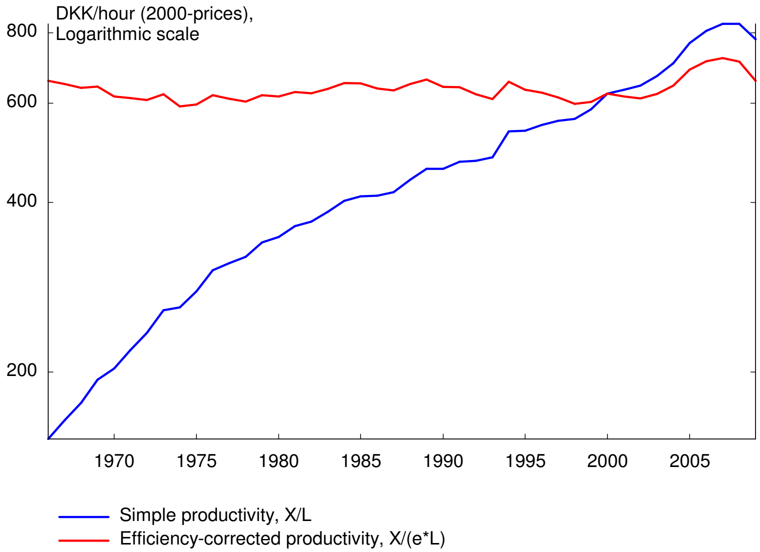
Efficiency indices capture trend shift in productivity

The efficiency indices of the factor block do not represent a constant increase in productivity, implying that the log value of an index is not a first-order polynomial in time. Instead, the efficiency indices represent sixth-order polynomials in time, so that the log of an efficiency index is not a straight line. The use of non-linear efficiency indices reflects the fact that it is impossible to explain all long-term movements in factor inputs by means of relative prices.

A typical example is that the annual increase of hourly productivity in manufacturing was high in the beginning of the estimation sample; cf. the concave shape of simple hourly productivity on a log scale in figure 6.4.

Figure 6.4

Hourly productivity in manufacturing



The concave shape cannot be explained by the relative price of labour. Instead, it is captured by a declining trend in the non-linear efficiency index. If the denominator of the hourly productivity measure, i.e. the input of man-hours, is multiplied by the non-linear efficiency index we obtain an adjusted measure without trend in the beginning of the estimation sample, see figure 6.4.

6.4 Inventory investments

We have now described the demand for five factors of production including machinery and buildings, but in addition most businesses also need a stock of goods, which can include both raw materials, intermediate and finished products.

ADAM does not determine the stock or inventory level but the inventory investment, which indicates the net flow into the inventory. If goods are taken from stock during a year, the industry has a negative inventory investment in that year.

On average, total investments in inventories correspond to just 0.2 per cent of GDP, but inventory investments can fluctuate considerably from year to year. Since 1980 inventory investments have fluctuated between plus 1.3 and minus 1.1 per cent of GDP.

Inventories comprise Danish and imported goods

In ADAM, inventory investments are divided into 21 components, representing 12 industries plus 9 import categories. This implies that the stocks of Danish goods are divided by industry and import stocks by import category.

Investment indicates adjustment to wanted inventory

The starting point for all inventory investment equations is that inventories adapt to a desired level and the inventory investment in a certain year indicates the change in inventory between the beginning and end of the year. This gives the following equation:

$$fIl = \alpha \cdot (K^{*} - K_{-1}) \qquad (6.11)$$

fIl Inventory investments
K^{*} Desired stock
K Actual stock

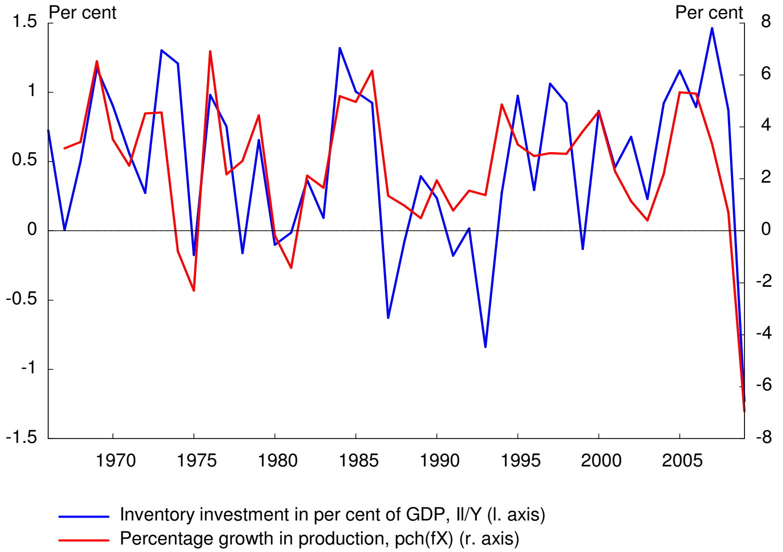
α is the adjustment parameter. The desired stock is assumed to be proportional to the expected sales

$$K^{*} = \kappa \cdot fA^{e} \qquad (6.12)$$

fA^{e} Expected sales
κ Desired stock ratio

The desired stock ratio represents the optimum ratio between inventory and sales. The higher this ratio is, the longer a product is stored in the inventory.

Figure 6.5 **Inventory investments and production growth**



Stock movements are cyclical

The inventory equations for ADAM implies that inventory investments are cyclical, and the positive correlation with the business cycle is confirmed by figure 6.5, where inventory investments as GDP share are compared with the percentage increase in total production.

Representing expected sales

In order to estimate the desired stock equations, expected sales are formulated as a weighted average of sales in the contemporaneous and the previous year. Sales are determined as total production minus inventory investments representing output not sold but added to the inventory. For import inventories, the corresponding import is used instead of production.

With the above expression for expected sales inserted in the desired stock equation, the inventory investments equation in (6.11) can be formulated as an equation in variables, which in principle could be observed:

(6.11)*
$$fIl = \alpha \cdot \left(\kappa \cdot (\beta \cdot fA + (1-\beta) \cdot fA_{-1}) - K_{-1} \right)$$

fA is sales and β is the weight in the expected sales expression.

However, we cannot just estimate the inventory investment equation in (6.11*) because of two problems that require a rewrite of (6.11*).

Transformation to differences

Firstly, it is a problem that there is no data for inventory, *K*, only for inventory investments, *fil*, representing the first order difference of the inventory. This problem is circumvented by transforming (6.11*) into the difference equation (6.13) where *K* is replaced by *fil*:

(6.13)

$$\text{Dif}(fIl) = \alpha \cdot \left(\kappa \cdot \text{Dif}(\beta \cdot fA + (1-\beta) \cdot fA_{-1}) - fIl_{-1}\right)$$

Inventory investments as chain index

Secondly, it is a problem that inventory investments at fixed prices, *file*, are difficult to interpret, when they are calculated with the chain index formula. Thus, we have chosen to introduce inventory investments measured in lagged prices, i.e. *fIl* multiplied by pil_{-1}. This quantity is called *fdil*, and the official inventory investments at fixed prices, *fil*, are replaced with *fdil* divided by lagged production prices, $fdil/px_{-1}$.

Inserting the above expression (6.13) for inventory investments, while α times κ is replaced by λ and 1 minus α by δ, will produce the following estimable equation for inventory investments:

(6.14)

$$fdIl\,/\,px_{-1} = \lambda \cdot \text{Dif}\left(\beta \cdot fA + (1-\beta) \cdot fA_{-1}\right) + \delta \cdot fdIl_{-1}\,/\,px_{-2}$$

Fifteen estimated inventory equations

There are three parameters to estimate, and the results for the 15 estimated inventory equations are shown in table 6.10. Adjustment parameter, δ, is typically set to zero, corresponding to full adjustment to the desired stock within each year. An expectation weight, β, of zero implies that the desired stock is proportional to previous year's sales.

Table 6.10

Estimated inventory investment equations

Stocks originating from	Variable	Stock ratio λ	Adjustment δ	Expectation β
Agriculture	$fdila/pxa_{-1}$	0.13441	0	0
Extraction of hydrocarbons	$fdile/pxe_{-1}$	0.02863	0	0
Energy supply	$fdilne/pxne_{-1}$	0.01501	0	0
Food processing	$fdilnf/pxnf_{-1}$	0.02166	0	0
Oil refineries	$fdilng/pxng_{-1}$	0.02482	0.54565	0
Manufacturing	$fdilnz/pxnz_{-1}$	0.17341	0	0,75
Private services	$fdilqz/pxqz_{-1}$	0.01389	0	1
Imports of food etc.	$fdilm01/pm01_{-1}$	0.07069	0	1
Imports of materials	$fdilm2/pm2_{-1}$	0.07580	0	0
Imports of coal and coke	$fdilm3k/pm3k_{-1}$	0.36725	0	0
Imports of crude oil	$fdilm3r/pm3r_{-1}$	0.03043	0	0
Imports of energy products	$fdilm3q/pm3q_{-1}$	0.16532	0.45471	0
Imports of manufacturing	$fdilm59/pm59_{-1}$	0.13696	0	1
Imports of cars	$fdilm7b/pm7b_{-1}$	0.25504	0	0,25

The 6 exogenous inventory investments relate to construction, financial services, sea transport, housing, public services, and to imports of ships, aircraft and drilling platforms.

It may be added that the equation for agricultural inventory investments includes a variable for yield from harvest, which indicates the difference between actual and normal harvest. A good harvest is often accompanied by a large inventory build-up, followed by a stock depletion over the next two years. The coefficient for the harvest variable implies that around two thirds of a good harvest is stored, while the rest is exported before the end of the year.

6.5 Labour force

ADAM determines the labour input and hence employment as a function of production and relative labour price. The model's labour force is primarily demographically determined, as it depends on the number of people in the working age groups. Especially in recent years, the labour force, i.e. the supply of labour, has responded to the economic cycle, and it appears that labour supply partly follows the fluctuations in employment. ADAM formalizes this cyclical reaction for some groups at the edge of the labour force.

Unemployment indicates the difference between the labour force and employment, and the cyclical response of the labour force implies that unemployment reacts less than employment. The aforementioned cyclical reaction is the only behavioural reaction in the labour supply.

Determination of the labour force

ADAM determines the labour force as the total population minus those outside the labour force. Outside the labour force are children under 15 years, most retirees and a number of benefit recipients etc. The labour market set-up in the model is illustrated in table 6.11 listing the groups that fall outside the labour force. The data for the labour market is calculated using a demographic model called Uadam, described in box 6.4 placed at the end of the present section 6.5 on the labour force.

Cyclical response in activation schemes and students

The cyclical response of the labour force does not involve all groups outside the labour force. Presumably, people in activation schemes and students are likely to respond to the job situation. If job opportunities improve, the activated find a job and quit the activation scheme, while students may rush to finish their study or get a job while studying.

Table 6.11 **ADAM's labour market (national accounts definition)**

		2000	2006	2009*
		— 1,000 persons, mid-year —		
1. Total population	*U*	5 340	5 437	5 523
2. Children	*Ub*	1 003	1 048	1 040
3. Population over 15	*U1599*	4 337	4 389	4 483
4. Old-age pensioners	*Ufpx+Upt*	650	739	778
5. Population of working age	*Uw*	3 675	3 641	3 695
6. Disabled pensioners	*Ufox*	228	214	214
7. Early retirement	*Uef*	156	144	131
8. Transitional allowance	*Uov*	25	1	0
9. Leave schemes	*Umf*	28	3	2
10. Maternity	*Usb*	33	52	50
11. Sickness benefit	*Usxa*	16	14	17
12. Holiday benefit	*Ufdp*	8	8	4
13. Students	*Uuxa*	117	135	136
14. Rehabilitation	*Urev*	29	19	18
15. Temporary benefit scheme	*Uly*	0	11	11
16. Local activation	*Ukak*	19	24	37
17. Activation allowance	*Uakx*	31	14	11
18. Social benefits	*Ukxa*	84	61	53
19. Other	*Ur*	53	48	89
20. Total outside the labour force (6-19)	*Uwxa*	827	749	773
21. Labour force (5 minus 20)	*Ua*	2 848	2 893	2 922
22. Employment	*Q*	2 712	2 783	2 824
23. Unemployment (21 minus 22)	*Ul*	136	109	98

In ADAM, the groups in table 6.11 lines 12, 13, 16 and 17 are endogenized. Holiday benefits in line 12 are for the previously unemployed, and the variable for recipients of holiday benefits follows with a lag the variable for unemployment benefit recipients. Equations for the students and the activated are both formulated as equation (6.15), which sets the relative increase in activated equal to a positive elasticity, α, multiplied by the relative increase in unemployment.

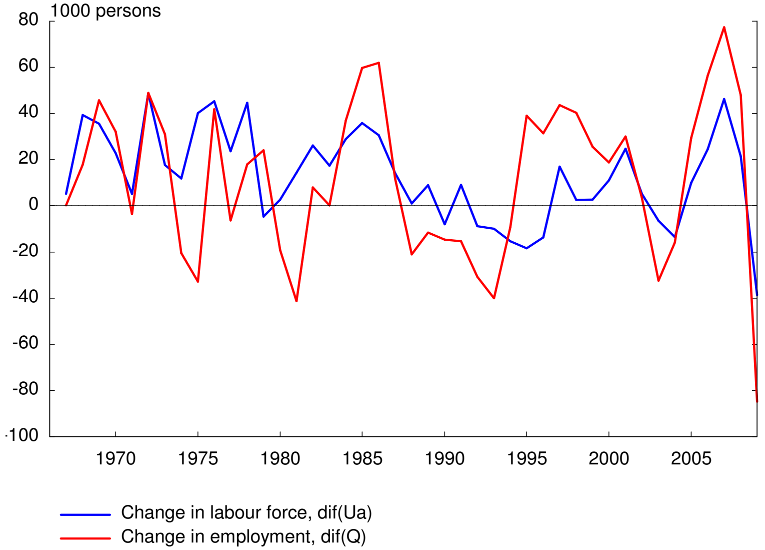
(6.15)
$$activated = (1 + \alpha \cdot (unemployed / unemployed_{-1} - 1)) \cdot activated_{-1}$$

This means that if employment increases and makes unemployment fall, the number of activated and of students outside the labour force will also fall, thereby increasing the labour force in ADAM.

Labour force and employment correlate

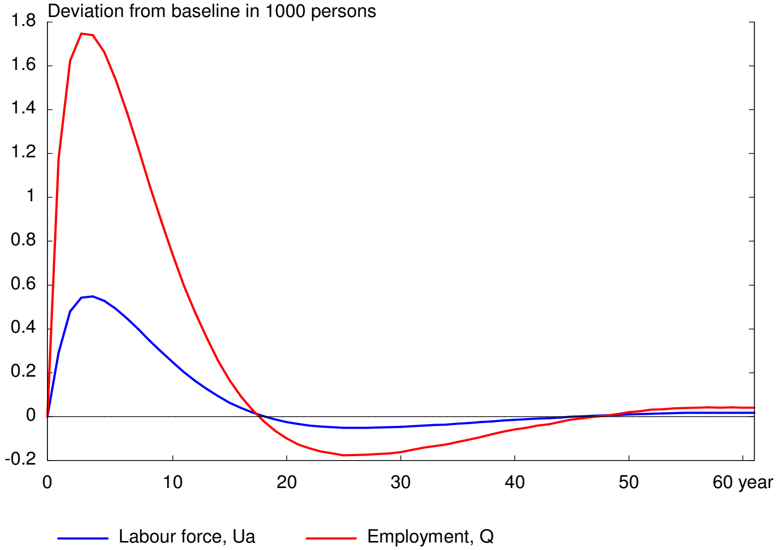
This implied cyclicality of the labour force is confirmed by the data, see figure 6.6, which illustrates the development in the labour force and employment and their co-variation since 1970.

Figure 6.6 **Labour force and employment**



Co-variation is particularly evident in later years of the period in which the labour force grew substantially during the economic upturn until 2008, even though demographics alone suggested that the labour force should stagnate. In recent years, the flexibility of the labour force has been increased by more cross-border workers, who are foreign citizens employed in Denmark or Danish citizens employed abroad. Border workers are included in the group of "Other" in line 19 of table 6.11.

Figure 6.7 **Labour market reaction, government purchase + 1%**



Labour force reaction in the model

ADAM's equations for the number of students and activated imply that an increase in employment increases the labour force, cf. figure 6.7, which shows the effect on employment and labour force if government purchase of goods is increased by 1 per cent. The effect on unemployment corresponds to the difference between employment and labour force reaction.

The pro-cyclical response of the labour force moderates the immediate reaction in unemployment. When the change in unemployment is smaller, the accompanying effect on wages and competitiveness is also moderated, and this will tend to stimulate the short- and medium-term reaction in employment. The pro-cyclicality of the labour force does not matter in the long term, where unemployment and thus also the labour force returns to the baseline after a demand shock.

Box 6.4 **Demographic model (Uadam)**

The auxiliary model, Uadam, puts together a number of sources: population comes from the official population statistics, employment is from the national accounts, and unemployment is the official national measure of unemployment. The number of old-age pensioners is from the normal social statistics, while the number of people in all the other schemes mainly comes from the new statistics on people without regular employment. The number of scheme participants etc. is measured as full-time persons to ensure a good correspondence to the national accounts employment and to the related government budget accounts.

Data in Uadam is split up into 1-year age groups. Age distribution relates to end-year, so that a given age group consists of people born in the same calendar year. All equations in Uadam follow the same basic outline

$$U_{kji} = b_{kji} \cdot U_i$$

The left-hand-side variable U_{kji} indicates the number of participants in scheme j aged i and calculated on the basis of statistics k. For example, variable U_{fs41} is number of 41-year-old sickness benefit recipients (scheme s), taken from the statistics *People without regular employment* (statistics f), formerly known as AMFORA.

U_i is the population aged i. The age-distributed population can be projected using Statistics Denmark's latest population forecast.

The ratio b_{kji} indicates the proportion of the population aged i participating in social or labour-market scheme j. Ratio b_{kji} is exogenous and may be used to analyse alternative scenarios for participation in scheme j.

All calculated variables relate to population ages. Uadam is an auxiliary model producing input to ADAM. There is no behaviour in Uadam and projections with Uadam will only give the demographic effect on participation in a particular scheme. In principle, it would be possible to incorporate behavioural and indirect effects in the ratios of

Uadam, but this type of analysis can also be performed by incorporating Uadam scenarios in ADAM, which automatically calculates secondary effects on, e.g. the number of unemployed.

Summary of chapter 6

The businesses' inputs of labour, capital and materials depend on production, on relative factor prices and on an efficiency index attached to each factor input. For given relative prices and given efficiency indices, the desired factor demand increases by 1 per cent, when industry production increases by 1 per cent. The estimated elasticities of substitution are numerically smaller than one, so there is a limited substitutability between labour and capital, and in particular inputs of materials, excl. energy are practically unaffected by the relative factor prices.

Inputs of production factors adjust to the desired level at different speeds. The price-insensitive materials input adapts the same year, while it takes more time for labour input and most time for building capital.

The factor block comprises five factor inputs, including two types of capital: machinery and buildings. In addition, most businesses have a stock of goods to facilitate production and sales. ADAM does not determine inventory levels, but the desired stock of goods is assumed proportional to demand and used to model the annual inventory investments related to industries and imports.

Among the factors of production, labour has a special status as non-produced factor, and the last section of the chapter explains how the labour supply is determined. ADAM's labour supply depends essentially on demographics and to a lesser extent on cyclical fluctuations, as the number of people outside the labour force, specifically students and activated in labour market schemes, decreases when unemployment falls.

7. Wage and prices

Denmark is a small open economy, and with a fixed exchange rate of the krone vis-à-vis the euro, Danish inflation is basically imported from abroad. This regime is reflected in the ADAM structure. The exchange rate is exogenous in the model, and if the increase of Danish prices is higher than the exogenous foreign price increase, the price of Danish products will rise relative to foreign prices. This leads to a loss of market share, so that employment falls and the higher unemployment reduces the wage increase and eventually also the price increase. The model is not in equilibrium until Danish prices increase at the same rate as foreign prices.

The wage equation is basically a Phillips curve, which makes wages cyclical by linking wage increases to unemployment. Prices are cost-based and cyclical, because labour costs respond to unemployment.

In the estimated Phillips curve, inflation has a coefficient of less than one. Thus, the Phillips curve is not vertical, but at a given foreign inflation, the model has an equilibrium rate of unemployment determined by structural factors. It can be added that it is difficult to estimate a stable wage equation, as evidenced by the discussion in the following.

Basically, the model's interaction between real economy and price formation implies that unemployment affects wages, wages affect prices, prices affect competitiveness, which affects the market share for Danish production and hence the size of employment and unemployment. This interaction is crucial to the model and its long-term equilibrium. In the following, we first look at the wage equation and then at the equations for production prices.

7.1 Wage

Basically, ADAM's wage equation determines the wage increase on the basis of the price increase and the deviation of unemployment from its long-term value, which we call structural unemployment. See equation (7.1) below, where coefficient α_1 is positive, and γ is negative.

(7.1)

$$\text{Dlog}(wage) = \alpha_0 + \alpha_1 \cdot \text{Dlog}(price) + \gamma \cdot (unemployment_{-1} - structuralunemployment_{-1})$$

ADAM's long-term unemployment included in the Phillips curve

By structural unemployment, we mean the unemployment that ADAM generates in the long term. ADAM's structural unemployment depends on the replacement ratio, which indicates unemployment benefit relative to wages. The equation of structural unemployment is shown in (7.2), where the coefficient β_1 is positive.

(7.2)

$$structuralunemployment = \beta_0 + \beta_1 \cdot replacementratio$$

Structural unemployment and thus wage formation is likely to depend on a number of labour market policy instruments and not just on the replacement ratio, for example, it may also depend on the length of the benefit period and on the design of activation schemes, but we have only estimated an effect of the replacement rate.

The level of structural unemployment reflects not only labour market policy but is also affected by other factors. Specifically, it can be mentioned that it has been attempted to make structural unemployment depend on the wage share in value added, and on the wedge between disposable hourly wage income and total hourly wage costs paid by the employer.

Using the wage share as explaining variable can capture that employers and employees negotiate on the wage share in value added when they negotiate wages. The wage share in manufacturing has been in ADAM's wage equation for some time, but it is statistically insignificant and has been omitted in the present model version. The negative result fits in with Nymoen and Rødseth (2003), which finds that Denmark is the only Nordic country where the wage share does not contribute significantly to explaining wage growth.

It can also be imagined that structural unemployment depends on the wedge between disposable hourly wage income received by the wage earner and total hourly wage costs paid by the employer. However, just as it is difficult to estimate the effect of the wage share, it is difficult to estimate the effect of the tax wedge.

Wage equation has changed over time

The complete wage equation, i.e. (7.1) with structural unemployment in (7.2) inserted, corresponds to a Phillips curve augmented by the price increase and by the replacement ratio. It is a standard result that the unemployment variable of the Phillips curve can explain part of the Danish wage formation; cf. Pedersen (1983), who also notes that the Phillips-curve coefficients are unstable and that the set of additional explanatory variables change over time.

Inflation is now lower than in the 70s

The wage equation determines manufacturing hourly wage, while other industries hourly wage is assumed to follow the manufacturing wage. The time series for manufacturing hourly wage starts in the 1960s and so does the possible estimation period for the wage equation. However, it is not obvious that the same wage equation can describe the wage growth over a period of this length, because a lot has happened with wage formation since the 1960s. Most striking is that the rate of wage increases has fallen from between 10 and 20 per cent in the 1970s to below 5 per cent since 1990, see figure 7.1.

A vertical Phillips curve explains the 70s ...

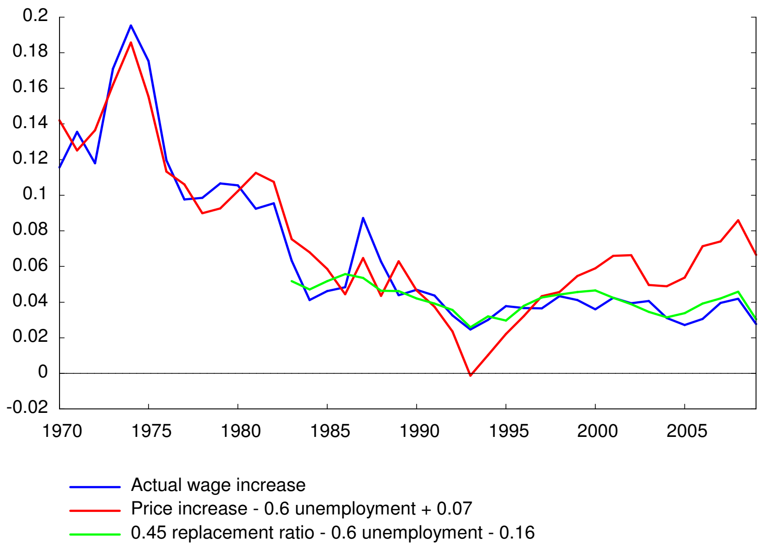
The high wage increases in the mid-1970s reflect, among other things, that the large increase in oil prices and the accompanying increase in consumer prices was passed on to wages. It is possible to explain a large part of the wage increase at the beginning of the displayed sample using a simple vertical Phillips curve with a coefficient of 1 for the contemporaneous price increase and a coefficient of -0.6 for unemployment, cf. figure 7.1.

However, the figure also shows that this type of wage equation cannot explain the wage development in recent years.

... and a Philllips curve with replacement ratio explains recent years

To explain the wage increase in recent years, it is better to drop the price increase and instead introduce the replacement ratio with a high coefficient, $-\gamma \cdot \beta_1$, of e.g. 0.45, see figure 7.1, where the result of this simple wage equation is shown, starting in 1983, cf. green curve in figure.

Figure 7.1 **Actual and explained wage increase**



Estimation period starts in 1983

We have argued that there appears to be a break in wage formation and it is decided to let the estimation period start in 1983. That is the year after 1982, when the price indexation of wages was abolished, and the fixed exchange rate policy introduced.[1]

The inflation decline in the 1980s is not an isolated Danish phenomenon, and there are also problems in the Phillips curves of other countries. It is not a general observation in other countries that the estimated coefficient for the price increase declines in the '80s, but there is evidence that the coefficient for the price increase declines when the monetary and exchange rate policy regime of a country begins to focus on price stability, cf. Benati (2008). The underlying argument is that if inflation expectations are anchored to say 2 per cent, it will not affect wage formation if the actual price increase differs from 2 per cent.

It does not solve all problems with the Danish wage equation to let the estimation sample start in 1983. Specifically, it is difficult to explain the large wage increase in 1987, which also stands out in Figure 7.1. The wage jump in 1987 may reflect the fact that employees were compensated for a

[1] The electronic annex on the wage equation estimates a vertical Phillips curve for the period 1967-1982.

reduction of the weekly working hours, and it may also have added to the wage jump in 1987 that wage increases in 1985 and 1986 had been moderated by government intervention. The estimated wage equation includes a dummy to explain wages in those years.

The problem of explaining wages in the mid-80s may reflect that it is a simplification to let the fixed exchange rate regime start in 1983. Probably, the first years constituted a period of transition.

7.2 Wage equation properties

Trivial to use only the wage equation

Seen in isolation, the wage equation determines the wage growth given unemployment and given the other explanatory variables of the equation. Wage levels do not appear directly in the wage equation. Thus, if the equation constant is raised by, e.g. 0.005, the wage growth immediately rises from, e.g. 3.5 to 4 per cent p.a. and the implied hourly wage level grows by 0.5 per cent p.a. relative to the baseline of the calculation.

This sounds like a trivial result, but when the wage equation interacts with the rest of ADAM, unemployment and prices will react, and the result of shocking the wage equation becomes more interesting. In the long term, it is not wage growth but unemployment that it is affected. The long-term wage increase is determined as a sum of the foreign price increase and Danish productivity growth, and in the long term unemployment will adjust to make the wage equation hold.

In ADAM wages affect prices and unemployment

Within the framework of ADAM, the explanatory variables of the wage equation are endogenous. For example, prices increase when labour costs increase, and more importantly, unemployment rises because market shares are lost when Danish wage and prices increase relative to foreign wages and prices.

Market shares will continue to decline and unemployment will continue to rise relative to the baseline scenario, as long as wages increase by more than in the baseline scenario. This process continues until the economy adjusts to a new equilibrium with the same wage increase as in the baseline scenario but with a higher unemployment rate and a higher wage level than in the baseline scenario.

For illustration the wage equation is supplemented ...

It is not necessary to involve all ADAM's equations to illustrate the recently described crowding-out process. It is enough to supplement the wage equation with an equation that makes the unemployment rate an increasing function of the ratio between Danish and foreign price, and an equation linking prices to wages. Thus, ADAM boils down to the three equations shown below as (7.3a) to (7.3c).

$$\text{Dlog}(wage) = 0.3 \cdot \text{Dlog}(price) - 0.55 \cdot (unemployment_{-1} - 3) + constant \quad (7.3a)$$

$$unemloyment = 0.2 \cdot \log(price_{-1} / foreignprice_{-1}) \quad (7.3b)$$

$$\text{Dlog}(price) = \text{Dlog}(wage) - \text{Dlog}(productivity) \quad (7.3c)$$

The wage equation (7.3a) is equivalent to the wage equation in (7.1) with ADAM coefficients for the price increase and replacement ratio inserted. For simplicity, structural unemployment is set to 3 per cent of the labour force, and the constant can be used to shock the equation.

... by an equation for unemployment ...

The equation in (7.3b) makes unemployment an increasing function of the ratio between domestic and foreign prices, reflecting that a large part of the Danish production is in competition with other countries

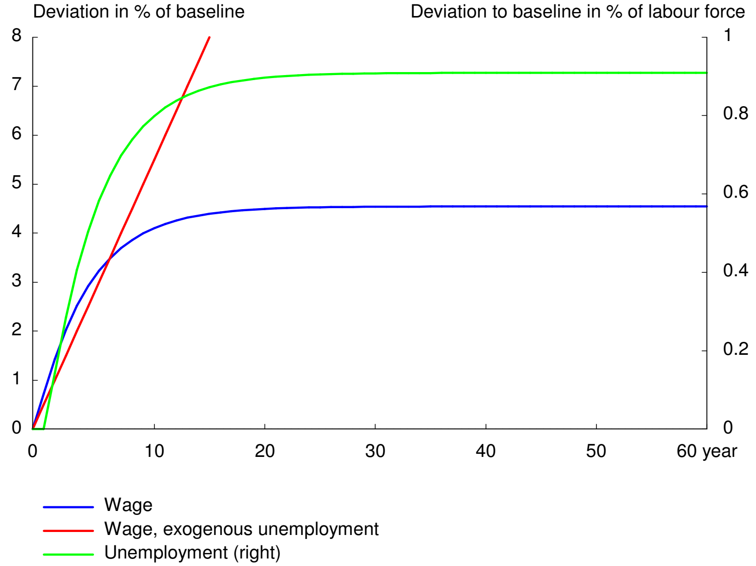
... and by a price equation

The price equation (7.3c) determines the price increase of Danish value added as wage growth minus productivity growth. Strictly speaking, we need a consumer price increase for the wage equation, and consumer prices in ADAM depend not only on wages and productivity but also on the price of imported goods. However, it makes no fundamental difference to include the import price in the analysis, so for simplicity, we use the price of value added instead of the consumer price. The stylized price equation does not describe the short term, but the equation holds in the long term, because ADAM produces a fixed long-term ratio between production price and production cost.

A permanent lift of the wage equation ...

There is a clear difference between looking at the wage equation alone or in a context with equations for unemployment and prices, and this difference is illustrated in figure 7.2. If unemployment and prices are exogenous and the wage equation is lifted permanently by adding 0.005 to the constant, the wage level will react by growing 0.5 per cent p.a. relative to its baseline. In the figure, this is illustrated by the red line that is cut off at 8 per cent higher wages, which is reached after 16 years.

Figure 7.2 **Increasing the constant in the wage equation by 0.5 per cent**



... increases long-term price level and unemployment

When the wage equation is supplemented by the proposed equations for unemployment and price increases in (7.3b) and (7.3c), wage and price levels find a new equilibrium, which is around 4.5 per cent higher than the baseline scenario, cf. the blue line in figure 7.2. In the new equilibrium, the wage increases are at the same rate as in the baseline, but unemployment has found a new and higher equilibrium that constitutes a new structural unemployment. The new equilibrium exceeds the baseline by around 0.9 per cent of the labour force, cf. the green curve. Thus, structural unemployment increases by 0.9 per cent of the labour force, and this happens because the wage equation coefficient for unemployment is -0.55, and -0.55 times 0.9 per cent balances the 0.5 per cent increase in the constant.

ADAM's wage equation error-corrects unemployment

The calculation illustrates that the crowding-out process makes ADAM's wage equation function as an error correction equation, which in the long term will adjust the unemployment rate to its equilibrium, while the wage level finds an equilibrium path parallel to its baseline scenario. The effect on the equilibrium for unemployment can basically be deducted from the coefficients in the wage equation, while the equilibrium level for the wage depends on coefficients in the equation that explains unemployment by the relative price between Denmark and competitors.

The calculation also illustrates that permanent shifts in the wage equation work as supply shocks with permanent effects on unemployment. It should be noted that an increase of the replacement ratio and hence of structural unemployment works like a higher constant in the wage equation.

A permanent change in inflation ...

With the estimated wage equation, a permanent change in the external inflation rate will also trigger a permanent change in unemployment. This effect of inflation arises, because the coefficient for the price increase is less than one. Thus, the wage equation does not correspond to a vertical Phillips curve, and if the Phillips curve is not vertical, a higher price increase can reduce long-term unemployment.

The question of non-vertical Phillips curves is analogous to the general question of adjusting behavioural equations, if the variables alter growth rates permanently, see box 3.1 on trend correction of the estimated consumption equation. The purpose of such a correction for change in growth trend is to preserve the long-term equilibrium of the equation.

... should not affect unemployment ...

It is obvious to consider such a correction in the wage equation, because the modest coefficient for the price increase must, as discussed, reflect the current fixed exchange rate regime. Abandoning the fixed exchange rate in order to adopt a regime of, e.g. creeping devaluation against the euro will no doubt affect the wage equation, and it is not likely that the long-term unemployment will be reduced by the higher inflation.

... so if inflation increases permanently ...

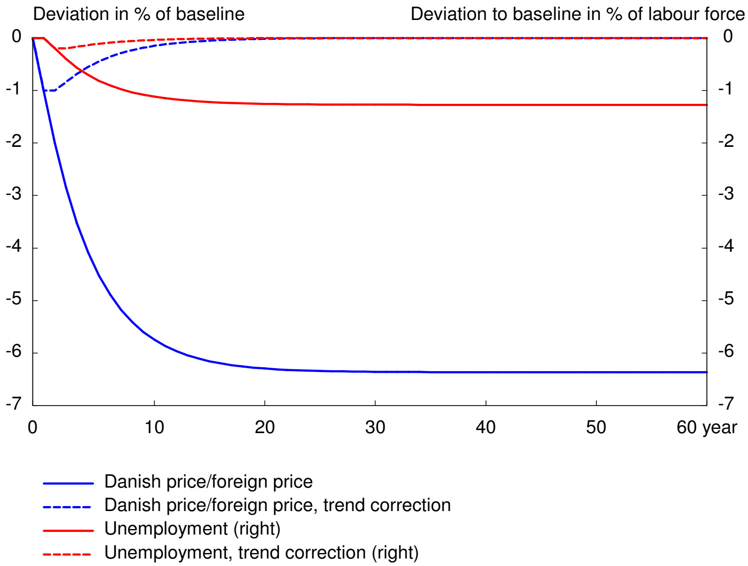
Based on the three equations (7.3a-7.3c) inflation can be raised by letting the foreign price in Danish currency increase by, e.g. 1 per cent more p.a. than it does in the baseline. With this assumption, the ratio between Danish and foreign price will start falling, which will increase the Danish market share and reduce unemployment. Lower unemployment raises the wage increase, and at some point, Danish prices will again be growing pari passu with foreign price.

The new equilibrium is characterized by an unemployment that is 1.27 per cent of the labour force lower than in the baseline scenario, while the Danish price has fallen relative to foreign price, see figure 7.3.

... wage equation constant should be increased

As already mentioned, unemployment is reduced permanently because the price increase enters wage equation (7.3a) with a coefficient of only 0.3. If the coefficient had been 1 instead, the one per cent higher price increase would have counterbalanced the one per cent higher wage increase on the left-hand side of the wage equation. Now, the coefficient for the price increase is only 0.3 but if the constant in wage equation (7.3a) is increased by 0.7 per cent, e.g. starting in year 2, the permanent effect on unemployment will disappear as it is expected to do in practice.[2]

Figure 7.3 **Unemployment and relative price, foreign price increase + 1% p.a.**



Effect of changes in productivity growth

There is usually less focus on the role of productivity than on the role of prices in the wage equation, partly because productivity growth cannot be controlled to the same degree as inflation. Productivity growth is, however, important for the basic wage formation, and higher productivity growth will permanently reduce unemployment in the three equations model (7.3a-7.3c), unless the wage equation is corrected for the new growth trend.

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[2] Correcting the constant term of the wage equation implies that the price-inflation term is perceived as a weighted average of Dlog(price) and a fixed inflation target of 2 per cent with the weights 0.3 and 0.7, i.e. 0.3·Dlog(price)+0.7·0.02. When the inflation target is increased by 1 per cent, the constant 0.02 is increased to 0.03.

An increase in productivity growth is beneficial to the economy because it makes real income grow faster. At the same time, the equilibrium rate of wage increase rises, and higher wage inflation will ceteris paribus make unemployment fall because the wage equation (7.3a) does not include productivity growth with a coefficient of 1.

If productivity growth increases permanently ...

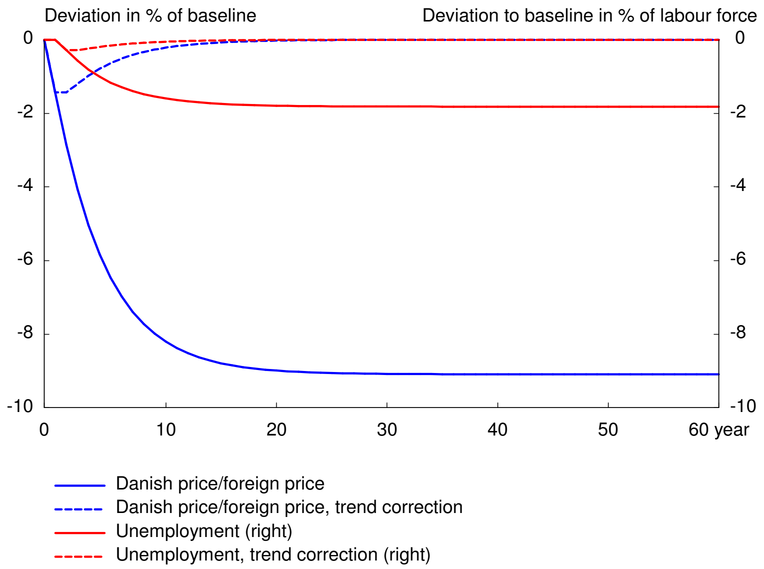
To illustrate the mechanism, we take again the three equations (7.3a-7.3c), where productivity growth is included in the price equation (7.3c). If, for instance, the annual productivity growth is set to increase by 1 percentage point, the price will start to grow 1 per cent slower. Thus, Danish price falls relative to foreign price, the market share increases and unemployment falls. Lower unemployment means higher wage growth, and when the annual wage growth has been raised by 1 percentage point, Danish and foreign price will again be growing at the same rate, and we have a new steady state.

The new equilibrium is characterized by a lower unemployment. The fall equals 1.8 per cent of the labour force, while the Danish price permanently drops by 9 per cent relative to foreign price, cf. figure 7.4.

... wage equation constant should be increased

Unemployment is reduced because the productivity increase is not included in the wage equation. If productivity growth were included with a coefficient of 1, because employees expect productivity growth to enter the wage increase, the higher productivity growth on the right-hand side of the wage equation would counterbalance the 1 per cent higher wage increase. Now, there is no productivity term in wage equation (7.3a), but if the wage equation constant is increased by 1 per cent, e.g. starting in year 2, the permanent effect on unemployment disappears, see figure 7.4.

Figure 7.4 **Unemployment and relative price, productivity growth + 1% p.a.**



7.3 Estimated wage equation

With ADAM's notation for the variables and augmented with a dummy, with the lagged wage acceleration and with the contemporaneous change in unemployment, equation (7.1) turns into (7.1*).

(7.1)*

$$\mathrm{Dlog}(lna) = \alpha_0 + \alpha_1 \cdot \mathrm{Dlog}(pcpn^{0.5} \cdot pyfbx^{0.5}) + \alpha_2 \cdot d8587 + \alpha_3 \cdot \mathrm{Dif}(\mathrm{Dlog}(lna1_{-1})) + \alpha_4 \cdot \mathrm{Dif}(bul) + \gamma \cdot (bul_{-1} - bulw_{-1})$$

lna	hourly wage in manufacturing
pcpn	consumer price excl. indirect taxes
pyfbx	value added deflator, private urban industries
d8587	dummy, -0.5 in 1985 and 1986, 1 in 1987,0 in other years
bul	unemployment rate
bulw	structural unemployment rate

The equation for structural unemployment rate (7.2) becomes (7.2*).

(7.2)*

$$bulw = \beta_0 + \beta_1 \cdot btyd$$

btyd unemployment benefit replacement ratio

Equation (7.2*) is inserted in (7.1), which is estimated using the least squares method. Estimated coefficients are shown in table 7.1.

The first explanatory variable in the table is the increase in the average of the applied consumer price and the value added deflator for private urban industries, i.e. manufacturing, construction and private services. It can be argued that both price indices affect wage formation, and it is chosen to use an average. The unrestricted-estimate of the coefficient for price increase is close to 0.1, but the coefficient is set to 0.3 in order to reduce the adjustment time in ADAM.

The second explanatory variable is a dummy, which is -0.5 in 1985 and 1986, 1 in 1987 and 0 otherwise, see the discussion of this period in relation to figure 7.1. This dummy removes an outlier in 1987 without significantly affecting the coefficients for the other explanatory variables. The third explanatory variable in the table is the lagged wage acceleration. Its positive coefficient implies that a higher wage increase stimulates the wage increase of the following year. It is difficult to give a specific reason for this mechanism, but the inclusion of the twice differenced wage in the estimated dynamics reflects that the wage increase is a sluggish variable.

The fourth explanatory variable is the change in unemployment, the coefficient is not significant but the variable is included anyway. The fifth explanatory variable is the difference between actual and structural unemployment. When actual unemployment is higher than structural, it dampens the wage increase. The corresponding coefficient, γ, is estimated to about -0.55, if the wage equation is estimated unrestricted. As already mentioned, the coefficient on the price increase is restricted to 0.3, and it is also chosen to restrict the coefficient on unemployment to -0.55. The simultaneous restrictions on the coefficient for the price increase and on the coefficient for unemployment are accepted.

Table 7.1 **Estimated wage equation**

Variable	ADAM-name		Coefficient	Std. Err.
Hourly wage	dlog(*lna*)			
Constant		α_0	-0.035	0.018
Price increase	dlog($pcpn^{0,5} \cdot pyfbx^{0,5}$)	α_4	0.3	-
Dummy for 1985-87	d8587	α_3	0.021	0.005
Hourly wage	Dif(Dlog(lna_{-1}))	α_1	0.320	0.091
Unemployment	dif(*bul*)	α_2	-0.250	0.134
Unemployment	bul_{-1}	γ	-0.55	-
Replacement ratio	$btyd_{-1}$	β_1	0.766	0.059
Constant in (7.2*)		β_0	-0.358	

Note: n=1983-2006 s=0.0059 R2=0.857 LM1=1.890 DF=1.57

The modest Dickey-Fuller statistic of 1.57 indicates that the unemployment rate and replacement ratio do not co-integrate. This suggests that long-term unemployment may depend on more than the replacement ratio, but as noted at the beginning of section 7.1, we have not found other explanatory variables.

Stylizing wage equation dynamics

The modest LM statistic suggests that the residual of the equation is free of auto-correlation. However, the estimated dynamics includes lagged wage acceleration, which can make the calculated wage increase fluctuate unintentionally, following a one-off shock to the wage equation. Thus, the estimated wage equation has a complicated dynamics, but for the model we would prefer a more simple and stylized dynamics.

If we estimate the equation with the dynamics restricted to be simple, it can affect the coefficients in a disadvantageous way and shift the steady state solution of the equation. In this situation, Baardsen and Nymoen (2008) propose to estimate the equation with an unrestricted and complicated dynamics, but to simplify the estimated dynamics when the equation is used in the model. The estimated dynamics must be simplified in a way that does not affect the steady state equation, and this is implemented in ADAM by exogenizing the variable with the wage acceleration.

Reducing the effect of the replacement ratio

The coefficient of 0.766 for the replacement ratio implies that long-term unemployment falls by 0.766 per cent of the labour force, if the replacement ratio falls by 1 percentage point. That sounds like a rather large effect, and it is likely that the estimated coefficient for the replacement ratio is artificially high because the fall in the replacement ratio over the past few decades have been accompanied by other tightening of labour market policies, such as a reduction of the period in which benefits can be claimed and additional activation measures for young unemployed persons. Thus, we assume that the real effect of changing the replacement ratio is less than the effect indicated by the estimated coefficient.

To address this issue in ADAM, we have duplicated the variable with the replacement ratio. The normal replacement ratio, *btyd*, has a coefficient of 0.1, which corresponds to a consensus estimate in the Labour Market Commission report, Arbejdsmarkedskommissionen (2009). The auxiliary variable, *btyde*, with the replacement ratio is exogenized and has a coefficient of 0.666, corresponding to the rest of the estimated coefficient. This means that changes in the benefit rate, i.e. the nominator of the replacement ratio, only affects long-term unemployment with a coefficient of 0.1.[3]

The short-run constant α_0 in the wage equation balances the effect of variables outside the error correction term, and this constant is adjusted when, for instance, the permanent effect on unemployment of higher inflation is eliminated; cf. the discussion in section 7.2 on wage equation properties.

7.4 Prices

While all producers in ADAM are assumed to be price-takers in the markets for factors of production, most of their production is sold on markets where producers are assumed to be price-setters. The price of production set by a given industry is determined on the basis of production costs, i.e. costs of: materials, energy, labour, machinery and buildings. In the long term, total unit cost passes through to the price.

The following relates to the determination of production prices. Prices of demand components are determined in technical price equations that weigh together production prices and import prices, as discussed in chapter 5 on the input-output system.

Price formation in ADAM makes ...

For that part of production, where producers are price-setters, price formation is assumed to take place under monopolistic competition. This means that producers in the long run set their production price to maximize profits. Demand for the production of an ADAM-industry reflects the price elasticities in, e.g. foreign trade and consumption system plus the breakdown of demand components on delivering industries.

... production price follow long-term unit cost

With the simplifying assumption of constant price elasticity in the industry-specific demand, the profit-maximizing price is given by a constant mark up on the long-term marginal cost. The latter corresponds to the long-term unit cost, since the production function has constant returns to scale in all industries.

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[3] The equation for long-term unemployment rate is $bulw = 0.666 \cdot btyde + 0.1 \cdot btyd - 0.358$. The first term, $0.666 \cdot btyde$, is seen as an exogenous background variable. Such variables are often represented by functions of time, e.g. the logistic trend in housing equations, but here it is assumed that the development in the replacement ratio represents the development in other labour market policy.

The mark up μ on long-term unit cost is a decreasing function of the numeric price elasticity ε of production demand, see the section on mark-up pricing in Varian (1999).[4]

$$1+\mu=\frac{1}{1-|\varepsilon|^{-1}}$$

In the borderline case where price elasticity is minus infinity corresponding to perfect competition, the mark up is 0. In that case, the profit-maximizing price equals the long-term unit cost, but in general, the price is above long-term unit cost.

The relationship between long-term price and the long-term unit cost is given by:

(7.4)
$$P_X^*=(1+\mu)\cdot\frac{L^*\cdot P_L+M^*\cdot P_M+E^*\cdot P_E+K^*\cdot P_K+B^*\cdot P_B+Spz}{X}$$

The notation is similar to what is used for presenting the factor block in chapter 6. That is, L is labour, E energy, M materials, K machinery, B buildings, X production, while P_i is the price of $i=X,L,M,E,K,B$. Spz is product-related taxes, and superscript * indicates long-term, i.e. the desired level.

The fraction in equation (7.4) represents the long-term unit cost. If long-term unit cost grows by 1 per cent p.a., the production price also ends up growing by 1 per cent p.a. Thus, in the long term inflation will correspond to the increase in unit cost, which reflects the increase of input prices, weighted by their respective cost shares.

As the long-term unit cost includes user cost on buildings and machinery, the price equation (7.4) implies that the production price is sensitive to the interest rate in the long term.

Furthermore, the constant mark-up of the price equation implies that profit per unit of output is proportional to the unit cost:

$$profit^*=\mu\cdot\frac{L^*\cdot P_L+M^*\cdot P_M+E^*\cdot P_E+K^*\cdot P_K+B^*\cdot P_B+Spz}{X}$$

profit profit per produced unit

At the same time, long-term unit cost does not depend directly on the production level, because the underlying production function has constant returns to scale.

---

[4] One plus markup, $1+\mu$, equals price above marginal cost in Varian.

The outlined price setting is estimated for five industries including the two major industries, manufacturing and private services, plus three industries, construction, energy supply and financial services. The production price in public services is by assumption set equal to the actual unit cost, and for the remaining six industries we have made the following assumptions:

Industries with special price formation

The production price in agriculture and in the food-processing industry is assumed to follow the world market and therefore tied to ADAM's agricultural export price. The price of sea transport is assumed exogenous and given from the world market. Prices of North Sea crude oil and of refinery production are also assumed to follow the world market, and the two production prices are tied to the import price of crude oil. The price of housing is a function of the price of housing investment.

It should be noted that the relation between production price and unit cost can fluctuate sharply in industries where the production price is determined on the world market, e.g. in extraction of hydrocarbons and in oil refineries. In these industries, profits react strongly when world prices fluctuate.

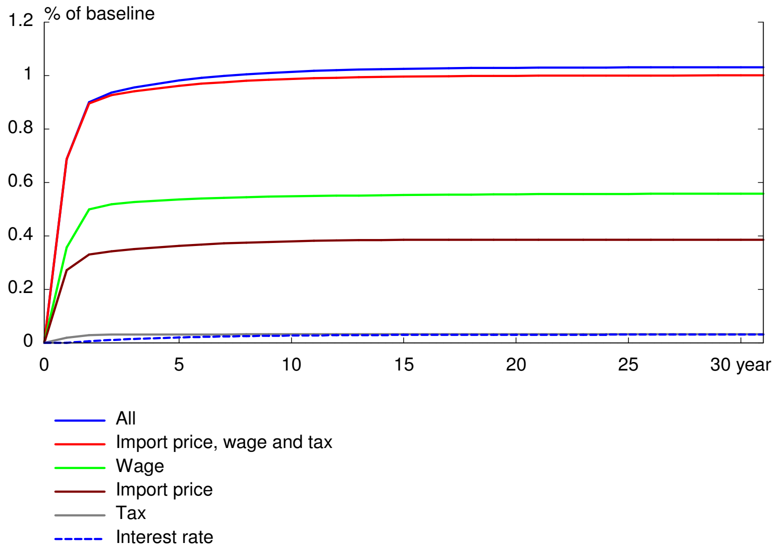
7.5 Price model properties

The flow of materials between ADAM's 12 industries and the domestic production of capital goods make most prices interdependent, because the production price of one industry enters the unit costs of the other industries. Independent shocks to production prices come from the primary factors of production, i.e. from changes in: the price of labour and the price of imports. There are also independent price shocks from changes in indirect tax rates and from changes in the interest rate entering the user cost of capital.

Illustrating price formation in production

To illustrate the price formation in ADAM's price-setting industries, we calculate the price effect of changes in wage, import price, tax and interest rate. The calculation applies a sub-model consisting of production price equations supplemented by auxiliary equations that make production prices interdependent by describing the flow of materials between industries. We calculate the effect on the average production price of 1 per cent increases in wage, import price, tax and interest rate, cf. figure 7.5.

Figure 7.5 **Production price effects, basic factor prices + 1%**



Production prices are homogeneous of degree 1 ...

The figure shows that the long-term wage elasticity of the production price is about 0.6, the long-term import price elasticity of the production price is 0.4, and the long-term tax elasticity of the production price is close to 0. The three elasticities sum to 1, reflecting that the production price model is long-term homogeneous of degree 1 in three prices, namely hourly wage, import price and tax per unit produced. If we also increase the interest rate, unit cost and production price will rise by slightly more than 1 per cent. This reflects that 1 per cent higher wage, import price and tax will make the price of capital goods and hence user cost increase by 1 per cent. Increasing the interest rate on top makes user cost increase by more than 1 per cent.

... and prices react relatively fast

Price adjustment is relatively fast because the effect of wage and material price is restricted to reach its equilibrium after two years. Price pass-through of user costs and production taxes come more gradually, as the actual production price error-corrects to desired price.

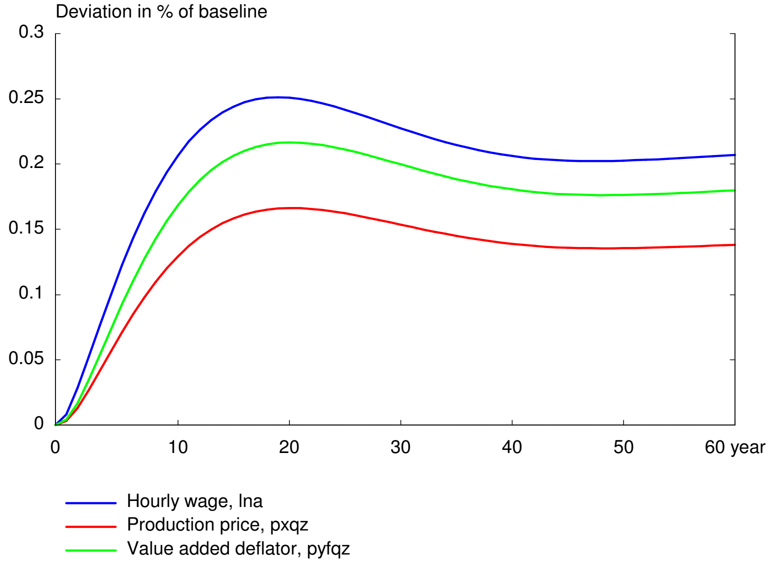
Expansive demand shock in ADAM

The stylized illustration in figure 7.5 relates to the production price model with exogenous wage and production. If, for instance, public purchases are increased by 1 per cent in the full ADAM model, the price reaction will primarily come via wages, while import prices remain unchanged.

Wage reacts more than production prices...

ADAM's wage variable responds to the fall in unemployment, and rising labour costs will pull production prices up. As the price of the imported materials are unchanged, the percentage long-term effect is lower for production prices than for the hourly wage, see figure 7.6a that exemplifies by showing the price effect in private services, *qz*.

Figure 7.6a **Prices in private services, public purchase + 1 per cent**



... and more than the price of value added

The price of value added in private services, *pyfqz* in ADAM, reacts more in the long term than the production price, but less than the wage. The price of value added depends not only on wage cost but also on the cost of capital, which rises less than wage cost because the import content in capital goods serves as a dead weight.

Desired wage cost per produced unit

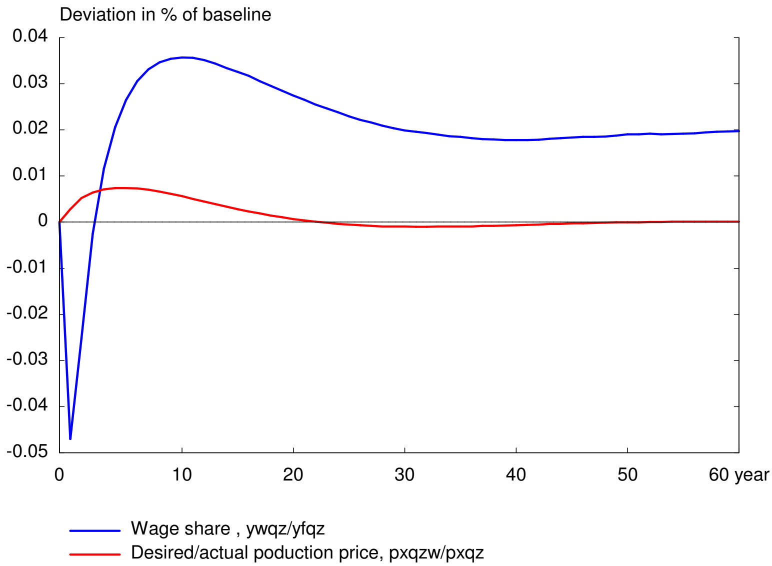
The full price pass-through of wage cost has been restricted to come within the first two years, and we note that the wage cost enters the restricted short-term dynamics in the form of desired wage cost per unit of production, i.e. as the desired hours worked L^* times hourly wage divided by output, $L^* \cdot P_L / X$.

It is decided to use the desired rather than the actual wage cost in order to avoid adverse cyclical effects in the production price. For example, ADAM normally indicates a drop in actual 'unit labour cost', $L \cdot P_L / X$, at the beginning of a cyclical upturn, because actual hours worked have a first-year production elasticity, which is less than 1. The desired hours worked have a production elasticity of 1 always, implying that the desired wage cost per unit of production increases slightly already in the first year because the hourly wage increases slightly.

Wage share falls in the short term ...

As mentioned, the sluggish response in labour input to a positive demand shock implies that L decreases relative to production X in the first year. In the short term, wage and price changes are too small to have any significant effect on the wage share, i.e. the compensation of employees as share of value added. What matters in the first year are volume changes, and when material inputs, M and E, have a first-year elasticity of 1 with respect to production, the compensation of employees will fall relative to value added in the first year.

Figure 7.6b **Wage share and desired/actual price, government purchase + 1%**



... and increases in the long term

The first-year decline in the wage share, $L{\cdot}P_L/(X{\cdot}P_x\text{-}M{\cdot}P_M\text{-}E{\cdot}P_E)$, is redressed in the following years, and the long-term effect on the wage share is positive, because the price of labour rises relative to the cost of capital, and the elasticity of substitution between the two factors is less than 1. The effect on the service industry's wage share is shown in figure 7.6b.

The long-term increase of the wage share is accompanied by a long-term fall in the share of capital cost in total cost. In the long term, the price model implies that the percentage effect on both the actual and desired production price will equal the percentage effect on total cost per unit of production, see figure 7.6b.

In the short term, the production price increases slower than the long term unit cost, which determines the desired price. Thus, in the first years, the desired production price increases relative to the actual production price, cf. figure 7.6b. At the same time, the actual production price increases relative to the actual unit cost, $(L{\cdot}P_L+M{\cdot}P_M+E{\cdot}P_E+K{\cdot}P_K+B{\cdot}P_B+Spz)/X$, which actually falls in the first year, when both labour and capital input fall relative to production.

7.6 Estimated price model

Production prices for price-setting industries are determined in error correction equations, where the price adjusts gradually to the desired production price:

(7.5)
$$\mathrm{Dlog}(px) = \alpha_1 \cdot \mathrm{Dlog}(pwvl) \cdot \frac{pwvl_{-1}}{px_{-1}} + \alpha_2 \cdot \mathrm{Dlog}(pwvl_{-1}) \cdot \frac{pwvl_{-1}}{px_{-1}}$$
$$+\alpha_3 \cdot \mathrm{Dlog}(pwvv) \cdot \frac{pwvv_{-1}}{px_{-1}} + \alpha_4 \cdot \mathrm{Dlog}(pwvv_{-1}) \cdot \frac{pwvv_{-1}}{px_{-1}}$$
$$-\gamma \cdot \left(\log(px_{-1}) - \log(pxw_{-1})\right) + \kappa$$

px	production price
pxw	desired production price
pwvl	desired unit labour cost
pwvv	desired unit cost, materials incl. energy

The equation applies to all price-setting industries. Variables have ADAM-names without industry-defining suffixes.

The desired production price *pxw* included in the error-correcting term corresponds to the right-hand side of the previously shown long-term cost equation (7.4). The two desired unit costs, *pwvl* for labour and *pwvv* for materials are defined in the following equations that apply the ADAM-notation without industry-suffix:

(7.6)
$$pwvl = \frac{l \cdot hqw}{fX} \qquad pwvv = \frac{pve \cdot fVew + pvm \cdot fVmw}{fX}$$

l	hourly wage
hqw	desired labour input
fX	production
pve	price of energy
fVew	desired energy input
pvm	price of materials
fVmw	desired input of materials

The short-term dynamics of the error correction equation (7.5) comprise only labour and material costs and not capital costs, which means that production price is insensitive to interest changes in the short term. In the short term, capital stock is assumed to serve as a fixed factor that does not affect the setting of prices.

As discussed in section 7.5 on price model properties, it is important to use the desired and not the actual unit cost of labour, if you want to avoid that the production price falls when the economy expands, and as indicated, we also use the desired unit costs of materials.

The desired unit labour cost is included in the price equation (7.5) with contemporaneous and 1-year lagged differences, and both differences are multiplied by the lagged labour cost share. The sum of the corresponding coefficients α_1 and α_2 is restricted to 1. This implies that the second year's price elasticity of labour cost equals the cost share of labour, which is also the long-term price elasticity of labour cost.

The desired unit cost for materials enters the price equation in the same way as unit labour cost, and the sum of the corresponding coefficients α_3 and α_4, are restricted to 1 similar to α_1 and α_2. This allows material cost to pass through to the production price within two years.

The price equation (7.5) is estimated for production prices in the five price-setting industries: Construction (*pxb*), manufacturing (*pxnz*), energy supply (*pxne*), private services (*pxqz*) and financial services (*pxqf*). Estimation results are shown in table 7.2.

Table 7.2 **Estimation result for price equations**

	α_1 Wage 1. year	α_2 Wage 2. year	α_3 Materials 1. year	α_4 Materials 2. year	γ Adjustment	κ Constant
B	0.81730	0.18270	0.95475	0.04525	0.20000	-0.00410
Nz	0.73267	0.26733	0.89554	0.10446	0.20000	0.00265
Ne	1.00000	0	1.00000	0	0.20000	0.02848
Qz	0.64666	0.35334	0.88594	0.11406	0.20000	0.00509
Qf	1.00000	0	1.00000	0	0.20000	0.04613

The first two columns with the heading 'Wage' show the first and second year's coefficient to the change in the desired unit cost of labour. As mentioned above, the coefficient sum $\alpha_1+\alpha_2$, is restricted to 1, and the same goes for $\alpha_3+\alpha_4$, related to the change in material costs. In addition, error correction parameter γ is set to 0.2. Estimated unrestrictedly, the error correction parameter is typically small and insignificant, and the restriction to 0.2 increases the speed of adaptation.

The estimated constant κ cannot be interpreted as the price mark-up introduced in the long-term price equation (7.4), because the constant also reflects the short-term dynamics in the estimated price equation. More generally, the proportionality factor between production price and unit cost reflects not only the theoretical relationship between mark up and price elasticity but also some measurement problems with both production and cost data.

Summary of chapter 7

ADAM's Phillips curve equation makes the rise in wages depend on the economic cycle, and the Danish wage increases more than the foreign wage, when unemployment is below its long-term equilibrium, and vice versa if unemployment is above. The relative price of Danish and foreign labour has an effect on the competitiveness and market shares, and the wage formation will thus push unemployment towards its equilibrium.

In price-setting industries, the production price is mainly determined by labour cost and import price, and in the longer term the production price also depends on the cost of capital. In industries such as agriculture, oil and sea transport the production price follows an exogenous world market price.

8. Public finances

Describing public finances is of great importance in ADAM. This reflects two things: the public sector constitutes a significant share of the Danish economy and ADAM is used to prepare and analyse fiscal policy. It is, therefore, important that the model is suitable for quantifying the interaction between public finances and the rest of the economy

Public employment represents around 30 per cent of total employment, and public value added over 20 per cent of total value added. The public production is used for public consumption, and a change in public consumption of 1 per cent will in the short term change the size of GDP by about ¼ per cent. In the long term, it is less the size and more the composition of GDP that changes.

Public consumption is provided to the private sector. At the same time, the public sector collects taxes and affects the private income distribution, both through the progression of the tax scale and by using a variety of income transfers to households. The public sector also gives subsidies to selected products or industries. Tax revenues come from both the direct taxation of private sector income and from indirect taxes that increase the prices of consumer goods.

Public expenditures and revenues are used to achieve a number of policy goals with regard to welfare and distribution, education, environment, infrastructure, growth, economic stability etc. Those goals must all be set under the condition that the overall fiscal policy is sustainable and ensures a stable development of public debt.

The following section provides an overview of ADAM's handling of public expenditures and revenues. It is followed by a section on public consumption and investment, a section on income transfers, a section on direct taxes and finally a section on indirect taxes.

8.1 Modelling of public finances

In public finances, we have three major categories to model: Public demand, i.e. consumption and investment, public transfer payments and tax revenues.

Public consumption

Consumption constitutes by far the largest share of public demand, and most of the public consumption is compensation of public employees. Leaving aside the rest, public consumption in current prices can be written as wage multiplied by employment:

(8.1) $$C_o = l_o \cdot Q_o$$

where C_o is public consumption, Q_o public employment, and l_o average annual wage. Public employment is an exogenous variable, while public wage is endogenous and follows private wage. The rest of public consumption and public investment is also modelled as a product of an exogenous volume and an endogenous price.

Transfer payments

The basic equation for modelling transfers is equally simple:

$$T = t_t \cdot U \qquad (8.2)$$

where T is the transfer payment, U is number of recipients of the allowance, and t_t is the allowance rate. The number of recipients is often exogenous or demographically determined but, for example, the number of recipients of unemployment benefit follows the number of unemployed, which also affects the number of people in public activation schemes. The allowance rate is modelled as an exogenous rate multiplied by an endogenous price or wage index. The latter index reflects the annual rate adjustment, which is also modelled in ADAM.

Taxes

On the revenue side, the basic equation for tax revenues is given by:

$$S = t_s \cdot Y \qquad (8.3)$$

where S is tax yield, Y represents tax base, and t_s is tax rate. The tax base is usually endogenous, while the rate is exogenous. The basic relation in (8.3) can be used for direct taxes, indirect taxes as well as for subsidies. Subsidies are treated as negative indirect taxes, see section 8.5 below.

The chapter deals with 90 per cent of public expenditures and revenues

Public consumption, public investment, transfer payments and subsidies constitute about 90 per cent of total public expenditures. While direct plus indirect taxes constitute about 90 per cent of total revenues. The expense and revenue categories covered by the chapter are shown in table 8.1.

The present chapter does not deal with, e.g. public interest payments, which are determined in the financial sub-model. Both interest payments and financial sub-model are discussed in chapter 9, where table 9.1 indicates all categories of public sector expenditure and revenue.

Table 8.1 **Public sector finances, selected items**

	ADAM-name	2000	2006	2009	2009
		DKK bn.			% of GDP
Public consumption	*Co*	325.1	422.6	496.4	32.1
Public investment	*Io*	21.6	31.7	33.4	2.2
Transfer payments	*Ty_o*	212.6	254.7	284.3	18.4
Subsidies	*-Spu_o*	30.9	34.8	43.4	2.6
Total expenditure	*Tf_o_z*	687.5	826.1	953.6	61.7
Tax on income and wealth	*Sy_o*	391.7	497.0	499.5	32.3
Tax on production and import	*Spt_o*	220.0	291.3	279.2	16.9
Other taxes	*Sa*	26.1	20.4	20.1	1.3
Total revenue	*Tf_z_o*	715.5	908.4	906.7	58.7

Detailed determination of public expenditure and revenue

For each expenditure or revenue item, ADAM's equation follows the principle of one of the three basic equations in (8.1) - (8.3), but the specific equation often expands the basic form. The breakdown of public finances is comparatively detailed, so it is often possible to include the actual rates in the model equation for an expenditure or revenue item. ADAM includes, for example, actual tax rates and brackets from the personal income tax scale, and the standard regulation of brackets is modelled.

Impact of rate-regulation

The standard regulation of income brackets and allowance rates etc. is instrumental in maintaining the relation between income and tax revenue as well as the relation between wage and transfer income. It may be added that the regulation of income brackets and that of allowance rates do not correspond entirely. Normally, 0.3 percentage points of the allowance rate regulation goes to various social measures so that transfer payments are adjusted slightly less than income brackets.

Fiscal instruments are exogenous

ADAM's equations make public expenditures and revenues endogenous, but the model has no fiscal reaction function, implying that fiscal policy is exogenous. Fiscal instruments include, in particular, public employment, purchases of goods and investment in real terms plus the exogenous rates of public transfers, income taxes and duties.

Public finances are cyclical

Endogenous public expenditures and revenues are often cyclical. An economic downturn will increase the expenditures for unemployment benefits and activation schemes, while the tax base and tax revenues fall. An economic upturn will have the opposite effect. Consequently, the public budget balance tends to be negative in periods of weak growth and positive in periods of strong growth.

The interaction of public finances with the economic cycle goes in both directions. While the business cycle affects public finances, public finances affect the business cycle.

The interaction can be illustrated by a calculation on ADAM. For example, we can give a temporary negative shock to the export market and weaken Danish exports. The decline in export dampens production and employment and increases unemployment. Tax revenues will fall and unemployment benefits increase, deteriorating the public budget balance.

At the same time, the increase in unemployment benefits to households partly offsets the decline in wage income. If we exogenize unemployment benefits, *Tyd* in ADAM, by dropping its equation, the fall in households' disposable income will be larger, see figure 8.1a.

Figure 8.1a **Effect on disposable income, *Ydk_h*, temporary 1% fall in export market,**

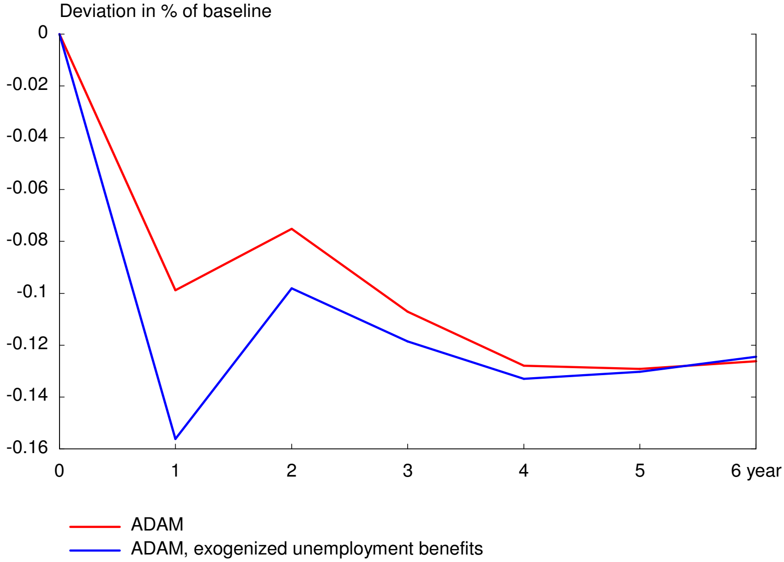
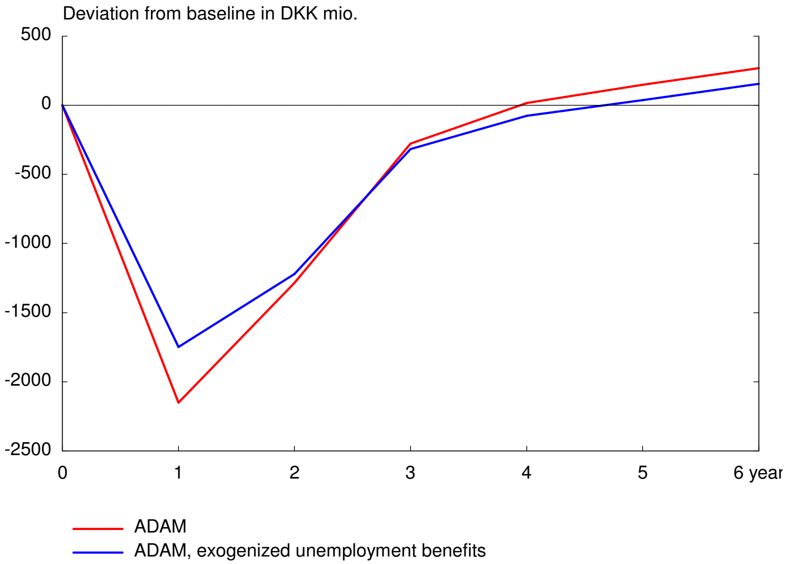


Figure 8.1b **Public budget balance, *Tfn_o, t*emporary 1% fall in export market**



The calculation illustrates that having an unemployment benefit system dampens the economic fluctuations in the private sector, while the public budget balance fluctuates more, see figure 8.1b. Unemployment benefits and, e.g. income taxes are important examples of automatic stabilizers.

8.2 Public consumption and public investments

Public consumption, *Co*, is the largest single item in public expenditures and it constitutes a major component of domestic demand.

Public consumption affects economic activity directly and is central to economic policy. The modelling of public consumption is basically simple, but some of the simplifying assumptions can be changed.

Consumption is supply-determined

In ADAM, public consumption is supply-determined. The central exogenous variable, and thus the central policy instrument, is public employment. This means employment in the industry, which mainly produces public services. It should be noted that this industry of public services, i.e. industry *o* in ADAM, does not include public corporations, e.g. the national railway company (DSB). The definition of public sector, which differs from public industry, is described in chapter 9.

Compensation of employees and imputed depreciations determine together gross value added, *Yfo*, in the public industry, because imputed depreciations is the only profit income in the public sector

(8.4) $$Yfo = wagerate \cdot Qo + Ivo + Spz\_xo$$

Qo is public employment, *Ivo* is public profit income, i.e. depreciation, and *Spz_xo* is other production taxes on public industry. The first additive term in the equation is the industry's payroll. The applied wage rate is affected by the composition of high- and low-paid public employees. Basically, the public wage follows private wage, but the public wage equation can be adjusted to give public wage another development.

Value added is cost-determined

A similar equation determines the gross value added in constant prices, *fYfo*. It is noted that when public value added is cost-determined, value added at constant prices cannot be used in measuring productivity. Public value added per employed, *fYfo/Qo*, primarily reflects the composition of employment, implying that an increase in the share of high-paid employed will appear as a productivity increase.

Input of materials controlled by model user

To get from value added to the production of public services, input of materials *Vo* must be added. This input is also referred to as “public purchase of goods” from private industries and from abroad. Basically, the input of materials follows gross value added, cf. equation (8.5) below, but the model user can adjust the proportional relation and change materials input relative to value added. Production *Xo* is determined in (8.6).

(8.5) $$fVo = kfvo \cdot fYfo$$

(8.6) $$Xo \ = \ Yfo \ + \ pvo \cdot fVo$$

The price of materials, *pvo*, is determined by the prices of production and imports, which enter the materials input in the public industry, see the model's input-output-based price determination, which is discussed in Chapter 5.

Public production is mainly used for public consumption

Production of public services is mainly supplied to public consumption. A smaller share goes to other demand components, such as private consumption of services, e.g. day-care services. The supply for public consumption is residually determined when the supply for other uses is deducted from public production. Supplies of public production are determined in ADAM's input-output model and volume identities, see chapter 5.

Also supplies from other industries

Not all of public consumption comes from the public services industry, for example, general practitioners are located in private services, *qz*. The private supplies for public consumption are made proportional to the rest of public consumption, but the model user can adjust the proportionality factor. When supplies from other industries are determined, the total public consumption at current prices can be determined:

(8.7) $Co = Xo$
- sales from industry o to private consumption
+ supplies from other industries to Co

Similar equations apply to public consumption in fixed prices, *fCo*.

Public investments

Public investments, *Io*, are divided into construction and other investments, and both categories are exogenous in real terms. The corresponding prices are determined by the prices of production and import included in investments.

It should be noted that many large public investments are neither made in ADAM's public sector nor in ADAM's public industry, but in publicly owned corporations, typically dealing with energy or transport, such as the national railway company DSB that is part of the *qz* industry.

8.3 Transfer payments

After public consumption, transfer payments represent the largest item of public expenditure.

Income transfers and recipients

It has always been crucial that ADAM can determine the cyclical income transfers, which depends on the number of unemployed. In recent years, it has also become important to describe how income transfers depend on demographics, cf. the discussion of withdrawal from the labour market.

The relation between number of recipients and total income transfer is illustrated for, respectively, early retirement benefits and old-age pensions in figures 8.2a and 8.2b.

Figure 8.2a **Early retirement, transfer and recipients**

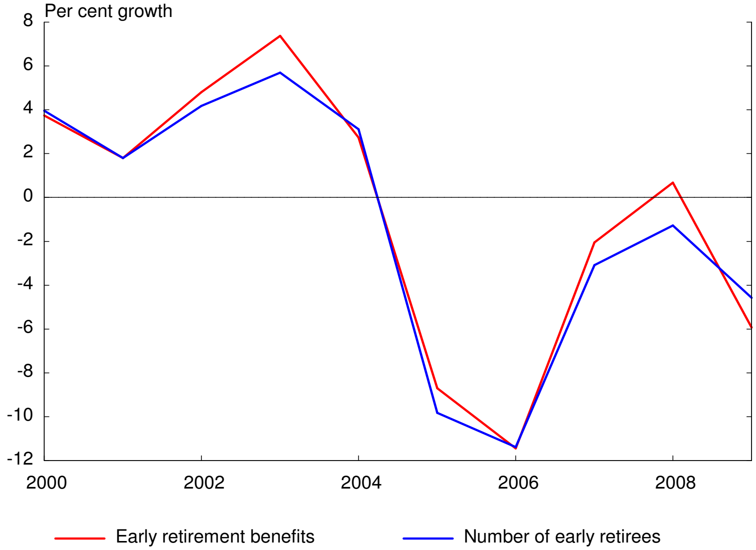


Figure 8.2b **Old-age pension, transfer and pensioners**

The sharp decline in early retirement recipients in the period 2005-2007 and the simultaneous sharp increase in old-age pensioners reflect the reduction of pension age from 67 to 65 over the same period.

The relationship between demographics and the number of benefit recipients is not modelled in ADAM but determined in the demographic auxiliary model Uadam, discussed in section 6.5 on the labour force.

Income transfer is a function of recipients, allowance rates and rate regulation

Most income transfers can be modelled as a product of a number of recipients and an allowance rate. The latter is split into a rate in real terms and a rate-regulating index, which follows the wage in the private sector. ADAM names for social transfers begin with Ty and have a suffix indicating which kind of allowance. For example, *Tyd* is unemployment benefits and *Typfp* is old-age pension. The equation for a transfer with suffix i, Ty_i, can be written as:

$$Ty_i = U_i \cdot tty_i \cdot ptty_i \quad (8.8)$$

Ty_i	income transfer
U_i	transfer recipients (base of transfer)
tty_i	real allowance rate (nominal rate deflated by $ptty_i$)
$ptty_i$	regulating index

Regulating index normally reflects wage

The regulating index $ptty_i$ reflects the formal rules for rate regulation. Most transfers are regulated by the private wage with a lag of 2 years. Some transfers are linked to a consumer price index.

The average real allowance rate, tty_i, constitutes a fiscal instrument. For example, a change in the replacement ratio of unemployment benefits can be implemented via a change in the related rate.

Allowance rate in real terms

Developments in tty_i reflect developments in the statutory allowance rate, before the rate is adjusted by $ptty_i$. In addition, the rate, tty_i will capture effects not covered by the specification. For example, measurement errors or data breaks can affect the implicitly calculated allowance rate, tty_i.

Table 8.2 summarizes the public transfer payments in ADAM. In the table, transfer variables are ranked starting with the most labour-market related benefits.

Most benefits reflect the demographics

Maternity allowance, family allowance, students grant, and the various retirement benefits constitute 85-90 per cent of all income transfers, and these transfers are entirely or predominantly determined by the demographic development.

10 per cent of the benefits are cyclical

Unemployment benefit, activation allowance etc. plus a share of social benefits amount to about 10 per cent of all public income transfers to households, and these transfers are to a large degree determined by the business cycle.

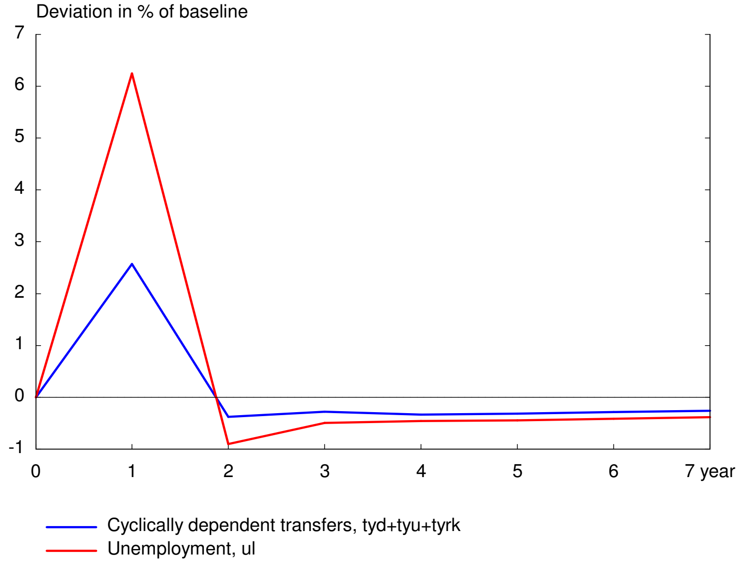
Table 8.2 **Public income transfers to households**

	ADAM-name	2000	2006	2009
		DKK bn.		
Unemployment benefit	*Tyd*	16	15	15
Entering labour force	*Tyu*	17	18	19
Temporary benefit and activation	*Tyuly+Tyuak*	4	4	4
Rehabilitation	*Tyurev*	5	4	3
Study grant	*Tyusu*	8	11	12
Temporarily out of labour force	*Tym*	17	22	26
Holiday allowance	*Tymfdp*	1	1	1
Leave, maternity, sickness	*Tyms+Tymb+Tymf*	16	21	25
Retirement	*Typ*	121	150	171
Old-age pension	*Typfp*	55	75	87
Invalidity pension	*Typfo*	28	34	38
Early retirement etc.	*Typef+Typov+Typfy*	21	21	21
Other pensions	*Typr*	16	21	24
Other transfers	*Tyr*	41	49	54
General social benefits	*Tyrk*	12	13	13
Housing benefit	*Tyrh*	9	11	12
Family allowance etc.	*Tyrbf+Trygc+Tyrr*	21	25	30
Total income transfers	*Ty_o*	213	255	284

Response of income transfers to unemployment

The impact of unemployment on income transfers can be illustrated by an ADAM-calculation on a temporary change in unemployment. The change in unemployment is created by a temporary increase in the labour force by 10,000 people in a single year.

Figure 8.3 **Income transfers, unemployment, labour force + 10,000 temporarily**



The one-off shock to labour supply increases the number of unemployed and the size of the cyclically dependent income transfers. Cyclical income transfers comprise unemployment benefits for the insured, *Tyd*, activation allowances and other benefits for those entering the labour market, *Tyu*, plus general social benefits, *Tyrk*, which includes unemployment benefits for the uninsured. The shock is temporary and in subsequent years both unemployment and transfers fall back and remains slightly below the baseline for a number of years, because the first year's high unemployment reduces wages and triggers a small positive effect on competitiveness and employment. Effects on unemployment and cyclical transfer payments are illustrated in figure 8.3.

8.4 Taxes on income and wealth

Direct taxes are a major revenue item in the government budget and important for private consumption as they reduce private disposable income.[1]

In ADAM, income taxes are broken down by components that relate to the annual fiscal Budget.

Table 8.3 **Direct taxes, overview**

	ADAM-name	2000	2006	2009
		DKK bn.		
Total personal PAYE tax	*Syk*	267.3	325.7	332.6
of which:				
- personal income tax	*Ssys+Ssysp*	249.0	296.4	309.0
- dwelling value tax	*Ssyej*	8.4	11.3	12.3
- share income tax	*Ssya*	6.7	12.3	8.7
- corporation tax scheme	*Ssyv*	1.8	3.7	1.2
- estate of deceased tax	*Ssyd*	1.3	1.9	1.4
Labour market contribution	*Sya*	56.7	71.6	79.7
Other personal taxes	*Syp*	7.6	8.5	29.7
Vehicle weight duty, households	*Syv*	5.5	7.6	7.9
Corporate tax	*Sy_c*	42.3	71.1	39.6
Pension yield tax	*Sywp*	12.4	12.5	10.0
Total direct taxes	*Sy_o*	391.7	497.0	499.5

The simple basic equation for tax revenues is as previously shown:

(8.9) $$S = t_s \cdot Y$$

where S is tax yield, Y is tax base and t_s is the tax rate in the basic equation. The equation for the yield of labour market contributions, *Sya*, is similar to (8.9).

(8.10) $$Sya = tsya \cdot Ysya \cdot ksya$$

The social contribution rate, *tsya*, equals the statutory rate, which has been 8 per cent since 1997. The formal income basis for the contribu-

[1] Direct taxes are used as a synonym for the national accounts taxes on income and wealth.

tion is a tax-related definition of earned income, which differs from ADAM's national accounts-related measure of earned income, *Ysya* that follows from ADAM national accounts variables. To determine the correct labour market contribution, ADAM's earned income, *Ysya*, is multiplied by a proportionality factor *ksya*, which is residually defined to make the product of the two equal the tax-related earned income. Several tax yield equations have the same simple form; e.g. equations for property value tax, share income tax, corporation tax scheme, and vehicle weight tax.

Revenues from personal income tax

Most PAYE tax revenues come from the personal income tax. Income tax on individuals is progressive and associated with two concepts of income: personal income and taxable income. In 2009, central government income tax had formally three progressive brackets: lower, medium and upper limit. Lower limit tax is levied on personal income exceeding the lowest income bracket called personal allowance, while medium and upper limit tax is levied on personal income exceeding the related income brackets. Municipal and church tax plus healthcare contribution are levied on taxable income exceeding personal allowance.

In ADAM, the personal income tax yield of 2009 can be divided into the above-mentioned five categories of tax summarized in table 8.4, which also shows the related income brackets and tax rates. Tax rates are official statutory tax rates, except the rate for municipal and church tax, which is a weighted national average.

Table 8.4 **Personal income tax, 2009**

Tax category	Income base	Income-limit	Tax rate	Yield
		DKK	Per cent	DKK bn.
Lower limit tax	*Ysp*	42,900	5.0 (*tsysp1*)	44.3 (*Ssysp1*)
Medium limit tax	*Ysp*	347,200	6.0 (*tsysp2*)	5.2 (*Ssysp2*)
Upper limit tax	*Ysp*	347,200	15.0 (*tsysp3*)	17.6 (*Ssysp3*)
Municipality and church tax	*Ys*	42,900	25.7 (*tsys1*)	184.3 (*Ssys1*)
Healthcare contribution	*Ys*	42,900	8.0 (*tsys2*)	57.5 (*Ssys2*)

Lower limit tax as example

The five tax categories are basically determined in the same way, so we can take the lower limit tax as an example. Lower limit tax yield, Ssysp1, is written as the product of rate and base:

(8.11) $$Ssysp1 = tsysp1 \cdot Ysp1$$

where the exogenous rate, *tsysp1*, is the official lower-limit tax rate in table 8.4, while the endogenous tax base, *Ysp1* requires a special calculation. First, ADAM determines total tax-related personal income, *Ysp*, which is the relevant income concept, and second, ADAM determines the share of *Ysp* taxed by the lower-limit rate.

Determining personal income

Tax-related personal income can be determined as the sum of: A-income (for wage and transfer recipients), profit of self-employed persons and other personal income, minus deduction for pension contributions etc., see table 8.5, which resembles the layout known from the annual income tax form. The table also illustrates that the so-called taxable income used to determine municipal tax is obtained by adding capital income and subtracting the employment allowance and other income deductions. The necessary tax-related income items are in ADAM determined by multiplying the closest corresponding national-accounting income item by a residually determined proportionality factor.

Table 8.5 **Personal and taxable income, 2009**

Income items	ADAM-name	DKK bn.
1. A-income etc.	*Yas*	981.9
2. Profit of self-employed etc.	*Yrpss*	49.3
3. Deduction for pension contributions etc.	*Tops+Syas*	88.4
4. Other personal income	*Ysprs*	16.3
5. Personal income (1+2-3+4)	*Ysp*	959.1
6. Interest income, net	*Tippps*	-48.7
7. Income deductions	*Ylws*	52.7
8. Other taxable income, net	*Ysrs*	-5.7
9. Taxable income (5+6-7+8)	*Ys*	852.0

Lower-limit tax base as share of personal income

After determining total personal income, *Ysp*, the next step is to include an assumption on the income distribution. The point of departure is the latest actual income distribution, which we use to calculate the share of *Ysp* taxed by the lower-limit rate. This share is called *bysp10*, and if income distribution by taxpayers remains unchanged; and if personal income grows pari passu with the regulation of the lower-limit income bracket, the lower-limit taxed share will remain *bysp10*.

In forecasts and model experiments, the overall development in personal income can easily deviate from the development in the lower-limit income bracket, which is normally referred to as the personal allowance. Consequently, the taxed share of personal income will change, but on the assumption of unchanged income distribution we can calculate the change in lower-limit taxed income share.

Equation for lower-limit tax base

The assumption about the sensitivity of the tax base is included in the equation that determines the lower-limit tax base, *Ysp1*:

$$Ysp1 = (bysp10 + 100 \cdot bysp11 \cdot kbysp1) \cdot Ysp \qquad (8.12)$$

kbysp1	difference in development of *Ysp* and personal allowance
bysp10	taxed share of *Ysp*, if *kbysp1* is 0
bysp11	change in taxed-share, if *kbysp1* is 1 per cent
Ysp	personal income

The equation formulates the tax base as a share of total personal income, and the share is a linear function of the difference between developments in personal income and personal allowance. If income and allowance have the same development, variable *kbysp1* is zero and the lower-limit taxed share of *Ysp* is *bysp10*. If *kbysp1* differs from zero, the taxed share of *Ysp* changes by *bysp11·kbysp1*.

The tax-share variables, *bysp10* and *bysp11*, in (8.12) are exogenous in ADAM, and both variables are calculated in ADAM's auxiliary model PSkat that relates to income distributions, tax brackets and tax bases. PSkat can also be used to analyse the effect of changing the tax rules, see box 8.1.

Notation and number of income-tax variables

As an example of the modelling of income tax on persons, we have used the lower limit tax. The yield of this tax is called *Ssysp1* in ADAM. Letter *p* stands for personal income, and number 1 indicates that the related income bracket, i.e. the personal allowance, is the smallest for the three central government taxes: lower, medium and upper limit. Medium-limit tax is called *Ssysp2* and upper limit tax is called *Ssysp3*, while municipal and church tax, which is not levied on personal but on taxable income, is called *Ssys1*. If two taxes on same income concept concern the same income bracket, which is the case for medium and upper limit tax in 2009, the numbering of the two will obviously be less important, but it facilitates, in general, the modelling that the ADAM name indicates the size of the income bracket.

For each of the two tax bases there are ADAM variables for 5 tax-categories, as the numerical suffix goes from 1 to 5. Thus, we have variables for a total of 10 personal income taxes, and in 2009, half of the 10 tax variables are superfluous and set to zero. However, we need more than 5 personal income tax variables to describe the historical period, and more than 5 variables may be needed in future years.

The income tax system is characterized by frequent changes, and it makes ADAM more robust to such changes that there are variables for 10 personal income tax categories and that tax categories are sorted out according to the base concept and size of income bracket, and not according to the official name. Changes in the tax system can, for instance, place an official tax category in different ADAM-variables in different years.

ADAM's tax calculation can be aggregated and disaggregated

ADAM's determination of personal income tax makes it possible to aggregate into fewer tax categories than, e.g. the five categories of 2009, and it is also possible to disaggregate and determine the tax revenues from a number of socio-economic segments in the population.

Aggregated tax calculation

The tax-aggregation option weighs together the five tax categories of 2009 into two: Tax on personal income and tax on taxable income. The two aggregate tax equations focus on the average tax rate on a macro level. For example, the aggregate equation for taxes on personal income is written:

$$Ssysp = (tssp0 + tssp1 \cdot kbysp) \cdot Ysp$$

where *tssp0* is a baseline estimate of the average tax rate, i.e. the ratio of tax, *Ssyp*, over income, *Ysp*. The baseline estimate for the tax rate is based on an assumed development in income and calculated using the auxiliary model Pskat.

The variable *kbysp* is zero if the income development generated in ADAM corresponds to the baseline used in PSkat. The variable *tssp1* indicates by how much the tax rate increases if *kbysp* is 1 per cent. The aggregate equation for tax on taxable income, *Ys*, is similar to the equation for tax on personal income.

Disaggregated tax calculation

The tax disaggregation option distributes the tax yield on six socioeconomic groups: Self-employed, employed, unemployed, early retirees, old-age pensioners and others.[2]

Disaggregating the tax yield on socio-economic groups makes it possible to analyse revenue effects of, e.g. changes in retirement rules or changes in the pension system. The disaggregation also makes it possible to estimate how a tax change will affect the various groups.

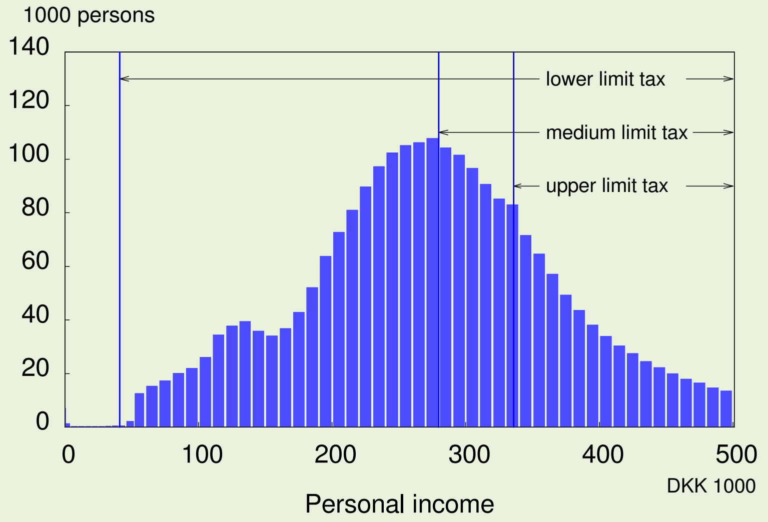
Box 8.1 **Income, tax brackets and tax base (PSkat)**

The auxiliary model PSkat determines the coefficients with prefix *bys* which is used in (8.12) to determine the tax base.

The starting point for PSkat-calculations is detailed information on income distribution and tax for taxpayers living in Denmark. The source is the tax-related part of official income statistics. PSkat works with both the income distribution for all taxpayers and the income distribution for the six socio-economic groups: Self-employed incl. assisting spouses, employees, unemployed, early retirement benefit recipients, old-age pensioners, and other taxpayers, which include invalidity pensioners.

The box figure shows the distribution of the personal income of employees in 2008. Vertical lines indicate the three central government tax brackets, and the figure illustrates the share of total income in each bracket.

[2] The disaggregation implies that the 5 categories of personal income tax in 2009 will become 30 categories of income tax, so we need 30 equations as in (8.11) to determine yields and 30 equations as in (8.12) to determine income shares.



The same illustration can be made for taxable income, where tax categories are municipal and church tax plus healthcare contributions, and of course also for the other five socio-economic groups and for taxpayers as a whole. The taxed share of income is called *bysp10* for the lower limit tax and *bys10* for the municipal and church tax.

Based on the latest income distribution, Pskat also calculates the effect on taxed income share, if there is a marginal change in income per taxpayer for fixed brackets in the tax system. It should be noted that reducing the income bracket by 1 per cent has the same effect on the taxed income share as increasing the income by 1 per cent. Thus, the effect on the taxed income share depends on the relative change in income and income bracket. The effect of 1 per cent additional income on the lower-limit taxed income share is called *bysp11*, and in equation (8.12) determining the tax base, the taxed income share is approximated by a first order polynomial in the difference between income and tax bracket development.

Among the other direct taxes, corporate tax and tax on pension returns are the two largest, and both fluctuate significantly from year to year.

The general corporate tax

It has proved useful to divide the yield of corporate tax called *Sy_c* into three parts, so that the hydrocarbon tax, *Sy_ck*, is determined in a special equation, while the general corporate tax on financial corporations, Sy_cf, and on other corporations, *Sy_cr*, is determined by the same type of equation. The equation for tax on financial corporations is:

(8.13)
$$Sy\_cf = ksy\_cf \cdot [tsy\_c \cdot (bsy\_c + ktsy\_c \cdot (1 - bsy\_c))] \cdot Ysy\_cf_{-1/2}$$

where the tax rate is represented by the expression within square brackets. The tax-rate expression models the transition between different corporate tax systems, cf. box 8.2. Thus, the equation makes tax on financial corporations proportional to the product of tax rate and taxed income, Ysy_cf, similar to the basic equation (8.9). The equation for tax on other corporations, Sy_cr, is identical to (8.13) with cf replaced by cr, so the two corporate sectors share the tax rate described in the square brackets.

Taxable income includes the net interest income of corporations, their dividend income and the part of their net profit income that does not cover personal taxes. Moreover, fiscal write-offs on investments are deducted, and so is the production tax (royalty) on North Sea oil production. The lag of half a year on income in (8.13) reflects that the financial year can differ from the calendar year, and corporations can use tax deductions to smooth out fluctuations in income.

Box 8.2 **Tax rate and transition between corporate tax regimes**

Corporations may pay their corporate tax in more than one way. In recent years, the PAYE arrangement has been the dominant system. The PAYE arrangement was originally introduced as an option for corporations to pay their corporate tax on a current basis, but at a reduced tax rate. Previously, corporations did not pay the tax until the following calendar year.

ADAM's corporate tax rate expression, *tsy_c*·(*bsy_c*+*ktsy_c*·(1-*bsy_c*)), approximates the transition between the two tax arrangements. Tax rate *tsy_c* is the statutory rate of the PAYE arrangement. Variable *bsy_c* describes the share of the PAYE arrangement, and *ktsy_c* is the additional rate charged in the traditional arrangement.

Hydrocarbon tax

The revenue from the hydrocarbon tax has increased since the change in 2004. The tax is levied on income from extraction of hydrocarbons and is in ADAM determined by:

(8.14)
$$Sy\_ck = ksy\_ck \cdot tsy\_ck \cdot [(1-tsy\_c)\cdot Yre - Tire\_o - Ivsk] \\ -(1-d4703)\cdot Spp\_xe$$

where *ksy_ck* is a proportional factor, *tsy_ck* the tax rate, and the square brackets contain the hydrocarbon tax base. The tax base includes gross profit, *Yre*, reduced by: general corporate tax, production tax *Tire_o* and tax-related depreciation, *Ivsk*. Furthermore, since 2004 the pipeline tax *Spp_xe* is offset against the hydrocarbon tax.

Pension yield tax

Also the pension yield tax *Sywp* is determined by a tax rate and the associated tax base. Interest earned and capital gains were previously taxed differently, so there is both a tax rate *tsywp* for interest income and a tax rate *tsywpa* for capital gains. In order to model the accumulation of wealth in the pension model, it is necessary to calculate the pension yield tax for a number of pension schemes. The tax yield equation has the general form:

(8.15)
$$Sywp_i = ksywp_i \cdot (tsywp \cdot Tip_i + tsywpa \cdot Owp_i)$$

Where $Sywp_i$ is the tax yield from scheme i. Interest income is called Tip_i and capital gain Owp_i.

Interest income and especially capital gain can fluctuate considerably between years, implying that the return of a pension fund may be negative in one year and positive in the next. This creates an asymmetry in the pension yield tax, which cannot be negative. Instead, the negative returns can be deducted from positive returns in the following years. The tax yield equation has an exogenous correction factor called $ksywp_i$, which among other things reflects this asymmetry.

Other direct taxes

Other direct taxes include motor vehicle duty on households, *Syv*, which is attached to the stock of cars. Other personal taxes *Syp* are divided into an exogenous and endogenous part. The latter concerns the 40 per cent tax on the lump-sum payments from capital pensions.

8.5 Taxes on production and import

The determination of production and import taxes is as comprehensive as the determination of direct taxes. Indirect taxes are important in economic policy, so indirect taxes are disaggregated and related to their tax bases, i.e. demand, production and import components, at the highest possible level of disaggregation in ADAM.[3]

Indirect taxes are characterized by entering the price formation, and the disaggregated treatment makes it possible to describe consistently the effect of rate changes on tax revenues and pricing.

Taxes on products

In the national accounts, taxes on products, i.e. taxes linked to specific products, are distributed on material inputs to industries and on different end uses. On this basis, each industry and each demand component in ADAM is assigned a set of tax rates used for the determination of tax revenues and prices. These implicitly calculated rates can also be used as fiscal instruments similar to the other tax rates in ADAM.

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[3] Indirect taxes are used as a synonym for national accounts taxes on production and import.

Table 8.6 **Production and import taxes**

	ADAM-name	2000	2006	2009
		DKK bn.		
Customs and import duties	*Spm*	2.4	3.5	2.6
Excise duties, net	*Spp*	42.5	56.0	52.2
General duty (VAT)	*Spg*	123.8	168.3	168.4
Vehicle registration duty	*Spr*	14.4	24.5	12.5
Total taxes on products, net		183.1	252.2	235.7
Other taxes on production, net	*Spz*	0.1	-1.8	-6.2
Total indirect taxes minus subsidies	*Sp*	183.2	250.4	229.5
Indirect taxes	*Spt*	222.7	294.8	282.2
Subsidies	*-Spu*	-39.5	-44.4	-52.7
Of which public sector:				
Indirect taxes minus subsidies		189.2	255.7	235.4
Indirect taxes	*Spt_o*	222.0	291.3	279.2
Subsidies	*-Spu_o*	-30.9	-35.6	-43.8
Of which EU				
Indirect taxes minus subsidies		-6.0	-5.3	-5.9
Indirect taxes	*Spt_e*	2.7	3.5	3.0
Subsidies	*-Spu_e*	-8.7	-8.8	-8.9

The revenue of excise duties from a consumption component is determined by:

(8.16) $$Spp\_c_j = fC_j \cdot tp_j$$

fC_j consumption component j, fixed prices
tp_j excise duty rate on consumption component j

Similarly, VAT yield to public sector is determined by:

(8.17) $$Spg\_c_j = (C_j - Spg\_c_j) \cdot tg \cdot btg_j$$

C_j consumption component j, current prices
tg VAT rate
btg_j VAT load on consumption component j

It can be seen that excise duties are levied on components in fixed prices rather than values. Thus, excise duties are modelled as taxes per unit, which is not completely accurate but a simplifying assumption. VAT is an ad valorem tax and levied on the relevant components in current prices.

Component-specific tax rates, tp_j and btg_j, are residually determined from equations (8.16) and (8.17), so that both equations hold in historical years with national accounts data. At ADAM's disaggregated level, taxable components are relatively homogeneous also in terms of taxation. This means that ADAM's implicit tax rates are relatively stable over time, as long as the related statutory rates are fixed.

In ADAM-calculated projections, we can as a starting point set all excise-duty rates equal to the rates of the last historical year corresponding to unchanged tax rules. It should be noted that this assumption makes the tax burden fall in a projection with inflation. Instead, we can choose to regulate the excise duties rates with the price development, which is done simply by activating a dummy *dtp*.

For ADAM-calculations on the impact of statutory tax changes, an auxiliary model can be used transforming budget estimates on revenues from statutory tax changes into changes in ADAM's tax rates. The auxiliary model called Basta is described in box 8.3.

Box 8.3 **Excise duty rates model (Basta)**

Auxiliary model Basta is intended for detailed analysis of excise duties. Basta uses a detailed description of each statutory tax and subsidy and their distribution on ADAM variables, such as consumption and investment components. The cornerstone is an excise duty matrix, ***A***, based on national accounts data:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \mathrm{K} & a_{1n} \\ \mathrm{M} & & \mathrm{M} \\ a_{m1} & \mathrm{L} & a_{mn} \end{bmatrix}$$

Index m indicates the type of tax, and n indicates the ADAM component on which it is levied. If the excise duty matrix is normalized by row sums, we obtain the matrix of distribution ***F***. Vector ***N*** contains the row sums.

$$\boldsymbol{F} = \begin{bmatrix} f_{11} & \mathrm{K} & f_{1n} \\ \mathrm{M} & & \mathrm{M} \\ f_{m1} & \mathrm{K} & f_{mn} \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} a_{1\cdot} \\ \mathrm{M} \\ a_{m\cdot} \end{bmatrix}$$

$$f_{ij} = a_{ij} / a_{i\cdot} \qquad a_{i\cdot} = \Sigma_{j=1}^{n} a_{ij}$$

Row sums indicate tax revenues by tax type, and if we multiply the transposed matrix of distribution, ***F'***, by ***N***, we get a vector ***S*** with tax revenues distributed on the n taxed components in ADAM:

$$\boldsymbol{S} = \boldsymbol{F}' \times \boldsymbol{N}$$

The matrix of distribution, ***F***, is assumed to remain unchanged in Basta, and on this assumption we can convert any vector, ***ΔN***, with statutory type-distributed revenue changes into revenue changes related to ADAM components. The conversion is done by multiplying by ***F'***.

$$\Delta\boldsymbol{S} = \boldsymbol{F}' \times \Delta\boldsymbol{N}$$

The corresponding changes in ADAM's exogenous tax rates are calculated by dividing changes in tax yield by ADAM components in real terms:

$$\Delta tp_i = \Delta Spp_i / fC_i, \quad \Delta tve_i = \Delta Sppve_i / fVe_i, \quad \Delta tvm_i = (\Delta Sppx_i - \Delta Sppve_i) / fVm_i$$

Thus, Basta can translate changes in official tax rates into changes in ADAM tax rates, provided that we have the immediate tax yield impact of each official tax change. The immediate, or ceteris paribus, tax yield impacts inserted in Basta do not take into account that a higher tax rate reduces the demand and hence also the tax base. By inserting the Basta-calculated tax rates in ADAM, we can estimate the actual change in tax yield.

Customs yield is determined in the same way as the yield from excise duties, as the rates are linked to import components in real terms. Vehicle registration duty relates exclusively to the purchase of vehicles, i.e. to consumption and investments, and the duty is determined as an ad valorem tax, similar to VAT.

Table 8.7 **Other taxes on production (taxes not related to specific products)**

	ADAM-name	2000	2006	2009
		DKK bn.		
Refunding trainee cost contribution etc.	*Spzaud*	2.3	3.2	4.2
Property tax	*Spzej*	13.6	18.3	22.6
Vehicle weight duty, producers	*Spzv*	1.9	2.3	2.5
Duty on wage and salary costs	*Spzam*	3.4	4.5	4.4
Miscellaneous production taxes	*Spzr*	0.1	0.1	0.1
Total other taxes on production		21.2	28.3	33.8
Other subsidies on production	*-Spzu*	-21.1	-30.2	-40.0
Total other production taxes, net	*Spz*	0.1	-1.8	-6.2

Other taxes on production

Indirect taxes not related to specific products are called “other taxes on production” in the national accounts, see table 8.7 for a breakdown by type.

Property taxes are levied on land value. In ADAM, the tax rate is an implicitly calculated land tax rate, while land value is approximated by multiplying the housing stock by a price index representing the assessed prices used for taxation.

All indirect taxes not related to specific products are related to industries. To this end, a set of parameters, α with subscript, is defined for each ADAM industry using national accounts data and the tax yield equation:

(8.18) $$Spz_j = \sum_i \alpha_{ij} \cdot Spz_i$$

where subscript *j* indicates industry and *i* indicates tax type.

Gross calculation of indirect taxes

ADAM's treatment of taxes is based on the input-output table. The input-output table shows indirect taxes net, i.e. indirect taxes minus subsidies, but to calculate public (and foreign) revenues and expenditures the modelling includes a gross calculation to determine the subsidies separately. Gross indirect taxes are determined as net taxes plus subsidies.

Table 8.8 **Subsidies**

	ADAM-name	2000	2006	2009
		DKK bn.		
EU-product-related subsidies	*Sppu_e*	7.8	1.2	0.5
Subsidy for public transport	*Sppukto*	5.8	7.9	8.1
Other product subsidies	*Sppur*	4.8	5.1	4.0
Product subsidies, total	*Sppu*	18.4	14.3	12.7
EU-production subsidies	*Spzuaa+Spzuz_eu*	0.9	7.6	8.4
Wage subsidy etc.	*Spzul+Spzuak*	5.8	10.0	13.5
Housing subsidies	*Spzuh*	4.2	4.7	4.2
Other production subsidies	*Spzuqr*	10.2	7.9	13.9
Production subsidies, total	*Spzu*	21.1	30.2	40.0
Subsidies, total	*Spu*	39.5	44.4	52.7

The subsidies in table 8.8 are determined in ADAM as the sum of a number of variables that are either exogenous or determined in accordance with the basic outline for tax equations. The only exception is other product-related subsidies, *Sppur*, which are determined by an equation with parameters reflecting the product balances in the national accounts.

Summary of chapter 8

ADAM is used by the economic ministries for preparing the economic policy, and it is chosen to have a detailed breakdown and modelling of public expenditures and revenues. Some public expenditures and revenues are cyclical, and the government budget balance is pro-cyclical in ADAM calculations.

Fiscal instruments in ADAM include, in particular, public employment, input of materials in the public sector and public investments in real terms plus the exogenous rates for transfers and taxes. There is no fiscal reaction function in ADAM, so all fiscal instruments are exogenous, and it is up to the model user to formulate a sustainable fiscal policy.

9. Income, savings surplus and financial wealth

Income and wealth formation is based on a number of definitional relations taken from the national accounts. There are three private sectors in ADAM (households, non-financial corporations and financial corporations) a public sector and a foreign sector.

For the domestic sectors, ADAM determines the main items: Disposable income *Yd*, savings (*Yd–C*) and savings surplus (*Yd–C–I*). ADAM's financial sub-model invests the savings surplus of a sector in financial assets or funds the savings deficit by issuing financial liabilities, such as loans. The financial sub-model is fully integrated with ADAM, and financial wealth affects the income formation through property income and through the wealth variable in the consumption function.

Savings in private pension schemes are modelled separately. Pension savings are special, because they are subject to special tax rules, and because of their special treatment in the consumption function.

The following section outlines ADAM's institutional national accounts and the transition from GDP to savings surplus. The second section concerns financial wealth and the financial sub-model; the third section illustrates the behaviour of the private savings surplus; while the fourth and final section describes the modelling of private pension schemes.

9.1 ADAM's institutional national accounts

The national accounts system has two perspectives on economic activity, a functional and an institutional one. The functional national accounts describe the product flows, i.e. production and imports, the use of products in industries, the end-uses: consumption, investment and exports; plus the input of labour and capital in industries as well as the associated business income. The institutional national accounts involve transactions of a distributional or financial nature and focus on ownership.

Sectors The owners in the institutional national accounts are grouped into sectors, and there are five main sectors in ADAM:

cr	Non-financial corporations
cf	Financial corporations
o	Public sector (general government)
h	Households etc.
e	Foreign sector

A business in a particular industry may be owned by the private sector, the public sector or have a foreign owner. Conversely, a household or a corporation may own companies in more than one industry. Thus, the institutional grouping into sectors cuts across the functional grouping into industries, and it should, for instance, be noted that ADAM has both a public sector and a public industry, see box 9.1.

Box 9.1 **Public sector versus public industry**

ADAM has both a sector *o*, which is equal to "general government" in the official national accounts, and an industry *o*, "production of public services". When necessary, the public sector variables have suffix *o1*, i.e. *Ywo1* for wage compensation in general government. This differs from *Ywo*, which is wage compensation in the public industry producing public services. For example, municipal gardeners are placed in ADAM's public sector and in ADAM's agricultural industry, not in ADAM's public industry.

Publicly owned corporations enter the industries matching their production, a large share is placed in the energy supply industry, *ne*, or in private services, *qz*. The publicly owned corporations are not part of the public sector *o* in ADAM, i.e. not part of general government, but part of the corporate sector. However, they fall within a wider definition of public sector comprising general government and publicly owned corporations. This wider public sector is not fully specified in ADAM.

Point of departure is GDP-equation

Value added in Danish production creates income for all five institutional sectors. The breakdown of value added by sector starts by considering the basic GDP-equation for an open economy, introduced in (5.1):

$$Y = C + I + E - M \Leftrightarrow Y + (M - E) = C + I \qquad (9.1)$$

Net imports, *M-E*, from abroad represent a part of foreign GDP, and with net imports placed on the left hand side, the equation says that Danish GDP plus the net use of foreign GDP equals the sum of consumption and investment. Just like income equals the sum of consumption and investment in a simple closed economy.

Box 9.2 **Calculating income and savings surplus for a sector**

	Contribution to GDP
+	Property income, net
+	Current transfers, net
=	**Disposable income**
–	Consumption
=	**Gross savings**
+	Capital transfers, net
–	Gross capital formation
–	Acquisition of non-produced real assets, e.g. land
=	**Savings surplus (Net lending)**

The transition from GDP to savings surplus in the national income and capital account is outlined in box 9.2 above and shown in more detail in table 9.1 using ADAM's variables and institutional sectors. The content of the table is discussed in the remainder of the present section.

GDP is split up into income categories

In table 9.1, net imports, *M-E*, are placed as a foreign GDP contribution in line 1. The Danish GDP is split up into gross operating surplus, wage compensation of employees and indirect tax:

$$Y = Yr + Yw + Spt - Spu \tag{9.2}$$

Table 9.1 **Institutional national accounts in ADAM, five sectors**

	Public, *o*	Foreign, *e*	Household, *h*	Non-fin. corporate, *cr*	Financial corporate, *cf*	Total
Net imports		*M-E*				*M-E*
Gross operating surplus	*Ivo1*		*Yr_h*	*	*Yrqt*	*Yr*
Wage compensation		*Ywn_e*	*Yw-Ywn_e*			*Yw*
Prod. and import taxes	*Spt_o*	*Spt_e*				*Spt*
Prod. and import subsidies	*-Spu_o*	*-Spu_e*				*-Spu*
GDP contributions, total						***Y+M-E***
Property income, net	*Tin_o*	*Tin_e*	*Tin_h*	*Tin_cr*	*Tin_cf*	0
Income and wealth taxes	*Sy_o*	*Syn_e*	*	*-Sy_cr*	*-Sy_cf*	0
Social contributions	*Stp_o* *+Tpt_o+Tp_h_o*	*Tpn_e*	*-(Stp_o+Tpn_e)* *-Tpt_o-Tp_h_o* *-Tp_h_cf*	0	*Tp_h_cf*	0
Social benefits not in kind	*-Ty_o*	*Tyn_e*	*Ty_o-Tyn_e* *+Typc_cf_h*		*-Typc_cf_h*	0
Other current transfers	*Tr_e_o-Tr_o_e* *+Tr_hc_o* *-Tr_o_hc*	*Tr_o_e-Tr_e_o* *+Tr_hc_e* *-Tr_e_hc*	*Trn_h*	*	*Trn_cf*	0
Disposable income	***Yd_o***	***(-Enl)***	***Yd_h***			
Adjustment, pension funds			*-Typc_cf_h* *+Tp_h_cf*		*Typc_h_cf* *-Tp_cf_h*	
Consumption	*-Co*		*-Cp*	0	0	*-C*
Gross savings						
Capital transfers etc., net	*Sk_h_o* *+Tk_e_o-Tk_o_e* *+Tk_hc_o* *-Tk_o_c-Tk_o_h*	*Tk_o_e-Tk_e_o* *+Tk_h_e* *+Tkn_c_e*	*-Sk_h_o* *-Tk_h_e* *+Tk_o_h*	*	*Tkn_cf*	0
Gross fixed investments	*-Io1*		*-I_h*	*	*-Iqf*	*-If*
Change in inventories	*-Ilo1*		*-Il_h*	*	*0*	*-Il*
Acq. non-produced assets	*-Izn_o*	*-Izn_e*	*-Izn_h*	*	*-Izn_cf*	0
Net lending	***Tfn_o***	***Tfn_e***	***Tfn_h***	***Tfn_cr***	***Tfn_cf***	**0**

* indicates that the cell represents a row residual. Blank indicates that ADAM has no variable for the cell. Box 9.3 below outlines the principles behind the notation of ADAM's variables.

Note. The table content is discussed on the following pages. The adjustment for pension funds affects the savings of households and financial corporations, cf. footnote 1 of the present chapter.

Gross operating surplus split up by sector

Gross operating surplus, *Yr*, is split up between ADAM's four domestic sectors. For the public sector, gross profits represent the depreciation of the capital stock. The household sector earns gross profits on sole proprietorships, while the gross profit of corporations is divided by the financial and non-financial corporate sector; see line 2 of table 9.1.

Wage compensation goes primarily to households

Wage compensation of employees, *Yw*, is primarily allotted to Danish households, but a little part of this is earned by foreign cross-border workers, similar to Danish cross-border workers receiving wage compensation from abroad, cf. line 3 of the table.

Box 9.3

Names of Variables in ADAM's institutional accounts and in table 9.1

Variables that specifically belong to the institutional system begin with either an S, which is the classifying letter for taxes, or a T, which denotes transfers other than taxes. Then follows typically three suffixes, which in turn indicate type, paying sector and receiving sector. The suffixes are separated by underscores. For example:

Tk_o_e

indicates capital transfers from public to foreign sector. If it is a net payment, an n is placed after the type indication, for example, net capital transfers from public to foreign sector is:

Tkn_o_e=Tk_o_e-Tk_e_o.

As a main rule, transfers to or from the public sector are calculated gross, while other transfers are calculated net. Transfers to or from the public sector are also, as a rule, split up by counterpart sectors

If a transfer is not broken down by counterparts, a *z* is used to denote the aggregate of all paying or receiving sectors, for example, *Tk_z_o* is total capital transfers from other sectors to public sector, while *Tk_o_z* denotes total capital transfers paid by the public sector.

If the *z* is omitted and only a single sector suffix is indicated, the variable will still denote a transfer between the sector indicated and all other sectors combined, but without the *z* it is not clear whether it is income or outlay for the sector indicated. For example, *Sy_o* denotes total income of current income and wealth taxes in sector *o* and enters public net lending with a positive sign, while *Sy_cf*, enters net lending of financial corporations with a negative sign and represents the sector's payment of income and wealth tax.

Indirect taxes go primarily to the public sector

Net *production and import taxes*, i.e. *Spt-Spu,* are primarily allotted to the public sector, but some indirect taxes and subsidies relate to EU law and are allotted to the foreign sector cf. line 4 of the table.

Property income, income transfers and disposable income

With the breakdown by sector of indirect taxes, *GDP contributions* can be calculated for all sectors, and when property net income and current income transfers are added and current taxes on income and wealth are deducted, we arrive at the disposable income of each sector.

Disposable income is used in the consumption function

The disposable income of households and corporations is the starting point for the determination of consumption in ADAM. Household disposable income is driving the short-term movements in consumption, while corporate disposable income is added for the long-term determination of consumption. The disposable income in the consumption function differs from the national accounts disposable income with respect to the treatment of transfers between households and private pension schemes. More specifically, the long-term disposable income in the consumption function excludes savings in pension schemes, see chapter 3, section 3.

Savings surplus (Net lending)

The savings surplus - also called net lending - is defined in textbooks as *Y–C–I*. In practice, this simple definition is modified by capital transfers, and in the case of households adjusted for savings in pension schemes, because these savings are perceived as compulsory savings. [1]

A positive savings surplus for a sector implies that the sector either acquires financial assets or reduces financial liabilities. It does not necessarily imply that the value of its financial net wealth increases, because assets and liabilities of the sector are also affected by price fluctuations.

Savings surpluses sum to zero

Institutional accounts variables that are not represented directly in the overall GDP identity (9.1) must sum to zero over the five ADAM-sectors. The five sectors include the foreign sector. Thus, there is no sector outside ADAM and transfer income in one sector must be transfer outlay in one of the other five sectors. This goes not only for all current and capital transfers, including taxes in table 9.1, but also for property income and acquisitions of non-produced real assets.

As a consequence of the zero-restriction on non-GDP related transactions, and of the overall GDP identity, savings surpluses must sum to 0 for all sectors combined, incl. the foreign sector, as illustrated in table 9.1:

(9.3)

$$Tfn\_cr + Tfn\_cf + Tfn\_o + Tfn\_h + Tfn\_e = 0$$

Determining non-GDP income components

As already mentioned, income components not represented in the overall GDP identity (9.1) sum to zero over all sectors of the economy. ADAM's determination of these "zero-sum components" is discussed in turn below.

The main component of *property income* is interest income, which is typically determined as the product of the interest rate and interest-bearing assets at the beginning of the year:

(9.4)

$$Tii = i \cdot W_{-1}$$

[1] Table 9.1 treats net contributions to pension institutions as a net transfer from households, *h*, to financial corporations, *cf*. This net transfer reduces the disposable income in sector *h* and increases disposable income in sector *cf*. When determining household savings, the net transfer is added and the savings surplus of households thus includes savings in the pension institutions. The same adjustment reduces the savings and savings surplus of financial corporations.

The interest income from bonds adjusts to this level with a considerable lag whenever interest rates are changed, because a large proportion of the bonds have a fixed interest rate.

Dividends on shares constitute another important component of property income. The dividend is typically determined as a fixed share of equity value at the beginning of the year:

(9.5) $$Tiu = btiu \cdot Ws_{-1}$$

Taxes are typically determined as a product of an exogenous tax rate and the associated tax base, and *social contributions* are typically given as an exogenous share of the wage compensation, see chapter 8 on public finances.

Social benefits not in kind, i.e. paid social benefits, include in particular public transfers, *Ty_o* such as pensions, unemployment benefits and general social benefits, discussed in chapter 8 on public finances. In addition, there are payments from private insurance, especially from pensions. The latter is determined as the product of a quota and the pension assets, see section 9.4 discussing the modelling of private pension schemes.

Other current transfers are a heterogeneous group. An important part relate to transfers to and from the EU, which are determined by the related rules involving, e.g. the GNP and VAT base. Development aid is also an important component determined as an exogenous share of GDP. Furthermore, a number of other current transfers are quite small and exogenous in ADAM.

Capital transfers and acquisitions of non-produced real assets are exogenous in the model, although some are defined as aggregates of subgroups, and total capital transfers to abroad are residually determined by the requirement that transfers must sum to 0 for the whole economy.

The *savings surplus* is determined as described for each sector. In practice, the non-financial corporate savings surplus is determined residually as minus the sum of savings surpluses in the other sectors.

9.2 Financial wealth and financial sub-model

Wealth placed in five financial instruments

The savings surpluses or deficits of the sectors must be placed in or financed by financial instruments, of which there are five in ADAM:

s	Shares
b	Other securities (”bonds”)
q	Other financial instruments, net (”loans”)
p	Pension reserves, life insurance and pension fund
cp	Pension reserves in banks

Pension reserves are typically placed in the other instruments, implying that the two pension instruments have a special status and appear mainly as a liability in the financial corporate sector, and as a corresponding asset in the household sector. For both pension instruments, there is also a minor asset in the foreign sector. Within the financial corporate sector, pension reserves are balanced by holdings of securities, as shown in table 9.4 placed at the end of the present chapter.

Transactions and revaluations

Placing the savings surplus in financial instruments is an example of a transaction, i.e. an agreement between a buyer and a seller to exchange an asset for payment. The net value of the financial instruments acquired by a sector constitutes the sector's net financial wealth. Net financial wealth is not only influenced by transactions, but also by revaluations due to, e.g. changes in bond or equity prices or changes in exchange rates. Financial wealth is also affected by so-called "other changes in volume" like cancellation of debt, theft, etc.

The relation between the value of a financial instrument at the beginning and at the end of a year is given by:

(9.6)

$$W = W_{-1} + Tf + Ow$$

where W is the end-year value of the instrument, Tf is the sum of transactions during the year, and Ow is the sum of revaluations and other changes in volume during the year.

Balance sheet

The components in ADAM's financial accounts are shown in table 9.2. Similar tables can be established for transactions and revaluations, but these are not shown here. The notation follows the same principles that apply to the flow variables.

Financial net wealth, net transactions and net revaluations must sum to 0 over all sectors of the economy.

For a given sector, the sum of financial instruments, calculated with a plus for assets and a minus for liabilities, equals net financial wealth, Wn.

Role of financial sub-model

ADAM's financial sub-model controls the relationship between savings surpluses and financial wealth, the use of financial instruments and the related revaluations. This is done for all sectors in the model. The financial sub-model also determines a couple of variables of special interest, such as central government debt and EDP debt.

Table 9.2 Financial accounts in ADAM

		Non-financial corporations cr		Financial corporations cf		General government o		Households h		Foreign sector e		Total
		Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	
Bonds	*b*	*Wnb_cr*		*Wnb_cf*		*Wnb_o*		*Wnb_h*		*Wnb_e*		0
Shares	*s*	*Ws_z_cr*	*Ws_cr_z*	*Ws_z_cf*	*Ws_cf_z*	*Ws_z_o*		*Ws_z_h*		*Ws_z_e*	*Ws_e_z*	0
Pension reserves, banks	*cp*				*Wcp_cf_z*			*Wcp_cf_h*		*Wcp_cf_e*		0
Pension reserves, life and pension	*p*				*Wp_cf_z*			*Wp_cf_h*		*Wp_cf_e*		0
Other financial instruments	*q*	*Wnq_cr*		*Wnq_cf*		*Wnq_o*		*Wnq_h*		*Wnq_e*		0
Total		*Wn_cr*		*Wn_cf*		*Wn_o*		*Wn_h*		*Wn_e*		0

Shares appear as assets and liabilities. Bonds and "other financial instruments" are net assets. Pension reserves are inherently gross figures. They only appear as liabilities in the financial corporate sector and as assets in household and foreign sector, cf. table 9.4.

Feedback to the rest of ADAM

The main feedback from the financial sub-model goes via private consumption. Consumption is influenced by property income, which depends on financial wealth and its distribution on instruments, and consumption is also influenced by the wealth variable in the consumption function. The wealth variable includes private financial wealth.

These two feedback mechanisms are relatively weak in the short term, but in the long term, the feedback via financial wealth is central to ADAM properties.

Outline of financial sub-model

A full outline of how the financial sub-model determines the financial accounts of a sector begins with the savings surplus of the sector and ends with the price-adjusted financial wealth variables at year-end:

- The savings surplus is the starting point.
- Sector borrowing is determined from ADAM's variables, e.g. mortgage loans are determined on the basis of housing stock.
- Net contributions to pensions are determined in ADAM's pension model, see section 9.4.
- Investment requirement, *Tfnf*, is defined as savings surplus plus borrowing minus contributions to pension schemes.
- The investment requirement is placed in bonds, equities or other instruments ("Loans"). The choice between instruments is based on either fixed shares adding up to 1, or on one of the placements being residually determined.
- When transactions in instruments are determined, shares and bonds are adjusted with price changes.

Determining the financial prices

Basically, only shares and bonds have endogenous prices in the financial sub-model. However, there is also derived price sensitivity in:

- Mortgage loans because they are based on mortgage bonds.
- Pension reserves invested in shares and bonds.

These wealth components "inherit" the market-value adjustment of the underlying assets.

The annual revaluation of stocks and bonds is determined for the total portfolio of shares and bonds. Then, the revaluation is distributed pro rata on sectors, so that all sectors holding a given instrument get the same relative revaluation.

The bond price for fixed-rate bonds depends on the interest rate and on the duration of the bonds.

There are three share prices in the model: A foreign price, *pws_lse*, a price for financial corporations, *pws_cf*, and a price for non-financial corporations, *pws_cr*. Danish share prices are modelled, so that the price in the long term converges to a fixed relationship between market value and book value. The book value is calculated as the sum of real capital and financial assets minus financial liabilities other than equity. The market-over-book value constitutes a ratio that is less for non-financial corporations than for financial corporations, and both ratios are less than 1 based on ADAM data.

9.3 Reaction of the private savings surplus

This section illustrates by means of an ADAM calculation how the savings surpluses, especially the private savings surplus, and the financial net wealth respond to a shock to the consumption function.

Consumption function controls private savings surplus

Many exogenous shocks to the model, for instance, a permanent increase in government purchases, have virtually no lasting effect on the private savings surplus. This stability of the savings surplus reflects that the consumption function reacts to private wealth, so that fluctuations in private savings tend to disappear. If, for example, private savings fall, financial wealth will fall as well, and this will dampen consumption and stimulate savings. Similarly, if private savings increase, financial wealth and consumption will increase, and savings will fall back.

1 per cent permanent boost of consumption function ...

This savings-stabilizing mechanism works through the consumption function, and we can produce a permanent change in the private savings surplus by shocking ADAM's consumption function, cf. figure 9.1 showing the reaction in the savings surpluses of the three main sectors. More specifically, we increase the adjustment term in the long-term consumption equation by 0.01. This is equivalent to increasing the equation constant, and the desired consumption will be 1 per cent larger for given income and wealth.

Figure 9.1 **Savings surpluses, desired consumption + 1%**

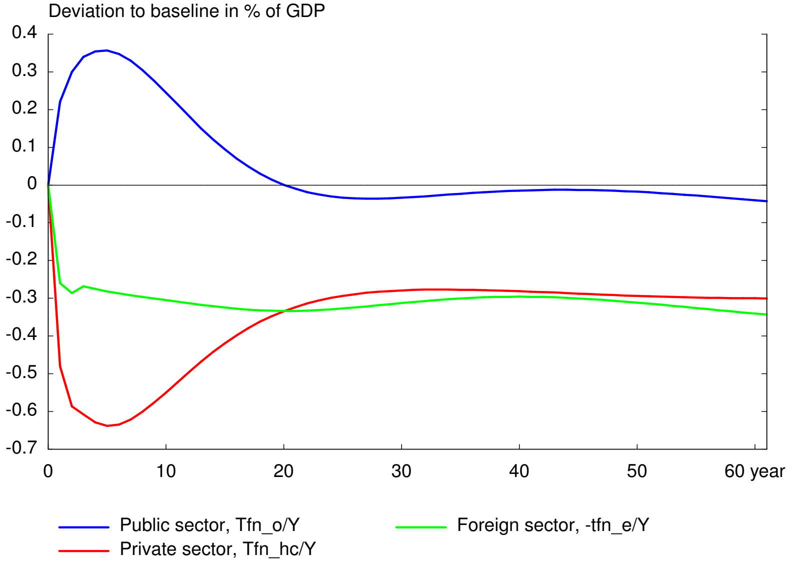
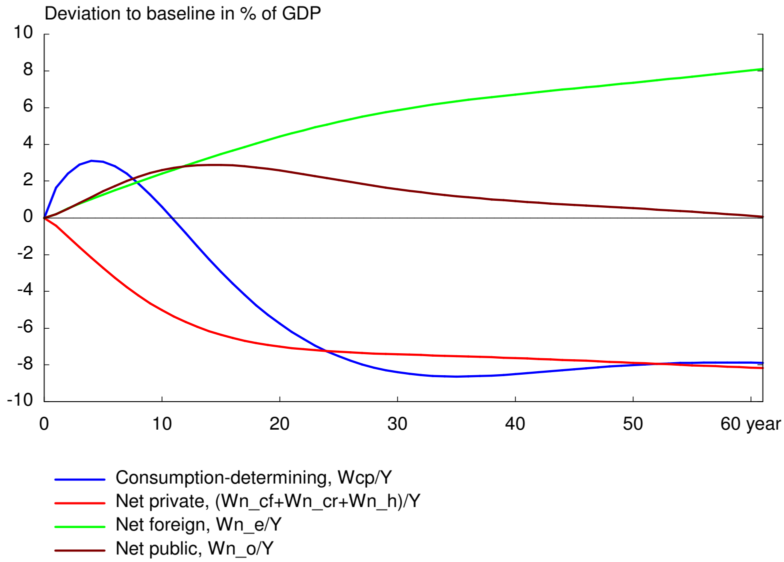


Figure 9.2 **Wealth variables, desired consumption + 1%**



... will reduce private wealth by 8 per cent of GDP ...

The immediate consequence is that consumption increases and savings fall. Smaller savings reduce the wealth of consumers relative to the baseline scenario, and in the long term, private wealth will fall relative to the baseline scenario. The long-term reduction in wealth amounts to about 8 per cent of GDP, see figure 9.2.

... and reduce private savings surplus by 0.3 per cent of GDP

With a permanently smaller wealth level, it requires smaller savings to make wealth grow in line with GDP; and in the long term private savings will fall because wealth falls. The long-term decline in private savings surplus is 0.3 per cent of GDP according to figure 9.1, and this magnitude can be related to the fall in wealth, to the rate of growth in GDP and to the effect on investment in real capital, see box 9.4.

Box 9.4 **The relation between wealth and savings effect**

The long-term decrease in consumption-determining wealth is 8 per cent of GDP, which reflects that wealth falls from being 209 per cent of GDP in the baseline scenario to being 201 per cent of GDP.

Like GDP, wealth increases by 3.5 per cent per annum in the long-term steady state, both before and after the shock to the consumption function. It requires an annual wealth increase equal to 7.32 per cent of GDP to make wealth grow by 3.5 per cent p.a. in the baseline ($0.035 \cdot 209 = 7.32$), and it requires a moderately smaller wealth increase of 7.04 per cent of GDP to make wealth grow 3.5 per cent p.a. after boosting the consumption function ($0.035 \cdot 201 = 7.04$).

Consumption-determining wealth is the sum of corporate and household net financial assets, *Wn_cf+Wn_cr+Wn_h*, minus a correction, which reduces pension wealth to its value after tax, and plus the value of the dwelling stock.

In the long term, where house prices follow investment prices, both dwellings stock and tax adjustment of pension wealth amounts to about the same share of GDP as in the baseline scenario. This means that the decline in consumption-determining wealth is roughly equivalent to the decline in private financial net wealth, see also figure 9.2. Thus, the change in wealth growth, i.e. 7.04 minus 7.32 per cent of GDP, mostly concerns financial net wealth, and this explains why the private savings surplus falls by 0.3 per cent of GDP.

Effect on public and foreign wealth accumulate

While the consumption function stabilizes the long-term effect on private wealth and savings balance, there is no equation in ADAM to stabilize the effect on public finances in the long term. To stabilize public finances, a fiscal reaction function must be added.

It appears from figure 9.1 that the effect on the government budget balance becomes negative in the long term, and when both private and public savings surpluses deteriorate, the balance of payments will also deteriorate, while foreign debt increases, see figure 9.2 where foreign debt is shown as an asset for the foreign sector. This implies that the increased propensity to consume is financed by increased foreign borrowing.

The effect on savings surpluses and wealth is changed markedly if the long-term effect on public debt is neutralized by a tax increase. This is discussed in connection with the shock to desired consumption in chapter 11, section 2.

9.4 Private pension schemes in ADAM

Pension assets in pension funds, life insurance corporations and banks represent a very large proportion of financial wealth in the household sector. In addition, pension savings are subject to special tax rules and often organized in collective schemes mandatory to the individual saver. Pension savings can have an important role in economic assessments, for instance, when the SP (special pension) savings were released in 2009. Moreover, the equations for pension savings influence the long-term properties of ADAM.

Development in pension reserves

Funded pensions are characterized by the existence of a pension wealth or pension reserve, *Wp*, which increases when contributions are paid to the pension scheme or when property income are added and decreases when pension yield tax or pensions are paid:

(9.7)
$$Wp = Wp_{-1} + contribution + interest - tax - pension\_payment$$

The general equation (9.7) is used to describe all pension schemes modelled in ADAM. The equation can be rewritten to illustrate that the relative growth in pension wealth is a function of contribution and payment rates, of the interest rate and of the income base that contribution rates are applied to:

(9.8)
$$\frac{\text{Dif}(Wp)}{Wp_{-1}} = btp \cdot \frac{Y}{Wp_{-1}} + (1-t) \cdot i - bty$$

Wp	pension wealth
btp	contribution rate, times *Y* gives *contribution*
Y	income base
$(1-t) \cdot i$	after-tax interest rate, times Wp_{-1} gives *interest - tax*
bty	payment rate, times Wp_{-1} gives *pension_payment*

The contribution rate reflects the rules of the pension scheme, while the payment rate – or payment quota - also depends on the age distribution and life expectancy of the pensioners and on the interest rate.

Pension wealth can be described with an error correcting equation

The equation (9.8) above implies that pension wealth follows an error correction form with the relative change in pension wealth on the left hand side, and lagged pension wealth on the right hand side. If contribution and payment rates and also the interest rate after tax are constant, pension wealth will end up being proportional to the income base of the pension scheme. For an annual growth rate of 3.5 per cent, i.e. $Wp = 1.035\ Wp_{-1}$, the solution to (9.8) is given by (9.9) below, where the proportionality factor between income and pension wealth is a fraction, which in addition to the assumed growth rate includes contribution rate, payment rate and interest rate:

(9.9)
$$Wp = \frac{1.035 \cdot btp}{0.035 - (1-t) \cdot i + bty} \cdot Y$$

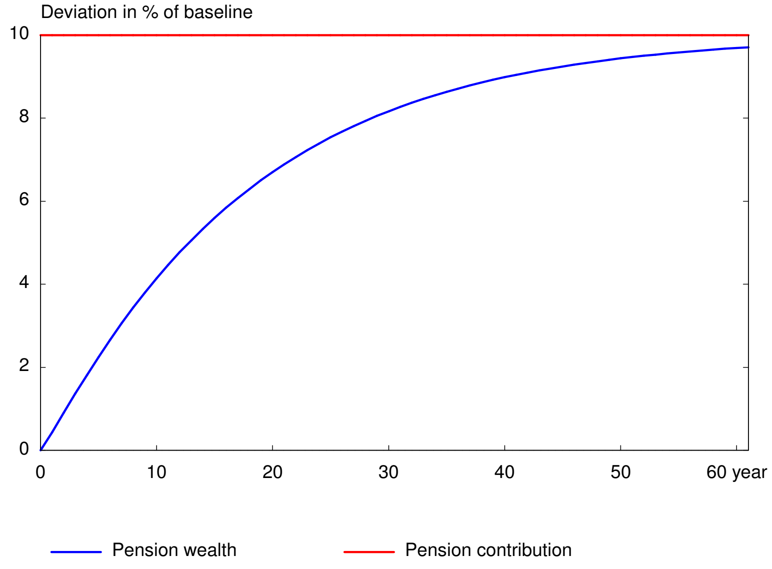
This is clearly a long-term relationship. When a pension scheme is new, the payment is relatively small, and pension wealth will build up and grow relative to income, but as the scheme approaches its steady state, pension wealth will grow slower and end up being constant relative to income.

Pension contributions and pension wealth

The relationship between pension contributions and pension wealth can be illustrated by a numerical example, in which the contribution rate, *btp*, in (9.9) is raised by 10 per cent. If payment rate and after-tax interest rate are kept exogenous, pension wealth will eventually increase by 10 per cent relative to income.

More specifically, the example concerns ADAM's pension wealth variable for mandatory schemes with one-off payment, *Wpco_bf*. The corresponding contribution variable, *Tpco_bf*, is exogenized and increased by 10 per cent, which corresponds to increasing the contribution rate by 10 per cent for fixed income. The higher contributions gradually increase pension wealth, until the wealth variable is 10 per cent above the baseline, see figure 9.3a.

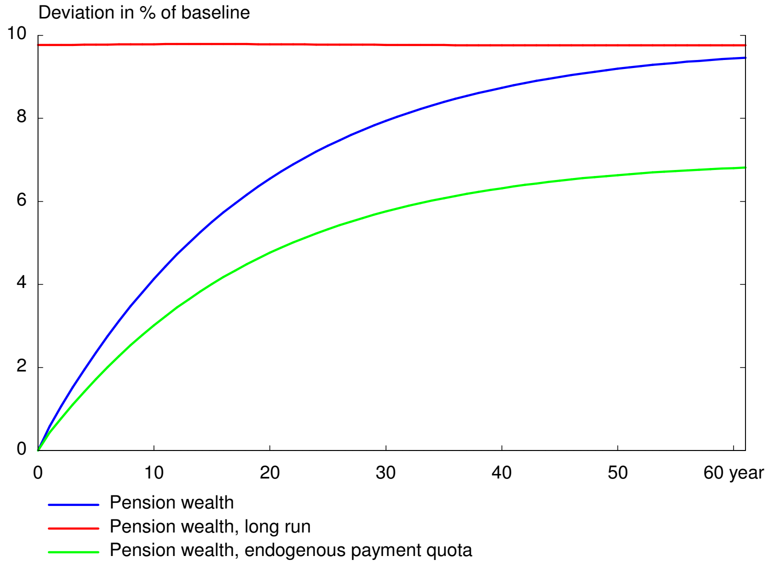
Figure 9.3a **Effect on pension wealth, pension contribution + 10%**



Interest rate and pension wealth

The relationship between interest rate and pension wealth can be illustrated by a numerical example, in which the interest rate rises, for example, from 3.5 to 4 per cent. The higher interest rate increases pension wealth by 9.8 per cent in the long term, conditioned on given contribution or payment rates and ignoring the tax on pension yield. This stylized long-term wealth effect amounting to 9.8 per cent can be derived from the long-term pension wealth equation in (9.9) where the payment, $btp \cdot Y$, is assumed exogenous, the tax rate, t, is assumed zero, and the payment rate, bty, is maintained at its baseline value of 0.056, while the interest rate, i, is changed from 0.035 to 0.04, (1.098=(0.035-0.035+0.056)/(0.035-0.04+0.056)).

Figure 9.3b **Effect on pension wealth, interest rate + 0.5 percentage point**



It is, however, unrealistic to maintain the payment rate if the interest rate increase. Because, when the return on assets exceeds the nominal growth rate of 3.5 per cent, pension savers' accounts will increase faster over time, and the mature accounts that are being paid out will consequently constitute a larger share of total wealth in the pension scheme. This is taken into account, as ADAM makes the payment rate an increasing function of the interest rate. This interest sensitivity of the payment rate moderates the interest sensitivity of pension wealth, see figure 9.3b. However, pension wealth will still increase when the interest rate rises, unless the pension scheme is changed.

At a given growth rate, a higher level of pension wealth implies a higher annual change in pension wealth. The change in pension wealth equals pension savings, and since pension savings is deducted from the income included in ADAM's consumption function, higher pension savings will work just like a reduction in the propensity to consume. Thus, via its effect on pension savings, the increase in interest rate will create an institutionally determined substitution effect on consumption; see the discussion on consumption function properties in chapter 3, section 2

ADAM's pension reserves

About 20 per cent of total pension wealth is administered by four institutions that are traditionally treated separately: a labour market supplementary pension fund (ATP) dating back to 1964, a closed wage earner fund (LD), a special pension fund (SP) and a temporary pension fund (DMP).

The remaining 80 per cent of pension wealth is placed in other pension funds, life insurance companies and banks. This large share of pension wealth is not divided by administering institution, but is instead divided by type of pension scheme. Specifically, pension schemes are divided into mandatory and not-mandatory, i.e. individual schemes and into schemes where the pension is paid in one lump sum and schemes paid in regular instalments, e.g. annuities. The two independent criteria produce four possibilities, which define four wealth variables.

This means that ADAM's total pension wealth, *Wp*, is a sum of eight wealth components.

(9.10)
$$Wp = Wpco\_bf + Wpcr\_bf + Wpio\_bf + Wpir\_bf + Watp + Wld + Wsp + Wdmp$$

The first right-hand-side variable, *Wpco_bf*, represents mandatory pension schemes with lump sum payments; *Wpcr_bf* is mandatory with instalments, *Wpio_bf* non-mandatory with lump sum, and *Wpir_bf* non-mandatory with instalments. These four variables represent claims on the financial corporate sector, excluding the special institutions ATP, LD, SP and DMP. The last four variables on the right hand side of (9.10) represent pension wealth in these special institutions.

Table 9.3 **Pension wealth in ADAM**

	Pension institutions and banks				Special institutions: ATP, LD etc.	Total
	Mandatory schemes		Individual schemes			
	Lump sum	Instalments	Lump sum	Instalments		
	Wpco_bf	*Wpcr_bf*	*Wpio_bf*	*Wpir_bf*	*Wpcr_atp+ Wpcr_ld+...*	*Wp*
	DKK bn.					
2005	186.7	1 082.3	108.8	140.1	472.2	1 990.1
2006	210.8	1 154.3	110.8	151.0	478.7	2 105.5
2007	218.9	1 206.9	111.3	153.8	468.1	2 158.9
2008	188.2	1 208.5	110.4	132.2	506.9	2 146.4
2009	219.9	1 357.4	111.3	150.4	472.0	2 311.0
2010	235.6	1 516.3	110.9	153.8	530.8	2 547.5
	pct.					
Share 2010	9.2	59.5	4.4	6.0	20.8	100.0

Source: ADAM's data bank, September 2011.

The interaction between the determination of private pension schemes and the rest of ADAM goes via the income and wealth variables in the consumption function and via taxes. For example, pension contributions reduce the income tax as they are deducted from the tax base. Households' property income from pension wealth is deducted from the income concept applied in the consumption function. Instead, net payment from pension schemes is added to the income in the consumption function. Thus, consumption depends on private disposable income, excluding pension savings.

All institutions, administering pension schemes, are placed in the corporate financial sector, *cf*, while the pension wealth and the related property income belong to the household sector. This is illustrated in table 9.4. Households (and foreigners) hold pension assets in the financial sector, and the financial sector has invested the pension wealth in shares or bonds. Thus, there is no net contribution to the financial wealth of the financial sector. Basically, the pension savers are holding the pension wealth invested in bonds and shares.

Table 9.4 **Pension-related assets and liabilities in ADAM sectors**

		Financial corporations with pension accounts share of *cf*		Households *h*		Foreign sector *e*		Total
		Asset	Liability	Asset	Liability	Asset	Liability	
Bonds	*b*	*Wpb_cf_z*						*Wpb_cf_z*
Shares	*s*	*Wps_cf_z*						*Wps_cf_z*
Pension reserves, banks	*cp*		*Wcp_cf_z*	*Wcp_cf_h*		*Wcp_cf_e*		*0*
Pension reserves, life and pension	*p*		*Wp_cf_z*	*Wp_cf_h*		*Wp_cf_e*		*0*
Total		*0*		*Wcp_cf_h+Wp_cf_h*		*Wcp_cf_e+Wp_cf_e*		*0*

Note: Similar tables can be made for transactions and revaluations.

The row with life insurance companies and pension funds corresponds to pension reserves in the national accounts.

Pension reserves in ADAM are larger than pension reserves in the national accounts, which only include holdings in life insurance companies and pension funds, i.e. *Wp_cf_z* in table 9.4. ADAM's pension wealth also includes pension accounts with banks.

Summary of chapter 9

Income and capital formation in ADAM reflects the national accounts definitions, and the associated identities for disposable income, savings and changes in net wealth are important, not least to the long-term properties of the model.

ADAM's pension model provides a stylized description of all private pension schemes. The pension wealth of the model includes pension accounts with banks, and the breakdown of total pension wealth reflects whether the pension scheme is mandatory and whether the pension will come as a lump sum or as regular instalments. This breakdown makes it easier to model the impact on taxes and household behaviour.

10. Setting up baseline scenarios

In order to calculate how ADAM responds to changes in an exogenous variable, we must solve the model twice. Once when the exogenous variable has its original values, and once when the exogenous variable is changed.

The solution with the initial values of the exogenous variable is usually referred to as the baseline or the baseline scenario. The word 'line' hints that ADAM is not just solved in a single year but in a multi-year period to illustrate the dynamic response to the exogenous shock.

In practice, ADAM is mostly used in relation to a specific projection. In the short term, the projection would constitute a forecast of the economic situation, and in the longer term, the projection would often be rather stylized and indicate a steady state in which a number of key ratios stay constant. For example, the consumption ratio, the wealth ratio, i.e. wealth relative to income, the import content in demand and the wage share in value added are often kept unchanged in stylized long-term projections. Such stylized projections are also called scenarios.

The many stylized assumptions and projections of exogenous variables are unlikely to hold, and the long-term baseline scenario is an uninteresting forecast, but it can be used to analyse the model. The purpose of the baseline scenario discussed here is to get a starting point for calculating the short- and long-term effects of shocking ADAM's exogenous variables.

The following section describes how to make the model variables grow with the same growth rate. This is followed by a section on how behavioural equations can be adjusted in the projection in order to repair a structural break or to neutralize the effect of changing the real growth rate or inflation. The last section illustrates the impact of the growth rate on the savings and investments.

10.1 Baseline scenario with steady-state growth

Effect of changing exogenous variables depends on the baseline

The baseline is normally not decisive for the outcome of changing an exogenous variable, but the model is not linear, so there is a link between the baseline and the effect of changing an exogenous variable. For instance, it is obvious that the effect of a change in interest rate depends on the size and sign of the private sector's net wealth. Another example is that the income effect of an exogenous shock to private net wealth will depend on the size of the interest rate.

The size of the growth rate can also play a role. If the economy is stationary, both net savings and net investment and also the savings surplus will be zero in equilibrium, regardless of the size of financial wealth and the stock of real capital. This means that the long-term effects on, e.g. net savings are always zero when the baseline is stationary, unless the exogenous shock is of a fundamental kind that makes the economy grow. If the economy is growing in the baseline, net savings are not tied to being zero in equilibrium, and a permanent reaction in wealth will be accompanied by a permanent reaction in savings.

A simple baseline can be produced by extending the roughly 1,000 exogenous variables in ADAM unchanged from the last historical observation. If all exogenous variables are constant, including the trend variables that represent technological progress, we will get a basically stationary baseline in which the endogenous variables may fluctuate in the beginning, but as they reach equilibrium, all variables will remain unchanged from year to year.

Baselines are non-stationary in practice

It is tempting to present ADAM’s properties based on a simple stationary baseline scenario without growth and inflation. Text book models on macroeconomics often have a stationary solution in order to simplify the presentation. The problem with a stationary baseline scenario is that it is slightly too stylized, when, for instance, net savings and savings surplus always end up being zero. The normal baseline for ADAM calculations is a non-stationary projection, where production and income increase from year to year, and where it requires savings to make wealth grow in line with income.

Growth rate in the baseline scenario

It is chosen to let volume variables in ADAM grow by 1.5 per cent per annum in the steady state of the baseline scenario; prices grow by 2 per cent p.a. and nominal values will consequently grow by a little over 3.5 per cent p.a. The selected growth rates for volumes and prices correspond to the average GDP growth and consumer price inflation over the past 20 years. Moreover, the 2 per cent price increase corresponds to the European Central Bank's upper limit for price stability in the euro area, so baseline inflation is consistent with the Danish fixed exchange rate policy. We note for the record that the baseline scenario is not a forecast of economic growth in coming years. The crucial aspect of the chosen volume growth and inflation is that they differ from zero, so we can illustrate the model properties with a non-stationary baseline.

Supply grows with productivity

To make total production grow by 1.5 per cent, labour supply and working hours are kept unchanged in the baseline, while the labour efficiency trend is given an annual growth rate of 1.5 per cent in all industries. Efficiency trends for capital and materials are kept unchanged, so the rise in productivity is on labour. This corresponds to Harrod-neutral productivity growth, where the relationship between capital and production is constant for unchanged interest rates.

Interest rate is determined abroad

Due to the fixed exchange rate policy, exchange rate and interest rate are exogenous variables in ADAM. The exchange rate is kept unchanged in the baseline and all interest rates are set equal to the nominal growth rate of 3.5 per cent implying that the real interest rate equals the real growth. The consistency of growth and interest rate is a simplifying assumption, which makes it easier to establish a baseline with constant wealth ratios.

Inflation is imported from abroad

The unchanged exchange rate implies that the Danish price increase is given from abroad. Thus, we determine the baseline inflation by making the exogenous foreign prices grow by 2 per cent p.a. If Danish prices grow less than foreign prices, the Danish market share will increase on both foreign and domestic markets; and if Danish prices grow by more than foreign price, Danish market shares will fall. Changes in market shares affect the activity, implying that unemployment and wage trend will react if the Danish price trend differs from the foreign. Changing the growth rate of wage costs affects the price trend, and the price of Danish products will end up growing by 2 per cent in the baseline.

Wage growth equals inflation plus productivity growth

With 2 per cent inflation and productivity growth of 1.5 per cent, the hourly wage must grow by 3.5 per cent p.a. in steady state. There is only one labour market in the model so the 3.5 per cent wage growth applies to all industries. This means that Danish costs per unit produced will increase by 2 per cent, similar to import prices. Consequently, all domestic prices will increase by 2 per cent, and all relative prices will eventually be constant in the baseline scenario.

Figure 10.1 **Price growth**

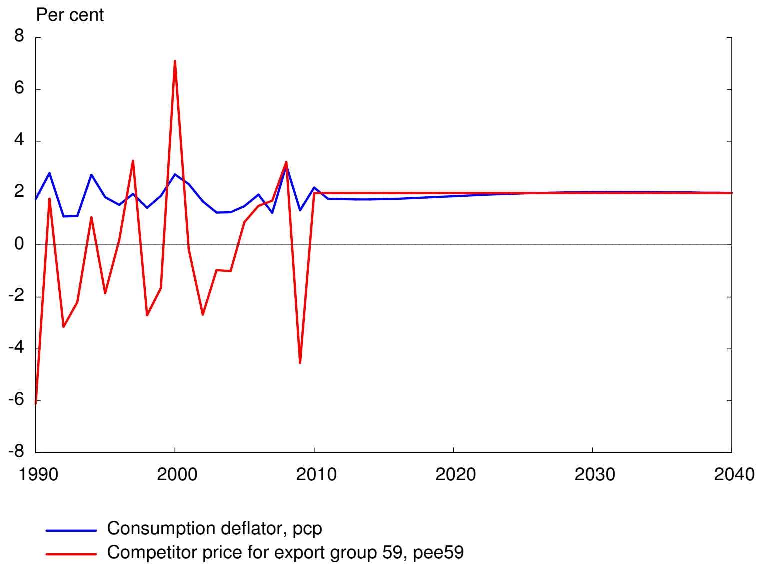
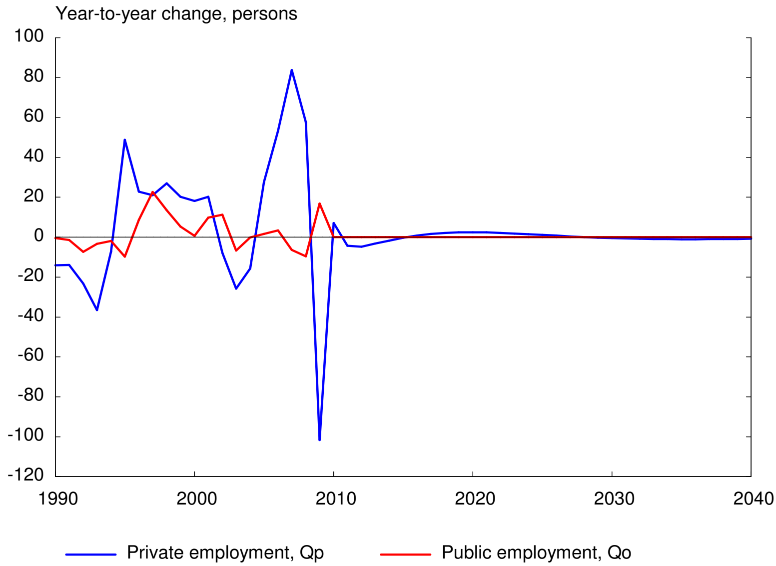


Figure 10.1 illustrates that the exogenous competitor price, *pee59*, on the market for manufacturing exports is set to grow by 2 per cent p.a. starting in the first baseline year 2010. However, it takes some years before the endogenous consumption deflator, *pcp*, reaches a steady state. In the first years, consumer prices grow by less than 2 per cent because unemployment starts slightly above its equilibrium level.

Employment unchanged in the long term

As already argued, wage growth in the steady state must reflect the assumed growth of prices and labour productivity. Consequently, ADAM's wage equation does not determine the steady state growth of the hourly wage, *lna*. Instead, the wage equation determines the equilibrium of the unemployment rate, *bul*, which will be constant in the long term, and when unemployment is constant, employment is also constant, as the long-term labour force is unchanged by assumption. Private employment is endogenous and fluctuates before reaching its equilibrium. The exogenous public employment is set to be unchanged from the first year of the baseline, see figure 10.2.

Figure 10.2 **Changes in employment**



Exogenous demand grows in line with supply

On the demand side, foreign demand is set to grow by 1.5 per cent, and with constant relative prices this will also become the growth rate of exports. It should be noted that if foreign and Danish demand do not grow in line, the resulting baseline scenario will be more complicated. For example, export price elasticity is not infinite in ADAM, so it would require a continuous decrease in the relative export price to make exports grow by more than foreign demand. Such a scenario would not represent a simple steady state with constant relative prices.

Public consumption and investment in fixed prices is also set to grow by 1.5 per cent. The nominal value of public spending on goods and services will grow by 3.5 per cent, and the same applies to expenditure on income transfers, as transfer rates are indexed to wages. The few exogenous expenditure items in ADAM are assumed to grow like indexed expenditures, and total public expenditures will grow in line with nominal GDP.

Constant public expenditure and tax levels

Public revenues consist mainly of taxes. Income tax rates and the VAT rate are constant in the baseline and excise duties are adjusted with prices. This makes public revenues grow in line with nominal GDP, and in a baseline with sustainable fiscal policy, public debt will also grow in line with nominal GDP. The interest rate equals the nominal growth rate, and in steady state the annual change in public debt will equal the interest outlay on the debt. This implies that the total public balance will equal the interest outlay, while the primary budget balance will be zero.

With these assumptions, fiscal policy is neutral and public finances will follow the general income and price growth without affecting the share of other sectors in the baseline scenario.

Total consumption increases in line with income

When tax level and terms of trade are unchanged, real disposable income will increase by 1.5 per cent similar to production, and in equilibrium, both private wealth and consumption will increase in line with income, see the illustration of central properties of the consumption function in chapter 3.

The consumption function makes total private consumption follow the general income growth; but the various components of consumption will, in principle, have different growth rates, since the components have different budget elasticity. Some budget shares will increase when per capita consumption increases, while other budget shares will fall, cf. the discussion of the consumption system in chapter 3.

Unchanged composition of consumption

In order to make all the industries grow in line, all demand components must grow in line and to achieve this, all budget elasticities in the consumption system are set to one, which is done by exogenizing the additional explanatory variables of the consumption system. These variables can make the budget elasticity differ from one, changing the budget shares in the baseline scenario. If the additional explanatory variables are kept constant, budget elasticities can be set to one.

A general budget elasticity of 1 is strictly speaking not a sufficient condition for balanced growth in consumption components. The budget shares also depend on the associated relative prices of the consumption components. However, the stylized price formation in the baseline make relative prices constant in the long term, ensuring that all consumption components grow in line with total consumption.

Investments increase in line with production

All investment components in fixed prices also end up growing by 1.5 per cent because in the steady state of the baseline, investment is proportional to capital stock, which grows in line with production when all relative factor prices are constant.

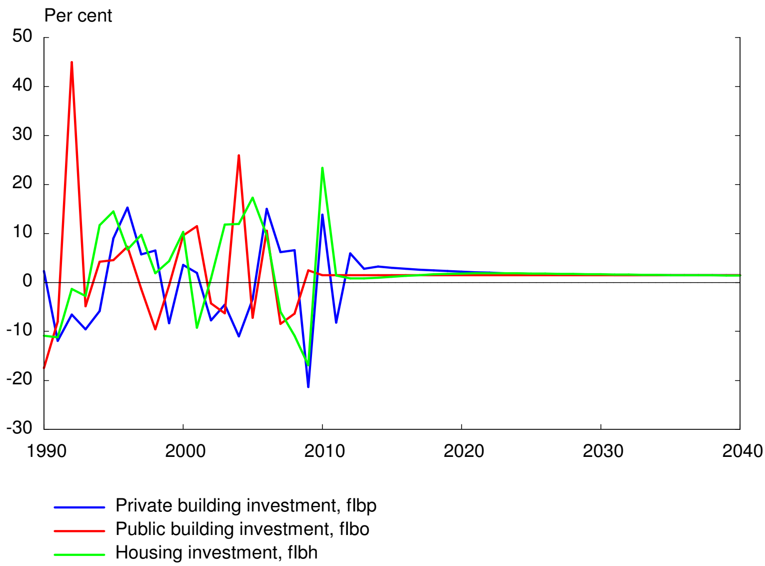
The overall price growth of 2 per cent also applies to the price of labour input measured in efficiency units. All productivity growth is related to labour as Harrod-neutral productivity growth, so that the labour cost per efficiency unit grows by the annual wage increase of 3.5 per cent minus the productivity increase of 1.5 per cent, i.e. by 2 per cent.

Imports increase in line with demand

Import components will grow with the overall growth rate of 1.5 per cent, given that the composition of demand and relative prices are unchanged, and given that the logistic trends in the import equations are horizontal and thus unchanged in the baseline scenario.

As an example of demand components in the baseline, figure 10.3 shows that it takes some years before the growth rate of private construction investments have reached the steady state, while the exogenous public building investments are set to grow by 1.5 per cent from 2010. Machinery investments and private consumption reach the steady state faster than building investments, but it must be noted that the model-generated fluctuations in building investments are smaller than the fluctuations in the historical period.

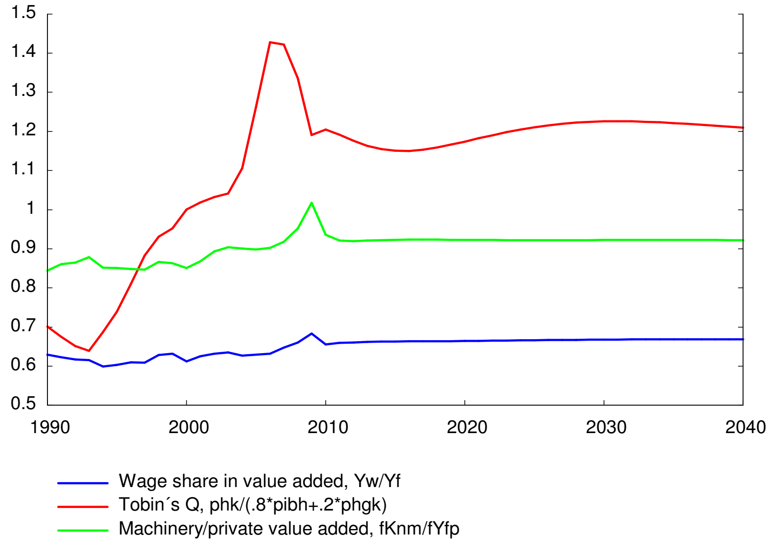
Figure 10.3 **Growth in building investments**



Wage share and other key ratios become constant

The unchanged relative factor prices imply that the wage share is constant in each industry, and as the composition of industries is constant as well, there is no composition effect on the overall wage share, which consequently remains unchanged in the long term, cf. figure 10.4. The same reasoning applies to the capital-output ratio represented by private machinery over private value added in the figure. On the whole, most key ratios in ADAM become constant, when the baseline scenario reaches the steady state. A third example is Tobin's q, i.e. the ratio of the market price of a house relative to the cost of building a new house. It takes quite some time for the housing market to adapt to its equilibrium, and Tobin's q is the last to reach its constant steady state among the three key ratios shown in figure 10.4.

Figure 10.4 **Wage share, capital-output ratio and Tobin's q**



10.2 Corrections of estimated behavioural equations

When the model is used in a projection, it is necessary to decide on the extrapolation of residuals in estimated behavioural equations. The natural starting point is to set the residual to zero when the equation is estimated under the assumption that the error term is random with mean zero. If the residual is set to zero in the projection, the equation is used as it is estimated, and if the residual is set to something else, the estimated equation is corrected.

In practice, it is rare to set the residuals of behavioural equations to zero in short-term forecasts, where you often have some information that is not captured by the equation. In fact, the residual of an equation is often referred to as the adjustment term of the equation.

In long projections with little focus on the first years of the projection period, it is more common to set an adjustment term to zero, if that is its mean in the estimation period. However, zero is not necessarily the best choice. You can choose to let the adjustment term differ from zero in a long projection, for example, if the later years of observations suggest that the equation has broken down, implying that the residual does not appear to be randomly distributed around zero.

For long-term projections with ADAM, it is also a problem that the estimated long-term relationship between variables may be affected by the growth rate of the variables, and these effects can be neutralized by correcting for changes in the trend of real variables and prices.

The rest of the present section on corrections of behavioural equations is divided into three sub-sections: the first sub-section discusses trend correction to preserve the long-term equation, the second sub-section exemplifies correction for structural breaks, and the third sub-section illustrates how the left-hand-side variable responds to the adjustment term of the equation.

10.2.1 Trend correction

ADAM's estimated equations are typically formulated in an error correction form, combining a long-term level equation with some short-term dynamics, where the variables appear in changes. However, this formal division into long- and short-term parts does not prevent that first order variables changes, such as growth and inflation rates, can affect the long-term solution of the error correction equation.

Strictly speaking, an undesirable dynamic effect on the equilibrium solution should be countered already in the estimation phase by imposing restrictions on the short-term coefficients in order to prevent the dynamics from affecting the long-term solution. For example, inflation does not affect long-term unemployment if inflation has a coefficient of 1 in the wage equation; cf. the discussion of wage equation properties in chapter 7.

In practice, it is difficult to remove the problems by restrictions without distorting the equation. Thus, it may be advantageous to refrain from imposing strong restrictions and instead use the adjustment term of the equation. For instance, in a fixed-exchange-rate regime expected inflation may be better represented by a constant than by the actual price increase. In this case, it seems reasonable to adjust the constant of the wage equation, if we change the baseline inflation rate.

The issue of trend correction, i.e. of adjusting equations in order to correct for changes in trend growth, is made clear by the explicit breakdown in long-term and short-term parts in ADAM's error-correcting equations, and the issue was discussed in the section on consumption equation estimation in chapter 3, box 3.2. The box showed that the transition, from the growth of the estimation period to the baseline growth trend, permanently increases actual consumption relative to the desired consumption, but only by 0.5 per cent. However, there are other error correction equations, where the transition to baseline growth has a larger impact.

Trend correction of manufacturing exports

When establishing the baseline scenario, it is chosen to trend-correct the bulk of ADAM's error correction equations, including the manufacturing export equation, which in (10.1) is given in a simplified version without: suffixes with SITC number, German reunification dummy and contemporaneous change in relative export prices. Thus, there is only one first order difference, Dlog(*fEe*), on the right hand side of the equation, *fEe* is export market and *fEw* is the desired export.

(10.1) $$D\log(fE) = 0.0233 + 0.62 \cdot D\log(fEe) - 0.15 \cdot \log(fE_{-1} / fEw_{-1})$$

In order to clarify the effect of short-term dynamics on the long-term solution, the error correcting equation in (10.1) is rewritten so that the export level equals the desired export minus an expression in square brackets with the first order differences and constant divided by the error correction parameter of 0.15:

(10.1)* $$\log(fE) = \log(fEw) - \left[\mathrm{Dlog}(fE_{+1}) - 0.0233 - 0.62 \cdot D\log(fEe_{+1})\right] / 0.15$$

In a steady state scenario with unchanged competitiveness, the market share is constant so that export growth, Dlog(*fE*), equals market growth, Dlog(*fEe*). The estimated coefficient for market growth is not 1 but 0.62, so the value of the square brackets in (10.1*) will vary with the selected growth rate. It is thus only by chance if the expression in square brackets is zero, and if this expression deviates from zero, the actual export *fE* will deviate from the desired export *fEw*, according to (10.1*).

Without trend correction export is 12 per cent above the desired level

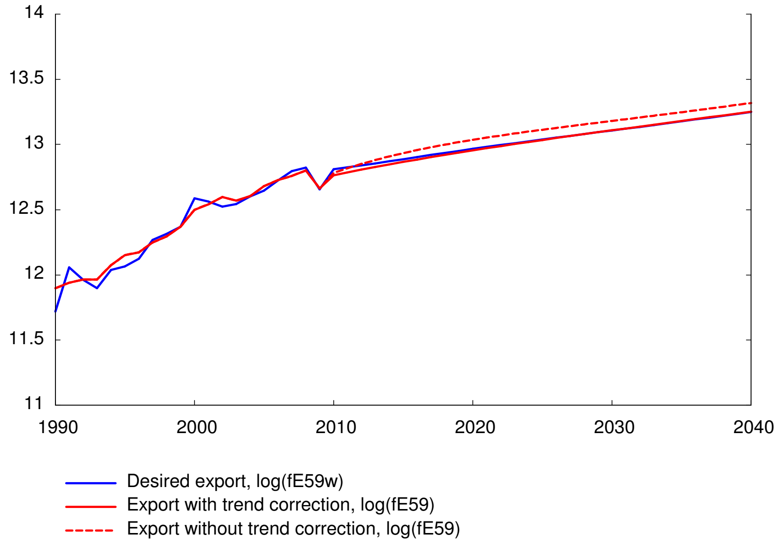
If the manufacturing export equation is not trend-corrected, the transition, from the export growth in the estimation period to the general baseline growth of 1.5 per cent, will lift actual export by 12 per cent[1] relative to desired export. This is illustrated in figure 10.5, where the desired manufacturing export, *fe59w* in ADAM notation, is compared to the actual manufacturing export, *fe59*, projected both with trend correction as in the baseline scenario and without trend correction.

The sample of figure 10.5 starts in 1990, so the figure illustrates that the baseline growth is lower than the export growth in the previous decades. The intuitive explanation, behind the tendency for the actual export to increase relative to the desired export, is that adaptation to the desired level is less delayed when the market is growing slower.

It must be emphasized that it can be chosen not to trend-correct manufacturing export and accept the export level resulting from the estimated error correction equation. The purpose of trend correction in the baseline scenario is very much to clarify that the dynamics of an error correction equation can affect the long-term solution, and that we can neutralize this effect by adjusting the equation.

1 In the baseline, volumes grow by 1.5 per cent p.a. in the long term. This gives the square bracket expression in (10.1*) a value of -0.0176 (=0.015-.0233-0.62·0.015) and not 0 as in the estimation period. Thus, log(fE) is increased by 0.117 (=0.0176/0.15) relative to log(fEw).

Figure 10.5 **Manufacturing exports in baseline scenario**



Trend correction of wage equation

The wage equation is probably the most central equation that should be considered for trend correction by a model user. ADAM's wage equation is formulated as a combination of a long-term equation for unemployment and an error correction equation that determines the increase in wages, *lna*, as a function of primarily the difference between actual and long-term unemployment, respectively *bul* and *bulw*, but also of e.g. the inflation rate with a coefficient of only 0.3.

The low coefficient of inflation implies that the estimated equation must be corrected, if a permanent effect on unemployment of a permanent change in inflation is to be avoided. This feature of the wage equation was illustrated in chapter 7 with a stylized numerical example, in which the wage equation was supplemented by a couple of auxiliary equations illustrating how exports and prices in ADAM are affected by wages.

It can now be illustrated by means of a calculation on the full ADAM model that the transition, from wage and price inflation in the estimation period to wage and price inflation in the baseline, will produce an unemployment rate above 4.5 per cent in the long term. This is 1 per cent more than the unemployment rate of 3.5 per cent that follows from the long-term equation. Box 10.1 explains why the long-term unemployment rate reacts by 1 per cent.

Box 10.1 **Trend correction in the wage equation**

ADAM's wage equation determines the increase in hourly wages, *lna*, as a function of a constant, four short-term variables and the deviation of unemployment from its structural value, *bulw:*

$$\text{Dlog}(lna) = 0.0347 + 0.3 \cdot \text{Dlog}(pcpn^{0.5} \cdot pyfbx^{0.5}) + 0.021 \cdot d8587$$
$$+ 0.320 \cdot \text{Dif}(\text{Dlog}(lna_{-1})) - 0.259 \cdot \text{Dif}(bul) - 0.55 \cdot (bul_{-1} - bulw_{-1})$$

In the wage equation, the inflation term is the increase in the geometric mean of "net" consumer prices, *pcpn*, (net of indirect taxes) and the value added deflator of private urban industries, *pyfbx*. Variable *d8587* is a dummy, cf. the description in chapter 7. The wage equation implies that the unemployment rate, *bul*, is given by:

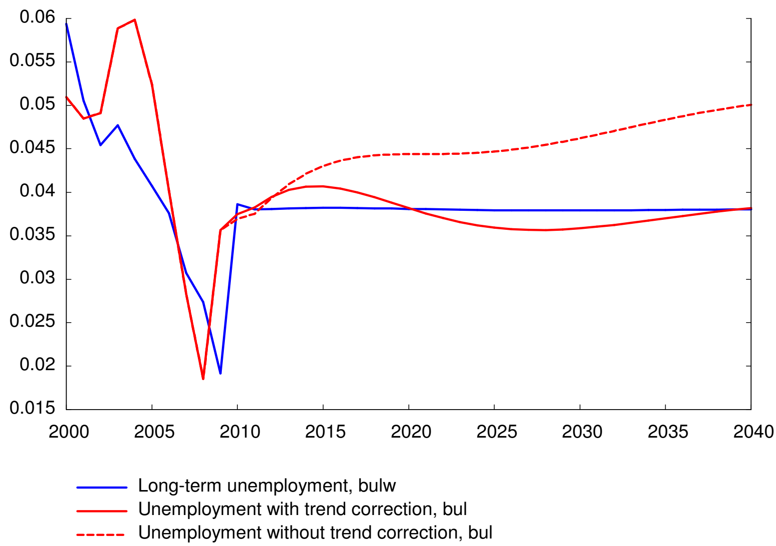
$$bul = bulw - [\text{Dlog}(lna_{+1}) - 0.0347 - 0.3 \cdot \text{Dlog}(pcpn_{+1}{}^{0.5} \cdot pyfbx_{+1}{}^{0.5})$$
$$- 0.021 \cdot d8587_{+1} - 0.320 \cdot \text{Dif}(\text{dlog}(lna)) + 0.259 \cdot \text{Dif}(bul_{+1})] / 0.55$$

The OLS estimation implies that the expression in square brackets has an average of 0 in the estimation period. In the baseline scenario, the expression has the value -0.0057 (=0.035-0.0347-0.3·0.02), as dummy and changes in wage growth and in unemployment are zero.

Consequently, the baseline unemployment rate, *bul*, equals the structural rate, *bulw*, plus approximately 0.01 (=0.0057/0.55). If, however, you trend correct by reducing the constant 0.0347 by 0.0057 to 0.029, the square bracket expression turns 0, and baseline unemployment, *bul*, will equal *bulw* of the long-term equation.

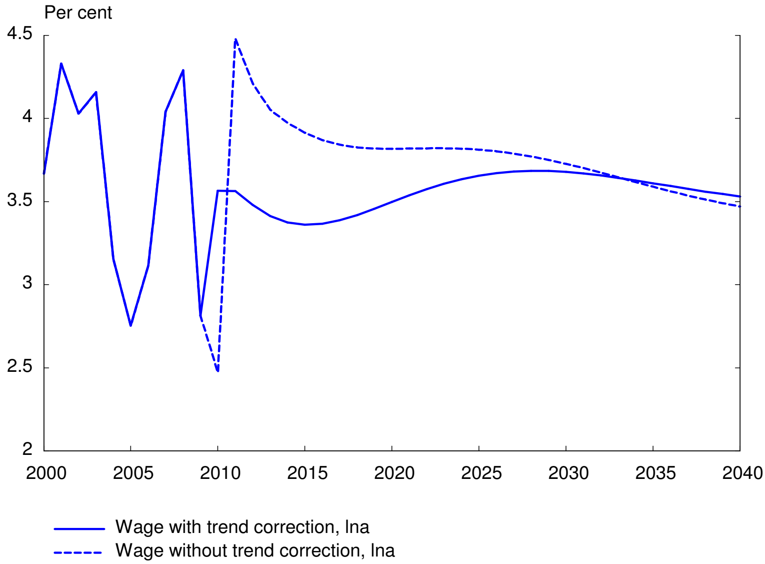
If the wage equation is trend-corrected according to the same principles as consumption and export equations, baseline unemployment will equal long-term unemployment, *bulw*, see figure 10.6 for illustration.

Figure 10.6 **Unemployment with and without trend correction**



In the calculation without trend correction, actual unemployment rises relative to the structural unemployment throughout the period shown in figure 10.6. This reflects the fact that it takes time to reach the long-term solution of *bulw* +0.01.

Figure 10.7 **Wage increase with and without trend correction**



It is noted that trend-correcting the wage equation affects only long-term unemployment and not long-term wage growth. Wage growth in the baseline scenario remains at 3.5 per cent regardless of the trend correction, as illustrated in figure 10.7. If we do not trend correct by reducing the constant, wage growth in the first years will exceed 3.5 per cent, and this will reduce market shares and increase unemployment. In the long term, we reach a new equilibrium with higher unemployment and an hourly wage that is higher than in the baseline scenario, but increases in line with the baseline scenario. The higher wage level fits with the smaller employment and smaller need for competitiveness when long-term unemployment is higher.

By trend-correcting the equations for wage and export, it can be achieved that the estimated long-term behavioural equations for unemployment and export hold, i.e. *bul*=*bulw* and *fE*=*fEw*, regardless of the growth trajectory chosen. The same can be achieved with respect to the other long-term behavioural equations by trend-correcting all behavioural equations. This procedure would make ADAM appear as a theoretically based model with clear long-term behavioural equations.

It is particularly relevant to consider a systematic trend correction of behavioural equations, if we are calculating the effect of changing the rate of growth or inflation. In relation to long projections like the baseline, focus is mainly on the issue of trend-correcting the wage equation, as this affects long-term unemployment, and trend-correcting selected equations, where the transition from historical to projected growth is important for the long-term level of the related variable.

The next sub-section discusses correction for structural breaks using the housing supply equation as an example.

10.2.2 Structural break correction

Housing stock equation as example of break correction

In the baseline scenario for ADAM, a few equation residuals are extended by repeating their last historical value. For example, the large residual in the housing stock equation is repeated in the baseline period. This equation determines the growth in the housing stock, *fKbh*, and the equation is shown below, except for a logistic trend variable that has turned into a constant at the end of the estimation sample.

(10.2)

$$\mathrm{Dlog}(fKbh) = 0.015 \cdot \mathrm{Dlog}\left(\frac{phk}{.8 \cdot pibh + .2 \cdot phgk}\right) + 0.862 \cdot \frac{nbs}{fkbh_{-1}}$$
$$+0.025 \cdot \log\left(\frac{phk_{-1}}{.8 \cdot pibh_{-1} + .2 \cdot phgk_{-1}}\right) + 0.0177 + Adterm$$

The increase in housing stock is mainly determined by the level of Tobin's q, $phk/(0.8 \cdot pib + 0.2 \cdot phgk)$, where *phk* in the numerator indicates the house price, while the weighted sum of investment price, *pibh*, and land price, *phgk*, indicates the cost of a new house. The effect of Tobin’s q is supplemented in (10.2) by the effect of: the change in Tobin's q, the number of supported new dwellings, *nbs*, and a constant. Besides, the adjustment term, *Adterm*, contains the equation residual.

The housing stock equation implies that the growth in housing stock mainly depends on Tobin's q, but as discussed in chapter 3 when presenting ADAM’s housing equations, it appears that the relationship has shifted in recent years, requiring a higher Tobin's q to achieve a particular growth in the housing stock. The apparent break in the relationship between Tobin's q and the growth in housing stock may reflect that the applied land price variable mainly describes the price of newly parcelled out land and underestimates how much the price of land lifted the total price, *phk*, of existing houses during the last surge in house prices. This measurement problem can, in principle, be dealt with by constructing another price index for land, but in the short term, we can only correct the induced break in the estimated equation.

Structural break indicated by housing equation residual

A structural break is indicated by the housing stock equation residual, which is clearly negative in the last years of the historical period. It has been decided to correct the equation by maintaining the level, which the residual reached by the end of the historical period. Thus, the adjustment term, *Adterm*, is -0.01 in the baseline scenario and not 0 as in in the estimation period. This type of correction is in accordance with Clements and Hendry (1996), who argue that the average bias of your forecast will be smaller, if the recent historical level of residuals is applied in your forecast, and a smaller bias can be an advantage, although the procedure may increase the standard error of the forecast.

Housing stock equation error-corrects the house price ...

ADAM's housing stock equation does not look like a traditional error-correcting equation, but it does work as an error-correcting equation determining the house price level in the long term, because deviations from the long-term value of Tobin's q affects the supply of housing stock and hence also the house price. This indirect error correction and determination of the house price is analogous to the determination of long-term unemployment in the wage equation.

The housing stock equation is not formulated in an error correction form. However, if the equation residual, *Adterm*, is reduced from its estimation average of 0 to -0.00078, this will balance the transition from historical growth rates to the baseline growth rate, and Tobin's q will in the long term attain its estimation-period average of 0.85. [2]

... and correcting for break shifts the long-term house price

However, it is not expedient that Tobin's q attains its estimation average, if the housing stock equation has broken down. As already mentioned, the equation adjustment term, Adterm, is set to -0.01 in the baseline scenario and not to -0.00078, corresponding to a pure trend correction of the equation constant. The difference of -0.00922 constitutes a negative correction of almost 1 per cent to the annual growth in the housing stock, and this reduction in the equation for housing supply implies that Tobin's q is about 40 per cent higher in the baseline scenario than in the estimation period.[3]

The impact of increasing the level of Tobin's q is illustrated in figure 10.8. The underlying negative correction for structural break in the housing stock equation works as a rise in the price of supplying a house.

All three price indices included in Tobin q are 1 in 2000, so Tobin's q is constructed to be 1 in 2000. Without the correction of the long-term level for Tobin's q, the house price and Tobin's q in 2000 appear to be around 20 per cent above the long-term equilibrium. With level correction, the house price and Tobin's q in 2000 appear to be 20 per cent below the long-term equilibrium.

Difficult to assess the long-term house price

It is particularly difficult to determine the long-term house price based on the estimated supply equation for housing. It is difficult because the equation seems to have a structural break, and it is also difficult because the slow adaptation of the housing stock, i.e. the moderate coefficient of 0.025 for Tobin's q in (10.2), implies that the long-term house price is sensitive to minor adjustments in the supply equation.

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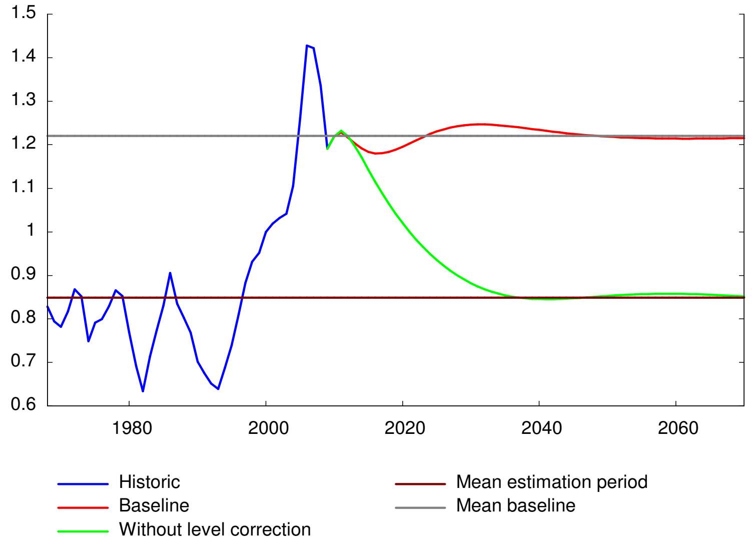
[2] Short-term dynamics in (10.2) comprises the left hand side variable and the first two variables on the right hand side as well and also the change in a logistic trend not shown in (2.10). The -0.00078 indicates the difference between the value of short-term dynamics in the estimation average and in the baseline.

[3] The effect of structural break correction on the long-term Tobin's q can be derived by normalizing the housing stock equation on the log of Tobin's q. The normalization implies that the residual, Adterm, on the right hand side, is divided by the coefficient of 0.025 for the log of Tobin's q. Consequently, the log of Tobin's q is changed by 0.369 (=0.00922/0.025).

Figure 10.8 **Tobin's Q with and without level correction**



10.2.3 Role of adjustment terms

Adjustment terms are often used in forecasts

We have just seen that the adjustment term in the housing stock equation may have a great impact on the house price in a long-term scenario. When establishing a normal forecast, the model user is more interested in the short-term effects of the adjustment terms. For example, it can generate a peculiar jump in the left-hand-side variable, if a systematic negative residual is set to zero in the forecast period.

Easy to adjust in first forecast year

When formulating a forecast, it can be chosen to deviate from the estimated equation whenever there are reasons to believe that, for instance, consumption will increase by more or less than the consumption equation indicates. Technically speaking, it is easy to make consumption in a particular year rise by, e.g. 1 per cent more than the equation says. You simply increase the adjustment term of the consumption equation by 0.01 in that year. It may, however, be harder to relate to the adjustment term in the following year.

In subsequent years adjustment and dynamics interact

If the 1 per cent adjustment in the consumption function is maintained in subsequent years, the effect on consumption, for given income and wealth, will increase and become larger than 1 per cent. This reflects the dynamics of the equation. Consumption equation (3.2a) from chapter 3 is replicated below with the constants of (3.2a), replaced by an adjustment term, *Adterm*, which also contains the equation residual.

$$\text{Dlog}(C) = 0.4 \cdot \text{Dlog}(Y) + 0.407 \cdot \log\left(\frac{Y_{-1}^{0.9} \cdot W_{-1}^{0.1}}{C_{-1}}\right) + Adterm$$

C is consumption, and the explanatory variables are income, *Y*, and wealth, *W*. Income and wealth are exogenous in the present context where we focus on the consumption equation and its adjustment term.

Maintained adjustment

If *Adterm* is increased from 0 to 0.01, consumption immediately becomes 1 per cent higher than its baseline. Maintaining the adjustment term of 0.01 in subsequent years will gradually increase the effect on consumption, and the long-term effect on consumption is almost 2.5 per cent (=0.01/0.407). This reflects that the negative error correction created by the additional consumption of 2.5 per cent will balance the adjustment term of 0.01.

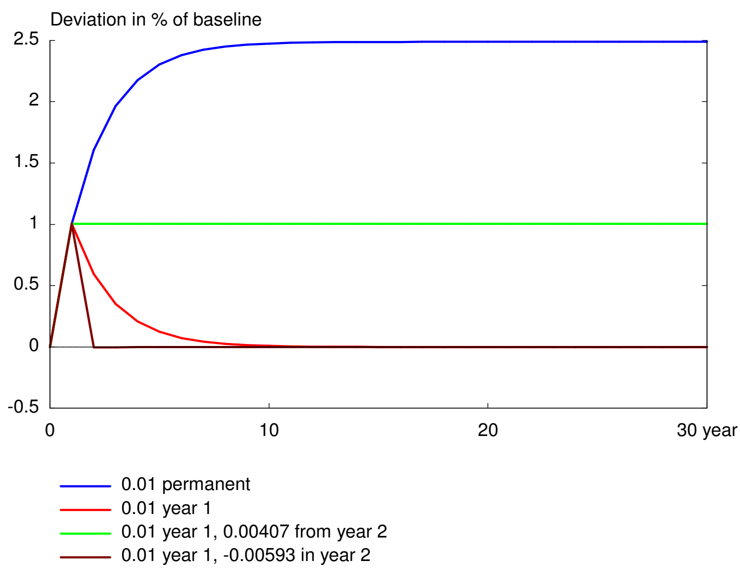
If you want to raise consumption by 1 per cent in any year, the adjustment term should still be raised by 0.01 in year 1, but only by 0.407·0.010 from year 2 and onwards. The latter adjustment is equivalent to adjusting the long term equation by 0.01 from year 1, as the long-term residual enters the consumption equation with a lag.

Temporary adjustment

If the adjustment term is only increased by 0.01 in year 1, the impact on consumption will gradually disappear in subsequent years, so the long-term effect of a temporary increase in the adjustment term is zero.

If you want the consumption effect to disappear in year 2, the adjustment term must be set to about minus 0.006 in year 2, because the error-correcting mechanism only creates a consumption decrease of 0.4 per cent in year 2. With this adjustment, consumption increases by 1 per cent in year 1 and returns to the baseline in year 2. See figure 10.9 for an illustration of the mentioned changes in the adjustment term.

Figure 10.9 **Consumption effect of adjusting the consumption equation**



This concludes the comments on the practical use of adjustment terms in forecasting, but the effect of a permanent 1 per cent adjustment in the consumption equation is analysed by an ADAM-calculation in chapter 11, section 2. Using ADAM involves the interaction with the modelled wealth and income formation, and it will also be discussed how the long-term consumption effect is influenced if public debt is restricted to constitute a constant share of GDP in the long term.

10.3 Savings and investments at different growth rates

By a general use of trend correction, we can avoid that a change in the growth rate will affect the long-term behavioural equations cf. section 10.2. This clarifies the long-term properties of the model, but the trend correction does not imply that the long-term solution is unaffected by the growth rate, because the growth rate affects the long-term solution via the dynamic identities in the model.

Long-term savings depend on rate of growth ...

As an example, we can take the basic mechanism that the savings ratio depends on growth rates. The mechanism can be illustrated by supplementing the consumption equation with the definitional equation of wealth.

... although the long-term consumption equation does not

The error correction term in the consumption equation is 0.009 in the baseline scenario, where volumes grow by 1.5 per cent, cf. box 1 in chapter 3.1. If the annual growth rate is raised to 3 per cent in a new baseline, the long-term consumption equation can be maintained by increasing the trend correction term to 0.018 (=0.03-0.4·0.03) and, if the growth rate is reduced to zero, the long-term equation is maintained by reducing the trend correction term to 0 (=0-0.4·0). With the mentioned trend corrections in force, long-term consumption will in all three baselines be given by the long-term equation included in (3.2a), i.e. by:

$$\log(C) = 0.9 \cdot \log(Y) + 0.1 \cdot \log(W) - 0.200$$

But although the long-term consumption equation is unchanged, both consumption and wealth ratios are affected by a transition to either 3 or 0 per cent growth.

The higher the growth, the lower the consumption and wealth ratios

Income is exogenous in a two-equation model with consumption and wealth equation, and if real income growth changes from 1.5 to 3 per cent, both consumption and wealth must eventually grow by 3 per cent. Higher wealth growth requires higher savings out of income, and eventually a new equilibrium will emerge with a lower consumption ratio, C/Y, compared to the initial baseline, see figure 10.10a.

The mechanism is that wealth will grow less than income in the first years of a new baseline where income growth is increased to 3 per cent. The decline in wealth ratio, W/Y, will moderate consumption and increase the savings ratio. Larger savings make wealth grow faster, and eventually both consumption and wealth will grow at the same rate as income, but both consumption and wealth ratios will become lower if income accelerates. If the growth rate falls, e.g. to zero, consumption and wealth ratios will increase, cf. figure 10.10.

Figure 10.10a **Stylized consumption ratios, three growth rates with trend correction**

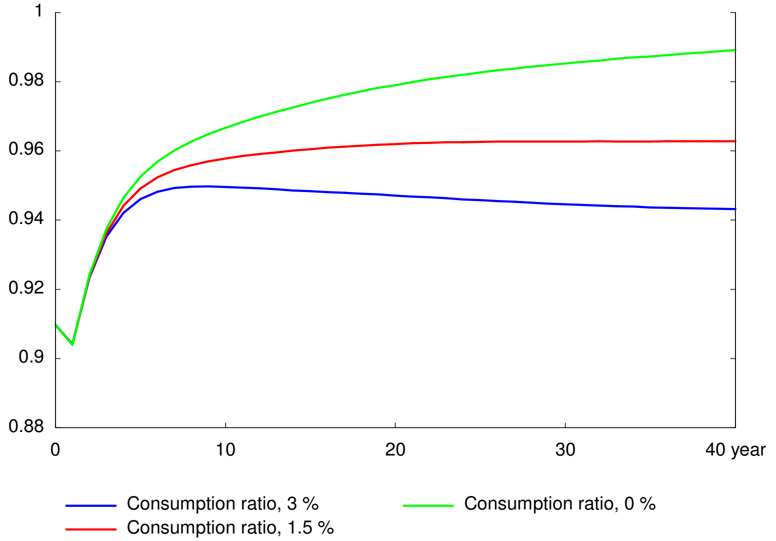
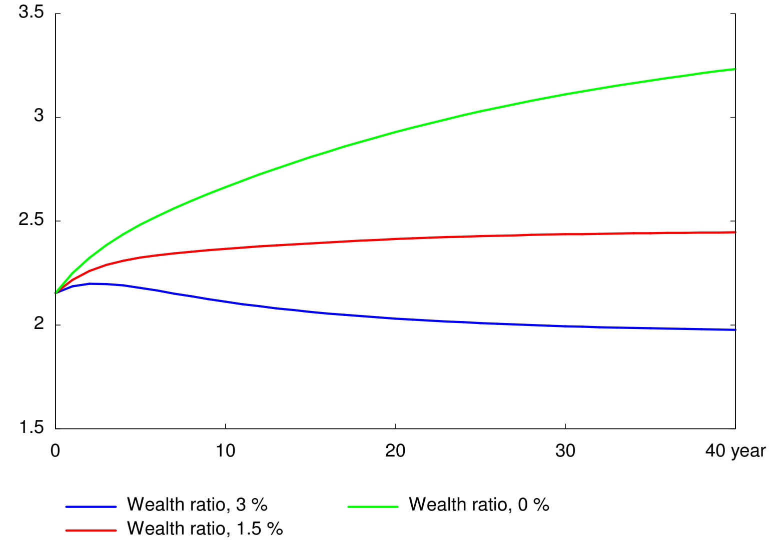


Figure 10.10b **Stylized wealth ratios, three growth rates with trend correction**



Zero growth gives consumption ratio 1

More specifically, the consumption ratio will become 1 in a stationary baseline scenario. In the baseline with 1.5 per cent growth, the wealth/income ratio stabilizes at almost 2.5, see Figure 10.10b. The savings ratio equals the growth rate multiplied by the wealth ratio, i.e.

0.037 (=0.015·2.5), implying that the consumption ratio is 0.963. In the baseline with 3 per cent growth, the wealth ratio falls in the long run to 2, and the consumption ratio falls to 0.94 (=1-.03·2). In the stationary baseline, the wealth ratio approaches a long-term level above 3.5. This wealth ratio is relatively far from the starting point of 2.5 at 1.5 per cent growth, and it takes a long time to reach the stationary equilibrium. It is noted that the size of the wealth ratio depends on the growth rate and on the long-term consumption equation.

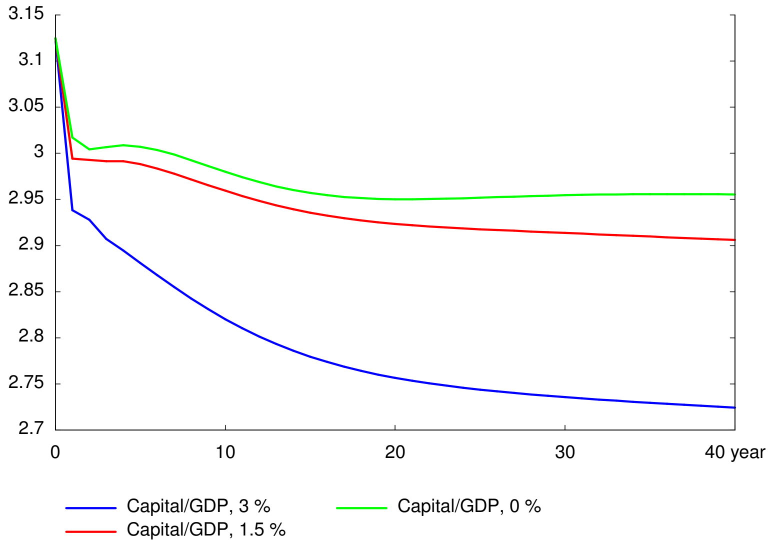
The effect of growth trend on user cost

As previously mentioned, the standard baseline assumes that the real interest rate equals real growth rate, so if the assumed growth rate is raised from 1.5 to 3 per cent, it is natural to raise the real interest rate equivalently, which raises the user cost. The higher cost of capital input implies that there will be less capital per employee, and the same labour force will produce slightly less. [4]

Effect on the capital-output ratio ...

The impact of the interest rate and thus of user cost on the use of capital can be illustrated by comparing the capital-output ratio in a baseline with 1.5 per cent growth with two baselines where the growth rate is, respectively, 0 and 3 per cent. Capital-output ratio is highest with a stationary baseline, where the real interest rate is zero, see figure 10.11.

Figure 10.11 **Total capital-output ratios, three baselines**

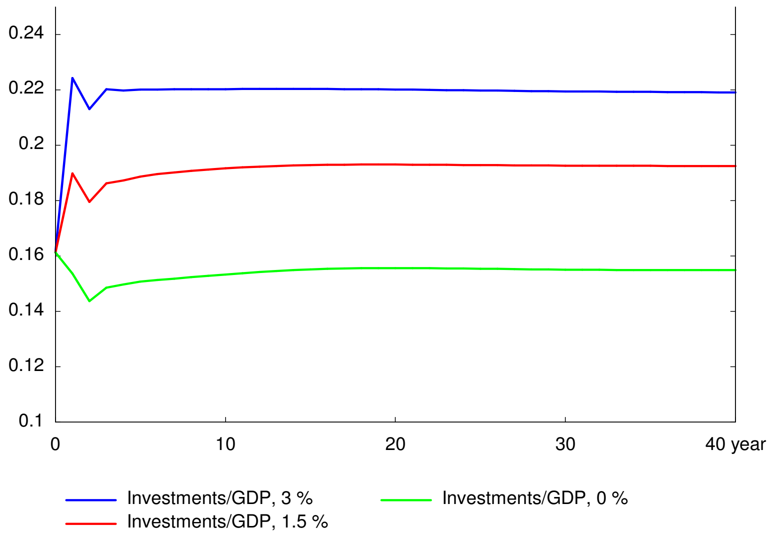


[4] It can be added that a change in baseline inflation also affects the cost of capital in ADAM. In a simple economy without taxes, user cost includes a simple real interest rate, and inflation is irrelevant to user cost, if the nominal interest rate equals nominal growth. However, the tax system is not neutral, and in ADAM both long-term inflation and tax rate affect user cost for a given real interest rate before tax. In addition, the user cost of housing capital assumes that half the consumers are credit rationed and will react to nominal rather than real interest rates.

... and on the level of investment

While the input of capital is highest in the stationary baseline, the lack of growth removes the need for investments to increase the capital stock. Net investment will thus be zero and the level of investment will be smaller than in scenarios with positive growth rates, cf. figure 10.12.

Figure 10.12 **Investment over value added, three baselines**



It may be added that the lower level of investment implies that investments react more strongly in percentage terms, when production changes relative to a stationary baseline than when it is a baseline with growth.

Summary of chapter 10

In the baseline scenario, all volumes grow pari passu in the long term and the same applies to prices. With a constant labour force, output growth is created by productivity growth, but to achieve a uniform growth rate, the productivity assumption must be supplemented with stylized assumptions about similar growth in foreign markets and in public demand, also the consumption system must be adjusted to give all consumption components a budget elasticity of 1 to make the components grow in line with the baseline rate of growth.

The adjustment terms in the estimated behavioural equations can be used to preserve the estimated long-term equations, so that the impact of changes in the growth trend of variables is balanced. It is also possible to balance the impact of structural breaks by continuing the latest historical level of adjustment terms in the relevant equations.

The higher the baseline growth, the higher the baseline savings and investment will be.

11. Model analysis

The easiest way to analyse the properties of ADAM is to make a series of model calculations and interpret the results. This chapter uses this approach and examines a number of model calculations that describe the effect of changing one or more selected exogenous variables. In one case, some key parameters of the model are changed in order to see the effect on the result of a model calculation.

The chapter starts with four shocks to the demand side in ADAM. In section 11.1, government purchases of goods are raised, in 11.2 the private propensity to consume is raised, in 11.3 interest rates are reduced, and in 11.4 all foreign prices are reduced. This is followed by a supply shock in section 11.5, where the supply of labour is increased.

The discussion of the first demand shock, the increase in government purchases, is relatively long as it introduces a number of features common to all model calculations presented in the chapter. For example, the effect of higher government purchases is also calculated under the assumption that an accompanying tax increase funds the additional purchases and neutralises the long-term effect on public debt.

All demand shocks have a zero effect on employment in the long term, but shocks differ as to what happens to the composition of demand and production. The supply shock results in a permanent increase of employment.

In order to illustrate the long-term properties of ADAM, the baseline and model calculations extend 60 years into the future. In the long term, ADAM works in many respects like a growth model that is kept going by labour input and productivity, while the Phillips curve ensures that employment is equivalent to labour force minus long-term unemployment, cf. the discussion of the stylized baseline scenario in the previous chapter. Long-term model effects are of course uncertain, since the estimated equations hardly remain unchanged for so many years. However, despite the uncertainty, long-term effects are of interest because they describe how ADAM works. The many equations in ADAM cannot be solved analytically, so it requires model simulations with a long time horizon to analyse the equilibrium of the model.

Section 11.6 examines how the adaptation to the public purchase shock depends on the coefficients of the behavioural equations. If, for instance, the export price elasticity is increased, the model reacts more quickly to all shocks, which move the model away from its equilibrium. On the other hand, the model will fluctuate more strongly. The section also discusses the effect of basing user cost on exogenous inflation expectations.

Section 11.7 compares the public purchase calculation with a similar calculation on the Economic Council SMEC model and also with the calibrated adaptation speed in Sorensen and Whitta-Jacobsen (2005).

11.1 Raising public purchase of goods by 1 per cent

Public purchase of goods from private industries is a traditional fiscal variable. It constitutes a component in total demand, and shocks to public purchase are often used to illustrate model properties.

The main results of increasing public purchases have been mentioned in connection with the presentation of ADAM in chapter 2. In the present section, we return to the public purchase shock and discuss the calculated effects in greater detail, but before discussing the effects, we start by commenting on the calculation design.

Public purchase of goods is increased relative to the baseline scenario

The baseline scenario used by the model calculations in this chapter is a stylized projection described in chapter 10. Production and demand, including public purchases, grow in real terms by 1.5 per cent in the baseline, making in the long run a difference whether a demand component is increased by a certain amount or by a certain percentage of its baseline value. It has been decided to do the latter and increase public purchase by 1 per cent in all years, corresponding to an increase by DKK 1.5 bn., 2005-prices, in the first year of the baseline period.

ADAM is dominated by non-linear equations

If ADAM were dominated by linear equations, it would be natural to illustrate the effect of increasing the public purchase by a specific amount in 2005-prices. The key equations are, however, often log-linear suggesting a more proportional relationship between, e.g. consumption and income, and this speaks in favour of formulating shocks in percentage terms.

It also speaks in favour of percentage shocks that there is growth in the baseline scenario. For example, the annual increase in labour productivity implies that the employment effect of a fixed shock in fixed prices would decrease throughout the calculation period. It is noted that the difference between shocks in relative or absolute terms vanish if the baseline is stationary, so that productivity and other variables, including public purchase have the same value in all years. As discussed in chapter 10, it has been chosen to make the model calculations in relation to a baseline with growth, because the normal starting point for ADAM calculations is a projection with growth.

Alternative scenarios and multipliers

When the public purchase variable, *fVmo*, is increased in the baseline scenario by 1 per cent and the model is solved, a new solution appears, called an alternative scenario. The difference between an endogenous variable in the alternative scenario and in the baseline scenario measures the effect on that variable of increasing the public purchase of goods. These effects are often referred to as multipliers.

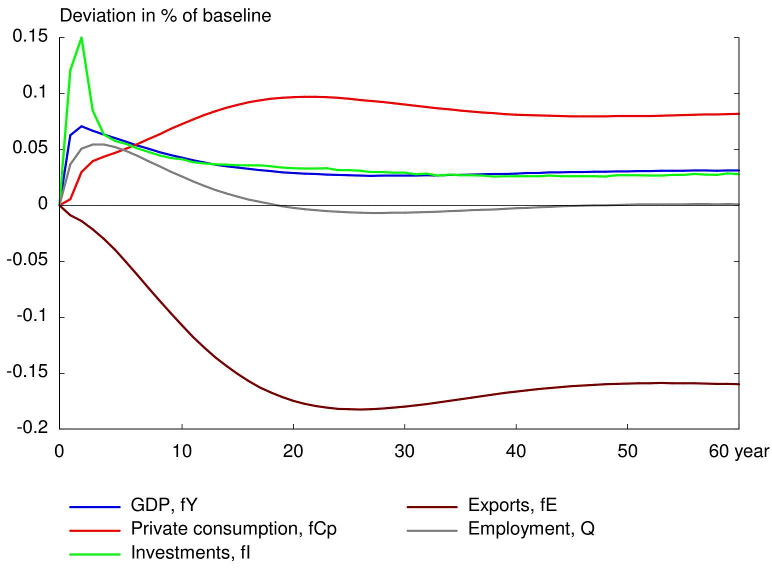
The effect on a variable can specify both simple and relative difference between the variable in alternative and baseline scenarios. For many volume and price variables, it is natural to indicate the relative difference when public purchases are increased by 1 per cent. In general, it depends on the nature of the variable and on the purpose of the calculation, whether it is chosen to specify effects as simple or relative differences. For example, it is easier to relate to the percentage change in GDP than to the percentage change in the public budget balance, especially if the balance crosses zero in the baseline or in the alternative scenario. In relation to the public budget balance, it is easier to use the simple difference, e.g. measured as a percentage of GDP.

Now, we turn to the ADAM-calculated effects of raising public purchase of goods by 1 per cent and start by looking at long-term effects.

11.1.1 Long-term effect

Figure 11.1 presents the multipliers calculated as relative deviations to the baseline scenario. Moreover, GDP and demand variables are measured in real terms in figure 11.1 and also in the other figures of the present chapter, unless otherwise stated. If the alternative scenario for a variable represents a new steady-state solution with the same growth rate as in the baseline scenario, the new values of the variable will eventually move in parallel to the baseline scenario, and the relative effects shown will be constant in the long term. The time axis of the figure is given in terms of years, and figure 11.1 covers a calculation period of 60 years in order to illustrate whether the effects become constant in the long term.

Figure 11.1 **Public purchase of goods + 1%**



Long-term employment effect is zero, figure indicates other long-term effects

It is not a coincidence that the long-term employment effect is zero. A long-term employment effect of zero is a general result for a small open economy with fixed exchange rate and a Phillips curve. For the other variables in the figure, and indeed for most variables in the model, it is difficult to quantify the long-term effect, a priori. For such variables, reliance must be made on model simulations and the long-term effect must be found in a figure like 11.1.

Investment reacts quickly, export slowly

It must be emphasized that the long-term effect can differ considerably from the short-term effect. For example, investments react quickly and strongly to higher public purchases, so that the long-term response in investments is less than the short-term response. For exports, it is vice versa. Exports are slow to react, and the long-term effect on exports is clearly larger than the short-term effect.

Crowding-out on the labour market ...

As already mentioned, the long-term effect on employment is zero in figure 11.1, reflecting that the unemployment rate returns to its long-term level in the baseline scenario. The underlying crowding-out process starts by a wage acceleration created by the initial fall in unemployment, and wages and prices will increase relative to the baseline scenario. When the Danish export price rises relative to the foreign market price, exports will begin to lose market shares and decline relative to the baseline scenario.

... significantly reduces exports ...

The crowding process stops only when exports have fallen so much that employment and unemployment return to the baseline scenario. When unemployment is back on baseline, the Danish price increase equals the price increase of foreign competitors, and this is a necessary condition for the economy to be in steady state with constant market shares.

... and increases consumption ...

The positive long-term effect on private consumption reflects that the higher Danish wage lifts the purchasing power of consumers, because the price of imported goods is exogenous.

... plus investment

The positive long-term impact on investment reflects the fact that capital requirements are increased, because the increase in labour costs has made capital relatively cheaper than labour, substituting labour for capital. The larger capital stock increases the need for investment to cover depreciation, while increasing production per employed.

GDP has increased permanently due to higher capital intensity ...

Although the long-term employment effect is zero, there is a positive long-term effect on GDP. The positive GDP effect is partly a substitution effect reflecting the fact that the same number of workers can produce more if the capital stock per employed grows, and partly a composition effect reflecting the way in which GDP is defined.

... and changed composition

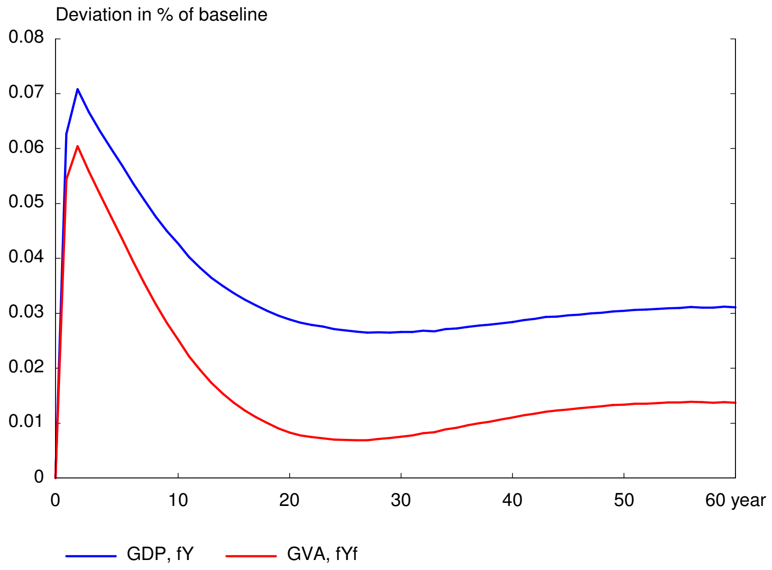
Specifically, the composition effect reflects that the indirect taxes share in GDP increases as consumption rises, and exports fall. Most indirect taxes are levied on private consumption, while the net tax on exports is negative because of agricultural export subsidies. Net indirect taxes are a transfer to the public sector, so this part of GDP does not affect the need for factors of production, which implies that the higher the share of indirect taxes in GDP, the higher is GDP per employed.

GDP increased more than GVA

The composition effect can be illustrated by comparing GDP with Gross Value Added, GVA, which is GDP excluding product-related net taxes. If all demand components, including their tax contents increase proportionally, GDP and GVA will also increase proportionally, so the difference between the GDP and GVA effects indicates the effect of the change in composition with more consumption and less exports. It is clear from figure 11.2 that GDP increases by more than GVA in the long term, as the higher consumption taxes add to GDP.

The positive long-term effect on GVA illustrates the substitution effect, which increases both GVA and GDP for given employment.

Figure 11.2 **GDP and Gross Value Added, public purchase of goods + 1%,**



The composition effect does not affect GVA

It is traditional to refer to the effect on GDP, which is a more familiar concept than gross value added, GVA. On the other hand, you are normally not interested in the just mentioned composition effect, and this speaks in favour of referring to GVA instead.

11.1.2 Role of wage formation

From short-term wage rigidity to long-term market clearing

As discussed in chapter 2, ADAM represents a traditional synthesis between, on the one hand, the Keynesian economic model that relates to the short term, where prices are sticky and markets do not clear, and on the other hand, the neoclassical long-term model, where prices adapt and markets clear, including the labour market.

Prices are driven by labour costs

In ADAM, output prices are driven by costs, and wage per hour is the crucial variable for the transition from Keynesian short-term effects to a more neoclassical long-term solution of the model. This reflects the role of the wage equation, which provides a link between the real economy and price formation by making the increase in hourly wage a decreasing function of unemployment, cf. chapter 7 on wage and price equations.

Level of wage reflects cumulated unemployment

The wage equation provides a relationship between the unemployment rate, *bul*, and the wage increase, dlog(*lna*). The relationship, which has a negative sign, is so close that the percentage effect on the wage level is closely correlated with the effect on cumulated unemployment, Σbul, see figure 11.3a, where the wage and the cumulated decline in unemployment is allocated on each axis. This means that the long-term effect on wage levels reflects by how much and for how long unemployment has deviated from the equilibrium of the baseline scenario; and the wage variable controls that the unemployment rare eventually equals long-term unemployment.

Figure 11.3a **Wage and cumulated unemployment, public purchase of goods +1%**

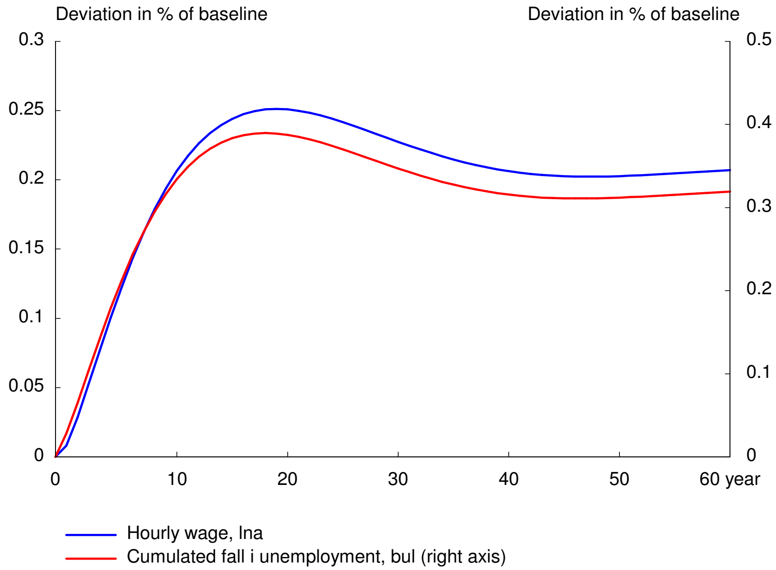
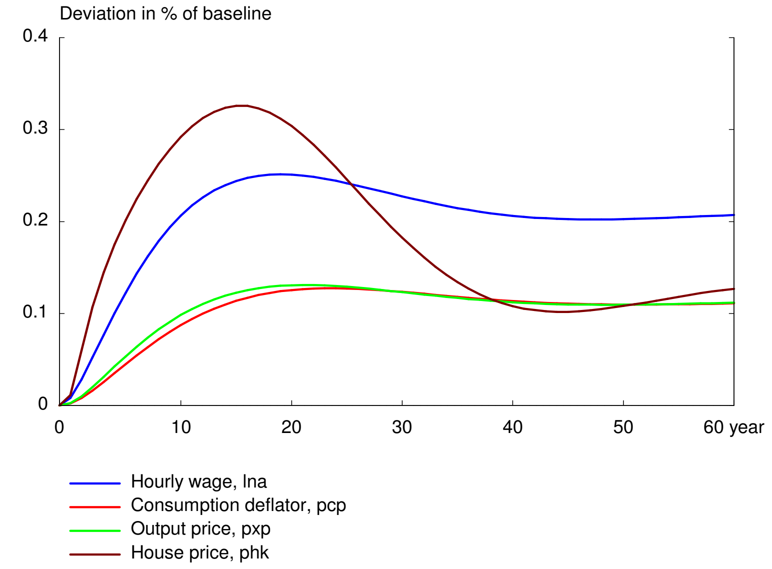


Figure 11.3b illustrates that both production and consumer prices react more weakly than wages. Only the demand-determined house price reacts faster and stronger than wages.

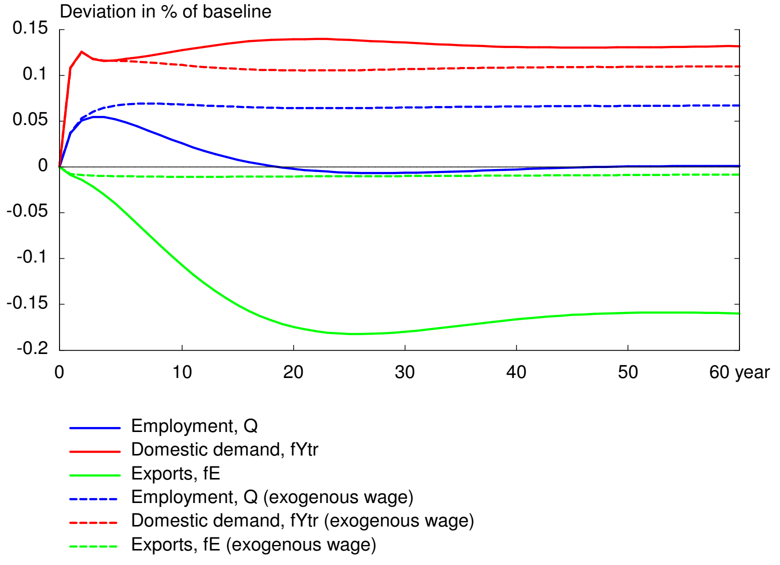
Figure 11.3b **Wage and prices, public purchase of goods + 1%**



With exogenous wage ADAM is purely demand-driven

The impact of the wage formation is illustrated in figure 11.4, which shows the effect of higher public purchases on employment, total domestic demand and exports calculated by, respectively, ADAM and ADAM excl. wage equation, i.e. ADAM with exogenous wage.

Figure 11.4 **ADAM and ADAM with exogenous wage, public purchase of goods + 1%,**



Positive long-term employment effect with exogenous wage

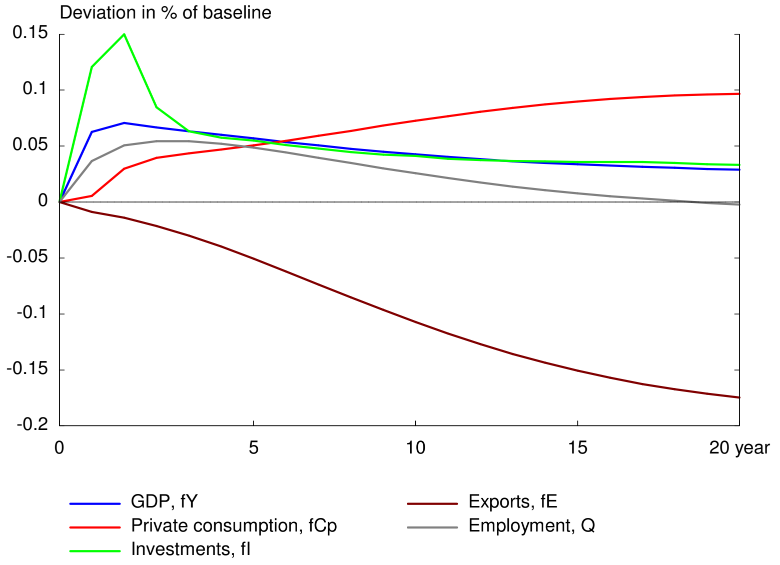
It appears that the wage response is critical to the long-term effects on employment and exports. Without wage equation, there is no crowding out mechanism to make the employment effect zero.

The wage response is less important for the effect on domestic demand as public purchases rise permanently, whether the wage is exogenous or not, cf. figure 11.4. Actually, the permanent effect on domestic demand is slightly higher when wages are endogenous, because the higher wage increases consumer purchasing power and private consumption.

11.1.3 Short- and medium-term effects

To clarify the short-term effects, the graphs in figure 11.1 are repeated in figure 11.5 with a shorter time scale, which includes the first twenty years and allows the employment effect to cross zero.

Figure 11.5 **Public purchase of goods + 1%**



Accelerator impact on investment

The percentage impact on investment peaks after a few years. This reflects the accelerator mechanism. If production increases by 1 per cent, businesses will demand 1 per cent more capital, and 1 per cent of the capital stock is equivalent to several per cent of the annual investment, because capital stock is larger than annual investment.

Large potential accelerator impact on construction …

The relative size of capital input and investment depends on the industry and more generally on the type of investment. A building is normally used for more years than a machine, and the ratio between capital stock and investment is higher for buildings than for machinery.

… but building investments respond with considerable lag

On the other hand, it is more time-consuming to increase building capital than machinery, so the addition to building capital is spread out over more years. The adjustment speed is determined by the estimated factor demand equations, which indicate that machinery capital and investment react faster than building capital and investment.

Machinery demand stimulated by substitution effect ...

Moreover, the demand for machinery increases a little more than the demand for buildings, as machinery is substituted for labour due to the higher wage. There is no equivalent substitution in building demand.

... and machinery reacts stronger

In summary, machinery responds stronger than commercial buildings, cf. figures 11.6a and 11.6b with the effects on investment and capital.

Figure 11.6a **Investments, public purchase of goods + 1%**

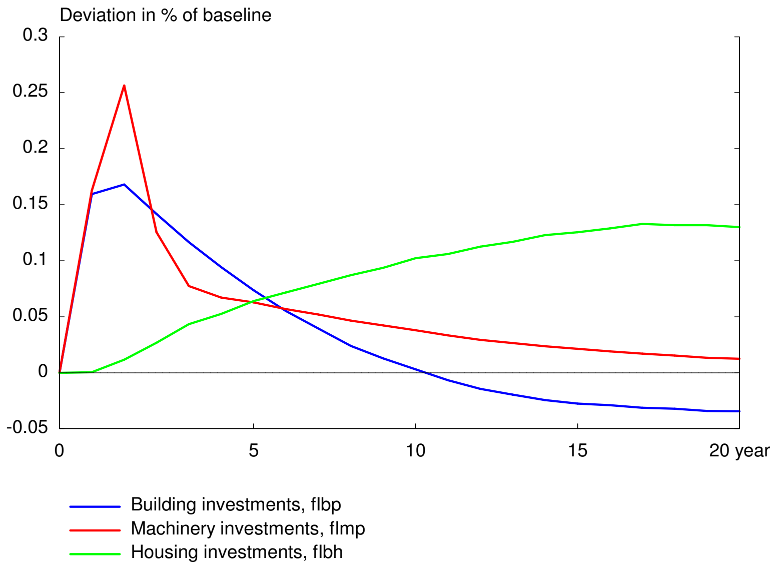
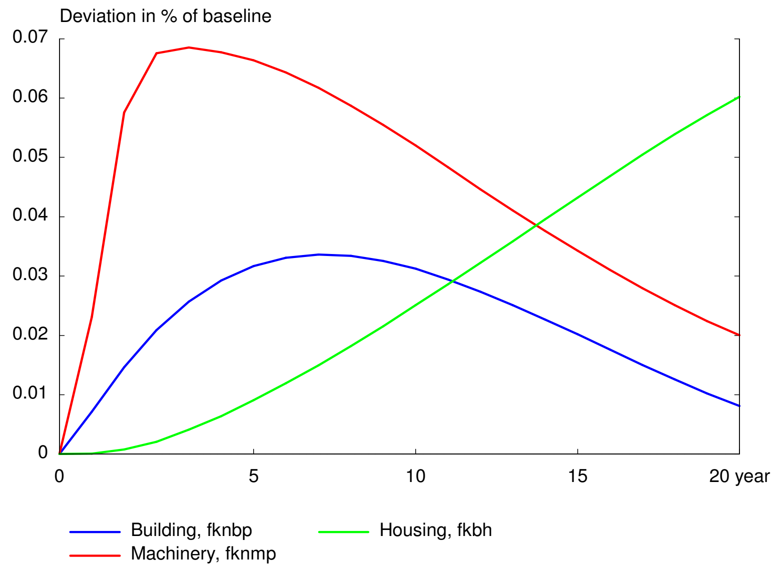


Figure 11.6b **Capital stocks, public purchase of goods + 1%**



Housing investment is sluggish

Housing investment reacts more slowly than business investment. The sluggish response of housing investment reflects mainly that housing demand is a function of private consumption and the increase in private consumption is spread out over more years than the increase in production, as already shown in figure 11.5. The consumption effect is rising throughout the 20 years sample shown in the figure, while the GDP effect peaks in the second year.

Accelerator impact on housing investment involves house prices

The growth in housing capital is a function of Tobin's q, the ratio between the market price of a house and the cost of a new house, so the increase in housing demand affects housing investment via the house price. This means that the accelerator impact on investment is channelled in another way, but the accelerator mechanism applies to housing investment as it does to business investment. Since the stock of dwellings is high relative to the annual investment, there will be a clear but delayed response in housing investment as housing demand increases gradually.

It can be added that the price elasticity of housing demand is restricted to be larger than suggested by the data, cf. the presentation of the house price equation in chapter 3. Without this restriction, the house price would respond stronger to demand changes, and a stronger reaction in the house price would create a stronger response in housing investment.

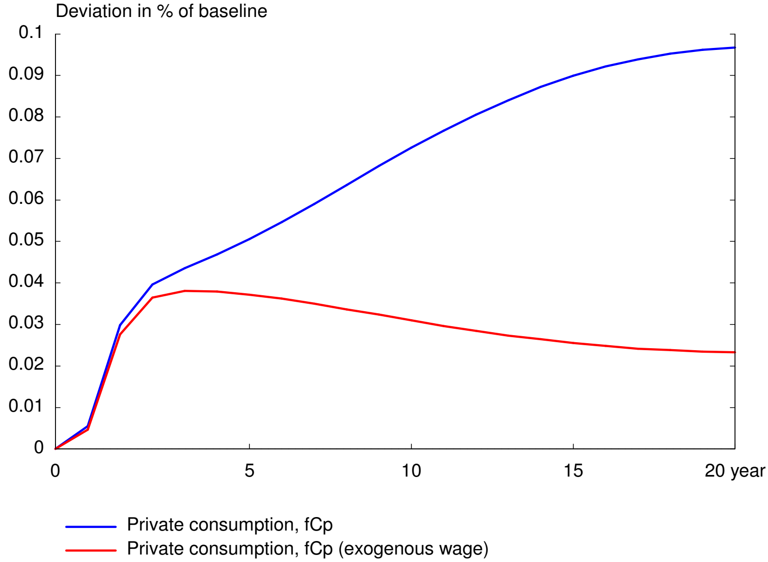
Investment effect is hump-shaped, consumption effect is not

The total investment effect appears to be hump-shaped with a peak in year two, see the previously shown figure 11.5. The hump-shaped time profile of investment is typical of a short-term cyclical response. It is also normal that private consumption increases when production and private income rise, but it differs from the textbook Keynesian model when consumption continues to rise relative to the baseline scenario after the initially strong investment impact has started to fall back. When the investment effect decreases, it will moderate the positive effect on private income, and if nothing else happens to income, the consumption effect will also fall back and turn hump-shaped.

Consumption effect keeps growing due to growing wages and purchasing power

When consumption continues to rise relative to the baseline scenario, it is because consumers' real income increases when wage increases, given that the price of imports is unchanged. If the wage is exogenized, ADAM will work as a simple Keynesian model, and the impact on consumption peaks after 4 years, see figure 11.7.

Figure 11.7 **ADAM and ADAM with exogenous wage, public purchase of goods + 1%**



11.1.4 Size of Keynesian multiplier

We have already mentioned that the ADAM-calculated effects can be referred to as multipliers, at least as long as the triggering exogenous change is simple.

Keynesian multiplier usually refers to GDP effect ...

More traditionally, the term multiplier does not relate to the effect on any endogenous variable, but mainly to GDP and specifically to the change in GDP relative to the exogenous demand change that has triggered the change in GDP. The word multiplier indicates that you are expecting the GDP change to be larger than the exogenous demand change. This traditional expectation reflects that the first-round increase in GDP will reinforce itself because higher GDP creates additional income and hence additional demand.

... divided by exogenous demand shock

The traditional Keynesian multiplier corresponds analytically to a first order derivative, where GDP is differentiated with respect to the exogenous demand, here represented by public purchases. ADAM cannot be solved analytically, but the traditional multiplier can be calculated as the ratio of the effect on GDP, *fY*, and the change in the public purchase, *fVmo*.

$$multiplier = \frac{\Delta fY}{\Delta fVmo}$$

The applied difference operator, Δ, indicates the difference between the variable in the alternative scenario and the variable in the baseline scenario. The multiplier shown can be made for any year in the calculation period year, but normally focus would be made on the Keynesian multiplier in the very short term, where changes in wages and competitiveness can be ignored.

It is tradition to focus on the GDP multiplier, which also represents the effect on total income, but a similar multiplier can be made for the effect on e.g. domestic demand, *fYtr*, by replacing *fY* with *fYtr* in the numerator. The Keynesian GDP multiplier of public purchases is shown in figure 11.8, together with similar multipliers for domestic demand and exports.

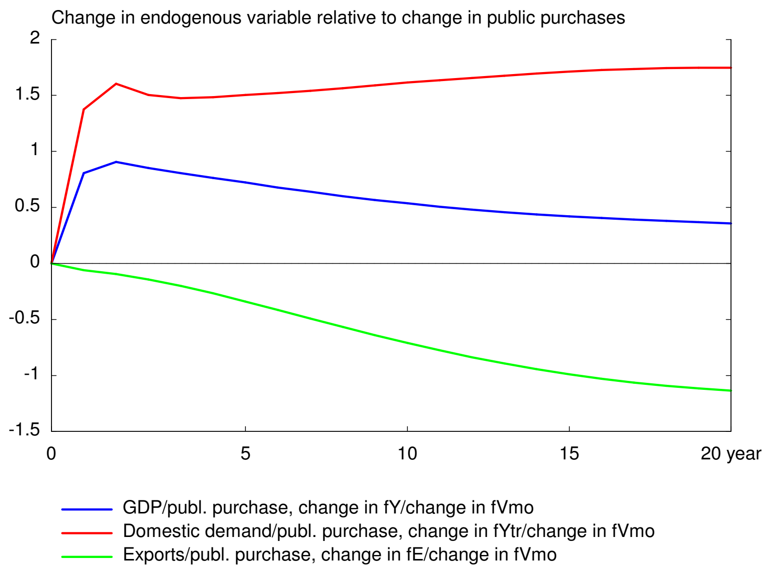
Domestic demand multiplier is larger than 1 ...

The multiplier for domestic demand is larger than 1 throughout the calculation period because the exogenous increase in government purchases of goods creates additional domestic demand in the form of more private investment and larger private consumption.

... while GDP multiplier is less than 1

The GDP multiplier, however, does not exceed 1 and peaks in year 2 with a value of slightly less than 1, so the Danish value added increases less in real terms than the public purchase of goods; notwithstanding that total domestic demand increases more than public purchases. The moderate GDP multiplier reflects that higher demand triggers not only GDP but also imports. Besides, the accompanying increase in taxes and decrease in unemployment benefits dampen the interaction between demand and income and the effect on private demand and on GDP.

Figure 11.8 **GDP, domestic demand and export multipliers, public purchase + 1%**



Factors explaining the GDP multiplier

In general, the traditional short-term GDP multiplier is a decreasing function of four factors: Import content in demand, tax rate on income, savings rate on disposable income and replacement ratio of unemployment benefits. These four factors relate to 'leaks' in the multiplier process. The leaks limit either the short-term effect of demand on GDP or the short-term effect of GDP on demand. In the longer term, the GDP multiplier of public purchases will moderate in any case, because the export multiplier falls.

At the end of the comments on Keynesian multipliers, it should be mentioned that the size of the multiplier reflects that the volume variables in ADAM represent chain indices, cf. box 11.1

Box 11.1 **Keynesian-multiplier when volumes are chain indices**

When variables in real terms are calculated with chain indices, the simple resources and uses identity for GDP, import and demand does not hold in real terms. Thus, it cannot be assumed that 1 krone additional purchase in real terms will result in exactly 1 additional krone of GDP, if the rest of demand and imports are kept constant.

The breakdown of the resources and uses identity in real terms makes it less relevant to compare the Keynesian multiplier with 1. Alternatively, the multiplier numerator and denominator can be measured in current prices, contemporary or lagged one year, because the identity for GDP, imports and demand hold in contemporary and one year lagged prices. Measuring in current prices will normally not affect the multipliers much, if we focus on the first few years where relative prices hardly change.

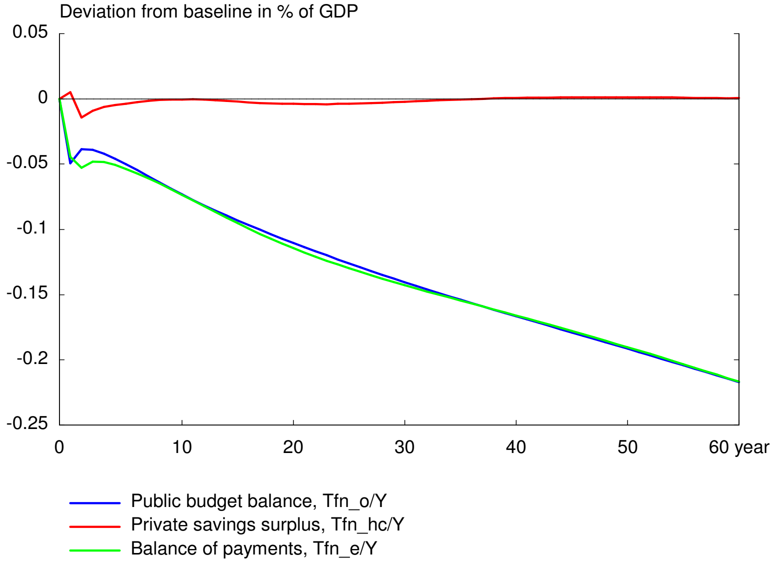
11.1.5 Savings surpluses of sectors

Increased public purchases deteriorate the public budget

The public budget balance deteriorates continuously when you permanently increase public purchases without funding. In the first years, the budget deterioration is moderated by lower expenditure on unemployment benefits and by a higher tax base, but as the immediate economic stimulus disappears, the budgetary deterioration will get full effect. In addition, the negative budgetary impact will accumulate as higher debt on which interest must be paid. This makes the budgetary effect self-perpetuating, and the negative effect on the public budget balance, incl. interest grows throughout the calculation period.

The public budget balance represents the savings surplus of the public sector. If we add the private sector savings surplus to the public sector, we get the total domestic savings surplus, which by definition corresponds to the balance of payments. This means that any deterioration of the public budget balance will be reflected in the private savings surplus or in the balance of payments.

Figure 11.9 **Savings surpluses, public purchase of goods + 1%**



Also the balance of payments deteriorates ...

The continuing deterioration of the public balance is first and foremost reflected in the balance of payments that also deteriorates continuously, while the private savings surplus remains largely unchanged in the long term, cf. figure 11.9. This means that the public budget balance and balance of payments plus public debt and external debt, all react to 1 per cent higher public purchase of goods by drifting away from the baseline. The ever increasing effect makes those four variables different from the many variables that react with a permanent level shift in the long term.

... while private savings surplus is held in place by the consumption function

Private savings surplus equals private savings minus private investments, and in the model the private savings surplus remains virtually unchanged in the long term, because fluctuations in the savings surplus are collected in private financial wealth, which is part of the wealth variable in the consumption function.

In the first years after the increase in public purchases, the private savings surplus falls, as the effect on house prices and housing wealth stimulates private consumption. The smaller savings surplus reduces private wealth and the negative wealth effect tends to reduce consumption and increase savings. Larger savings will restore the savings surplus, and in the long term, the private savings surplus remains virtually unchanged as stated.

There is no similar mechanism to control the public budget balance and public debt in ADAM, but the continuing deterioration of the public balance and debt is unsustainable in the long term. If you want to increase the public purchase of goods permanently, you also have to increase public revenues in order to keep public finances sustainable.

11.1.6 Tax funding

It is chosen to tax-finance the increased public purchases

Increasing the public purchase of goods can be financed by reducing other public outlays or by increasing revenues. For example, the tax base can be expanded in the long term by increasing participation rates. In other words, there are several possibilities, but it is here chosen to finance the higher spending by higher taxes, so that public debt in the long term remains unchanged as a share of GDP.

Funding neutralize the budgetary effect in the long term

The public budget balance automatically responds to cyclical fluctuations, implying that it is not realistic to introduce a set of tax changes, which keeps the public budget unaffected in all years. It is better to increase the tax rates once and for all, so that the additional public purchases are financed by additional revenues for unchanged tax base, while accepting the cyclical budget fluctuations that will be created by the short-term response in the tax base and employment.

With tax funding included, the ADAM calculation resembles a simple textbook example of the balanced budget multiplier. In the ADAM calculation it is, however, only in the long term that the debt ratio is stabilized and the additional revenue fully corresponds to the additional expenditure. The tax increase that balances the budget in the long term will only balance the budget in the short term, if public expenditures and revenues are exogenous, and they are not exogenous in ADAM.

Funding includes permanent and temporary tax rises

As mentioned under the presentation of ADAM in chapter 2, the long-term effect on public budget balance is neutralized with a combination of a permanent increase in central government income tax and a temporary increase in capital tax.

The permanent increase in income tax is to ensure that the public budget balance remains unaffected in the long term, where employment is unaffected by the demand shock. The permanent income tax increase is the crucial element in the funding.

The temporary increase in capital tax is to remove the debt effect created in the short term by the income-tax financed increase in public purchase of goods. This combined income tax and public purchase shock has a long-term budget multiplier of zero, but before the long term begins it has, on average, a negative effect on the business cycle and thus also on the budget balance and public debt. The size and the calculation of the two tax changes are explained in box 11.2.

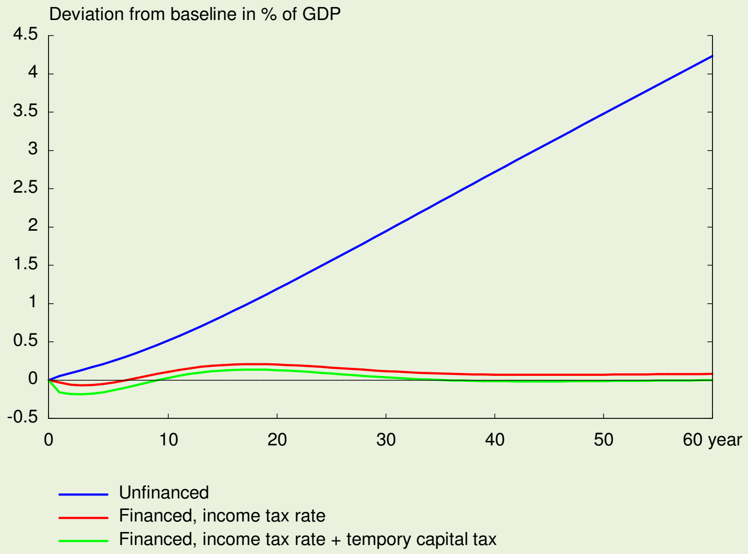
Box 11.2 **Tax funding the public purchase of goods**

The starting point is the growing negative impact on public finances, resulting from a permanent increase in public purchases of goods by 1 per cent. The negative effect on the budget balance accumulates in ADAM to a negative effect on the public net asset called *wn_o*. The decrease in net asset reflects that the public debt is increasing relative to the baseline scenario, and the task is through higher taxes to neutralize the negative budgetary development and make public debt return to baseline in the long term.

In order to remove the rising trend in the effect on the public debt, the increase in purchases of goods is supplemented by a permanent relative increase in the state income tax rates by 1.5 per cent, so that a tax rate of, say, 15 per cent is increased to about 15.2 per cent. This results in a constant long-term effect on the public debt ratio (debt/GDP), as depicted by the red curve running parallel to the x-axis in the box figure below. When the effect on the debt ratio is constant, debt will increase every year with the interest outlays on the debt because the interest rate equals the nominal growth rate. At the same time, there is zero effect on the primary public budget balance, as the additional public purchases are funded each year by the extra tax revenue.

The long-term effect on the debt ratio is 0.2 per cent of GDP, which can be seen as a net effect of the economic fluctuations created by the income tax-financed increase in government purchases. When the fluctuations are over, public debt has increased by 0.2 per cent of GDP. In the first year, public debt falls cf. the red curve, because the increase in purchases of goods affects demand more directly than the accompanying increase in income tax, so the economy is stimulated to begin with. However, the decline in private consumption triggers a negative accelerator adjustment in the housing market and the implied negative economic impact is stronger than the positive economic impact during the first years of the calculation period. Thus, after 8 years, public debt is higher than in the baseline scenario, see the red curve in box figure.

Effect on public debt, + 1% public purchase of goods



It is not possible to remove the constant debt effect with a permanent tax increase, at least not when the interest rate equals growth. A temporary increase in taxes is needed, and it is decided to increase capital tax with a lump sum of just over 0.2 per cent of GDP in the first year and only in the first year of calculation. This makes the long-term effect on public debt zero, so the higher public purchases are fully funded, see the box figure.

The additional capital tax proceeds of just over 0.2 per cent of GDP trigger a modest economic dip, which deteriorates public debt slightly, so that the long-term debt effect of the temporary capital tax is only 0.2 per cent, which exactly shifts the debt curve down to the x-axis.

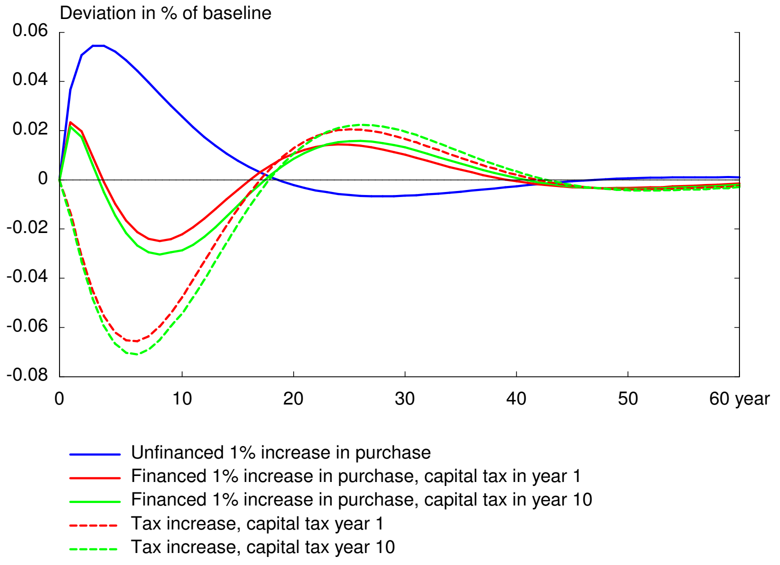
Capital tax is chosen for the temporary tax increase, because capital taxes directly affect private savings surplus and wealth but not disposable income, which excludes capital transfers. Private wealth enters the consumption equation with a lag, so the one-off increase in capital taxes does not affect consumption in the first year of the calculation period, but the implied wealth reduction will have a temporary impact on consumption in the subsequent years.

In practice, the temporary tax increase could hardly be placed already in the first year where public purchases are increased. It is more likely that you at first settle for the permanent tax increase and postpone the one-off funding until it becomes obvious that the debt ratio has risen.

Tax funding dampens effect on demand ...

Tax funding dampens the positive demand effect of higher public purchases, but in the first years of the calculation there is still a positive effect on demand and employment. The initially positive effect reflects that private consumption responds to tax increase with a lag. For example, the first-year consumption elasticity with respect to disposable income is only 0.4, and the temporary increase in capital tax only affects consumption through the lagged wealth variable and has no first-year impact on consumption.

Figure 11.10 **Employment effects, public purchase and tax increases**



... and is instrumental in redressing effect on employment

However, the decline in disposable income and consumption triggers a negative adjustment in the housing market, cf. chapter 2, and after five years, the initially positive effect on employment turns negative. The employment impact of the funded increase in public purchases is the sum of the positive employment effect of public purchases and the negative employment effect of the necessary tax increases, see the illustration of employment effects in figure 11.10.

The temporary capital tax increase placed at the beginning of the calculation period is, as discussed in box 11.2, a technical assumption to ensure that public debt is unaffected in the long term. The role of the capital tax increase can be illustrated by means of an alternative calculation, in which it takes 10 years before capital taxes are increased temporarily. In this case, the employment effect will be as depicted by the green curve in figure 11.10. At first, the capital tax barely affects employment, but after a few years, the employment effect is more negative with a temporary capital tax increase in year 1. In the long term, the employment effect is zero in any case.

The necessary one-off increase in capital tax is, as mentioned, in the box 0.2 per cent of GDP when the public purchase of goods is increased by 1 per cent, but other shocks may require a larger one-off change in capital taxes to eliminate the long-term impact of public debt.

11.2 Raising the propensity to consume by 1 per cent

After having illustrated a permanent shock to public consumption in 11.1, we now illustrate a permanent shock to private consumption. Private consumption can be shocked indirectly by changing taxes, cf. the tax funding of public purchases. We will, however, not use fiscal interventions to increase disposable income, but introduce a change in behaviour, which makes households increase their consumption excl. housing by 1 per cent for a given income and wealth. Shocking the propensity to consume in this way can illustrate how the model's consumption formation works, and it can also help to explain the effect of a lower interest rate in paragraph 11.3.

Shock via adjustment term

The shock to the propensity to consume is made with the adjustment term in the consumption equation, cf. box 11.3.

Box 11.3 **Shocking the private propensity to consume**

The total error correction equation for actual consumption was presented in chapter 3 in equation (3.2a).

The shock to the propensity to consume combines a permanent increase of 0.01 in the long-term equation constant -0.200 with a one-off increase of 0.01 in the error correction equation constant 0.011 in (3.2a). Increasing the constant terms is implemented by increasing the adjustment term of the long-term equation with 0.01 in the entire calculation period and increasing the adjustment term of the error-correcting equation with 0.01 in the first year of the calculation period.

These adjustments lift consumption by 1 per cent relative to the result of equation (3.2a) without adjustments. It should be noted that this consumption, *fCpuxh* in ADAM notation, excludes housing consumption, which is proportional to the housing stock. There is no shock to housing consumption, so the immediate effect on total private consumption, *fCp*, is less than 1 per cent.

Shocks to consumption function were discussed in chapter 3 ...

The shock to the consumption equation corresponds to one of the numerical examples used to illustrate the consumption equation properties in chapter 3. In chapter 3, the consumption equation was supplemented by two simple definitional equations to determine wealth and income including property income from wealth, and it turned out that the effect of increasing the propensity to consume depends on the relationship between growth and interest rates.

... and are now repeated using the entire ADAM model

By using the entire ADAM model, we can illustrate the interaction between the consumption equation and the model's income and capital formation, including the spill over on public tax revenues. The calculation on ADAM shows that the long-term consumption effect is affected by the interest rate after tax being less than the growth rate, and the consumption effect also depends on whether the accompanying effect on public debt is neutralized by a tax increase or not.

Short-term effects of a higher propensity to consume

The shock to the consumption equation works initially in the same way as an increase in public purchases. Production and incomes are rising, and this creates additional demand for investment and consumption. Consumption excluding housing consumption and, to a lesser extent, also overall consumption increase by more than 1 per cent for a short period, cf. figure 11.11.

Higher production makes employment rise. The lower unemployment stimulates wages, weakening competitiveness and exports. In this way, the first five years of the reaction pattern reminds of the crowding-out process that accompanies an increase in public purchases.

In the longer term, the positive consumption effect is reduced

About five years into the calculation period, the expansionary effect on private consumption begins to decline because private wealth begins to fall, and as consumption falls back toward the baseline scenario, the positive effects on activity and employment are reduced. The reduced pressure on the labour market redresses the effect on competitiveness. Thus, the turnaround in the expansive consumption effect implies that the crowding out process stops, so that exports stop falling relative to the baseline and begin to return to the baseline.

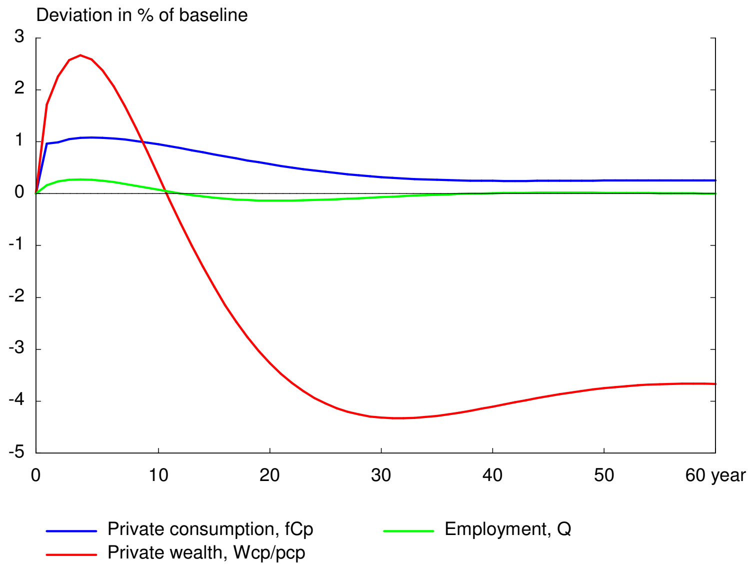
The initially positive wealth effect falls and becomes ...

The positive consumption effect begins to decline as consumer wealth begins to fall relative to the baseline scenario. In the first years of the calculation period, the wealth effect is positive because house prices and thus housing wealth increases when consumption increases. This immediate increase in housing wealth can during the first years offset the fall in financial wealth that follows when the positive shock to the consumption function reduces savings, but after a few years the decline in financial wealth will dominate.

... negative in the long term

The fall in total wealth effect reduces the positive effect on consumption. With a smaller consumption effect, savings are stimulated and there will be a long-term equilibrium, where the wealth effect has stopped falling and stabilized at a negative level, see figure 11.11.

Figure 11.11 **Propensity to consume + 1%**



The positive long-term effect on consumption ...

It is natural that the long-term wealth effect is negative when we reduce the propensity to save, but it is noted that the long-term consumption effect is positive. In order to have a positive long-term consumption effect, it is not enough that the propensity to consume has increased, as illustrated in chapter 3 on consumption equation properties.

... reflects that interest rate after tax is less than growth rate

Based on the illustration in chapter 3, the long-term consumption effect of a higher propensity to consume will only be positive if the return on assets is less than income growth. In the present ADAM calculation, the pre-tax interest rate equals long-term income growth. Consequently, the interest rate after tax is less than income growth, and this difference basically explains the positive long-term effect on consumption.

It may be noted that the positive long-term effect on consumption will increase itself by triggering a crowding-out process that permanently increases the terms of trade and real income, but the underlying explanation for the permanent consumption effect is as stated that the interest rate after tax is lower than income growth and that the increase in the propensity to consume reduces financial wealth.

The relationship reflects that financial wealth pays less return after tax than what is needed to keep financial wealth growing in a steady state. Thus, there is room for more consumption when wealth falls, cf. box 11.4, extending the argument from chapter 3.

Box 11.4 **Long-term relation between financial wealth and consumption**

ADAM's consumption-determining wealth, W, consists primarily of housing wealth, W_h, and financial wealth. If we ignore changes in bond and share prices and ignore the correction for tax value of pension assets, the change in financial wealth equals the private savings surplus, Y-C-I, where Y is private disposable income, C is private consumption, and I is private investment - both residential and non-residential. This makes W move in accordance with the equation:

(11.1)

$$W = W_{-1} + D(W_h) + (Y - C - I)$$

In the short term, the cyclical housing wealth gives a special contribution to the wealth variable. However, we here focus on the long term, i.e. on the steady state, where house prices develop in line with other prices. Thus, the change in both wealth variables can be represented by steady-state growth rate times lagged wealth, and (11.1) can be written as:

(11.1)*

$$Y - C - I = growth \cdot (W_{-1} - W_{h,-1})$$

The variable in brackets, W-W_h, is financial wealth, so the equation says that in steady state, where variables grow in line, the savings surplus equals the product of growth rate and financial wealth.

The equation works as a budget constraint, which is met in steady state through the interaction between consumption and financial wealth. If the savings surplus is too large according to (11.1*), financial and thus also total wealth, W, will increase by 'too much', and this extra wealth will via the consumption function enhance consumption and reduce the savings surplus, until (11.1*) holds. Thus, the financial wealth variable controls that actual consumption equals potential consumption.

Income variable, Y, is affected by financial wealth through interest income, which is taxed at rate t. With $(1\text{-}t)\cdot interestrate\cdot(W_{-1}\text{-}W_{h,-1})$ as disposable interest income and $Y_{exinterest}$ as the rest of Y, (11.1*) can be written:

*(11.1**)*

$$C = Y_{exinterest} - I + ((1-t) \cdot interestrate - growth) \cdot (W_{-1} - W_{h,-1})$$

This is the dynamic equation for private wealth, W, formulated as an equation for consumption at constant wealth growth. The simple equation (11.1**) implies that the model generates a steady state scenario for all relevant variables, and in this scenario private wealth, W, is determined by the estimated consumption equation, which has wealth as an argument, and financial wealth in (11.1**) is determined by deducting housing wealth, Wh, from private wealth, W.

It may be added that gross income from the entire real capital stock is included in income variable, $Y_{exinterest}$, from which gross capital formation, I, is deducted. This means that the steady-state consumption equals wage income plus non-financial transfer income minus taxes plus growth-adjusted return on net financial assets plus 'investment-adjusted' return on real capital.

The baseline assumptions imply that the after-tax interest rate is lower than the steady state growth rate, so (11.1**) confirms that in the long term consumption will increase when financial wealth falls, unless the private sector compensates the public sector for its loss of tax revenues. Financial wealth falls, for example, when the propensity to consume or the housing stock increases.

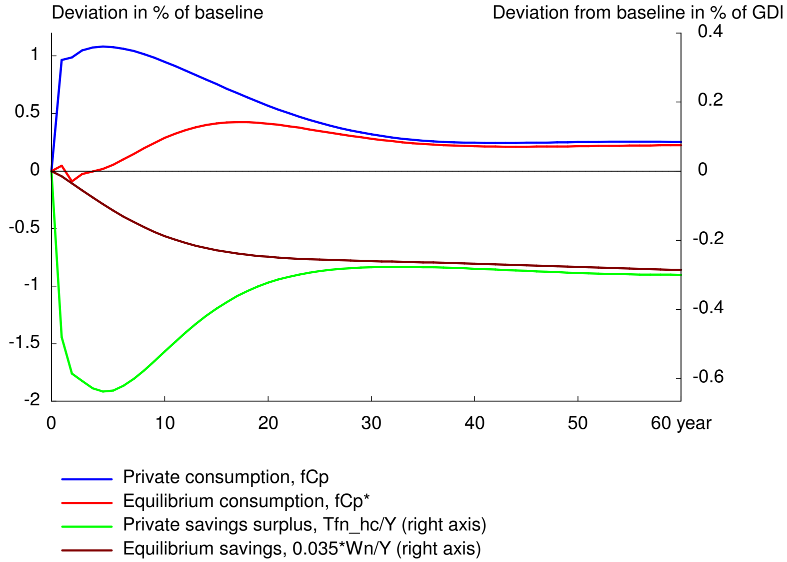
In ADAM, private financial wealth is the sum of financial wealth of households, financial corporations and the rest of the corporate sector, with ADAM notation: *Wn_h+Wn_cf+Wn_cr*. This sum is referred to as *Wn* in the following. Ignoring changes in asset prices, the change in Wn equals the private savings surplus, *Tfn_hc*. Ignoring capital transfers, *Tfn_hc* equals normal national accounts private disposable income minus consumption and investment, i.e. *Yd_hc-Cp-Ip*. We thus have:

$$Wn - Wn_{-1} = Tfn\_hc = Yd\_hc - Cp - Ip$$

In steady-state, savings surplus is a fixed share of financial wealth

As already illustrated in figure 11.11, the wealth in the consumption function falls in the long term, and as the positive consumption effect actually increases housing stock and also housing wealth, financial wealth, *Wn*, falls in the long term. Consequently, a smaller savings surplus is required in order to keep financial wealth growing by 3.5 per cent, which is the nominal growth in steady state. After 30 years, the effect on the private savings surplus, *Tfn_hc*, is negative and close to 3.5 per cent of the negative effect on financial assets, see figure 11.12, where 3.5 per cent of financial wealth is called equilibrium savings.

Figure 11.12 **Propensity to consume + 1%**



The relationship between financial wealth and the steady state of consumption is illustrated by figure 11.12, which compares the effect on actual private consumption with the effect on 'equilibrium consumption'. The latter, called fCp^*, indicates the level of consumption that private disposable income and investment leaves room for when private financial wealth is growing at its steady-state growth of 3.5 per cent p.a. The figure illustrates that actual consumption corresponds to equilibrium consumption in the long term, when also the private savings surplus corresponds to equilibrium savings and the model is in steady state. The expression for equilibrium consumption, which is not an ADAM-variable, is set up in box 11.5.

Box 11.5 **Equilibrium consumption in the ADAM calculation**

The equilibrium consumption in real terms, fcp^*, from figure 11.12 is derived by setting the private savings surplus equal to 3.5 per cent of private financial wealth, move nominal consumption, Cp^*, to the left hand side and divide everything by the consumer price.

$$Tfn\_hc = 0.035 \cdot Wn \Leftrightarrow$$

$$Yd\_hc - Cp^* - Ip = 0.035 \cdot Wn \Leftrightarrow$$

$$fCp^* = \frac{Yd\_hc - Ip - 0.035 \cdot Wn}{pcp}$$

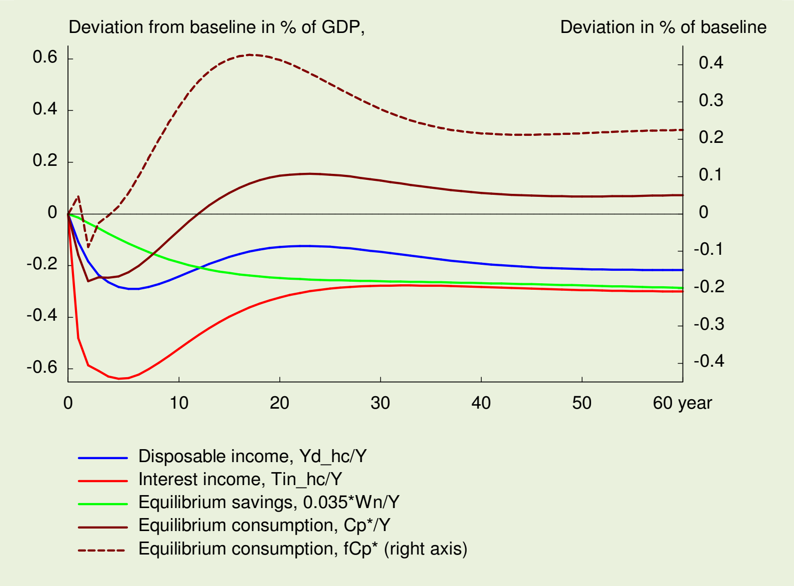
Tfn_hc	Private savings surplus,= *Yd_hc-Cp-Ip* as capital transfers = 0
Wn	Private financial wealth, =*Wn_h*+*Wn_cr*+*Wn_cf*
Yd_hc	Private disposable income
Cp	Private consumption in current prices
Ip	Private investment in current prices
fCp	Private consumption in fixed prices
pcp	Price of private consumption

* on *Cp* og *fCp* indicates equilibrium consumption

In steady state, equilibrium consumption at current prices corresponds to the consumption determined in equation (11.1**) in box 11.4.

The box figure confirms, firstly, that the decline in 'equilibrium savings' as a share of GDP, i.e. the decline in 0,035·*Wn/Y*, in the long term corresponds to the decrease in interest income before tax, *Tin_hc/Y*.

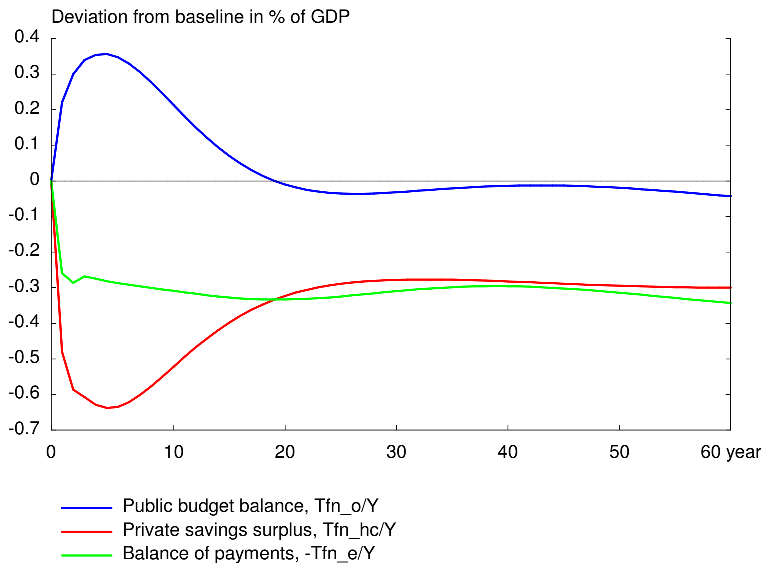
Secondly, it appears that the decline in equilibrium savings is larger than the decline in disposable income, *Yd_hc/Y*. Investments do not change significantly relative to GDP, so the difference between the decline in equilibrium savings and the decline in income comes close to representing the increase in equilibrium consumption. As a share of GDP, Cp^*/Y, equilibrium consumption increases in the long term by 0.07 per cent, and the relative increase in fCp^* is just over 0.2 per cent.



Positive consumption effect stimulated by lower tax payments

The taxation of interest income implies that consumers will save tax when the financial wealth and hence interest income falls, and this tax saving has a positive long-term effect on consumption.

Figure 11.13 **Propensity to consume, + 1%**



Tax savings can be neutralized

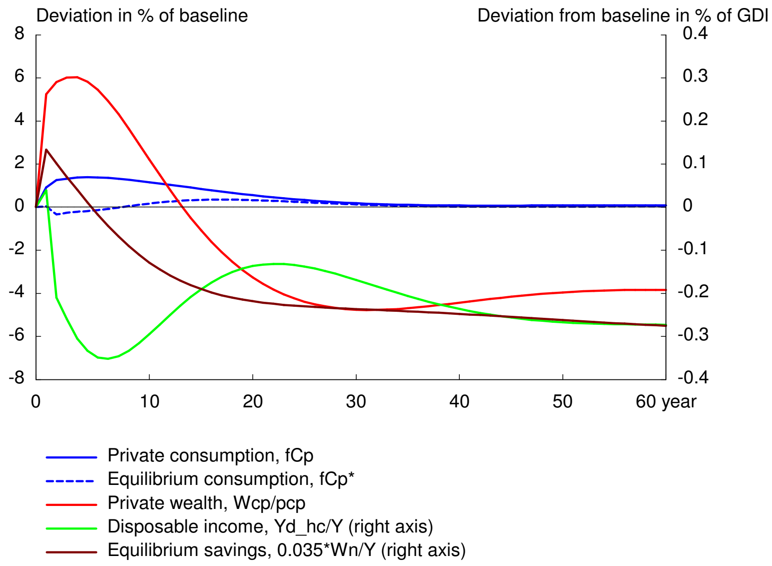
The counterpart of the decline in private savings surplus is mainly that the balance of payments worsens, but the tax saving of consumers reduces the tax revenues of the public sector and creates a tendency for the government budget balance to deteriorate in the long term, see figure 11.13. This means that the increase in the propensity to consume has made public finances unsustainable, and as with the increase in government purchases, it is natural to introduce a fiscal tightening to remove the long-term effect on public debt.

In order to eliminate the long-term upward drift in public debt, central government income tax must be raised by 1 per cent, which corresponds to raising, e.g. upper limit tax rate from 15 to 15.15 per cent. This modest tax increase does not prevent the higher propensity to consume from stimulating the economy in the beginning of the calculation period. When the positive cyclical impact is played out, public debt has been reduced by 3 per cent of GDP, so in order to keep public debt unchanged, the permanent increase in income tax must be supplemented by a one-off capital transfer from public to private sector.

When the tax saving on the lower interest income is offset by higher income taxes, disposable income will in the long term fall by the same amount as savings surplus, see figure 11.14. This will reduce the positive effect on consumption, and the long-term consumption effect will be close to zero, cf. figure 11.14.

The difference between effects calculated with and without restrictions on long-term public debt shows that if the effect on public finances is not neutralized, the long-term effect on private consumption will be affected because the new equilibrium triggers a transfer between public and private sector.

Figure 11.14 **Propensity to consume + 1%, public debt unchanged**



It must be emphasized that the long-term effect on consumption in figure 11.14 is based on scenarios where the interest rate before tax corresponds to the long-term income growth of 3.5 per cent. If the interest rate was higher, the long-term effect on consumption would be negative, as the consumption increase at the beginning of the calculation period would be paid by lower consumption in the long term, and if interest rates were smaller than income growth, the long-term effect on consumption would be positive. The latter situation arises in the following section 11.3, where the interest rate falls.

Summary on raising propensity to consume

In summary, the propensity to consume shock discussed in this section reminds in the short term of a shock to public demand. In the long term, there is a clear difference between shocking public demand and private consumption. For example, the long-term increase in private consumption is always less than the immediate increase when the propensity to consume is increased. Instead, private wealth will fall in the long term when the propensity to consume rises. If the long-term impact on public debt is removed, the long-term consumption effect is close to zero in the present ADAM calculation, where the interest rate corresponds to the long-term growth rate.

11.3 Fall of 0.5 per cent in interest rates

Due to the Danish fixed exchange rate policy against the euro, the Danish interest rate is largely determined by conditions abroad, so the interest rates in ADAM are basically exogenous, similar to foreign prices and foreign demand. Formally, the banks' lending rate is modelled as a function of the discount rate and bond yield, but since both the discount rate and bond yield reflect the corresponding foreign interest rates, the lending rate is de facto also given from abroad.

Danish interest rates and interests on foreign loans fall 0.5 per cent.

The calculation is based on a reduction of interest rates by 0.5 per cent, i.e. from 3.5 per cent in the baseline scenario to 3.0 per cent. The decline in interest rates includes both ADAM's domestic and foreign interest rate. The latter determines the interest payments on foreign debts and loans, but the calculation does not include that a general foreign decline in interest rates would stimulate foreign markets and foreign prices. Thus, the calculation describes a situation where Denmark can borrow 0.5 per cent cheaper abroad, while domestic interest rates fall by 0.5 per cent.

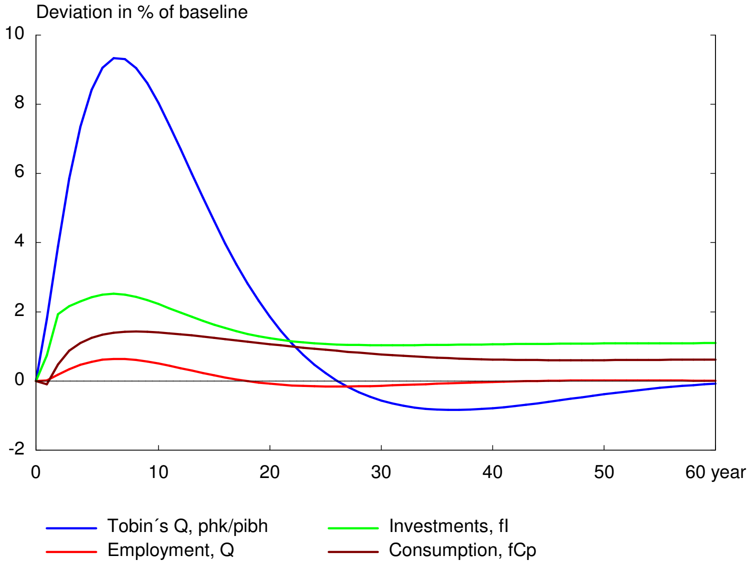
There is more than one interest rate in ADAM, but all key interest rates are 3.5 per cent in the baseline scenario and fall to 3 per cent in the alternative scenario. It is noted that the equation for the banks' lending rate implies that the lending rate falls by less than 0.5 per cent. The lending rate affects investments in machinery, and in order to simplify the interest rate shock we have chosen to exogenize the lending rate and reduce it by exactly 0.5 per cent.

The decline in interest rates affects public finances, but we shall start by calculating the effect of lower interest rates without neutralizing the long-term impact on public debt.

Lower interest rates increase both investments ...

Lower interest rates reduce the cost of capital and increase the demand for capital goods. Larger capital demand will drive investment and house prices, and through housing wealth it will also stimulate private consumption. Private consumption responds to the increase in wealth with a 1-year delay, since wealth has no short-term effect in the consumption equation. In the first year, consumption thus hardly reacts to the lower interest rate, but from the second year onward, the consumption effect is positive.

Figure 11.15 **Interest rate falls by 0.5%**



... and private consumption

There is no interest rate in the consumption equation, so the lower interest rate does not create a formal substitution from future to current consumption. But although there is no formal substitution effect, the consumption impact of the increase in housing wealth resembles the impact of consumers substituting current for future consumption, cf. Pedersen and Rasmussen (2001). Pedersen and Rasmussen compare the effect of lowering interest rates in, respectively, ADAM and the general equilibrium model DREAM. In contrast to ADAM, DREAM has a consumption equation with inter-temporal substitution and forward-looking expectations. However, despite formal differences the consumption effect is similar in the two models, because the induced reaction in house prices drives private consumption in ADAM.

Employment and wage increase, while costs of production start to decline

Higher demand will make production and employment rise, so unemployment falls and wages rise. Despite the wage increase, output prices fall in the short term because capital costs drop when interest rates drop. The lower export price makes exports rise at the beginning of the period.

Effect on competitiveness reflects assumptions

It should be noted that the effect of interest rates on competitiveness reflects our assumptions. We have assumed that the lower international interest rate only applies to the Danish foreign assets and liabilities. If the general foreign interest rate level was to fall by 0.5 per cent, we would not expect Danish prices to fall relative to import and foreign competitor prices, and the lower interest rate would in this case not improve competitiveness.

With the immediate positive effect on competitiveness included, both domestic and exports increase in the first years of the calculation period. However, in the longer term, the wage reaction will dominate, and the immediate positive effect on competitiveness will disappear and turn into a negative effect.

Export effect is negative in the long term

This means that there is a negative long-term effect on exports of a simple reduction in interest rates, because exports are ‘crowded out' to make room for the positive long-term effect on domestic demand.

Fall in interest rates reduces notably the cost of long-term investments

In the long term, the effect of lower interest rates is the highest for housing investments and the smallest for businesses building investments, see figure 11.16a. The user cost is based on a smaller depreciation rate for buildings than for machinery, so user costs of buildings fall more in percentage terms. On the other hand, there are only small substitution possibilities for buildings in ADAM’s production functions, and machinery increases by more than building capital, see figure 11.16b.

Figure 11.16a **Investments, interest rate falls by 0.5%**

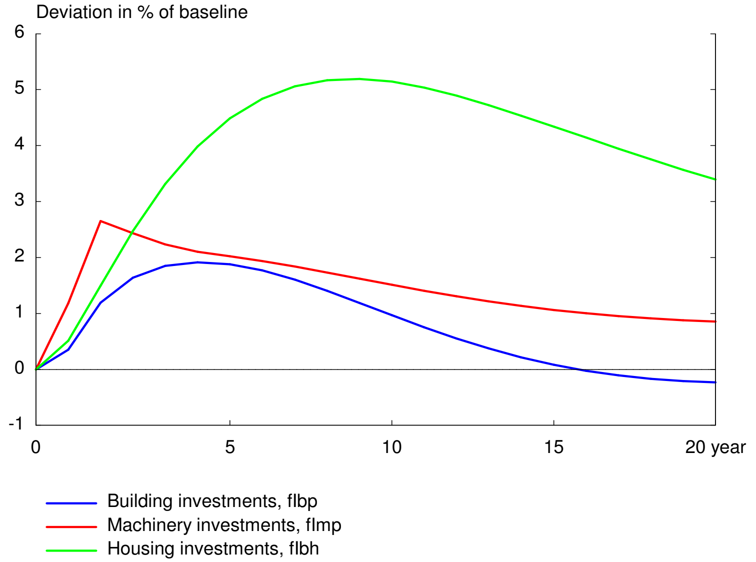
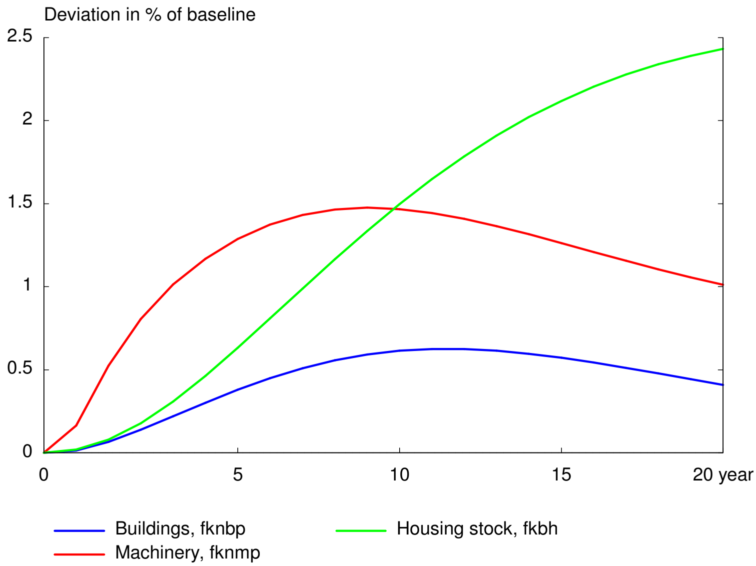


Figure 11.16b **Capital stocks, interest rate falls by 0.5%**



Investment effect remains positive

Total investment effect remains positive in the long term. The increased capital and investment requirement reflects that the capital stock remains relatively cheaper because interest rates are lower, and the wage rate is higher than in the baseline scenario. For example, a 1 per cent larger capital stock growing parallel to the baseline requires both a 1 per cent higher reinvestments and a 1 per cent higher net investment.

The higher capital intensity increases labour productivity and total output produced by equilibrium employment. Thus, the lower interest rate has a permanently positive effect on production and value added.

Positive long-term consumption effect, negative long-term wealth effect

The consumption effect increases during the first years of the model calculation and remains positive in the long term, cf. the previously shown figure 11.15. The positive long-term effect on consumption is accompanied by a negative long-term effect on private wealth used in the consumption function.

As we have seen, private wealth increases in the first part of the calculation period due to the strong short-term reaction in house prices and in housing wealth. Thereafter, the wealth effect starts to fall, as the short-term impact on housing wealth is redressed, and as financial wealth falls because consumption has increased more than income.

Pension savings decrease when interest rates fall

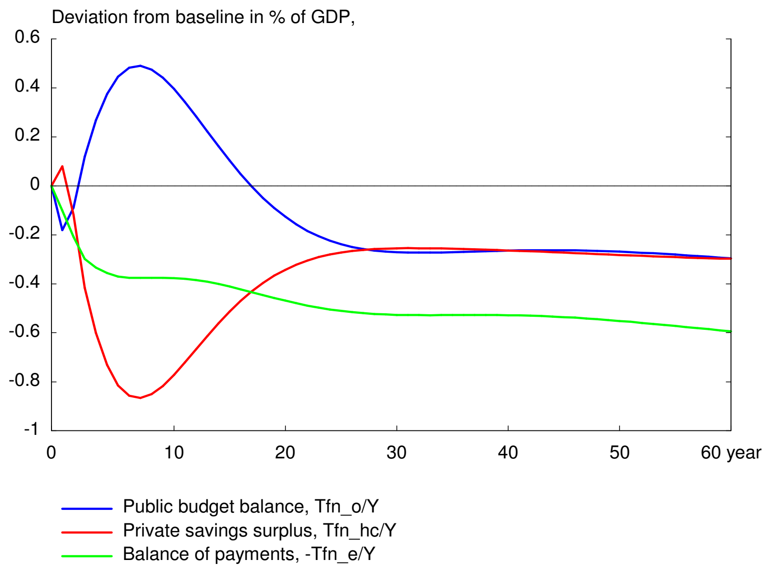
There is no interest rate in the consumption function, so the long-term negative effect on private wealth is not due to a substitution effect having reduced the propensity to save. The negative wealth effect rather reflects a negative institutional savings effect, which arises when the lower interest rate reduces the growth in private pension assets.

The income concept applied in the long-term equation for the desired consumption is calculated, exclusive of savings in pension assets. Thus, when pension savings fall due to lower accrued interest on pension wealth, the applied income variable, *ydl_hc*, will rise and stimulate consumption. The accompanying increase in the consumption equation's income, *ydl_hc*, relative to total private disposable income, *yd_hc*, will work like a higher propensity to consume, as discussed under consumption equation properties in section 3.2. A higher propensity to consume makes private wealth fall, cf. section 11.2.

Public finances deteriorate in the long term

A fall in the interest rate also resembles a rise in the propensity to consume by having a negative long-term effect on public finances, as the lower interest income on private wealth means lower tax revenues in the public sector, see figure 11.17 with effects on savings surpluses.

Figure 11.17 **Savings surpluses, interest rate falls by 0.5%**



Removing effect on public debt reduces positive effect on consumption

This means that the 0.5 per cent fall in interest rates must be accompanied by a tax increase if you want to neutralize the long-term effect on public debt. More specifically, central government income tax rates must be increased by 3 per cent, e.g. from 15 to 15.5 per cent, and this is supplemented by a one-off capital transfer at 1.4 per cent of GDP from public to private sector.

With the income tax increase included, the positive long-term effect on private consumption is reduced, and in the long term consumption is only somewhat higher than its baseline scenario, see figure 11.18. Compared to the calculation without tax financing, the long-term consumption effect is reduced significantly from about 0.6 to about 0.1 per cent. It can be added that the latter effect, 0.1 per cent of total private consumption, is the combined result of an increase of one and a half per cent in the interest-sensitive housing consumption and a decrease of one quarter per cent in consumption excl. housing.

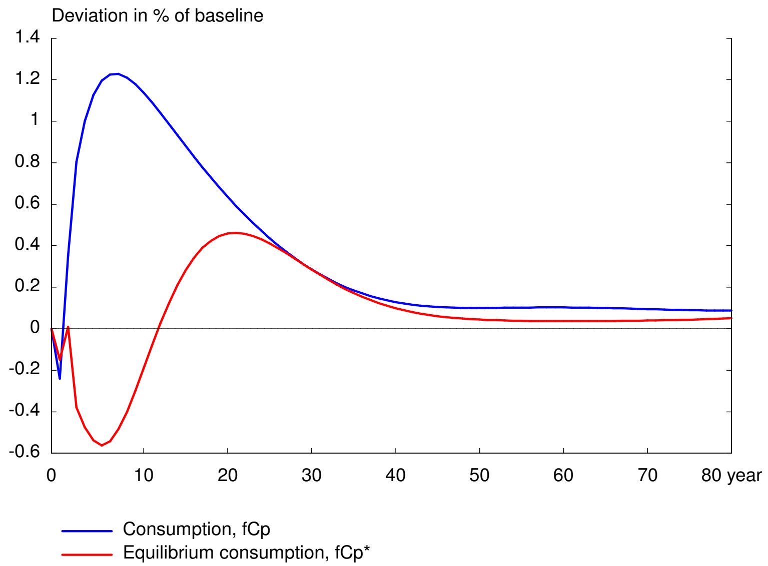
Consumption effect approaching equilibrium ...

The small positive long-term consumption effect of lower interest rates (for constant public debt ratio) is confirmed by the small positive long-term effect on the expression for equilibrium consumption, fCp^*, formulated in the previous section 11.2 on higher propensity to consume. When actual consumption corresponds to equilibrium consumption, financial wealth will be growing with the nominal steady-state rate of 3.5 per cent. It is clear from figure 11.18 that the effect on actual consumption in the long term ends up being close to the effect on equilibrium consumption. The two curves also come close when the propensity to consume is shocked, but it makes a difference that the adaptation takes much more time when interest rates change, so figure 11.18 has an x-axis covering 80 years.

. ... but the adaptation period is long

It takes a long time for consumption to reach equilibrium following a drop in interest rates. The long adaptation period reflects that the lower rate of interest reduces the steady state equilibrium of pension wealth, and it takes a long time for pension wealth to adapt to its new equilibrium. It also takes a long time for housing capital to adapt to the permanently higher level, which results from the fall in user cost. In general, it makes a difference to the calculation in 11.2 on higher propensity to consume that more relative prices change in the long term when interest rates are reduced.

Figure 11.18 **Interest rate falls by 0.5%, public debt unchanged**



Sensitivity analysis of the consumption effect

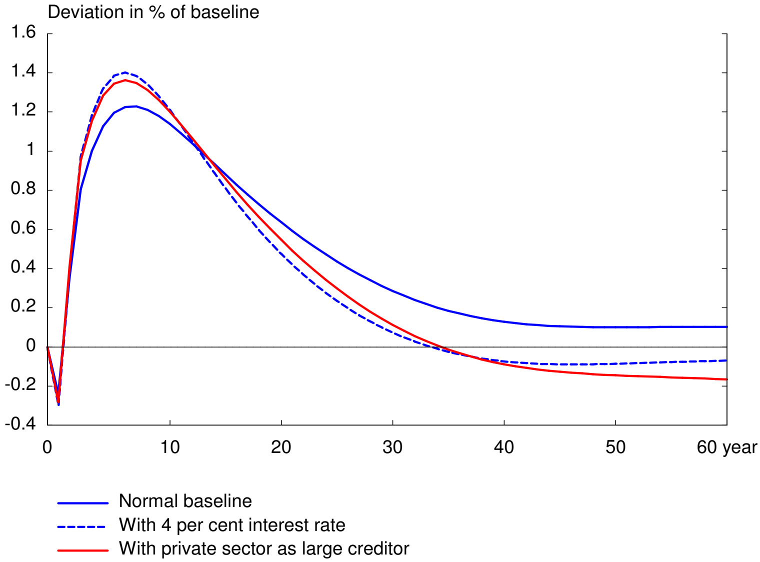
The consumption effect of lower interest rates can be illustrated a little further with a sensitivity analysis on the applied baseline scenario. The effect of lower interest rates depends on the baseline on two points. First, the size and sign of private interest income in the baseline scenario determines the size and sign of the income effect of lower interest rates. Second, the difference between the interest rate and nominal income growth determines the long-term impact on consumption of the implied effect on the propensity to consume.

Positive interest income reduces the long-term consumption effect

On the first point, the private sector has a positive interest income in the baseline scenario, so the income effect of a fall in interest rates is negative. The relationship can be illustrated by using a baseline with an even more positive private interest income, corresponding to the private sector being a larger creditor with a higher financial wealth.

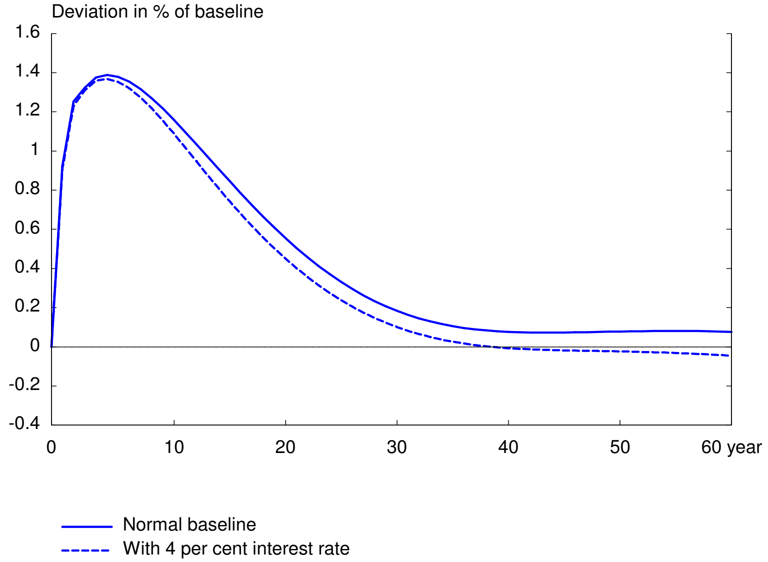
With this alternative starting point, a fall in interest rates has a larger negative effect on income, and there is a negative long-term effect on consumption from a fall in interest rates accompanied by tax increases to keep public debt unchanged, see the red curve in figure 11.19a.

Figure 11.19a **Private consumption, interest rate falls by 0.5%, public debt unchanged**



It should be noted that we are discussing private interest income, including interest accrued on pension wealth. Excluding the interest on pension wealth, private interest income has been negative since the data starts in 1966, but incl. interest on pension wealth, private interest income has mainly been positive for the last 20 years. It is against this background that private interest income is assumed to remain positive in the baseline scenario. It may be added that the consumption effect also reflects the size of public debt in the baseline scenario. If public debt and interest outlays were higher, public interest outlays would fall by a larger amount when interest rates fall, and it would take a smaller tax increase to neutralize the impact on public debt.

Figure 11.19b **Priv. consumption, propensity to consume + 1%, public debt unchanged**



Difference between interest and growth rate has an effect on consumption

The second point about the role of the baseline concerns the difference between interest rate and growth rate. In the baseline, this difference is set to zero before tax. However, when interest rates are lowered from 3.5 to 3 per cent, the interest rates becomes 0.5 per cent less than the growth rate, which remains at 3.5 per cent. It may be noted that the payment of tax on interest income makes the interest rate after tax less than growth, but the restriction on public debt implies that the tax savings on interest income are replaced by a general tax increase, so we may as stated compare the interest rate before tax with the growth rate.

Fall in interest rates increases private debt ...

We have seen that the fall in interest rates makes private wealth fall relative to the baseline scenario, and as the lower interest rates make housing wealth increase, there will be a significant fall in private financial wealth, as total private wealth falls while non-financial wealth rises. This decline in financial wealth tends to increase consumption when the interest rate is only 3 per cent and growth is 3.5 per cent.

... and a higher interest-rate growth differential makes the consumption effect more negative

The relationship was discussed in section 3.2 on consumption function properties, and its importance is illustrated by the fact that long-term consumption effect becomes more negative, if we use a baseline with an interest rate of 4 per cent instead of 3.5 per cent, see figure 11.19a. With this baseline, the interest rate equals growth after being reduced by 0.5 per cent, so there is no positive consumption effect of lower financial wealth. If the baseline interest rate is increased even further to 4.5 per cent, the interest-rate growth differential will remain positive in the model calculation, and lower interest rates will have a clearer negative consumption effect in the long term.

It is noted that the long-term consumption effect is negative, although the fall in interest rates increases value added and GDP. The higher GDP goes largely to the higher investments created by the fall in interest rates, and if the consumption effect is negative because interest rates exceed growth, the lower consumption is balanced by a positive effect on net exports, which is used to pay the foreign lenders. When public debt remains unchanged, lower private financial wealth has its counterpart in higher foreign debt.

It may be added that Pedersen and Rasmussen (2001) assume a positive interest-rate growth differential in their analysis of ADAM and DREAM. This premise contributes to the finding of a clear negative long-term consumption effect from reducing the interest rate, regardless of the model. Besides, Pedersen and Rasmussen also include a tax change to ensure that public debt is unaffected by the fall in interest rates.

Interest-rate growth differential impacts effect of higher propensity to consume

The importance of the interest-rate growth differential can be further illustrated by reference to the calculation that raises the propensity to consume for fixed interest rates. In section 11.2, long-term consumption effect is close to zero when we increase the propensity to consume, while keeping public debt unchanged by a tax increase. If this calculation is repeated on a baseline, where the interest rate is 4 per cent and thus 0.5 per cent larger than growth, the long-term consumption effect becomes negative, see figure 11.19b.

Summary on fall in interest rates

In summary, we have seen that a fall in interest rates has an expansionary effect on investment in both the short and long term. The short-term effect on consumption is also positive, and a fall in interest rates has on the whole a short-term expansionary effect on the economy. In a longer term, the expansionary effect is balanced by a loss of market share and the initial expansion of domestic demand is reduced, as the initial boost to housing wealth disappears. If public finances are not compensated for the loss of tax revenues on interest income, we get a positive long-term effect on consumption relative to the standard baseline scenario from chapter 10. If long-term public debt ratio is kept unchanged by a tax increase, the long-term effect on consumption is reduced significantly. If we also have a baseline where the private sector is a net creditor and the interest-rate growth differential is positive, the long-term consumption effect of a fall in interest rates is likely to be negative

11.4 Fall of 1 per cent in foreign prices

Foreign prices are here import prices, competitor prices in the export equations, plus world-market determined prices, e.g. oil prices. There are 14 of these prices in ADAM. They are all exogenous, and to shock foreign prices we reduce all 14 by 1 per cent relative to the base line scenario. As the interest rate shock in the previous section, it is a shock that comes from abroad. The change in exogenous variables is specified in box 11.6, which also mentions that excise duties are price indexed in order to have the same relative price change before and after tax.

Real income rises, competitiveness falls

The immediate effect of lower foreign prices is that the real income of consumers rises, while the competitiveness of businesses falls.

In the short term, market share and employment fall

Lower import prices reduce Danish consumer prices, but the first-year increase in consumer spending cannot compensate for the loss of Danish market shares. This means that unemployment begins to rise already in the first year, and the higher unemployment makes wages fall relative to the baseline scenario. This will redress the fall in competitiveness and market shares, and eventually unemployment will stop rising and return to the baseline scenario. The adaptation to the new equilibrium implies that the wage and domestic prices will end up being 1 per cent below the baseline scenario, similar to foreign prices.

In the long term only the price level is affected ...

If both foreign and domestic prices fall by 1 per cent in the long term, relative prices are not affected by the fall in foreign prices. Thus, there is no effect on the volume variables in the model, provided that volumes in the long term only respond to relative prices and not to price levels. This traditional property has been incorporated in ADAM's demand equations, which comprise the consumption system, housing model, factor block plus export and import demand, cf. the respective chapters.

... so the fall in foreign prices works like a monetary shock

Reducing foreign prices in the model does seem to illustrate that there is a long-term classical dichotomy between ADAM prices and volumes. In any case, the long-term effect on the price variables is close to minus 1 per cent, while the long-term effect on volume variables is close to zero, see figures 11.20a and 11.20b. This means that the permanent reduction of foreign prices in Danish kroner works as a monetary shock, which in the long term only affects the price level. On the volume side, exports take a long time to return to the base line scenario.

Figure 11.20a **Volume variables, foreign prices fall by 1 per cent**

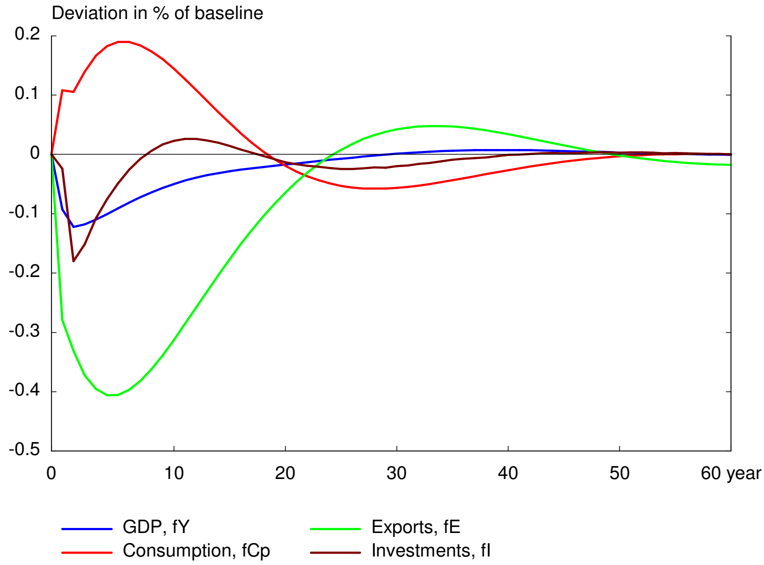
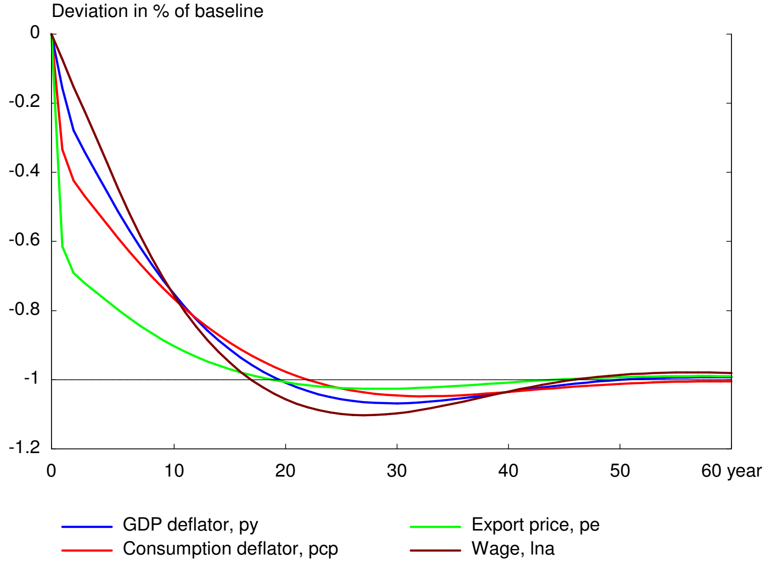


Figure 11.20b **Price variables, foreign prices fall by 1 per cent**



The result relies on public revenues and expenditures being price-indexed

Traditionally, the issue of dichotomy between prices and volumes refers to the properties of behavioural equations, but the long-term zero-effect on volumes can, for instance, be broken by the tax system. If taxes in the long term do not move pari passu with income, the lower prices and wages will produce a permanent change in the tax level and thus in real disposable income. This will affect private consumption and change the composition of demand, so that not all demand components will return to the baseline. In general, the 1 per cent reduction of foreign prices will affect volumes permanently, if the fall in price level permanently changes the relation between gross value added and disposable income.

As already mentioned, excise duties are assumed to be price indexed, and with the statutory wage indexation of allowances and tax brackets modelled in ADAM, also transfer payments and income taxes will be proportional to earned income in the long term. Other capital taxes are exogenous in ADAM and thus represent a small exception to the indexing principle. However, by reducing other capital taxes by 1 per cent, cf. box 11.6, we come close to a situation where domestic prices follow the foreign price level in the long term, while volumes in the long term are unaffected.

Box 11.6 **Input for calculation on 1 per cent fall in foreign prices**

In the calculation, 14 foreign prices are reduced by 1 per cent relative to their baseline. The 14 exogenous prices are shown below.

pm01	import price on foods etc.
pm2	import price on materials
pm59	import price on manufacturing
pm7b	import price on cars and trucks

pm7y	import price on ships etc.
pmt	price on tourist spending
pms	price on services excl. tourists
pe01	export price on foods etc.
pee2	competitor price on export of materials
pee59	competitor price on manufacturing export
peet	competitor price on tourists revenues
pxqs	production price of sea transport
peesq	competitor price on services excl. sea transport and tourists
boil	oil price

In addition, the prices of coal and other energy, *pm3k* and *pm3q*, are exogenized and reduced by 1 per cent. The calculation is made with price indexation of excise duty rates; see the discussion of this option in chapter 8.5. Besides, two minor exogenous transfer variables (capital taxes, *Sk_h_o*, and compensation of employees working abroad, *Ywn_e*) are both reduced by 1 per cent. Thus, *Sk_h_o* follows the other taxes, and *Ywn_e* follows the foreign wage.

In the calculation on the temporary wage shock, the wage variable, *lna*, is changed by increasing the adjustment term, *jrlna*, in year 1, so that the related proportionality factor, 1+*jrlna*, rises by 1 per cent. If *jrlna* is 0 in the first year of the baseline, it is raised to 0.01.

One-off shock of 1 per cent to the wage equation

The model-calculated effects of reducing foreign prices permanently by 1 per cent can be compared with the effects of shocking the wage equation by 1 per cent in just the first year of the calculation period. The wage equation determines the wage increase and the described shock makes the wage increase by an additional 1 per cent in the first year. For example, by 4.5 per cent instead of 3.5 per cent, and this makes the wage level 1 per cent higher in the first year of the calculation period.

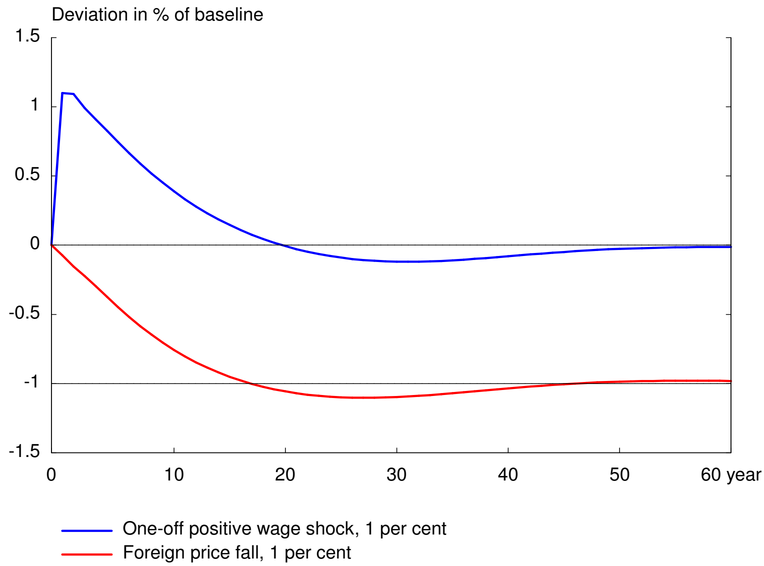
In the short term, market share and employment fall

Higher wages deteriorate competitiveness in the same way as lower foreign prices deteriorate competitiveness, and through the accompanying negative effect on foreign trade the wage shock will be crowded out in the same way as a permanent reduction of foreign prices will be balanced by a reduction in the Danish wage and price level. Crowding out the one-off wage shock of 1 per cent means that wage growth for some years must be slightly below the baseline scenario.

In the long term wages return to the baseline scenario

While the wage variable ends up 1 per cent below its baseline, when foreign prices fall by 1 per cent; the wage variable returns to the baseline, when the adjustment term of the wage equation is shocked by 1 per cent in year one. The differences and similarities between the wage effects of the two shocks are illustrated in figure 11.21.

Figure 11.21 **Hourly wage, foreign prices -1%, one-off wage shock +1%**



As discussed in chapter 7, the wage equation is formulated as a Phillips curve where unemployment affects wage growth. The initial wage shock and loss of competitiveness increase unemployment and reduce wage growth for some years, until the accumulated effect on the wage level equals minus 1 per cent, which cancels the initial wage shock.

Faster adaptation to foreign price fall

Although the crowding out mechanism is similar for the foreign price and the domestic wage shocks and the long-term effects look symmetrical, there is a minor difference in the adjustment speed. The wage variable, *lna*, adapts faster to its steady state, when foreign prices are changed. This is because lower foreign prices directly affect the relative price terms in foreign trade equations. The wage shock does not attain its full impact on relative prices until the wage change is passed through to prices, e.g. to the production price in manufacturing, *pxnz*, that enters the relative price term in the export equation for SITC 5-9.

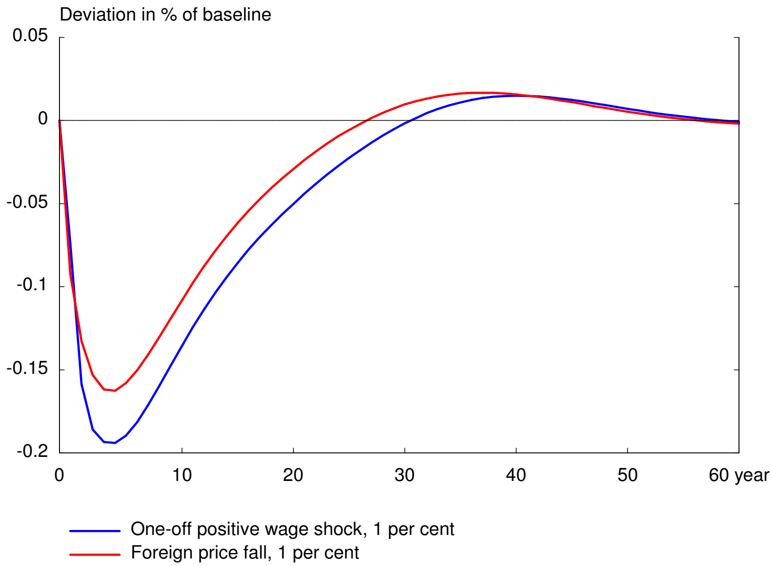
Figure 11.22 illustrates that employment is faster at returning and crossing the baseline, when foreign prices are changed.

Public budget balance deteriorates in the adaptation period

Both the fall in foreign prices and the one-off shock to the wage equation creates an economic downturn, which deteriorates the public budget balance, but the negative budget impact is not permanent. In the long term, the effect on the primary budget balance will disappear as production and employment return to the baseline.

However, the effect on public debt does not disappear, not even in the case of a temporary wage shock, where both prices and volumes return to the baseline. It requires a temporary increase in public revenues to eliminate the increase in public debt. The counterpart to higher public debt is higher foreign debt. Thus, in the absence of a fiscal reaction function to stabilize public debt, we have hysteresis in public and foreign debt, and in the related interest flows. It is noted that the same two debt variables will keep increasing relative to the baseline, if the public purchase of goods is raised permanently, cf. subsection 11.1.5 discussing the effect of public purchases on the savings surpluses.

Figure 11.22 **Employment, foreign prices -1%, one-off wage shock +1%**



A permanent shock to the wage equation would work as a supply shock

We have only shocked the wage equation in a single year, so there is no permanent effect on unemployment. If you permanently increase the adjustment term of the wage equation, it will work as a negative supply shock, which permanently increases the long-term unemployment of the model. A permanent reduction in the adjustment term of the wage equation works as a positive supply shock, as illustrated in chapter 2.

11.5 Expanding the labour force by 0.5 per cent

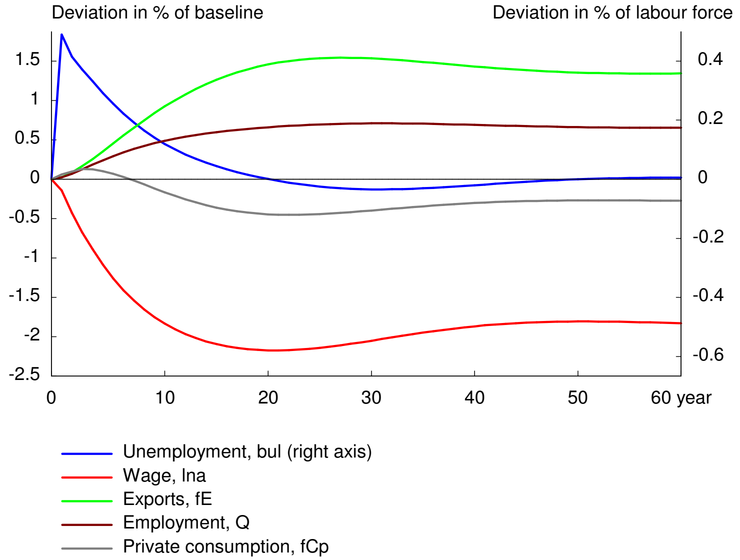
Labour force increased by reducing a group without transfer income

In this model calculation, the labour force is increased by 0.5 per cent equivalent to almost 15,000 people. The labour force in ADAM is increased by reducing the number of people outside the labour force. People outside the labour force include recipients of public transfers, such as the early retired, activated, students and persons on maternity leave. It is here chosen to reduce the group of people who do not receive transfers. This residual group is referred to as 'other' in table 6.11 in chapter 6. If the additional people drawn into the labour force had lost a public allowance, the immediate impact on private consumption would be more negative, but the scope for private and public consumption will always grow when there are more people in work.

Unemployment rises immediately but is unchanged in the long term

When the supply of labour is increased, the newcomers to the labour force are not automatically employed so unemployment rises at once. Higher unemployment triggers a lower wage increase, and the decline in wages relative to the baseline scenario will increase competitiveness and exports and thus gradually pull the extra labour force into employment, while unemployment will fall and return to the baseline. Figure 11.23 shows some key effects of increasing the labour force.

Figure 11.23 **Labour force +0.5%**



Consumption falls in the long term ...

Private consumption rises to begin with, because the new members of the labour force receive unemployment benefits. In a longer term, the consumption effect becomes negative because lower wages will reduce the real income of consumers as import prices remain unchanged.

... but there is room for higher government spending or lower taxes

When the additional labour supply has been employed, the government will receive more tax revenue, and public debt will be falling compared to the baseline scenario. This means that the larger labour force provides the basis for financing either higher public spending or lower taxes. Both higher public spending and lower taxes will boost domestic demand and make it easier to employ the expanded labour force

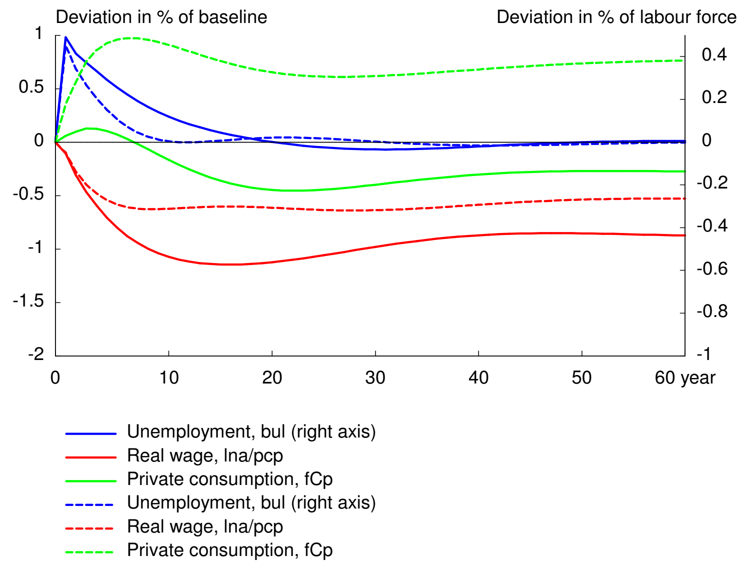
Alternative to increasing labour supply may be to increase taxes

If the alternative to a higher labour supply is to improve public finances by higher taxes, it is natural to accompany the increase in labour force with a tax reduction to keep public debt unchanged in the long term. If the increase in labour supply is accompanied by such a tax reduction, the model calculation will indicate the difference between closing a gap in public finances by means of either labour supply or taxes.

With lower tax, long-term consumption effect is positive

More specifically, the central government tax rates and healthcare contributions are reduced by 4.5 per cent, so that e.g. the upper limit tax rate drops from 15 to 14.3 per cent. The induced increase of private consumption expands the activity and makes unemployment return faster to its long-term baseline scenario, cf. figure 11.24.

Figure 11.24 **Labour force +0.5%, dashed curves represent public debt unchanged**



When consumption increases, exports need to increase less, and consequently, the real wage has to fall by less in the long term when long-term public debt is constant. It is noted that the real wage before tax, *lna/pcp*, falls in both calculations, but the real wage after tax rises in the calculation with unchanged public debt. The long-term rise in real wage after tax is around 0.3 per cent, which corresponds to the increase of private consumption.

11.6 Sensitivity analysis of adjustment pattern

A sensitivity analysis describes how sensitive a model-calculated result is to changes in the parameters of the model. This analysis can help to interpret the model properties by indicating which parameters and which equations are important for the model effects.

The point of departure is the unfunded shock to public purchases discussed in section 11.1. The present section examines two factors in the adjustment pattern when public purchases are increased. First, we examine how the adjustment pattern depends on key coefficients in the behavioural equations. The focus is on how long it takes to reach the long-term equilibrium, and on how much the model fluctuates on the way to equilibrium. Second, we examine how it affects the adjustment pattern that user costs are based on exogenous inflation expectations.

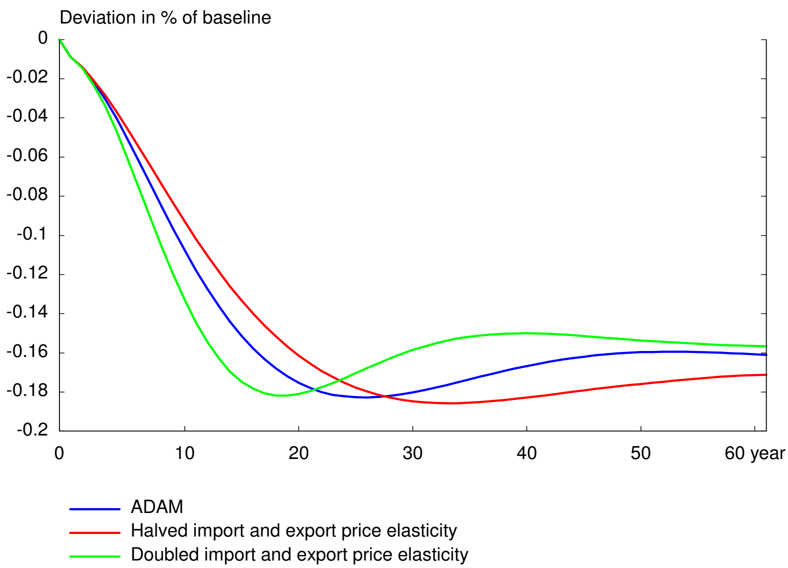
11.6.1 Role of key coefficients

In the sensitivity analysis of the adjustment pattern it is chosen to focus on the price elasticity of foreign trade, the coefficient for unemployment in the wage equation and the price elasticity of housing demand.

Speed of adjustment grows with price elasticity of foreign trade

In the purchase of goods calculation, domestic prices rise relative to foreign prices as part of the crowding-out process, which makes the price elasticity of exports crucial for the reaction of exports to a given change in relative prices. The higher the price elasticity, the faster the export volume, *fe*, will react, cf. figure 11.25.

Figure 11.25 **Public purchase of goods + 1%, effect on export volumes, *fE***



The figure shows that the difference primarily relates to the dynamic adjustment of exports, while the long-term export effects are less different. In the short term, it is obvious that higher price elasticity will increase the reaction of exports to the cyclical response in prices. In the long term, exports will fall by an amount that balances the higher domestic demand and keeps unemployment at its long-term level. This means that the crucial long-term role of the export price elasticity is to determine by how much export prices should increase in order to create the necessary fall in exports, while the role of price elasticity for the long-term effect on export volumes is more indirect and less obvious.

Long-term export effect depends only slightly on price elasticity

The (modest) difference between long-term export effects in figure 11.25 suggests that the larger the price elasticity is, the smaller is the long-term decline in exports. This inverse relationship between price elasticity and long-term export decline reflects that the positive effect on domestic demand will be higher when the foreign trade price elasticity is lower. Lower price elasticity implies that export prices and hence also the terms of trade will have to increase by more in order to produce a certain decrease in exports; and the more the terms of trade increases, the larger will be the accompanying positive effect on real income and private consumption.

Export price effect depends on price elasticity

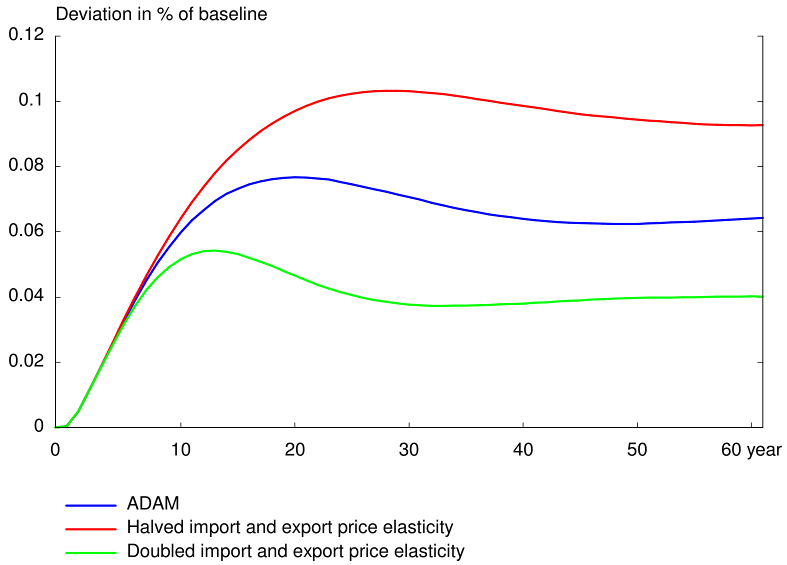
While the long-term effect on export volumes depends only modestly on the foreign trade price elasticity, it is as stated obvious that the long-term effect on export prices reflects the price elasticity, see figure 11.26.

Export volume effect is affected by import price elasticity

Export and import price elasticity is changed proportionally in the sensitivity analysis of exports illustrated in figures 11.25 and 11.26. If the import price elasticity is instead held constant, the relation between price elasticity and export effect changes. With constant import price elasticity, the long-term fall in exports will become larger if export price elasticity becomes larger.

This positive correlation between export price elasticity and long-term export fall appears with constant import price elasticity, because the induced rise in domestic prices always becomes smaller when the export price elasticity is raised. Consequently, imports increase by less if the import price elasticity is held constant, and exports will have to fall by more to compensate for the smaller import effect. Thus, with constant import price elasticity exports fall by more in the long term when the export price elasticity is raised, because imports rise by less.

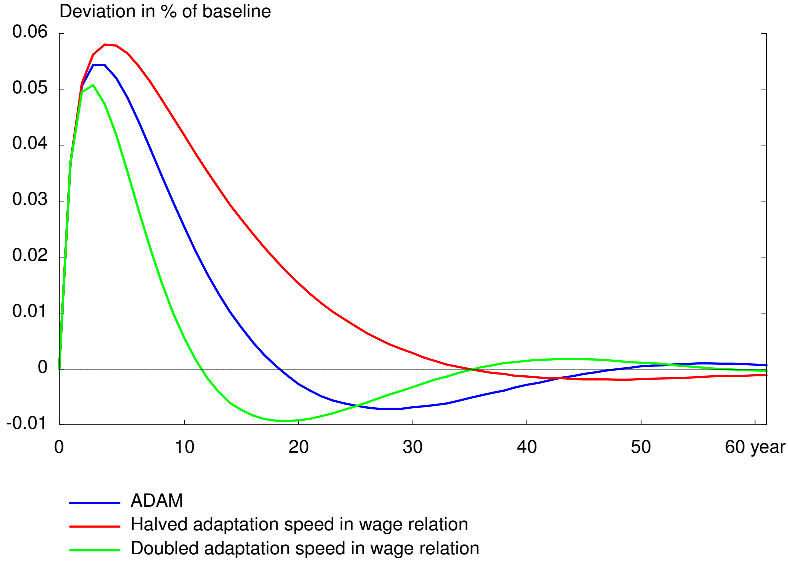
Figure 11.26 **Public purchase of goods + 1%, effect on export prices, *pe***



Price elasticity is larger for exports than for imports

Import price elasticity has the same qualitative role as export price elasticity, but part of the import does not compete with Danish production and the average price elasticity of imports is smaller than that of exports. Thus, it means less to double or halve the price elasticity of imports than to double or halve the price elasticity of exports.

Figure 11.27 **Public purchase of goods + 1%, effect on employment, *Q***



Wage response to unemployment rate is important for adjustment pattern

It is the wage equation that creates the transition from a short-term model with cyclical unemployment to a long-term model with equilibrium unemployment. In the wage equation, it is the coefficient for actual minus equilibrium unemployment, i.e. for *bul-bulw*, which is crucial for the speed of transition from short- to long-term model. The coefficient is called γ in the wage equation (7.1) discussed in chapter 7. The higher this coefficient is numerically, the stronger wages respond to the short-term decline in unemployment created by higher public purchases, and the faster employment will return to its baseline scenario, representing the equilibrium, see figure 11.27.

With a high coefficient for unemployment, wages will overreact and for a period overshoot the long-term effect on wages. This overshooting of wages and competitiveness makes employment overreact as well, illustrating that a higher coefficient for unemployment makes employment fluctuate more around its equilibrium, cf. figure 11.27.

Price elasticity of housing demand determines fluctuations in house prices

In ADAM, the house price is modelled as the price that clears the housing market, while the housing stock adapts at a slower pace. This means that house prices react quickly and strongly to income and consumption changes, and the accompanying impact on consumer wealth and housing investment can potentially make both production and employment overreact and fluctuate. The response of house prices to higher income hinges on the price elasticity of housing demand. The higher the price elasticity, the smaller the price response is.

Higher price elasticity means less volatility in house prices

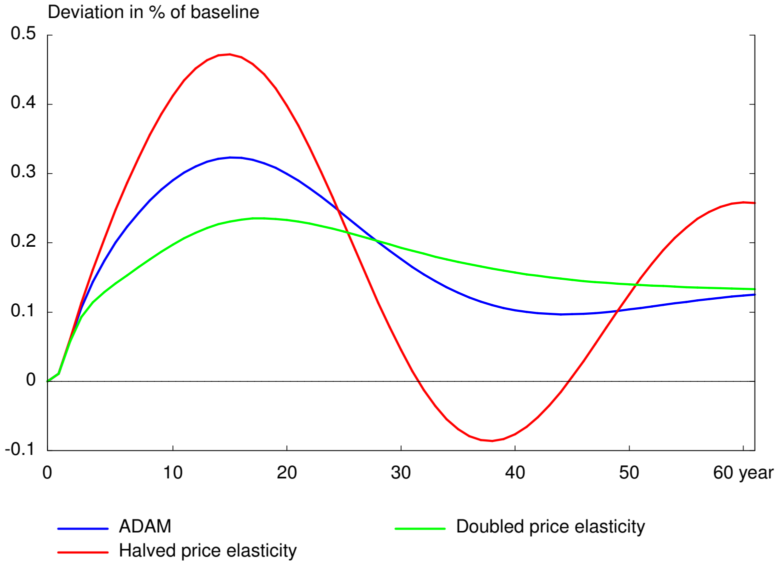
This inverse relation between the price elasticity of housing demand and the market-clearing response of house prices can be seen from equation (3.9) that determines the desired housing stock, as discussed in chapter 3. The price elasticity is called β_1 in equation (3.9), which is reproduced below, with the house price moved to the left hand side as explained variable, and with housing stock on the right hand side.

$$houseprice = \beta_0^{\frac{-1}{\beta_1}} \cdot \left(\frac{consumption}{housingstock}\right)^{\frac{-1}{\beta_1}} \cdot \frac{consumptionprice}{usercostrate}$$

Private consumption excl. housing is the income variable in housing demand, so consumption is included on the right hand side of the equation divided by housing stock. House price elasticity β_1 is minus 0.3 in ADAM, so the implied elasticity of house prices with respect to consumption over housing stock is around 3.3. This means that 1 per cent higher consumption triggers a house price increase of 3.3 per cent for a given housing stock. If you halve the house price elasticity of 0.3 to 0.15, you will get twice the house price increase, i.e. 6.7 per cent.

The role of the house price elasticity is illustrated in figure 11.28, which shows that house prices react stronger to higher public purchases if the price elasticity is halved. The stronger reaction includes not only that house prices respond more in the short term, house prices also fluctuate significantly for years and for a period they undershoot the baseline.

Figure 11.28 **Public purchase of goods + 1%, effect on house price, *phk***



Price elasticity of demand does not determine long-term equilibrium of house prices

In the long term, the house price is given from the supply side and, regardless of the price elasticity on the demand side, house prices will eventually increase proportionally with the price of housing investment. The house price affects the scale of housing investments, so the smaller the price elasticity, the stronger the response in both house prices and housing stock will be. Thus, high price elasticity in housing demand prolongs the adjustment of the housing stock to equilibrium.

Fast crowding-out with stable adjustment ...

To make ADAM's key variables, including output and employment, adapt quickly and smoothly to equilibrium requires new and different estimated coefficients for exports, imports and wages.

... requires a highly price sensitive and responsive foreign trade ...

To increase the speed of the adjustment to equilibrium, both price elasticities and adjustment coefficients in the foreign trade equations must be increased. The average adjustment coefficient of 0.154 for exports in ADAM indicates that 15.4 per cent of the discrepancy between the actual and desired exports will be closed within a year. Imports adapt faster than exports in ADAM, and it is chosen to multiply export's and import's adjustment coefficients by, respectively, 4 and 2. The price elasticities are increased by a factor of 10

... plus a sluggish wage formation

With the significantly increased price elasticity of foreign trade, the long-term effect on wages is clearly smaller. A smaller long-term effect on wages increases the tendency for wages to overreact in the short term, and the increased degree of overshooting makes both wages and employment fluctuate stronger before they reach equilibrium. The tendency to over-reaction in wages and employment can be avoided by reducing the coefficient for unemployment in the wage equation. You can, for instance, reduce the coefficient for unemployment by about 2/3 as shown in table 11.1, which gives an overview of parameter changes.

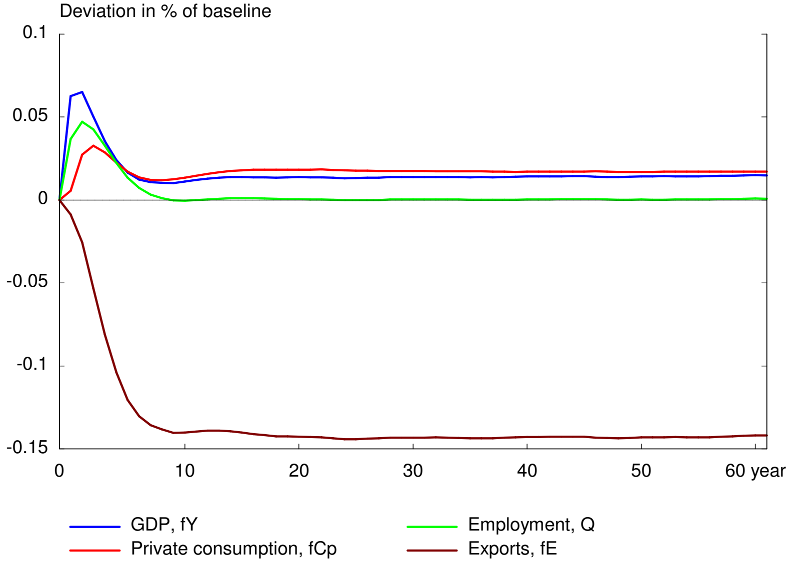
Table 11.1 **Illustrative change in key parameters**

	ADAM	Factor	New value
Avg. export price elasticity SITC 2, 59 and service	-2.020	10	-20.200
Avg. import price elasticity SITC 01, 2 and 59	-0.889	10	-8.890
Avg. export adjustment SITC 2, 59 and service	0.154	4	0.616
Avg. import adjustment SITC 01,2 and 59	0.455	2	0.910
Coefficient for unemployment, wage equation	-0.550	0.35	-0.193

Figure 11.29 shows the effect of higher public purchases on selected variables, with the alternative parameter values from table 11.1 inserted in the model.

The time it takes before the employment effect crosses zero is reduced from 18 years with the standard ADAM to 10 years with the alternative key parameters inserted. Moreover, the effects shown are stabilized within a shorter span of years, so the model reaches its equilibrium much faster with the changed parameters.

Figure 11.29 **Public purchase of goods + 1%, alternative ADAM cf. table 11.1**



Larger price elasticity of foreign trade reduces effect on private consumption

With 10 times the normal price elasticity in foreign trade, the necessary increase in export prices and wages is significantly lower. Thus, the terms of trade will improve by less and the accompanying permanent increase in private consumption will also be smaller than with standard ADAM, see figure 11.30. The figure compares the effects of the official ADAM, solid curves, with the alternative model, dashed curves, for GDP and employment and also for private consumption and exports.

Figure 11.30a **Public purchase of goods + 1%, ADAM solid, alternative ADAM dashed**

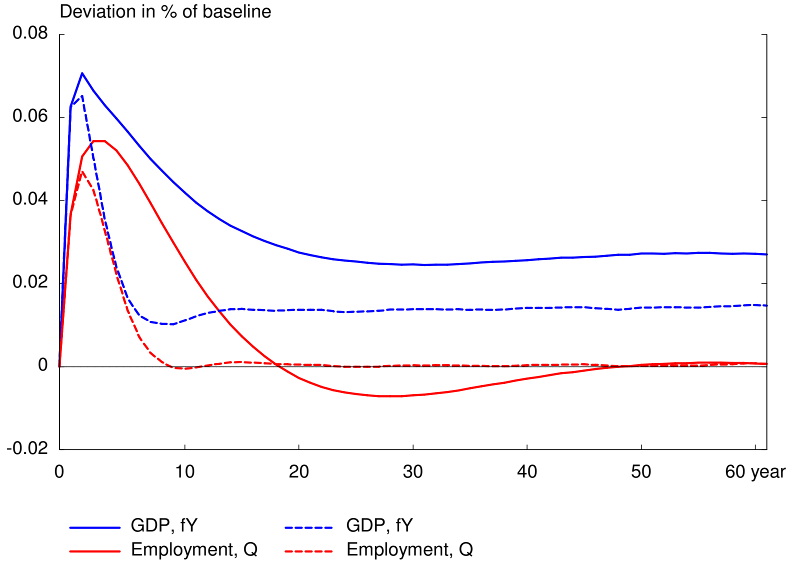


Figure 11.30b **Public purchase of goods + 1%, ADAM solid, alternative ADAM dashed**

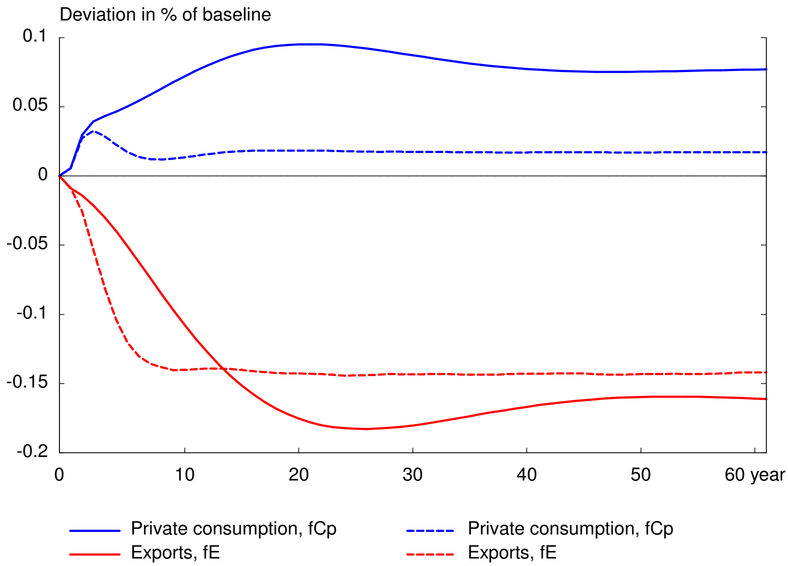


Figure 11.30 shows, for example, that the extra high price elasticity in foreign trade makes the consumption effect hump-shaped, so that the effect on consumption peaks in the beginning of the calculation period, together with the effect on investment. It can also be noted that the smaller long-term effect on wages and private consumption in the alternative model reduces the long-term effects on both labour productivity and indirect taxes in real terms, and this contributes to a lower long-term effect on real GDP.

In the alternative model, only coefficients relating to lagged variables have been changed. Thus, there is no difference in the first-year effects, and only modest differences in the immediately following years. The difference between standard and alternative ADAM concerns primarily the longer term, and for several variables the long-term effect is changed as well. However, not all long-term effects are changed; for example, the long-term effect on employment is zero in both models.

Alternative parameters are contrary to empirical evidence

The sensitivity analysis has illustrated that by changing a few model coefficients the adjustment to equilibrium can be speeded up. It can be technically easier to evaluate economic policies and exogenous shocks using a model that reaches equilibrium within a short time span, but it is, of course, problematic to use a model with fast adjustment when its coefficients cannot be estimated from the data.

11.6.2 Role of expected inflation

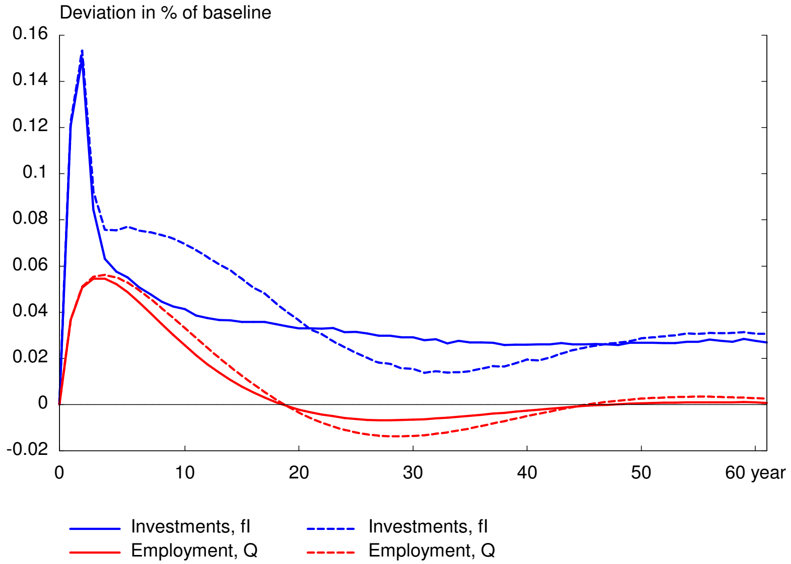
Exogenous inflation in user cost

As mentioned in the discussion of the model residential and businesses investments, it is decided to exogenize inflation expectations in user cost. This is done with the argument that the fixed exchange rate policy anchors the investor expectations. To illustrate the role of exogenous inflation expectations, the calculation on increased public purchases is repeated with adaptive inflation expectations. For example, the expected house price increase, *rpibhe*, is calculated with the formula:

$$rpibhe = 0.8 \cdot rpibhe_{-1} + 0.2 \cdot (pibh / pibh_{-1} - 1)$$

which makes the expected price increase a function of contemporaneous and lagged increases in the investment price, *pibh*. The expected house price increase, *rpibhe*, is included in the user cost of housing capital and has affected the estimated parameters, but when using ADAM it is standard to exclude both the equation above and similar equations for expected inflation in other user cost variables.

Figure 11.31 **Public purchases + 1%, expected inflation exogenous (solid) and adaptive (dashed)**



Adaptive inflation expectations increase cyclical response of investments ...

In the short term, where prices barely respond, it makes no difference whether inflation expectations are exogenous or adaptive, but after a few years, investments react more with endogenous inflation expectations. When higher public purchases stimulate domestic wages and prices, endogenous inflation expectations will increase. Higher inflation expectations will reduce user costs and thus reinforce the expansive effect on investments, cf. figure 11.31. The figure shows the effect of higher public purchases on total investment and employment, with both exogenous and endogenous inflation expectations.

... housing investments in particular

Housing investments are especially sensitive to the response of price expectations. Machinery investments have high import contents, so there is little response in the price of machinery investment. Besides, the large depreciation rate on machinery works as a dead weight in the user cost term. The user cost on building capital is more responsive, as both import contents and depreciation rates are smaller than for machinery. However, the substitution possibilities are much smaller for buildings, so buildings user cost has a minor impact on investments.

The pro-cyclical effect of adaptive inflation expectations is not permanent. In the long term, price increases will return to the baseline scenario and during the adjustment to the long term, price increases of all goods, including capital goods, will for a period be lower than in the baseline scenario. During this period user costs will be larger than in the baseline scenario, and this will dampen the effect on investments.

Model is more stable with exogenous inflation expectations

In summary, adaptive inflation expectations tend to make investment and employment fluctuate when the model is shocked. The response of the model is more stable with exogenous inflation expectations.

11.7 Comparison with SMEC and textbook

This section compares the public-purchases calculation on ADAM with a similar calculation on the Economic Council's SMEC model and with a numerical example on the response of output gaps in open economies with fixed exchange rate, see Sørensen and Whitta-Jacobsen (2005).

11.7.1 SMEC

Raising the public purchase of goods

The public purchases calculation on SMEC is taken from Grinderslev and Smidt (2007). They raise the public purchase of goods permanently by DKK 1 bn. 2000-prices without tax funding, and we compare the result of this calculation on SMEC with a similar calculation on ADAM.

Employment responds faster in SMEC

It turns out that SMEC's GDP effect is systematically higher and peaks later than ADAM's, see figure 11.32. At the same time, the employment effect in SMEC is only temporarily larger than in ADAM and quicker at crossing the x-axis. The employment effect crosses zero after 10 years in SMEC and after 17 years in ADAM[1], see figure 11.33.

In SMEC, wages are more sensitive to unemployment and employment is more sensitive to wages

We have not identified all significant differences between ADAM and SMEC, but some of the difference between GDP and employment effects in the two models can be explained by SMEC's stronger reaction in wages when unemployment changes and stronger substitution effect in employment when the relative price of labour changes.

[1] In the imitated SMEC calculation, the percentage increase in public goods is slightly reduced over time, and this reduces the amount of time before employment crosses zero to 17 years from the 18 years mentioned in section 11.6 on sensitivity analysis.

Figure 11.32 **Public purchases + DKK 1 bn., SMEC and ADAM - GDP**

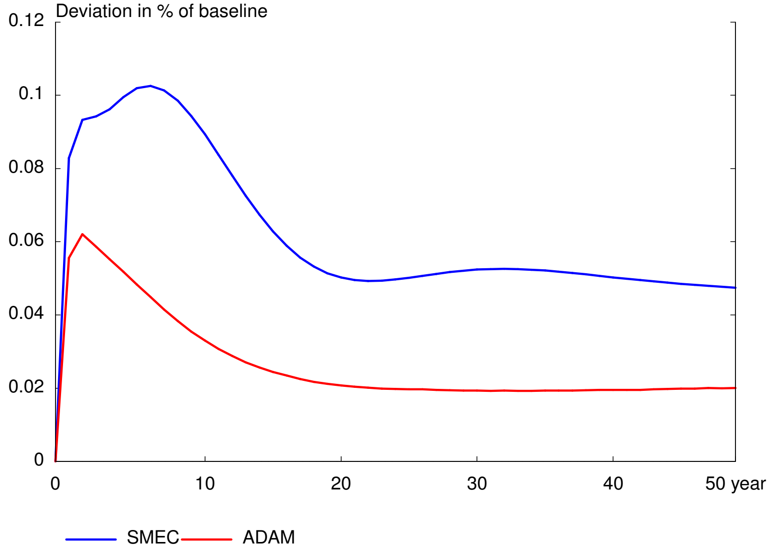
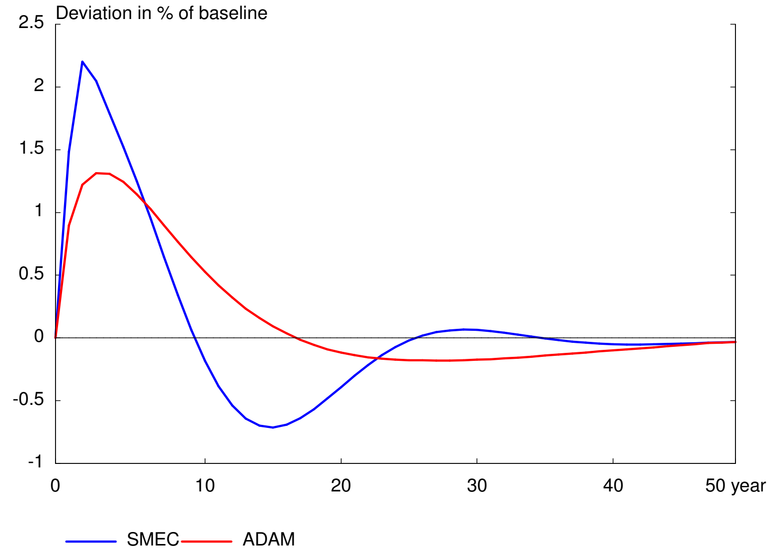


Figure 11.33 **Public purchases + DKK 1 bn., SMEC and ADAM - employment**



More specifically, the coefficient for unemployment in the wage equation is -0.89 in SMEC compared to -0.55 in ADAM, while SMEC's production functions are Cobb-Douglas with unitary elasticity of substitution between labour and capital. The average elasticity of substitution in ADAM's CES functions is clearly less than 1, cf. the presentation of factor demand in chapter 6

Wage response increases SMEC's consumption ...

The stronger wage response in SMEC, as illustrated in figure 11.34, gives a stronger improvement in the terms of trade and helps to explain that private real income and consumption rises by more than in ADAM.

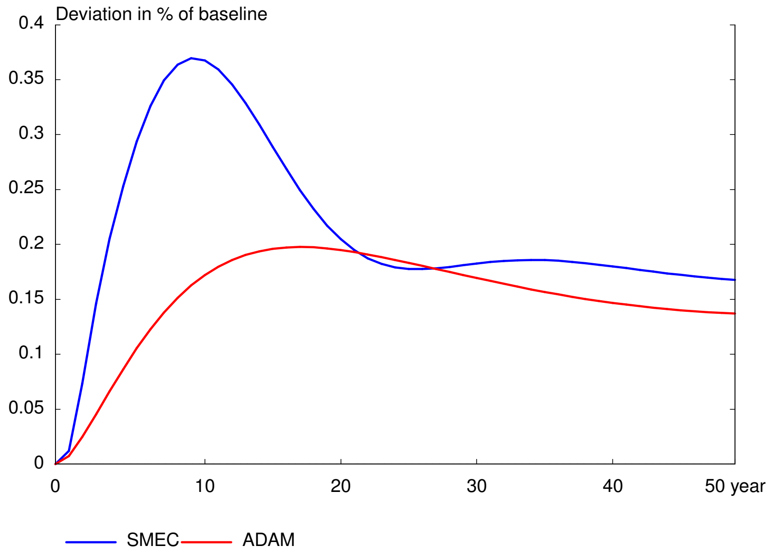
... and investment response

Moreover, the strong wage response in SMEC plus the inflation's-impact on user cost makes capital cheaper relative to labour and as SMEC's capital demand is price sensitive, investments increase by more than in ADAM. Thus, both consumption and investments respond stronger than in ADAM, and also labour productivity increase by more in SMEC due to the larger substitution effect. This helps to explain why GDP responds more than in ADAM, both in the short and in the long term.

Productivity effect is stronger in SMEC

Although production increases by more in SMEC, the employment effect turns negative at an earlier stage than in ADAM, see the previous figure 11.33 with the two employment effects. The faster crowding out of employment in SMEC reflects a stronger replacement of labour with capital, cf. the stronger increase in labour productivity.

Figure 11.34 **Public purchases + DKK 1 bn., SMEC and ADAM – hourly wage**



Comparing the two models demonstrates the point from the sensitivity analysis in section 11.6 that the crowding out of employment effects can be speeded up by using a wage equation that responds more to unemployment. It is also shown that higher elasticity of substitution in factor demand pulls in the same direction.

11.7.2 Textbook

Stylized model with 5-year half-life in output gap

ADAM's adjustment to equilibrium given a permanent shock to the public purchase of goods can also be compared to the description of economic adjustment in an applied macroeconomic textbook. Sørensen and Whitta-Jacobsen (2005) estimate in chapter 24, on open economies with fixed exchange rate, that it takes five years to crowd out half of a permanent demand shock to output. In the stylized textbook model, the output response to demand shocks is illustrated by an output gap following a simple autoregressive process that writes down the gap with an annual factor of 0.87. This corresponds to a half-life of five years ($0.87^5=0.5$).

$$ouputgap_t = ouputgap_0 \cdot 0.87^t$$

The estimate of the five-year half-life reflects some assumptions with respect to size and elasticity of foreign trade and with respect to the response of inflation to output gap. The latter response has the same role as the reaction of wage increases to unemployment in ADAM, so these assumptions relate to the same sort of factors included in the sensitivity analysis of ADAM's key coefficients in section 11.6.

Output gap is the difference between actual and potential output

Output gaps represent the difference between actual and potential output. Potential output is the output that can be produced at full potential employment, where potential employment equals labour force minus long-term unemployment. Normally, the output gap is measured on the basis of actual GDP and a potential GDP, which is calculated using a simple aggregate production function with capital and potential labour as inputs.

Output gaps can be calculated from the desired employment in the factor block

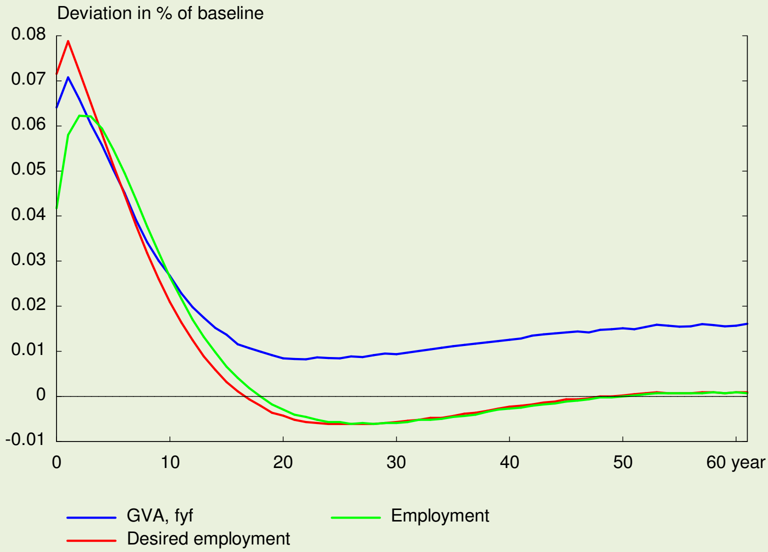
This way of measuring the overall output gap cannot be readily applied in ADAM due to an aggregation problem. There are 12 industries in ADAM and thus more than one production function. In relation to ADAM, the output gap can instead be represented by using the desired employment that enters the factor demand equations. The effect on the desired employment is proportional to the output effect in the short term and zero in the long term if potential employment is unchanged, see box 11.7 on output gaps in ADAM.

Box 11.7 **ADAM's output gap response to increased public purchases**

There is no variable for potential production in ADAM, but unemployment has a long-term equilibrium, and the corresponding potential employment is obtained by subtracting the long-term unemployment from the labour force. ADAM has 12 industries, so there is no clear linkage from employment and capital stock to total GDP, but for a given shock to the model, e.g. increased public purchases, an effect on the output gap can be estimated.

A quick estimate is to let the model-calculated effect on either GDP or gross value added, GVA, represent the effect on the output gap, but this is not a convincing estimate because neither the GDP nor the GVA effect return to zero, like a normal output gap does. As discussed in section 11.1, the positive long-term GDP effect reflects the sum of a composition effect and a substitution effect. The long-term GVA effect reflects only the substitution effect, which is positive because higher wages increase capital input per employed.

Basically, the long-term substitution effect on GVA represents an increase in potential GVA. Thus, the corresponding output gap does return to zero, but it is unclear how quickly the ADAM-calculated GVA effect is to be written down to zero in order to represent the reaction of the output gap.



It is much easier to make an employment gap that returns to zero in the long term. The baseline employment equals potential employment and employment returns to the baseline in the long term, so the employment gap can be represented by the model-based employment effect in the box figure.

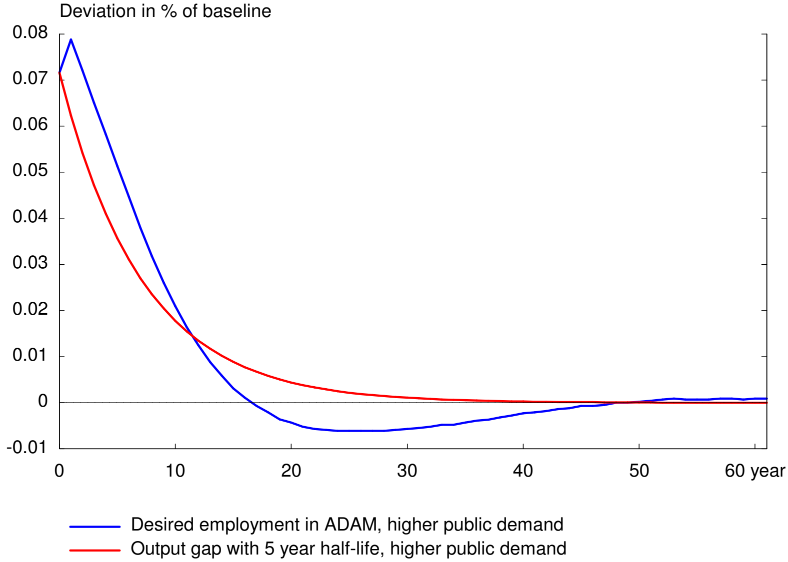
However, it is not satisfactory to represent the output gap by the employment effect because employment reacts more slowly in the short term than output. Instead, we can use the effect on the total desired employment calculated from the desired labour inputs in ADAM's industries. The desired employment depends, cf. chapter 6, on actual production and relative factor prices.

In the short term, the desired employment is proportional to production because relative factor prices remain practically unchanged. In the long term, the desired employment will be reduced by the increase in wage over user cost and thus not reproduce the positive substitution effect on production. The long-term increase in wage relative to user cost, makes the long-term effects on both the actual and desired employment zero, cf. the box figure. It is therefore chosen to let the effect on the desired employment represent the effect of the demand shock on the output gap.

Response of output gap to higher public purchases

Figure 11.35 compares an output gap, which is halved every five years, with an output gap that reproduces the ADAM-calculated effect on desired employment in case of an unfunded 1 per cent increase in the public purchase of goods. The output gap with fixed half-life is set to start at 0.072 per cent like the ADAM-calculated gap, so the two output gaps can only be compared from year 2 and onwards.

Figure 11.35 **ADAM-calculated output gap and output gap with fixed half-life**



Accelerator effect in ADAM

The ADAM-calculated output gap peaks in year 2, because of the accelerator effect on investment and remains above the gap with fixed half-life for about a dozen of years. There is no accelerator effect in the stylized model of Sørensen and Whitta-Jacobsen, which helps to explain that the output gap follows a simple autoregressive process in the absence of further shocks.

ADAM calculation relates to unfunded increase in public purchase of goods

It can be added that Sorensen and Whitta-Jacobsen assume that shocks to public demand are funded, so their stylized effect on the output gap represents the balanced budget multiplier. In their stylized model, the funding assumption does not influence the profile in the output gap response, but in ADAM, the response becomes more complicated and volatile, if the increase in public purchases is financed with higher taxes. As discussed in chapter 2, it will only take a few years before you run into a temporary downturn with negative output gap, because the tax-driven reduction of private income and housing demand triggers a negative accelerator response in house prices and housing investments.

So even if the textbook formally describes a funded increase of public purchases, the effect on the output gap with fixed half-life is more comparable to an unfunded increase of public purchases in ADAM.

Protracted response in output gaps

Both output gaps in figure 11.35 describe a rather lengthy output response that does not resemble a normal cyclical fluctuation, and neither is it normal to have only a single and permanent demand shock in the course of 20-30 years.

The textbook explains cyclical fluctuations by stochastic shocks

Sorensen and Whitta-Jacobsen explain cyclical fluctuations by assuming that the economy is hit by a series of more or less random shocks to the simple autoregressive output gap equation. The series of shocks will make the output gap oscillate around zero in a manner that resembles cyclical fluctuations, cf. the analysis of business cycles in chapter 19 of Sørensen and Whitta-Jacobsen.

Something similar in relation to ADAM can be imagined, where some or all residuals in behavioural equations could vary randomly. This kind of stochastic simulation would certainly produce a more fluctuating response of output and employment than what follows from a single permanent increase in public purchases. The calculation has not been made on the present ADAM, but the problem is not new and stochastic simulations have been tried in the past; for example, on one of the first American macro models, see Adelman and Adelman (1959). The conclusion was then that random shocks could make the model generate fluctuations, reminiscent of cyclical movements.

So even when it takes a long time for the model to close the output gap after a permanent change in the exogenous variables, and even when the interaction between accelerator and multiplier only generates a weak cyclical effect, short-term cyclical fluctuations can be explained by random shocks to the model. However, the presence of random shocks does not change that it takes a long time for systematic shocks to obtain their final effect.

Summary of chapter 11

In summary, ADAM is a Keynesian demand model in the short term, and in the long term it works as an equilibrium model with growth. The response of wages to unemployment is crucial for ADAM to return to its long-term equilibrium when it has been exposed to shocks. If wages are exogenous, demand shocks will have lasting effects on employment.

It takes 18 years before the positive employment effect of an unfunded increase in public purchases crosses the zero line, and it takes even longer for the model to reach its equilibrium. If the higher public purchases are financed by a tax increase, the employment effect turns negative after 4 years, but it still takes a long time for the model to reach equilibrium.

Similar to higher government purchases, a higher private propensity to consume stimulates demand in the short run. In the long term, the consumption effect will be reduced by the fall in private wealth, and especially if the effect on public debt is neutralized, the consumption effect will be close to zero in the long term.

A fall in the interest rate stimulates: private consumption through its effect on housing wealth, and private investments through its effect on user costs. In the long term, lower interest rates affect the composition of demand, because businesses capital and housing capital become relatively cheaper to use. The long-term effect on total private consumption depends on private financial wealth in the baseline scenario. If the private sector is a net-creditor, it will tend to make the long-term consumption effect negative, especially if the interest rate is larger than the growth rate in the baseline scenario.

A fall in the foreign price level reduces competitiveness and Danish production in the short term. In the long term, the impact on the real economy disappears when the domestic wage and price level has fallen by the same percentage as the foreign price level. Thus, a change in the foreign price level works like a monetary shock. A temporary shock to ADAM's wage equation makes the model respond in a similar way, i.e. without long-term effects on the real economy when the wage has returned to the baseline.

An expansion of the labour force increases unemployment at first, but the accompanying reduction of wage growth will gradually increase Danish market shares and bring unemployment back to its equilibrium. The higher employment and larger income basis improve public finances and increase consumption possibilities in the long term. The short-term unemployment effect of the supply shock will disappear more quickly if public debt is held constant in the long term by a tax cut.

The adjustment pattern of the model depends on the response of foreign trade to competitiveness and on the response of wages to unemployment. A quick and stable adjustment to equilibrium would require a substantial increase in the price elasticity of foreign trade in ADAM plus a reduction of the coefficient for unemployment in the wage equation. The coefficient for unemployment must be reduced to prevent wages from overshooting, as this will create fluctuations and delay the adjustment to equilibrium.

ADAM is a model of the same type as SMEC, but some of the key coefficients differ. For example, the wage equation in SMEC has a larger coefficient for unemployment and there is more substitution in SMEC's production functions, so the positive employment effect of higher public purchases disappears more quickly than in ADAM.

Many of ADAM's properties, including the difference between short and long term, are similar to what can be found in a standard curriculum in macroeconomics. As with other applied models, ADAM can, however, also produce results that look quite different from the results of more stylized and aggregated textbook models, and for which an interpretation is therefore required.

Electronic annex

The table below lists the model group papers referred to in the book's chapters. The papers can be found at
http://www.dst.dk/TilSalg/ADAM/Dokumentation/Modelgruppepapirer/Oversigt.aspx

Chapter 3:	
Ny formulering af forbrugssystemet	GRH20110
Egenskaber og estimation af CES-forbrugssystemet	AIV28910
Ligningerne for makroforbruget	RBJ10111
Boligligningerne	RBJ11111
Chapter 4:	
Mere om multivariat analyse af eksporten (II)	JAO06695
New export aggregation and re-estimation	DSI30609
Chapter 6:	
Ny formulering af faktorblokken	GRH10510
Dynamiske identiteter med kædeindeks	GRH02307
Forslag til ny kapitalligninger	GRH08O08
Estimation af faktorblokken	GRH09710
Chapter 7:	
Lønligningen	RBJ20d10
Chapter 11:	
Beskæftigelsesgab som outputgab	RBJ01311
Multiplier tables	DSI01313

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