Danmarks Statistik
MODELGRUPPEN
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An I(2) Analysis of Danish Manufactured Exports: A Different Perspective[1]

We make an I(2) analysis of Danish manufactured exports. We take a different perspective and model exports using wages. It is found that nominal wages are best characterized as integrated of order two. The long run relations are determined to be an export demand relation and wage inflation cointegrating with the rate of unemployment in a dynamic steady state relation (supply relation). The long run relations are found to be empirically constant over the periods covered. The short run structure shows wage-to-market effect.

Key words: Cointegrated VAR, Manufactured exports, Wage rates, Long run identification, Short run structure.

[1] This paper has been presented at the University of Copenhagen on a seminar-cointegration analysis of economic policy.

0. Introduction

Manufactured exports, comprising about half the total exports, are vital to the Danish economy. It is, therefore, not surprising a great deal of effort has been put into modeling Danish manufactured exports. As part of the effort, for instance, two of the Danish Macroeconomic Models – MONA of Danmarks Nationalbank and ADAM of Danmarks Statistic – have an estimated behavioral relation for manufactured exports at the heart of the models, see ADAM (1995) and MONA (2003). Increased market share improves balance of payment and reduce unemployment, this can reduce conflict of objectives in macroeconomic policies such as the tradeoff between inflation and unemployment, as in the standard Phillips curve. Exports can thus play a central role in the business cycle and help to pull an economy out of a recession (Nielsen , 1999).

A popular approach for modeling exports is based on a seminal article of Armington (1969). This methodology has been used several times for the Danish economy, see e.g. Jensen and Knudsen (1992), Kongsted (1998), Nielsen (2002), Anderson (2009) and Sisay (2009). Most of the studies so far tend to relate the responsiveness of volume of exports to price changes. The estimates for the long run price elasticities on the macro level lie somewhere between 2 and 4 for Danish manufactured exports. However, price elasticities tend to be quite sensitive to changes in model specification even when similar data, methods and sample periods are used, see Sisay (2009).

In this paper, we try to model Danish Manufactured exports from a different perspective, which is to relate the volume of exports to relative wages. Why might this be interesting? In the past few years, Danish wages have increased more than the wages in Denmark's major trading partners. For instance, wages in the Danish manufacturing industry rose by 3.7% and by an average of 2.8% in the trading partners from 2007:4 to 2008:4 (Confederation of Danish Industry, DI, 2009). Needless to say wage increases affect Danish competitiveness. "Very low German wages, in particular, have put pressure on Danish competitiveness over several quarters..." (quoted on the web, Steen Nielsen, Director of wage statistic, DI, 2009). Most importantly, wages are central in the crowding out mechanism. Pressure and overheating in the domestic labor market entails higher wages increases in the domestic economy than abroad, these restores equilibrium by reducing exports so that the pressure from total demand subsides. The *responsiveness* of volume of exports to changes in relative wages is key to this crowding out mechanism (MONA, 2003). Often calculating price index is not as straightforward as it is computing wages, the wage relative to other OECD countries could be a better indicator of competitiveness than export price.

Manufactured exports have been modeled based on relative wages in MONA. The long run wage elasticity of Industrial exports is estimated at 1.2 (MONA, 2003). The relation is estimated using Engle and Granger two step procedure. However, a single equation analysis does not replace a multivariate analysis; this is why in this paper we develop an econometric framework based on a cointegrated vector autoregressive (VAR) model, see Johansen (1996) and Juselius (2006). We apply an I(2) analysis similar to Nielsen (2002), the difference is that we model exports based on wages and Nielsen 's (2002) work is based on export prices. A major development in Nielsen (2002) unlike previous works (see e.g. Kongsted, 1998), was the questioning of the assumption that the first differences of the nominal price and cost variables are stationary. He generalized the statistical framework by applying a multivariate cointegration analysis, that allows variables to be integrated up to second order, I(2). Similarly nominal wages can better be characterized as integrated of order two, this is why we motivate using I(2) models.

Developing an I(2) analysis, We found two long run relations – one for export demand relation and another for supply relation. We are not aware of other authors who modeled wages and exports using a multivariate cointegrated I(2) model. The rest of the paper is organized as follow. In the next section we present our theoretical framework, in part 2 we explain the dataset and time series interpretations, in part 3 we elucidate our methodology, we present our results and discussions in part 4, and conclude in part 5.

1. The Theoretical Framework

The economic framework is based on the work of Armington (1969). Armington introduced a technique to model the export market based on the assumption that competing products are imperfect substitutes and a constant elasticity of substitution (ρ>1, in numerical value) is assumed.[2] The same framework has been applied to the Danish economy as in Kongsted (1998), Nielsen (2002), Andersen (2009) and Sisay (2009). Usually Armington model is used to specify an inverse relationship between the export market share and relative prices. A novelty in this paper is to use relative wages. An inverse relationship between the export market share and relative wage can be specified as[3]

$$x_t - x_{ft} = -\rho\,(w_t - w_{ft} - e_t), \quad \rho > 1 \qquad (1)$$

Where; X_t *and* W_t are the volume of Danish manufactured exports and hourly wages in the industrial sector in Danish Krone, respectively. X_{ft} is the size of the export market in volume terms, which is defined as the weighted average of imports from Denmark's trading partners. W_{ft} is the foreign hourly wage rate in the industrial sector, denominated in foreign currency which is calculated as a weighted average of hourly wages in Denmark's largest trading partners. E_t is the effective exchange rate denominated as Danish krone per foreign currency unit, calculated by weighting together the currencies of a number of Denmark's largest trading partners as in W_{ft}. And ρ is interpreted as the long run wage elasticity of exports. Equation (1) completes the demand relation for Danish manufactured exports. A demand relation with relative prices instead can be derived from a utility maximization (cf. Armington, 1969).

Expansion of exports can reduce unemployment without affecting wages, in turn overheating and friction in the labour market raises wages and reduces exports. A simple way of characterizing this conflict of macroeconomic policy objectives and the crowing out mechanism is to consider a standard Phillips curve relation. A.W. Phillips in the 1950s documented a statistical inverse relationship between wage inflation and unemployment in the UK. Since then some variants of the Phillips curve has been developed. The standard Phillips curve is written as

$$\Delta w_t = -\omega U_t, \quad \omega > 0 \qquad (2)$$

Here, U_t is the rate of unemployment. If we assume nominal wages are I(1) the coefficient ω can not be identified hence (2) cannot be stationary. If, however, nominal wages are assumed to be I(2) the rate of unemployment can cointegrate with wage inflation in a dynamic steady state relation. Nominal variables are best characterized as I(2) processes, see Juselius (1999). This is why in this paper we apply I(2) analysis for Danish manufactured exports. Equation (1) and (2) are candidates for long run cointegrating relations.

2. The Data and Time Series Interpretations

The theoretical discussion suggests an information set given by five dimensional vector, $Y_t = (x_t, x_{ft}, w_t, (w_f{+}e)_t, U_t)'$. W_f+e simply transforms the foreign wage in Danish Krone comparable with the domestic wage, see below. The data are quarterly and seasonally adjusted and log transformed with average 2000 = 0. The sample covers 1975:1 to 2007:4.[4] Figure 1 (A – F) shows the data and important linear combinations. (A) Shows the two wage rates, they tend to move together in most of the sample periods. The volume of Danish manufactured exports and the export market are shown in (B). (C) and (D) show the rate of

[2] This theory has been well developed in Dornbusch (1987), Hooper and Mann (1989), Hung et al. (1993).

[3] Lower case variables are log transformed.

[4] A description of the data and how they are calculated can be found in MONA (2003).

unemployment and the effective exchange rate, respectively. The rate of unemployment has generally been rising till 1994 and falling since then. The Danish Krone was subjected to a series of devaluations during the late 1970s and early 1980s, but after the commitment to the fixed exchange rate regime the Krone has generally been appreciating. (E) shows the relative wages and market share. Generally, the higher the foreign wage relative to Danish wage, the higher is Danish export. In the beginning of the 1990s, however, this positive relationship between competitiveness $(w_f+e-w)_t$ and market share $(x-x_f)_t$ does not seem to hold. After 1990 the Danish export shares continue to grow despite worsening in competitiveness. We follow Nielsen (1999 and 2002) and interpret the export gain as a result of the German reunification in 1990. Nielsen (2002) breaking Danish exports by country of destination, has shown that the Danish market share in Germany has grown by 40% between 1990 and 1993. In the empirical analysis the German reunification will be captured by a shift dummy which equals 1 after 1990:3 and zero elsewhere, see below. Wage inflation (Δw_t) and the unemployment rate (U_t) are reported in (F), which are inversely correlated in line with the prediction of (2).

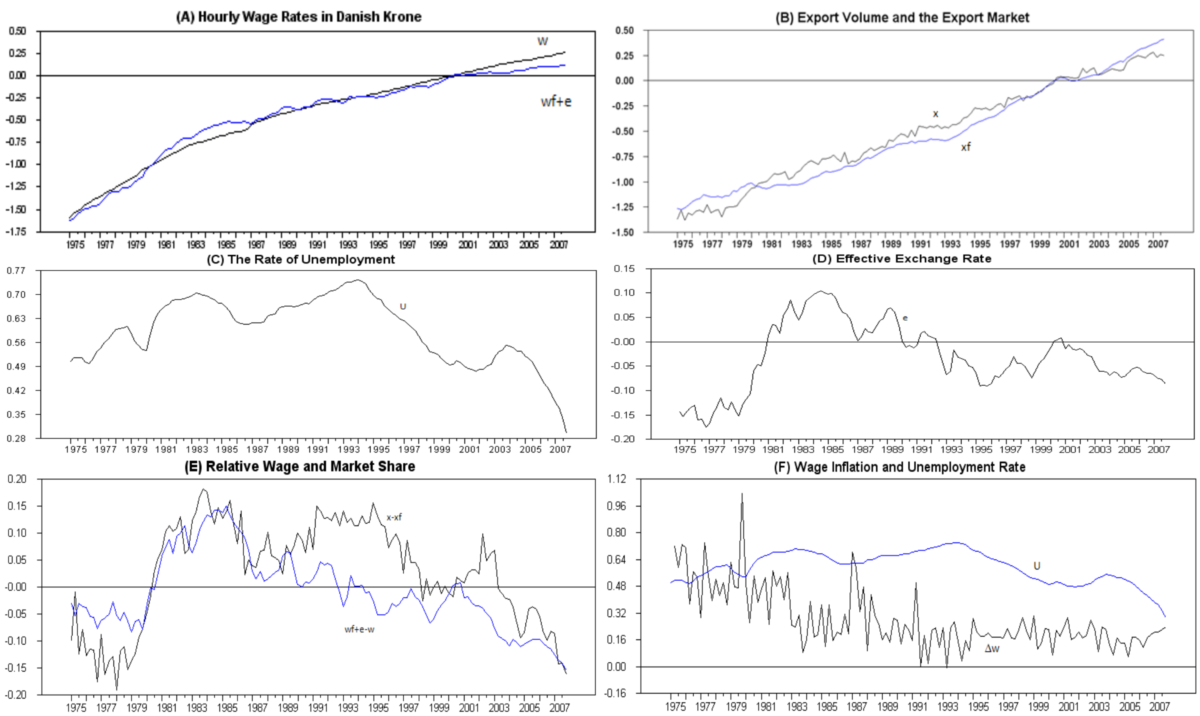

Figure 1 Data and linear combinations in logs, 2000 = 0.

A central feature of our analysis is that the nominal wages are taken to be integrated of order two, I(2), for the purposes of modeling. In other words, the first difference of the nominal wages display persistent behavior over the samples investigated, i.e. the first difference of nominal wages can better be characterized as I(1) process (Juselius, 1999 and 2006). A simpler case is the existence of one I(2) trend affecting the two wage rates identically with loadings proportional to β_2 = (0,0,1,1,0); a long run homogeneity between domestic and foreign wages (w_t-w_{ft}-e_t) would cancel the I(2) trend. In this case we say the relative wages cointegrate from I(2) to I(1), CI(2, 1). From the graphs above we can see that the relative wages are less persistent than the wage rates in level. A more complicated case is the existence of two or more I(2) trend in which case it is difficult to uncover stationary long run relations. With an assumption of one I(2) trend if x_t and x_{ft} are I(1), then they could cointegrate CI(1, 1) with the relative wages to form a stationary demand relation. If the unemployment rate is I(1), then we could uncover a dynamic stead state relation between wage inflation and unemployment as in the classical Phillips curve.

3. The Econometric Approach

The empirical analysis will be based on a VAR(k) I(2) model for the p-dimensional vector Y_t, which can be re-parameterized in acceleration rates, changes and levels as:

$$\Delta^2 Y_t = \sum_{i=1}^{k-2} \Psi_i \Delta^2 Y_{t-i} + \Pi Y_{t-1} - \Gamma \Delta Y_{t-1} + \mu_0 + \mu_1 t + \mu_2 t_2 + \emptyset D_t + \varepsilon_t \qquad (3)$$

Where $\varepsilon_t \sim iid(0, \Omega)$, $t = 1, \ldots, T$, and Ω is the covariance matrix of ε_t,$Y_t = (x_t, x_{ft}, w_t, (w_{ft} + e_t), U_t)'$, and the initial values $Y_{-k+1}, \ldots, Y_o$, are considered fixed. The k matrices of autoregressive coefficients $(\Pi, \Gamma, \Psi_1, \Psi_2, \ldots, \Psi_{k-2})$ are each of dimension p x p. μ_0 and μ_1 are a vector of constants and linear drift terms. D is a vector of dummy variables (step dummy, permanent impulse dummy and transitory impulse dummy) and ϕ is the corresponding coefficient, finally t_2 is a broken linear trend which corresponds to the step dummy to be specified below, and all parameters are unrestricted, see Frydman et.al (2008), Johansen (1996), and Juselius (2006).

The cointegrated I(1) model, H_r, is formulated as a reduced rank restriction on Π as $\Pi = \alpha\beta'$, *where* α, β *are p x r of rank r < p.* Implicitly assuming that Γ is unrestricted. The cointegrated I(2) model, $H_{r,s1}$, imposes additional restriction on Γ, i.e. the Γ matrix is no longer unrestricted in the I(2) model, the I(2) model in addition need to satisfy $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$, where ξ and η are $(p-r)x\ s_1\ of\ rank\ s_1 < p - r$. The intuition is that the differenced process has unit root when data is I(2). $\alpha_\perp$ and $\beta_\perp$ are orthogonal complements to α and β, respectively; in turn can be decomposed into I(1) and I(2) directions as $\alpha_\perp = [\alpha_{\perp 1}, \alpha_{\perp 2}]$ and $\beta_\perp = [\beta_{\perp 1}, \beta_{\perp 2}]$.[5] The matrices $\alpha_{\perp 1}$ and $\beta_{\perp 1}$ are of dimension $p \times s_1$, and $\alpha_{\perp 2}$ and $\beta_{\perp 2}$ are of dimension $p \times s_2$; which are defined by $\alpha_{\perp 1} = \alpha_\perp (\alpha'_\perp \alpha_\perp)^{-1}\xi$, $\beta_{\perp 1} = \beta_\perp (\beta'_\perp \beta_\perp)^{-1}\eta$, $\alpha_{\perp 2} = \alpha_\perp \xi_\perp$ and $\beta_{\perp 2} = \beta_\perp \eta_\perp$, where $s_2 = (p - r) - s_1$ is the number of I(2) trends, and $\xi_\perp$ and $\eta_\perp$ are orthogonal complements to ξ and η, respectively.

A correct specification of deterministic components is even more important in the I(2) model than in the I(1) model, and it has to be restricted in certain ways to avoid undesirable consequences. In the I(2) model, unrestricted constant cumulate once to a linear trend and twice to a quadratic trend; unrestricted trend cumulates once to a quadratic trend and twice to a cubic trend. We would need to restrict the deterministic components appropriately so as to avoid quadratic and cubic trends as higher order trends are not important for our analysis (Juselius, 2006). In the empirical analysis (as in Nielsen , 2002) we restrict the constant and the linear trend in order to allow linear trends in all components of the model (including the cointegration relation) and higher order trends are excluded. The specification of the dummy variables in the I(2) model is as important as the constant and linear drift terms. Unrestricted permanent blip dummy is consistent with a blip in the acceleration rates, $\Delta^2 Y_t$, a level shift in ΔY_t and broken linear trends in Y_t. It is not always clear from the outset whether the level shift and broken linear trend cancel in the cointegration relation, alternatively the long run exclusion can be tested. The effect of including a step dummy is similar to unrestricted constant; it has to be restricted in a way that allows level shifts in all directions of the model (including the cointegration relation) and higher order trends has to be excluded. Similarly, the broken linear trend has to be restricted so as to avoid quadratic and cubic trends and allow the cointegration relation to be stationary around a broken linear trend. Unrestricted transitory blip dummies will cumulate once to a blip in ΔY_t and twice to a level shift in Y_t, these effects are not serious and can often be ignored, see Juselius (2006) for a discussion.

The empirical analysis is performed using the full ML procedure derived in Johansen (1997), see also Juselius (2006) for a discussion. As opposed to the two step procedure, the space spanned by $\tau = (\beta, \beta_{\perp 1})$ can be determined by solving one reduced rank regression. The I(2) model contains *p-s_2* cointegration

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[5] The matrices $\alpha_{\perp 1}, \alpha_{\perp 2}, \beta_{\perp 1}$, and $\beta_{\perp 2}$ are also defined as $\alpha_1, \alpha_2, \beta_1$, and β_2 in many literatures on I(2) model.

relations of which the r relation can become stationary by polynomial cointegration, $\beta^{*\prime}Y_t^* - \delta\beta_2' \Delta Y_t \sim I(0)$, and the s_1 relations, $\beta'_{\perp 1}x_t$, can become stationary by differencing $\beta'_{\perp 1}\Delta x_t \sim I(0)$ The model with a reduced rank restriction on Π and a level shift and trend restricted to the cointegration space can be rewritten as

$$\Delta^2 Y_t = \sum_{i=1}^{k-2} \Psi_i \Delta^2 Y_{t-i} + \alpha\beta^{*'} Y_{t-1}^* - \Gamma\Delta Y_{t-1} + \emptyset\Delta D903s_t + \sum_{i=0}^{k-2} \phi_i \Delta^2 D903s_{t-i} + \phi_p D_{p,t} + \phi_{tr} D_{tr,t} + \mu_0 + \epsilon_t \quad (4)$$

Where, $D903s_t = \begin{cases} 0 & for\ 1975{:}1\ to\ 1990{:}2 \\ 1 & for\ 1990{:}3\ to\ 2007{:}4 \end{cases}$, $Y_{t-1}^* = (Y_{t-1}', D903s_{t-1}, t)'$, $\beta^{*\prime} = (\beta', \gamma_0', \beta_0')$, and $D_{p,t}$ and $D_{tr,t}$ are a permanent blip and transitory blip dummies to be specified below in the empirical analysis. A different formulation to (4) is a model with a broken linear trend restricted to the cointegration space, see Juselius (2006) and Frydman et.al (2008) for a discussion.

4. The Empirical Analysis

The first step in the empirical analysis is to get a well specified model with appropriate lag length.[6] That, among other things, requires appropriate specification of deterministic components and the inclusion of dummies for outliers and level shifts. However, handling blip dummies and step dummies in I(2) model is not straightforward. Asymptotic distributions for rank test in CATS are not correct when level shift and broken linear trend are included in the model, unfortunately CATS is the only package available to us. In the following, to reduce undesirable consequences in the trace test statistic, we estimate (4) without a broken linear trend and level shift, we include a trend in the cointegrating relation and unrestricted constant. We will consider level shift in the statistically well developed I(1) model.

The Schwarz and Hannan-Quinn information criterion point to k = 2. Based on a likelihood ratio test, we can restrict k = 4 to k = 3 but not to k = 2. In practice, a well specified model seldom needs a lag length above 2, it's not either obvious from the outset whether significant residual correlation is because of few lags or misspecification. k = 2 maintained, we scrutinized the residuals and included blip dummies for outlying observations.[7] Still k = 2 is preferred. In the following a lag length of two will be maintained. Table 1 reports a battery of misspecification test results, the model have sound statistical properties. Multivariate normality cannot be rejected. VAR is robust to ARCH effects (Juselius, 2006) which can be ignored. Similar inference can be made about the rest statistics.

Table 1 Test for misspecification of the unrestricted VAR(2)

Univariate tests			Multivariate Tests		
Equation	ARCH(2)	Normality	Normality $\chi^2(10)$	=	13.133 [0.216]
Δx_t	7.583 [0.023]	2.060 [0.357]	AR(1) χ^2 (25)	=	37.804 [0.068]
Δx_{ft}	1.807 [0.405]	0.365 [0.833]	AR(2) χ^2 (25)	=	19.283 [0.783]
Δw_t	7.676 [0.022]	2.532 [0.282]	ARCH(1) χ^2 (225)	=	328.247 [0.000]
$\Delta(w_f\text{-}e)_t$	0.097 [0.953]	3.891 [0.143]	ARCH(2) χ^2 (450)	=	517.873 [0.015]
ΔU_t	1.475 [0.478]	2.559 [0.278]			

Note: AR is the test of autocorrelation order 1 and 2, ARCH tests 1st and 2nd order, figures in square bracket are p-values according to $\chi^2(v)$.

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[6] The empirical analysis is made using Cats in Rats (Dennis, 2006) and PcGive (Doornik and Hendry, 2001a).

[7] We motivated and included 4 permanent blip dummies, one can consult the economic calendar and find out what economic (or any other) activity in those years caused the outlying observations, we do not intend to do that here.

4.1. I(2) analysis

I(2) symptoms in the I(1) model

As a motivation to the consequent I(2) analysis, we first estimate VAR(2) I(1) model for Y_t = (x_t, x_{ft}, w_t, $(w_f+e)_t$, U_t)' and check for any symptoms of I(2)ness. We allow a level shift and trend in the cointegration space and unrestricted constant.[8] One way of detecting I(2)ness in I(1) model is to see the characteristic roots of the model. If one or several large roots remain in the model for any reasonable choice of r, then it is a sign of I(2)ness in at least one of the variables. (cf. Juselius, 2006). Table 1 in the appendix reports the modulus of the characteristic roots. Starting from the unrestricted model, restricting the first largest root to unity introduces additional root close to unity, this suggests the presence of I(2) trend in the nominal data. Another diagnostic tool for detecting I(2) behavior is to look at the graphs of the cointegrating relations. If the X-form and R-form (the model when the short run regressors and deterministics are concentrated out) graphs are significantly different, that is one additional sign of I(2)ness (Juselius, 2006). Figure 1 in the appendix presents the cointegrating relations, in the first cointegrating relation the X-form is drifting in a non stationary manner whereas the R-form is quite stationary, this clearly is a sign of I(2)ness. We will proceed to the I(2) analysis of the nominal data and continue with I(1) analysis of the real data after determining the rank and testing nominal to real transformation in the I(2) framework.

Rank determination

The rank indices of the I(2) model is determined using the trace test. The trace test is calculated for all possible combinations of r and s_1, then the joint hypothesis, H_{r,s_1}, is tested against the unrestricted model H_5. Table 2 reports the trace test statistic, s_{r,s_1}. The distribution of s_{r,s_1} is non-standard and depends on the deterministic specification of the model. To avoid undesirable consequences in the asymptotic distribution, we estimated the I(2) model without a broken linear trend and level shift. A broken linear trend has the effect of shifting the table to the right, which implies the test will be undersized if one ignores the broken trend (Juselius, 2006). To facilitate comparison, table 2 in the appendix reports the trace test statistics when a broken linear trend and level shift are included in the model.

Table 2 The trace test for the rank indices of the I(2)model

p-r	r	s_{r,s_1}					
5	0	590.006 [0.000]	394.168 [0.000]	285.666 [0.000]	216.522 [0.000]	181.237 [0.000]	162.626 [0.000]
4	1		253.321 [0.000]	170.174 [0.000]	110.906 [0.002]	81.004 [0.022]	76.419 [0.002]
3	2			107.018 [0.001]	**67.503** **[0.072]**	**51.807** **[0.078]**	47.125 [0.016]
2	3				**45.221** **[0.105]**	**26.442** **[0.331]**	**22.061** **[0.140]**
1	4					**14.013** **[0.297]**	**5.56** **[0.528]**
	s_2	5	4	3	2	1	0

Note: figures in square bracket are p-values

The test procedure starts with the most restricted model, $H_{0,0}$, then continues to the end of the first row at $H_{0,5}$, and proceeds similarly row-wise from $H_{1,0}$ to $H_{1,4}$ until the first acceptance. The first acceptance is at (r=2, s_1=1, s_2 = 2). The case of two I(2) trends is complicated, it's not straight forward how stationary and economically interpretable long run relations could be revealed. It is also mentioned, the presence of broken trend has the effect of shifting asymptotic tables to the right, thus if the asymptotic tables where

[8] Dummies for outlying observations are included as well.

simulated with broken trend, $H_{2,1}$ could easily be rejected. The case of two I(2) trends is not appropriate choice. Accordingly, the next acceptance is $H_{2,2}$. This is consistent with the theoretical framework presented above, in which the number of I(2) trends was assumed to be one, $s_2 = 1$. If $H_{2,2}$ is rejected, the next best choice will be $r = 3$ and $s_2 = 2$, however, the empirical analysis with two I(2) trends and three cointegrating relations is even far from straightforward, in addition to that, the information set and the theoretical setup is not enough to support three stationary relations which makes the long run identification of stationary relations complicated. Consistent with the theoretical setup, the model, $H_{2,2}$, is maintained. From table 2 in the appendix the choice of $H_{2,2}$ is more clear when a broken linear trend and shift dummy are included. For reasons mentioned already we stick with the I(2) model without broken linear trend.

Rank determination is the most important and often difficult part in a VAR analysis; accordingly, all available information has to be utilized. Figure 2 below shows the polynomially cointegrating relations, $\beta^{*\prime}Y_t^* - \delta\beta_2'\Delta Y_t$, for the preferred model $H_{2,2}$. Both stationary relations are mean reverting. Another way to look at the rank determinations is to scrutinize the eigenvalues of the companion matrix. For the preferred model to be stationary all eigenvalues are required to be strictly inside the unit circle. Each I(1) trend generates one unit root and each I(2) trend generates double unit roots. For r=2 and $s_1 = 2$, we expect 4 = 2 + (2 x 1) unit roots. The eigenvalues in the maintained model are given by

1, 1, 1, 1, 0.784, 0.651, 0.332, 0.332, 0.261, 0.183

The unrestricted eigenvalue is quite large, which indicates the borderline nature of the second stationary relation. In line with the theoretical setup, $H_{2,2}$ is maintained.

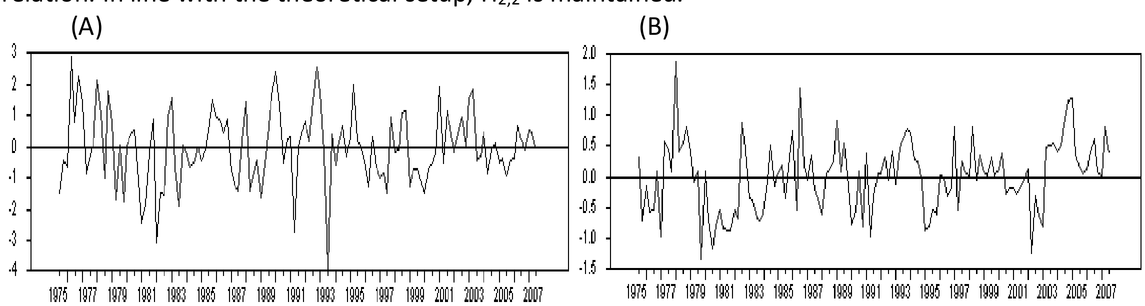

Figure 2 the polynomially cointegrating relations, first and second stationary relation, respectively.

Nominal-to-real transformation

The I(1) model is statistically more developed than the I(2) model. When the nominal data is found to be I(2), the model can be transformed into the simpler I(1) model. Provided the nominal to real transformation is accepted, inference about the key parameters in the I(2) model can be conducted in the I(1) model. If however, nominal to real transformation is not valid, there will be loss of information and violation of the I(1) properties in the transformed model (Juselius, 2006).

The demand relation discussed above is homogeneous in wages. If w_t and $(w_f+e)_t$ contain the same I(2) trend with same coefficient, then $(w-w_f-e)_t$ could be made I(1), and the I(2) model can be transformed to an I(1) model without loss of information as: $Y_t = (x_t, x_{ft}, w_t, (w_{ft} + e_t), U_t)' \sim I(2)$ to $Z_t = (x_t, x_{ft}, (w-w_f + e)_t, \Delta w_t, U_t)' \sim I(1)$. Given the long run homogeneity between the two wage rates, the VAR analysis based on either data is identical. Δw_t is included in the transformed data not to lose information from the I(2) trend, and to allow deviations from homogeneity in the short run and to give the possibility of polynomially cointegrating relation. The transformation is valid if the hypothesis of long run homogeneity is accepted in all cointegrating relations, $\tau = (\beta, \beta_{\perp 1})$; this is the case when the I(2) trends affect the nominal variables with loadings $B_{\perp 2}$ proportional to b. $B_{\perp 2}$ are given in table 3, the estimated loadings to the two nominal wage rates are nearly the same. The long run homogeneity can be tested as a linear hypothesis on τ expressed as

$b'\tau_i = 0$, i = 1,2,..,p-s_2. The test result is given in table 4. The test of homogeneity cannot be rejected with a likelihood ratio test statistic of 1.472 (p-value of 83.2%) asymptotically distributed as $\chi^2(4)$.

Table 3 Estimated loadings to I(2) trend

	x_t	x_{ft}	w_t	$(w_f+e)_t$	U_t
$\hat{\beta}'_{\perp 2}$	0.038	0.152	0.833	1	-0.15
b'	0	0	1	1	0

Table 4 Test of long run homogeneity

Hypothesis	*v*	$\chi^2(v)$	*p-value*
$b'\tau_4$	4	1.472	0.832

Finally, we introduce the concept of weak exogeniety. Denmark is a small open economy. It's true that the export market has influence on Danish exports, however, the vice versa is unlikely. Hence, one can a priori condition on the export market, x_{ft}. By conditioning on a weakly exogenous variable, a partial system with relatively stable parameters can be achieved, which is the same as the full model from a likelihood perspective. In the subsequent I(1) analysis we consider the transformed information set $Z'_t = [(x-x_f)_t, (w-w_f-e)_t, \Delta w_t, U_t, x_{ft}]$. The linear combination in the two export volumes is of convenience, since both x_t-x_{ft} and x_{ft} are included, it's not a restriction on the likelihood function. It simply facilities long run and short run identification and conditioning on the foreign variable, see Kongsted (1998). The transformed I(1) model with accepted long run homogeneity and weak exogeniety of the export market can be written as

$$\Delta\tilde{Z}_t = \tilde{\alpha}\tilde{\beta}^{*\prime}\tilde{Z}^*_{t-1} + \Gamma\Delta\tilde{Z}_{t-1} + \sum_{i=0}^{1}\tilde{\varphi}_i\,\Delta D903s_{t-i} + \sum_{i=0}^{1}\tilde{\theta}_i\,\Delta x_{ft-i} + \phi D_t + \mu_0 + \tilde{\varepsilon}_t \qquad (5)$$

Where, $\tilde{Z}_t' = [(x\text{-}xf)_t, (w\text{-}wf\text{-}e)_t, \Delta w_t, U_t]$, $\tilde{Z}^*_{t-1} = (\tilde{Z}'_{t-1}, D903s_{t-1}, xf_{t-1}, t)$, $\tilde{\beta}^{*\prime} = (\beta', \gamma_0', \delta_0', \beta_0')$, D_t is permanent and transitory blip dummies, μ_0 is a constant and $\tilde{\varepsilon}_t$ is error term. This model will be maintained in the consequent I(1) analysis.

4.2. I(1) Analysis

The I(1) model is statistically more developed than the I(2) model, dealing with deterministic components and dummies for outliers and level shifts is easier in the I(1) model. It was mentioned above that the German reunification was not modeled in the I(2) model for statistical reasons. It is now time to consider the effect of the German reunification on Danish exports. We also revisited misspecification and lag length determination after including a step dummy. The choice of k = 2 is more clear in the I(1) framework. We scrutinized the residuals for VAR(2) and included two permanent blip dummies. The battery of misspecification test results (not reported) show the model has desirable statistical properties.

Identification of the long run structure

Guided by the theoretical discussion, we now continue to identify the two cointegrating relations. The demand relation and supply relation are the theoretical candidates for long run relations. The long run structure is identified by imposing restrictions on each of the cointegrating relations. As a starting point we estimate the unrestricted cointegrated relation and normalize on $(x\text{-}x_f)_t$ for the first relation and on Δw_t for the second relation, this is reported as H_1 in table 5 below. We then impose restrictions on $\hat{\beta}_1$and $\hat{\beta}_2$ consistent with the demand relation (1) and the supply relation (2), this is reported under H_2. Finally, the trend from $\hat{\beta}_1$in H_2 can be removed to have the structure reported under H_3. Both H_2 and H_3 are accepted with a p-value of 10% and 9%. Both structures are generically and empirically identified, see Johansen (1996) or Juselius (2006). There is not much difference between H_2 and H_3, we simply continue with H_3. The long run demand relation can be written as

$$x_t - xf_t = -1.698\,(w\text{-}wf\text{-}e)_t + 0.136 D903s_t \qquad (6)$$

The wage elasticity of industrial exports is estimated at 1.7, which is higher than the single equation estimate of 1.2 in MONA (2003). Both the market share and relative wages are significantly adjusting. The effect of the German reunification in Danish manufactured export is a 13 per cent increase in Danish export, which is close to the 11 per cent estimate in Nielsen (2002). The second relation is consistent with the standard Phillips curve relation (2) augmented with a trend, the polynomially cointegrating relation is given by

$$\Delta w_t = -0.041U_t - 0.001t \qquad (7)$$

This is just a standard Phillips curve interpretation. Wage inflation is significantly adjusting as in the standard Phillips curve. It represents the trade off the policy makers face between higher inflation and higher output, and vice versa. It can also be interpreted as a supply relation, which determines the price of domestic labor input. Competitiveness gain in the international market boosts trade balance and this can reduce the conflict of objective between higher inflation and lower unemployment which is empirically consistent for the Danish economy. For all these the responsiveness of export is decisive.

Table 5 Identification of the long run structure

	$(x\text{-}xf)_t$	$(w\text{-}w_f\text{-}e)_t$	Δw_t	U_t	x_{ft}	$D903s_t$	t	v	LR	P
			H_1							
$\hat{\beta}_1$	1	1.957	-42.515	-1.816	-0.199	-0.094	-0.006			
$\hat{\beta}_2$	-15.639	-15.492	1	6.936	-1.468	0.487	0.039			
$\hat{\alpha}_1$	-0.037	0.013	0.023	-0.001						
	[-2.269]	[1.608]	[7.837]	[-0.317]						
$\hat{\alpha}_2$	0.015	0.006	0.001	-0.003						
	[2.896]	[2.349]	[1.214]	[-2.222]						
			H_2							
$\hat{\beta}_1$	1	1.683	0	0	0	-0.093	-0.001	5	9.147	0.103
		[9.728]				[-2.409]	[-1.572]			
$\hat{\beta}_2$	0	0	1	0.043	0	0	0.001			
		[3.511]		[9.776]			[10.053]			
$\hat{\alpha}_1$	-0.141	-0.065	0.018	0.012						
	[-2.331]	[-2.194]	[1.642]	[0.807]						
$\hat{\alpha}_2$	1.709	-0.489	-0.973	0.042						
	[2.415]	[-1.423]	[-7.692]	[0.246]						
			H_3							
$\hat{\beta}_1$	1	1.698	0	0	0	-0.136	0	6	10.988	0.089
		[9.441]				[-5.048]				
$\hat{\beta}_2$	0	0	1	0.041			0.001			
				[6.662]			[11.176]			
$\hat{\alpha}_1$	-0.154	-0.061	0.016	0.003						
	[-2.544]	[-2.048]	[1.460]	[0.203]						
$\hat{\alpha}_2$	1.625	-0.557	-0.970	0.064						
	[2.322]	[-1.631]	[-7.729]	[0.372]						

Note: Likelihood ratio test are distributed $\chi2(v)$, figures in square bracket are t-values, p is the p-value.

Finally, Table 3 in the appendix reports a battery of misspecification test results for the identified structure H_3 and we can see the model has acceptable properties. Recursive estimation for parameter non-constancy (figure 2 in the appendix) show the model does not seem to suffer from parameter non-constancy.

4.3. The Short Run Structure

The identification of the short run structure is facilitated by keeping the identified long run relations fixed and treating $\tilde{\beta}'Z_{t-1}$ as predetermined stationary regressors as $\Delta Z_{t-1.}$ Keeping the cointegration structure H_3, we first estimated a multivariate dynamic equilibrium error correction model, achieved by premultiplying the reduced form with *p x p matrix* A_0 = I. The system is estimated with Full Information Maximum Likelihood, which is exactly identified by the *p-1* zero restrictions on each row of A_0. Further zero restrictions are over identifying. By imposing overidentifying restrictions, we achieved the parsimonious system reported in table 4 in the appendix. Economic theory is not precise about the short run structure; we therefore relied on simplification search and empirical evidence when imposing overidentifying restrictions. A likelihood Ratio test for 32 overidentifying restrictions cannot be rejected with a significance level of 0.60. The battery of misspecification tests (not reported) show the system has desirable properties.

The contemporaneous change in the export market, Δx_{ft}, is excluded from the market share equation. This is an important theoretical feature as it would mean expansion in import market *ceteris paribus* are equally distributed on all suppliers, this is not realistic for a small open economy like Denmark, see Nielsen (2002). For market share equation the coefficient to $\Delta(x\text{-}x_f)_{t-1}$ and Δx_{ft-1} are restricted to be equal so that only Δx_{t-1} enters in the dynamic adjustment. The coefficients in the simplified system have appropriate sign and significance. All current effects are accounted for by the residual covariance matrix $\hat{\Omega}$, reported at the bottom of the table. These are only correlations and do not say anything about causality. Most of the correlations are small and can be ignored with the exception of the residuals from $\Delta(x\text{-}x_f)_t$ and $\Delta(w\text{-}w_f\text{-}e)_t$ equations correlated with a coefficient of -0.25; and the residuals from $\Delta^2 w_t$ and $\Delta(w\text{-}w_f\text{-}e)_t$ equations correlated with a coefficient of 0.33.[9]

6. Identified short run adjustment structure

	$\Delta(x\text{-}x_f)_t$	$\Delta(w\text{-}w_f\text{-}e)_t$	$\Delta^2 W_t$	ΔU_t
$\Delta(w\text{-}w_f\text{-}e)_t$	-0.65	0	0	0
	[0.090]			
$\Delta(x\text{-}x_f)_{t-1}$	-0.36111	0	0	0
	[0.000]			
Δx_{ft-1}	-0.36111	0	0	0
	[0.000]			
$\Delta(w\text{-}w_f\text{-}e)_{t-1}$	0	0.359	0	0
		[0.000]		
ΔUR_{t-1}	0	0	0	0.788928
				[0.000]
$ECM1_{t-1}$	-0.13029	-0.093	0	0
	[0.043]	[0.000]		
$ECM2_{t-1}$	0	-0.672	-1.06827	0
	0	[0.003]	[0.000]	
D794pt	0	0.025789	0.026245	0
		[0.143]	[0.000]	
D802pt	0	0	-0.01061	0.048647
			[0.038]	[0.000]

$\hat{\Omega}$	$\Delta(x\text{-}x_f)_t$	$\Delta(w\text{-}w_f\text{-}e)_t$	$\Delta^2 W_t$	ΔU_t
$\Delta(x\text{-}x_f)_t$	0.029			
$\Delta(w\text{-}w_f\text{-}e)_t$	0.060	0.014		
$\Delta^2 W_t$	0.044	**0.331**	0.005	
ΔU_t	-0.025	0.141	-0.020	0.007

$ECM1_{t-1} = (x - x_f)_t + 1.698(w\text{-}w_f\text{-}e)_t - 0.136D903s_t$

$ECM2_{t-1} = \Delta w_t + 0.041U_t + 0.001t$

Note: the Column headings are the dependent variables in each of the model equation and the raw headings are the predetermined regressors. ECM1t-1 and ECM2t-1 are the long run identified demand and supply relations, equation (8) and (9), respectively. Constant terms not reported.

[9] A correlation is said to be large if it is greater than $2/\sqrt{T}$, where T is sample size. In our case we have $2/\sqrt{129}$ = 0.176.

Determining causal order is not straightforward. This is particularly difficult for the correlation between $\Delta(w\text{-}w_f\text{-}e)_t$ and $\Delta^2 w_t$. In the following we ignore this and focus on the correlation between $\Delta(x\text{-}x_f)_t$ and $\Delta(w\text{-}w_f\text{-}e)_t$. This is an interesting case as a successful reformulation of the market share equation where $\Delta(w\text{-}w_f\text{-}e)_t$ enters as a regressor identifies the short run wage elasticity of exports. The short run structure with $\Delta(w\text{-}w_f\text{-}e)_t$ entering the market share equation is reported in table 6. A likelihood Ratio test for 32 overidentifying restrictions cannot be rejected with a significance level of 0.55. This final structure resembles the reduced form model in table 4. The zero restriction on $ECM2_{t\text{-}1}$ in equation $\Delta(x\text{-}x_f)_t$ and the corresponding non-zero coefficient in the equation for $\Delta(w\text{-}w_f\text{-}e)_t$ are both identifying generically and empirically. The contemporaneous correlation is highly reduced and becomes positive; however, this is not a problem as it is highly insignificant. In addition, this could be an explanation to the lack of positive relationship between competitiveness and market share in recent years.

The short run wage elasticity of exports although borderline significant is estimated at 0.65. The error correction coefficient to the demand relation has the appropriate sign and has become more significant. The equation for relative wage adjusts to both the demand and supply relations significantly with appropriate sign. The equation for unemployment rate reflects an autoregressive pattern with its own lagged value, this could be a reflection of the persistence nature exhibited from the graphical analysis above; besides this not much can be inferred from the equations.

5. Conclusion

This paper applied I(2) analysis, a technique developed by Johansen (1995), to Danish manufactured exports. We modeled exports from a different perspective i.e. related exports to wages. It was found that nominal wages can better be characterized as I(2) processes. A nominal to real transformation of the system was accepted which facilitated the I(1) analysis of the transformed real vector. We found two long run cointegrating relations: a demand relation with long run wage elasticity of 1.7 and a supply relation. The short run wage elasticity of exports is estimated at 0.65. Our analysis obtains a higher and more robust long run wage elasticity of manufactured exports than the single equation analysis in MONA (2003). The identified structure was shown to be stable in the sample periods covered.

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Appendix

Table 1 Moduli of the companion matrix for the I(1) model (Nominal vector Y_t)

VAR(p)	0.919	0.919	0.883	0.785	0.785	0.733	0.341	0.317	0.317	0.288
r =4	1	0.950	0.858	0.858	0.687	0.687	0.356	0.342	0.290	0.244
r =3	1	1	0.927	0.927	0.692	0.692	0.692	0.333	0.265	0.265
r =2	1	1	1	0.936	0.856	0.665	0.354	0.354	0.254	0.254
r =1	1	1	1	1	0.952	0.757	0.395	0.342	0.275	0.186

Table 2 The trace test for the rank indices of the I(2)model

p-r	r			s_{r,s_1}			
5	0	585.641	395.369	286.232	217.570	182.390	161.650
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
4	1		257.490	177.920	119.330	88.568	86.665
			[0.000]	[0.000]	[0.000]	[0.004]	[0.000]
3	2			106.562	77.064	**45.671**	44.589
				[0.001]	[0.009]	**[0.235]**	[0.032]
2	3				**41.672**	**27.365**	26.081
					[0.208]	**[0.282]**	[0.045]
1	4					**13.722**	**8.910**
						[0.319]	**[0.191]**
	s2	5	4	3	2	1	0

Note: figures in square bracket are p-values

Table 3 Misspecification test results for the cointegrated structure H_3

Univariate Tests			Multivariate Tests
Equation	ARCH(2)	Normality	Normality Chi2(8) = 13.196 [0.105]
$\Delta(x\text{-}x_f)_t$	2.341[0.310]	2.337[0.311]	AR(1) CHI2(16) = 15.844 [0.464]
$\Delta(w\text{-}w_f\text{-}e)_t$	6.095[0.047]	1.002[0.606]	AR(2) CHI2(16) = 8.389 [0.936]
$\Delta^2 w_t$	18.259[0.000]	8.845[0.012]	ARCH(1) CHI (225) = 126.969 [0.036]
ΔU_t	1.942[0.379]	2.440[0.295]	ARCH(2) CHI (450) = 217.986 [0.182]

Note: AR is test of autocorrelation order 1 and 2, ARCH tests 1st and 2nd orders, figures in square bracket are p-values.

4. A parsimonious Multivariate Equilibrium Correction Model

	$\Delta(x-x_f)_t$	$\Delta(w-w_f-e)_t$	$\Delta^2 W_t$	ΔU_t
$\Delta (x-x_f)_{t-1}$	-0.339	0	0	0
	[0.000]			
Δx_{ft-1}	-0.339	0	0	0
	[0.000]			
$\Delta (w-w_f-e)_{t-1}$	0	0.335	0	0
		[0.000]		
ΔU_{t-1}	0	0	0	0.788
				[0.000]
$ECM1_{t-1}$	-0.100	-0.093	0	0
	[0.076]	[0.000]		
$ECM2_{t-1}$	0.807	-0.636	-1.064	0
	[0.086]	[0.005]	[0.000]	
$D794p_t$	0	0.025	0.027	0
		[0.064]	[0.000]	
$D802p_t$	0	0	-0.010	0.048
			[0.038]	[0.000]

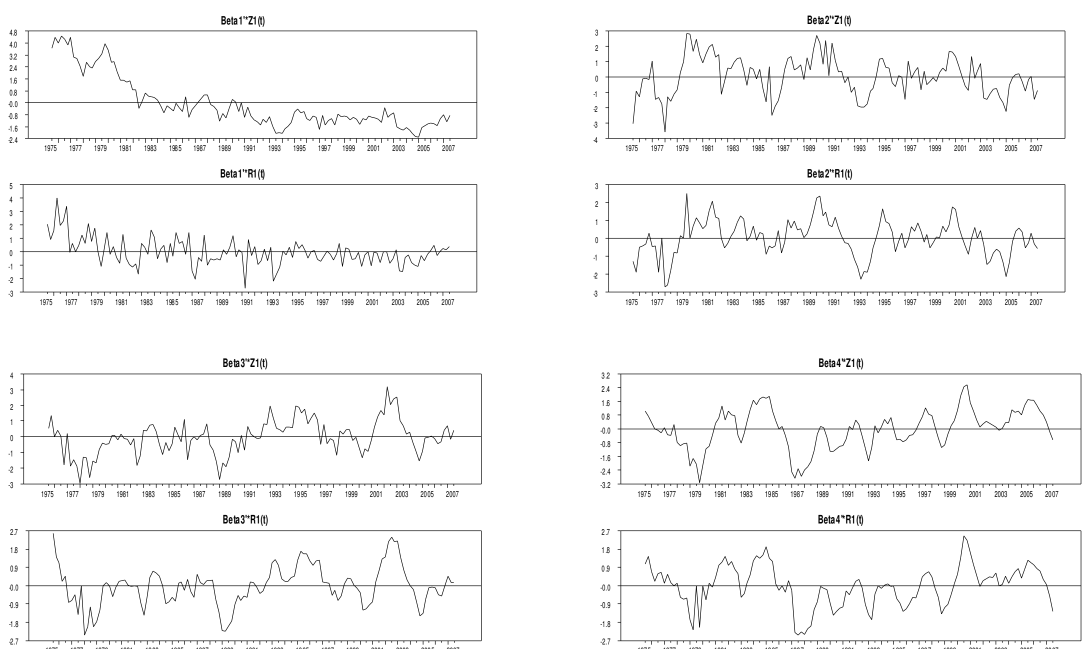
	$\Delta (x-x_f)_t$	$\Delta(w-w_f-e)_t$	$\Delta^2 W_t$	ΔU_t
$\Delta (x-x_f)_t$	0.030			
$\Delta (w-w_f-e)_t$	**-0.252**	0.014		
$\Delta^2 W_t$	-0.045	**0.329**	0.005	
ΔUR_t	-0.064	0.141	-0.020	0.007

$ECM1_{t-1} = (x - x_f)_t + 1.698\ (w-w_f-e)_t - 0.136 D903s_t$

$ECM2_{t-1} = \Delta w_t + 0.041 U_t + 0.001t$

Note: the Column headings are the dependent variables in each of the model equation and the raw headings are the predetermined regressors. ECM1t-1 and ECM2t-1 are the long run identified demand and supply relations, equation (8) and (9), respectively. Constant term included in all equation but not reported.

Figure 1 Graphs of the cointegration relation for the I(1) Model (nominal vector Yt)



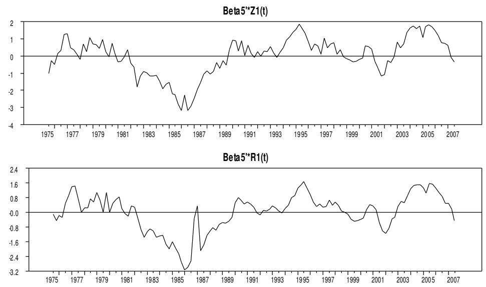
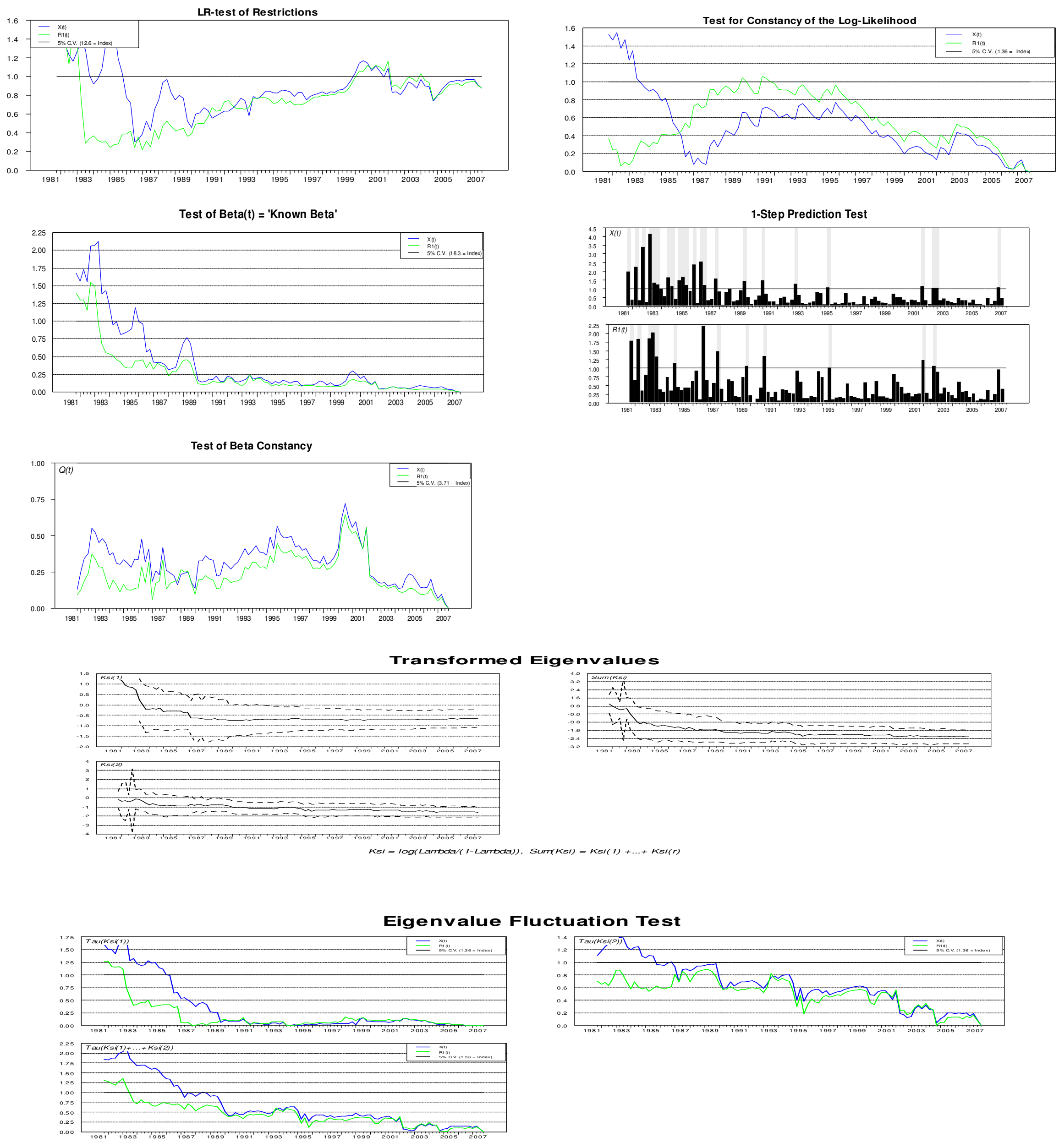
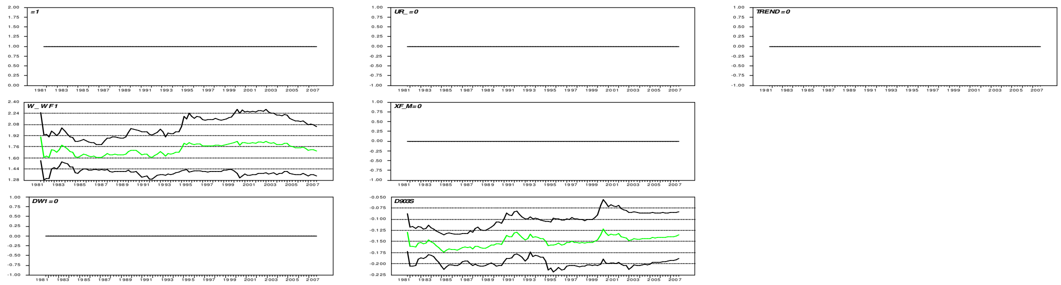


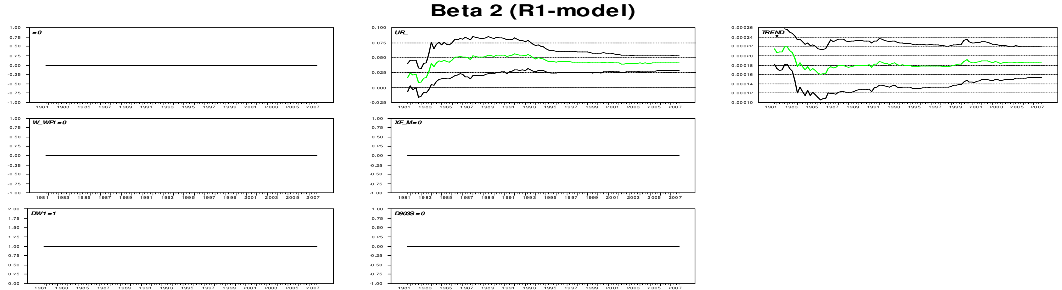
Figure 2 recursively estimated parameter constancy test results



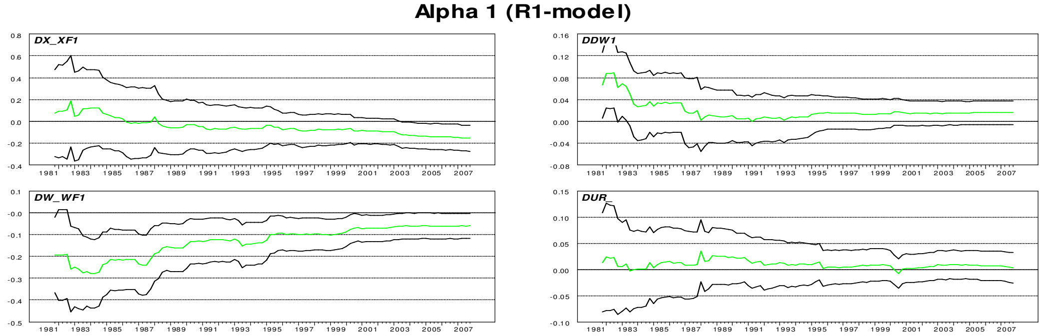
Beta 1 (R1-model)



Beta 2 (R1-model)



Alpha 1 (R1-model)



Alpha 2 (R1-model)