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Producer Price Index Manual

Theory and Practice

International Labour Organization

International Monetary Fund

Organisation for Economic Co-operation and Development

United Nations Economic Commission for Europe

The World Bank

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Foreword

This *Producer Price Index Manual* replaces the United Nations' *Manual on Producers' Price Indices for Industrial Goods* issued in 1979 (Series M, No. 66). The development of the *PPI Manual* has been undertaken under the joint responsibility of five organizations—the International Labour Organization (ILO), International Monetary Fund (IMF), Organisation for Economic Co-operation and Development (OECD), United Nations Economic Commission for Europe (UNECE), and World Bank—through the mechanism of an Inter-Secretariat Working Group on Price Statistics (IWGPS). It is published jointly by these organizations.

The *Manual* contains detailed, comprehensive information and explanations for compiling a PPI. It provides an overview of the conceptual and theoretical issues that statistical offices should consider when making decisions on how to deal with the various problems in the daily compilation of a PPI, and it is intended for use by both developed and developing countries. The chapters cover many topics; they elaborate on the different practices currently in use, propose alternatives whenever possible, and discuss the advantages and disadvantages of each alternative. Given the comprehensive nature of the *Manual*, we expect it to satisfy the needs of many users.

The main purpose of the *Manual* is to assist producers of the PPI, particularly countries that are revising or setting up their PPI. The *Manual* draws on a wide range of experience and expertise in an attempt to describe practical and suitable measurement methods. It should also help countries to produce their PPIs in a comparable way, so that statistical offices and international organizations can make meaningful international comparisons. Because it brings together a large body of knowledge on the subject, the *Manual* may be used for self-learning or as a teaching tool for training courses on the PPI.

Other PPI users, such as businesses, policymakers, and researchers, make up another targeted audience of the *Manual*. The *Manual* will inform them not only about the different methods that are employed in collecting data and compiling such indices, but also about the limitations, so that the results may be interpreted correctly.

The drafting and revision process has required many meetings over a five-year period, in which PPI experts from national and international statistical offices, universities, and research organizations have participated. The *Manual* owes much to their collective advice and wisdom.

The electronic version of the *Manual* is available on the Internet at www.imf.org. The IWGPS views the *Manual* as a "living document" that it will amend and update to address particular points in more detail. This is especially true for emerging discussions and recommendations made by international groups reviewing the PPI, such as the International Working Group on Service Sector Statistics (the Voorburg Group) and the International Working Group on Price Indices (the Ottawa Group).

The IWGPS welcomes users' comments on the *Manual*, which should be sent to the IMF Statistics Department (e-mail: TEGPPI@imf.org). They will be taken into account in any future revisions.

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Preface

The ILO, IMF, OECD, UNECE, and World Bank, together with experts from a number of national statistical offices and universities, have collaborated since 1998 in developing this *Producer Price Index Manual*. In addition, these organizations have consulted with a large number of potential users of the *PPI Manual* to get practical input. The developing organizations endorse the principles and recommendations contained in this *Manual* as good practice for statistical agencies in conducting a PPI program. *Because of practical constraints, however, some of the current recommendations may not be immediately attainable by all statistical offices and, therefore, should serve as guideposts for agencies as they revise and improve their PPI programs*. In some instances, there are no clear-cut answers to specific index number problems such as specific sample designs, the appropriate index estimation formula to use with given data inputs, making adjustments for quality changes, and handling the appearance of new products. Statistical offices will have to rely on the underlying principles laid out in this *Manual* and economic and statistical theory to derive practical solutions.

A. Producer Price Indices

PPIs measure the rate of change in the prices of goods and services bought and sold by producers. An *output* PPI measures the rate of change in the prices of products sold as they leave the producer. An *input* PPI measures the rate of change in the prices of the inputs of goods and services purchased by the producer. A *value-added* PPI is a weighted average of the two.

The *PPI Manual* serves the needs of different audiences. On the one hand are the compilers of PPIs. This *Manual* and other manuals, guides, and handbooks are important to compilers for several reasons. First, there is a need for countries to compile statistics in comparable ways so they can make reliable international comparisons of economic performance and behavior using the best international practices. Second, statisticians in each country should not have to decide on methodological issues alone. The *Manual* draws on a wide range of experience and expertise in an attempt to outline practical and suitable measurement methods and issues. Such measurement methods and issues are not always straightforward, and the *Manual* benefits from recent theoretical and practical work in the area. Third, much of the written material in some areas of PPI measurement covers a range of publications. This *Manual* brings together a large amount of what is known on the subject. It may therefore be useful for reference and training. Fourth, the *Manual* provides an independent reference on methods against which a statistical agency's current methods, and the case for change, can be assessed. The *Manual* should serve the needs of users. Users should be aware not only of the methods employed by statistical offices in collecting data and compiling the indices, but also of the potential such indices have for errors and biases, so that users can properly interpret the results. For example, index number theory presents many issues on formula bias, and the *Manual* deals extensively with the subject.

Collecting data for PPIs is not a trivial matter. In practical terms, PPIs require sampling, from a representative sample of establishments, a set of well-defined products whose overall price changes are representative of those of the millions of transactions taking place. Statistical offices then monitor the prices of these same products on a periodic basis (usually monthly) and weight their price changes according to their net revenue. However, the quality of the commodities produced may be changing, with new establishments and commodities appearing and old ones disappearing on both a seasonal and permanent basis. Statistical offices need to closely monitor potential changes in quality. Yet the index compilers must complete the task of producing a representative index monthly, in a timely manner.

It is also important to have a well-developed theoretical basis for compiling such indices that is readily accessible for practitioners and users alike. There should be a firm understanding of user needs and how the index delivered fits both. Fortunately, there is a great body of research in this area, much of which is fairly recent. This *Manual* covers the theoretical basis of index numbers to help support some of the practical considerations.

This *Manual* provides guidelines for statistical offices or other agencies responsible for compiling a PPI, bearing in mind the limited resources available. *Calculating a PPI cannot be reduced to a simple set of rules or a standard set of procedures that can be mechanically followed in all circumstances.* Although there are certain general principles that may be universally applicable, the procedures followed in practice have to take account of particular circumstances. Statistical offices have to make choices. These include procedures for the collection or processing of the price data and the methods of aggregation. Other important factors governing methodology are the main use of the index, the nature of the markets and pricing practices within the country, and the resources available to the statistical office. The *Manual* explains the underlying economic and statistical concepts and principles needed to enable statistical offices to make their choices in efficient and cost-effective ways and to recognize the full implications of their choices.

The *Manual* draws on the experience of many statistical offices throughout the world. The procedures they use are not static but continue to evolve and improve, for a variety of reasons. First, research continually refines the economic and statistical theory underpinning PPIs and strengthens it. For example, recent research has provided clearer insights about the relative strengths and weaknesses of the various formulas and methods used to process the basic price data collected for PPI purposes. Second, recent advances in information and communications technology have affected PPI methods. Both theoretical and data developments can impinge on all the stages of compiling a PPI. New technology can affect the methods used to collect prices and relay them to the central statistical office. It can also improve the processing and checking, including the methods used to adjust prices for changes in the quality of the goods and services covered. Finally, improved formulas help in calculating more accurate higher-level indices.

B. Background to the Present Revision

Some international standards for economic statistics have evolved mainly to compile internationally comparable statistics. However, standards may also be developed to help individual countries benefit from the experience and expertise accumulated in other countries. All countries stand to gain by exchanging information about index methods. The UN published the existing *Manual on Producers' Price Indices for Industrial Goods* (United Nations, 1979) over 25 years ago. The methods and procedures presented then are now outdated. Index number theory and practice and improvements in technology have advanced greatly over the past two decades.

B.1 Concerns with current index methods

The *PPI Manual* takes advantage of the wealth of recent research on index number theory. It recommends many new practices instead of just codifying existing statistical agency practices. There are a number of reasons for this.

First, the standard methodology for a typical PPI is based on a Laspeyres price index with fixed quantities from an earlier base period. The construction of this index can be thought of in terms of selecting a basket of goods and services representative of base-period revenues, valuing this at base-period prices, and then repricing the same basket at current-period prices. The target PPI in this case is defined to be the ratio of these two revenues. Practicing statisticians use this methodology because it has at least three practical advantages. It is easily explained to the public, it can use often expensive and untimely weighting information from the date of the last (or an even earlier) survey or administrative source (rather than requiring sources of data for the current month), and it need not be revised if users accept the Laspeyres premise. One notable advantage of the Laspeyres approach under the ease of explanation heading is its consistency in aggregation. It produces various breakdowns or subaggregates related to one another in a particularly simple way.

Statistical agencies implement the Laspeyres index by putting it into price-relative (price change from the base period) and revenue-share (from the base period) format. In this form, the Laspeyres index can be written as the sum of base-period revenue shares of the items in the index times their corresponding price relatives. Unfortunately, simple as it may appear, there still are a number of practical problems with producing the Laspeyres index exactly. Consequently, statistical agency practice has introduced some approximations to the theoretical Laspeyres target.

- Until recently it has been impossible to get accurate revenue shares for the base period down to the finest level of commodity aggregation, so statistical agencies settle for getting base-period revenue weights at the level of 100 to 1,000 products.

- For each of the chosen product aggregates, agencies collect a sample of representative prices for specific transactions from establishments rather than attempting to enumerate every possible transaction. They use equally weighted (rather than revenue-weighted) index formulas to aggregate these elementary product prices into an elementary aggregate index, which will be used as the price relative for each of the 100 to 1,000 product groups in the final Laspeyres formula. Practitioners recognize that this two-stage procedure is not exactly consistent with the Laspeyres methodology (which requires weighting at each stage of aggregation). However, for a number of theoretical and practical reasons, practitioners judge that the resulting elementary index price relatives will be sufficiently accurate to insert into the Laspeyres formula at the final stage of aggregation.

The above standard index methodology dates back to the work of Mitchell (1927) and Knibbs (1924) and other pioneers who introduced it about 80 years ago, and it is still used today.

Although most statistical agencies have traditionally used the Laspeyres index as their *target index*, both economic and index number theory suggest that some other types of indices may be more appropriate target indices to aim for: namely, the Fisher, Walsh, or Törnqvist-Theil indices. As is well known, the Laspeyres index has an upward bias compared with these target indices. Of course, these target indices may not be achievable by a statistical agency, but it is necessary to have some sort of theoretical target to aim for. Having a target concept is also necessary, so that the index that is actually produced by a statistical agency can be evaluated to see how close it comes to the theoretical ideal. In the theoretical chapters of this *Manual*, it is noted that there are four main approaches to index number theory:

(1) Fixed-basket approaches and symmetric averages of fixed baskets (Chapter 15);
(2) The stochastic (statistical estimator) approach to index number theory (Chapter 16);
(3) Test (axiomatic) approaches (Chapter 16); and
(4) The economic approach (Chapter 17).

Approaches 3 and 4 will be familiar to many price statisticians and expert users of the PPI, but perhaps a few words about approaches 1 and 2 are in order.

The Laspeyres index is an example of a fixed-basket index. The concern from a theoretical point of view is that it has an equally valid “twin” for the two periods under consideration—the Paasche index, which uses quantity weights from the current period. If there are two equally valid estimators for the same concept, then statistical theory tells us to take the average of the two estimators in order to obtain a more accurate estimator. There is more than one way of taking an average, however, so the question of the "best" average to take is not trivial. The *Manual* suggests that the "best" averages that emerge for fixed-base indices are the geometric mean of the Laspeyres and Paasche indices (Fisher ideal index) or the geometric average of the quantity weights in both periods (Walsh index). From the perspective of a statistical estimator, the "best" index number is the geometric average of the price relatives weighted by the average revenue share over the two periods (Törnqvist-Theil index).

There is one additional result from index number theory that should be mentioned here—the problem of defining the price and quantity of a product that should be used for each period in the index number formula. The problem is that the establishment may have sales for a particular product specification in the period under consideration at a number of different prices. So the question arises, what price would be most representative of the sales of this transaction for the period? The answer to this question is obviously the *unit value* for the transaction for the period, since this price will match up with the quantity sold during the period to give a product that is equal to the value of sales.[1]

Now consider concerns about the standard PPI methodology. There are *six main areas of concern* with the standard methodology:[2]

(1) At the final stage of aggregation, the standard PPI index is *not* a true Laspeyres index, since the revenue weights pertain to a reference base *year* that is different from the base *month* (or quarter) for prices. Thus the expenditure weights are chosen at an annual frequency, whereas the prices are collected at a monthly frequency. To be a true Laspeyres index, the base-period revenues should *coincide* with the reference period for the base prices. In practice, the actual index used by many statistical agencies at the last stage of aggregation has a weight reference period that precedes the base-price period. Indices of this type are likely to have some upward bias compared with a true Laspeyres index, especially if the expenditure weights are price-updated from the weight reference period to the Laspeyres base period. It follows that they must have definite upward biases compared with theoretical target indices such as the Fisher, Walsh, or Törnqvist-Theil indices.

(2) At the early stages of aggregation, unweighted averages of prices or price relatives are used. Until relatively recently, when enterprise data in electronic form have become more readily available, it was thought that the biases that might result from the use of unweighted indices were not particularly significant. However, recent evidence suggests that there is potential for significant upward bias at lower levels of aggregation compared with results that are generated by the preferred target indices mentioned above.

(3) The third major concern with the standard PPI methodology is that, although statistical agencies generally recognize that there is a problem with the treatment of quality change and new goods, it is difficult to work out a coherent methodology for these problems in the context of a fixed-base Laspeyres index. The most widely received good practice in quality-adjusting price indices is "hedonic regression," which characterizes the price of a product at any given time as a function of the characteristics it possesses relative to its near substitutes. In fact, there is a considerable amount of controversy on how to integrate hedonic regression methodology into the PPI's theoretical framework. The theoretical and practical chapters in the *Manual* devote a lot of attention to these methodological problems. The problems created by the disappearance of old goods and the appearance of new models are now much more severe than they were when the traditional PPI methodology was developed some 80 years ago (then, the problem was mostly ignored). For many categories of products, those priced at the beginning of the year are simply no longer available by the end of the year. Thus, there is a

[1]Note that the *Manual* does *not* endorse taking unit values over *heterogeneous* items at this first stage of aggregation; it endorses only taking unit values over *identical* items in each period.

[2]These problems are not ranked in order of importance; they all seem equally important.

tremendous concern with *sample attrition*, which impacts on the overall methodology; that is, at lower levels of aggregation, it becomes necessary (at least in many product categories) to switch to chained indices rather than use fixed-base indices. Certain unweighted indices have substantial bias when chained.

(4) A fourth major area of concern is related to the first concern: the *treatment of seasonal commodities*. The use of an annual set of products or the use of annual revenue shares is justified to a certain extent if one is interested in the longer-run trend of price changes. If the focus, however, is on short-term, month-to-month changes (as is the focus of central banks), then it is obvious that the use of annual weights can lead to misleading signals from a short-run perspective, since monthly price changes for products that are out of season (i.e., the seasonal weights for the product class are small for the two months being considered) can be greatly magnified by the use of annual weights. The problem of seasonal weights is a big one when the products are not available at all at certain months of the year. There are solutions to these seasonality problems, but the solutions do not appeal to traditional PPI statisticians because they involve the construction of *two* indices: one for the short-term measurement of price changes and another (more accurate) longer-term index that is adjusted for seasonal influences.

(5) A fifth concern with standard PPI methodology is the general exclusion of services from the PPI framework. A typical PPI will include mining, manufacturing, electricity, gas supply, and water supply activities, normally referred to as an industrial PPI. Many countries may also include agricultural prices. Thus, PPI coverage includes many more goods-producing activities than services. In a way, this just reflects the historical origins of existing PPI theory. National PPIs have essentially been concerned with coverage of goods for 80 years, but 80 years ago goods were much more significant than services. Hence, there was not much focus on the problems involved in measuring services. It is only over the past 30 or so years that the shift to services has caused services output to exceed output of goods. In addition to inertia, there are some serious conceptual problems involved in measuring the prices of many services. Some examples of difficult-to-measure services are insurance, gambling, financial services, advertising services, telecommunication services (with complex plans), entertainment services, and trade. In many cases, statistical agencies simply do not have appropriate methodologies to deal with these difficult conceptual measurement problems. Thus, output prices for these service sector PPIs are not widely measured.[3]

(6) A final concern with existing PPI methodology is that it tends not to recognize that more than one PPI may be required to meet the needs of different users. There are three basic types of PPIs that users might want: *gross output* price indices, *intermediate input* price indices, and *value-added* price indices. Most countries concentrate on producing output price indices by product and industry, with little attention given to input price indices. Another example for multiple indices is gross output indices versus net sector indices. Aggregating industry or product gross output indices includes double-counting the effects of input price changes—the input price change effects are included in both the originating sector and the using sector indices. Net sector indices exclude interindustry price effects and are, therefore, a better gauge of the evolution of inflation through the production chain. In addition, some users may require information on the month-to-month movement of prices in a very timely fashion. This requirement leads to a fixed-weight PPI along the lines of existing PPIs, where current information on weights is not necessarily available. However, other users may be more interested in a

[3]The Voorburg Group, which meets annually, has included the expansion of PPIs to services as part of its work program. The OECD, as part of its contribution to this program, conducts periodic surveys on the extension of PPIs in services activities. The latest survey results along with developments in services statistics are available at http://www.oecd.org/document/43/0,2340,en_2649_34355_2727403_1_1_1_1,00.html.

more accurate or representative measure of price change and may be willing to sacrifice timeliness for increased accuracy. Thus, statistical agencies might produce one of the theoretical target indices (e.g., Fisher, Walsh, or Törnqvist-Theil) that uses current- and base-period weight data with a delay of a year or two. These are entirely reasonable developments, recognizing that different users have different needs. Since all three approaches have strong support, it would be reasonable for a statistical agency to pick one approach for its flagship index but make available the other two treatments as "analytical series" for interested users. Another example where multiple indices would be useful occurs in the context of seasonal products. The usual PPI is a month-to-month index, and it is implicitly assumed that all products are available in each month. As noted in item (4) above, this assumption is not warranted. In this context, a month-to-month PPI will not be as "accurate" as a year-over-year PPI that compares the prices of products in this month with the corresponding products in the same month a year ago. Again, the need emerges for multiple indices to cater to the needs of different users.

Many of the above areas of concern are addressed in this *PPI Manual*. Frank discussions of these concerns should stimulate the interest of academic economists and statisticians to address these measurement problems and to provide new solutions that can be used by statistical agencies. Public awareness of these areas of concern should lead to a willingness on the part of governments to allocate additional resources to statistical agencies so that economic measurement will be improved. In particular, there is an urgent need to fill in some of the gaps that exist in the measurement of service sector outputs.

B.2 Efforts to address the concerns in index number methods

Several years ago it became clear that the outstanding and controversial methodological concerns related to price indices needed further investigation and analysis. An expert group consisting of specialists on price indices from national and international statistical offices and universities from around the world formed to discuss these concerns. It met for the first time in Ottawa in 1994. During six meetings between 1994 and 2001, the Ottawa Group presented and discussed over a hundred research papers on the theory and practice of price indices. While much of the research related to consumer price indices (CPIs), many of the issues carried across to PPIs. It became obvious there were ways to improve and strengthen existing PPI and CPI methods.

In addition, the Voorburg Group on Service Sector Statistics, with members from many national statistical offices, has held annual meetings for over a decade. Many agenda topics of the Voorburg Group related to expanding country PPIs to cover service industries and products. The Group has provided many technical papers on concepts and methods for compiling service PPIs. These papers serve as documentation that other countries can follow.

At the same time, the control of inflation had become a high-priority policy objective in most countries. Policymakers use both the CPI and PPI widely to measure and monitor inflation. The slowing down of inflation in many parts of the world in the 1990s, compared with the 1970s and 1980s, increased interest in PPI and CPI methods rather than reduce it. There was a heightened demand for more accurate, precise, and reliable measures of inflation. When the rate of inflation slows to only 2 or 3 percent each year, even a small error or bias becomes significant.

Recent concern over the accuracy of price indices led governments and research institutes in a few countries to commission experts to examine and evaluate the methods used, particularly for the CPI. The methods used to calculate CPIs and PPIs have been subject to public interest and scrutiny of a kind and level that were unknown in the past. One conclusion reached is that existing methods might lead to some upward bias in both the CPI and PPI. One reason for this was that many goods and services had inadequate allowance for improvements in their quality. The direction and extent of such bias will, of course, vary between commodity groups, and its total effect on the economy will vary among countries. However, the upward bias has the potential to be large, so this *Manual* addresses adjusting prices for changes in quality in some detail, drawing on the most recent research in this area. There are other sources of bias including that arising from no allowance, or an inappropriate one, made for changes in the bundle of items produced, when production switches between commodities with different rates of price change. Further, different forms of bias might arise from the sampling and price collection systems. Several chapters deal with these subjects, with an overall summary of possible errors and biases given in Chapter 11.

CPIs are widely used for the index linking of social benefits such as pensions, unemployment benefits, and other government payments. The cumulative effects of even a small bias could have notable long-term financial outcomes for government budgets. Similarly, a major use of PPIs is as an escalator for price adjustments to long-term contracts. Agencies of government, especially ministries of finance, and private businesses have taken a renewed interest in price indices, examining their accuracy and reliability more closely and carefully than in the past.

In response to the various developments outlined above, the need to revise, update, and expand the UN manual was gradually recognized and accepted during the late 1990s. The joint UNECE/ILO meeting of national and international experts on CPIs held at the end of 1997 in Geneva made a formal recommendation to revise *Consumer Price Indices: An ILO Manual* (ILO, 1989). The main international organizations interested in measuring inflation have taken responsibility for the revision. The United Nations Statistical Commission in 1998 approved this strategy and agreed to set up the *Inter-secretariat Working Group on Price Statistics* (IWGPS).

C. Organization of the Revision

C.1 Agencies responsible for the revision

The following international organizations—concerned with measuring inflation, with policies designed to control inflation, and with measurement of deflators for national accounts—have collaborated on revising the *CPI* and *PPI Manuals*:

- The International Labour Organization (ILO);
- The International Monetary Fund (IMF);
- The Organisation for Economic Co-operation and Development (OECD);
- The Statistical Office of the European Communities (Eurostat);
- The UN Economic Commission for Europe (UNECE); and
- The World Bank.

These organizations have provided, and continue to provide, technical assistance on CPIs and PPIs both to developing countries and to countries in transition from planned to market economies. They joined forces for the present revision of the *CPI* and *PPI Manuals*, setting up the IWGPS for this purpose. The group's role was to organize and manage the process rather than act as an expert group.

The responsibilities of the IWGPS were as follows:

- To appoint the various experts on price indices who shared in the revision either as members of the Technical Expert Group (who provided substantive advice on the contents of the *Manual*) or as authors of the various chapters;
- To provide the financial and other resources needed;
- To arrange meetings of the Technical Expert Group, prepare the agendas, and write the reports on the meetings; and
- To arrange for the publishing and disseminating of the two *Manuals*.

Members of the IWGPS were also members of the Technical Expert Groups. The experts taking part in the Technical Expert Groups were invited in their personal capacity as experts and not as representatives, or delegates, of the national statistical offices or other agencies that employed them. Participants were able to give their expert opinions without in any way committing the offices from which they might have come.

C.2 Links with the new *Consumer Price Index Manual*

One of the first decisions of the IWGPS was to produce a new international *PPI Manual* at the same time as the *CPI Manual*. There have been international standards for CPIs for over 70 years, but the UN's 1979 PPI manual was the first international manual on producer prices. Despite the importance of PPIs for measuring and analyzing inflation, the methods used for compiling them have been comparatively neglected, at both national and international levels.

The IWGPS set up two Technical Expert Groups, one for each *Manual*, whose membership overlapped. *The two manuals have similar contents and are fully consistent with each other conceptually, sharing common text when suitable.* The two groups worked in close liaison with each other. The PPI and CPI methods have a lot in common. Both use essentially the same underlying economic and statistical theory, except that the CPI draws on the economic theory of consumer behavior, whereas the PPI draws on the economic theory of production. However, the two economic theories are isomorphic and lead to the same kinds of conclusions about index number compilation. The *Manuals* have practical and operational applications (Chapters 1–13 and the Glossary) that are supported by their theoretical underpinnings (Chapters 14–22).

Most members of the Technical Expert Groups on CPIs and PPIs also engaged as active members of the Ottawa Group. The two *Manuals* were able to draw on the contents and conclusions of all the numerous papers presented at meetings of the Ottawa and Voorburg Groups.

D. Acknowledgments

The *PPI Manual* is the result of a five-year process that involved multiple activities. The first activities were the development of the *Manual* outline and the recruitment of individuals to draft the various chapters. Next, members of the Technical Expert Group on the PPI (TEG-PPI), the IWGPS, and others refereed the draft chapters. Then came the posting of the draft chapters on a *PPI Manual* website for comment by interested individuals and organizations. The final steps were consultation with a focus group of selected users from national statistical offices. Final copyediting of the *Manual* was coordinated in the IMF External Relations Department by James McEuen. The editor wishes to thank Mbaye Gueye for assistance in the final review of the *Manual* and all of those involved in the process, with special recognition for the following:

- *The author, or authors, of the chapters (with their affiliations).*

Preface	Paul Armknecht (*PPI Manual* editor, IMF), W. Erwin Diewert (University of British Columbia), Peter Hill (*CPI Manual* editor, expert)
Reader's Guide	Paul Armknecht (IMF), Peter Hill (expert)
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Chapter 2	Andrew Allen (U.K. Office of National Statistics), Paul Armknecht (IMF), David Collins (Australian Bureau of Statistics)
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Glossary Appendix	Bert Balk (Statistics Netherlands, Appendix).

- *The individual members of the IWGPS and the TEG-PPI.*

IWGPS: Organizational membership is as follows: Eurostat, ILO, IMF, OECD, UNECE, and World Bank. During the revision of the *Manual*, the *CPI Manual* editor (Peter Hill), TEG-CPI chairperson (David Fenwick), and *PPI Manual* editor and TEG-PPI chairperson (Paul Armknecht) were observers. The ILO was the Secretariat for the Group, and Sylvester Young the chairperson of the IWGPS.

The IWGPS met formally four times: September 24, 1998 (Paris), February 11, 1999 (Geneva), November 2, 1999 (Geneva), and March 21–22, 2002 (London). Informal meetings were held on several occasions.

TEG-PPI: Andrew Allen (U.K. Office of National Statistics), Paul Armknecht (chair, IMF), Bert Balk (Statistics Netherlands), Matthew Berger* (Australian Bureau of Statistics), David Collins* (Australian Bureau of Statistics), W. Erwin Diewert (University of British Columbia), Yoel Finkel (Israel Central Bureau of Statistics), Dennis Fixler (U.S. Bureau of Economic Analysis), Irwin Gerduk (U.S. Bureau of Labor Statistics), Jan Karlsson (UNECE), Robin Lowe (Statistics Canada), Richard McKenzie* (Australian Bureau of Statistics), David Roberts (OECD), Paul Schreyer (OECD), Mick Silver (Cardiff University), and Kimberly Zieschang (IMF). The IMF was the Secretariat for the Group.

The TEG-PPI met five times: November 2–3, 1999 (Geneva), September 20–22, 2000 (Madrid), October 29–30, 2001 (Geneva), March 19–21, 2002 (London), and February 25–27, 2003 (Washington, D.C.).[4]

[4]Individuals with an asterisk (*) after their name served for only part of the period.

- *The participants of a focus group seminar on the* PPI Manual *in Pretoria, South Africa.*

The IMF Statistics Department and Statistics South Africa, supported by funding from the government of Japan through the Administered Account for Selected Fund Activities—Japan and the OECD Centre for Co-operation with Non-Member Countries, held a seminar with selected user agencies during June 23–27, 2003. Participants provided excellent feedback on the usefulness of the new *Manual* and made many good suggestions for improvements. The participants in the seminar and their affiliated agencies were Adnan Badran (Jordan Department of Statistics), Langa Benson (Statistics South Africa), Gustavo Javier Biedermann (Central Bank of Paraguay), Bikash Bista (Nepal Central Bureau of Statistics), Juleeemun Dhananjay (Mauritius Central Bureau of Statistics), Istvan Kölber (Hungarian Central Statistics Office), Inga Kunstvere (Latvia Central Bureau of Statistics), Phaladi Labobedi (Botswana Central Bureau of Statistics), Guergana Maeva (Bulgarian National Institute of Statistics), Moffat Malepa (Botswana Central Bureau of Statistics), Gopal Singh Negi (Indian Ministry of Commerce and Industry), Ali Rosidi (Statistics Indonesia), Matti Särngren (Statistics Sweden), Joy Sawe (Tanzanian National Bureau of Statistics), Soon Teck Wong (Statistics Singapore), Harry Thema (Statistics South Africa), and Bouchaib Thich (Morocco Direction de la Statistique).

Reader's Guide

International manuals in economic statistics have traditionally provided guidance about concepts, definitions, classifications, coverage, valuation, recording data, aggregation procedures, formulas, and so on. They have mainly aided compilers of the relevant statistics in individual countries. This *Manual* shares this same principal objective.

The *Manual* will benefit users of PPIs, such as government and academic economists, financial experts, and other informed users. The PPI is a key statistic for policy purposes. It attracts much attention from the media, governments, and the public in most countries. The PPI is a sophisticated concept that draws on a great deal of economic and statistical theory and requires complex data manipulation. This *Manual* is therefore also intended to promote greater understanding of the properties of PPIs.

In general, compilers and users of economic statistics must have a clear view of what the statistics measure, in principle. Measurement without theory is unacceptable in economics, as in other disciplines. This *Manual* therefore contains a thorough, comprehensive, and up-to-date survey of relevant economic and statistical theory. This makes the *Manual* self-contained in both the theory and practice of PPI measurement.

The *Manual*, consequently, is large. Because different readers may have different interests and priorities, it is not possible to devise a sequence of chapters that suits all. Indeed, users do not read international manuals from cover to cover in that order. Manuals also serve as reference works. Many readers may have interest in only a selection of chapters. The purpose of this Reader's Guide is to provide a map of the contents of the *Manual* that will aid readers with different interests and priorities.

A. An Overview of the Sequence of Chapters

As mentioned in the preface, the chapters of this *Manual* are arranged so that practical and operational issues (Chapters 1–13 and the Glossary) are supported by theoretical underpinnings (Chapters 14–22). Specifically, the *Manual* is divided into four parts:

- Part I (Chapters 1–3) examines PPI methodology, uses, and coverage;
- Part II (Chapters 4–11) covers compilation issues;
- Part III (Chapters 12–13) considers operational matters; and
- Part IV (Chapters 14–22) explores conceptual and theoretical issues.

The remaining paragraphs in this section give synopses of the individual chapters.

A.1 Part I: Methodology, uses, and coverage

Chapter 1 is a general introduction to the theory and practice of PPIs. It is intended for all readers. It provides the basic information needed to understand the later chapters and a summary of index number theory, as explained in much more detail in Chapters 15–20. It then provides a summary of the main steps involved in compiling a PPI, drawing on material in Chapters 3–9. It does not provide a summary of the *Manual* as whole nor does it cover specific topics or special cases that are not of general relevance.

Chapter 2 outlines the history of price indices and how PPIs have changed in response to the demand for broader measures of price change. Chapter 3 presents a few basic concepts, principles, classifications, and the scope or coverage of an index. The scope of a PPI can vary significantly from country to country.

A.2 Part II: Compilation issues

Chapters 4–9 form an interrelated sequence of chapters describing the various steps involved in compiling a PPI, from collecting and processing the price data through calculating the final index. Chapter 4 discusses deriving the value weights attached to the price changes for different goods and services. Establishment censuses or surveys supplemented by data from other sources typically provide the weight data.

Chapter 5 deals with sampling issues. A PPI is essentially an estimate based on a sample of the prices of products produced by a sample of establishments. Chapter 5 considers sampling design and the pros and cons of random versus purposive sampling. Chapter 6 describes the procedures used to collect the prices from a selection of establishments and products. It deals with topics such as questionnaire design, specifying the transactions selected, and methods for collecting data, including the use of electronic media.

Chapter 7 addresses the difficult question of how to adjust prices for changes over time in the quality of the goods or services selected. Changes in value due to changes in quality count as changes in quantity not price. Disentangling the effects of quality change poses serious theoretical and practical problems for compilers. Chapter 8 addresses two closely related questions: first, how to deal with goods and services that disappear from the sample; second, how new goods or services not previously produced can enter the sample.

Chapter 9 gives a step-by-step description of editing procedures, calculating elementary price indices from the raw prices collected for small groups of products, and the resulting averaging of the elementary indices to obtain indices at various levels of aggregation up to the overall PPI itself. The chapter also provides a description of the process for the periodic update of the value weights.

Chapter 10 deals with a few cases that need special treatment. For example, it presents methods for handling seasonal agricultural and clothing products, petroleum refining, steel mills, electronic computers, motor vehicles, shipbuilding, construction, retail trade, telecommunication services, some financial services, legal services, and medical hospitals. Chapter 11 provides an overview of the errors and biases to which PPIs may be subject.

A.3 Part III: Operational issues

Chapter 12 deals with issues of organization and management. Conducting the price surveys and processing the results make for a massive operation that needs careful planning, organization, and efficient management. Chapter 13 addresses publication and dissemination standards for the PPI results.

A.4 Part IV: Conceptual and theoretical issues

Chapter 14 marks a break in the sequence of chapters because it is not concerned with compiling a PPI. Its purpose is to examine the place of the PPI in the general system of price statistics. The PPI is not a set of independent, isolated statistics. The flow of producer goods and services to which it relates is only one of a larger set of interdependent flows within the economy as a whole. The analysis of inflation requires more than one index, and it is essential to know exactly how the PPI relates to the CPI and to other price indices, such as indices of export and import prices. The supply and use matrix of the *System of National Accounts 1993* (Commission of the European Communities and others, 1993) provides the proper conceptual framework for examining these interrelationships.

Chapters 15–18 provide a systematic and detailed exposition of the index number theory underlying PPIs. These chapters examine different approaches to index number theory. Collectively, they provide a comprehensive and up-to-date survey of index number theory, including recent methodological developments as reported in journals and conference proceedings.

Chapter 15 provides an introduction to index number theory, focusing on breaking up value changes into their price and quantity components. Chapter 16 examines the axiomatic and stochastic approaches to PPIs. The axiomatic, or test, approach lists many properties that are desirable for index numbers to have and tests specific formulas to see whether they have them.

Chapter 17 explains the economic approach, using the economic theory of producer behavior. In this approach, an output PPI is defined as a "fixed-input" economic price index that assumes fixed technology. Changes in the index arise solely from changes in the output prices between two periods. An input PPI is defined as a "fixed-output" economic price index that also assumes fixed technology. Changes in the index arise solely from changes in the input prices between two periods. Although these economic indices cannot be calculated directly, a certain class of index numbers, known as "superlative" indices, can be expected to approximate them in practice. From an economic perspective, the ideal index for PPI purposes should be a superlative index, such as the Fisher index. The Fisher index also is a very desirable index on axiomatic grounds.

Chapter 18 deals with aggregation issues. Chapter 19 presents a constructed data set to explain the numerical outcomes of using different index number formulas. It shows that, in general, the choice of index number formula can make a notable difference, but that different superlative indices all approximate one another.

Chapter 20 addresses the important question of what is the theoretically most appropriate elementary price index formula to use at the first stage of PPI compilation if no information is available on quantities or values. This has been a comparatively neglected topic until recently, even though the choice of formula for an elementary index can have a significant impact on the overall PPI. The elementary indices are the basic building blocks used to construct higher-level PPIs.

Chapters 21 and 22 conclude the *Manual*. They address two conceptually difficult issues. Chapter 21 considers the theoretical issues of adjusting for quality change on the basis of the hedonic approach. Chapter 22 examines the treatment of seasonal products.

A glossary of terms and a bibliography appear at the end of the sequence of chapters.

B. Alternative Reading Plans

Different readers may have different needs and priorities. Readers interested mainly in compiling PPIs may not wish to pursue all the finer points of the underlying economic and statistical theory. Conversely, readers more interested in the use of PPIs for analytic or policy purposes may not be interested in the details of the conduct and management of price surveys. Not all readers will want to read the entire *Manual*, or even want to follow the same reading plan.

However, all readers, whether users or compilers, will find it useful to read the first three chapters. Chapter 1 provides a general introduction to the whole subject by providing a review of the PPI theory and practice appearing in the *Manual*. It provides the basic knowledge needed for understanding later chapters. Chapter 2 explains the need for PPIs and their uses. Chapter 3 examines many basic conceptual issues and the scope of a PPI.

B.1 A compiler-oriented reading plan

Chapters 4–13 are mainly for compilers. They follow a logical sequence that roughly matches the various stages of compiling a PPI. They start with deriving the value weights and collecting the price data and finish with publishing the final index. Chapter 12, on organization and management, is intended for both managers and compilers. It discusses many important issues on the structure and mechanisms that statistical offices need to monitor, control, and ensure the quality of the PPI and to be efficient in the use of resources.

Chapter 14 is for both compilers and users of PPIs. It places PPIs in perspective within the overall system of price indices.

The remaining chapters, Chapters 15–22, are mainly theoretical. Compilers may find it necessary to follow certain theoretical topics in greater depth, in which case they have immediate access to the relevant material. It would be desirable for compilers to acquaint themselves with at least the basic index number theory set out in Chapter 15 and the numerical example developed in Chapter 19. The material in Chapter 20 on elementary price indices is also important for compilers.

B.2 A user-oriented reading plan

Although all readers should find Chapters 1–3 useful, and Chapters 4–13 are mainly for compilers, several topics have aroused great interest among many users.

Chapters 7 and 8 discuss the treatment of quality change, item substitution, and new products. Users may also find Chapter 9 helpful because it provides a concise description of the various stages of compiling a PPI.

Chapter 11, "Errors and Bias in the PPI," and Chapter 14, "The System of Price Statistics" are also of interest to both users and compilers.

Chapters 15–22 cover the economic and statistical theory underlying the PPI, and they are likely to be of interest to many users, especially professional economists and students of economics.

C. A Note on the Bibliography

In the past, international manuals on economic statistics have not usually provided references to the associated literature. It was not helpful to cite references when the literature was confined mostly to printed volumes, including academic journals or proceedings of conferences, found only in university or major libraries. Compilers working in many statistical offices were unlikely to have ready access to such literature. However, this has changed with the Internet and the World Wide Web, which make all such literature readily accessible. Therefore, this *Manual* breaks with past tradition by including a comprehensive bibliography to the large literature that exists on index number theory and practice that many readers are likely to find useful. In addition, websites are referenced that contain specialist papers on index number theory and practice, including those of the Ottawa Group and the Voorburg Group.

Abbreviations

ABS	Antilock brake system; Australian Bureau of Statistics
AF	Acre foot
ANZIC	Australian and New Zealand Standard Industrial Classification
ATM	Automated teller machine
BPM5	*Balance of Payments Manual, Fifth Edition*
BEA	Bureau of Economic Analysis
BLS	U.S. Bureau of Labor Statistics
CAPI	Computer-assisted personal interviews
CATI	Computer-assisted telephone interviews
CD	Compact disk
CD-ROM	Compact disk-read-only memory
CD-RW	Compact disk-rewritable
c.i.f.	Cost-insurance-freight
CIR	Current Industrial Report
COFOG	Classification of the Functions of Government
COICOP	Classification of Individual Consumption by Purpose
COL	Cost of living
COPNI	Classification of the Purposes of Nonprofit Institutions Serving Households
COPP	Classification of the Purposes of Producers
CPA	Classification of Products by Activity, also known as PRODCOM (Eurostat)
CPC	Central Product Classification
CPI	Consumer price index
CSWD	Carruthers, Sellwood, Ward, Dalén price index
DRAM	Dynamic random-access memory
DRG	Diagnostic-Related Group
DRP	Disaster Recovery Plan
e-	Electronic (e-business, e-commerce, e-mail, etc.)
EC	European Commission
ECB	European Central Bank
ECI	Employment cost index
EDI	Electronic data interchange
EFQM	European Foundation for Quality Management
ESMR	Enhanced specialized mobile radio
EU	European Union
Eurostat	Statistical Office of the European Communities
FEPI	Final expenditure price index
FIOPI	Fixed-input output price index
FISIM	Financial Intermediation Services Implicitly Measured

f.o.b.	Free on board
FOIPI	Fixed-output input price index
FPI	Final uses price index
FPPI	Farm product price index
GB	Gigobytes
GDDS	General Data Dissemination System (IMF)
GDP	Gross domestic product
GPI	Global price index; government price index
GPS	Global positioning system
HBS	Household Budget Survey
HICPs	Harmonized Indices of Consumer Prices (Eurostat)
HP	Hodrick- Prescott; horsepower
HPI	Household consumption price index
HS	Harmonized Commodity Description and Coding System
ICP	Implicit characteristic price
ICPI	Intermediate consumption price index
IDI	Implicit deflator index
ILO	International Labour Office /International Labour Organization
IMF	International Monetary Fund
I/O	Input/output
IPP	International Price Program
ISIC	International Standard Industrial Classification of All Economic Activities
ISO	International Standards Organization
IT	Information technology
IWGPS	Inter-Secretariat Working Group on Price Statistics
KPI	Fixed capital formation price index
LIFO	Last in, first out
LKAU	Local kind of activity unit
LPG	Liquefied propane gas
MHz	Megahertz
MPI	Import price index
MSA	Metropolitan Statistical Area
NACE	General Industrial Classification of Economic Activities within the European Communities
NAFTA	North American Free Trade Association
NAICS	North American Industrial Classification System
1993 SNA	Commission of the European Communities (Eurostat), International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations, and World Bank, 1993, *System of National Accounts 1993* (Brussels/Luxembourg, New York, Paris, and Washington)
NPI	Inventory price index
NPISH	Nonprofit institution serving households
OECD	Organisation for Economic Co-operation and Development
OLS	Ordinary least squares
Ottawa Group	International Working Group on Price Indices

P_C	Carli price index
P_{CSWD}	Carruthers, Sellwood, Ward, and Dalén price index
P_D	Dutot price index
P_{DR}	Drobisch index
P_F	Fisher price index
P_{GL}	Geometric Laspeyres price index
P_{GP}	Geometric Paasche price index
P_H	Harmonic average of price relatives
P_{IT}	Implicit Törnqvist price index
P_J	Jevons price index
P_{JW}	Geometric Laspeyres price index (weighted Jevons index)
P_{KB}	Konüs and Byushgens price index
P_L	Laspeyres price index
P_{LM}	Lloyd-Moulton price index
P_{Lo}	Lowe price index
P_{ME}	Marshall-Edgeworth price index
P_P	Paasche price index
P_{RH}	Ratio of harmonic mean prices
P_T	Törnqvist price index
P_W	Walsh price index
P_Y	Young price index
PC	Personal computer
PCE	Personal consumption expenditures
PCS	Personal communications service
PDA	Personal digital assistant
PMC	Profit-maximizing center
PPI	Producer price index
PPP	Purchasing power parity
PPS	Probability proportional to size
PR	Price relative
PRODCOM	Product/commodity classification system for the European Community
RAM	Random-access memory
R_H	Ratio of harmonic average prices
RMSE	Root mean square error
ROSC	Reports on the Observance of Standards and Codes
rpm	Revolutions per minute
RSA	Residential Service Area
SAF	Seasonal adjustment factors
SDDS	Special Data Dissemination Standard (IMF)
SEHI	Superlative and exact hedonic indices
SIC	Standard Industrial Classification
SITC	Standard International Trade Classification
SMI	Supply markup index
SNA	*System of National Accounts*
SPI	Supply price index
SSR	Structured Schedule Review
SUT	Supply and use table
TEG-PPI	Technical Expert Group for the Producer Price Index

UN	United Nations
UNECE	UN Economic Commission for Europe
VAT	Value-added tax
Voorburg Group	International Working Group on Service Sector Statistics
VPI	Valuables price index
WD	Wheel drive
WLS	Weighted least squares
WPI	Wholesale price index
XPI	Export price index
YPI	Output price index

PART I

Methods, Uses, and Coverage

1. An Introduction to PPI Methodology

1.1 A price index is a measure of the proportionate, or percentage, changes in a set of prices over time. PPIs measure changes in the prices of domestic producer goods and services. Such measures need to distinguish between changes in the volume of domestic production and such changes in nominal terms. Because the prices of different goods and services do not all change at the same rate, a price index can reflect only their average movement. A price index typically assumes a value of unity, or 100, in some base period. The values of the index for other periods of time show the average proportionate, or percentage, change in prices from the base period. Price indices can also measure differences in price levels between different cities, regions, or countries at the same point of time.

1.2 Two basic questions are the focus of this *Manual* and the associated economic literature on price indices:

- Exactly what set of prices should be covered by the index?
- What is the most suitable way in which to average their movements?

1.3 The answer to the first question depends largely on the purposes for which the index is to be used. Separate price indices can be compiled for different flows of goods and services, such as household production, government production, investment, or foreign trade flows. *Output* PPIs, which measure changes in the prices of goods and services produced by *businesses*, are the primary concern of this *Manual*. However, businesses do not all sell the same set of goods and services. Thus, there can be more than one output PPI depending on the particular set of goods and services selected. As well as considering the problems involved in measuring output prices, this *Manual* will also consider the problems associated with constructing *input* PPIs, used for deflating the value of intermediate inputs used in production. An intermediate input is an input that is used by one establishment or production unit but is the output of another establishment. Of interest to economists is deflating changes in value added over time, and weighted averages of the differences between output and intermediate input price indices, *value-added* PPIs, may ideally serve this purpose.

1.4 Once the appropriate set of prices (and, if weights are available, related quantities and revenue information) are collected, the second question concerns the choice of formula to average the price movements. Two standard methods are available to measure sectoral and overall price changes over time: compile an average of price changes or compile a ratio of average prices. This is summarized below and considered in detail in Chapters 15–20.

1.5 This chapter provides a general introduction to, and review of, the methods of PPI compilation. It provides a summary of the relevant theory and practice of index number compilation that helps reading and understanding the detailed chapters that follow, some of which are inevitably quite technical. The chapter describes the various steps involved in PPI compilation, starting with the basic concepts, definitions, and purposes of PPIs. It then discusses the sampling procedures and survey methods used to collect and process the price data, and finishes with the eventual calculation and dissemination of the final index.

1.6 In an introductory presentation of PPI methods of the kind given in this chapter, it is necessary to start with the basic concept of a PPI and the underlying index number theory. This includes the properties and behavior of the various kinds of index numbers that might be used for PPI purposes. Only after deciding the type of index and its coverage based on these theoretical considerations is it possible to go on to determine the best way in which to estimate the index in practice, taking account of the resources available. As noted in the Reader's Guide, however, the detailed presentation of the relevant index theory appears in later chapters of the *Manual* because the theory can become technically complex when pursued in some depth.

The exposition in this chapter does not therefore follow the same order as the chapters in the *Manual*.

1.7 The main topics covered in this chapter are as follows:

- The uses and origins of PPIs;
- Basic index number theory, including the axiomatic and economic approaches to PPIs;
- Elementary price indices and aggregate PPIs;
- The transactions, activities, and establishments covered by PPIs;
- The collection and processing of the prices, including adjusting for quality change;
- The actual calculation of the PPI;
- Potential errors and bias;
- Organization, management, and dissemination policy; and
- An appendix providing an overview of the steps necessary for developing a PPI.

1.8 Not all of the topics treated in the *Manual* are included in this chapter. The objective of this general introduction is to provide a summary presentation of the core issues with which readers need to be acquainted before tackling the detailed chapters that follow. It is not the purpose of this introduction to provide a comprehensive summary of the entire contents of the *Manual*. Some special topics, such as the treatment of certain products whose prices cannot be directly observed, are not considered here because they do not affect general PPI methodology.

A. The Uses and Origins of PPIs

1.9 Four of the principal price indices in the system of economic statistics—the PPI, the CPI, and the export and import price indices—are well known and closely watched indicators of macroeconomic performance. They are direct indicators of the purchasing power of money in various types of transactions and other flows involving goods and services. As such, they are also used to deflate nominal measures of goods and services produced, consumed, and traded to provide measures of volumes. Consequently, these indices are important tools in the design and conduct of the monetary and fiscal policy of the government, but they are also of great utility in informing economic decisions throughout the private sector. They do not, or should not, comprise merely a collection of unrelated price indicators but provide instead an integrated and consistent view of price developments pertaining to production, consumption, and international transactions in goods and services.

1.10 In the system of price statistics, PPIs serve multiple purposes. The precise way in which they are defined and constructed can very much depend on by whom and for what they are meant to be used. PPIs can be described as indices designed to measure the average change in the price of goods and services either as they leave the place of production or as they enter the production process. A monthly or quarterly PPI with detailed product and industry data allows monitoring of short-term price inflation for different types or through different stages of production. Although PPIs are an important economic indicator in their own right, a vital use of PPIs is as a deflator of nominal values of output or intermediate consumption for the compilation of production volumes and for the deflation of nominal values of capital expenditure and inventory data for use in the preparation of national accounts.[1]

1.11 Beyond their use as inflationary indicators or as deflators, certain frameworks for PPIs provide insight into the interlinkages between different price measures. One such framework is aggregation of stage-of-processing indices. This concept classifies goods and services according to their position in the chain of production—that is, primary products, intermediate goods, and finished goods. This method allows analysts to track price inflation through the economy. For example, changes in prices in the primary stage could feed through into the later stages, so the method gives an indicator of future inflation further down the production chain. However, each product is allocated to only one stage in the production chain even though it could occur in several stages. This topic will be considered in Chapter 2 and again in Chapter 14.

1.12 A further method for analysis is to aggregate by stage of production, in which each product is allocated to the stage in which it is used. This differs from stage of processing because a product is included in each stage to which it contributes and is not assigned solely to one stage. The classification of products to the different stages is usually

[1]PPIs are used for this purpose because the volumes underlying the nominal values are not directly measurable.

achieved by reference to input-output tables, and, in order to avoid multiple counting, the stages are not aggregated. There is a growing interest in this type of analysis. For example, these types of indices are already compiled on a regular basis in Australia.[2] This topic will also be considered in Chapters 2 and 14.

1.13 As explained in Chapter 2, PPIs have their beginnings in the development of the wholesale price index (WPI) dating back to the late 19th century. Laspeyres and Paasche indices, which are still widely used today, were first proposed in the mid-19th century. They are explained below. The concepts of the fixed-input *output* price index and the fixed-output *input* price index were introduced in the mid- to late 20th century. These two concepts provide the basic framework for the economic theory of the PPI presented in Chapters 15 and 17.

1.14 Initially, one of the main reasons for compiling a WPI was to measure price changes for goods sold in primary markets before they reached the final stage of production at the retail market level. Thus the WPI was intended to be a general purpose index to measure the price level in markets other than retail. The WPI has been replaced in most countries by PPIs because of the broader coverage provided by the PPI in terms of products and industries and the conceptual concordance between the PPI and the *System of National Accounts*, discussed in more detail in Chapter 14. It is this concordance that makes components of the PPI useful as deflators for industrial outputs and product inputs in the national accounts. In addition, the overall PPI and PPIs for specific products are used to adjust prices of inputs in long-term purchase and sales contracts, a procedure known as "escalation."

1.15 These varied uses often increase the demand for PPI data. For example, using the PPI as an indicator of general inflation creates pressure to extend its coverage to include more industries and products. While many countries initially develop a PPI to cover industrial goods produced in mining and manufacturing industries, the PPI can logically be extended to cover all economic activities, as noted in Chapters 2 and 14.

B. Some Basic Index Number Formulas

1.16 The first question is to decide on the kind of index number to use. The extensive list of references given at the end of this *Manual* reflects the large literature on this subject. Many different mathematical formulas have been proposed over the past two centuries. Nevertheless, there is now a broad consensus among economists and compilers of PPIs about what is the most appropriate type of formula to use, at least in principle. While the consensus has not settled for a single formula, it has narrowed to a very small class of *superlative* indices. A characteristic feature of these indices is that they treat the prices and quantities in both periods being compared symmetrically. They tend to yield very similar results and behave in very similar ways.

1.17 However, when a monthly or quarterly PPI is first published, it is invariably the case that there is not sufficient information on the quantities and revenues in the current period to make it possible to calculate a symmetric, or superlative, index. It is necessary to resort to second-best alternatives in practice, but in order to be able to make a rational choice between the various possibilities, it is necessary to have a clear idea of the target index that would be preferred, in principle. The target index can have a considerable influence on practical matters such as the frequency with which the weights used in the index should be updated.

1.18 The *Manual* provides a comprehensive, thorough, rigorous, and up-to-date discussion of relevant index number theory. Several chapters from Chapter 15 onward are devoted to a detailed explanation of index number theory from both a statistical and an economic perspective. The main points are summarized in the following sections. Many propositions or theorems are stated without proof in this chapter because the proofs are given or referenced in later chapters to which the reader can easily refer in order to obtain full explanations and a deeper understanding of the points made. There are numerous cross-references to the relevant sections in later chapters.

[2]See, for example, Australian Bureau of Statistics (ABS) (2003 and other years); available via the Internet: www.abs.gov.au.

B.1 Price indices based on baskets of goods and services

1.19 The purpose of an index number may be explained by comparing the *values* of producer's revenues from the production of goods and services in two time periods. Knowing that revenues have increased by 5 percent is not very informative if we do not know how much of this change is due to changes in the *prices* of the goods and services and how much to changes in the *quantities* produced. *The purpose of an index number is to decompose proportionate or percentage changes in value aggregates into their overall price and quantity change components.* A PPI is intended to measure the price component of the change in producer's revenues. One way to do this is to measure the change in the value of an aggregate by holding the quantities constant.

B.1.1 Lowe indices

1.20 One very wide, and popular, class of price indices is obtained by defining the index as the percentage change between the periods compared in the total cost of producing a fixed set of quantities, generally described as a "basket." The meaning of such an index is easy to grasp and to explain to users. This class of index is called a *Lowe* index in this *Manual*, after the index number pioneer who first proposed it in 1823: see Section B.2 of Chapter 15. Most statistical offices make use of some kind of Lowe index in practice. It is described in some detail in Sections D.1 and D.2 of Chapter 15.

1.21 In principle, any set of goods and services could serve as the basket. The basket does *not* have to be restricted to the basket actually produced in one or other of the two periods compared. For practical reasons, the basket of quantities used for PPI purposes usually has to be based on a survey of establishment revenues conducted in an earlier period than either of the two periods whose prices are compared. For example, a monthly PPI may run from January 2000 onward, with January 2000 = 100 as its price reference period, but the quantities may be derived from an annual revenue survey made in 1997 or 1998, or even spanning both years. Because it takes a long time to collect and process revenue data, there is usually a considerable time lag before such data can be introduced into the calculation of PPIs. The basket may also refer to a year, whereas the index may be compiled monthly or quarterly.

1.22 Let there be n products in the basket with prices p_i and quantities q_i. Let period b be the period to which the quantities refer and periods 0 and t be the two periods whose prices are being compared. In practice, it is invariably the case that $b \leq 0 < t$ when the index is first published, and this is assumed here. However, b could be any period, including one between 0 and t, if the index is calculated some time after t. The Lowe index is defined in equation (1.1).

$$(1.1)\ P_{Lo} \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \equiv \sum_{i=1}^{n} \left(p_i^t / p_i^0 \right) s_i^{0b} ,$$

$$\text{where} \quad s_i^{0b} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} .$$

The Lowe index can be written, and calculated, in two ways: either as the ratio of two value aggregates, or as an arithmetic weighted average of the price ratios, or *price relatives*, p_i^t / p_i^0, for the individual products using the hybrid revenue shares s_i^{0b} as weights. They are described as *hybrid* because the prices and quantities belong to two different time periods, 0 and b, respectively. The hybrid weights may be obtained by updating the actual revenue shares in period b, namely $p_i^b q_i^b / \sum p_i^b q_i^b$, for the price changes occurring between periods b and 0 by multiplying them by the price relative between b and 0, namely p_i^0 / p_i^b. The concept of the *base period* is somewhat ambiguous with a Lowe index, since either b or 0 might be interpreted as being the base period. To avoid ambiguity, b is described as the *weight reference period* and 0 as the *price reference period.*

1.23 Lowe indices are widely used for PPI purposes.

B.1.2 Laspeyres and Paasche indices

1.24 Any set of quantities could be used in a Lowe index, but there are two special cases that figure prominently in the literature and are of considerable importance from a theoretical point of view. When the quantities are those of the first of the two periods whose prices are being compared—

that is, when b = 0—the *Laspeyres* index is obtained, and when quantities are those of the second period—that is, when $b = t$,—the *Paasche* index is obtained. It is necessary to consider the properties of Laspeyres and Paasche indices, and also the relationships between them, in more detail.

1.25 The formula for the Laspeyres price index, P_L, is given in equation (1.2).

$$(1.2)\ P_L = \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \equiv \sum_{i=1}^{n} \left(p_i^t / p_i^0 \right) s_i^0 \ ,$$

where s_i^0 denotes the share of the value of product i in the total output of goods and services in period 0: that is, $p_i^0 q_i^0 / \sum p_i^0 q_i^0$.

1.26 As can be seen from equation (1.2), and as explained in more detail in Chapter 15, the Laspeyres index can be expressed in two alternative ways that are algebraically identical: first, as the ratio of the values of the basket of producer goods and services produced in period 0 when valued at the prices of periods t and 0, respectively; second, as a weighted arithmetic average of the ratios of the individual prices in periods t and 0 using the value shares in period 0 as weights. The individual price ratios, (p_i^t/p_i^0), are described as price *relatives*. Statistical offices often calculate PPIs using the second formula by recording the percentage changes in the prices of producer goods and services sold and weighting them by the total value of output in the base period 0.

1.27 The formula for the Paasche index, P_P, is given in equation (1.3).

$$(1.3)\ P_P = \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} \equiv \left\{ \sum_{i=1}^{n} \left(p_i^t / p_i^0 \right)^{-1} s_i^t \right\}^{-1} ,$$

where s_i^t denotes the actual share of the expenditure on commodity i in period t: that is, $p_i^t q_i^t / \sum p_i^t q_i^t$. The Paasche index can also be expressed in two alternative ways, either as the ratio of two value aggregates or as a weighted average of the price relatives, the average being a *harmonic* average that uses the revenue shares of the later period t as weights. However, it follows from equation (1.1) that the Paasche index can also be expressed as a weighted arithmetic average of the price relatives using hybrid expenditure weights in which the quantities of t are valued at the prices of 0.

1.28 If the objective is simply to measure the price change between the two periods considered in isolation, there is no reason to prefer the basket of the earlier period to that of the later period, or vice versa. Both baskets are equally relevant. Both indices are equally justifiable, or acceptable, from a conceptual point of view. In practice, however, PPIs are calculated for a succession of time periods. A time series of monthly Laspeyres PPIs based on period 0 benefits from requiring only a single set of quantities (or revenues), those of period 0, so that *only the prices* have to be collected on a regular monthly basis. A time series of Paasche PPIs, on the other hand, requires data on *both* prices *and* quantities (or revenues) in each successive period. Thus, it is much less costly, and time consuming, to calculate a time series of Laspeyres indices than a time series of Paasche indices. This is a *decisive practical* advantage of Laspeyres (as well as Lowe) indices over Paasche indices and explains why Laspeyres and Lowe indices are used much more extensively than Paasche indices. A monthly Laspeyres or Lowe PPI can be published as soon as the price information has been collected and processed, since the base-period weights are already available.

B.1.3 Decomposing current-value changes using Laspeyres and Paasche indices

1.29 Laspeyres and Paasche quantity indices are defined in a similar way to the price indices, simply by interchanging the ps and qs in formulas (1.2) and (1.3). They summarize changes over time in the flow of quantities of goods and services produced. A Laspeyres quantity index values the quantities at the fixed prices of the earlier period, while the Paasche quantity index uses the prices of the later period. The ratio of the values of the revenues in two periods (V) reflects the combined effects of both price and quantity changes. When Laspeyres and Paasche indices are used, the value change can be exactly decomposed into a price index times a quantity index only if the Laspeyres price (quantity) index is matched with the Paasche quantity (price) index. Let P_L and Q_L denote the Laspeyres price and quantity indices and let P_P and Q_P denote the Paasche price and quantity indices. As shown in Chapter 15, $P_L \times Q_P \equiv V$ and $P_P \times Q_L \equiv V$.

1.30 Suppose, for example, a time series of industry output in the national accounts is to be deflated to measure changes in output at constant prices over time. If it is desired to generate a series of output values at constant base-period prices (whose movements are identical with those of the Laspeyres volume index), the output at current prices must be deflated by a series of Paasche price indices. Laspeyres-type PPIs would not be appropriate for the purpose.

B.1.4 Ratios of Lowe and Laspeyres indices

1.31 The Lowe index is transitive. The ratio of two Lowe indices using the same set of q^bs is also a Lowe index. For example, the ratio of the Lowe index for period t + 1 with price reference period 0 divided by that for period t also with price reference period 0 is:

$$(1.4)\quad \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b \Big/ \sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} = P_{Lo}^{t,t+1} .$$

1.32 This is a Lowe index for period t + 1, with period t as the price reference period. This kind of index is, in fact, widely used to measure short-term price movements, such as between t and t + 1, even though the quantities may date back to some much earlier period b.

1.33 A Lowe index can also be expressed as the ratio of two Laspeyres indices. For example, the Lowe index for period t with price reference period 0 is equal to the Laspeyres index for period t with price reference period b divided by the Laspeyres index for period 0 also with price reference period b. Thus,

$$(1.5)\quad P_{Lo} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b} = \frac{P_L^t}{P_L^0} .$$

B.1.5 Updated Lowe indices

1.34 It is useful to have a formula that enables a Lowe index to be calculated directly as a chain index in which the index for period t + 1 is obtained by updating the index for period t. Because Lowe indices are transitive, the Lowe index for period $t+1$ with price reference period 0 can be written as the product of the Lowe index for period t with price reference period 0 multiplied the Lowe index for period t + 1 with price reference period t. Thus,

$$(1.6)\quad \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}\right]\left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}\right] = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}\right]\left[\sum_{i=1}^{n}\left(\frac{p_i^{t+1}}{p_i^t}\right) s_i^{tb}\right],$$

where the revenue weights s_i^{tb} are hybrid weights defined as:

$$(1.7)\quad s_i^{tb} \equiv p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^t q_i^b .$$

1.35 Hybrid weights of the kind defined in equation (1.7) are often described as *price-updated* weights. They can be obtained by adjusting the original revenue weights $p_i^b q_i^b / \sum p_i^b q_i^b$ by the price relatives p_i^t / p_i^b. By price updating the revenue weights from b to t in this way, the index between t and t + 1 can be calculated directly as a weighted average of the price relatives p_i^{t+1} / p_i^t without referring back to the price reference period 0. The index can then be linked on to the value of the index in the preceding period t.

B.1.6 Relationships between fixed-basket indices

1.36 Consider first the interrelationship between the Laspeyres and the Paasche indices. A well-known result in index number theory is that if the price and quantity changes (weighted by values) are *negatively* correlated, then the Laspeyres index exceeds the Paasche. Conversely, if the weighted price and quantity changes are *positively* correlated, then the Paasche index exceeds the Laspeyres. The proof is given in Appendix 15.1 of Chapter 15.

1.37 This has different implication for consumers and producers. The theory of consumer behavior indicates that consumers typically react to price changes by substituting goods or services that have become *relatively* cheaper for those that have become *relatively* dearer. Thus they purchase smaller quantities of the higher-priced products and more of lower-priced ones. This is known as the *substitution effect*, and it implies a negative correlation between the price and quantity relatives. In this case the Laspeyres CPI would be greater than the Paasche CPI, with the gap between them tending to widen over time.[3] That the Laspeyres tends to rise faster than the Paasche is a matter of concern to many analysts and CPI users because it suggests that the widely used Laspeyres index may have an upward bias.

1.38 The theory of the firm indicates the opposite behavior on the part of producers. As prices for particular products begin to rise, producers will shift production away from lower-priced, less profitable products toward the higher-priced, more profitable ones. This type of substitution by producers implies a positive correlation between price and quantity relatives. In this case the Paasche PPI would be greater than the Laspeyres PPI, with the gap between them widening over time. That the Paasche tends to rise faster than the Laspeyres is a matter of concern to many analysts and PPI users because it suggests that the widely used Laspeyres index may have a downward bias, a point taken up later.

1.39 In practice, however, statistical offices often do not calculate Laspeyres or Paasche indices but instead calculate Lowe indices as defined in equation (1.1). The question then arises of how the Lowe index relates to the Laspeyres and Paasche indices. It is shown in Section D.1 of Chapter 15 that *if there are persistent long-term trends in relative prices and if the substitution effect for purchasers is dominant, the Lowe index will tend to exceed the Laspeyres, and therefore also the Fisher and the Paasche.* Assuming that the time period b is prior to the time period 0, the ranking under these conditions will be:

Lowe $\geq$ Laspeyres $\geq$ Fisher $\geq$ Paasche.

Moreover, the amount by which the Lowe exceeds the other three indices will tend to increase, the further back in time period b is in relation to period *0*.

1.40 The positioning of period b is crucial. Given the assumptions about long-term price trends and substitution, a Lowe index will tend to increase (decrease) as period b is moved backward (forward) in time. While b may have to precede 0 when the index is first published, there is no such restriction on the positioning of b as price and quantity data become available for later periods with the passage of time. Period b can then be moved forward. If b is positioned midway between 0 and t, the quantities are likely to be equirepresentative of both periods, assuming that there is a fairly smooth transition from the relative quantities of 0 to those of t. In these circumstances, the Lowe index is likely to be close to the Fisher and other superlative indices and cannot be presumed to have either an upward or a downward bias. These points are elaborated further below and also in Section D.2 of Chapter 15.

1.41 It is important that statistical offices take these relationships into consideration in deciding upon their policies. There are obviously practical advantages and financial savings from continuing to make repeated use over many years of the same fixed set of quantities to calculate a PPI. However, the amount by which such a PPI exceeds some conceptually preferred target index, such as the economic index discussed in Section E below, is likely to get steadily larger the further back in time the period b to which the quantities refer. Most users are likely to interpret the difference as an upward bias.[4] A large bias may undermine the credibility and acceptability of the index.

[3]If the revenue shares—that is, the weights associated with the price relatives—happen to be the same in both periods, the Laspeyres must be greater than the Paasche because a weighted arithmetic average is always greater than a harmonic average with the same weights. In order to maintain the revenue shares intact, the substitution of the quantities in response to changes in relative prices must be perfect.

[4]Of course, if producers are price takers from the market and the demand shifts dominate, then producers will respond by increasing the quantities produced of goods with higher relative prices. The correlation between prices and quantities in this instance will be positive, and the relationship among the indices will be:

Paasche $\geq$ Fisher $\geq$ Laspeyres $\geq$ Lowe,

and the bias interpreted as downward.

B.2 The Young index

1.42 Instead of holding constant the quantities of period b, a statistical office may calculate a PPI as a weighted arithmetic average of the individual price relatives, holding constant the revenue shares of period b. The resulting index is called a *Young* index in this *Manual*, again after an another index number pioneer. The Young index is defined in Section D.3 of Chapter 15 as follows:

$$(1.8)\ P_{Yo} = \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0} \right), \quad \text{where } s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}.$$

1.43 In the corresponding Lowe index, equation (1.1), the weights are hybrid revenue shares that value the quantities of b at the prices of 0. As already explained, the price reference period 0 usually is more current than the weight reference period b because of the time needed to collect and process and the revenue data. In that case, a statistical office has the choice of assuming that *either* the quantities of period b remain constant *or* the revenue shares in period b remain constant. Both cannot remain constant if prices change between b and 0. If the revenue shares actually remained constant between periods b and 0, the quantities would have had to change inversely in response to the price changes. In this case the elasticity of substitution is 1; for example, the proportionate decline in quantity is equal to the proportionate increase in prices.

1.44 Section D.3 of Chapter 15 shows that the Young index is equal to the Laspeyres index plus the covariance between the difference in annual shares pertaining to year b and month 0 shares $\left(s_i^b - s_i^0\right)$ and the deviations in relative prices from their means $(r - r_i^*)$. Normally, the weight reference period b precedes the price reference period 0. In this case, if the elasticity of substitution is larger than 1—for example, the proportionate decline in quantity is greater than the proportionate increase in prices—the covariance will be positive. Under these circumstances the Young index will exceed the Laspeyres index.[5] Alternatively, if the elasticity of substitution is less than 1, the covariance will be negative and the Young will be less than the Laspeyres.

1.45 As explained later, the Young index fails some critical index number tests discussed in Section C of this chapter and in Chapter 16, Section C.

B.2.1 Geometric Young, Laspeyres, and Paasche indices

1.46 In the geometric version of the Young index, a weighted geometric average is taken of the price relatives using the revenue shares of period b as weights. It is defined as:

$$(1.9)\ P_{GYo} \equiv \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b},$$

where s_i^b is defined as above. The geometric Laspeyres is the special case in which $b = 0$: that is, the revenue shares are those of the price reference period 0. Similarly, the geometric Paasche uses the revenue shares of period t. Note that these geometric indices cannot be expressed as the ratios of value aggregates in which the quantities are fixed. They are not basket indices, and there are no counterpart Lowe indices.

1.47 It is worth recalling that for any set of positive numbers the arithmetic average is greater than, or equal to, the geometric average, which in turn is greater than, or equal to, the harmonic average, the equalities holding only when the numbers are all equal. In the case of unitary cross-elasticities of demand and constant revenue shares, the geometric Laspeyres and Paasche indices coincide. In this case, the ranking of the indices must be:

> ordinary Laspeyres ≥ geometric Laspeyres and Paasche ≥ the ordinary Paasche.

1.48 The indices are, respectively, arithmetic, geometric, and harmonic averages of the same price relatives that all use the same set of weights.

1.49 The geometric Young and Laspeyres indices have the same information requirements as their

[5]This occurs because products with the large relative price increases ($r - r_i^*$ is positive) would also experience declining shares between periods b and 0 ($s_i^b - s_i^0$ is positive), thus having a positive influence on the covariance. In addition, products with small relative price increases ($r - r_i^*$ is negative) would experience increasing shares between b and 0 ($s_i^b - s_i^0$ is negative), thus having a positive influence on the covariance.

ordinary arithmetic counterparts. They can be produced on a timely basis. Thus, these geometric indices must be treated as serious practical possibilities for purposes of PPI calculations. As explained later, the geometric indices are likely to be less subject than their arithmetic counterparts to the kinds of index number biases discussed in later sections. Their main disadvantage may be that, because they are not fixed-basket indices, they are not so easy to explain or justify to users.

B.3 Symmetric indices

1.50 When the base and current periods are far apart, the index number spread between the numerical values of a Laspeyres and a Paasche price index is liable to be quite large, especially if *relative* prices have changed a lot (as shown Appendix 15.1 and illustrated numerically in Chapter 19). Index number spread is a matter of concern to users because, conceptually, there is no good reason to prefer the weights of one period to those of the other. In these circumstances, it seems reasonable to take some kind of *symmetric average* of the two indices. More generally, it seems intuitive to prefer indices that treat both of the periods symmetrically instead of relying exclusively on the weights of only one of the periods. It will be shown later that this intuition can be backed up by theoretical arguments. There are many possible symmetric indices, but there are three in particular that command much support and are widely used.

1.51 The first is the *Fisher price index, P_F*, defined as the *geometric* average of the Laspeyres and Paasche indices; that is,

$$(1.10)\quad P_F \equiv \sqrt{P_L \times P_P}\ .$$

1.52 The second is the *Walsh price index, P_W*, a pure price index in which the quantity weights are *geometric* averages of the quantities in the two periods; that is

$$(1.11)\quad P_W \equiv \frac{\sum_{i=1}^{n} p_i^t \sqrt{q_i^t q_i^0}}{\sum_{i=1}^{n} p_i^0 \sqrt{q_i^t q_i^0}}\ .$$

The averages of the quantities need to be *geometric* rather than arithmetic for the *relative* quantities in both periods to be given equal weight.

1.53 The third index is the *Törnqvist price index, P_T*, defined as a *geometric* average of the price relatives weighted by the average *revenue* shares in the two periods:

$$(1.12)\quad P_T = \prod_{i=1}^{n} \left(p_i^t / p_i^0\right)^{\sigma_i}\ ,$$

where σ_i is the arithmetic average of the share of revenue on product *i* in the two periods, and

$$(1.13)\quad \sigma_i = \frac{s_i^t + s_i^0}{2}\ ,$$

where the s_i s are defined as in equation (1.2) and above.

1.54 The theoretical attractions of these indices become apparent in the following sections on the axiomatic and economic approaches to index numbers.

B.4 Fixed-base versus chain indices

B.4.1 Fixed-basket indices

1.55 This topic is examined in Section F of Chapter 15. When a time series of Lowe or Laspeyres indices is calculated using a fixed set of quantities, the quantities become progressively out of date and increasingly irrelevant to the later periods whose prices are being compared. The base period whose quantities are used has to be updated sooner or later, and the new index series linked to the old. Linking is inevitable in the long run.

1.56 In a chain index, each link consists of an index in which each period is compared with the preceding one, the weight and price reference periods being moved forward each period. Any index number formula can be used for the individual links in a chain index. For example, it is possible to have a chain index in which the index for $t + 1$ on t is a Lowe index defined as $\sum p^{t+1}q^{t-j} / \sum p^t q^{t-j}$. The quantities refer to some period that is j periods earlier than the price reference period *t.* The quantities move forward one period as the price reference period moves forward one period. If $j = 0$, the chain Lowe becomes a chain Laspeyres, while if $j = -1$, [that is, $t - (-1) = t + 1$], it becomes a chain Paasche.

1.57 The PPIs in some countries are, in fact, annual chain Lowe indices of this general type, the quantities referring to some year, or years, that precedes the price reference period 0 by a fixed period. For example,

- The 12 monthly indices from January 2000 to January 2001, with January 2000 as the price reference period could be Lowe indices based on price updated revenues for 1998;
- The 12 indices from January 2001 to January 2002 are then based on price updated revenues for 1999; and so on with annual weight updates.

The revenues lag behind the January price reference period by a fixed interval, moving forward a year each January as the price reference period moves forward one year. Although, for practical reasons, there has to be a time lag between the quantities and the prices when the index is first published, it is possible to recalculate the monthly indices for the current later, using current revenue data when they eventually become available. In this way, it is possible for the long-run index to be an annually chained monthly index with contemporaneous annual weights. This method is explained in more detail in Chapter 9. It is used by one statistical office.

1.58 A chain index between two periods has to be "path dependent." It must depend on the prices and quantities in all the intervening periods between the first and last periods in the index series. Path dependency can be advantageous or disadvantageous. When there is a gradual economic transition from the first to the last period with smooth trends in relative prices and quantities, chaining will tend to reduce the index number spreads among the Lowe, Laspeyres, and Paasche indices, thereby making the movements in the index less dependent on the choice of index number formula.

1.59 However, if there are fluctuations in the prices and quantities in the intervening periods, chaining may not only increase the index number spread but also distort the measure of the overall change between the first and last periods. For example, suppose all the prices in the last period return to their initial levels in period 0, which implies that they must have fluctuated in between, a chain Laspeyres index does not return to 100. It will be greater than 100. If the cycle is repeated, with all the prices periodically returning to their original levels, a chain Laspeyres index will tend to "drift" further and further above 100 even though there may be no long-term upward trend in the prices. Chaining is therefore not advised when the prices fluctuate. When monthly prices are subject to regular and substantial seasonal fluctuations, for example, monthly chaining cannot be recommended. Seasonal fluctuations cause serious problems, which are analyzed in Chapter 22. While a number of countries update their revenue weights annually, the 12 monthly indices within each year are not chain indices but Lowe indices using fixed annual quantities.

B.4.2 The Divisia index

1.60 If the prices and quantities are *continuous functions of time,* it is possible to partition the change in their total value over time into price and quantity components following the method pioneered by Divisia. As shown in Section E of Chapter 15, the Divisia index may be derived mathematically by differentiating value (that is, price times quantity) with respect to time to obtain two components: a relative value-weighted price change and relative value-weighted quantity change. These two components are defined to be price and quantity indices, respectively. The Divisia index is essentially a theoretical index. In practice, prices can be recorded only at discrete intervals even if they vary continuously with time. A chain index may, however, be regarded as a discrete approximation to a Divisia index. The Divisia index itself offers no practical guidance about the kind of index number formula to choose for the individual links in a chain index.

C. The Axiomatic Approach to Index Numbers

1.61 The *axiomatic approach* to index numbers is explained in Chapter 16. It seeks to decide the most appropriate formula for an index by specifying a number of axioms, or tests, that the index ought to satisfy. It throws light on the properties possessed by different kinds of indices, some of which are by no means intuitively obvious. Indices that fail to satisfy certain basic or fundamental axioms may be rejected completely because they are liable to behave in unacceptable ways. The axiomatic approach is also used to rank indices on the basis of their desirable, and undesirable, properties.

1.62 Twenty axioms or tests (T) are initially considered in Chapter 16. Only a selection of them are given here by way of illustration.

- T1—*Positivity*: The price index and its constituent vectors of prices and quantities should be positive.

- T3—*Identity Test*: If the price of every product is identical in both periods, then the price index should equal unity, no matter what the quantity vectors are.

- T5—*Proportionality in Current Prices*: If all prices in period t are multiplied by the positive number λ, then the new price index should be λ times the old price index; that is, the price index function is (positively) homogeneous of degree 1 in the components of the period *t* price vector.

- T10—*Invariance to Changes in the Units of Measurement* (commensurability test): The price index does not change if the units in which the products are measured are changed.

- T11—*Time Reversal Test*: If all the data for the two periods are interchanged, then the resulting price index should equal the reciprocal of the original price index.

- T12—*Quantity Reversal Test*: If the quantity vectors for the two periods are interchanged, then the price index remains invariant.

- T14—*Mean Value Test for Prices*: The price index lies between the highest and the lowest price relatives.

- T16—*Paasche and Laspeyres Bounding Test*: The price index lies between the Laspeyres and Paasche indices.

- T17—*Monotonicity in Current Prices*: If any period *t* price is increased, then the price index must increase.

1.63 Some of the axioms or tests can be regarded as more important than others. Indeed, some of the axioms seem so inherently reasonable that it might be assumed that any index number actually in use would satisfy them. For example, test T10, the commensurability test, says that if milk is measured in liters instead of pints, the index must be unchanged. One index that does not satisfy this test is the ratio of the arithmetic means of the prices in the two periods (the *Dutot* index). This is a type of elementary index that is widely used in the early stages of PPI calculation. This is discussed in more detail in Chapter 20, Sections C and F.

1.64 Consider, for example, the average price of salt and pepper. Suppose it is decided to change the unit of measurement for pepper from grams to ounces while leaving the units in which salt is measured (for example, kilos) unchanged. Because an ounce is equal to 28.35 grams, the absolute value of the price of pepper increases by over 28 times, which effectively increases the weight of pepper in the Dutot index by over 28 times. When the products covered by an index are heterogeneous and measured in different physical units, the value of any index that does not satisfy the commensurability test depends on the purely arbitrary choice of units. Such an index must be unacceptable conceptually. However, when the prices refer to a strictly homogeneous set of products that all use the same unit of measurement, the test becomes irrelevant. In practice, products may differ in terms of their quality characteristics, and there is a sense in which this variation in quality is similar to variation in the units of measurement. While the quality of individual products may not change, the price changes of the higher-price varieties of, say, types of pepper, when aggregated, will be given more emphasis in the calculation.

1.65 Another important test is T11, the time reversal test. In principle, it seems reasonable to require that the same result should be obtained whichever of the two periods is chosen as the price reference period: in other words, whether the change is measured forward in time, from 0 to *t,* or backward in time, from *t* to 0. The Young index fails this test because an arithmetic average of a set of price relatives is not equal to the reciprocal of the arithmetic average of the reciprocals of the price relatives. This follows from the general algebraic result that the reciprocal of the arithmetic average of a set of numbers is the *harmonic* average of the reciprocals, not the arithmetic average of the reciprocals. The fact that the *conceptually* arbitrary decision to measure the change in prices forward from 0 and *t* gives a different result from measuring backward from *t* to 0 is seen by many users as a serious disadvantage. The failure of the Young index to satisfy the time reversal test needs to be taken into account by statistical offices.

1.66 Both Laspeyres and Paasche indices fail the time reversal test for the same reasons as the Young index. For example, the formula for a Laspeyres calculated backward from t to 0, P_{BL}, is:

$$(1.14)\ P_{BL} = \frac{\sum_{i=1}^{n} p_i^0 q_i^t}{\sum_{i=1}^{n} p_i^t q_i^t} \equiv 1/P_P\ .$$

This index is identical with the reciprocal of the (forward) Paasche, not with the reciprocal of the forward Laspeyres. As already noted, the (forward) Paasche tends to register a smaller increase than the (forward) Laspeyres, so that the Laspeyres index cannot satisfy the time reversal test. The Paasche index also fails the time reversal test.

1.67 On the other hand, the Lowe index satisfies the time reversal test *provided* that the quantities q_i^b remain fixed when the price reference period is changed from 0 to t. However, the quantities of a Laspeyres index are those of the price reference period, *by definition*, and *must change* whenever the price reference period is changed. The basket for a forward Laspeyres is different from that for the backward Laspeyres, and the Laspeyres fails the time reversal test as a consequence.

1.68 Similarly, the Lowe index is transitive whereas the Laspeyres and Paasche indices are not. Assuming that a Lowe index uses a fixed set of quantities, q_i^b, whatever the price reference period, it follows that

$$P_{Lo}^{0,\,t} = P_{Lo}^{0,\,t-k} \cdot P_{Lo}^{t-k,\,t}$$

where $P_{Lo}^{0,\,t}$ is the Lowe index for period t with period 0 as the price reference period. The Lowe index that compares t directly with 0 is the same as that calculated indirectly as a chain index through period $t–k$.

1.69 If, on the other hand, the Lowe index is defined in such a way that quantities vary with the price reference period, as in the index $\sum p^{t+1}q^{t-j}$ / $\sum p^t q^{t-j}$ considered earlier, the resulting chain index is not transitive. The chain Laspeyres and chain Paasche indices are special cases of this index.

1.70 In reality, quantities do change, and the whole point of chaining is to enable the *quantities* to be continually updated to take account of the changing universe of products. Achieving transitivity by arbitrarily holding the quantities constant, especially over a very long period of time, does not compensate for the potential biases introduced by using out-of-date quantities.

C.1 Ranking indices using the axiomatic approach

1.71 In Section B.6 of Chapter 16, it is shown not only that the Fisher price index satisfies all the 20 axioms initially listed in the chapter but also, more remarkably, that it is the *only* possible index that can satisfy all 20 axioms. Thus, on the basis of this set of axioms, the Fisher clearly dominates other indices.

1.72 In contrast to Fisher, the other two symmetric indices defined in equations (1.11) and (1.12) above do not emerge too well from the 20 tests. In Section B.7 of Chapter 16, it is shown that the Walsh price index fails four tests, whereas the Törnqvist index fails nine tests. Although the Törnqvist index does not perform well on these tests, especially compared with Fisher, it should be remembered that the Törnqvist index and Fisher index may, nevertheless, be expected to approximate each other quite closely when the data follow relatively smooth trends, as shown in Chapter 19.

1.73 The Lowe index with fixed quantities emerges quite well from the axiomatic approach. In particular, in contrast to the Laspeyres, Paasche, and Young indices, it satisfies the time reversal test. As already explained, however, the attractiveness of the Lowe index depends very much on the positioning of period b that supplies the quantity weights, rather than its axiomatic properties.

1.74 One limitation of the axiomatic approach is that the list of axioms itself is inevitably arbitrary to some extent. Some axioms, such as the Paasche and Laspeyres bounding test failed by both Törnqvist and Walsh, could be regarded as contrived and dispensable. In particular, many of the test properties have an arithmetic basis, whereas the Törnqvist index is a geometric average. Additional axioms or tests can be envisaged, and two further axioms are considered below. Another problem with a simple application of the axiomatic approach is that it is not sufficient to know which tests are failed. It is also necessary to know how badly an index fails. Badly failing one major test, such as the commensurability test, might be considered sufficient to rule out an index, whereas failing several minor tests marginally may not be very disadvantageous.

C.1.2 Some further tests

1.75 Consider a further symmetry test. It is reasonable that reversing the roles of prices and quantities in a price index should yield a quantity index of the same formula as the price index. A formula that is good enough for a price index should be equally good for a quantity index. The *factor reversal test* requires that the product of such a quantity index and the original price index should be identical with the change in the value of the aggregate in question. This test is important if, as stated at the outset of this chapter, price and quantity indices are intended to enable changes in the values of aggregates over time to be factored into their price and quantity components in an economically meaningful way. Another remarkable result derived from the axiomatic approach, and given in Section B.6 of Chapter 16, is that the Fisher index is the only price index to satisfy four minimal tests: T1 (positivity), T11 (time reversal), T12 (quantity reversal), and T21 (factor reversal).[6] Because the factor reversal test implicitly assumes that the prices and quantities must refer either to period 0 or to period *t*, it is not relevant to a Lowe index in which three periods are involved, *b*, 0, and *t.*

1.76 It was shown earlier that the product of the Laspeyres price (quantity) index and the Paasche quantity (price) index is identical with the change in the total value of the aggregate in question. Because Laspeyres and Paasche have different functional forms, this implies that they fail the factor reversal test. However, Laspeyres and Paasche indices may be said to satisfy a weak version of the factor reversal test in that dividing the value change by a Laspeyres or Paasche price index does lead to a meaningful quantity index, even though its formula is not identical with that of the price index.

1.77 Another test, discussed in Section C.8 of Chapter 16, is the *additivity test*. A good property for an index is that the changes in the subaggregates add up to the changes in the totals. This is more important from the perspective of quantity indices than it is for price indices. Price indices may be used to deflate value changes to obtain implicit quantity changes. The results may be presented for subaggregates such as output by industry or product groups. Just as output aggregates at current prices are, by definition, obtained simply by summing individual output values or revenues, it is reasonable to expect that the changes in the subaggregates of a quantity index should add up to the changes in the totals—the additivity test. Quantity indices that use a common set of prices to value quantities in both periods must satisfy the additivity test. Similarly, if the Lowe quantity index is defined as $\sum p^j q^t / \sum p^j q^0$ it is also additive. The Geary-Khamis quantity index used to make international comparisons of real consumption and GDP between countries is an example of such a Lowe quantity index. It uses an arithmetically weighted average of the prices in the different countries as the common price vector $\boldsymbol{p}^j$ to compare the quantities in different countries.

1.78 An alternative solution is to use some *average* of the prices in two periods to value the quantities. If the quantity index is also to satisfy the time reversal test, the average must be symmetrical. The *invariance to proportional changes in current prices test* (which corresponds to test T7 listed in Chapter 16 except that the roles of prices and quantities are reversed) requires that a quantity index depend only on the *relative,* not the absolute, level of the prices in each period. The Walsh quantity index satisfies this test, is additive, and satisfies the time reversal test as well. It emerges as a quantity index with some very desirable properties.[7]

1.79 Although the Fisher index itself is not additive, it is possible to decompose the overall *percentage change* in a Fisher price, or quantity, index into additive components that reflect the percentage change in each price or quantity. A similar multiplicative decomposition is possible for a Törnqvist price or quantity index.

[6]See Funke and Voeller (1978, p. 180).

[7]Additivity is a property that is attractive in a national accounts context, where many aggregates are actually defined by processes of addition and subtraction. It is also useful when comparing national accounts data for different countries using purchasing power parities (PPPs), a type of international price index. (See *CPI Manual*, International Labour Organization and others [2004, Annex 4].)

D. The Stochastic Approach

1.80 The stochastic approach treats the observed price relatives as if they were a random sample drawn from a defined universe whose mean can be interpreted as the general rate of inflation. However, there can be no single unique rate of inflation. There are many possible universes that can be defined, depending on which particular sets of industries, products, or transactions the user is interested in. Clearly, the sample mean depends on the choice of universe from which the sample is drawn. The stochastic approach does not help decide on the choice of universe. It addresses issues such as the appropriate form of average to take and the most efficient way to estimate it from a sample of price relatives, once the universe has been defined.

1.81 The stochastic approach becomes particularly useful when the universe is reduced to a single type of product. When there are market imperfections, there may be considerable variation within a country in the prices at which a single product is sold in different establishments and also in their movements over time. In practice, statistical offices have to estimate the average price change for a single product from a sample of price observations. Important methodological issues are raised, which are discussed in some detail in Chapter 5 on sampling issues and Chapter 20 on elementary indices. The main points are summarized in Section I below.

D.1 The unweighted stochastic approach

1.82 In Section C.2 of Chapter 16, the unweighted stochastic approach to index number theory is explained. If simple random sampling has been used to collect prices, equal weight may be given to each sampled price relative. Suppose each price relative can be treated as the sum of two components: a common inflation rate and a random disturbance with a zero mean. Using least-squares or maximum likelihood estimators, the best estimate of the common inflation rate is the unweighted *arithmetic* mean of price relatives, an index formula known as the *Carli* index. This index can be regarded as the unweighted version of the Young index. This index is discussed further in Section I below on elementary price indices.

1.83 If the random component is multiplicative, not additive, the best estimate of the common inflation rate is given by the unweighted *geometric* mean of price relatives, known as the *Jevons* index. The Jevons index may be preferred to the Carli on the grounds that it satisfies the time reversal test, whereas the Carli does not. As explained later, this fact may be decisive when deciding on the formula to be used to estimate the elementary indices compiled in the early stages of PPI calculations.

D.2 The weighted stochastic approach

1.84 As explained in Section F of Chapter 16, a *weighted* stochastic approach can be applied at an aggregative level covering sets of different products. Because the products may be of differing economic importance, equal weight should not be given to each type of product. The products may be weighted on the basis of their share in the total value of output, or other transactions, in some period or periods. In this case, the index (or its logarithm) is the expected value of a random sample of price relatives (or their logarithms), with the probability of any individual sampled product being selected being proportional to the output of that type of product in some period or periods. Different indices are obtained depending on which revenue weights are used and whether the price relatives or their logarithms are used.

1.85 Suppose a sample of price relatives is randomly selected, with the probability of selecting any particular type of product being proportional to the revenue of that type of product in period 0. The expected price change is then the Laspeyres price index for the universe. However, other indices may also be obtained using the weighted stochastic approach. Suppose both periods are treated symmetrically, and the probabilities of selection are made proportional to the arithmetic mean revenue shares in both periods 0 and t. When these weights are applied to the logarithms of the price relatives, the expected value of the logarithms is the Törnqvist index. From an axiomatic viewpoint, the choice of a symmetric average of the revenue shares ensures that the time reversal test is satisfied, while the choice of the arithmetic mean, as distinct from some other symmetric average, may be justified on the grounds that the fundamental proportionality in current prices test, T5, is thereby satisfied.

1.86 The examples of the Laspeyres and Törnqvist indices just given show that the stochastic approach in itself does not determine the form of the index number. There are several stochastic indices to choose from, just as there are many possible universes. However, as already noted, the elementary prices from which most aggregate price indices are constructed usually have to be based on samples of prices, and the stochastic approach may provide useful guidance on how best to estimate them.

E. The Economic Approach

1.87 The economic approach differs from the previous approaches in an important respect: quantities are no longer assumed to be independent of prices. If, for example, it is assumed that firms behave as revenue maximizers, it follows that they would produce more of products with above-average price changes in, say, period 1 compared with period 0. As a result, the revenue shares in period 1 from such products will increase, and therefore, their weights. This behavioral assumption about the firm, as it switches production to higher-priced products, allows something to be said about what "true" indices should be and the suitability of different index number formulas. For example, the Laspeyres index uses fixed period-0 revenue shares to weight its price relatives and ignores the substitution of production toward products with higher relative price changes in period 1. It will thus understate aggregate price changes—be biased downward against its true index. The Paasche index uses fixed period 1 weights and ignores the initial revenue shares in period 0. It will thus overstate aggregate price changes—be biased upward against its true index.

1.88 The economic approach can be seen to be very powerful, since it has identified a type of bias in Laspeyres and Paasche indices not apparent from other approaches: *substitution bias*. Laspeyres and Paasche indices ignore the change in weights as producers substitute their production toward products with above-average price increases. Yet the nature of the bias arises from an assumption about the behavior of producers—that they are revenue maximizers. Consider an alternative assumption: that producers respond to demand changes prompted by purchasers buying less of products with relatively high price changes. Products whose price increases are, for example, above average will see a falloff in demand leading to a falloff in production. In this case the revenue shares or weights of products with above-average price increases will fall in period 1, and the fixed period-0 weighted Laspeyres will overstate aggregate price changes—it will be upward biased. This compares with the Paasche index, which will understate aggregate price changes—it will be downward biased. It is shown in Chapter 17 that Laspeyres and Paasche indices can under certain conditions act as bounds on a more generally applicable "true" economic theoretic index. The axiomatic approach in Section C led to an index number formula that used an average of the Laspeyres and Paasche indices, and, even at this early stage in the discussion, the economic approach seems to provide further support.

1.89 The economic approach also identifies the circumstances under which the conventionally used Laspeyres index is appropriate. This would require that the firm does not change its production configuration in response to relative price changes, at least over the short term of the price index comparisons. Economic theory thus argues that the Laspeyres index may be appropriate for industries in which quantities are known not to respond to relative price changes over the period of the price comparisons. But it is more likely that this will be the exception rather than the norm, and the theory points to a requirement for a more generally applicable index number formula.

1.90 The PPI indices considered here include output, input, and value-added price indices (deflators), and different assumptions arise in their formulation from economic theory. In the output case, an assumption is made that firms act to maximize revenues, from a given input base. Firms substitute toward products with relatively high price increases. For the input price index, the concern is to minimize the costs of purchased intermediate goods. Firms substitute away from input products with relatively high price increases. For the value-added deflator, the unusual use of negative weights for the inputs is considered. The economic approach, as shown in Chapter 17, demonstrates that:

- A substitution bias can exist when using Laspeyres and Paasche formulas.
- The nature of the bias depends on the behavioral assumptions of the firm, which will vary between industries and the type of PPI index required—input or output PPI.
- Laspeyres and Paasche indices act as bounds on their true indices and, under certain conditions,

also are bounds for a more generally applicable true index.

It follows that some symmetric average of these bounds is justified from economic theory.

1.91 The approach from economic theory is thus first to develop theoretical index number formulas based on what are considered to be reasonable models of economic behavior by the producer. This approach is very different from the others considered here. A mathematical representation of the production activity—whereby capital and labor conjoin to turn intermediate inputs into outputs—is required. Also, an assumption of optimizing behavior (cost minimization or revenue maximization), along with other assumptions, is required so that a theoretical index can be derived that is "true" under these conditions. The economic approach then examines practical index number formulas such as Laspeyres, Fisher, and Törnqvist, and considers how they compare with "true" formulas defined under different assumptions. Three theoretical formulations will be examined—each, in principle, requiring different assumptions about the optimizing behavior of the firm. None can be practically calculated (for reasons that will be explained). The first approach to an economic theoretical producer price index is the concept of the fixed-input *output price* index. This index is a ratio of hypothetical revenues over the two periods being compared, say periods 0 and 1, that the revenue-maximizing establishment could realize, where the technology and inputs to work with were fixed to be the same for both of the periods. An establishment that, for example, doubles its revenue using a fixed technology and inputs, effectively doubles its prices. The theoretical index is a ratio of revenues, so it incorporates substitution effects as more revenue is obtained as firms substitute toward higher-priced products. The theoretical index wishes to have as its period 1 quantities the results of the firm changing the mix of output it produces in response to relative price changes. But there is a dilemma: only price changes should be reflected, and by allowing quantities to change in this way pure price changes would not be measured. So the theoretical index fixes the amount that can be produced by holding the technology and inputs at some constant level. The firm can change its output mix but must use constant inputs and technology. Note that there is an entire *family* of theoretical price indices depending on which period's reference technology and inputs are held constant: fixed period-0 technology and primary inputs, fixed period-1 technology and primary inputs, or some average of the two.

1.92 Theoretical fixed-output *input price* indices may also be defined. These are the ratio of hypothetical intermediate input costs that the cost-minimizing establishment must pay in order to produce a set of outputs, again with technology and primary inputs fixed to be the same for the comparison in both periods.

1.93 The measurement of GDP using the production approach involves calculating the *value added* by the industry. Value added is the difference between the value of output produced by industries and the value of the intermediate inputs used. The value added by each industry is then summed along with taxes less subsidies on products to provide an estimate of GDP. An important use of the PPI is to deflate the values of outputs and inputs at current-period prices to estimate value added at constant prices. In Chapter 17 the economic approach is first used to define a theoretical *output price index*, *intermediate input price index*, and *value-added deflator* for *a single establishment.* Aggregation is then undertaken over establishments in order to define national counterparts to these establishment price indices in Chapter 18.

E.1 Theoretical output price indices

1.94 The theoretical *output price index* between periods 0 and 1 is the ratio of the maximum revenues that the establishment could attain when faced with period 0 and 1 prices using a fixed, given technology and a fixed set of inputs. Consider a theoretical index in which period 0 technology and inputs are held constant, the theoretical counterpart to the Laspeyres index. What is required for the numerator of the ratio is to generate what the period 1 quantities would be, holding the production process and inputs constant in period 0 after the change in relative prices from the period 0 technology and inputs. This in turn requires a mechanism to generate these hypothetical period 1 quantities from the fixed period-0 technology and inputs. In the economic approach the technology of a firm or industry is described in terms of a production (possibility) function, which tells us the maximum amount of output(s) that can be produced from a given set of inputs. If the values of all the inputs to a firm or industry were given, the production function would be able to generate all possible combinations of output of products from the technology—it would be a mathematical representation of the technology that converts inputs to outputs. The prevailing relative prices would dictate exactly how much of each product is produced. The economic approach to the PPI relies on the assumption of *optimizing behavior* on the part of producers in competitive, price-taking markets so that they respond to relative price changes. In this approach, while actual prices are considered for both periods, the quantities in each period may not be the observed ones. They are generated from a given period's production function (with fixed technology) and level of inputs, using assumptions of maximizing behavior and dictated by relative prices, which may be the ones in another period. This is a powerful analytical framework because it allows us to consider, at least in theory, how quantities would respond to different price regimes (say, period 1 prices) under constant (say, period 0) reference technologies and inputs. They are hypothetical quantities that cannot be observed but are generated in a mathematical model so that their formulation can be compared with real index number formulas based on observable prices and quantities.

1.95 "Pure" price index number formulas (based on observed data) and theoretical indices have in common that they may both be defined as the ratios of revenues in two periods. However, by definition, while the quantities are fixed in pure price indices, they vary in response to changes in relative prices in theoretical indices. In contrast to the axiomatic approach to index theory, the economic approach recognizes that the quantities produced are actually dependent on the prices. In practice, rational producers may be expected to adjust the *relative* quantities they produce in response to changes in *relative* prices. A theoretical PPI assumes that a producer seeking to maximize revenues will make the necessary adjustments. The baskets of goods and services in the numerator and denominator of a theoretical PPI are not, therefore, exactly the same.

E.2 Upper and lower bounds on a theoretical output price index

1.96 The theoretical price index between periods 0 and 1 is the ratio of revenues in those periods using fixed technology and inputs. Consider an index that held the technology and inputs constant in period 0. The revenue generated in period 0 from period 0 prices using period 0 technology and inputs is what actually happened: the denominator of the theoretical ratio is the observed revenue, assuming the producer was optimizing revenue. The numerator is period 1 prices multiplied by the hypothetical quantities that would have been produced using the same period 0 technology and inputs, had period 1 prices prevailed. It is **not**, as in the Laspeyres index, period 1 prices multiplied by the actual quantities produced at period 0 prices using period 0 technology and inputs. Both the theoretical and the Laspeyres indices use the same period 0 technology and inputs, but the theoretical index generates quantities from it as if period 1 prices prevailed, whereas the Laspeyres index uses the actual period 0 quantities. In practice, relative prices may change between the two periods, so the quantities generated will be different. Higher revenue could be achieved by substituting, at least marginally, some products that have relatively high price changes for some that have relatively low ones. The theoretical index based on period 0 technology and inputs will take account of this and will increase by more than the Laspeyres index. The theoretical index will be at least equal to or greater than the Laspeyres, since the producer has the possibility of, at worst, producing the same set of products as in period 0. Being a revenue maximizer, it is assumed the producer will substitute products with relatively high price changes—the Laspeyres index thus incurs a "substitution bias."

1.97 By a similar line of reasoning, it can be shown that when relative prices change, the theoretical output price index based on period 1 technology and inputs will increase by less than the Paasche index. In other words, as shown in Chapter 17, Section B.1, the Laspeyres index provides a lower bound to its (period 0) theoretical index, and the Paasche an upper bound to its (period 1) theoretical index. Note that these inequalities are in the opposite direction to their CPI cost-of-living index counterparts. This is because the optimization problem in the cost-of-living theory is a cost minimization problem as opposed to the present revenue *maximization* problem.

1.98 The practical significance of these results stems from the fact that the Laspeyres and Paasche indices can be calculated directly from the observed prices and quantities, whereas the theoretical indices cannot, thus giving some insight into the bias involved in the use of these two formulas. Suppose the official objective is to estimate a base-period theoretical output price index, but that a Laspeyres index is calculated instead for practical reasons. One important conclusion to be drawn from this preliminary analysis is that the PPI may be expected to have a downward bias. Similarly, a series of Paasche PPIs used to deflate a series of output values at current prices generates a series of values at constant period 0 prices (Laspeyres volume index), which in turn will also suffer from a downward bias. The approach informs us that there are *two* equally valid theoretical economic price indices, and that the bound, while useful, shows only how Laspeyres and Paasche indices compare with their own theoretical counterparts. What we require are *two-sided* bounds on the theoretically justified index.

E.3 Estimating theoretical output indices by superlative indices

1.99 The next step is to establish whether there are special conditions under which it may be possible to exactly measure a theoretical PPI. In Section B.2 of Chapter 17 theoretical indices based on weighted "averages" of the period 0 and period 1 technology and similarly weighted averages of the period 0 and 1 inputs are considered. These theoretical indices deal adequately with *substitution effects*; that is, when an output price increases, the producer's supply increases, holding inputs and the technology constant. Such theoretical indices are argued to generally fall between the Laspeyres (lower bound) and Paasche (upper bound) indices. The Fisher index, as the geometric mean of the Laspeyres and Paasche indices, is the only symmetric average of Laspeyres and Paasche that satisfies the *time reversal test.* Thus, economic theory was used to justify Laspeyres and Paasche bounds, and axiomatic principles led to the Fisher price index as the best symmetric average of these bounds.

1.100 In Section B.3 of Chapter 17 the case for the Törnqvist index number formula is presented. It is assumed that the revenue function takes a specific mathematical form: a translogarithmic function. If the price coefficients of this translog form are equal across the two periods being compared, then the geometric mean of the economic output price index that uses period 0 technology and the period 0 input vector, and the economic output price index that uses period 1 technology and the period 1 input vector, are *exactly equal* to the Törnqvist output price index. The assumptions required for this result are weaker than other subsequent assumptions; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period. The ability to relate an actual index number formula (Törnqvist) to a specific functional form (translog) for the production technology is a powerful analytical device. Statisticians using particular index number formulas are in fact replicating particular mathematical descriptions of production technologies. A good formula should not correspond to a restrictive functional form for the production technology.

1.101 Diewert (1976) described an index number formula to be *superlative* if it is equal to a theoretical price index whose functional form is flexible—it can approximate an arbitrary technology to the second order. That is, the technology by which inputs are converted into output quantities and revenues is described in a manner that is likely to be realistic of a wide range of forms. Relating a class of index number formulas to technologies represented by flexible functional forms is another powerful finding, since it gives credence to this class of index number formulas. Note also that the translog functional form is an example of a *flexible* functional form, so the Törnqvist output price index number formula is *superlative.* In contrast to the theoretical indices, a superlative index is an actual index number that can be calculated. The practical significance of these results is that they give a theoretical justification for expecting a superlative index to provide a fairly close approximation to the unknown, underlying theoretical index in a wide range of circumstances.

1.102 In Section B.4 the Fisher index is revisited from a purely economic approach. An additional assumption is invoked, that outputs are homogeneously separable from other commodities in the production function: if the input quantities vary, the output quantities vary with them, so that the new output quantities are a uniform expansion of the old output quantities. It is shown that a homogeneous quadratic utility function is flexible and corresponds to the Fisher index. The Fisher output price index is therefore also *superlative*. This is one of the more famous results in index number theory. Although it is generally agreed that it is not plausible to assume that a production technology would have this particular functional form, this result does at least suggest that, in general, the Fisher index is likely to provide a close approximation to the underlying unknown theoretical PPI—and certainly a much closer approximation than either the Laspeyres or the Paasche indices can yield on their own.

1.103 This intuition is corroborated by the following line of reasoning. Diewert (1976) noted that a homogeneous quadratic is a flexible functional form that can provide a second-order approximation to other twice-differentiable functions around the same point. He then described an index number formula that is exactly equal to a theoretical one based on the underlying aggregator function as *superlative* when that functional form is also flexible—for example, a homogeneous quadratic. The derivation of these results, and further explanation, are given in detail in Section B.3 of Chapter 17. In contrast to the theoretical index itself, a superlative index is an actual index number that can be calculated. The practical significance of these results is that they provide a theoretical justification for expecting a superlative index to provide a fairly close approximation to the unknown underlying theoretical index in a wide range of circumstances.

E.3.1 Superlative indices as symmetric indices

1.104 The Fisher index is not the only example of a superlative index. In fact, there is a whole family of superlative indices. It is shown in Section B.4 of Chapter 17 that any quadratic mean of order r is a superlative index for each value of $r \neq 0$. A quadratic mean of order r price index P^r is defined as follows:

$$(1.15)\ P^r \equiv \frac{\sqrt[r]{\sum_{i=1}^{n} s_i^0 \left(\frac{p_i^t}{p_i^0}\right)^{r/2}}}{\sqrt[r]{\sum_{i=1}^{n} s_i^t \left(\frac{p_i^0}{p_i^t}\right)^{r/2}}},$$

where s_i^0 and s_i^t are defined as in equation (1.2) above.

1.105 The symmetry of the numerator and denominator of equation (1.15) should be noted. A distinctive feature of equation (1.15) is that it treats the price changes and revenue shares in both periods symmetrically whatever value is assigned to the parameter r. Three special cases are of interest:

- When $r = 2$, equation (1.15) reduces to the Fisher price index;
- When $r = 1$, it is equivalent to the Walsh price index;
- In the limit as $r \to 0$, it equals the Törnqvist index.

1.106 These indices were introduced earlier as examples of indices that treat the information available in both periods *symmetrically*. Each was originally proposed long before the concept of a superlative index was developed.

E.3.2 The choice of superlative index

1.107 Section B.5.2 of Chapter 17 addresses the question of which superlative formula to choose in practice. Because each may be expected to approximate to the same underlying theoretical output index, it may be inferred that they ought also to approximate to each other. That they are all symmetric indices reinforces this conclusion. These conjectures tend to be borne out in practice by numerical calculations. It seems that the numerical values of the different superlative indices tend to be very close to each other, but only so long as the value of the parameter r does not lie far outside the range 0 to 2. However, in principle, there is no limit on the value of the parameter r, and in Section B.5.1 of Chapter 17, it is shown that as the value of r becomes progressively larger, the formula tends to assign increasing weight to the extreme price relatives, and the resulting superlative indices may diverge significantly from each other. Only when the absolute value of r is very small, as in the case of the three commonly used superlative indices—Fisher, Walsh, and Törnqvist—is the choice of superlative index unimportant.

1.108 Both the Fisher and the Walsh indices date back nearly a century. The Fisher index owes its popularity to the axiomatic, or test, approach, which Fisher (1922) himself was instrumental in developing. As shown above, it appears to dominate other indices from an axiomatic viewpoint. That it is also a superlative index whose use can be justified on grounds of economic theory suggests that, from a theoretical point of view, it may be impossible to improve on the Fisher index for PPI purposes.

1.109 However, the Walsh index has the attraction of being not merely a superlative index, but also a conceptually simple *pure* price index based on a fixed basket of goods and services. That the Walsh index is both a superlative and a pure index throws light on the interrelationships between the theoretical output price index and pure price indices. The distinctive feature of a Walsh index is not just that the basket of goods and services is a simple (geometric) average of the quantities in each of the two periods; by being a geometric average, it also assigns equal importance to the *relative*, as distinct from the absolute, quantities. Such an index clearly treats both periods symmetrically.[8] Pure price indices do not have to diverge from the theoretical output price index and are not inherently biased as estimators of the theoretical index. Bias is likely to arise only when the relative quantities used in a pure price index favor one of the periods at the expense of the other, as in a Laspeyres or Paasche index.

E.3.3 Representativity bias

1.110 That the Walsh index is a Lowe index that is also superlative suggests that the bias in other Lowe indices depends on the extent to which their quantities deviate from those in the Walsh basket. This can be viewed from another angle.

1.111 Because the quantities in the Walsh basket are *geometric* averages of the quantities in the two periods, equal importance is assigned to the *relative*, as distinct from the absolute, quantities in both periods. The Walsh basket may therefore be regarded as being the basket that is most representative of *both* periods.[9] If equal importance is attached to the production patterns in the two periods, the optimal basket for a Lowe index ought to be the most representative basket. The Walsh index then becomes the conceptually preferred target index for a Lowe index.

1.112 Suppose that period b, whose quantities are actually used in the Lowe index, lies midway between 0 and t. In this case, assuming fairly smooth trends in the relative quantities, the actual basket in period b is likely to approximate the most representative basket. Conversely, the farther away that period b is from the midpoint between 0 and t, the more the relative quantities of b are likely to diverge from those in the most representative basket. In this case, the Lowe index between periods 0 and t that uses period b quantities is likely to exceed the Lowe index that uses the most representative quantities by an amount that becomes progressively larger the farther back in time period b is positioned. The excess constitutes "bias" if the latter index is the target index. The bias can be attributed to the fact that the period b quantities tend to become increasingly unrepresentative of a comparison between 0 and t the farther back period b is positioned. The underlying economic factors responsible are, of course, exactly the same as those that give rise to bias when the target index is the economic index. Thus, certain kinds of indices can be regarded as biased without invoking the concept of an economic index. Conversely, the same kinds of indices that tend to emerge as being preferred, whether or not the objective is to estimate an economic index.

1.113 If interest is focused on short-term price movements, the target index is an index between consecutive time periods t and $t + 1$. In this case, the most representative basket has to move forward one period as the index moves forward. Choosing the most representative basket implies chaining. Similarly, chaining is also implied for the target economic index t and $t + 1$. In practice, the universe of products is continually changing as well. As the most representative basket moves forward, it is possible to update the set of products covered as well as take account of changes in the relative quantities of products that were covered previously.

E.3.4 Data requirements and calculation issues

1.114 Because superlative indices require price and revenue data for both periods and revenue data are usually not available for the current period, it is not feasible to calculate a superlative PPI, at least at the time that a PPI is first published. In practice, it may be necessary for the official index to be a Laspeyres-type index. However, in the course of time more revenue data may become available, enabling a superlative PPI to be calculated subsequently. Some statistical offices may find it useful to do so, without necessarily revising the original

[8]The Marshall-Edgeworth index (see Chapter 15) uses a simple arithmetic average of the quantities, but the resulting basket will be dominated by the quantities for one or other of the periods if the quantities are larger, on average, in one period than the other. The Marshall-Edgeworth is not a superlative index.

[9]The Walsh basket is the one that minimizes the sum of the squares of the logarithmic deviations between the quantities in the two actual baskets and those in the index basket.

official index. Comparing movements in the official PPI with those in a subsequently calculated superlative version may be helpful in evaluating and interpreting movements in the official PPI. It may be that revenue data can be collected from establishments alongside price data, and this is to be encouraged so that Fisher PPI indices may be calculated in real time for at least some industrial sectors. To the extent that revenue data are available on an annual basis, annual chain Laspeyres indices could be produced initially and Fisher or Törnqvist indices produced subsequently as the new revenue weights become available. The advantage of the annual updating is that chaining helps to reduce the spread between the Laspeyres and Paasche indices.

1.115 Section B.7 of Chapter 17 notes that, in practice, PPIs are usually calculated in stages (see Chapters 9 and 20) and addresses the question of whether indices calculated this way are consistent in aggregation—that is, have the same values whether calculated in a single operation or in two stages. The Laspeyres index is shown to be exactly consistent, but superlative indices are not. However, the widely used Fisher and Törnqvist indices are shown to be approximately consistent.

E.4 Allowing for substitution

1.116 Section B.8 of Chapter 17 examines one further index proposed recently, the Lloyd-Moulton index, P_{LM}, defined as follows:

$$(1.16)\quad P_{LM} \equiv \left[\sum_{i=1}^{n} s_i^0 \left(\frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad \sigma \neq 1 .$$

The parameter σ, which must be nonpositive for the output PPI, is the elasticity of substitution between the products covered. It reflects the extent to which, on average, the various products are believed to be substitutes for each other. The advantage of this index is that it may be expected to be free of substitution bias to a reasonable degree of approximation, while requiring no more data, except for an estimate of the parameter σ, than the Laspeyres index. It is therefore a practical possibility for PPI calculation, even for the most recent periods. However, it is likely to be difficult to obtain a satisfactory, acceptable estimate of the numerical value of the elasticity of substitution, the parameter used in the formula.

E.5 Intermediate input price indices and value-added deflators

1.117 Having considered the theory and appropriate formula for *output* price indices, Chapter 17 turns to *intermediate input* price indices (Section C) and to *value-added* deflators (Section D). The behavioral assumption behind the theory of the output price index was one of producers maximizing a revenue function. An input price index is concerned with the price changes of intermediate inputs, and the corresponding behavioral assumption is the minimization of a conditional cost function. The producer is held to minimize the cost of intermediate inputs in order to produce a set of outputs, given a set of intermediate inputs prices and that primary inputs and technology are fixed. These are fixed so that hypothetical input quantities can be generated from a fixed setup that allows the input quantities in period 1 to reflect the producer buying more of those inputs that have become cheaper. Theoretical intermediate input price indices are defined as ratios of hypothetical intermediate input costs that the cost-minimizing producer must pay in order to produce a fixed set of outputs from technology and primary inputs fixed to be the same for the comparison in both periods. As was the case with the theory of the output price index, theoretical input indices can be derived on the basis of either fixed period-0 technology and primary inputs, or fixed period-1 technology and primary inputs, or some average of the two. The observable Laspeyres index of intermediate input prices is shown to be an *upper bound* to the theoretical intermediate input price index based on period-0 technology and inputs. The observable Paasche index of intermediate input prices is a *lower bound* to its theoretical intermediate input price index based on period-1 fixed technology and inputs. Note that these inequalities are the reverse of the findings for the output price index but that they are analogous to their counterparts in the CPI for the theory of the true cost-of-living index, which is also based on an expenditure (cost) minimization problem.

1.118 Following the analysis for the output price index, a family of intermediate input price indices can be shown to exist based on an *average* of period-0 and period-1 technologies and inputs leading to the result that Laspeyres (upper) and Paasche (lower) indices are bounds on a reasonable theoretical input index. A symmetric mean of the two bounds is argued to be applicable given that

Laspeyres and Paasche indices are equally justifiable, with the Fisher index having support on axiomatic grounds. If the conditional intermediate input cost function takes the form of a translog technology, the theoretical intermediate input price index is exactly given by a Törnqvist index, which is superlative. If separability is invoked, Fisher and Walsh indices are also shown to be superlative, and the three indices closely approximate each other.

1.119 The third index is the *value-added* deflator. The analysis is based on the maximization of a net revenue function, a function that relates output revenue less intermediate input costs to sets of output prices, input prices, and given primary inputs and technology. The results follow those from using a revenue function for the output price index. Laspeyres and Paasche indices are lower and upper bounds on their respective theoretical value-added deflators, and a family of theoretical value-added deflators could be defined that lie between them. The Fisher index again has some support as a symmetric average on axiomatic grounds, although the Törnqvist index is shown, using fairly weak assumptions, to correspond to a flexible translog functional form for the net revenue function and is, therefore, superlative. This finding requires no assumption of the more restrictive constant returns to scale that are necessary for Fisher and Walsh indices, analogous to those for the output price index.

F. Aggregation Issues

1.120 It has been assumed up to this point that the theoretical PPI is based on the technology of a single representative establishment. Chapter 18 examines the extent to which the various conclusions reached above remain valid for PPIs that are actually compiled for industries or the overall economy. The general conclusion is that essentially the same relationships hold at an aggregate level, although some additional issues arise that may require additional assumptions.

1.121 That there are three possible PPIs requires an examination of how they relate to each other. It is thus necessary to consider how the value-added deflator is related to the output price and the intermediate input price indices, and how the output price index and the intermediate input price index can be combined in order to obtain a value-added deflator. It is shown in Chapter 18 that when the Laspeyres output price index is used to separately deflate outputs and the Laspeyres input price index is used to separately deflate inputs—*double deflation*—at each stage of aggregation, the results are the same as when Laspeyres is used to aggregate in one single stage. The separate deflation of inputs by the input price index and outputs by the output price index make up the components of the double-deflated value-added index. The same applies for the Paasche index. However, if superlative price indices are used, there are some small inconsistencies. It was noted previously that unlike superlative indices, Laspeyres and Paasche indices may suffer from serious substitution bias. They may add up, but not to the right number. A value-added deflator equivalent to the separate Laspeyres (Paasche) deflation of output and input indices is shown as a weighted "average" of the Laspeyres (Paasche) output price index and the Laspeyres (Paasche) intermediate input price index, although the weights used to combine the input and output deflators are rather unusual.

1.122 But how do we derive estimates of double-deflated value added? There is an equivalence between a number of methods. Using the product rule, a value ratio divided (deflated) by a Laspeyres value-added deflator generates a Paasche value-added quantity index; or, correspondingly, a value ratio divided by a Paasche value-added deflator generates a Laspeyres value-added quantity index. An alternative approach yielding equivalent results is to take value added in, say, period 0 at period-0 prices and escalate (multiply) it by a series of Laspeyres value-added quantity indices. The resulting series of value added at constant period-0 prices will be identical to the results from separately escalating the value of inputs and outputs by their respective Laspeyres input and output quantity indices and subtracting the (escalated) former from the latter. More usually, estimates of value added at constant prices are derived by deflation. Deflating a series of nominal current-period value added by a series of Paasche value-added indices yields a series of value added at constant prices. This is equivalent to double deflation: the separate deflation of the inputs and output current-period values by their respective input and output separate Paasche price indices, subtracting the former from the latter. Similar equivalence results can be found using the less well-known approach for a comparison between periods 0 and 1 of deflating the period-1 values by a Paasche *quantity* index to provide a measure of current period-0 quantities at period-1 prices. These can be compared with the nominal period-1 value at current prices to provide, for bilateral comparisons, an estimate of quantity change at constant period-1 prices. These results were devised for the establishment, and it is also shown in Chapter 18 that they hold on aggregation for Laspeyres and Paasche indices and fairly closely for the three main superlative indices: Fisher, Törnqvist, and Walsh.

G. Illustrative Numerical Data

1.123 Chapter 19 presents numerical examples using an artificial data set. The purpose is not to illustrate the methods of calculation as such, but rather to demonstrate how different index number formulas can yield very different numerical results. Hypothetical but economically plausible prices, quantities, and revenues are given for six products over five periods of time. In general, differences between the different formulas tend to increase with the variance of the price relatives. They also depend on the extent to which the prices follow smooth trends or fluctuate.

1.124 The numerical results are striking. For example, the Laspeyres index over the five periods registers an increase of 44 percent, whereas the Paasche falls by 20 percent. The two commonly used superlative indices, Törnqvist and Fisher, register increases of 25 percent and 19 percent, respectively, an index number spread of only 6 points compared with the 64-point gap between the Laspeyres and Paasche. When the indices are chained, the chain Laspeyres and Paasche register increases of 33 percent and 12 percent, respectively, reducing the gap between the two indices from 64 to 21 points. The chained Törnqvist and Fisher register increases of 22.26 percent and 22.24 percent, respectively, being virtually identical numerically. These results show that the choice of index formula and method does matter.

H. Choice of Index Formula

1.125 By drawing on the index number theory surveyed in Chapters 15–19 it is possible to decide on the type of index number in any given set of circumstances. However, there is little point in asking what is the best index number formula for a PPI. The question is too vague. A precise answer requires a precise question. For example, suppose that the principal concern of most users of PPIs is to have the best measure of the *current rate* of factory gate inflation. The precise question can then be posed: what is the best index number to use to measure the change between periods $t-1$ and t in the prices of the producer goods and services leaving the factory between periods $t-1$ and t?

1.126 The question itself determines both the coverage of the index and the system of weighting. The establishments in question have to be those of the country in question and not, say, those of some foreign country. Similarly, the question refers to establishments in periods $t-1$ and t, not to establishments five or ten years earlier. Sets of establishments five or ten years apart are not all the same, and their inputs and production technologies change over time.

1.127 Because the question specifies goods and services produced in periods $t-1$ and t, the basket of goods and services used should include *all* the quantities produced by the establishments in periods $t-1$ and t, and *only* those quantities. One index that meets these requirements is a pure price index that uses a basket consisting of the total quantities produced in both periods $t-1$ and t. This is equivalent to an index that uses a simple arithmetic mean of the quantities in the two periods, an index known as the Marshall-Edgeworth index. However, this index has a slight disadvantage in that if domestic production is growing, the index gives rather more weight to the quantities produced in period t than those in $t-1$. It does not treat both periods symmetrically. It fails tests T7 and T8 listed in Chapter 16 on the axiomatic approach, the invariance to proportional changes in quantities tests. However, if the arithmetic mean quantities are replaced by the geometric mean quantities, as in the Walsh index, both tests are satisfied. This ensures that the index attaches equal importance to the *patterns* of production, as measured by *relative* quantities produced in both $t-1$ and t.

1.128 The Walsh index therefore emerges as the pure price index that meets all the requirements. It takes account of every single product produced in the two periods. It utilizes all the quantities produced in both periods, and only those quantities. It gives equal weight to the patterns of production in both periods. In practice, it may not be feasible to calculate a Walsh index, but it can be used as the standard by which to evaluate other indices.

1.129 The index theory developed in Chapters 15–17 demonstrates that the Fisher and the Törnqvist indices are equally good alternatives. Indeed, the Fisher may be preferred to the Walsh on axiomatic grounds, given that the two indices will tend to give almost identical results for comparisons between successive time periods.

1.130 As already noted, for practical reasons the PPI is often calculated as a time series of Laspeyres indices based on some earlier period 0. In this case, the published index between $t-1$ and t may actually be the monthly-change version of the Laspeyres index given in equation (1.4) above. Given that some substitution effect is operative, which seems extremely likely on both theoretic and empirical grounds, it may be inferred, by reasoning along lines explained in Chapter 15, that the monthly-change Laspeyres index will tend to be less than the Walsh index between $t-1$ and t. If the PPI is intended to measure producer inflation, therefore, the monthly-change Laspeyres could have a downward bias, a bias that will tend to get worse as the current period for the Laspeyres index moves further away from the base period. This is the kind of conclusion that emerges from the index theory presented in Chapters 15 and 16. It is a conclusion with considerable policy and financial implications. It also has practical implications because it provides an argument for rebasing and updating a Laspeyres index as often as resources permit, perhaps on an annual basis as many countries are now doing.

1.131 If the objective of the PPI is to measure the current rate of change in revenues for a fixed, given technology and set of inputs, to be used for output deflation, this translates into asking what is the best estimate of the change in producer output prices. The theory elaborated in Chapter 17 shows that the best estimate will be provided by a superlative index. The three commonly used superlative indices are Fisher, Törnqvist, and Walsh. One or the other of these indices emerges as the theoretically most appropriate formula, whether the objective is to measure the current rate of factory gate inflation or as a deflator. A monthly-change Laspeyres is likely to have the same bias whatever the objective.

1.132 If the objective were to measure price changes over long periods of time—say, 10 or 20 years—the main issue for long-term comparisons is whether to chain or not, or at least how frequently to link.

I. Elementary Price Indices

1.133 As explained in Chapters 9 and 20, the calculation of a PPI typically proceeds in two or more stages. In the first stage, *elementary price indices* are estimated for the *elementary aggregates* of a PPI. In the second stage, these elementary indices are combined to obtain higher-level indices using the elementary aggregate indices with revenue weights. An elementary aggregate consists of the revenue for a small and relatively homogeneous set of products defined within the industrial classification used in the PPI. Samples of prices are collected within each elementary aggregate, so that elementary aggregates serve as strata for sampling purposes.

1.134 Data on the revenues, or quantities, of the different goods and services may not be available within an elementary aggregate. Since it has been shown that it is theoretically appropriate to use superlative formulas, data on revenues should be collected alongside those on prices whenever possible. Given that this may not possible, that there are no quantity or revenue weights, most of the index number theory outlined in the previous sections is not applicable. An elementary price index is a more primitive concept that relies on price data only. It is something calculated when there is no explicit or implicit quantity or revenue data available for weights. Implicit quantity or revenue data may arise from a sampling design whereby the selection of products is with probability proportionate to quantities or sales revenue.

1.135 The question of what is the most appropriate formula to use to estimate an elementary price index is considered in Chapter 20. This topic was comparatively neglected until a number of papers in the 1990s provided much clearer insights into the properties of elementary indices and their relative strengths and weaknesses. Since the elementary indices are the building blocks from which PPIs are constructed, the quality of a PPI depends heavily on them.

1.136 As explained in Chapter 6, compilers have to select *representative products* within an elementary aggregate and then collect a sample of prices for each of the representative products, usually from a sample of different establishments. The individual products whose prices are actually collected are described as the *sampled products*. Their prices are collected over a succession of time periods. An elementary price index is therefore typically calculated from two sets of matched price observations. It is assumed in this section that there are no missing observations and no changes in the quality of the products sampled, so that the two sets of prices are perfectly matched. The treatment of new and disappearing products, and of quality change, is a separate and complex issue that is discussed in detail in Chapters 7, 8, and 21 of the *Manual*.

I.1 Heterogeneity of products within an elementary aggregate

1.137 If a number of different representative products are selected for pricing, the set of products within an elementary aggregate cannot be homogeneous. Even a single representative product may not be completely homogeneous, depending upon how tightly it is specified. This topic is considered in more detail in Chapters 5–7. The degree of heterogeneity of the sampled products must be explicitly taken into account in the calculation of an elementary index.

1.138 When the quantities are not homogeneous, *they cannot be meaningfully added from an economic viewpoint, and their prices should not be averaged*. Consider again the example of salt and pepper, which might be representative products within an elementary aggregate. Pepper is an expensive spice sold in very small quantities such as ounces or grams, whereas salt is relatively cheap and sold in much larger quantities, such as pounds or kilos. A simple arithmetic average of, say, the price of a gram of pepper and the price of a kilo of salt is an arbitrary statistic whose value depends largely on the choice of the quantity units. Choosing the same physical unit of quantity, such as a kilo, for both does not resolve the problem, because both the average price and the change in the average price would be completely dominated by the more expensive product, pepper, even though producers may obtain more revenue from salt. In general, arithmetic averages of prices should be taken only when the corresponding quantities are homogeneous and can be meaningfully added.

I.2 Weighting

1.139 As already noted, it is assumed in this section that there are no quantities or revenues available to weight the prices, or the price relatives, used to calculate an elementary index. If they were available, it would usually be preferable to use them to decompose the elementary aggregate into smaller and more homogeneous aggregates.

1.140 However, some system of weighting may have been implicitly introduced into the selection of the sampled products by the sample design used. For example, the establishments from which the prices are collected may have been selected using probabilities of selection that are proportional to their sales or some other variable.

I.3 Relationships between different elementary index formulas

1.141 Valuable insights into the properties of various formulas that might be used for elementary price indices may be gained by examining the numerical relationships between them, as explained in Section D of Chapter 20. There are two basic options for an elementary index:

- To average the price relatives—that is, the ratios of the matched prices;
- To calculate the ratio of average prices in each period.

1.142 It is worth recalling that for any set of positive numbers the arithmetic average is greater than or equal to the geometric average, which in turn is greater than or equal to the harmonic average, the equalities holding only when the numbers are all equal. Using these three types of average, the ranking of the results obtained by the second method are predictable. It should also be noted that the ratio of geometric averages is identical with the geometric average of the ratios. The two methods give the same results when geometric averages are used.

1.143 As explained in Section C of Chapter 20, there are several elementary price indices that might possibly be used. Using the first of the above options, three possible elementary price indices are:

- The arithmetic average of the price relatives, known as the *Carli* index, or P_C; the Carli is the unweighted version of the Young index.
- The geometric average of the price relatives, known as the *Jevons* index, or P_J; the Jevons is the unweighted version of the geometric Young index.

- The harmonic average of the price relatives, or P_H.

As just noted, $P_C \geq P_J \geq P_H$.

1.144 Using the second of the options, three possible indices are:

- The ratio of the arithmetic average prices, known as the *Dutot* index, or P_D;
- The ratio of the geometric averages, again the Jevons index, or P_J;
- The ratio of the harmonic averages, or R_H .

The ranking of *ratios* of different kinds of averages is not predictable. For example, the Dutot, P_D, could be greater or less than the Jevons, P_J.

1.145 The Dutot index can also be expressed as a weighted average of the price relatives, in which the prices of period 0 serve as the weights:

$$(1.17) \quad P_D \equiv \frac{\sum_{i=1}^{n} p_i^t \Big/ n}{\sum_{i=1}^{n} p_i^0 \Big/ n} = \frac{\sum_{i=1}^{n} p_i^0 \left(\frac{p_i^t}{p_i^0} \right)}{\sum_{i=1}^{n} p_i^0} .$$

As compared with the Carli, which is a simple average of the price relatives, the Dutot index gives more weight to the price relatives for the products with high prices in period 0. However, it is difficult to provide an economic rationale for this kind of weighting. Prices are not revenues. If the products are homogeneous, very few quantities are likely to be purchased at high prices if the same products can be purchased at low prices. If the products are heterogeneous, the Dutot should not be used anyway, since the quantities are not commensurate and not additive.

1.146 Noting that $P_C \geq P_J \geq P_H$, it is shown in Section D of Chapter 20 that the gaps between these indices widen as the variance of the price relatives increases. The choice of formula becomes more important the greater the diversity of the price movements. Moreover, both P_D and P_J can be *expected* to lie *approximately* halfway between P_C and P_H. While it is useful to establish the interrelationships between the various indices, they do not actually help decide which index to choose. However, because the differences between the various formulas tend to increase with the dispersion of the price relatives, it is clearly desirable to define the elementary aggregates in such a way as to try to minimize the variation in the price movements within each aggregate. The less variation there is, the less difference the choice of index formula makes. Since the elementary aggregates also serve as strata for sampling purposes, minimizing the variance in the price relatives within the strata will also reduce the sampling error.

I.4 The axiomatic approach to elementary indices

1.147 One way to decide between the various elementary indices is to exploit the axiomatic approach outlined earlier. A number of tests are applied to the elementary indices in Section E of Chapter 20.

1.148 The Jevons index, P_J, satisfies all the selected tests. It dominates the other indices in the way that the Fisher tends to dominate other indices at an aggregative level. The Dutot index, P_D, fails only one, the commensurability test. This failure can be critical, however. It reflects the point made earlier that when the quantities are not economically commensurate, their prices should not be averaged. However, P_D performs well when the sampled products are homogeneous. The key issue for the Dutot is therefore how heterogeneous are the products within the elementary aggregate. If the products are not sufficiently homogeneous for their quantities to be additive, the Dutot index should not be used.

1.149 The Carli index, P_C, is widely used, but the axiomatic approach shows that it has some undesirable properties. In particular, as the unweighted version of the Young index, it fails the commodity reversal, the time reversal, and the transitivity tests. These are serious disadvantages, especially when month-to-month indices are chained. A consensus has emerged that the Carli may be unsuitable because it is liable to have a significant upward bias. This is illustrated by numerical example in Chapter 9. Its use is not sanctioned for the Harmonized Indices of Consumer Prices (HICPs) used within the European Union. Conversely, the harmonic average of the price relatives, P_H, is liable to have an equally significant downward bias, although it does not seem to be used in practice anyway.

1.150 On the axiomatic approach, the Jevons index, P_J, emerges as the preferred index. However, its use may not be appropriate in all circumstances.

If one observation is zero, the geometric mean is zero. The Jevons is sensitive to extreme falls in prices, and it may be necessary to impose upper and lower bounds on the individual price relatives when using the Jevons.

I.5 The economic approach to elementary indices

1.151 The economic approach, explained in Section F of Chapter 20, seeks to take account of the economic behavior of producers and their economic circumstances. Price differences may be observed at the same point of time for two quite different reasons:

- Exactly the same product may be sold by different categories of producers at different prices.
- The sampled products are not exactly the same. The different prices reflect differences in quality.

Both phenomena may occur at the same time.

1.152 *Pure* price differences can occur when the products sold at different prices are exactly the same. Pure price differences imply differing technologies or market imperfections of some kind, such as local monopolies, price discrimination, consumer or producer ignorance, or rationing. If all consumers had equal access, were well informed, and were free to choose, and all producers produced using the same technologies in price-taking markets, all sales would be made at a single price, the lowest on offer.

1.153 On the other hand, if markets were perfect, producers would be prepared to supply at different prices only if the products were qualitatively different. Included in the term "product" are the terms and conditions surrounding the sale, including the level of service and convenience. It is tempting to assume, therefore, that the mere existence of different prices implies that the products *must* be qualitatively different in some way. For example, even units of the same physically homogeneous product produced at different locations or times of the day may be qualitatively different from an economic viewpoint. For example, a service supplied in the center of town in the evening may carry a price premium, due to higher labor costs, even though it is essentially the same service. In this instance the higher price is arguably not a pure price difference. However, the relative prices in different establishments do not necessarily have to match differences in producer inputs and technologies and consumers' preferences and may be, in part, pure price differences. In practice, almost all markets are imperfect to some extent, and pure price differences cannot be assumed away a priori.

1.154 If there is only a single homogeneous product produced by an establishment on a "normal" day, the price differences must be pure. The average price is equal to the *unit value*, defined as the total value sold divided by the total quantity. The unit value is a quantity-weighted average of the different prices at which the product is sold. It changes in response to changes in the mix of quantities sold at different prices as well as to any changes in the prices themselves. In practice, however, the change in the unit value has to be estimated from a sample of prices only. Unit values exist at two levels. The first is for a production run i at the establishment level where a batch of, say, q_i products may be sold for revenue $p_i q_i$, the price recorded being the unit value. There may be more than one production run at different batch sizes, and the unit values may vary with batch size. The recorded "price" for these products may then be the revenue from several batches divided by the quantity supplied, $\Sigma\, p_i q_i \,/\, q_i$. If the mix of batch sizes varies over time, then there will be unit-value bias when dividing the unit value in one period by that in a preceding period. The second aggregation of unit values is across establishments producing the same commodity. Again, any difference in the relative quantities sold from different establishments will lead to unit-value bias if the commodities are not strictly homogeneous.

I.5.1 Sets of homogeneous products

1.155 The economic approach views the products as if they were a sample from a basket produced by a group of rational, revenue-maximizing producers. One critical factor is how much product variation there is within an elementary aggregate, bearing in mind that it should be as narrowly defined as possible, possibly even consisting of a set of homogeneous products.

1.156 If the sampled products are all identical, the observed price differences must be due to establishments using different production technologies and market imperfections such as price discrimination, consumer ignorance, or rationing, or some kind of temporary disequilibrium. Informed producers with unrestricted production possibilities would not sell at a lower price if they had the opportunity to sell exactly the same product at a higher price. It is tempting to assume, therefore, that the products are not really homogeneous and that the observed price differences *must* be due to quality differences of some kind or another, but imperfections in producer and consumer markets are widespread and cannot be assumed away a priori.

1.157 As explained in Section B of Chapter 20, when a single product is sold at different prices, the price of that product for PPI purposes is the unit value, defined as total sales divided by total quantities: that is, the quantity-weighted average price. The price relative for the product is the ratio of the unit values in the two periods. This may be affected by a change in the pattern of products that sell at high and low prices as well as by changes in the individual prices.

1.158 If the representative sampled products are selected with probabilities proportional to the quantities sold at the different prices in the first period, a simple (unweighted) arithmetic average of their prices will provide an estimate of the unit value in the first period. The Dutot index is the ratio of the simple arithmetic average prices in the two periods. However, given that the two sets of prices are perfectly matched—that is, geared to the pattern of production in the first period only—the Dutot cannot take account of any changes in the patterns of production between the two periods and may not provide an unbiased estimate of the ratio of the unit values. As shown in Section F of Chapter 20, the sample Dutot with probabilities proportional to quantities sold in the first period may be expected to approximate to a Laspeyres-type index in which the quantity weights are fixed, by definition. It does not provide a satisfactory estimate of a unit-value index in which the relative quantities do change. Moreover, this approximated Laspeyres-type index is not a conventional Laspeyres index because the quantities do not refer to different products, or even different qualities, but to different quantities of exactly the same product sold at different prices.

1.159 In practice, even though producers' choices may be restricted because of their production technology, buyer-seller relationships, market ignorance, and other market imperfections, they may switch production toward products sold at high prices and away from those at low prices, as market conditions change and restrictions on choice are eased. The Dutot index, based on matched prices, cannot take account of such switches and may tend to understate the *rise* in the unit values for this reason. Alternatively, it may be that the demand side dictates market behavior, with establishments responding to demand by increasing production of low-priced products. When the ratio of the unit values changes because purchasers, or at least some of them, succeed in switching from establishments selling at high prices to establishments selling at low prices, the failure of PPIs to take account of such switches leads to the Dutot index overstating the *fall* in the unit-value index.

I.5.2 Heterogeneous elementary aggregates

1.160 In practice, most elementary aggregates are likely to contain a large number of products that are similar but not identical. Assuming producers are informed and have a perfectly flexible set of production possibilities, the relative prices may then be expected to reflect producers' marginal rates of substitution. Within the same elementary aggregate, the different products will often be close substitutes for each other, often being no more than marginally different qualities of the same generic product, so that the quantities produced may be expected to be quite sensitive to changes in relative prices.

1.161 Using an economic approach, it is possible to ask what is the best estimate of the "true" economic index, for the elementary aggregate. Bearing in mind, however, that no information on quantities and revenues is available within the aggregate, it is necessary to resort to considering certain hypothetical special cases. Suppose that producers react to purchasers' preferences; as demand increases for a relatively low-priced product, producers produce more of it. Assume purchasers have so-called Cobb-Douglas preferences, which imply that the cross-elasticities of substitution between the different products are all unity. The quantity relatives vary inversely with the price relatives, so that their revenue shares and the establishment's revenues remain constant. The true economic index can then be shown to be a weighted geometric average of the price relatives, the weights being the revenue shares—which, as just noted, are the same in both periods. Now, suppose that the products whose prices are sampled are randomly selected with probabilities proportional to their revenue shares in the first period. As shown in Section F of Chapter 20, with this method of selection, the simple geometric average of the sample price relatives—that is, the Jevons index—may be expected to provide an approximation to the underlying economic index.

1.162 However, for PPIs the assumption of unit cross-product elasticities of substitution with equal revenues in both periods is *not* consistent with producer economic theory. Revenue-maximizing producers will produce *more* of the sampled products with above-average price increases, so their share of revenue cannot be expected to be constant. Indeed the Jevons index, in assuming constant revenue shares, will understate price changes under such revenue-maximizing behavioral assumptions. The Jevons index allows implicit quantities to fall as relative prices increase, to maintain equal revenue share, rather than allowing an increase. There is not an accepted unweighted price index number formula that incorporates such substitution behavior, although the Jevons index has been shown to be unsuitable under producer revenue-maximizing assumptions.

1.163 Alternatively, suppose that the production technology is such that, at least in the short term, there is no substitution in response to relative price changes, and the relative quantities remain fixed. In this case, the true economic index would be a Laspeyres-type index. If the products were sampled with probabilities proportional to the revenue shares in the first period, a simple arithmetic average of the price relatives—that is, the Carli index—would approximate to it.[10] However, assuming no substitution is unreasonable and counterfactual in general, although it may occur exceptionally.

1.164 Thus, using the economic approach, under one set of conditions the Jevons index would provide an approximation to the underlying economic index, while under another set of conditions the Carli index would do so. In most cases, the actual conditions seem likely to be closer to those required for the Jevons to estimate the underlying index than for the Carli, since the cross-elasticities of substitution seem much more likely to be close to unity than zero for industries whose pricing behavior is demand driven. Thus, the economic approach provides some support for the use of Jevons rather than Carli, at least in most situations. However, if producer revenue-maximizing behavior is believed to dominate an industry, use of the Jevons index is not supported.

1.165 Another alternative is suggested in Section G of Chapter 20. If products are sampled according to fixed revenue shares in each period, then the resulting sample can be used with the Carli formula (P_C) to estimate the Laspeyres index, and the harmonic mean formula (P_H) to calculate the Paasche index. By taking the geometric average of these two formulas, as suggested by Carruthers, Sellwood, Ward (1980), and Dalén (1992a), a Fisher index would result:

$$(1.18)\ P_{CSWD} = \sqrt{P_C \times P_H}\ .$$

1.166 However, since statistical offices would not have the revenue shares for the current period, an approximation to the Fisher index is obtained by assuming they are not too different from those used in the base period 0. A similar assumption would justify the use of a Jevons index (P_J,) as an approximation to a Törnqvist index. Again recall, that these approximations result when the observations are sampled in proportion to revenue shares.

1.167 One lesson to be drawn is that, when trying to decide on the most appropriate form of the price index for an elementary aggregate, it is essential to pay attention to the characteristics of the products within the aggregate and not rely on a priori generalizations. In particular, the Dutot index should be used only when the products are homogeneous and measured in exactly the same units. When the products are heterogeneous, the choice between the Carli and the Jevons indices turns on the extent to which, and the nature of, substitution behavior that is likely to occur in response to relative price changes. In many cases, the Jevons is likely to be preferred. Because Jevons is also the preferred index on axiomatic grounds, it seems likely to be the most suitable form of elementary index in most situations, although the circumstances underlying its use should be carefully established.

[10]Notice that the Dutot index cannot be used when the products are not homogeneous, since an arithmetic average of the prices of different kinds of products is both arbitrary and economically meaningless. If a Laspeyres index is estimated as a simple average of the price relatives—that is, assuming equal revenue shares—the implied quantities cannot be equal because they vary inversely with the prices.

J. Seasonal Products

1.168 As explained in Chapter 22, the existence of seasonal products poses some intractable problems and serious challenges for PPI compilers and users. Seasonal products are products that are either:

- Not available during certain seasons of the year, or
- Are available throughout but their prices or quantities are subject to regular fluctuations that are synchronized with the season or time of the year.

1.169 There are two main sources of seasonal fluctuations: the climate and custom. Month-to-month movements in a PPI may sometimes be so dominated by seasonal influences that it is difficult to discern the underlying trends in prices. Conventional seasonal adjustment programs may be applied, but these may not always be satisfactory. However, the problem is not confined to interpreting movements in the PPI; seasonality creates serious problems for the compilation of a PPI when some of the products in the basket regularly disappear and reappear, thereby breaking the continuity of the price series from which the PPI is built up. There is no panacea for seasonality. A consensus on what is best practice in this area has not yet been formed. Chapter 22 examines a number of different ways in which the problems may be tackled using an artificial data set to illustrate the consequences of using different methods.

1.170 One possibility is to exclude seasonal products from the index, but this may be an unacceptable reduction in the scope of the index, since seasonal products can account for a significant proportion of total household consumption. Assuming seasonal products are retained, one solution is to switch the focus from month-to-month movements in the index to changes between the same month in successive years. In some countries, it is common for the media and other users, such as central banks, to focus on the annual rate of inflation between the most recent month and the same month in the previous year. This year-over-year figure is much easier to interpret than month-to-month changes, which can be somewhat volatile, even in the absence of seasonal fluctuations.

1.171 This approach is extended in Chapter 22 to the concept of a rolling year-on-year index that compares the prices for the most recent 12 months with the corresponding months in the price reference year. The resulting *rolling-year indices* can be regarded as seasonally adjusted price indices. They are shown to work well using the artificial data set. Such an index can be regarded as a measure of inflation for a year that is centered around a month that is six months earlier than the last month in the rolling index. For some purposes, this time lag may be disadvantageous, but in Section F of Chapter 22 it is shown that under certain conditions the current month's year-over-year monthly index, together with the previous month's year-over-year monthly index, can successfully predict the rolling-year index that is centered on the current month. Of course, rolling-year indices and similar analytic constructs are not intended to replace the monthly or quarterly PPI but to provide supplementary information that can be extremely useful to users. They can be published alongside the official PPI.

1.172 Various methods of dealing with the breaks in price series caused by the disappearance and reappearance of seasonal products are examined in Chapter 22. However, this remains an area in which more research needs to be done.

K. Concepts, Scope, and Classifications

1.173 The purpose of Chapter 3 of the *Manual* is to define and clarify a number of basic concepts underlying a PPI and to explain the scope, or domain, of the index: that is, the set of products and economic activities that the index is intended to cover. The chapter also discusses the various price concepts and types of prices that are used in PPI compilation and examines the structure of the classification systems used in the PPI for products and industries.

1.174 The general purpose of an index of *producer* prices is to measure changes in the prices of goods and services produced by businesses. However, an operational definition of a PPI requires a decision about, first, whether the index will cover output prices or input prices (or both); second, whether the index is meant to cover all production, that is, all economic activities and/or products, or just particular industries and/or product groups; third, for the economic activities included, whether the index should cover just market activities; and, finally, what is the geographic boundary in which the defined production is included. The scope of a PPI is inevitably influenced by what is intended or believed to be its main use, although it should be borne in mind that the index may also be used as a proxy for a general price index and used for purposes other than those for which it is intended.

K.1 Population coverage

1.175 Many decisions must be made to define the scope and coverage of the PPI. These include the economic activities, products, and the types of buyers and sellers to include in the index. The PPI could cover all economic activities in a country, which could be the ultimate goal of the price index. In many countries the PPI is limited to a few industrial activities such as agriculture, mining, manufacturing, and energy supply. These activities represent a good starting point. However, the share of such activities in national economies is becoming smaller, and services such as transport, communication, medical care, trade, and business services are becoming increasingly more important. If the primary purpose of the PPI is an inflation indicator or a deflator for national accounts aggregates, a broad coverage of economic activity is needed.

1.176 A PPI can be compiled and classified both by industry and product. For example, the food slaughtering industry produces meat and leather. Generally, the industrial coverage of the PPI is limited to specific industrial sectors such as mining and manufacturing, and this in turn limits the product coverage. If broad product coverage is a goal, then the PPI would have to cover a larger number of goods- and service-producing sectors. The PPI can also identify products by stage of processing and produce measures of products for final demand, those for intermediate consumption, and those that are primary products.

1.177 A PPI also could cover all domestic production, including exports, or be limited to production for domestic markets only. If it covers all domestic production, then products for export could be separately identified, and an export price index developed. Imports are usually not within the scope of the output PPI because their production is not domestic, but they could be covered in an input price index. (Foreign trade price indices will be the subject of a separate manual.) In addition, the PPI is usually limited to marketed products and thus excludes nonmarket goods and services.

K.2 Price coverage

1.178 The PPI should measure actual transaction prices reflecting revenue received by the producer for goods and services actually sold to customers. These prices would not necessarily be "list" or "book" prices because they should reflect any applicable discounts, rebates, surcharges, etc. that may apply to their customers for the sampled transactions. These would include contract prices, where they exist, and spot market prices. Care must be taken to make sure the prices reflect those at the time the transaction occurs and not those at the time of order, particularly for major durable goods such as airplanes and ships, which have a long production period between order and delivery.

1.179 Average prices are acceptable in the PPI if they represent a strictly *homogeneous* set of product transactions and are for the current time period. Often these two criteria for an average price cannot be met. If average prices are calculated over a large number of transactions with differing quality and/or terms of sale, they are not acceptable in the PPI. Changes in such prices will reflect any changes in the mix of quality characteristics of the products sold as well as any changes in terms of sale. Such changes in the heterogeneous mix of transactions lead to what is often referred to as *unit-value bias* in the measurement of price changes.

1.180 Special care needs to be taken with subsidized prices and intracompany transfer prices. The prices used in the PPI should reflect the revenue received by producers from transactions. Prices for products on which subsidies are received will not reflect the revenue to the producer unless the subsidies are included. This involves making adjustments to the prices as discussed in Section B.3 of Chapter 3. Also, intracompany transfer prices may not reflect actual market prices and may require special treatment as outlined in Section B.4 of Chapter 3.

K.3 The treatment of some specific types of transactions and prices

1.181 The price concept is not always as clear-cut as that for simple homogeneous goods sold in day-to-day transactions. There are a number of conceptually difficult products and industries that present particular problems—agriculture, clothing, steel, ships, automobiles, and banking services, to name a few. Pricing concepts and strategies for these and other special cases are covered in more detail in Chapter 10.

K.4 Statistical units

1.182 The statistical unit in the PPI is usually a single, homogeneous, output-generating entity such as the *establishment*, a concept outlined in the *1993 SNA*. Separate auxiliary, sales, or administrative units are not included. This unit is the decision-making unit for all production operations and maintains records on prices and production activities. In some cases records from a clustering of establishments are sent to a single record-keeping unit, the enterprise, from which prices will have to be collected.

1.183 The rapid rise in electronic commerce (e-commerce), globalization, and outsourcing of production is making the identification of the statistical unit, the producing establishment, more difficult. This is particularly the case with the formation of *virtual corporations*. A virtual corporation is the creation of a partnership among several companies sharing complementary expertise and producing a product with a very short life cycle. With the conclusion of the product's life span, the corporation is disbanded. Also, a considerable volume of business undertaken among corporations is being transacted on the Internet, which is difficult to monitor. These activities will require new approaches to identify and capture such transactions in the PPI.

K.5 Classification

1.184 The classification system provides an organizing structure for the PPI and is the first step in sample surveying. It forms the index structure and defines which industries, products, and aggregate levels will be included. It also determines the publication scheme for the PPI results. International standard classification systems, discussed in Section E.2 of Chapter 3, are available and should be used to group economic activities and products. The use of these classifications provides meaningful series for policymaking and analysis, as well as facilitating international comparisons.

1.185 Industrial classifications group producer units according to their major kind of activity, based mainly on the principal class of goods or services produced—that is, by an output criterion. At the most detailed level of industrial coding (the four-digit level), categories are delineated according to what is in most countries the customary combination of activities undertaken by the statistical units, the establishments. The successively broader levels of classification (three-digit, two-digit, single-digit) combine the statistical units according to character, technology, organization, and financing of production. The major international industrial classifications are the *International Standard Industrial Classification of all Economic Activities (ISIC)*, the *General Industrial Classification of Economic Activities within the European Communities (NACE)*, the *North American Industrial Classification System (NAICS)*, and the *Australian and New Zealand Standard Industrial Classification (ANZIC)*.

1.186 Product classifications group products into somewhat homogeneous categories on the basis of physical properties and intrinsic nature, as well as the principle of industrial origin. Physical properties and intrinsic nature are characteristics that distinguish the product. These include raw materials from which the goods are made, the stage of production and way in which the goods are produced or service rendered, the purpose or use of the products, and the prices at which they are sold. The product categories should be exhaustive and mutually exclusive so that a product belongs to only one category.

1.187 The categories of products (coded, for example, to five digits) can be aggregated to higher-level groupings (four, three, two, and single digits) of products with similar characteristics and uses. There are two primary international product classifications used for PPIs: the *Central Product Classification (CPC)* and the *Eurostat Classification of Products by Activity (CPA* or *PRODCOM)*. In general, each five-digit subclass of the CPC consists of goods and services that are predominantly produced in one specific four-digit class or classes of ISIC Revision 3.

L. Sampling and Collection of Price Data

1.188 As explained in Chapter 9, there are two levels of calculation involved in a PPI. At the lower level, samples of prices are collected and processed to obtain lower-level price indices. These lower-level indices are the elementary indices whose properties and behavior are explained in Chapter 20 and are summarized in Section I above. At the higher level, the elementary indices are averaged to obtain higher-level indices using the relative value of output or revenue as weights. All the index number theory elaborated in Chapters 15–18 comes into play at this higher level.

1.189 Lower-level indices are calculated for elementary aggregates. Depending on the resources available and procedures adopted by individual countries, these elementary aggregates could be subclasses of the industry and product classifications as described in the previous section. If it is desired to calculate PPIs for different regions, the subclasses have to be divided into strata referring to the different regions. In addition, in order to improve the efficiency of the sample estimator, it will usually be desirable, if feasible, to introduce other criteria into the definitions of the strata, such as the size of the establishment. When the subclasses are divided into strata for data collection purposes, the strata themselves become the elementary aggregates. Because a weight needs to be attached to each elementary aggregate in order to calculate the higher-level indices, an estimate of the quantity or value of output for each elementary aggregate should be available, albeit in a preceding period, from separate surveys of establishments, as outlined in Chapter 4 and Section N below. On the other hand, quantity data may not be readily available for all elementary aggregates and may have to be estimated using allocation methods like those described in Chapter 4, Section E.1. It is preferable that the lower-level indices also be compiled using quantity or value weights and that such data are collected at the time of price collection in the initial base period, or, if possible, also in each successive period. This would allow Laspeyres or Fisher indices to be compiled at the lower level, which is advisable for theoretical reasons outlined in Section I above and in Chapter 20. If no weights can be derived, the elementary indices have to be estimated from price data alone, as explained in Chapter 20.

1.190 Chapter 5 is concerned with sampling strategies for price collection. Chapter 6 is concerned with the methods and operational procedures actually used to collect prices. The sampling and collection of prices is at the lower level, sampling being considered first.

L.1 Random sampling and purposive sampling

1.191 Prices are collected for products from establishments in particular industries. The sampling process involves multiple stages of selection. Once the purpose and scope for the PPI have been decided (for example, which single-digit industrial activities will be included), then decisions can be made about the four-digit industries to be included. After the industries have been chosen, then the establishments within industries must be selected and sampled, and then individual (representative) products must be selected or sampled. Finally, individual transactions that represent the sampled products in each sample establishment must be selected. The procedures used for selecting the sample at each stage are important.

1.192 In designing the sample for price collection purposes, due attention should be paid to standard statistical criteria to ensure that the resulting sample estimates are not only unbiased and efficient in a statistical sense, but also cost effective.[11] There is a large literature on sampling survey techniques to which reference may be made and which need not be summarized here. In principle, it would be desirable to stratify the establishments and products by criteria that differentiate them according to their relative price changes, and to further select both establishments and products using random sampling with known probabilities of selection. This ensures that the sample of products selected is not distorted by subjective factors and enables sampling errors to be calculated. However, many countries continue to rely heavily on the purposive selection of establishments and products because random sampling may be too difficult and too costly. Purposive selection is believed to be more cost-effective, especially when the sampling frames available are not comprehensive and not well-suited for PPI purposes. It may also be cost effective to use "cutoff" sampling procedures, discussed in Chapter 5, Section D.1.2, which are more objective than purposive sampling. Cutoff sampling first establishes a targeted threshold value, and then all establishments/products

[11]There are two types of bias encountered in the literature on index numbers: *sampling bias,* as understood here, and the *nonsampling biases* in the form of substitution bias or bias due to inadequate adjustments for quality change, as discussed in Chapters 11 and 7 of the *Manual*. It is usually clear in context which type of bias is meant.

above this value are selected for the sample. It is a simple means, for example, of selecting the representative four-digit industries within a single-digit category, or products within an establishment.

1.193 The representative sampling of establishments and products requires comprehensive and up-to-date sampling frames. Two separate frames are usually needed for PPI purposes, one listing the universe of establishments and the other listing the universe of products. Examples of possible sampling frames for establishments are business registers, establishment censuses, and central or local government administrative records. When the sampling frames contain the requisite information, it may be possible to increase the efficiency of the sample estimate by selecting samples of establishments using probabilities that are proportional to the size of some relevant economic characteristic, such as the total value of output or sales. Sampling frames for products are usually available from establishment or business censuses and may be supplemented by telephone or price survey visits.

1.194 Depending on the information available in the sampling frame, it may be possible to group the establishments into strata on the basis of region, in addition to industrial activity, to form the elementary indices. When there is information about size, a random sample of establishments may be selected with probabilities proportional to size. An example of this approach is presented in Chapter 5, Section E. Price relatives from preceding periods may further be used as part of the sample allocation, with larger samples being drawn from industrial groups whose variance of price relatives is larger. All of this increases the efficiency of the sample estimate. It would also be possible to use cutoff sampling procedures as a simpler, though less efficient, procedure. Cutoff sampling, unlike random sampling, is open to bias, if the excluded smaller establishments have different price changes to the included larger ones. The extent of the bias depends on the threshold cutoff value and the level of aggregation; some of the bias will be offsetting.

1.195 In most countries, the selection of the individual products to be priced within the selected establishments tends to be purposive, being specified by the central office responsible for the PPI. The central office draws up lists of products that are deemed to be *representative* of the products within an elementary aggregate. However, if detailed output or sales by product are available from a census of establishments, these data can be used to select the sample through probability proportional to size or cutoff sampling.

1.196 It has been argued that the purposive selection of products is liable to introduce only a negligible amount of sampling bias, but this may be no more than speculation or conjecture. In principle, random sampling is preferable, but it may not be feasible for many countries given the additional costs that may be involved. For example, the U.S. Bureau of Labor Statistics (BLS) and the U.K. Office of National Statistics make extensive use of random selection procedures to select both establishments and products within establishments. The last stage of sampling is to select the individual transactions within the establishment to represent the price movements of the selected products. Procedures for selecting transactions are presented in Chapter 5, Section E.3. At this level many countries consult with an official from the establishment to select the most representative transactions for each product. Often selecting those with the largest volume of output or sales does this. Such a procedure is analogous to using cutoff sampling. It is also possible to select a probability sample of transactions if the officials can provide estimates of the relative importance of the transactions.

1.197 As explained in Chapter 5, Section F, the universe of establishments and products, from which the sample is taken, has several dimensions. That the universe is changing over time is a major problem not only for PPIs but also for most other economic statistics. Products disappear, to be replaced by other kinds of products, and establishments close while new ones open. This creates both conceptual and practical problems, given that the measurement of price changes over time requires some continuity in the products priced. The matched-models method requires that the price changes recorded should refer to matched products that are identical in both time periods, so that price changes are not tainted by quality changes. But this matching creates a new problem: new products and new establishments are not introduced, and the sample deteriorates. There are further problems created when products are no longer produced or establishments close, and these are considered in some detail in Chapters 7 and 8, and are outlined in sections L.2.4, L.2.5, and M below.

L.2 Regular price collection

1.198 The previous section focused on the sampling issues that arise when prices have to be collected for a large number of products from a large number of establishments. This section is concerned with some of the operational issues relating to price collection, which are discussed in detail in Chapter 6.

L.2.1 Frequency and timing

1.199 Calculating the PPI entails collecting prices from businesses relating to particular products and time periods. Decisions must be made about the frequency of collection (monthly or quarterly) and the time period covered for the prices (a single point in time, several times during the month, or a monthly average). Usually, price collection is monthly and covers the entire month. However, resource considerations may limit collection to a single point in time.

L.2.2 Product specifications

1.200 For each product in the sample, a detailed list of the specifications needs be collected. These specifications are those that are important in identifying and determining the price and quality characteristics of the detailed transaction. Details such as product name, serial number, description or features, size, units of measure, class of customer, discounts, etc. should be included. The collection of data on such quality characteristics is important to the matched-models method, but it will be seen from Section M below that they can serve as a data source for hedonic regressions, which have a similar function—to price-adjust replacement products of different quality.

L.2.3 Price collection methods

1.201 The aim of survey collection techniques is to facilitate the transmission of price data from businesses to the statistical office in a secure and cost-effective manner, while minimizing the administrative burden of the respondent. In principle, the relevant prices for a PPI should be the basic prices actually received by the establishment. For some products, the prices collected may be *estimated* transaction prices because the transaction sampled did not have sales during the reference period. In addition, it is generally neither practical nor cost-effective to try to collect prices each month or quarter directly from establishments by personal visits. Data can effectively be collected using mail questionnaires, telephone contacts, fax, and electronic media. A range of approaches to PPI data collection are presented in Chapter 6: postal survey, automated telephone response, personal interview, telephone interview, and Internet data provision. All of these methods rely on good questionnaire design, good respondent relations, and good interviewing techniques. The exact methods chosen by countries for particular industries will depend on the special circumstances applicable to each form of collection in their industry/country.

L.2.4 Continuity of price collection

1.202 A PPI is intended to measure *pure price changes*. The products whose prices are collected and compared in successive time periods should ideally be perfectly *matched*—that is, they should be identical in respect of their physical and economic characteristics. Identical economic characteristics include the terms and conditions of sale. When the products are perfectly matched, the observed price changes are *pure* price changes. When selecting representative products, it is therefore necessary to ensure that enough of them can be expected to remain on the market over a reasonably long period of time in exactly the same form or condition as when first selected. Without continuity, there would not be enough price changes to measure.

1.203 Having identified the products whose prices are to be collected, the normal strategy is to ask the respondent to continue pricing exactly those same products for as long as possible. The respondents can do this if they are provided with very precise, or tight, specifications of the products to be priced. Alternatively, they must keep detailed records themselves of the products that they have selected to price.

1.204 The ideal situation for a price index would be one in which all the products whose prices are being recorded remain on the market indefinitely without any change in their physical and economic characteristics, except of course for the timing of their sale.[12] Most products, however, have only a

[12]It is worth noting that many theorems in index number theory are derived on the assumption that exactly the same (continued)

limited economic life. Eventually, they disappear from the market to be replaced by other products. Because the universe of products is continually evolving, the representative products selected initially may gradually account for a progressively smaller share of output and sales. As a whole, they may become less and less representative. Since a PPI is intended to represent all products, some way has to be found to accommodate the changing universe of products. In the case of producer durables whose features and designs are continually being modified, some models may have very short lives indeed, being on the market for only a year or less before being replaced by newer models.

1.205 At some point the continuity of the series of price observations may have to be broken. It may become necessary to compare the prices of some products with the prices of other new ones that are very similar, but not identical. Statistical offices must then try to eliminate from the observed price changes the estimated effects of the changes in the characteristics of the products whose prices are compared. In other words, they must try to adjust the prices collected for any changes in the quality of the products priced, as explained in more detail below. In the limit, a completely new product may appear that is so different from those existing previously that quality adjustment is not feasible, and its price cannot be directly compared with that of any previous product. Similarly, a product may become so unrepresentative or obsolete that it has to be dropped from the index because it is no longer worth trying to compare its price with those of any of the products that have displaced it. Similar issues of course arise for establishments, although the focus here is on products.

L.2.5 Resampling

1.206 One strategy to deal with the changing universe of products would be to resample, or reselect, at regular intervals the complete set of products to be priced. For example, with a monthly index, a new set of products could be selected each January. Each set of products would be priced until the following January. Two sets have to be priced each January in order to establish a link between each set of 12 monthly changes. Resampling each year would be consistent with a strategy of updating the revenue weights each year.

1.207 Although resampling may be preferable to maintaining an unchanged sample or selection, it is not used much in practice. Systematically resampling the entire set of products each year would be difficult to manage and costly to implement. Moreover, it does not provide a complete solution to the problem of the changing universe of products because it does not capture price changes that occur at the moment of time when new products or new qualities are first introduced. Many producers deliberately use the time when products are first marketed to make significant price changes. A more practical way in which to keep the product sample up to date is to rotate it gradually by dropping certain products and introducing new ones. Products may be dropped for two reasons:

- The product is believed by the respondent or central office to be no longer representative. It appears to account for a steadily diminishing share of the total revenue within the product group or industry in question.
- The product may simply disappear from the market altogether. For example, among other reasons, it may have become obsolete due to changing technology, or unfashionable due to changing tastes.

1.208 At the same time, new products or new qualities of existing products appear on the market. At some point, it becomes necessary to include them in the list of products priced. This raises the general question of the treatment of quality change and the treatment of new products.

M. Adjusting Prices for Quality Changes

1.209 The treatment of quality change is perhaps the greatest challenge facing PPI compilers. It is a recurring theme throughout the *Manual*. It presents both conceptual and practical problems for compilers of PPIs. The whole of Chapter 7 is devoted to the treatment of quality change, and Chapter 8 addresses the closely related topic of new goods and product substitution.

1.210 When a sampled product is no longer produced or is unrepresentive and is dropped from the list of products priced in some establishment, the

set of goods and services is available in both the time periods being compared.

normal practice is to find a new product to replace it. This is in order to ensure that the sample, or selection, of sampled products remains sufficiently comprehensive and representative. If the new product is introduced specifically to replace the old one, it is necessary to establish a link between the series of past price observations on the old transaction and the subsequent series for the new transaction. The two series of observations may, or may not, overlap in one or more periods. In many cases, there can be no overlap because the new quality, or model, is introduced only after the one that it is meant to replace is discontinued. Whether or not there is an overlap, the linking of the two price series requires some estimate of the change in quality between the old product and the product selected to replace it.

1.211 However difficult it is to estimate the contribution of the changed quality to the change in the observed price, it must be clearly understood that *some estimate has to be made either explicitly or, by default, implicitly*. The issue cannot be avoided or bypassed. All statistical offices have limited resources and many may not have the capacity to undertake the more elaborate explicit adjustments for quality change described in Chapter 7. However, even though it may not be feasible to undertake an explicit adjustment through lack of data or resources, it is not possible to avoid making some kind of implicit adjustment. Even apparently "doing nothing" necessarily implies some kind of adjustment, as explained below. Whatever the resources available to them, statistical offices must be conscious of the implications of the procedures they adopt.

1.212 Three points are stressed in Section A of Chapter 7:

- The pace of innovation is high, and possibly increasing, leading to continual changes in the characteristics of products.
- There is not much consistency among countries in the methods they use to deal with quality change.
- A number of empirical studies have demonstrated that the choice of method does matter, since different methods can lead to very different results.

M.1 Evaluation of the effect of quality change on price

1.213 It is useful to try to clarify why one would wish to adjust the observed price change between two products that are similar, but not identical, for differences in their quality. A change in the quality of a good or service occurs when there is a change in some, but not most, of its characteristics. For purposes of a PPI, a quality change must be evaluated from the producer's perspective with regard to the revenue received. As explained in Section B of Chapter 7, the evaluation of the quality change is essentially an estimate of the per-unit change in revenue that a producer will receive for the new characteristics possessed by the new quality using the same technology. This amount is not a price change because it represents the monetary value of the change in the value of production that is involved to produce the new quality. The value can either be estimated on the basis of the value to the user of the new quality, or the production costs from the producer.

1.214 In many cases the concern can be seen with a need to make a quality adjustment to either the original or replacement product's price. The prices of two products need to be compared. They differ in quality, so some of the difference in price is due to quality differences. A quality adjustment in this instance is seen as an adjustment to the price (or price change) of the original or replacement product to remove that part due to quality differences. A quality adjustment can be seen as a coefficient that multiplies the price of, say, the replacement product to make it commensurate, from the producer's point of view, with the price of the original. To take a simple example, suppose that the quantity of some product and its replacement are variable and that quantity k of the replacement is produced using the same technology at the same cost and sold for the same price as quantity j of the original. The producer is indifferent between selling one unit of the original and j/k units of the replacement. To make the price of one unit of the replacement commensurate with the price of one unit of the original, it must be multiplied by k/j. This is the required quality adjustment.

1.215 For example, if two units of the replacement product are equivalent to three of the original, the required quality adjustment to be applied to the price of the replacement product is ⅔. Suppose the revenue from one unit of the replacement is the same as one unit of the original, then the price of the replacement, after adjusting for the change in quality, is only ⅔ that of the price of the original. If one unit of the replacement sells for twice the price of the original, then the quality-adjusted price is (2 × ⅔ =) 1⅓ that of the original: the price increase is 33 percent, not 100 percent. The PPI seeks to record the change between the price of the original and the quality-adjusted price of the replacement.

1.216 Of course, it is difficult to estimate the quality adjustment in practice, but the first step has to be to clarify conceptually the nature of the adjustment that is required in principle. In practice, producers often treat the introduction of a new quality, or new model, as a convenient opportunity in which to make a significant price change. They may deliberately make it difficult for purchasers to disentangle how much of the observed difference in price between the old and the new qualities represents a price change.

1.217 For PPI purposes, an explicit quality adjustment is often possible using differences in the costs of production between the two qualities. This approach works as long as production costs are based on the establishment using the same technology. Another alternative is to make an implicit adjustment by making an assumption about the pure price change: for example, on the basis of price movements observed for other products. The discussion below examines the implicit methods first and then the explicit methods. These approaches are examined in some detail in Sections D and E of Chapter 7.

1.218 When the technology changes, there is no comparable basis for comparing costs between the two qualities, and these procedures break down. An alternative approach would be to use hedonic regression techniques, which are also discussed below and in more detail in Section G of Chapter 7.

M.2 Implicit methods

M.2.1 Overlapping qualities

1.219 Suppose that the two qualities overlap, both being produced at time *t*. If both are produced and sold in a competitive market, economic theory suggests that the ratio of the prices of the new to the old quality should reflect their relative cost to producers and value to purchasers. This implies that the difference in price between the old and the new qualities does not indicate any change in price. The price changes up to period *t* can be measured by the prices for the old quality, while the price changes from period *t* onward can be measured by the prices for the new quality. The two series of price changes are linked in period *t*, the difference in price between the two qualities not having any impact on the linked series.

1.220 When there is an overlap, simple linking of this kind may provide an acceptable solution to the problem of dealing with quality change. In practice, however, this method is not used very extensively to deal with noncomparable replacements because the requisite data are seldom available. Moreover, the conditions may not be consistent with those assumed in the theory. Even when there is an overlap, the market may not have had time to adjust, particularly when there is a substantial change in quality. When the new quality first appears, the market is liable to remain in disequilibrium for some time. The producers of new qualities may price strategically over the product life cycle to, for example, price-discriminate in the early periods following introduction. There is a case in which the overlap method is used extensively in spite of these difficulties: when the index is rebased or products are rotated. The advantage of refreshing the sample is deemed to outweigh such disadvantages.

1.221 There may be a succession of periods in which the two qualities overlap before the old quality finally disappears from the market. If the market is temporarily out of equilibrium, the relative prices of the two qualities may change significantly over time, so that the market offers alternative evaluations of the relative qualities depending on which period is chosen. When new qualities that embody major new improvements appear on the market for the first time, it may be that their prices fall relatively to older qualities, before the latter eventually disappear. In general, if the price series for the old and new qualities are linked in a single period, the choice of period can have a substantial effect on the overall change in the linked series.

1.222 The statistician has then to make a deliberate judgment about the period in which the relative prices appear to give the best representation of the relative qualities. In this situation, it may be preferable to use a more complex linking procedure that uses the prices for both the new and the old qualities in several periods in which they overlap. Such information may be available from the respondent's records, although this requires a good relationship with the respondent and good record-keeping and retrieval systems by the respondent. In this case, the timing of the switch from the old to the new can have a significant effect on the long-term change in the linked series. This factor must be explicitly recognized and taken into consideration.

1.223 If there is no overlap between the new and the old qualities, the problems just discussed do not arise because no choice has to be made about when to make the link. However, other and more difficult problems take their place.

M.2.2 Nonoverlapping qualities

1.224 In the following sections, it is assumed that the overlap method cannot be used because there is a discontinuity between the series of price observations for the old and new qualities. Adopt the notation that the actual price of the new quality is P_t in period t and the price of the old quality is p_{t-1} in the previous period. Since the new quality is not available in period t, an imputation is made for its price in period t (p^*_t). In order to make the comparison between the prices in periods $t-1$ and t, a comparison between products of equal quality in the eyes of the producer is needed. The ratio p^*_t / P_t is the required quality adjustment since this ratio provides the estimate of the quality differences at the same point in time. Using lowercase ps for the old quality and uppercase Ps for the new, it is assumed that the price data available to the index compiler take the following form:

$$\ldots, \quad p_{t-3}, \quad p_{t-2}, \quad p_{t-1}, \quad P_t, \quad P_{t+1}, \quad P_{t+2}, \ \ldots$$

The problem is to estimate the pure price change between $t-1$ and t in order to have a continuous series of price observations for inclusion in the index. Using the same notation as above,

- Price changes up to period $t-1$ are measured by the series for the old quality;
- The change between $t-1$ and t is measured by the ratio p^*_t / p_{t-1}, where p^*_t is equal to P_t *after* adjustment for the change in quality; and
- Price changes from period t onward are measured by the series for the new quality.

1.225 The problem is to estimate p^*_t. This may be done explicitly by one of the methods described later. Otherwise, one of the implicit methods has to be used. These may be grouped into three categories.

- The first solution is to assume that $p^*_t / p_{t-1} = P_t / p_{t-1}$ or $p^*_t = P_t$. No change in quality is assumed to have occurred, so that the whole of the observed price increase is treated as a pure price increase. In effect, this contradicts the assumption that there has been a change in quality. The noncomparable replacement is deemed comparable.
- The second is to assume that $p^*_t / p_{t-1} = 1$, or $p^*_t = p_{t-1}$. No price change is assumed to have occurred, the whole of the observed difference between p_{t-1} and P_t being attributed to the difference in their quality.
- The third is to assume that $p^*_t / p_{t-1} = I$, where I is an index of the price change for a group of similar products, or possibly a more general price index.

1.226 The first two possibilities cannot be recommended as default options to be used automatically in the absence of any adequate information. The use of the first option could be justified only if the evidence suggests that the extent of the quality change is negligible, even though it cannot be quantified more precisely. "Doing nothing"—that is, ignoring the quality change completely—is equivalent to adopting the first solution. Conversely, the second could be justified only if evidence suggests that the extent of any price change between the two periods is negligible. The third option is likely to be much more acceptable than the other two. It is the kind of solution that is often used in economic statistics when data are missing.

1.227 Elementary indices are typically based on a number of series relating to different sampled products. The particular linked price series relating to the two qualities is therefore usually just one out of a number of parallel price series. What may happen in practice is that the price observations for the old quality are used up to period $t-1$ and the prices for the new quality from t onward, the price change between $t-1$ and t being omitted from the calculations. In effect, this amounts to using the third option: that is, estimating the missing price change on the assumption that it is equal to the average change for the other sampled products within the elementary aggregate.

1.228 It may be possible to improve on this estimate by making a careful selection of the other sampled products to include only those whose average price change is believed to be more similar to the product in question than the average for the group of sampled products as a whole. This procedure is described in some detail in Section D.2 of Chapter 7, where it is illustrated with a numerical example and is described as "targeting" the imputation or estimation.

1.229 The general method of estimating the price on the basis of the average change for the remaining group of products is widely used. It is sometimes described as the "overall" mean method. The more refined, targeted version is the "targeted" or "class" mean method. In general, one or other method seems likely to be preferable to either of the first two options listed above, although each case must be considered on its individual merits.

1.230 Although the overall mean method superficially seems a sensible practical solution, it may nevertheless give biased results, as explained in Chapter 7. It needs to be repeated that the introduction of a new quality is precisely the occasion on which a producer may choose to make a significant price change. Many of the most important price changes may be missed if, in effect, they are assumed to be equal to the average for products not subject to quality change.

1.231 It is necessary, therefore, to try to make an *explicit* adjustment for the change in quality, at least when a significant quality change is believed to have occurred. Again there are several methods that may be used.

M.3 Explicit quality adjustments

M.3.1 Quantity adjustments

1.232 The quality change may take the form of a change in the physical characteristics of the product that can easily be quantified, such as change in weight, dimensions, purity, or chemical composition of a product. It is generally a considerable oversimplification to assume that the quality of a product changes in proportion to the size of some single physical characteristic. For example, it is very unlikely to rate a refrigerator that has three times the capacity of a smaller one as worth three times the price of the latter. Nevertheless it is clearly possible to make some adjustment to the price of a new quality of different size to make it more comparable with the price of an old quality. There is considerable scope for the judicious, or commonsense, application of relatively straightforward quality adjustments of this kind. A discussion of quality adjustments based on size is given in Section E.2 of Chapter 7.

M.3.2 Differences in production/ option costs

1.233 An alternative procedure may be to try to measure the change in quality by the estimated change in the costs of producing the two qualities. The method is explained in Section E.3 of Chapter 7. The estimates can be made in consultation with the producers of the goods or services, if appropriate. This method, like the preceding one, is likely to be satisfactory only when the quality changes take the form of relatively simple changes in the physical characteristics of the good, such as the addition of some new feature, or option, to an automobile. It is not satisfactory when a more fundamental change in the nature of the product occurs as a result of a new discovery or technological innovation. It is clearly quite unacceptable, for example, when a drug is replaced by another more effective variant of the same drug that also happens to cost less to produce.

1.234 Another possibility when the quality change is more complex or subtle is to seek the advice of technical experts, especially when the respondent may not have the knowledge or expertise to be able to assess or evaluate the significance of all of the changes that may have occurred, at least when they are first made.

M.3.3 The hedonic approach

1.235 Finally, it may be possible to systematize the production/option cost approach by utilizing econometric methods to estimate the impact of observed changes in the characteristics of a product on its price. The market prices of a set of different qualities or models are regressed on what are considered to be the most important physical or economic characteristics of the different models. This approach to the evaluation of quality change is known as *hedonic analysis*. When the characteristics are attributes that cannot be quantified, they may be represented by dummy variables. The regression coefficients measure the estimated marginal effects of the various characteristics on the prices of the models and can therefore be used to estimate the effects on price of changes in those characteristics.

1.236 The hedonic approach to quality adjustment can provide a powerful, objective, and scientific method of estimating the effect on price of changes in quality for certain kinds of products. It has been particularly successful in dealing with computers. The economic theory underlying the hedonic approach is examined in more detail in Chapter 21. The application of the method is explained in some detail in Section E.4 of Chapter 7. Products can be viewed as bundles of tied characteristics that are not individually priced because the producer sells the bundle as a single package. The objective is to try to "unbundle" the characteristics to estimate how much they contribute to the total price. In the case of computers, for example, three basic characteristics are the processor speed, the size of the random-access memory (RAM), and the hard drive capacity. An example of a hedonic regression using these and other characteristics is given in Section E.4 of Chapter 7, the actual numerical results being given in Table 7.3.

1.237 The results obtained by applying hedonics to computer prices have had a considerable impact on attitudes toward the treatment of quality change in PPIs. They have demonstrated that for goods where there is rapid technological change and improvements in quality, the size of the adjustments made to the market prices of the products to offset the changes in the quality can largely determine the movements of the elementary price index. For this reason, the *Manual* contains a thorough treatment of the use of hedonics. Reference may be made to Section G of Chapter 7 for further analysis, including a comparison showing that the results obtained by using hedonics and matched models can differ significantly when there is a high model turnover.

M.4 Conclusions on quality change

1.238 It may be concluded that statistical offices must pay close attention to the treatment of quality change and try to make explicit adjustments whenever possible. The importance of this topic can scarcely be overemphasized. Failure to pay proper attention to quality changes can introduce serious biases into the PPI.

N. Product Substitution and New Goods

N.1 Replacement products

1.239 As noted in the previous section, price indices would, ideally, seek to measure pure price changes between matched products that are identical in the two periods compared. However, as explained in Chapter 8, the universe of products that a PPI has to cover is a dynamic universe that is gradually changing over time. Pricing matched products constrains the selection of products to a static universe of products given by the intersection of the two sets of products existing in the two periods compared. This static universe by definition excludes both new products and disappearing products, and in both cases their price behavior is likely to diverge from that of the matched products. Price indices have to try to take account of the price behavior of new and disappearing products so far as possible.

1.240 A formal consideration and analysis of these problems is given in Appendix 8.1 in Chapter 8. A replacement universe is defined as one that starts with the base-period universe but allows new products to enter as replacements as some products disappear. Of course, quality adjustments of the kind discussed in the previous section are needed when comparing the prices of the replacement products with those of the products that they replace.

1.241 One way in which to address the underlying problem of the changing universe is by sample rotation. This requires a completely new sample of products or establishments to be drawn to replace the existing ones. The two samples must overlap in one period that acts as the link period. As noted in Section B.2 of Chapter 8, this procedure can be viewed as a systematic exploitation of the overlap method of adjusting for quality change. It may not, therefore, deal satisfactorily with all changes in quality that occur, because the relative prices of different goods and services at a single point in time may not provide satisfactory measures of the relative qualities of all the goods and services concerned. Nevertheless, frequent sample rotation helps by keeping the sample up to date and may reduce the extent to which explicit quality adjustments are required. Sample rotation is, however, expensive.

N.2 New goods and services

1.242 The difference in quality between the original product and the one that replaces it may become so great that the new quality is better treated as a new good, although the distinction between a new quality and a new good is inevitably somewhat arbitrary. As noted in Section D of Chapter 8, a distinction is also drawn in the economic literature between evolutionary and revolutionary new goods. An evolutionary new good or service is one that meets existing needs in much more efficient, or new, ways; a revolutionary new good or service provides completely new kinds of services or benefits. In practice, an evolutionary new good can be fitted into some subclass of the product or industry classification, whereas a revolutionary new good will require some modification to the classification in order to accommodate it.

1.243 As explained in Section D.2 of Chapter 8, a major concern with new goods or services relates to the timing of the introduction of the new product into the index. It is often the case that new goods enter the market at a higher price than can be sustained in the longer term, so that their prices typically tend to fall over the course of time. Conversely, the quantities sold may be very small initially but may increase significantly over time. These complications make the treatment of new products particularly difficult, especially if they are revolutionary new goods. Because of the tendency for the price of a new good to fall even after it has been introduced, it is possible that important price reductions may fail to be captured by PPIs because of the technical difficulties created by new products. The issues are examined in some detail in Section D of Chapter 8. The chapter concludes by expressing concern about the capacity of PPIs to deal satisfactorily with the dynamics of modern markets. In any case, it is essential that statistical offices are alert to these issues and adopt procedures that take account of them to the maximum extent possible, given the data and resources available.

O. Revenue Weights

1.244 Once the price data have been collected and adjusted as necessary, the next step in the calculation of a PPI is to combine, or average, the elementary price indices to arrive at price indices at higher levels of aggregation up to the overall PPI itself. For this purpose, revenue weights are needed for the various elementary aggregates. These weights are needed whatever index number formula is used for aggregation purposes. Chapter 4 is concerned with the derivation and sources of the revenue weights.

O.1 Establishment censuses and surveys

O.1.1 Establishment or business censuses

1.245 The establishment or business census covers all establishments that have productive activity within the geographic borders of the country. These censuses may be conducted over a span of years, with different economic activities covered at different times during the cycle. For example, a census of agriculture would be conducted one year, a census of industrial activities (mining, manufacturing, and energy supply) completed during the next year, followed by a census of services. In some instances there may be a size cutoff to exclude very small establishments. For example, some countries exclude establishments with fewer than five employees or with some low threshold of annual production, or complete the census using only a sample of small establishments.

1.246 A detailed accounting of annual output in value (at basic prices) and quantity terms by detailed product classification is typically obtained at the enterprise or establishment level. This would include sales and inventories by product, as well as value and quantity of inputs at the prices paid by producers. These data can be used to derive the revenue weights by detailed product classification and establishment. This is an excellent source of weight data, assuming that the coverage of economic activity is essentially complete.

O.1.2 Enterprise or industry surveys

1.247 These surveys differ from censuses primarily in three respects:

- The coverage is limited to a sample of establishments rather than a full enumeration;
- The product detail is limited to higher aggregate levels such as groups, and
- The types of data requested are generally more limited than those requested in a census.

1.248 For example, product information in the census may be obtained at the eight-digit product code level using PRODCOM, with complete detail on product sales and inventories, while in the industry survey data are reported at the six-digit level and are requested only for sales. Also, data may be reported only for the enterprise rather than broken down by establishment.

1.249 Thus, for enterprise or industry surveys, the weights that are available will generally be for higher levels in the aggregation structure, such as product group and industry, rather than detailed product and establishment. The use of these weights for the PPI will depend on how the PPI aggregation structure has been established. If multitier weights (for example, one set of weights for the industry level and above, and another set of weights at the establishment level and below) have been set up, the survey results could be used for aggregation at higher levels, while the weights at lower levels are determined separately. For example, the survey weights could be used for aggregating from the four-digit industry level to higher levels, while sampling weights (that is, sampling fractions from probability selection procedures) could be used at the establishment and product level. In this scheme, the weights at the higher levels would be updated periodically from the industry survey data, while the weights at the lower levels would be updated as the samples of establishments and products are refreshed. This process is discussed in more detail in Chapter 5.

O.2 Other sources for estimating revenue weights

O.2.1 National accounts

1.250 Although much of the same source data described above would also be used in developing the output data for the production account in the national accounts, there can be significant differences. In a number of countries, there may be significant undercoverage in annual industry surveys owing to the exclusion of informal activities. National accountants often make adjustments from a variety of sources for this type of undercoverage or for known biases in the survey data. In such instances, the adjusted national accounts information on output by industry may prove to be a better source of weight information at the industry level than the original survey data.

1.251 The national accounts often provide additional detail on weights particularly if supply and use tables or input/output tables are available. The information on commodity flows for various industries and commodities by type of use is an excellent source of net weight information for development of stage-of-processing indices. One drawback of national accounts data is that the estimates include imputations for nonmarket activities, and such imputed data may not be appropriate for use as weights in an index whose coverage is primarily market activity.

O.2.2 Business register

1.252 Most countries maintain a business register that provides a list of firms that are involved in productive activities. Such registers usually contain information on location, economic activity, size (for example, employment, payrolls, value of annual production, or turnover), contact persons, tax information, etc. The business register could be an alternative source of weight information, particularly if business censuses are not conducted on a regular basis or if annual surveys do not provide sufficient information for establishing weights. This is particularly true if there is an ongoing system for updating and maintaining the information contained in the register, and it contains data at the establishment level.

1.253 There are several shortcomings to the use of these registers for weight information. Often the business register is updated only when a firm begins operations. Unless the register is maintained by purging firms that are no longer in business, it will have superfluous information. The information on the size of the firm also needs to be updated on a regular basis. Much of the information may relate to the time at which the firm was introduced into the register. Also, the business register may comprise a list of *enterprises*, which is not completely suitable for the PPI, where the concern is to obtain information at the *establishment* level. The register will usually be devoid of information on products, which means that additional data collection will be necessary before weights can be established at the product level.

O.2.3 Additional sources of weights

1.254 A wide variety of administrative data on production values may be available from public agencies charged with regulating or monitoring certain economic activities. For example, national, regional, or local governmental bodies regulate many public utilities, communication, and transport activities. Typically, these agencies require detailed annual reports that provide information on production value and/or turnover. These sources also have records of all regulated enterprises/establishments, which can be used as a source for a sampling frame.

1.255 In many countries, data on retail and wholesale turnover are produced on a regular basis from separate surveys. Such data, if maintained at a detailed economic activity level, could serve as a source of weights for wholesale and retail economic activities. This, of course, would depend on whether wholesale and retail trade will be included in the PPI and if the survey information is deemed reliable for use as weights.

1.256 Customs records are a source of information on exports by product and enterprise. If detailed customs records are maintained and available for statistical purposes, information on detailed products by shipping enterprise should be available and provide a source for weights as well as a potential sampling frame for sampling export products.

P. Basic Index Calculations

1.257 Chapter 9 provides a general overview of the ways in which PPIs are calculated in practice. The methods used in different countries are by no means all the same, but they have much in common. There is clearly interest from users as well as compilers in knowing how most statistical offices set about calculating their PPIs. The various stages in the calculation process are illustrated by numerical examples.

1.258 Chapter 9 is descriptive and not prescriptive, although it does try to evaluate the strengths and weaknesses of existing methods. It makes the point that, because of the greater insights into the properties and behavior of indices gained in recent years, it is now recognized that not all existing practices are necessarily optimal.

1.259 Because the various stages involved in the calculation process have, in effect, already been summarized in the preceding sections of this chapter, it is not proposed to repeat them all again in this section. However, it may be useful to give an indication of the nature of the contents of Chapter 9.

P.1 Elementary price indices

1.260 Chapter 9 describes how the elementary price indices are calculated for the elementary aggregates. It reviews the principles underlying the delineation of the elementary aggregates themselves. Elementary aggregates are relatively small groups of products that are intended to be as homogeneous as possible, not merely in terms of the physical and economic characteristics of the products covered, but also in terms of their price movements. They may also be broken down by location and establishment type. Samples of prices are collected for a number of representative transactions across establishments within each elementary aggregate in order to estimate the elementary price index for that aggregate, with each elementary price index providing a building block for the construction of the higher-level indices.

1.261 Section B of Chapter 9 considers the consequences of using alternative elementary index formulas to calculate the elementary indices. It proceeds by means of a series of numerical examples that use simulated price data for four different products within an elementary aggregate. The elementary indices and their properties have been explained in some detail in Section I above. An elementary price index may be calculated either as a chain index or as a direct index: that is, *either* by comparing the price each month, or quarter, with that in the immediately preceding period *or* with the price in the fixed-price reference period. Table 9.1 uses both approaches to illustrate the calculation of three basic types of elementary index, Carli, Dutot, and Jevons. It is designed to highlight a number of these indices' properties. For example, it shows the effects of "price bouncing," in which the same four prices are recorded for two consecutive months, but the prices are switched among the four products. The Dutot and Jevons indices record no increase, but the Carli index registers an increase. It also illustrates the differences between the direct and the chain indices. After six months, each of the four prices is 10 percent higher than at the start. Each of the three direct indices records a 10 percent increase, as also do the chained Dutot and Jevons indices because they are transitive. The chained Carli, however, records an increase of 29 percent, which is interpreted as illustrating the systematic upward bias in the Carli formula resulting from its failure to satisfy the time reversal test.

1.262 Section B.3 of Chapter 9 notes that the chaining and direct approaches have different implications when there are missing price observations, quality changes, and replacements. It concludes that the use of a chain index can make the estimation of missing prices and the introduction of replacement products easier from a computational point of view.

1.263 Section B.5 of Chapter 9 examines the effects of missing price observations, distinguishing between those that are temporarily missing and those that have become permanently unavailable. Table 9.3 contains a numerical example of the treatment of the temporarily missing prices. One possibility is simply to omit the product whose price is missing for one month from the calculation of indices that compare that month with the preceding and following months and also with the base period. Another possibility is to impute a price change on the basis of the average price for the remaining products using one or other of the three types of average. The example is a simplified version of the kind of examples that are used in Chapter 7 to deal with the same problem.

1.264 The possibility of using other elementary index formulas is considered in Section B.6. The harmonic mean of the price relatives, P_H, and the ratio of the harmonic means, R_H, are examined. The P_H has the inverse properties of the Carli index, P_C, and can therefore be assumed to have an opposite bias. As it is also a rather difficult concept to explain, it is not recommended. The Jevons index, P_J, has attractive axiomatic properties but is advised only when particular patterns of substitution are expected. The geometric mean of the P_C and the P_H, a kind of elementary Fisher index, remains a possibility with some theoretical attractions, though because it provides close results to the Jevons index, P_J, is advised only under the substitution possibilities discussed in Chapter 20.

1.265 Section C of Chapter 9 discusses the issue of consistency in aggregation between lower- and higher-level indices that may arise if different formulas are used at different levels. Consistency of aggregation means that if an index is calculated stepwise, by calculating intermediate indices that are themselves subsequently aggregated, the same result should be obtained as if the calculation were made in a single step without the intermediate indices. This can be an advantage for purposes of presentation. If a Young or Laspeyres index is used for the higher-level indices, including the overall PPI itself, then the Carli index is the required form of elementary index that is consistent with it.[13] Given that the Carli does not emerge as the preferred elementary index from the axiomatic and economic approaches to elementary indices, this creates a dilemma when the Laspeyres or Young formula is used. It is suggested that consistency in aggregation may not be so important if there are different degrees of substitution within elementary aggregates at the lower level, as compared with the degree of substitution between products in different elementary aggregates at the higher.

1.266 It is not necessary to use the same index formula for every elementary index. The characteristics of the price behavior within each elementary aggregate should be examined to identify the most appropriate formula. However, it may be decided to use a single formula throughout if resources are limited and computational procedures need to be kept as simple as possible.

P.2 Calculation of higher-level indices

1.267 Section C of Chapter 9 considers the calculation of the higher-level indices utilizing the elementary price indices and the weights provided for the elementary aggregates. In many instances statistical offices do not use a true Laspeyres index but rather a Lowe or Young index (discussed in Section B.1 above). These two indices use price reference periods and weight reference periods that differ, while in the Laspeyres index the price and weight reference period are one and the same. Typically the weight reference period precedes the price reference period in the version of the Young and Lowe indices used by statistical offices, owing to the time it takes to develop revenue weights from establishment surveys in earlier periods. It is at this stage that the traditional index number theory discussed in Chapters 15–17 comes into play. Since this theory has been explained in detail and in depth in these chapters, which are also summarized in Sections B–E of this chapter, it is not repeated here.

1.268 At the time the monthly PPI is first calculated, the only revenue weights available must inevitably refer to some earlier period or periods. As

[13]Also recall that the Jevons index would be consistent with a geometric Laspeyres at higher levels.

mentioned above, this predisposes the PPI to some form of fixed-basket index (Laspeyres, Lowe, or Young index, or chained Laspeyres index). However, at some later date estimates must become available of the revenues in the current period, so that retrospectively it becomes possible to calculate a Paasche-type index and superlative indices such as Fisher or Törnqvist.[14] There is some interest in calculating such indices later, if only to see how the original indices compare with the superlative indices. Some countries may wish to calculate retrospective superlative indices for this reason. Thus, although most of the discussion in Chapter 9 is based on the assumption that some type of fixed-basket index is being calculated, this should not be interpreted as implying that this is the only possibility in the long term.

P.3 Production and maintenance of higher-level indices

1.269 In practice, the higher-level indices up to and including the overall PPI are often calculated as Young indices: that is, as weighted averages of the elementary price indices using weights derived from revenues in some earlier weight reference period. This is a relatively straightforward operation, and a numerical example is given in Table 9.5 of Chapter 9, in which, for simplicity, the weight and price reference periods are assumed to be the same. Table 9.6 illustrates the case in which weight and price reference periods are not the same and the weights are price-updated between weight reference period b and the price reference period 0. This yields a Lowe index with quantities fixed for period *b*. It illustrates the point that statistical offices have two options when a new price reference period is introduced: they can either preserve the relative quantities of the weight reference period or they can preserve the relative revenues. They cannot do both. Price updating the revenue weights preserves the quantities and produces a Lowe index. A Lowe index with quantities fixed in period *b* might be preferred, because it has better axiomatic properties compared with a Young index with revenue shares from period *b*.

[14]In fact, if a Laspeyres index is used and the revenue shares do not change much through time, a geometric Laspeyres index will approximate a Törnqvist index (Chapter 9, Section C.6).

1.270 The weights in the PPI need to be updated periodically or problems will result when a fixed set of weights is used for a very long period of time. For example, the prices of consumer durables, especially when quality-adjusted, have been falling relative to other goods, although the quantities purchased and revenue share have increased. An out-of-date set of weights would give insufficient weight to these falling prices. In the presence of rapid changes in technology or tastes, the weights need to be updated frequently and not allowed to continue for too long.

1.271 Section C.7 of Chapter 9 notes that the introduction of new weights is a necessary and integral part of the compilation of a PPI over the long run. Weights have to be updated sooner or later, and some countries actually update their weights each year. Whenever the weights are changed, the index on the new weights has to be linked to the index on the old weights so that the PPI inevitably becomes a chain index over the long term. Chapter 9 also discusses the techniques for linking series together by developing a set of linking factors (coefficients) that can be used for either forward linking or backward linking. This involves calculating the higher-level indices on both the old and new weights during an overlap period.

1.272 Apart from the technicalities of the linking process, the introduction of new weights, especially if carried out at intervals of five years or so, provides an opportunity to undertake a major review of the whole methodology. New products may be introduced into the index, classifications may be revised and updated, and even the index number formula might be changed. Annual chaining facilitates the introduction of new products and other changes on a more regular basis, but in any case some ongoing maintenance of the index is needed whether it is annually chained or not.

P.4 Data editing

1.273 Chapter 9 concludes with Section D on data editing. It is included in Chapter 9 because data editing is a process that is closely linked to the actual calculation of the elementary prices indices. Data editing involves two steps: the detection of possible errors and outliers, and the verifying and correction of the data. Effective monitoring and quality control are needed to ensure the reliability of the basic price data fed into the calculation of the elementary prices indices on which the quality of the overall index depends.

Q. Organization and Management

1.274 The collection of the price data is a complex operation involving extensive work by a large number of statistical office staff and respondents. The whole process requires careful planning and management to ensure that data collected conform to the requirements laid down by the central office with overall responsibility for the PPI. Appropriate management procedures are described in Chapter 12 of the *Manual*.

1.275 Price collectors should be well trained to ensure that they understand the importance of selecting the right transactions for pricing on initiation of the sample. Inevitably, price collectors are bound to use their own discretion to a considerable extent. As already explained, one issue of crucial importance to the quality and reliability of a PPI is how to deal with the slowly evolving set of products. Products may disappear and have to be replaced by others, but it may also be appropriate to drop some products before they disappear if they have become quite unrepresentative. Price collectors and product analysts need appropriate training and very clear instructions and documentation about how to proceed. Clear instructions are also needed to ensure that price collectors and respondents report the correct prices when there are discounts, special offers, or other exceptional circumstances.

1.276 The price data reported also have to be subjected to careful checking and editing. Computers using standard statistical control methods can carry out many checks. It may also be useful to send out auditors to verify the quality and accuracy of reported price data. The various possible checks and controls are explained in some detail in Chapter 12.

1.277 Improvements in information technology should obviously be exploited to the fullest extent possible. For example, collectors may use establishment websites for price information; establishments can use some form of electronic data transfer to report their prices or use an Internet-based reporting system set up by the statistical office.

R. Publication and Dissemination

1.278 As noted here and in Chapter 2, the PPI is an extremely important statistic whose movements can influence the central bank's monetary policy, affect stock markets, influence wage rates and contract settlements, and so on. There must be public confidence in its reliability and the competence and integrity of those responsible for its compilation. The methods used to compile it must therefore be fully documented, transparent, and open to public scrutiny. Many countries have an official PPI advisory group consisting of both experts and users. Its role is not just to advise the statistical office on technical matters but also to promote public confidence in the index.

1.279 Users of the index also attach great importance to having the index published as soon as possible after the end of the each month or quarter, preferably within two or three weeks. On the other hand, most users do not wish the index to be revised once it has been published, and there can be some trade-off between timeliness and the quality of the index. For example, it would be possible to revise the index subsequently—by calculating a Fisher index when the requisite revenue data become available—without affecting the timeliness of the current index.

1.280 Publication must be understood to mean the dissemination of the results in any form. In addition to publication in print, or hard copy, the results should be released electronically and be available through the Internet on the website of the statistical office.

1.281 As explained in Chapter 13, good publication policy goes beyond timeliness, confidence, and transparency. The results must be made available to all users, within both the public and the private sectors, at the same time and according to a publication schedule announced in advance. There should be no discrimination among users in the timing of the release of the results. The results must also not be subject to governmental scrutiny as a condition for their release, and the results must be seen to be free from political or other pressures. There are many decisions to be taken about the degree of detail in the published data and the alternative ways in which the results may be presented. Users need to be consulted about these questions. These issues are discussed in Chapter 13. As they do not affect the actual calculation of the index, they need not be pursued further at this point.

Appendix 1.1: An Overview of Steps Necessary for Developing a PPI

1.282 This appendix provides a summary overview of the various steps involved in designing a PPI, deriving the index structure and weighting pattern, designing the sample, establishing price collections, calculating indices, and disseminating the results. It also outlines procedures for ensuring that the price samples, index structure, and weighting pattern remain representative. These issues are discussed in more detail in subsequent chapters.

1.283 In following the steps described below, it is important to be mindful of the practical experience of national statistical agencies, which has led to the identification of several important prerequisites for the construction and compilation of an accurate PPI. That is:

- The prices recorded in the index over time must relate to:

  (i) product specifications that are representative indicators of price change;
  (ii) constant-quality products with fixed specifications; and
  (iii) actual market transactions inclusive of all discounts, rebates, etc;

- The weights need to be representative of the relevant pattern of transactions over the period for which they are used for index aggregation; and
- The aggregation formulas used must be appropriate to the needs of the particular index and not yield significant bias or drift.

Basic Steps in PPI Development

1.284 Ten basic steps can be defined for the design, construction, dissemination, and maintenance of a producer price index. These steps are:

1. Determining the objectives, scope, and conceptual basis of the index;
2. Deciding on the index coverage and classification structure;
3. Deriving the weighting pattern;
4. Designing the sample;
5. Collecting and editing the prices;
6. Adjusting for changes in quality;
7. Calculating the index;
8. Disseminating the indices;
9. Maintaining samples of businesses and product specifications; and
10. Reviewing and reweighting the index.

1.285 A summary of the issues involved with each of these steps is provided in the rest of this appendix.

Step 1. Determining the objectives, scope, and conceptual basis of the index

1.286 Decisions made following close consultation with users (both external users and internal national statistical agency users such as national accounts) about the objectives of the proposed PPI, and hence its scope, have a fundamental influence on the determination of the conceptual basis of the index.

1.287 Uses range from economic policy (for example, inflation analysis), to business applications such as contract price escalation and monitoring of relative performance, industry policy formulation, and volume estimation (for example, national accounts growth estimates). All key stakeholders need to be consulted early in the index design stage to ascertain what their needs are (that is, what are the questions they are aiming to answer and, hence, the characteristics of the required statistics).

1.288 After the objectives have been determined, informed decisions need to be made about the economic scope of the index—that is, what is the domain of price transactions that the index is aiming to measure.

1.289 As discussed earlier and in Chapter 2, it is necessary to determine whether the index is to be demand based (an input index) or supply based (an output index).

1.290 Assuming it is to be supply based (which is the most common form of PPI compiled by national statistical agencies), an important consideration in defining the scope of the index is whether it should be a *net* or *gross* output index (see Chapter 2). The scope of a gross output index is broader than that of a net output index in that it also includes *intrasectoral* transactions. That is, taking as an example a manufacturing sector output index, transactions between different manufacturers would be in scope (for example, sales of refined sugar for the production of soft drinks), not just sales outside the manufacturing sector.

1.291 A further consideration is whether the scope of the index should be confined to domestic transactions only, or be broadened to include transactions with the rest of the world (exports).

1.292 Having decided on the objectives and scope of the new PPI, it is then necessary to formulate the detailed conceptual basis of the measure, again in consultation with users as necessary. Conceptual characteristics to be determined include the point of pricing, the valuation basis, coverage, and classification structure.

1.293 Decisions on the point of pricing and on the valuation basis of the index largely fall into place once the objectives and scope have been determined. As a rule of thumb, for an output (supply-based) index, the pricing point is ex-producer (for example, ex-factory, ex-farm, ex-service provider) with a valuation basis of "basic prices" (that is, reflecting the amount received by the producer exclusive of any taxes on products and transport and trade margins). On the other hand, for an input (demand-based) index, the pricing point is "delivered into store" with a valuation basis of "purchasers' prices" (that is, reflecting the amount paid by the purchaser inclusive of any taxes on products and transport and trade margins).

Step 2. Deciding on the index coverage and classification structure

1.294 The issue of the actual coverage of the domain of transactions defined by the economic scope of a PPI can be viewed from several perspectives.

1.295 Choices need to be made as to whether *nonmarket* transactions should be included or excluded. The decision will be based on a consideration of the primary objective of the index and on practical pricing considerations such as the following.

1.296 For example, for an index that aims to reflect changes in actual market transaction prices, prices of notional transactions such as changes in stocks and imputed dwelling rents have no place (in contrast to the national accounts, where conventions provide for the valuation of certain nontraded goods and services so that economic activity is not omitted). Further, it can be argued that for a price index designed primarily for the purpose of analyzing inflation, prices of commodities that are not determined on the basis of buyers and sellers interacting (that is, as a result of supply and demand forces) should be excluded because they do not provide signals of market-driven inflation. Examples include the nominal prices sometimes charged by providers of general government services (for example, health and education) and prices that are heavily subsidized through government funding or regulated by government policy.

1.297 Similarly, practical decisions need to be made about whether efforts should be expended on trying to capture price changes of goods and services transacted in the nonobserved ("hidden") economy. Issues such as the relative size of the nonobserved economy and its accessibility for price measurement need to be considered.

1.298 Other coverage issues include the treatment of intracompany transfer prices and capital formation on own account. A decision needs to be made whether these flows are to be included or excluded. If they are to be included, an assessment needs to be made about whether the book entry valuations recorded in the company accounting records are realistic in terms of being contemporary market-based estimates, or are merely notional estimates. If the latter, the preferred approach would be to assign the weight associated with these transfers to the prices obtained from businesses engaging in arms-length trading.

1.299 There are also issues of geographic coverage, in particular whether international transactions should be priced for the PPI. That is, should prices for direct imports and foreign purchases of residents used as inputs to productive activity be priced for an input index, and, on the other hand, should prices for exports and domestic purchases of nonresidents be included in an output index.

1.300 An output index can be constructed under alternative classification structures. The most common constructs are based on industry, commodity, or stage of processing. International industry classifications (for example, ISIC) and commodity classifications (for example, CPC) are available for use in index construction to ensure adherence to accepted statistical standards and facilitate international comparisons. Many countries or regions have developed local adaptations of these classifications that still conform to the underlying principles.

1.301 Formal classifications are hierarchical in nature. For example, ISIC covers the entire economic activity of an economy and provides for the progressive aggregation of data from a fine level of detail (for example, soft drink manufacturing), through successively broader levels of aggregation (for example, manufacturing of beverages; food, beverage, and tobacco manufacturing; total manufacturing). In designing an index classification structure, it is important to consider issues such as:

- *Publication goals*. In particular, the level of detail to be released, whether the indices will be national only or include regional series, and the needs of internal users;
- *Potential bias in the index due to product replacement and new goods*. There are opportunities to minimize such bias through grouping products that are close substitutes.

1.302 Having determined the index classification structure, the weighting pattern needs to be derived and issues of sample design and price collection addressed.

Step 3. Deriving the weighting pattern

1.303 A price index can be considered as being built up from samples of prices of individual (or price relatives) that are progressively weighted together through successive levels of aggregation within a classification framework.

1.304 In considering the development of an index weighting pattern, two different categories of indices need to be considered: lower-level indices (sometimes referred to as elementary aggregates) and upper-level indices.

1.305 The lower-level indices are built up by combining together the individual prices using one of a range of available price index formulas. At this initial level of aggregation, the internal weighting can be either *explicit* or *implicit*. If *explicit* weights are used, then, as part of the price collection activity, it is necessary to obtain relevant value data (for example, product sales). This is discussed further under Step 5 below. On the other hand, if *implicit* weights are used, then the design features of the sampling techniques employed to select the product specifications for pricing need to result in the prices being "self-weighted." Such a result would be achieved, for example, by using probability sampling proportional to size.

1.306 Upper-level indices are formed through weighting together lower-level indices through progressive levels of aggregation defined by the classification structure, usually employing weights that are fixed for a period (say one, three, or five years) between index reweighting.

1.307 The selection of the level in the index hierarchy at which the structure and weights are fixed for a period is particularly important. The main advantage of setting the level relatively high (for example, at the four-digit industry or product group level) is that the price statistician then has greater discretion to update the lower-level price samples (at the establishment and product level), their structure, and their internal weighting on a needs basis as market activity changes. New products and establishments can be introduced easily into the samples, and the weights at the lower level reestablished on the basis of more recent market conditions. That is, there is greater opportunity to keep the index representative through an ongoing program of sample review (see Step 9).

1.308 On the other hand, if the level is set relatively low in the index structure, there is less freedom to maintain the representativeness of the index on an ongoing basis, and there will be a greater dependence on the periodic index review and reweighting process (see Step 10). In such circumstances, the argument for frequent reweighting becomes stronger.

1.309 Assume a manufacturing output index is to be developed with the broad index structure based on ISIC. In order to derive the upper-level weighting pattern, a data source is required; potential sources include industry surveys, economic censuses, input-output tables, and international trade statistics.

1.310 The relevant values need to be assigned to each of the industry groupings, taking a top-down approach. It may be appropriate to assign the values associated with industry output that is not going to be directly priced in the index (either because it is too small, or because of practical pricing difficulties) to a related industry in order to maintain the correct broad weighting relativities. The assumption underlying this practice is that the price movements of the unpriced products are more likely to be similar to those of related products than to those of the aggregate of all the products priced in the index.

1.311 Weights aim to be representative of the pattern of transactions expected to prevail during the period for which they are used in the index construction (perhaps one year, or five years, depending on the frequency of reweighting). It may therefore be necessary to adjust some of the values to *normalize* them and overcome any irregularities in the data for the particular period from which it is being sourced (for example, as a result of a one-off increase in production of a product in response to a temporary increase in demand). Alternatively, the weights may be *smoothed* by basing them on data from a run of years (say, three years). Other adjustments may be needed to overcome problems of seasonality that are discussed in Chapter 22.

1.312 If the time reference (base price) period of the index is different from the period from which the value weights are derived, then it is important to *revalue* the weights to the prices of the time reference (base price) period using relevant price indices in order to ensure that the weights are effectively based on the underlying quantities or volumes.

1.313 Having assigned weights to the upper-level index structure that are to be fixed for a period of one or more years, the next step is to consider the lower-level index construct and the sample design.

1.314 If explicit lower-level weighting of price samples is to be incorporated, then output or sales data[15] will need to be obtained directly from manufacturing businesses during the process of establishing price collections (Step 5).

Step 4. Designing the sample

1.315 Take the example of soft drink manufacturing used in Step 2, and assume that this is an *index regimen item* with a fixed weight of, say $100 million, within the upper-level index structure. It is now necessary to choose techniques for selecting samples of businesses (statistical units) to provide transaction prices of a selection of representative products on an ongoing basis. The prices, or price movements, collected from different businesses will be aggregated to form indices.

1.316 To select a sample of businesses (for example, manufacturers of soft drinks), the first step is to identify the sample frame (that is, the population of units from which to select). Possible frame sources include registers of businesses maintained by national statistical agencies, commercially maintained lists (for example, as used for marketing mail outs), company registers, taxation records, telephone directory "yellow pages," etc., or some combination of such sources.

1.317 Either probability (scientific) sampling or nonprobability (judgmental) sampling techniques can be used,[16] and the choice may be based largely on practical considerations such as resources to develop sampling frames, data sources like a business register to develop sampling frames, and data collection resources to undertake the required intensive efforts to recruitment establishments. Some agencies use a combination of techniques, for example scientific sampling to select the businesses and judgment sampling to select the product specifications for pricing.

1.318 In deciding how to select the sample of businesses, the degree of industry concentration is a relevant consideration. For example, in a highly concentrated industry dominated by, say, three businesses producing over 90 percent of the output, it may be acceptable to aim for high, rather than complete, coverage and to select only the three largest businesses.

1.319 However, as the degree of concentration decreases, the greater is the need for the sample to include a selection of smaller businesses. If, for example, the three largest businesses account for less than 70 percent of the industry output, with the remaining 30 percent being produced by a large number of small businesses, it may not be possible to

[15]As discussed in Chapter 4 and subsequently in Chapter 5, output data, which include sales and inventories, are preferred for weights. However, in many countries data on output are not readily available, and for practical reasons a close proxy such as sales or value of shipments may be used.

[16]Judgmental sampling should be avoided, if possible. Often cutoff sampling, as discussed in Chapter 4, Section D, can be used in place of judgmental sampling.

achieve adequate representation of price movements by relying only on prices reported by the three largest businesses. That is, it may not be reasonable to assume that the pricing behavior of the small businesses mirrors that of the large ones, because, for example, they may target separate niche markets and direct their pricing strategies accordingly. Therefore, it would be prudent to select a sample of the small businesses to represent the markets they serve.

1.320 The less concentrated is the industry structure, the stronger is the case for using probability sampling techniques. Experience has shown that, although many manufacturing and mining industries may be dominated by a few large businesses, many service industries have a very large number of small businesses, and, if there are any large businesses, they produce a relatively small proportion of the output. An added advantage of probability sampling techniques is that they enable sampling errors to be calculated, which provide some guide to the accuracy of the resultant indices.

1.321 Procedures need to be implemented to ensure that samples of businesses remain representative through, for example, regularly augmenting the sample by enrolling a selection of new businesses as they enter the market. Also, a sample rotation policy needs to be considered in order to spread the business reporting load.

1.322 Once the sample of businesses has been selected, they need to be contacted to agree on a sample of representative product specifications for ongoing price reporting. This is discussed further under Step 5.

Step 5. Collecting and editing the prices

1.323 The main source of ongoing price data is usually a sample of businesses. The sample can relate to either buyers or sellers, or a combination of both. The choice will be influenced by the pricing point of the index (input or output) and practical considerations such as the relative degree of concentration of buyers, and of sellers, and the implications for sample sizes and costs.

1.324 The statistical units to be sampled may be head offices reporting national data, establishments reporting regional data, or a mixture. Decisions on the units to be surveyed may be based largely on pragmatic grounds such as efficiency of collection, location of relevant business records, etc.

1.325 The aim of the price collection is to enable the calculation of reliable indicators of period-to-period—say, monthly—price change. As such, choices need to be made as to the type and frequency of pricing. For example, point-in-time prices may be the easiest to collect and process (for example, transaction prices prevailing on a particular day, say the 15th of the month) and commonly prove to be reliable indicators. For workload management, it may be decided to spread pricing over the reference period with, say, three or four pricing points and different commodities priced on different days.

1.326 For commodities with volatile prices, it may be necessary to price them on several different days of the month and calculate time-weighted averages; alternatively, businesses can be asked to provide weighted average monthly prices (usually derived by dividing the monthly value of product sales by the quantity sold). This approach should be avoided because it is susceptible to the unit value "mix" problem, where products of different qualities are included.

1.327 The most appropriate pricing methodology to use is *specification pricing*, under which a manageable sample of precisely specified products is selected, in consultation with each reporting business, for repeat pricing. In specifying the products, it is particularly important that they are fully defined in terms of all the characteristics that influence their transaction prices. As such, all the relevant technical characteristics need to be described (for example, make, model, features) along with the unit of sale, type of packaging, conditions of sale (for example, delivered, payment within 30 days), etc. This technique is known as *pricing to constant quality*. When the quality or specifications change over time, adjustments must be made to the reported prices (see Step 7).

1.328 Another important consideration in establishing and maintaining price collections is to ensure that the prices reported are *actual market transaction prices*. That is, they must reflect the net prices received (or paid) inclusive of all discounts applied to the transactions whether they be volume discounts, settlement discounts, or competitive price-cutting discounts, which are likely to fluctuate with market conditions. Any rebates also need to be considered. The collection of nominal list prices, or book prices, is not reflective of actual transactions, is unlikely to yield reliable price indices, and may result in quite misleading results because fluctuations in market prices are not captured.

1.329 The principles underlying the selection of the sample of product specifications from a particular business, whether using probability or nonprobability sampling, are similar. That is, the outputs of the business and the markets are stratified into categories with similar price-determining characteristics. For example, in selecting a sample of specific motor vehicles in consultation with the manufacturer, the first dimension may be the broad category of vehicle (for example, four-wheel-drive recreational vehicles, luxury cars, family cars, and small commuter cars). These categories will reflect different pricing levels as well as different pricing strategies and market conditions. A further dimension may be to cross-classify by the type of market (for example, sales to distributors, fleet sales, and exports).

1.330 Then, from each of the major cells of the matrix of vehicle category by market, a sample of representative vehicles can be selected, with each one representing a broader range of vehicles.

1.331 If *explicit* internal weights are to be used in the construction of the lower-level indices (for example, for motor vehicles), then the relevant sales data for (i) the individual vehicles in the sample, (ii) the wider range of vehicles being represented (that is, as defined in the matrix of vehicle category by market), and (iii) all vehicles should be collected from the business for a recent period. This will enable internal weights to be calculated for combining the prices of individual product specifications and the prices of different producers.

1.332 Ideally, initialization of a collection from a business will be undertaken through a personal visit. However, this is an expensive exercise, and budgetary considerations may necessitate compromise. Alternative, though less effective, approaches to initialization include the use of telephone, Internet, fax, and mail, or some combination of approaches. At a minimum, the larger businesses and those producing complex (for example, high-tech) products and operating in changing markets should be visited.

1.333 In cases where the products are unique and not reproduced over time—for example, construction industry output and many of the customized business services—specification pricing is not feasible, and alternative pricing techniques must be used, often involving compromise. Possibilities include model pricing, collecting unit values for reasonably homogeneous components of a good or service, input pricing, and the collection of charge-out rates (for example, for a legal service).

1.334 Most national statistical agencies use mail questionnaires to collect their producer prices. Collection procedures include the design of tailored forms incorporating the particular product specifications for each sampled business and collection control to facilitate the dispatch, mark-in, and follow-up with the reporting businesses.

1.335 It is important that rigorous *input editing* techniques are employed, and that any price observations that do not appear credible are queried (usually by telephone) and either confirmed with an acceptable reason or amended. Input editing involves analyzing the prices reported by an individual business and querying large changes (editing tolerances may be built into processing systems) or inconsistent changes across product lines. An important objective of the editing process is to ensure that actual transaction prices are reported, inclusive of all discounts, and to detect any changes in the specifications.

1.336 If the price of a product has not changed for, say, six months, it may be appropriate to contact the business to make sure the prices reported are not being automatically repeated.

1.337 *Output editing*, which is often an integral part of calculating the lower-level indices (see Step 6), involves comparing the price levels, and price movements, of similar products between different businesses and discretely querying any outliers.

1.338 In undertaking these editing processes, reference to other supporting price information is often valuable. Examples include international commodity prices (for example, London Metal Exchange), exchange rates, press and wire service reports, and general market intelligence obtained during the sample maintenance activities described under Step 9.

1.339 Alternatives to the traditional mail questionnaire include telephone, e-mail, Internet, telephone data entry, fax, and the use of electronic data transfer from company databases. Several national statistical agencies have had experience with at least some of these alternatives. Important factors to be considered are data security, the convenience of reporting for the business, cost, and effectiveness.

Step 6. Adjusting for changes in quality

1.340 The technique of *specification pricing* was outlined under Step 5. The objective is to *price to constant quality* in order to produce an index showing *pure price change*. This is the most common technique employed by national statistical agencies in compiling PPIs.

1.341 To the extent that pricing is *not* to constant quality, then, over time, the recorded prices can incorporate a nonprice element. For example, if a product improves in quality and its recorded price does not change, there is an effective price *fall* because an increased volume of product is being sold for the same price. Conversely, if the quality of a product declines without a recorded price change, there is an effective price *rise*. In such circumstances, the recorded price of the new product of changed quality needs to be adjusted so that it is directly comparable with that of the old product in the previous period.

1.342 Failure to make such adjustments can result in biased price indices and consequently biased constant price, or volume, national accounts estimates.

1.343 It is possible to identify fairly readily the main price-determining characteristics of many goods (for example, a washing machine) that are mass produced to fixed technical specifications and can be readily described in terms of brand names, model codes, etc. However, specification pricing cannot be used for customized goods such as the output of the construction industry. Nor can it be used for much of the output of business service industries (such as computing, accounting, and legal services) because it is unique in nature (each transaction is commonly tailored to the needs of an individual client). Further, it is far more difficult to identify all the price-determining characteristics of many services because some are intangible.

1.344 In such cases, other approaches to pricing to constant quality must be employed—for example, model pricing—using narrowly defined unit values or collecting charge-out rates (see Step 5).

1.345 Even in areas that do lend themselves to specification pricing, problems arise when there are *changes* to the specifications, and hence the quality, of the products over time. Examples of possible product changes would include:

- Presenting it in new packaging;
- Selling it in different size lots (for example, 1 kg packets of sugar replaced with 1.2 kg packets); and
- Replacing it with a product with different technical and design characteristics (for example, a new model of motor vehicle).

1.346 The first step, in consultation with the provider, is to fully identify the changes and assess whether they are, in fact, quality changes.

1.347 The first example above (new packaging) may be deemed to be cosmetic only; alternatively, it could be assessed as being substantive if, for example, it led to a reduction in the damage to the contents. In the latter case, a value would need to be placed on the improvement on the basis of some estimate of the value of reduced damage.

1.348 The second example (change in size lot) would be likely to involve an office adjustment based on matching the new and old prices per a common unit of measure (for example, price per kilogram).

1.349 The third example (new model of motor vehicle) is the most complex. Possible techniques include using an assessment of the difference in the production costs of the old and new models to adjust the price of the new model. Alternatively, the different product characteristics can be identified and a value placed on them. The valuation can be based on consultation with the producer or, if the new model has features that were available as options on the old one, market prices will exist for those options. In cases where the old and new model are sold (in reasonable volume) in parallel, the difference in the overlapping transaction prices may be taken as a guide to the value of the quality difference.

1.350 Increasingly, national statistical agencies are researching and selectively implementing hedonic regression techniques as a means of placing a market value on different characteristics of a product—for example, the value of an additional unit of RAM on a personal computer. When the characteristics of a particular product change, these techniques enable its price to be adjusted to make it directly comparable with that of the old model. Unfortunately, hedonic techniques tend to be very costly, involving extensive research and analysis and the collection of large volumes of data.

Step 7. Calculating the index

1.351 Under Step 3, the two categories of indices were described: lower-level and upper-level indices. Having established the structure and weighting pattern of the index, constructed a processing system, and established the regular price collection, the first step in the routine production cycle is to aggregate the input-edited prices to form the lower-level indices. There is a range of micro-level index formulas available for use, each being based on different assumptions about the relative behavior of prices and quantities in the economy (see Chapters 15 and 17).

1.352 The initially compiled lower-level indices should be scrutinized for credibility in terms of the latest period movement, the annual movement, and the long-term trend. *Output editing*, involving comparisons of price levels and movements between different businesses, is an integral part of the credibility checking. Reference to the type of supporting information described under Step 5 will be valuable for this analysis.

1.353 Despite the most rigorous collection processes, there are often missing prices that need to be imputed. Prices may be missing either because the provider failed to report on time or because there were no transactions in that product specification in the relevant period. Imputation techniques include applying the price movements of like products to the previous period price observations. The like products may either be reported by the same business or by other businesses. Another approach is to simply repeat the previous period prices, but this approach should be used only if there is reasonable certainty that the prices have not changed.

1.354 Once the price statistician is satisfied with the lower-level index series, the series should be aggregated to form the hierarchy of upper-level indices, including the total measure. This aggregation is undertaken using the classification structure and weighting pattern, determined in Step 2, and an appropriate index formula.

1.355 Extensive studies have concluded that the theoretically optimal formulas for this purpose satisfy a range of tests and economic conditions, and as a class are known as superlative formulas (Chapter 15). A basic characteristic of such formulas is that they employ weights based on volume data from both the current period and the period of index comparison. In practice, since the volume data from the current period are not available at the time of index construction, the use of a superlative formula would necessitate the estimation of the current-period volume data in order for timely indices to be produced. When the current-period volumes subsequently became available, the index numbers would need to be recompiled using the actual volumes, and the earlier index numbers revised. This ongoing cycle of recompilation and revision of published index numbers would create a high degree of uncertainty among users (as explained under Step 8) and is highly undesirable. Therefore, most national statistical agencies compromise and use a base-weighted formula such as Laspeyres.

1.356 The upper-level indices are aggregated across industries, commodities, and/or stages at the national or regional level, as defined in Steps 2 and 3, to produce the aggregates required for publication (Step 8).

1.357 Finally, annual average index numbers and the suite of publication and analytical tables should be produced and the commentary on main features prepared for publication (see Step 8). It is prudent to apply broad credibility checks to the aggregates before release. Are the results sensible in the context of the prevailing economic conditions? Can they be explained?

Step 8. Disseminating the indices

1.358 During the initial user consultation phase described under Step 1, and the formulation of the index classification structure under Step 2, broad publication goals will have been formulated. It is now time to refine and implement these goals, probably involving further user contact.

1.359 As well as releasing time series of index numbers for a range of industries or commodities or stages, and aggregate measures (for example, all groups), user analysis can be enhanced by the release of time series of percentage changes, as well as tables presenting the contribution that individual components have made to aggregate index point changes. This latter presentation is particularly important to help gain an understanding of the sources of inflationary pressure.

1.360 Different tabular views of the data can be provided. For example, classification by:

- Source—imported or domestically produced;
- Economic destination—consumer or capital goods; and
- Industry and/or commodity.

1.361 Some form of analysis of the main movements and, ideally, the causes of those movements, should be provided. These will be based on the percentage change and point-contribution tables described above.

1.362 In addition to the summary tables, analytical tables, and detailed tables, explanatory notes should outline the conceptual basis of the index, including the objectives, scope, coverage, pricing basis, sampling techniques, and data sources. The weighting patterns should also be published. Any caveats or limitations on the data should be included to caution users.

1.363 As well as release in hard copy form, electronic delivery and access through the Internet website of the national statistical agency should form part of the overall dissemination strategy.

1.364 In terms of timeliness of release, there will be a trade-off between accuracy and timeliness. In general, the faster the release, the lower the accuracy of the data, and hence its reliability, as the need for revisions increases. Price index users—whether they be public policy economists, market analysts, or business people adjusting contract payments—place a high value on certainty (that is, the nonrevisability of price indices). Accordingly, some compromise in the timeliness of release will probably need to be made in order to achieve a high degree of certainty and user confidence.

1.365 Policies need to be developed in relation to:

- Security of data through the uses of a strict embargo policy;
- Publication selling prices and electronic access charges based on relevant principle—for example, commercial rates, cost recovery, or rationing of demand; and
- Community access to public interest information—for example, through free provision to public libraries.

1.366 Ongoing consultation with users should be maintained to ensure that the indices, and the way they are presented, remain relevant. The establishment of a formal user group, or advisory group, should be considered.

Step 9. Maintaining samples of businesses and product specifications

1.367 As noted in Section R above, some of the necessary prerequisites for the production of an accurate price index are to incorporate prices that, over time, relate to:

- Product specifications that are representative indicators of price change;
- Constant quality products with fixed specifications; and
- Actual market transactions inclusive of all discounts, rebates, etc.

1.368 Step 5 above expands on these principles and outlines the methodology for selecting the sample of product specifications from a business at initialization, preferably by personal visit.

1.369 Given the dynamics of many marketplaces in terms of changing product lines and marketing strategies, it is important that procedures are put in place to ensure that the product samples remain representative and have fixed specifications, and that the prices reported incorporate all discounting.

1.370 Further, if explicit internal weighting is used in the lower-level index aggregation, these weights need to be monitored and updated as necessary, on a component-by-component basis.

1.371 Ideally, a rolling program of regular interviews of the sampled businesses would be established to undertake these reviews on a fairly frequent basis. Costs may prohibit regular visits to all of the businesses, so it may be necessary to prioritize them according to factors such as their weight in the index, the extent of technical change in the industry, and the volatility of the markets. A program may be devised such that the high-priority businesses are visited frequently and the lower-priority ones visited less frequently and/or contacted by telephone. Many national statistical agencies have such structured programs in place.

1.372 In addition to these structured proactive reviews, resources should be made available to enable a quick reaction to changed circumstances in relation to a particular commodity or industry and to undertake specific reviews on a needs basis. For example, competitive pressures resulting from deregulation of a particular industry may quickly, and radically, transform the product lines and methods of transacting and produce substantial market volatility. Examples in recent years include the deregulation of the electricity supply, telecommunications, and transport industries in many countries.

1.373 The samples of businesses also need to be reviewed, either through a formal probability-based sampling process incorporating a rotation policy, or some more subjective approach that includes initialization of price collections with substantial new businesses as they enter the market.

Step 10. Reviewing and reweighting the index

1.374 Other necessary prerequisites for the production of an accurate and reliable price index that were listed in Section R above are that:

- The weights need to be representative of the relevant pattern of transactions over the period for which they are used for index aggregation; and
- The aggregation formulas used must be appropriate to the needs of the particular index and not yield significant bias or drift.

1.375 Studies have concluded that, in practice, price indices are often not highly sensitive to small errors in weighting patterns. However, the greater is the variation in price behavior across different commodities, the more important are the weights in the production of an accurate measure of aggregate price change.

1.376 Assuming that a rolling sample review program is in place for the maintenance of price samples and the lower-level internal weights (see Step 9), then the question of the frequency of reweighting of the upper-level indices (which were established under Step 3) needs to be considered. Alternatively, if no such sample review program is in place, a strategy needs to be put in place for the periodic reweighting of the entire index (lower and upper levels) along with a complete review of the product samples.

1.377 Practices in this regard vary among national statistical agencies. Some agencies update the upper-level weights on an annual basis and link the resultant indices at the overlap period such that there is no break in continuity of the series. That is, if the link was at June 2000, then the “old” weights would be used to calculate the index movements between May and June, and the new weights used to calculate the index movements between June and July (and subsequent months), with the July movements “linked” onto the June level. This process is termed annual *chaining* or *chain linking*.

1.378 The more common national practice is to reweight and chain on a less frequent basis, perhaps once every three or five years. Considerations in making decisions on the frequency of reweighting include:

- Changes over time in the pattern of transactions covered by the index:

  (i) The greater the volatility in the transaction patterns, the greater the need for frequent reweighting to maintain the representativeness of the weights. If the trading patterns are highly volatile, it may be desirable to “normalize” or smooth them by using data from a run of years in order to mitigate against *chain-linking bias* or *drift*,
  (ii) If the trading patterns are relatively stable and tend to shift on a trend basis, very frequent reweighting is of little benefit, and it may be assessed that reweighting every three, five, or more years is adequate;

- The availability of reliable and timely weighting data sources; and
- Resource constraints.

1.379 If reweighting is done on an infrequent basis using data from a single year, it is important that a *normal* year is selected in terms of providing weights that can be expected to be representative of the period (say, five years) for which they are used in the index. Again, the use of data from a run of years may be prudent.

1.380 In addition to developing a reweighting strategy, it is desirable to undertake thorough periodic (say, every five or ten years) reviews of the PPIs to ensure that the conceptual basis is still relevant to the needs of users.

Summary

1.381 Early consultation with users and decisions on the scope and conceptual basis of a PPI are fundamental to the production of a *relevant* index. In order for the index to be *accurate*, it must be constructed using indicative transaction prices (measured to constant quality) and representative weights.

1.382 The issue of reporting burden is an important consideration in seeking the cooperation of businesses and, along with resource constraints facing national statistical agencies, heavily influences decisions on sampling strategies and other methodological matters. Ensuring the security of often commercially sensitive price data is another essential prerequisite to building good business relationships.

1.383 A dissemination strategy that meets the needs of the wide variety of users must be developed, and ongoing consultation maintained, to ensure to insure that users' requirements continue to be met.

1.384 It is important to appreciate that a price index seeks to provide contemporary information in relation to dynamic markets. As such, it is not sufficient to develop a new index framework, establish the collection of the price samples, and simply aggregate them over time. Mechanisms need to be put in place to ensure the ongoing integrity and representativeness of the measure. That is, the price samples and weights need to be systematically reviewed and updated periodically.

2. Background, Purpose, and Uses of Producer Price Indices

2.1 PPIs are a key economic indicator in most countries. This chapter provides background information on the development of price indices, discusses the role of national and international agencies in price index development, identifies the variety of ways in which PPIs can be compiled, and explains the uses of these different variations.

A. Background and Origins of Price Indices

2.2 PPIs are used for a variety of different purposes (see Section E below). There is a general public interest in knowing the extent to which the prices of goods and services have risen. Also, it has long been customary in many countries to adjust levels of wages, pensions, and payments in long-term contracts in proportion to changes in relevant prices, a procedure known as index linking or contract escalation. Price indices have a long history for this reason.

2.3 A very early example is a simple index compiled by William Fleetwood in 1707, which was intended to estimate the average change in the prices paid by Oxford University students over the previous two and a half centuries. Another 18th-century example is an index compiled by the legislature of Massachusetts in 1780 in order to index the pay of soldiers fighting in the Revolutionary War against England (see Diewert, 1993a, for an account of the early history of index numbers).

2.4 During the 19th century, interest in price indices gathered momentum. In 1823 Joseph Lowe published a study on agriculture, trade, and finance in which he developed the concept of a price index as the change in the monetary value of a selected set, or basket, of goods and services, an approach still used today. He also noted the various uses for a price index, such as the linking of wages and rents, and the calculation of real interest. Diewert (1993a) argues that Lowe can be considered "the father of the consumer price index." Later in the 19th century further important contributions were made, including those of Laspeyres (1871) and Paasche (1874), whose names are associated with particular types of price indices that are still widely used. Marshall (1887) advocated the use of chain indices, in which indices measuring price movements from one year to the next are linked together to measure price movements over longer periods of time.

2.5 During the 1920s several important developments occurred. In 1922, Irving Fisher published his monumental work, *The Making of Index Numbers*. This was prompted by Fisher's interest in inflation and his advocacy of the Quantity Theory of Money, in which changes in the money supply were held to lead to corresponding changes in the price level. A good measure of changes in the price level was needed—that is, a good price index—which led him into a systematic investigation of the properties of hundreds of different kinds of possible formulas for price indices.

2.6 Fisher's preferred index, the geometric average of the indices advocated by Laspeyres and Paasche, is now known as the Fisher index. As explained in detail in Chapters 1 and 17 of this *Manual*, the Fisher index (or the closely related Törnqvist index) remains the preferred measure from a theoretical point of view for most purposes. From the perspective of the *economic approach to index number theory*, these indices have been shown in most circumstances to provide an unbiased estimate of changes in the cost of living for consumers and for price changes for firms that maximize revenue and minimize costs. The full details of the economic approach to the PPI are discussed in Chapter 17. The Fisher index number formula can also be justified from the perspective of averaging two equally plausible fixed-basket index number formulas (the Laspeyres and Paasche formulas), and this justification is presented in Chapter 15. The Fisher index also has a strong justification from the viewpoint of the *test approach to index number theory*, which is discussed in Chapter 16. The Törnqvist formula can also be justified from the viewpoint of the *stochastic approach to index number theory*, which is also discussed in Chapter 16.

2.7 In 1924, Konüs published a seminal paper laying down the foundations for the economic theory of the cost-of-living index, or COL index. A COL index is designed to measure the change in the cost of maintaining a given standard of living (or utility or welfare) as distinct from maintaining sufficient purchasing power to buy a fixed set of goods and services. In reality, consumers do not go on purchasing the same set of goods and services over time but adjust their expenditures to take account of changes in relative prices and other factors. The producer counterpart to the consumer's cost-of-living index is the *fixed-input output price index*. This *economic approach* to the theoretical foundations for the PPI was not fully developed until the 1970s: see Fisher and Shell (1972), Samuelson and Swamy (1974), and Archibald (1977). This approach is pursued in Chapter 17.

2.8 In 1926, Divisia published a paper in which he proposed price and quantity indices that factor the change in the monetary value of some aggregate flow of goods and services over time continuously and instantaneously into its price and quantity components. While Divisia's approach to index number theory is not immediately applicable, since price and quantity data are not available on a continuous basis, the Divisia index is useful conceptually when one has to choose between fixed-base indices or chained indices. The Divisia index and its connection with the chain principle for constructing index numbers are discussed in Chapter 15.

2.9 Thus, by 1930 the theoretical foundations (from all of the above perspectives) for the compilation of price indices, including PPIs, had been laid. While there have been many refinements to index number theory from both an economic and statistical viewpoint during the mid- and late 20th century, the essential elements were already in place early in the century. Developments in index number theory and practice over the past few decades are dealt with in detail in various chapters in this *Manual* and will not be summarized here, except to note that all of the above approaches led to a very small number of index number formulas being designated "best." In particular, the Fisher formula emerges as being "best" from the perspectives of the economic, test (axiomatic), and averaging of fixed-basket indices approaches, whereas the Törnqvist formula emerges as being "best" from the perspectives of the economic and stochastic approaches. The purpose of this brief historical survey has been to place the contents of this *Manual* in a longer-term perspective and to show that the measurement of price changes, or inflation, has long been recognized to be theoretically challenging as well as practically important.

B. Official Price Indices

2.10 As noted, there has always been considerable interest in, and demand for, price indices from the general public as well as governments. The 1780 index, referred to in the previous section, was specifically commissioned by a government agency in order to adjust the pay of soldiers in its employment. It is now generally acknowledged that governments have an obligation to provide the community and not merely themselves with information about price movements in the economy. A price index is a public good.

2.11 The practice of index-linking wages has a long history. Index linking means that the wage rate or material costs are adjusted in proportion to the change in some specified price index, the purpose being to maintain the real purchasing power of wages over the kinds of goods and services typically consumed by wage earners. As explained later in this chapter, a major use of the PPI is to make adjustments in long-term contracts for changes in material costs. For such applications the specification of the index that is to be used can be a matter of some controversy. Whatever the exact formula used, index linking has important financial implications both for those making and receiving the payments in question. This in turn implies that there is a need for impartial, independent, objective, reliable, and credible price indices. The responsibility for compiling price indices must therefore be entrusted to a statistical agency that has both sufficient resources and the necessary independence from pressure groups of various kinds. This provides a second reason why governments find themselves under an obligation to compile and publish price indices, or to supervise and monitor whatever agency is entrusted with the responsibility.

2.12 In practice, the government agency that is given the responsibility to compile and publish PPIs is usually either the statistics office or bureau, or the central bank. The reason why the central bank has been entrusted with the task of compiling PPIs is that the PPI is seen as a major indicator of domestically induced inflation, which most central banks want to control using instruments of monetary policy.

2.13 Price indices for industrial commodities also have a long history. In Canada a wholesale price index (WPI) of 89 commodities was compiled using an unweighted geometric mean for the period 1867–90. After that the index was expanded to cover more commodities and to use a Laspeyres index. The first industrial commodities index in the United States was produced in 1902 (covering the period 1890–1901), using an unweighted average of price relatives for about 250 commodities. This index was developed in response to a U.S. Senate Finance Committee request for an investigation into the effects of tariff laws on prices of domestic and foreign agricultural and manufactured products. A system of weighting was first used in 1914. The original index was also referred to as the WPI because it covered commodity prices before they reached retail markets.

2.14 In Europe, the first WPI for the United Kingdom was prepared by the Board of Trade and presented to Parliament in 1903. The price reference year was 1871, and the series covered the years from 1871 to 1902. The prices were mainly derived from the trade accounts, with weights estimated from different commodities used, or consumed, in the country between 1881 and 1890. The index covered 45 commodities, mainly basic materials and foodstuffs. Following World War II, a number of countries also began collection of data on wholesale prices of commodities in an effort to measure price changes at an earlier level in the production process. Around 1970, Eurostat, the Statistical Office of the European Union (EU), began a systematic program to encourage members to collect industrial output prices in an effort to get information on prices as products left producers' factories. These price indices were thus called producer price indices—PPIs—because they attempted to measure the change in prices producers received at the factory gate. In the past 5–10 years, many national statistical agencies have been progressively extending coverage of their national PPIs to measure changes in service industry prices, which in many countries now account for nearly two-thirds of GDP.

2.15 PPIs are usually compiled monthly, although some countries compile them only quarterly. Countries also try to publish them as soon as possible after the end of the month to which they refer, sometime within two weeks of the reference month. Moreover, most countries prefer not to revise them once they have been published. In contrast to many other kinds of statistics, most of the required data, at least on prices, can be collected at the same time.

2.16 PPIs have two characteristics that users find important. They are published *frequently*, usually every month but sometimes every quarter. They are available *quickly*, usually about two weeks after the end of the month or quarter. PPIs tend to be closely monitored and attract a lot of publicity. In many countries the PPI is not revised once it is published, which is viewed as an advantage by many users.[1]

C. International Standards for Price Indices

2.17 Once some statistic is accorded official status and given some prominence, the establishment of international standards usually follows. International standards are needed for several reasons—and not merely in order to compile internationally comparable statistics. The first international standards for PPIs were promulgated in 1979 by the United Nations. The UN Statistical Commission at its 19th session requested the preparation of manuals on the practical aspects of collecting and compiling price and quantity statistics within the overall framework of the *Guidelines on Principles of a System of Price and Quantity Statistics*, which was issued in 1977. The *Manual on Producers' Price Indices for Industrial Goods* was released in 1979 by the UN Statistical Office to provide practical guidance on the preparation of industrial PPIs.

[1]In most countries both the PPI and CPI are not subject to revision once published unless an error is discovered in price collection or compilation. In a number of countries, however, it is standard practice to revise the PPI once more complete information is available. For example, in the United States the PPI is revised with a three-month lag; that is, the most recent three months are preliminary (subject to revision), while the fourth month's data are final.

2.18 This *Manual* discusses revised and updated methods for PPI compilation based on current practice and recent developments in price index number theory.

2.19 Some international statistical standards are developed primarily to enable internationally comparable data to be collected and published by international agencies such as the statistical offices of the UN, the ILO, the IMF, or the OECD. The publication of such data by an international agency is often seen as a guarantee that the data conform to internationally accepted standards even though this may not always be the case in practice. Although national statistical offices actually supply the data to the international agencies, their publication by the international agencies is often interpreted as a public endorsement of their reliability, which enhances their status and credibility even within their own country.

2.20 However, international standards are not developed simply to enable internationally comparable data to be compiled. Many countries choose to use them as norms or standards for their own statistics. In this way, small national offices with limited resources of their own benefit from the collective views and experience of experts from a wide range of countries on which the international standards are based.

C.1 The current revision

2.21 This *Manual* has been developed in response to several factors. A considerable amount of work on the methodology of price indices, covering both theoretical issues and optimal methods of calculation, was undertaken at an international level during the 1990s as a result of the formation of international group of price experts. This group, the International Working Group on Price Indices, established under the auspices of the UN Statistical Commission, met for the first time in Ottawa in 1994 (and is therefore called the "Ottawa Group"). It brought together leading experts on price indices from national statistical offices and universities from around the world. During the course of its seven meetings through 2002, well over a hundred papers on the theory and practice of price indices have been presented and discussed. This collective activity at the international level has inevitably led to some rethinking about, and elaboration of, the current international standards on both CPIs and PPIs as embodied in *Consumer Price Indices: An ILO Manual* (Turvey and others, 1989) and the *Manual on Producers' Price Indices for Industrial Goods* (UN, 1979). The current *PPI Manual* also incorporates approaches to the measurement of output prices in the services sector and, as such, has benefited from review by the International Working Group on Service Sector Statistics (the Voorburg Group).

2.22 Another factor is the high priority accorded to the control of inflation as a policy objective in most countries, after the experience of high, or even hyper, inflation in the past three decades of the 20th century. The slowing down of inflation in many parts of the world in the 1990s, compared with the 1970s and 1980s, far from reducing interest in its measurement, has stimulated a demand for more accurate and reliable measures of inflation. Whereas an error or bias of 1, or even 2, percentage points in the annual rate of inflation may not be considered so important when inflation is running at 10, 20, or more percent, it becomes very significant when the rate of inflation itself is estimated to be only 1 or 2 percent. Inflation may slow down to the point at which it is not even clear whether prices are rising or falling, on average.

2.23 Users of PPIs in some countries have become convinced that the indices are subject to an upward bias, mainly as a result of their failing to make proper allowance for improvements in the quality of many goods and services, especially newer goods, such as computers, that are subject to rapid technological progress. The treatment of changing quality has long been recognized as particularly difficult on both conceptual and practical grounds. This topic has been intensively investigated, with numerous new studies on the subject appearing in the 1990s.

2.24 It has also been realized that, because of the widespread use of price indices for the index linking of social benefits such as pensions and other government payments and as an escalator for price adjustments to long-term contracts, the cumulative effects of even small potential biases can have considerable financial consequences for government budgets and private industry purchases over the long term. This has led to governments themselves scrutinizing the accuracy and reliability of price indices more intensively than in the past.

2.25 Within the EU, the convergence of inflation was deemed to be an important prerequisite for the formation of a monetary union. This requires precisely defined price indices that are comparable among countries. An intensive and prolonged review of all aspects of the compilation of CPIs was undertaken during the 1990s by all the national statistical offices of the member countries of the EU in collaboration with Eurostat. This work culminated in the elaboration of a new set of international standards for the 29 member and candidate countries of the EU and led to the development of the EU's Harmonised Indices of Consumer Prices, or HICPs. Work on the HICPs proceeded in parallel with that of the Ottawa Group, many of whose experts also participated in the development of the HICPs.

2.26 The need for revising the ILO manual to incorporate these new developments was one of the major recommendations at the 1997 joint UN-ECE/ILO meeting on the CPI. Similarly, the IWGPS came to the conclusion that a new PPI manual was long overdue, as was a manual on external trade price indices.

2.27 Significant developments have taken place in the practice of PPI construction that now necessitate a revision of the 1979 UN manual. Among these are emergence of the economies in transition, increased inflation, reality that PPIs may overstate inflation even when international standards are followed, the need for constructing and publishing more than one index to meet specific requirements, the need for separate PPIs for different stages in the production process, etc.

C.2 Responsibilities of the international agencies

2.28 The traditional practice of index-linking wages and contracts in many countries has meant that, at both a national and an international level, ministries or departments concerned with economic policies and statistics have taken responsibility for PPIs. However, many government departments—especially ministries of finance, economics, industry and trade, and of course central banks—are concerned about inflation and have acquired an interest in a variety of PPIs as key indicators of inflation. The experience of inflation in the past three decades has also increased general public awareness of, and concern about, the PPI.

2.29 Similarly, all the international agencies concerned with general economic policy now attach importance to the PPI and its movements. In addition, the IMF, the World Bank, the regional UN Economic Commissions, the OECD, and the Commission of the EU all have a strong interest in PPIs. All of these agencies have provided technical assistance in the compilation of PPIs to countries in transition as well as to developing countries. The agencies therefore agreed to pool their resources and collaborate in the present *PPI Manual*, establishing an Inter-Secretariat Group to manage the process.

C.3 Links between the new *CPI* and *PPI Manuals*

2.30 One of the first decisions of the IWGPS was to produce a manual on PPIs parallel to the one on CPIs. Movements in producer prices are clearly important for the measurement of inflation and the analysis of the process of inflation within an economy. PPIs have been comparatively neglected, however. Whereas there has been an international manual on CPIs for over 70 years, there has been a manual on the PPI covering industrial output only for about 20 years.

2.31 This new *PPI Manual* was therefore developed and written in conjunction with the *CPI Manual* (ILO and others, 2004). PPIs and CPIs have a lot of methodology in common. Both draw on the same theoretical literature pertaining to index numbers. Whereas CPIs also draw on the economic theory of consumer behavior, PPIs draw on the economic theory of production and the short-term rigidities in the production process. However, the two economic theories are isomorphic and lead to the same kinds of conclusions with regard to index number compilation. It was therefore decided that the two *Manuals* should be similar in form and as consistent as possible, sharing common text when appropriate.

C.4 The Inter-Secretariat Group and the Technical Expert Groups

2.32 Responsibility for the production of both the *CPI* and the *PPI Manuals* rested with the same Inter-Secretariat Group consisting of staff from the statistical offices, departments, or divisions of the ILO, the IMF, the World Bank, the UN, the OECD, and the EU. Expert advice on the contents of the two *Manuals* was provided by two parallel technical expert groups consisting of invited experts on

CPIs or PPIs from national statistical offices and universities together with experts from the international agencies themselves. To ensure consistency, there was overlap of membership between the two expert groups.

2.33 Most members of the two technical expert groups also participated in meetings of the Ottawa Group, which supported the decision to revise the *CPI Manual* and to produce a new *PPI Manual*. The *Manuals* draw on the contents and conclusions of the papers presented at meetings of the Ottawa Group and the Voorburg Group, thus providing the outlets through which the conclusions of the Groups can exert an influence on the actual compilation of price indices.

D. Purpose of a Producer Price Index

D.1 Background

2.34 The PPI provides a weighted average of the price changes in a group of products between one time period and another. The average price change over time cannot be directly observed and must be estimated by measuring actual prices at different points in time. Price index numbers are compiled from the collected price observations through time; their significance lies in a series of index numbers referencing the comparison prices between a particular period and a reference base. For an index to provide information on price changes, at least two index numbers from the same series need to be available, and these index numbers must relate to the same basket of goods.

2.35 The PPI does not attempt to measure the actual level of prices but is limited to the measurement of the average change in prices from one period to another. The PPI does not measure the value of production or cost of production, but it can be used to measure either the change in output prices owing to changes in the basic prices received by producers or, alternatively, the change in prices paid by producers for inputs of goods and services used in the production of output.

2.36 There is no unique PPI, since the prices of different combinations of goods and services do not all change at the same rate. Relative prices are changing all the time, with some prices rising and others falling. Because price changes can vary considerably from product to product, the value of the price index will be dependent on the precise set of goods and services selected. It will also depend on the weights attached to the different kinds of product within the set.

2.37 In general terms a PPI can be described as an index designed to measure the average change in the price of goods and services either as they leave the place of production or as they enter the production process. Thus, producer price indices fall into two clear categories: input prices (that is, at purchaser prices) and output prices (that is, at basic or producer prices). The *1993 SNA* (paragraph 6.205, page 151) defines basic and producer prices as follows:

> The *basic price* is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy receivable, on that unit as a consequence of its production or sale. It excludes any transport charges invoiced separately by the producer;
>
> The *producer's price* is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any VAT, or similar deductible tax, invoiced to the purchaser. It excludes any transport charges invoiced separately by the producer.

The difference between basic and producer prices is generally the per unit subsidy that the producer receives and taxes on production. While basic prices are preferred in the PPI because they represent the per unit revenue received by the producer, producer prices may have to be used when information on subsidies is not available. In most cases producers do not receive subsidies, so the basic and producer prices will be the same.

2.38 Thus, *output prices* should be the basic prices received by the producer. The output price index measures the average price change of all covered goods and services resulting from an activity and sold on the domestic market and also on export markets. In constructing a family of output PPIs, export prices are usually collected from a separate source to produce a separate export price index.

2.39 PPI prices should be actual transaction prices, which can be directly recorded. The price should be recorded at the time when the transaction occurs (ownership changes) rather than when the goods are ordered, which in certain cases can be significantly different. This topic is discussed in more detail in Section B of Chapter 3 and Section B.5 of Chapter 6. Intercompany transfer prices should be used with caution.

2.40 *Input price indices* measure the change in the prices of all intermediate inputs used in production by a specified sector of the economy. *Intermediate inputs* are inputs into the production process of an establishment that are produced elsewhere in the economy or are imported.[2] Thus, an input PPI measures changes in the cost of the basket of purchases required as inputs into the production process, but these inputs must not be primary inputs like land, labor, or capital. Producer input prices should exclude deductible taxes on products (that is, value-added tax, or VAT) but include the retail or wholesale margins of the supplier, since they measure the actual cost of the good or service to the producer. In constructing a family of input PPIs, import prices are usually collected from a separate source to produce a separate import price index.

2.41 The industrial coverage of a producer price index can vary across countries. In some countries, producer price indices refer only to indices related to input and output of the industrial sector, whereas in others the producers of services are also included. For example, in many countries the aggregate PPI includes only industrial activities such as mining, manufacturing, and public utilities (gas, electric, and water supply). In others, agriculture is included along with transport and telecommunication services. Ideally, the PPI would cover all economic activities as presented in Chapter 14 on the framework for price statistics. Many countries are progressively developing service industry PPIs for incorporation within their larger PPI frameworks. These developments are discussed among interested statistical agencies at the annual Voorburg Group meetings.

[2]By convention, purchases of durable capital inputs are not treated as intermediate inputs. A durable input is one that lasts longer than the time period being used in the index. In practice, durable inputs are inputs that last longer than two or three years.

D.2 Sources of inflationary pressure and price change

2.42 Prices for the PPI can be measured at two different points: as inputs into the production process and as outputs produced by the production process. Therefore, the PPI can be split into two key groups. Input prices measure the prices of products purchased for use in the production process at purchasers' prices. Output prices measure the price of products as they are sold to the next stage in the production chain—which could be to a wholesaler, retailer, or another production enterprise—at basic prices. These are often referred to as "factory gate" prices and represent basic prices as defined in the *1993 SNA*. The essential difference between input and output PPIs is that an input PPI measures potential inflation, by indicating the price pressures that producers are facing. The producer faces many other costs such as labor and capital costs and also has to consider how much of any overall price change the market will bear, so it is unusual to see the full effect of an intermediate input price rise being transmitted directly to the output price. Output prices measure the price change that actually takes place and are therefore a more direct measure of inflation. Output prices themselves, however, can also be an input further along in the production process, and as such they represent a measure of potential inflation in further stages of production (for example at the wholesale and retail levels).

2.43 Output prices are usually directly collected, whereas detailed input prices are often a mix of directly collected and output proxies, the structure of which is determined by the aggregation required. Output proxies are often used to avoid having to collect input prices for manufacturers' purchases from other parts of manufacturing—the assumption being that there is a stable profit margin. While all output prices generally measure prices for sale to the domestic market, input prices will also include import prices for imports that are used in the manufacturing process, such as crude oil and agricultural produce.

D.3 Net versus gross indices

2.44 PPIs can be produced on two different weighting concepts—*gross*, or *net of intersectoral sales*. The concept can be more clearly explained by an example. The aggregate weight for gross sectoral output of the motor vehicles industry would include both the sales of the parts and the sales of the finished cars, even though the value of the parts are included in the value of the finished cars. The net sectoral output of the motor vehicles industry would measure only the sales of motor vehicles to other sectors of the economy and would exclude the sales of parts.

2.45 It is desirable to produce aggregated PPIs on a net sectoral basis. When using gross sectoral indices, there is a problem with multiple counting of price change as products flow through the different production processes—this occurs where the output of one industry is used as an input into another industry within the same sector of PPI aggregation. The net sectoral approach is the measure that best reflects the impact of inflation in a sector, such as manufacturing, on the rest of the economy. Gross sectoral indices, however, also provide valuable information and are useful for deflating the total turnover of industries, which by definition is on a gross basis. To avoid this problem of multiple counting, net sectoral weights are calculated, which involves weighting the index together with sales weights that have intrasectoral sales removed. This is usually achieved through an input-output framework.

2.46 In addition to output and (intermediate) input PPIs, there is another type of PPI that can be constructed from establishment or industry output and input PPIs. This is a sectoral PPI that will act as an appropriate deflator for the *net output* or *value added* of the sector. The value added of a sector is the value of its outputs less the value of the intermediate inputs used to produce those outputs. The issues involved in constructing this type of PPI are considered in Chapter 17.

D.4 Effect of tax switching between direct and indirect taxes

2.47 Taxes on products are generally excluded from PPIs because these are usually deductible as an expense by businesses as they are paid to the government. Taxes such as excise duty on imports are sometimes included, since these are not deductible and have to be paid by the producer. This can lead to changes in the price level owing to changes in tax procedures as import duties are imposed or restricted. To remove this potential inconsistency in an index, it is possible to produce ex-tax/ex-duty indices. These have all of the tax effects removed to allow a clearer comparison of product price changes over time. This is done by weighting revenues with the tax cost removed.

D.5 Export/import prices

2.48 Export and import prices are an important extension of domestic PPIs. These are used in the deflation of external trade. Also, import prices feed into the producer input index, since these are an important contribution to producer costs. In theory, price and quantity data on exports produced and imports used by an establishment could be collected at the establishment level. In practice, this is very difficult to do, and so price and quantity data on imports into the economy and exports produced by the economy are collected by other surveys. Foreign trade price indices will be the subject of a separate manual.

D.6 PPI versus WPI

2.49 The PPI historically is an outgrowth of programs developed to measure wholesale prices. The WPI attempts to measure price changes as they occur at one stage prior to final demand—the wholesale level. The WPI would normally cover the price of products as they flow from the wholesaler to the retailer. It includes products from domestic wholesalers and factories as they are delivered to retailers. As such, the WPI differs from the PPI because it includes both domestically produced products sold in the home market (included in the PPI) and imported products (excluded from the PPI), while excluding prices of exported products. In addition, the WPI measures transactions at purchasers' prices, which include delivery charges and taxes on products such as sales taxes and VAT.

2.50 As Chapter 14 on the system of price statistics explains, the PPI concepts are much more consistent with the *1993 SNA* than are the WPI concepts. The PPI system can be used to develop price indices for all domestically produced products, both for home distribution and export. Within the PPI framework, an index for the output of the wholesaling industries would be the most comparable in coverage to the WPI. However, the differences in pricing concepts still remain. The PPI for the wholesaler would be a double-deflation price index for the gross margin between the wholesaler's revenue at basic prices and its cost of goods bought at purchasers' prices. When the gross margin can be identified on each product (selling price less purchase cost), the PPI is, equivalently, a price index of product gross margins. WPI prices are, in contrast, the purchasers' prices received by the wholesaler.

E. PPI Uses

2.51 Price instability introduces uncertainty into economic analysis and decision making, so the main uses of the PPI relate to efforts to minimize this uncertainty. The PPI therefore has the following main uses:

- Short-term indicator of inflationary trends;
- National accounts deflators;
- Indexation in legal contracts in both the public and private sectors, particularly for more detailed PPI components;
- Required by international organizations such as Eurostat, the OECD, IMF, and European Central Bank (ECB) for economic monitoring and comparison;
- Current cost accounting;
- Compilation of other inflation measure such as the final expenditure price index (FEPI); and
- Analytical tool for businesses/researchers.

E.1 Short-term indicator of inflationary trends

2.52 A monthly or quarterly PPI with detailed product and industry data allows short-term price inflation to be monitored through different stages of production and is a key use of the PPI. The key users of the PPI as a short-term indicator are central banks and government finance ministries or departments. Also, many companies (including investment banks and brokerage firms) and government agencies require the data for macroeconomic forecasting. These users also need the data to build models to look at the price pressures that different sectors of the economy are facing, with the aim of helping their investment clients to achieve better stock market returns.

E.2 National accounts deflator

2.53 Although PPIs are an important economic indicator in their own right, a vital use of the PPIs is as a deflator of output or sales data for the compilation of production volumes and the deflation of capital expenditure and inventory data for use in the national accounts. As a result, the concepts underlying the PPI are often conditioned by those underlying the national accounts. This can lead to conflicts in the requirements; for example, for contract escalation, users would like weights to be fixed for a long period. However, for deflation of national accounts, current-weighted indices and fine aggregations are required, since in theory deflation is best done at the lowest level of disaggregation, possibly using Paasche price indices. (See paragraphs 16.16–16.19 of the *1993 SNA*, pages 382–83.) Only a few countries are actually able to use pure Paasche indices for this purpose. In many countries the objective of getting as close as possible to a Paasche index is achieved by using chain-linked indices. Chain linking is discussed in more detail in Chapters 9 and 15.

E.3 Indexation of contracts

2.54 Indexation of contracts is a procedure whereby long-term contracts for the provision of goods and services include an adjustment to the value of monetary amounts for the goods or services based on the increase or decrease in the level of a price index. The purpose of the indexation is to take the inflationary risk out of the contract. A PPI offers an independent measure of the change in prices of the good or service being considered. Indexation is common in long-term contracts, where even relatively small levels of inflation can have a substantial effect on the real value of the revenue flows (such as from the building of ships and aircraft).

2.55 It is important that parties to the contract understand the exact makeup of the index to ensure that it is suitable for the purpose. Also, parties should be aware of the impact of rebasing on the long-term index values. Often users expect the same product weights to apply throughout the length of the contract, even if this spans several rebasing periods.

E.4 International organizations

2.56 Members of the EU are required to provide PPI data under the Short-Term Indicators Regulation, which specifies monthly delivery and at a detailed level of aggregation. Other international organizations using PPIs include the ECB, IMF, and OECD. The PPI is a required indicator for countries subscribing to the IMF Special Data Dissemination Standard (SDDS), and it is recommended as a useful extension of inflation measurement to all member countries participating in the IMF General Data Dissemination System (GDDS).[3]

E.5 Current cost accounting

2.57 Current cost accounting is a method of accounting for the use of assets in which the cost of using the assets in production is calculated at the current price of those assets rather than by using the historic cost (the price at which the asset was originally purchased). The price index used should not be a general price index but should be specific to the asset being used. Although current cost accounting is no longer commonly used in low-inflation countries, these data are still relevant to the needs of high-inflation countries, in which there are still users requiring indices for estimating the current value of their capital assets.

E.6 Analytical tool for business researchers

2.58 Detailed PPIs can be useful to businesses and researchers looking at specific products and markets. Companies can use PPIs to compare the growth rate of their own prices with those of the representative index for the industry or the commodity. This can be done at a very detailed level, where fine PPI aggregations are published. Researchers looking at specific markets can also gain an understanding of conditions in the market by examining PPIs. This can be done in conjunction with other economic data such as output figures to identify pressures on margins, for example. Similarly, competition and monopoly authorities can use PPIs as a tool in examining whether competitive pressures are evident or not.

F. A Family of PPIs

2.59 PPIs can be calculated in a number of different combinations. As already mentioned, PPIs can represent either input and output prices, with differing levels of aggregation. They can also be calculated as net output price indices by industry, which adjust for intraindustry use of products to avoid the effects of double-weighting both final output and intermediate usage. These net output PPIs can be used in order to deflate the nominal value added of an industry, thus constructing an index of *real value added*. PPIs can also be calculated by the stage of the production cycle to which they relate—such as raw materials, intermediate products, and products for final demand. PPIs can be calculated for the country as a whole or on a regional basis, if significant price differences occur among regions. This topic is considered in more detail in Chapter 3.

F.1 Industry aggregation

2.60 The most basic indices are output indices classified by a standard industrial classification system. A range of aggregation possibilities exists for different users. The lowest level of index form is determined by the level of sampling. In the United Kingdom, for example, the sample is based on the six-digit CPA (Classification of Products by Activity) codes, and indices are calculated at this level. The indices are then weighted up into ISIC or NACE four-digit, two-digit group, or higher-level totals. Classification systems are discussed in more detail in Chapter 3.

F.2 Macroeconomic aggregations

2.61 High-level aggregations, such as all manufacturing, are important for monitoring macroeconomic trends. To aid interpretation it is possible to produce high-level series with certain industries excluded—for example, all manufacturing excluding food, drink, tobacco, and petroleum. This enables users to analyze trends without the influence of the most volatile industries. Another possibility is to produce indices with and without excise duty. This is done by developing separate weights to reflect the lower value of ex-duty sales and by either collecting data with and without duty or estimating the duty content of prices. This enables analysts to monitor inflationary trends before government intervention and also to identify the direct effect of government intervention.

F.3 Commodity analysis

2.62 Input prices at manufacturers' purchaser prices can be aggregated and analyzed by commodity. Analysis by commodity reveals the impact of inflationary pressure from raw materials, which are often priced on international markets and are out-

[3]Required data series for the two data standards can be found in the *Guide to Data Dissemination Standards, Module 1: The Special Data Dissemination Standard* and *Module 2: The General Data Dissemination System*. A brief overview of these standards can be found on the IMF Dissemination Standards Bulletin Board (http://dsbb.imf.org/).

side the control of domestic agencies. A particularly important example is the price change of crude oil. Aggregations of commodities can also be constructed to show the total impact of commodity price change on the economy.

F.4 Stage of processing

2.63 Another method for analysis is to aggregate by stage-of-processing indices. This concept classifies goods and services according to their position in the chain of production—that is, primary products, intermediate goods, and finished goods. This method allows analysts to track price inflation through the economy—for example, changes in prices in the primary stage could feed through into the later stages, so the method gives an indicator of future inflation further down the production chain. However, each commodity is allocated to only one stage in the production chain even though it could occur in several stages. This topic is considered further in Chapter 14.

F.5 Stage of production

2.64 A further method for analysis is to aggregate by stage of production, in which each commodity is allocated to the stage in which it is used. This differs from stage of processing because a product is included in each stage to which it contributes and not assigned solely to one stage. The classification of products to the different stages is usually achieved by reference to input-output (I/O) tables in order to avoid multiple counting of the stages that are not aggregated. There is a growing interest in this type of analysis—for example, these types of indices are already compiled on a regular basis in Australia.[4] This topic is also considered in Chapter 14.

F.6 Final expenditure price index

2.65 A further variant is the FEPI. This measures prices paid by consumers, businesses, and government for final purchases of goods and services. Intermediate purchases are excluded. PPIs are used as proxies for the final prices paid for investment goods by businesses and government in the FEPI model used in the United Kingdom and Australia. This is because most PPIs reflect changes in basic prices or producers' prices (not purchasers' prices). This topic is further considered in Chapter 5.

F.7 Regional PPIs

2.66 In general, states and provinces of countries are very interested in having regional measures of domestic products and also prefer to measure changes in the real output of the state or province. For this reason, it is possible to produce regional PPIs within a country to use as deflators. Countries would generally develop regional PPIs only if they are particularly meaningful—for example, when there is regional price dispersion and regional markets for produced goods.[5] The main difficulty in many countries is that producers are unlikely to be producing just for users within their region but are likely to be selling to the whole domestic economy for which there is usually a single market. In a competitive marketplace the purchaser will look to achieve the lowest price per given quality, and so producers have to be able to sell at a competitive price regardless of location, except for products or regions with high transport and distribution costs.

2.67 Regional PPIs are produced in Thailand, for example, where certain industries and products such as food production and construction materials exhibit significant variability across regions and for which regional information is available through the regional offices of the national statistical office. In this instance the information is informative to the authorities and is done at relatively low incremental cost by regional offices with the same software package used by the national office. Indonesia also produces regional indices.

F.8 Productivity analysis

2.68 A final use for PPIs is in deflating the nominal value added of an industry into a real value added. These industry measures of real value added are then divided by labor input to the industry to form estimates of *industry labor productivity*, or are

[4]See, for example, Australian Bureau of Statistics (2001b).

[5]If a country is producing regional accounts, regional PPIs would be used for deflators assuming they are available in enough industry and product detail.

divided by an index of industry primary input usage to form estimates of *industry total factor productivity*. Productivity increases act as a primary driver of increases in the standard of living of a country, so it is of some interest to try to determine which Industries are the main drivers of productivity improvements. (See OECD [2001] for additional material on productivity.)

3. Coverage and Classifications

A. Population Coverage

A.1 Economic activities included in the population coverage

3.1 Although the scope of a PPI may include all domestic goods- and service-producing establishments, traditionally the PPI has been compiled as a measure of price change for the goods-producing sectors of the domestic economy. These include agriculture, forestry, and fishing; mining; manufacturing; and public utilities.

3.2 The construction sector has generally not been considered a component of the traditional goods-based PPI. This may be less a matter of definitional consistency and more a function of the difficulty in constructing meaningful and accurate price measures for this sector. Chapter 10 discusses in detail the problems associated with price measurement in this sector, largely due to the uniqueness and complexity of any particular construction project. Successful approaches to surveying construction have relied on techniques that differ significantly from methods employed in manufacturing.

3.3 The services sectors that are in scope for a PPI vary across countries. Many countries are interested in creating a corporate services price index. This restricts coverage to business services, including professional services, finance, insurance, real estate, accommodation and food, information, communications, and the transportation of goods. A more expansive definition could include all services transactions that are in intermediate demand. This would encompass wholesale trade and the intermediate demand component of retail trade, transportation of people, and educational services. Finally, a number of countries are working toward an economy-wide PPI. This would bring all non-goods-producing sectors for both final demand and intermediate demand within the domain of a single PPI index.

A.2 Class of buyer coverage

3.4 Practices among countries differ greatly on whether all or only some final demand transactions are within scope of the PPI. From a practical perspective, the decision on whether to include transactions directly to consumers, the personal consumption expenditures (PCE) portion of GDP, relates to whether the PPI program is industry-based or commodity-based. An industry-based sample design readily lends itself to including both intermediate and final demand transactions. The sample unit and survey respondent almost always can provide data for both classes of transactions. Often it is difficult for the respondent to report only on intermediate demand transactions. In the case of air passenger transportation, records are generally not available to distinguish between a business traveler and a vacation traveler.

3.5 Chapter 5 provides more insight into sampling frame issues and the inclusion or exclusion of direct sales to consumers. Ultimately, the decision to include any or all sales to final demand is a scope question. Frame sources and sample designs could be made to support either choice. If the CPI covers personal consumption expenditures, it is duplicative and costly to similarly cover these transactions in the PPI. This is an issue to some extent in the goods sectors, such as electric utility sales to households. But it is much more significant in the services sectors, where direct sales to households are frequently encountered. An alternative could be to coordinate surveying activities across the CPI and PPI programs. This issue should be resolved prior to undertaking any expansion of the PPI into the service sector.

3.6 For deflation of the national income accounts, it appears to be highly desirable to calculate and publish indices that are differentiated by GDP category. A semifinished good, such as a semiconductor wafer, sold as an export would belong in the export component of final demand. A similar good sold in the domestic market would appear in intermediate demand. A passenger car sold to a corporation for internal use would belong in the fixed domestic investment component of final demand. The same vehicle designated for sale to households would appear in the PCE component of final demand. Tax accounting services provided directly to households would appear in PCE, a final demand component. Similar services provided business-to-business would appear in intermediate demand.

3.7 Resource and respondent burden constraints and data availability may well preclude the PPI program from publishing along GDP categories. Given adequate sample size, index accuracy in this instance is enhanced by calculating indices by type of buyer to conform to GDP categories. Where this is not possible, the alternative is to calculate a single index, such as passenger cars, and apportion the weight among the various passenger car indices published using input/output data from the national accounts. For example, a single tax accounting index could be calculated. That same index could be mapped into a stage-of-processing structure by using I/O weights to apportion the total commodity output weight between intermediate demand and final demand.

A.3 Nonmarket goods and services

3.8 Most countries have defined nonmarket activities as falling outside of the scope of the PPI. Examples of these activities include general government services such as national defense and the value of owner-occupied structures. Situations may exist where one class of customer may receive the service with no charge while another class of customer may pay a market price. That is the case with local government-run hospitals in the United States, where payment is determined by family income exceeding a legislated value.

3.9 A different issue is whether to include in the scope of the PPI any revenue-generating activity, even if it is a small portion of the economic activity of the establishment. For example, should the gift shop sales of a state-owned museum (let us assume a free admissions policy) fall within the scope of the PPI? Or should the establishment be deemed to be out of scope because most of its activities are supported by general tax revenues? The decision on what to include in the PPI is generally made on the basis of program resources. Establishments and/or entire industries with few market-priced activities are generally excluded from the PPI. It is deemed too expensive to survey the establishment for such a small return.

A.4 Import and export coverage

3.10 On a conceptual basis, the inclusion of exports and exclusion of imports conforms to the measurement of output price change consistent with index use for the purpose of GDP deflation. In contrast, the inclusion of imports and the exclusion of exports is consistent with demand-based index use. Both formulations are highly meaningful for a variety of important data users. Please refer to Chapter 2 for a discussion of PPI uses and major aggregations. Resources permitting, both import and export price data could be incorporated into different PPI aggregations to form different families of indices. However, the PPI concept is generally associated with output measurement. This is inclusive of exports and exclusive of imports.

3.11 Overlap considerations with an import and export price index program raise considerations about whether nonduplication of export pricing is feasible. The PPI need not include export pricing to still meet some needs of GDP deflation, since exports are a separate GDP category. Identification of goods destined for export, however, may be a problem. Where reimbursement of VAT occurs, such identification may be straightforward. In other cases price discrimination between domestic and export sales may exist but may involve dealing with very different respondents within the enterprise to secure survey data. The use of I/O table weights at least solves the weighting problem.

A.5 Globalization and e-commerce considerations

3.12 The e-commerce revolution, coupled with globalization, is having a substantial impact on determinations of population coverage. The outsourcing of production, and globalization, are redefining the role of many business enterprises. An enterprise that had been a major manufacturer may now outsource all production to establishments based in other countries. The enterprise may not even provide the material inputs to the production entity because it is more cost-effective to allow the production entity to arrange its own inputs utilizing just-in-time inventory techniques. If the fabricated good is repatriated before marketing, this leaves the domestic enterprise only a wholesale trade margin output-generating activity. However, the enterprise is busily engaged in new product development and prototyping. These are the main wealth-generating activities for the modern corporation. But the enterprise has its output valued as wholesale margin rather than as manufacturing with a gross sales output valuation.

3.13 A related phenomenon is the establishment of virtual corporations to manufacture a new product with a quite short expected life span. The virtual corporation may be production facilities that can be quickly converted to different manufacturing activities to produce items on a contract basis. The virtual corporation can be established by a consortium of firms with different skills coming together briefly to manufacture a new product with a short expected life.

3.14 In both cases, the PPI program is challenged to review its concepts of domestic production and manufacturing. Criteria for manufacturing may need to be revised to give primary weight to new product design and prototyping, while discounting the importance of actual production. The boundary between manufacturing and wholesale trade may need to be reestablished in recognition of this. Finally, the statistical agency can be expected to be challenged by the speed with which these partnerships are formed and dissolved. Traditional surveying methods may be too slow and cumbersome to permit inclusion of short-lived virtual corporation partnerships in the PPI program. New surveying methods may need to be developed in order to ensure coverage of this most dynamic part of the economy.

B. Price Coverage

B.1 Order prices and shipment prices

3.15 The appropriate price to obtain from a theoretical perspective should be the price at the time there is a change in ownership from the producer to the buyer. Unfortunately, it may be too difficult to adhere to this theoretical requirement uniformly in practice. Therefore, statistical agencies have generally used the concept of shipment price for the actual transaction occurring as close to the survey pricing date as possible. In most circumstances the shipment price is final at the time of delivery to the customer. There are situations where the shipment price cannot be finalized until well after shipment. An example of this is the case of cumulative volume discount. This could necessitate using an estimation procedure, such as reliance on the previous-period cumulative volume discount, to best approximate the current transaction price. The construction sector presents special problems in that prices are often renegotiated upon completion of the activity. Often, unforeseen circumstances encountered in performing the activity require renegotiation. Chapter 10 further discusses problems associated with measuring price change in the construction sector.

3.16 Order prices refer to the price quoted at the time the customer places an order. There usually will not be any difference between order and shipment prices. For some classes of goods, however, such as aircraft and ships, there may be a period of months or even years between the placing of the order and the actual shipment. No output would have been generated when the order was placed, and the final shipment price would likely reflect some form of price escalation to adjust the order price to account for subsequent cost increases. These considerations may make the use of order prices questionable for such goods that have an extended production period given the uses to which the PPI is put such as GDP deflation.

B.2. Net transaction prices

3.17 Net transaction prices are actual shipment prices received by the producer for the sales transaction of a good or service to a customer. The price includes the impact of all discounts, surcharges, rebates, etc. for a unique customer or unique class of customer. The statistical agency is not always able to obtain a transaction price net of all discounts and inclusive of all surcharges. Of greatest concern is the ability to secure a type of price whose movement closely proxies the movement of a net transaction price. The inability to include a cash discount will not affect the measure of price movement if it is a constant. But the failure to include competitive discounts, which can be expected to vary considerably over time, may well compromise the accuracy of the index.

3.18 There are a variety of different types of price that may meet the definition of the net transaction price. These include contract prices, spot market prices, average prices, and intracompany transfer prices. The different types of price are treated separately below, and limitations and problems associated with their use are also discussed.

B.2.1 Contract prices

3.19 Contract pricing generally refers to a written sales instrument that specifies both the price and the shipment terms. The contract may include arrangements for a single shipment or multiple shipments. The contract usually covers a period of time in excess of one month. Contracts are often unique in that all the price-determining characteristics in one contract cannot be expected to be repeated exactly in any other contract. The challenge is to maintain a constant quality methodology over time, especially when the contract expires and item substitution is necessary.

3.20 Contract terms may be unique to each agreement in terms of customized product features, negotiated price tied to the unique buyer-seller relationship, or quantity differences. In addition, contracts reflect supply and demand conditions at the time the contract is entered into. At best, price adjustments for long-term contracts can be made for input cost changes. But a long-term contract does not reflect current-period market conditions for new transactions.

3.21 To attain an accurate index where contract pricing is widespread, especially in the short term, a larger item sample is necessary. This is to reflect the proper proportion of new contracts or renegotiated contracts being entered into each pricing period. Assume that all contracts for the purchase of new machine tools are for three years' duration. If only one item is tracked in the PPI sample, it would be three years before any price change due to new supply and demand conditions would affect the price (at contract renegotiation). With an item sample of ten contracts, all with different contract expiration dates, the index would better reflect actual price conditions for machine tool contract sales because contract renegotiations would be encountered much more frequently than once every three years.

B.2.2 Spot market prices

3.22 Spot market price is a generic term referring to any short-term sales agreement. Generally, this refers to single-shipment orders with delivery expected in less than one month. Goods sold on this basis usually are off-the-shelf and, therefore, are not subject to any customization. These prices are subject to discounting and directly reflect current market conditions.

3.23 Spot market prices can be extremely volatile. The pricing methodology employed can be critical in minimizing the volatility encountered from these phenomena. It is advisable to take several measures during the current month and average them. Crude petroleum and agricultural products are particularly prone to extreme short-term volatility. Of course, it becomes extremely difficult to interpret aggregate data when highly weighted subaggregations, such as food and energy, are subject to high price volatility. We do not know if a measured change from one month to the next is due to the particular point in the month that the price(s) was/were collected rather than being a change sustained over most or all of the month. With volatile prices, a better solution may be to collect average prices, although these have their own weaknesses as well.

3.24 A more subtle index distortion can be caused by a nonrepresentative mix of contract and spot market prices. If the goal of the index is to accurately reflect price movement for the population of transactions in the current period, the proportion of index items falling into each category must be accurate. Contract prices cannot be expected to move similarly to spot market prices in the short term. Business users of the PPI may well prefer an index of spot market prices because these best reflect current market supply and demand conditions. This is quite useful for new purchase decisions. However, GDP deflation requires a price measure reflective of all transactions in that product area. The PPI cannot hope to meet all user needs and must focus on its primary goal of accurately representing all transactions.

B.2.3 Average prices

3.25 Average prices reflect multiple shipments of a given product within a consistently defined time period. Often, reporters can readily provide such data on a weekly or monthly basis. Usually, average pricing is possible for commodities or very simple and standard manufactured goods. The advantage of average pricing is that it very effectively increases the number of price observations used to calculate the index, thereby reducing variance. An average price should meet two requirements:

- The price is reflective of the current time period, and
- The price relates to homogeneous transactions.

3.26 It is often impossible to secure a price that meets both requirements. Companies often compute average prices on a monthly basis. By the time they are computed and provided to the statistical agency, they become one-month lagged prices. If the product area is characterized by extreme volatility, a one-month lagged price may be unacceptable.

3.27 The problem of product mix can also be difficult to overcome. Machinery and equipment often are sold with a choice of relatively expensive options. Automobiles could include as options air conditioning, traction control, antilock brakes, and leather upholstery. If the manufacturer were to provide an average price for all cars sold of a particular model, it would include a mix of optional equipment. The options mix could vary significantly from month to month. This causes significant variance in the index and largely compromises the ability to perform meaningful short-term price analysis.

3.28 The problems associated with average prices tend to compromise the short-term analytical capability of the index. However, in certain circumstances they may enhance the accuracy of long-term index movement. For example, in the United States the index for telecommunication services relies on average prices with a known product mix problem. The industry is characterized by the frequent introduction of new calling plans, which any customer can switch to at the customer's discretion. These new plans are competitive discounts. The average pricing methodologies capture these discounts on a current-month basis. Any other pricing methodology, such as pricing a specific bill, would not capture this discounting. Thus, this price index on a long-term basis is more indicative of industry pricing trends using average prices, but there is significant variability on a month-to-month basis. This average price approach assumes that the product mix fluctuations are some fixed long-term proportion. Where this assumption does not hold, which is often the case, the average price alternative would not be an acceptable alternative because of the shifts in product mix that are likely to occur in long-term comparisons. The statistical agency needs to carefully explore the characteristics of an average price before establishing such a pricing mechanism with a new reporter.

3.29 Average prices have the advantage of representing the entire population of transactions for a particular good or service. Therefore, the concern when pricing a single transaction of holding transaction terms constant does not apply.

B.3 Subsidized prices

3.30 Subsidized prices are considered to differ from market prices in that some significant portion of variable and/or fixed costs are covered by a revenue source other than selling price. The following subsidies can be encountered:

- Fixed or variable subsidy on a per unit sold basis. For example, a monthly rental subsidy per apartment determined by the family income of the tenant.
- Budget subsidy where a service provider, such as a government-owned hospital, receives both an operating budget and capital budget annual allocation. Patients with a demonstrated ability to pay may be charged an economic price. Less-fortunate patients are either charged reduced rates or receive free service.
- Cross-subsidy where activity A of the service provider generates sufficient revenue to allow activity B to charge a noneconomic price. Tuition charges at a university may well subsidize research activities.[1]

3.31 Subsidized prices must be researched to determine whether they proxy market prices and should be used directly in the index or whether they must be adjusted to best reflect proxies for market prices.

3.32 In the case of fixed or variable subsidies directly on the sale of a good or service, the price could reflect the price to the customer plus the subsidy amount. Budget subsidies could be apportioned on a per unit sales basis if the respondent's accounting system is designed to support this calculation. This is much more problematic.

3.33 Cross-subsidization either requires reliance on the reporter's accounting system to allow for price adjustment or requires bundling of the different services into a more broadly defined index.

[1]Cross-subsidies within a sample unit are generally excluded for PPI purposes.

B.4. Intracompany transfer prices

3.34 Intracompany transfer prices are of increasing importance as globalization progresses, as discussed in Section A.5. Intracompany transfer prices are defined as the value assigned on a per unit or per shipment basis to goods shipped from one establishment of an enterprise to another. Ownership of the good does not change hands, so the value assigned to the shipment is not a market price. Where there is a vertically integrated enterprise, these shipments cross industry lines and account for revenue within that product line. Therefore, they are reflective of output-generating activity in the domestic economy.

3.35 One of the primary goals of the PPI is to help determine the magnitude and direction of price movement on both a macro- and microeconomic level. Price movements at earlier stages of processing or within intermediate demand are often of the greatest interest to policymakers concerned with price inflation. For such a use, any index containing nonmarket prices not paralleling market price movement is of dubious value. Intracompany transfer prices may well distort price analysis of market trends in the domestic economy.

3.36 It is generally recognized that the statistical agency must research the basis for setting intracompany transfer prices to determine how closely they proxy market prices. Often, vertically integrated companies establish separate profit-maximizing centers (PMCs) and allow the use of market measures to determine the performance of each unit. In such instances, intracompany transfer prices generally meet the test as good market price proxies.

3.37 Where tax considerations are important in price setting, intracompany transfer prices are generally poor proxies. Internationally traded goods might have valuations set to minimize import tariffs and corporate taxes. The statistical agency may decide to exclude such intracompany transfer prices from the index when they are judged to be accounting entries with no relation to market prices or values sensitive to taxation. On the other hand, to the extent that such activity is a significant portion of an industry's output, it is important to get the best proxy prices available because they will be needed to derive the industry PPI for use as a deflator in compiling GDP. In the case of exported goods, these may be the only prices available, and they will reflect the actual export values.

B.5 Discounts and surcharges

3.38 Discounts and surcharges are adjustments to the list price available to specific customers under specific conditions. The list price may not be a market price because no goods are ever sold at that price, or only a subset of customers purchase goods at that price. All or most transactions may occur with adjustments to the list price that reflect specific market conditions that may or may not be of long duration. Changes in discounts are a major problem for the accurate reflection of price movement. These adjustments often affect various customers differently and are of major importance in calculating an accurate and representative price measure. Sole reliance on list or catalog prices generally invalidates the price measures even for long-term analysis. Because prices posted on the Internet are usually real offered prices and transactions made through the Internet will be at those prices, it will be interesting to see if e-business activity mitigates against this problem and causes companies to advertise transactions prices generally. This could largely eliminate the difference between list and actual prices for most price adjustments (except, perhaps, quantity and prompt payment discounts) and still allow for special terms to valued customers.

3.39 Discounts generally fall into the following categories:

- Competitive discounts reflect unique supply or demand conditions, generally in specific markets for the good. These discounts are generally of short duration in any specific market area but may be applicable in at least one market area on a frequent basis.
- Prompt payment discount for remitting payment within a fixed time period such as ten days. These discounts are generally of small magnitude, remain unchanged for long periods of time, and are available to all customers.
- Quantity discounts are generally tied to specific order sizes and increase with the size of the order. These discounts are generally available to all customers.
- Class-of-customer discounts are specific to certain classes of buyer. Trade discounts are available to wholesalers to help cover their selling expenses. Advertising discounts are available

to retailers to help cover their promotional expenses. These tend to be expressed as percentages and remain unchanged for long periods of time.
- Financing discounts relate to providing assistance to customers to pay for the good they are purchasing. They may serve as a buy-down on the bank loan interest rate for the customer's borrowing to pay for the good. Floor plan allowances for automobile sales from the manufacturer to the dealer involve a rebate to the dealer to effectively buy down the interest rate charged the dealer by a bank lender.
- Price rebates are discounts paid to the customer after the actual transaction has occurred.
- Cumulative volume discounts are offered to customers who purchase a certain amount of an item in units or sales in several shipments over a specific period of time.

3.40 Surcharges are additions to the listed price. These are generally of short duration and reflect unusual cost pressures affecting the manufacturer. Examples include fuel surcharges for trucking companies.

3.41 The constant quality assumption underlying any index requires that the statistical agency must hold transaction terms constant. Quality adjustment would be required for any change in discounts included in the pricing specification similar to a change in product characteristics. A related problem would be a change in discount terms, such as changing the shipment size intervals for a quantity discount. One way to guard against this problem is to specify the exact quantity shipped in the item specification. Similarly, if a particular class of buyers is specified, any subsequently encountered type of buyer discount can be treated as a price change not requiring quality adjustment.

3.42 The inclusion of all appropriate discounts and surcharges is essential for ensuring index accuracy and utility. Certain discounts tend to remain unchanged for long periods, such as trade discounts and prompt payments discounts. Other discounts and most surcharges are highly sensitive to changes in input costs, competitive conditions, and interest rates. Often, manufacturers leave list prices unchanged while discreetly discounting for preferred customers or more astute buyers. Lack of information by purchasers can often greatly affect pricing strategies. It is entirely possible for a list price index and a net transaction price index to move in different directions.

B.6 Agricultural prices

3.43 For many agricultural products the prices collected should be "farm gate" prices—that is, the per unit prices received by the farmer for each product sold as it leaves the farm. In most cases this will represent an average price for each product. Such average prices are usually acceptable because they represent the unit cost of a single, homogeneous product. Often the price may include transport costs of the product by the farmer to a delivery point designated by purchasers. Such costs, to the extent that they are not separately billed by the farmer to the recipient, would be included in the price of each product. This follows the same principle as that of the *1993 SNA* regarding transport costs—to the extent that shipping costs are not separately billed but are included as part of normal business practice, they are a component of the basic price. The product description should include this as part of the product specifications.

B.7 Structured product descriptions

3.44 For each product transaction that is selected for price collection, the statistical office should maintain a detailed description of the important characteristics associated with the product and type of transaction. These should include all the characteristics that the establishment uses to determine the price. Chapter 6 on price collection advises documenting complete descriptions for each product in the PPI, each description containing the most important price-determining characteristics of the product. Chapters 7 and 21 make strong cases for setting up this documentation in a structured way, allowing product characteristics to be coded into binary and continuous variables. Coded or structured descriptions enable systematic tracking of product specifications and easier discovery and identification of changes in specification when establishments discontinue or modify the products they sell. They also are a prerequisite for statistical analysis of the impacts of product characteristics on product prices and, thus, for using hedonic techniques, among others, for quality adjustment. Structured product descriptions are discussed in more detail in Chapter 6.

C. Geographic Coverage

C.1 Treatment of imports and exports

3.45 Because the output PPI is a measure of price change for marketed domestically produced output, import prices are excluded and export prices are included. See Section A.4.

3.46 By definition, this requires that foreign purchases of residents (imports) should be excluded. In addition, domestic purchases by nonresidents (exports) should be included.

3.47 If the PPI is constructed as a purchaser's index, it is largely impossible to adhere to the geographic coverage parameters. Much of the intermediate demand sales flow through the wholesale trade sector. It becomes increasingly difficult to distinguish import items when inputs are purchased from wholesalers. It is also difficult to apply appropriate domestic expenditure weights in sampling. Exports would be missed entirely for shipments direct from manufacturers to overseas buyer. These are rather convincing reasons to construct the PPI as a producer's index.

3.48 Regional indices are generally unnecessary in the PPI. Production facilities may well be spread throughout the nation. But often they are aggregated within a single profit-maximizing unit, which sets a single selling price. Regional price differentiation is quite rare. Some sectors, such as power generation in public and private utilities and construction, evidence regional repricing. But these are exceptions.

D. Statistical Units

D.1 Characteristics of statistical units

3.49 A statistical unit in the PPI should refer to a single output-generating entity. Separate auxiliary establishments, such as sales offices or administrative offices, are important to the extent that they may be a record center or reporting unit for activities of several entities.

3.50 A statistical unit, analogous to the SNA establishment concept, is organized as a single decision-making unit. All operations within the statistical unit are coordinated to accomplish the goal and objectives of the unit. This could encompass activities such as price setting and the setting of production limits.

3.51 The statistical unit may consist of one or many operating establishments organized to utilize inputs efficiently and effectively, compartmentalize production activities, and generate output.

3.52 For sampling purposes, a clustering of units may occur when the various physical locations report to a single record center. The record center is expected to house sufficient production, engineering, accounting, and marketing data to permit full ongoing participation in the PPI. This would include product and transaction data and data to permit repricing and quality adjustment.

D.2 Operational problems in identifying sample units

3.53 E-commerce is causing a shortening of production life cycles for new products. Computerized networks that control all phases of the production of products permit the formation of virtual corporations to come together expressly to produce a product with a short prospective life span. The virtual corporation is the creation of a partnership among several companies sharing complementary expertise. With the conclusion of the product's life span, the corporation is disbanded.

3.54 Traditionally, PPI programs have relied on administrative records or surveys of output for a sampling frame. Industrial sectors are resampled on a periodic basis, and resampling activities require a considerable time period to be completed. Thus, the traditional approach is largely unsuited for the timely inclusion of virtual corporations into the index. They are unlikely either to be identified in a sample frame or to be amenable to providing prices over a period of time. New approaches are needed to identify and incorporate these entities into the PPI.

3.55 Another problem associated with e-commerce involves incorporating Internet and electronic data interchange (EDI)[2] sales into the PPI.

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[2]Electronic data interchange is a secure method for computerized communications between two unrelated parties. A great deal of e-business is done on the more secure EDI (continued)

Historically, the PPI captured transactions consummated by a physical exchange of paper. Now the statistical agency must determine if e-business transactions exist and which PMC or record center would provide PPI survey-related data encompassing these transactions. This may involve the identification of new record centers in the same corporation.

3.56 A related concern is whether the corporate structure has been altered to accommodate e-business. The enterprise might establish an e-commerce sales corporation to handle all e-business transactions. Should the PPI collect the intracompany transfer price between the manufacturing PMC and the sales corporation or the sales corporation price to the customer? As a general rule, mentioned previously, the price most reflective of the net transaction market price should be used.

3.57 A final concern relates to the outsourcing of production activities. What activities must be conducted by the originating company for it to remain as the producing sample unit? If the sample unit designs and prototypes the product, outsources production, and then markets the product, is it still in manufacturing? Has it become the wholesaler? What if the material inputs are not purchased and owned by the designer/marketer? What if the outsourcing is done by an overseas company, and the product is not repatriated before it is marketed? Traditional definitions of an output-generating sample unit appear to be deficient in offering guidance on how to handle pricing for many of the new industrial organizational relationships now in common use.

E. Classification

E.1 The role of classification

3.58 The classification structure largely determines the scope of price collection for the sample unit. The sample unit is chosen to provide data for a particular economic sector as defined by the classification system. If the sample frame covers a four-digit ISIC (International Standard Industrial Code) industry, the statistical agency is concerned with selecting representative items falling within the four-digit product scope.

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rather than the Internet. Although, as security increases on the Internet, EDI may become less utilized.

3.59 The classification system provides an organizing structure and is the first step in surveying. Once the subaggregation within the classification system is selected, an appropriate frame can be found from which to select representative industries or products for inclusion in the index.

3.60 Similarly, the classification structure forms the index structure and defines which product line or industry or aggregate weights are needed. Not coincidentally, the classification system serves as the basic language allowing the turnover survey and PPI survey to have a direct concordance. This, of course, is of great benefit to the PPI, the national income accountant, and the sophisticated data user.

3.61 A classification system must meet certain criteria to be useful to the PPI practitioner. The classifications must largely reflect the realities of industrial structure and reflect current-period production. The classifications must be relevant for long time periods to permit time-series analysis. The classifications must be mutually exclusive, easy to interpret and communicate, conform to real world categories, and be all-inclusive. The aggregation structure employed in the classification system must conform to the real world.

3.62 Individual product specifications selected for inclusion in the survey must map into one and only one classification category. This lowest level of classification ideally should conform to an economic product line definition (it would be assigned a unique product code). This would relate to homogeneity in use and price behavior. While this lowest level of classification (a product line composed of relatively homogeneous products) would most likely be too detailed for publication purposes, it would meet a variety of needs. The detailed product line level would define the class of goods available for any needed item substitution to replace discontinued goods. Also, any major classification system revisions generally would not affect such a detailed level. So remapping data to a different structure can be greatly expedited if product line assignments were already done. Finally, the relevant product characteristics are defined by the product line definition and permit automated mapping of items to product lines by characteristics.

3.63 The level of publication is driven by a number of factors. First, there is the issue of weight availability. One must be able to accurately weight every publication category. Second, adequate coverage is a critical concern to ensure accuracy, minimize variance, and ensure continuous publishability. Published indices, to meet user needs, must be fit for use and continuously available for extended periods of time. Third, the level of publication should meet user needs. PPI data are wanted at quite detailed levels for most major uses, including GDP deflation, contract escalation, economic analysis, and inventory valuation.

3.64 The aggregation structure employed by the classification system should meet major user needs. If a four-, three-, two-, or one-digit ISIC industry structure is used, this should meet the primary uses of the index. Alternative aggregation structures can be used if that is the best way to meet all major needs. This might include one set of indices following a hierarchic industry structure and another family of indices following a stage-of-processing structure.

E.2 International standard classification systems

3.65 This subsection presents the major international classification systems that are important to the PPI survey. Many individual country systems are adapted from these classification systems and are generally in concordance with one or more of them. These adaptations of international classifications reflect local circumstances by adding further detail or reducing detail by grouping some items. Such modifications of international reference classifications result in derived systems. Other countries develop more fundamentally different structures but allow for cross-classification at a reasonably detailed level of aggregation in the reference standard. Such structures are referred to as related classification systems. There is much to be gained internationally by adopting existing standards and contributing to international working groups, maintaining those standards in order to update them and make them as widely applicable as possible. That should increase the degree of applicability of the standard system and reduce the need for local variations that inhibit international comparisons.

Table 3.1. ISIC and NACE

ISIC, Revision 3	NACE, Revision 1
17 sections	17 sections
	31 subsections (detail within sections C and D)
60 divisions	60 divisions
159 groups	222 groups
292 classes	503 classes

E.2.1 Production activity

International Standard Industrial Classification of All Economic Activities

3.66 The ISIC classifies producer units according to their major kind of activity, mainly on the basis of the principal class of goods produced or services rendered; that is, ISIC classifies principally by an output-type criterion. The categories of the ISIC at the most detailed level (classes) are delineated according to what is in most countries the customary combination of activities described in statistical units. The groups and divisions, the successively broader levels of classification, combine the statistical units according to the character, technology, organization, and financing of production. Wide use has been made of the ISIC, both nationally and internationally, in classifying data according to kind of economic activity.

The General Industrial Classification of Economic Activities within the European Communities

3.67 NACE is the standard industrial classification of the EU. NACE maps into the ISIC but generally adds detail where needed for classifying establishments in the EU. Specifically, NACE is identical with ISIC at the top level, represented by the letters A through Q. NACE then subdivides the ISIC A letter divisions further in mining and quarrying (C) and manufacturing (D) by using a second letter character. The ISIC second, third, and fourth levels, represented at each level by digits 0 through 9, are also used by NACE, but NACE subdivides detailed ISIC codes at the three- and four-digit levels. The NACE structure is compared with the ISIC in Table 3.1.

Table 3.2. ISIC and NAICS

	ISIC, Revision 3/NACE	NAICS
A.	Agriculture, hunting, and forestry	11 Agriculture, Forestry, Fishing, and Hunting *except* 1141 Fishing
B.	Fishing	1141 Fishing
C.	Mining and quarrying	21 Mining
D.	Manufacturing	31–33 Manufacturing 511 Publishing industries 56292 Materials recovery facilities 811212 Computer and office machine repair and maintenance 811213 Communication equipment repair and maintenance 811219 Other electronic and precision equipment repair and maintenance 8113 Commercial and industrial equipment repair and maintenance *except* automotive
E.	Electricity, gas, and water supply	22 Utilities
F.	Construction	23 Construction
G.	Wholesale and retail trade, repair of motor vehicles, motorcycles, and personal and household goods	42 Wholesale trade 44–45 Retail trade 8111 Automotive repair and maintenance 811211 Consumer electronics repair and maintenance 8114 Personal and household goods repair and maintenance
H.	Hotels and restaurants	72 Accommodation and food services
I.	Transport, storage, and communications	48–49 Transportation and warehousing 513 Broadcasting and telecommunications
J.	Financial intermediation	52 Finance and insurance
K.	Real estate, renting, and business activities	514 Information services and data processing services 53 Real estate and rental and leasing 54 Professional, scientific, and technical services 55 Management of companies and enterprises 561 Administrative and support services
L.	Public administration and defense	92 Public administration
M.	Education	61 Educational services
N.	Health and social work	62 Health care and social assistance
O.	Other community, social, and personal service activities	562 Waste management and remediation services *except* 56292 Materials recovery facilities 71 Arts, entertainment, and recreation 81 Other services (*except* 92 Public administration) *except* 811 Repair and maintenance 81411 Private households
P.	Private households with employed persons	81411 Private households
Q.	Extraterritorial organizations and bodies	99 Unclassified establishments

The North American Industrial Classification System

3.68 NAICS was developed for adoption by the members of the North American Free Trade Association (NAFTA): Canada, Mexico, and the United States. It represents a significant departure from the existing industrial classification system in the United States, with one-to-one mappings possible for about only half of the existing four-digit codes of the outgoing Standard Industrial Classification (SIC). The number of major sectors has increased from 10 to 20, coverage of service industries has improved, and certain detailed categories of industries have been reclassified.

3.69 Unlike the predominantly output-based criterion underlying ISIC and NACE, the NAICS system is based on a process-oriented principle. It attempts to group all establishments with like production processes, whether or not the majority of output is in the same detailed product category.

3.70 NAICS can be mapped into the ISIC, Revision 3, for 60 high-level (generally ISIC division) groupings. A rough idea of the relationship between ISIC and NAICS at the ISIC section level is shown in Table 3.2 (on preceding page), which was derived from published sources.[3] It is evident from the variety of NAICS codes included in the ISIC/NACE sections in this table that NAICS has a rather different structure at the top level compared with ISIC, Revision 3, and NACE. Thus, while NACE is an elaboration and slight reorganization (within mining and manufacturing) of ISIC, NAICS is a substantive, if mappable, reorganization of ISIC. The two regional systems thus represent contrasting approaches to providing international comparability of national data.

3.71 NAICS does not adhere as closely as NACE to the international ISIC standard and is not as uniform across the member states of NAFTA as NACE is across the EU. On the other hand, it is a very modern system in the prominence and detail given to information and other service activities.

Australian and New Zealand Standard Industrial Classification

3.72 The ANZSIC was developed between the Australian Bureau of Statistics (ABS) and New Zealand Department of Statistics for use in the collection and publication of statistics in both countries. It is related to the ISIC in concept and contains a hierarchical structure of divisions (17), subdivisions (53), groups (158), and classes (465). The ABS has developed a concordance between ANZSIC and ISIC, Revision 3.

[3]A detailed concordance between NAICS and ISIC, Revision 3, is still in preparation by the NAICS working party.

E.2.2 Product classification

Central Product Classification

3.73 The CPC extends the Harmonized Commodity Description and Coding System (HS) used in the classification of traded goods to cover services and nontraded goods. It is designed to correlate to some extent with the ISIC, which, in turn, is based on the type of product a producer unit or establishment principally produces. It is, therefore, integrated with both of these international standards.

3.74 Specifically, the CPC coding system consists of five digits indicating 9 sections, 70 divisions, 305 groups, 1,167 classes, and 2,092 subclasses. Each of the 2,092 subclasses is an aggregate of one or more headings or subheadings of the HS. Integration with ISIC, Revision 3, has been brought about to some extent by grouping CPC subclasses according to the ISIC activities for which they are the principal products. In general, each five-digit subclass of the CPC consists of goods and services that are predominantly produced in one specific four-digit class or classes of ISIC, Revision 3.

3.75 However, since CPC is a product classification, it cannot be used to uniquely identify the industry of a product's origin: a given detailed CPC code may identify products originating from establishments classified in different ISIC activity categories. However, identification of product type by originating activity would be possible in principle merely by recording both the ISIC and CPC codes for each product record collected in the business surveys providing source data.

Eurostat Classification of Products by Activity (CPA and PRODCOM)

3.76 The CPA is designed to correlate with, and thus derives from, NACE, the EU specialization of ISIC. The motivation for developing CPA is that the CPC is not sufficiently detailed to be the single central product classification system for a comprehensive system of economic statistics, and that European users of the product classification preferred that it be derived from the industrial activity system. For coding of industrial statistics, CPA has been specialized in the PRODCOM product coding system, either by adding detail to CPA or aggregating some of its components, following the rule that no PRODCOM aggregation violate a broader CPA grouping.

3.77 As noted above, the desire for identification of product type and originating activity on the same product record could also be achieved by entering both the ISIC (NACE) and CPC codes, rather than creating a new (CPA or PRODCOM) product code. The latter may reduce the number of coding characters and thus database size, and could simplify staff training and coding operations to some extent. However, these conveniences are purchased at the expense of precisely what CPC has been designed to provide, which is the ability to group first on the basis of physical and intrinsic product characteristics rather than originating activity.

North American Product Class System

3.78 There is no product classification correlating with the NAICS activity structure, and there is, as yet, none planned. A new North American product classification system is under development, driven generally by a market- or demand-based grouping principle rather than a process-grouping principle. By implication, the new product system to be developed for the NAFTA countries can be expected to be distinct from the NAICS, more fundamentally than the CPC is distinct from the ISIC. Under distinct industry and product coding systems, there is less homogeneity required of individual producer units within the same activity in the sense of having very similar detailed products and being very specialized in the production of those products. Further, the activity classification of a given establishment may well be more stable, though of course subject to change as the establishment adopts or significantly revises its production process.

3.79 Like the CPC, the prospective North American product classification would provide data on product by activity, when desired, by requiring both NAICS and product codes on each product record collected in the industry surveys supplying source data for the national accounts and other economic statistics. In view of the existing CPC, which is based on the now almost universally adopted HS product coding system for internationally traded items, it can be hoped that the North American system will strongly resemble the CPC or be mappable to it at a detailed level.

PART II

Compilation Issues

4. Weights and Their Sources

A. Introduction

4.1 As an index number, the PPI is computed as an average of the price relatives of the many products for which prices are collected. The average is weighted to reflect the importance of each priced product in terms of its share of total output of the establishment.[1] Ideally a weight should be attached to each price collected. However, as noted in Chapter 5, this is not always feasible or cost-effective.[2] This chapter explores the statistical issues underlying the determination of weights. It outlines the objectives and criteria for determining weights, describes and evaluates the varying data sources that are traditionally used to generate the weights, and suggests some additional sources and methods for deriving the weights. Finally, it describes how the weighting might be accomplished in practice.

B. Role of Weights

4.2 The PPI is calculated from many prices collected from all types of establishments, covering the selected economic activities and products. The collected prices are first combined to compile indices for each individual product. For example, 10 prices for different types of transactions for a product may be collected from an establishment, and these prices are combined to produce the index for the product from that establishment. Weights are usually not available for these individual transactions, and the establishment's product index is thus computed as an unweighted average of the prices collected for the various transactions. Once this has been done, the establishment product indices are combined to produce the subgroup and group indices, and eventually the all product index (see Figure 4.1 in Section C.4). Because some products have greater production or sales than others, each product is given a weight to represent its importance in total output or sales during the reference (base) period for the weights. To arrive at the aggregate index figure, the price relatives of the individual products are multiplied by these weights to derive a weighted average aggregate index.

4.3 Thus, the weights are key elements in the construction of a PPI. They determine the impact that a particular price change will have on the overall index. For example, in some countries, a 5 percent rise in the price of milk products would have a much greater impact on the average rate of price change in the producer sector than a 5 percent increase in the price of tea products because the output value of milk is higher than that for tea. Without weights, relative price changes for all commodities in the PPI basket would be given equal importance in the calculation of the index above. Of course, if there is no dispersion of price changes, then weights would be unimportant.

4.4 Over time, establishment production levels shift in response to economic conditions. Some products and industries become more important while others become less important. Statistical offices periodically should update the weights in the PPI to reflect these changes in market structure. Best practice suggests that this be done at least once every five years. Details on how to introduce new weights into the PPI appear in Chapter 9, Section C.

[1]As noted in Chapter 5, when it is feasible to implement probability sample designs for selection of elementary products, the weight may also reflect the fraction of total output a sampled product represents among the totality of transactions in an economic activity or product class that are produced by businesses.

[2]Referencing the previous footnote, probability samples generally are not implemented except in large, advanced statistical systems. In the absence of such probability designs, elementary product weights may be judgmentally determined. In the interest of adopting transparent, reproducible procedures under judgmental sampling, elementary product weights generally are taken to be equal within an industry or product classification that is to be represented in the index. Equal weighting may also be implied by certain simple randomized sample designs, such as simple random sampling with replacement.

C. Appropriate Weights and Structure for PPIs

C.1 Value weights

4.5 As discussed in Chapter 14, the value aggregate from the national accounts framework that aligns with the basic price received by the producer of goods and services is the value of production. Thus, when estimating the PPI using the weighted average of long-term relatives formula (that is, the current price divided by the base-period price as in equation [4.2] of Section C.2), the best approach would be to have value of production weights at basic prices for all levels of index aggregation (from the elementary aggregate level of product/commodity within the establishment to the total output index by industry or product).

4.6 Since the PPI can also be used to measure the change in intermediate input prices, the value weights for the input index would be the cost of the input products to the producer. In the supply-and-use framework presented in Chapter 14, this value would be the cost of intermediate inputs valued at purchaser prices.

4.7 The use of values to weight long-term price relatives (that is, the current price divided by the base-period price) maintains the fixed-quantity relationship that existed in the base period. The value weight multiplied by the long-term price relative provides the estimate of what it would cost at today's prices to produce the quantity of product in the price reference period.

4.8 The value of production comprises the receipts from sales of all output by establishments and the change in value of inventories of finished goods on hand at the end of the period. If the value of production is unavailable or questionable because of concerns about the estimation of inventories, total sales (turnover) may be used. An analogous measure would be the value of shipments (that is, value of goods shipped at basic prices).

C.2 Quantity weights

4.9 In the traditional Laspeyres formula, base-period quantities can be used as weights to value base-period production volume at current-period prices. Consider the following:

$$(4.1)\quad I_L^{c,m} = \frac{\sum P_i^m q_i^0}{\sum p_i^0 q_i^0},$$

where $I_L^{c,m}$ is the Laspeyres price relative for subcategory "*c*" in month "*m*,"
P_i^m is the average price of product "*i*" in month "*m*,"
Q_i^0 is the quantity of product "*i*" purchased or sold in the base period "0," and
P_i^0 is the average price of product "*i*" in the base period "0."

The value in the numerator is often referred to as the current value of base-period production. It reflects what the cost would be at current prices to produce the quantity of output in the base period. This current value of base-period production is compared with the base-period value of production in the denominator to derive the long-term price relative.

4.10 The use of quantity weights is appropriate as long as the same specific product was produced as in the base period, that is, there is no qualitative difference between the current product produced and the base-period product. If the price-determining characteristics among the various transactions that are priced differ, then we have a dissimilarity, and the transactions with different characteristics should have separate weights.

4.11 Quantity weights are feasible only at the detailed product level. At higher levels of aggregation, such as at the product group level or industry level, a value aggregate is more appropriate for calculating the index because there are no unique, meaningful quantity levels available that apply to different products.[3] Thus, the index at the aggregate level would be the ratio of the sum of the base-period quantities valued at current prices to the sum of the base-period values, as in equation (4.1), but the values in the numerator are those summed from the calculation of values for each of the products at current prices. Alternatively, the simpler formula-

[3]This holds true unless one is willing to accept a notional or implicit quantity measure that is a representative aggregate of the different quality products being compiled. The problem with this approach is that the implicit quantity measure then must assume some type of average quality that should be comparable over time.

tion is to use a base-period value weighted average of price relatives such as

$$(4.2)\quad I_L^{c,m} = \sum \left(\frac{p_i^m}{p_i^0} \right) \frac{p_i^0 q_i^0}{\sum p_i^0 q_i^0} .$$

C.3 Net output weights

4.12 The output of one activity is often used as input to another activity within the same industrial grouping, as discussed in Chapters 2 and 17. The use of gross value weights for both activities would result in double-counting because the value of output in the first activity (for example, raw materials) is an input to the second (assembled goods). The value of output of the second activity, therefore, includes that of the first. If the two activities are aggregated to produce a group index, the importance of the first activity is counted twice in the group index. To eliminate this double-counting effect, net weights can be derived.

4.13 One of the principal uses of price indices is to analyze the price change faced by buyers of particular commodities. Such analysis may not be a problem at a detailed level because product price indices are particularly useful for this purpose. For example, a change in the index for "primary aluminum ingot, alloyed" is easily interpreted by a buyer of this product. However, the interpretation of price changes that involve various products or different industries may not be straightforward if they include the effects of overweighting. For example, if the basic price of aluminum increases, how should one interpret a metals products index that includes various types of aluminum at different stages of production? To interpret this aggregate index correctly, it is necessary to know how the various elements in the index have been combined, including specifically how these elements have been weighted together to form the higher-level index.

4.14 Using a weighting scheme based on net output weights eliminates double-counting when aggregating. However, before the net output weight can be defined, it is necessary to define the aggregation structure. It is the aggregation structure that determines which prices should be counted. Only then can the weight structure be identified and the value for each component determined. Thus, the process of constructing net output weights involves two steps:

(i) Define the aggregation of interest in such a way that it is possible to identify the portion of the products produced within the aggregation that is sold to buyers outside of the aggregation.

(ii) Assign weights to the products produced within the aggregation that reflect only the value of products sold to buyers outside of the aggregation. These weights are termed **net output weights** because they include only the value of output for products exiting the aggregation, that is, the net output.

4.15 When this type of weight structure is used, price movements of products are included only to the extent that the products are sold outside of the aggregation structure. Thus, each aggregate index can be viewed as a measure of price change for buyers of the final products from enterprises included in the aggregation structure.

4.16 In many countries, net output weights are used to develop aggregate indices by the processing stage. In such aggregations, the weights used for products sold for final demand exclude the value of goods used as intermediate inputs. This approach avoids the problem of giving too much importance to price changes of intermediate goods as they wend their way through the production process.

C.4 Classification issues

4.17 For the purpose of applying the weights, products are grouped either because they have a common end-use or because they are considered substitutes for one another.[4] These families of products are joined at different levels to form a hierarchy in a classification system. Every product has a unique place in the classification used. Such criteria were used when the International Standard Industrial Classification of All Economic Activities and other classifications were established.

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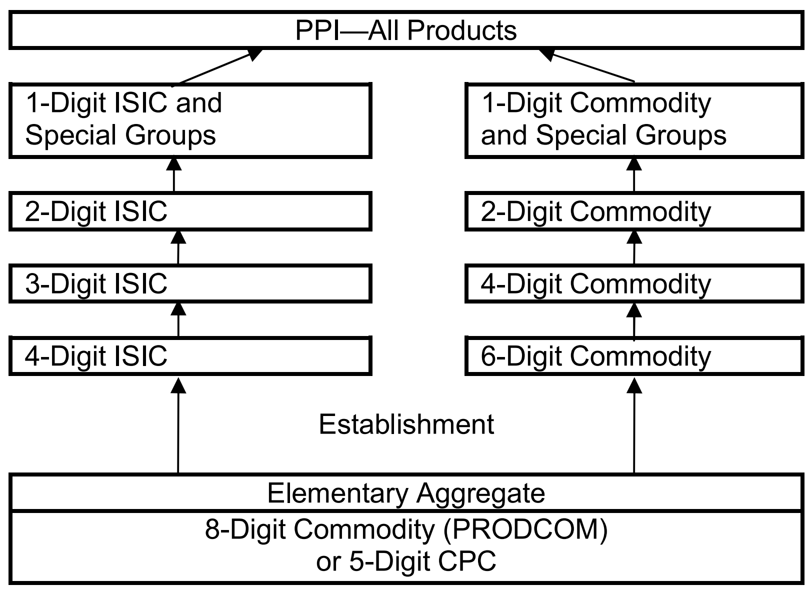
[4]Alternatively, some groupings are made for products that exhibit common price trends. Such groupings are important when a product index or group index is used to make imputations for missing products, which is discussed in Chapter 9.

Figure 4.1. Typical PPI Aggregation Structure



4.18 For the purposes of international comparison and internal consistency, the classification scheme of goods and services should be in line with the most recent version of the Central Product Classification, version 1.1 (CPC), or the Classification of Products by Activity. In terms of economic activities, the establishments should be classified using ISIC, Revision 3, the General Industrial Classification of Economic Activities within the European Communities (NACE), Revision 1, or a derivative of these industrial classification systems. From an individual country's perspective, it is also desirable that the classification used be consistent across all enterprise and production statistics (for example, establishment census and industrial survey statistics).

4.19 Each product selected for inclusion in the PPI is assigned a product code in accordance with the product classification system. Likewise, each establishment selected in the sample is assigned an industry code according to the industrial classification system. Subindices by product are computed from groupings of the selected transactions in accordance with the product classification system. The selected sample transactions can also be aggregated according to the industrial classification system to produce indices by economic activity. These subindices are further aggregated following the hierarchy of the classification systems to arrive at major groups or divisions and, finally, the all products index as shown in Figure 4.1. Because inputs may be overweighted in the aggregation to derive higher-level indices, the statistical agency may choose to use net output weights as discussed in the previous section.

4.20 This aggregation starts with the sample of specific product transactions selected within establishments. The transaction prices or price relatives are combined using the price index formula to arrive at the first level of index aggregation, which is referred to as the elementary aggregate or the elementary index. Weights are often not collected below the first level of aggregation. This aggregate is usually for specific types of products within the product classification. In our example, we use the eight-digit product code level. At all subsequent levels (establishment, six-digit product code, etc.), it is necessary to obtain a consistent set of aggregation weights. For example, the weights for the sample of establishments should cover the entire four-digit industry even though not all establishments were selected. This means that the weights for the nonsampled establishments must be assigned to those selected. Also, the weights for the products selected within an establishment should include the entire weight for the sampled establishment. Once the weights have been established at these levels, it is a relatively straightforward procedure to aggregate by industry or product to higher-level aggregates.

C.5 Unimportant industries and products

4.21 Some industries and products will be of little importance in terms of their share of total production. For example, an industry that represents less than 0.1 percent of production within the industrial or service sectors could be excluded from the sample. In such cases, the output for the industry that is excluded should be distributed across those that were selected, or it should be assigned to a closely related industry. It may also be possible to make meaningful combinations of smaller industries producing related products that meet the criteria for minimum sizes. A similar procedure would also be applied to products that are insignificant. In either case, the weight for the nonsampled component needs to be included somewhere in the weighting structure.

4.22 A situation that will occur is having an important industry or product that falls below the size threshold chosen. In such an instance, if no meaningful combinations are apparent, the industry or product may have to be published on its own. This often is the case for growth sectors where industries and products are expected to become more important over time. The statistical office will want to include them because their contribution to economic activity will become significant before the next scheduled weight update.

C.6 Time period covered by weights (weight reference period)

4.23 The weight reference period is the time period—usually a whole year—to which the weights relate. The accuracy and reliability of a PPI are determined, in large part, by the weighting structure. For this reason, the choice of the period covered by the weights is crucial. The period chosen as weight reference period should be (i) reasonably normal/stable and (ii) not too distant from the price reference period.

4.24 The weight reference period and the price reference period used in the index formula should refer to the same period. When they differ, the weights should be updated for price changes between the weighting period and the price reference period. For example, if the weights refer to calendar year 2001 and the base price is for December 2001, the weights should be adjusted for the change in prices between the average price for the calendar year and the price in December. This is discussed further in Chapter 9.[5]

4.25 The weights may be chosen from multiple periods depending on the formula used to calculate the index. In Chapter 15, it is recommended that a symmetric index be used, which requires weights for the base period and the current period. In practice, weights are often not available for the current period on a sufficiently timely basis; base-period weights, therefore, are normally used. For example, the weights may represent (i) the value of output produced during the price reference period (Laspeyres index), (ii) the value of output produced during the current period (Paasche index), or (iii) a geometric average of the values in base and current period (Fisher or Törnqvist index). An index computed by using quantity or value weights for the current period can be produced only with a time lag, because it takes time to collect and process current production data. That is why most statistical offices adopt a Laspeyres-type index, which requires quantity or value weights for the price reference period only.

4.26 The weights that are used typically refer to a single calendar year. In some instances, a single year's data may not be adequate either because of unusual economic conditions or insufficient sample

[5] There is also an index reference period, which is the period when the price index is equal to 100. In many countries, the weight reference period, the price base period, and the index reference period are the same. More and more frequently, however, countries are introducing chained indices in which the weights are updated on an annual basis. In such cases, the three periods can be different. For example, weight reference period could be the previous year (2001), the base price period could be the previous December (December 2001), and the index reference period could be maintained as December 2000 = 100. This is discussed in more detail in Chapter 9.

sizes from survey data. An average of several years' data may provide the best weight reference period because it reduces the sampling and seasonal variance of the production or sales data for a given size of the annual sample.[6]

4.27 For seasonal products (as discussed in Chapter 22), it may be preferable to develop separate weights by month or by quarter to calculate indices at the elementary aggregate level. In addition to information for each period within the year, this approach may require additional data for the same period in a number of previous years.

D. Elementary Aggregate or Stratum-Level Weights

D.1 Coverage of weights

4.28 The calculation of the all industry or all products indices starts with the measurement of the relative price change for an elementary aggregate, which represents the first level at which price observations are combined to calculate an index. At this level, sometimes referred to as a stratum, weights are needed to calculate higher-level indices. This typically involves combining individual-product-level or establishment-level indices to derive product groups industry indices. The elementary aggregate index covers all prices collected for one product in one stratum. The stratification may be by product, industry, size of establishment, or some combination of these.

4.29 It is important that the weight for each elementary index represents the production value of *all* products produced within the stratum, not just the value for the selected sample of particular products at particular establishments chosen to represent this aggregate. (Chapter 5 deals with ensuring that the sample of elementary products is representative.)

4.30 Below the elementary index level, individual transaction prices may not have weights because the statistical office has not collected additional data on production or sales for sampled products within establishments.[7] If no weights are available, depending on the formula used (see Chapter 9), it is assumed that all the weights are equal (an average of price ratios approach) or the weights are proportional to their base-period prices (a ratio of average prices approach).[8] The former means that each price quote within the elementary aggregate is as important as any other price, that is, the shares of production value are equal. In the case of the ratio of average prices, the importance of each price quote depends on its price level in the base period and the fact that all the quantities produced are equal. This is appropriate if the production value in the base period is proportional to relative price levels in the base period. Thus, items with higher prices in the base period have more importance.

4.31 Once the price indices for the elementary aggregates are computed, the product/industry indices are obtained as weighted combinations of the indices for each elementary aggregate. Then the product indices are combined following the hierarchy of the classification, with appropriate weights applied along the way. For instance, assume the elementary aggregate is established at the eight-digit product code level (as in Figure 4.1). All transactions within this classification are used to estimate the eight-digit product index. Each eight-digit product index has an assigned weight, and the indices are aggregated to produce the six-digit product group level index. All six-digit product group indices are further aggregated using production weights at the six-digit level to obtain four-digit level indices and so on, until the all products index is obtained. In addition, the eight-digit product indices can be aggregated to derive industry-level indices, and industry indices can be aggregated according to the industrial classification structure to derive group- and division-level indices.

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[6] During periods of high inflation, multiple-year weights should be calculated by averaging value shares rather than averaging actual value levels. Averaging the value levels will give more importance to the more recent years' data.

[7] The situation in the United States is somewhat different since compilers use probability sampling, where the weight within the elementary aggregate is determined by the inverse of the probability of being selected in the sample.

[8] The average of price ratios formula is $\frac{1}{n}\sum\frac{p_i^t}{p_i^0}$. The ratio of average prices is: $\frac{\sum p_i^t}{\sum p_i^0}$.

D.2 Sources for weights

4.32 The primary sources of weight information for the PPI are business- or establishment-based censuses, annual industrial surveys, and business registers.

D.2.1 Business or establishment censuses

4.33 The business census covers all establishments that have productive activity within the geographic borders of the country. These censuses may be conducted over several years with different economic activities covered at different times during the cycle. For example, a census of agriculture would be conducted one year, a census of industrial activities (mining, manufacturing, and energy supply) completed during the next year, and a census of services the year after that. In some instances, there may be a size cutoff to exclude small establishments. For example, some countries exclude establishments with fewer than five employees or with a low threshold of annual production. Alternatively, those countries might complete the census using a sample of small establishments only.

4.34 A detailed accounting of annual output in value (at basic prices) and quantity terms by detailed product classification is typically obtained at the enterprise or establishment level. This would include sales and inventories by product, as well as value and quantity of inputs at the prices paid by producers. These data can be used to derive the value weights by detailed product classification and establishment. This is an excellent source of weight data, assuming that the coverage of economic activity is essentially complete.

D.2.2 Enterprise or industry surveys

4.35 These surveys differ from censuses primarily in three respects: (i) the coverage is limited to a sample of establishments rather than a full enumeration, (ii) the product detail is limited to higher aggregate levels such as groups, and (iii) the types of data requested are generally more limited. For example, product information in the census may be obtained at the eight-digit product code level with complete detail on product sales and inventories. In the industry, however, survey data are reported at the six-digit level and are requested only for sales. Also, data may be reported only at the enterprise level rather than broken down by establishment.

4.36 In these cases, the weights that are available will generally be for higher levels in the aggregation structure such as product group and industry, rather than detailed levels like product and establishment. The use of these weights for the PPI will depend on how the PPI aggregation structure has been established. If multitiered weights (for example, one set of weights for the industry level and above, another set of weights at the establishment level and below) have been set up, the survey results could be used for aggregation at higher levels, while the weights at lower levels are determined separately. For example, the survey weights could be used for aggregating from the four-digit industry level to higher levels, while sampling weights (that is, sampling fractions from probability selection procedures) could be used at the establishment and product level. In this scheme, the weights at the higher levels would be updated periodically from the industry survey data, while the weights at the lower levels would be updated as the samples of establishments and products are refreshed. This process is discussed in more detail in Chapter 5.

D.2.3 National accounts

4.37 Although much of the same source data described above would also be used in developing the output data for the production account in the national accounts, there can be significant differences. In a number of countries, there may be significant undercoverage in annual industry surveys because of the exclusion of informal activities. National accountants often make adjustments from a variety of sources for this type of undercoverage or for known biases in the survey data. In such instances, the adjusted national accounts information on output by industry may prove to be a better source of weight information at the industry level than the original survey data.

4.38 The national accounts often provide additional detail on weights, particularly if supply and use tables or input-output tables are available. The information on commodity flows for various industries and products by type of use is an excellent source of net weight information for developing stage-of-processing indices.[9]

D.2.4 Business register

4.39 Most countries maintain a business register, which provides a list of firms that are involved in productive activities. Such registers usually contain information on location, economic activity, size (for example, employment, payroll, value of annual production, or turnover), contact persons, tax information, and so on. The business register could be an alternative source of weight information, particularly if business censuses are not conducted on a regular basis or if annual surveys do not provide sufficient information for establishing weights. This is particularly true if there is an ongoing system for updating and maintaining the information contained in the register, and it contains data at the establishment level.

4.40 There are several shortcomings in the use of these registers for weight information. Often the business register is updated only when a firm begins operations. Unless the register is maintained by purging firms that are no longer in business, it will be outdated. The information on size of the firm also needs to be updated regularly. Much of the information may relate to the time when the firm was introduced into the register. Also, the business register may comprise a list of enterprises that is not completely suitable for the PPI, where the goal is to obtain information at the establishment level. The register will usually be devoid of information on products, which means that additional data collection will be necessary before weights can be established at the product level.

D.2.5 Other sources of weights

4.41 A variety of administrative data on production values may be available from public agencies charged with regulating or monitoring certain economic activities. For example, many public utility, communication, and transport activities are regulated by national, regional, or local governmental bodies. Typically, these agencies require detailed annual reports that provide information on production value and turnover. These sources also have records of all regulated enterprises/establishments, which can be used as a source for a sampling frame.

4.42 Another source for weight data is industry associations. Many associations conduct surveys of their membership that include detailed information on value of sales by product. Alternatively, in industries dominated by one or two large firms, the market shares for these firms can be a source of weight data.

4.43 In many countries, data on retail and wholesale turnover are produced regularly. Such data, if maintained at a detailed economic activity level, could serve as a source of weights for wholesale and retail economic activities. This would depend on whether wholesale and retail trade will be included in the PPI and if the survey information is deemed reliable for use as weights.

4.44 Customs records are an alternative source of information on exports by product and enterprise. If detailed customs records are maintained and available for statistical purposes, information on detailed products by shipping enterprise should be available and provide a source for weights, as well as a potential frame for samples of export products.

4.45 *Statistical offices should make certain that data from any alternative sources conform to the definitions of the PPI.* Whichever weight concept the PPI uses (output, production, sales, or value of shipments), the data from these alternative sources should conform to that definition. For example, data on retail and wholesale turnover may be available at a detailed product level. One problem with these data is that they measure sales at purchasers' prices, which is inconsistent with other weights based on output at basic prices. The statistical office would need to adjust the sales information for taxes on products (for example, VAT) and separately invoiced transport charges. This adjustment derives turnover at basic prices; but, to derive an output measure, the statistical office would have to also make estimates of inventories for each product. In the example, if value of shipments were the weighting concept, only the adjustment used to derive basic prices is required.

4.46 Statistical offices also need to adjust primary source data for any known inconsistencies or errors. It often happens that reporting errors and in-

[9] Use of output data from the national accounts supply and use tables will provide weights that include nonmarket activities (see Chapter 14). Users must be aware of this fact if they intend to exclude nonmarket activity from the PPI.

consistencies are uncovered in censuses and surveys after final results are available. Statistical offices need to make sure appropriate adjustments are made to these source data when deriving PPI weights. For example, an establishment survey provides weights on total output by product, which includes the total value of inventories. The statistical office realizes that output values should only include the change inventories. It will be necessary to go back to the source data and adjust the final inventory figure to take out the value of inventories at the beginning of the period.[10]

E. Product and Transaction Weights

4.47 The selection of transactions to observe the price movements for each industry or product in the classification systems is a sampling issue discussed in detail in Chapter 5. The value weights at the industry or product level will generally be obtained from one of the sources discussed in the previous section. As soon as these results are available, one must determine what specific transactions in goods and services should constitute each elementary aggregate of the PPI. The data from industrial censuses are preferred because they provide a much larger coverage of goods and services than can possibly be observed in most surveys of enterprises. However, even the census will not contain details for each transaction that has transpired. For this reason, each elementary aggregate of the PPI must be represented by selected goods and services that are considered either important or representative of typical changes in relative prices for their class. The relative price changes of these particular goods and services are then monitored, and their average is subsequently used as a measure of relative price changes for that elementary aggregate.

E.1 Explicit and implicit weights

4.48 When the sample of representative transactions has been selected, a determination must be made about whether explicit weights can be derived. If probability sampling techniques are used, the inverse of the sampling fractions (or the sampling intervals)[11] are used as the weights.

4.49 In the case of judgmental samples, the weights for the selected industry and establishment should be adjusted to incorporate the weights of transactions not selected for the sample. Thus, the weight for small industries not selected should be allocated to those that were selected. For establishments, the same approach is used; the weight for the nonselected establishments within an industry must be allocated to those that were selected. Within the establishment, the total weight for the establishment can be distributed to the representative products in proportion to their share of sales. Finally, for each representative product, the weight for the product can also be distributed to each selected transaction in proportion to the selected transaction's sales. In this fashion, the weight for each establishment would be allocated to each price observation.

4.50 Alternatively, if certain products in an elementary aggregate are judged more important, higher weight may be assigned judgmentally or on the basis of secondary information from administrative or industry sources.

4.51 If no weights are available for the selected transactions, the formula used for averaging price observations will assign implicit weights to individual transactions. If the average of price ratios formula is used, as discussed in Section D.1, the implicit assumption is that relative price changes for each transaction within the elementary aggregate are equally important in terms of base-period quantities.[12] If the ratio of average prices is used, we assume that the importance of each observation is proportional to its base price.[13] The latter approach makes the strong assumption that production values are proportional to the base prices. In the ratio of average prices formula, transactions with

[10]This assumes that there has been no change in prices between the start and end period. If there has been such a change in prices, an inventory valuation adjustment must be made. See Bloem, Dippelsman, and Maehle (2001, pp. 60–63) or Shrestha and Fassler (2003) for techniques to make such an adjustment.

[11]For example, if total output for an industry is 10,000 and five establishments are to be selected, then the sampling fraction is 1 in 2,000, the sampling interval is 2,000, and the weight for each establishment selected is 2,000.

[12]This uses the first formula in footnote 8. For each transaction, the current price is divided by the base price, then the average of these price relatives is calculated.

[13]This uses the second formula in footnote 8. The current average price of the selected transactions divided by the base-period average price of the selected transactions yields the price relative.

higher prices receive more importance than those with lower prices. Often these differences in price levels occur because of the nature of the transaction specifications rather than real differences in the relative importance of transactions within the establishment.

4.52 Another alternative formula is the geometric average.[14] The geometric average of price relatives and the ratio of geometric average prices yield the same result. The use of this formula assumes that the weight of each observation is equal to its share of base-period production value (not its share of base-period quantities). Thus, as relative prices change, the assumption is made that there is an inverse relationship between the change in prices and the quantity produced consistent with a unitary elasticity of substitution so that a 1 percent rise in price results in a 1 percent decline in quantity produced. For the PPI, this inverse relationship between price and quantity may not be a valid assumption under some circumstances. See Chapter 20 for a detailed presentation of this issue.

E.2 Sources of product and transaction weights

E.2.1 Business censuses and surveys

4.53 As discussed previously, the censuses of business and the establishment census[15] are good sources for value of production or sales information to use as weights at the establishment and product level. Usually, such censuses would also contain information about products within establishment that is the most valuable source for obtaining weights by product classification within establishment. These censuses will not provide information by transaction because such information would place a heavy burden on reporting units.

[14]The geometric average formula is $\prod_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)^{1/n}$.

[15]The censuses of business usually are conducted by economic activities such as agriculture, mining and manufacturing, trade, services, and so on. These are usually collected in a cyclical fashion with one or two censuses per year over a five- or seven-year period. The establishment census covers all establishments at one time, regardless of their economic activity. Thus, the establishment census has broader economic coverage than the individual economic censuses.

4.54 Annual surveys by industry will often provide information at higher levels of aggregation, such as estimates of production by industry or key product lines within industrial activity. However, information at the establishment level is generally limited to the sampled establishments and will not contain full product detail within those establishments. Establishment and detailed product weights will be available from these surveys only to the extent that there is an overlap in the samples of establishments and products between the PPI survey and the industrial survey.

4.55 If a multitiered weighting system is used, such surveys would be a good source for updating weights at higher levels of the aggregation structure. They could also be used as a source for updating weights when producing annually chained indices (see Chapter 9).

E.2.2 Business registers

4.56 If the business register contains production or sales data, it forms a potential source of establishment weights. If the register is updated frequently, the weight information could be more current than census data. However, the business register is not likely to contain data on products produced within individual establishments. In addition, the weight information in the register may have differing reference periods for the establishments depending on procedures for updating information. If this is the case, the value weights will need to be adjusted for the differences in the weight reference period so that they are standardized across establishments.

E.2.3 Weights obtained from the probability sampling process

4.57 Sampling fractions or sampling intervals developed when the samples are drawn can be used as weights at the establishment and product level as appropriate. Individual weights at both the product level (if not available during the initial sampling phase) and the transaction level can be obtained through a sample disaggregation process using probability sampling techniques at the establishment level.

4.58 Disaggregation within establishment is accomplished by working with a knowledgeable respondent to determine probabilities of selection from production or sales data available at the establishment level as described in Chapter 5. By applying this technique at various levels, products and transactions are selected and the sampling factors ultimately determine the weights for the products and transactions.

E.2.4 Internal product and transaction weights obtained from establishments

4.59 When judgmental selection of products is used, the weights for each product are adjusted proportionally upward to represent all products within the establishment or product classification as discussed in Section E.1 of this chapter. Similarly, when judgmental selection of transactions is used, the weights for each transaction within the selected product can be adjusted proportionally upward to represent all transactions for the product.

E.2.5 Other sources

4.60 Data from administrative and regulatory sources can also serve as a source of weights if there is reporting of product and transaction information in sufficient detail. In the last few years, some countries have started to use electronic databases maintained by enterprises, marketing firms, and trade organizations to derive weights at the establishment, product, and transaction level. The databases consist of electronic data records that are maintained by or collected from producer enterprises. These data sets include information on the quantity sold, inventories, and the corresponding values for each. They also include the individual transactions, their prices, and the specifications for the transaction. This information can be used to derive PPI weights at the product and transaction level more frequently than otherwise would be possible. However, one should bear in mind the limitations of this source of information: the data usually represent only large producers. This may be adequate in highly concentrated industries, but it is less useful where small enterprises are prevalent.

4.61 Additional data may be available from tax revenue sources. Many countries have value-added or gross sales tax schemes that provide detailed information on sales revenue for a variety of enterprises and economic activities. Electronic scanner data on sales collected at the point of purchase are also available and used by a number of countries to derive weight information for detailed classifications.

E.2.6 New revolutionary products

4.62 As discussed in Chapter 8, new products should be introduced into the PPI as soon as possible to avoid potential bias in the index. The chapter discusses two types of new products: *evolutionary* products that represent continuous improvement over existing products and *revolutionary* products that represent a break from products previously available and are a new genre. Traditional surveys and sources of weight information typically will not provide the statistical office with any usable data. Some examples of revolutionary products include video recording devices and mobile phones.

4.63 If the new product falls within the existing classification structure, it can be introduced into the PPI calculation system by adding the product within an existing class. The weight for the product class remains the same, but the weights for the individual products will have to be recalculated. Since information on output of the revolutionary product will not be available from existing establishment surveys, the statistical office will have to seek production data from other sources. If only a few enterprises are involved in the production or distribution of the product, the statistical office can do a special survey to collect the value of output data directly from the enterprises. Other alternatives are to contact a trade association that represents the industry and product or, if it is subject to regulation, to contact the regulatory authority. The statistical office will also have to determine the specific sample transactions to price on an ongoing basis.

4.64 Once a production value is obtained, the temporal value of the new product weights is aligned with that of the other products in the class. For example, if the new weights refer to calendar 2002, but the weights for the other products in the class are for 2000, the new product weights should be deflated to a price level reflecting calendar year 2000 prices. The statistical office can use the product class PPI to deflate the 2002 production value. For example, a new mobile phone system is introduced in addition to the traditional land line system. The enterprise offering the new system can provide data on total revenue received during 2002, its first year of operations. The other weight information in the telephone class has a weight reference period of 2000. Mobile phone data for 2002 reflect average prices in 2002 and can be adjusted to 2000 price levels by dividing the 2002 value by the price change in the telephone class between 2000 and 2002.[16] The new weights for land line and mobile are then used to calculate the aggregate index for telephone services using the old elementary index for land line phones and the new index for mobile phones.

E.2.7 Household enterprises

4.65 Household enterprises engaged in economic activity should be included in the PPI. Often statistical offices will exclude establishments below a certain size, for example, those that have fewer than 10 employees. Such size cutoffs are made because of the lack of good source data for weights and the relative unimportance of such establishments in most industries. This will exclude most unincorporated household enterprises; but, in many industries, this type of establishment dominates. For example, small establishments dominate many home craft industries and agriculture. It is important that they be included in the PPI for these industries.

4.66 In many countries, statistical offices can identify these establishments as part of their establishment censuses where data on value of production or turnover can be obtained. Such censuses can be used to develop sampling frames for industries with significant concentrations of small establishments. In other countries, government tax authorities maintain records on such establishments for administrative purposes. As mentioned earlier, the tax records may not have adequate information to derive PPI weights, but they can serve as a sampling frame for identifying units. The statistical office may have to derive the value of the weights for these establishments as part of a separate survey. For example, a random sample of small establishments can be drawn from the tax files to collect information on production, products, and prices. This information would then be used to estimate production weights for the establishments and products in the PPI price survey.

[16]The index compilers would have to make a similar adjustment to the 2002 prices for the selected mobile phone transactions to estimate base prices for calendar 2000. Current prices would then be compared with the estimated base-period prices to derive the elementary index for mobile phones on a 2000 reference period.

F. Practical Steps for Selecting and Determining Weights

4.67 The process for determining weights in the PPI structure can be viewed in a variety of ways. The following represents an overview of the steps required so that readers of this *Manual* have a sense of the milestones involved in developing a full set of PPI weights.

F.1 Determine sources for weights of economic activities and products that are in scope

4.68 The initial scope of the PPI is established in terms of the economic sectors (manufacturing, mining, construction, agriculture, transport, etc.) and the products that are produced in those sectors. The sources of the weights for each sector must be determined from those sources discussed in Section D. A review of these sources may indicate that additional surveys or censuses are needed.

F.2 Determine weights for sampled industries and products

4.69 Samples with a cutoff are used to determine the activities and products that will be included in the index for each sector. Such samples exclude activities and products that fall below a certain threshold. For the selected industries and products, weights must be established from those sources presented in Section D.

F.3 Determine weights for sampled establishments

4.70 The establishment weight could be taken directly from information provided in the sources discussed in Section E.2. However, if probability or judgmental sampling techniques must be used, the weight for each sampled establishment must be derived. If establishments are selected using probability techniques, the weights will be derived from the sampling probabilities. If judgmental techniques are used, the weights for selected establishments must be adjusted upward to include the weight for the remaining establishments.

F.4 Determine weights for sampled products

4.71 The product weight could be taken directly from information provided in the business census if that information is available as discussed in Section E.2. However, if probability or judgmental sampling techniques must be used, the weight for each sampled product must be derived. If products are selected using probability techniques, the weights will be derived from the sampling probabilities. If judgmental techniques are used, the initial weights for selected products must be adjusted upward to include the weight for the remaining products. When all products are selected within an establishment, the final product weight can be determined by distributing the establishment weight to each selected product using the product's relative importance among all selected products.

F.5 Determine weights for the sample of transactions

4.72 If no weights are used, the index formula for combining the transaction prices will determine the implicit weights as discussed in Section E.1. If transaction weights are used, they will be determined as discussed in Section E.2.

F.6 Adjust weights on the basis of sample yield

4.73 After all the establishments have been brought into the sample, the weights must be further adjusted for sample losses during the recruitment phase. Establishments that refused to participate must have their weight allocated to those that did respond, and other adjustments, as discussed in Chapter 5, may be necessary for establishments that were out of scope or out of business. When establishment weights are adjusted, the product and transaction weights may also have to be adjusted if they represent actual values rather than proportions.

F.7 Update weights for price changes due to differences between weight reference date and price base period

4.74 When the weights are introduced into the monthly or quarterly processing system, they must represent the same time period as the price base period used in the index calculation. If the weight reference period and the price base period differ, the weight should be adjusted for price changes (see Chapter 9).

F.8 Adjust weights as sample augmentation occurs

4.75 Sample attrition results in a continual decrease in the number of products and establishments in the sample. In addition, new products are produced by enterprises in response to customer demand. The PPI should have a cycle that augments the sample; doing so maintains the size and representation of the sample as discussed in Chapter 5. When new samples of establishments and products are introduced, the weights for establishments and products within establishments will need to be adjusted.

4.76 Portions of Chapter 10 deal in greater detail with determining the weights for some specific economic activities such as insurance, financial services, and retail trade.

5. Sampling Issues in Price Collection

A. Introduction

5.1 In an ideal world, it would always be possible to use statistically sound sampling techniques to produce price indices with a high degree of accuracy and within given resource constraints. Reality, however, is usually very far away from this ideal. It is almost always impossible to achieve efficient samples because (i) accurate estimates of population variances, required for allocation of sample units to strata, are rarely available; (ii) sampling frames are always deficient to some extent, missing some key information, such as births of new establishments, or desired stratification variables; and (iii) response rates are unpredictable and may prove to be deficient, which affects the accuracy of the price index levels and measured price changes.

5.2 The aim of the sampling statistician is, therefore, to make the best use of what is available and to apply the principles of sampling theory in a commonsense and practical way. Arguably the most important steps in sampling are to establish and understand fully what the survey is trying to estimate, the limitations of the sampling frame, and the environment in which the survey will be conducted, that is, likely response rates, data quality, and levels of resources.

5.3 There is a direct relationship among the uses of the PPI, the scope of the PPI survey coverage, and the requirements for sampling frames. Two of the major uses of the PPI are as a general indicator for inflation and a deflator in the national accounts. The broader the coverage of the PPI in terms of economic activities, the more useful it is in inflation analysis and compiling constant price GDP measures. But broad coverage requires the ability to develop sampling frames for a wide range of economic activities, including both goods-producing and service-producing activities. These sampling frames also must be kept up to date by recording both the births and deaths of enterprises in each sector.

5.4 Once coverage and uses have been established, a sample design can be drawn up, with decisions made about stratification, sample size, and allocation. Random sampling techniques may be employed in countries where large amounts of data are available and reasonable estimates of variance can be made. In many country situations, only limited details of sampling parameters are available, and the statistician may have to fall back on procedures that use expert knowledge at many stages in the selection process. To the extent possible, acceptable, practicable sampling procedures should be used. Judgmental approaches should be used only as a last resort.

5.5 As with most panel samples collected through time, price surveys suffer from problems associated with a changing population. Any sample of establishments and products will become increasingly unrepresentative over time, and it is likely to be depleted as establishments cease the sale or the production of selected products or cease operations altogether. Some form of panel rotation or supplementation for the samples is advised to minimize any bias caused by sample attrition, non-coverage of new products, new establishments, and new production technologies.

B. Common Problems in Price Survey Sampling

5.6 There may be many reasons why price surveys are thought to be unrepresentative and thus liable to lead to inaccurate results. All national price surveys suffer from problems to some extent. The following are some examples:

- Samples are selected purposively rather than using probability sampling methods, increasing the chances of bias. For example, establishments may be selected for their convenient geographical location or because they are known to be good respondents;

- Without probability selection methods, estimates of statistical accuracy cannot be made (but without some initial estimate of variance, a randomly selected sample cannot be optimized—that is, lowest variance given cost constraints—either. This is a difficult problem that is dealt with later);
- The sample size for an industry or commodity may have become outdated if the industry or commodity has grown or contracted since the base period (period when sample was selected);
- New products may not be identified or included in the survey. This problem may be relieved to some extent by rotating the sample of establishments;
- The sampling frame may be out of date or may not include certain groups of the target population. For example, a common problem in the PPI is that information on small producers is unreliable because this group often is volatile and difficult for administrative authorities to track, resulting in the weight for small producers being wrong (typically they are underrepresented); and
- Surveys may be voluntary, increasing the chance of nonresponse bias that results when those who do not respond have different price experiences than those who do respond.

C. Starting Position

5.7 Before starting to design a price survey, it is vital to understand the reasons for the survey and its uses. This will determine the format of the outputs required and help decide what data should be collected for the inputs. It is essential to assess and understand the environment in which the survey will be conducted—for example, what response rates might be expected and how good the data quality might be. Obviously, some of the most important decisions to be made concern the level of available resources. So all of the following parameters will affect the sample design and the future success of the survey.

5.8 It is vital to establish the **objectives** of the survey, by consulting survey users and answering questions such as the following:

- *Will the price indices be used for deflation of output, and/or as a measure of inflation?*

If output deflation is the goal, then reliable, detailed industry and product indices will have a high priority in the PPI, and detailed item indices will be required in the CPI. If, on the other hand, inflation indicators are required, then more emphasis will be placed on aggregate indices, and a range of indicators may be required using different prices and weights—for example, input, output, wholesale, and retail price indices.

- *What will the geographical coverage be? National or regional?*

The geographical coverage is usually national for the PPI, but in a few countries with regional differences in price movements, regional indices may be important. In addition, a number of countries compile regional GDP estimates. There may be a need for regional PPI estimates for use as deflators, particularly if there are regional differences in price movements.

- *Do we want a monthly or quarterly time series?*

Typically, the PPI is collected monthly as an inflation indicator, but in many countries the PPI may be quarterly because of cost considerations and because its primary use is as a deflator for national accounts usually produced on a quarterly basis.

- *Which prices are we trying to estimate? Basic prices, producer prices, wholesale prices, or purchasers' prices?*

The pricing concept will vary depending on the type of index produced. For the output PPI, the pricing concept is the basic price, that is, the per-unit revenue received by the producer from production. For an input PPI, the pricing concept is the purchasers' price, that is, the per unit cost paid by the producer for material and energy inputs to the production process.

- *Assuming that a choice has to be made (for cost reasons), are industry PPIs of a higher priority than product PPIs, or vice versa?*

If industry PPIs are of a higher priority, then a two-stage sampling scheme is used to derive reliable industry and product estimates; whereas if product PPIs have priority, reliable product samples should be compiled and then aggregated to yield industry PPIs whose reliability may not be quite as accurate.

- *Will separate indices be compiled for export and domestic market prices?*

The PPI should cover all production of domestic producers, including products for domestic use and those for exports. Often countries collect information only on products for domestic use, although the PPI could be used to produce export price indices also.

- *Which industries and products should be covered? At what level of detail?*

In the PPI, the industrial sector (mining and manufacturing) and public utilities are the primary sectors typically covered. Services, however, are becoming much more important in terms of economic importance and growth, and should be covered in the PPI through future expansions.

5.9 The **data to be collected** must be identified and understood:

- *What is the type of price to be collected, and can we collect actual transaction prices rather than list prices?*

It can be difficult to define and collect prices for many goods and services. Often the quoted list or book price does not represent the price received by the establishment. Ideally, we want to collect actual prices received for a representative sample of establishment transactions. For goods, this can be achieved quite regularly. This is also the case with most services. However, for some services—for example, banking and insurance services—the service and price of financial intermediation are not clear-cut, and the actual price may have to be derived from transaction information. (Additional information on prices for these services is provided in Chapter 10.) In addition, if the main use of the indices is to deflate output, then the prices collected should be actual transaction prices.

- *Will we collect basic prices (excluding taxes on products, including subsidies, and excluding transport costs separately invoiced)?*

According to the *1993 SNA*, output of goods and services ideally should be valued at basic prices, and so should PPIs, if they are to be used as deflators. If output PPIs used prices other than basic prices, their subsequent deflation may give spurious results.

- *At what time should prices be recorded?*

In line with the valuation of output in the *1993 SNA*, accrual accounting rules should be followed as far as possible, so that in the PPI, sales prices are recorded at the time of shipping or delivery. Although country practices often differ—for example, prices may be recorded at the time of purchase or order—the preferred timing is at the time of shipping or delivery. Prices could be an average of several observations during the month or the price on a particular day of the month; both approaches are used and are acceptable.

- *How should a price (transaction) be described?*

The price-determining characteristics of each product or variety should be identified so that transaction specifications can be sufficiently detailed. For example, the price per liter of paint will depend on the number of cans to be shipped, type and quality of paint, terms of payment (net 30 days), type of customer, and any special discounts that may apply.

- *Are there likely to be periods of seasonal nonavailability? If so, how will these missing prices be dealt with?*

Seasonal nonavailability has a direct impact on the quality of the index because the sample size will be predictably lower during these periods. This should be taken into consideration in the design of sample strata, so that several similar products included within the strata have year-round availability. Also, sample sizes for these strata should be increased because of the higher variability in price movements among seasonal products.

5.10 A decision should be made about the **level of accuracy** required:

- *Ideally, a maximum acceptable sampling error should be identified for each published index.*

Sampling error can be assessed, however, only if probability sampling techniques have been used. This often means starting with some estimates of variance for the component index to determine initial sample sizes. Then, once samples have been collected and variances calculated, the sample can be optimized based on the new variance information. However, the calculation of variances and sampling errors is very difficult to accomplish (Leaver, Johnstone, and Archer, 1991; Leaver and

Swanson, 1992; Cope and Freeman, 1998; and Morris and Birch, 2001).[1]

- *In practice, there is a trade-off between cost and accuracy.*

A high level of accuracy that would be desirable requires larger sample sizes that may not be affordable. In such cases, costs often determine the sample sizes, and the level of accuracy may suffer somewhat.

5.11 Once the coverage is decided, the **population** to be sampled should be identified and the sampling frame reviewed to determine whether the existing frame needs to be supplemented.

- *Does the frame contain all of the units in the target population? Does it cover all of the industries that are in the scope and all of the establishments in the targeted industries? Will separate frames have to be developed for each industry, group, or division?*

Most business registers have a cutoff (threshold) below a certain size (number of employees or value of sales) and probably some industries that are less well covered, for example, construction and retail trade. Also, there is a need to identify establishments separately from parent enterprises.

- *How are units defined in the frame? There are probably borderline units where it is uncertain if they belong in the population.*

A separate sample frame will need to be developed for PPI industries or products in order to facilitate the selection of the sample of establishments for those industries and products. For example, ancillary or auxiliary units of an enterprise may be out of scope, or certain products that are secondary to the industry should be included in the frame for another industry.

[1]The United States has estimates of variance for its CPI, and the United Kingdom has estimates of variance for its PPI. In both cases, the sample design was set up first without information on variances. The resulting variances are greater than if they had been known in advance. Once these first variances have been calculated, they can then be used to improve the efficiency of the sample design by reallocation of sample strata and the number of price observations in each.

- *Are units mutually exclusive?*

There could be double counting, which occurs when an establishment could be included both in its own right and as part of its parent enterprise.

- *Is there information available to allow stratification?*

We need certain data elements that will serve as stratification variables—for example, industrial classification, production or sales, number of employees, and location of establishment—in order to select the sample.

- *Is there information available to allow weighting for probability proportionate to size (PPS) selection?*

We will need measures of size, such as output, total sales, and value of shipments. If such measures of value are not available, employment may have to be used as a proxy.

5.12 The level of available **resources** should be decided:

- *This will be a constraint on sample sizes.*

It is generally more expensive to increase the number of establishments sampled, as opposed to increasing the number of prices collected from each establishment. Simply increasing the second may add little to accuracy, when intraestablishment (within an establishment) variance is low compared with interestablishment (between establishments) variance.

- *And this may dictate the methods of measurement.*

For example, whether personal visits can be used in addition to telephone collection or postal or electronic questionnaires.

5.13 **Legislative issues** may affect the sample design.

- *Will the survey be voluntary or statutory?*

This will affect response rates, which, in turn, have implications for accuracy and sample sizes. Statutory surveys will have higher response rates, although they may result in lower data quality.

- *Are there rules concerning confidentiality?*

This may impose a lower limit on sample sizes—for example, a minimum of four units per stratum may be required.

D. Sample Design

5.14 Given information about what the PPI survey is intended to achieve, the format of the inputs and outputs, desired level of accuracy, and available resources, the process of designing the sample can begin.[2] Again, decisions need to be made, but the main objective of the design process is clear—to maximize efficiency—that is, to minimize sampling and nonsampling errors, and to minimize costs.

5.15 Decisions will need to be made about:

- Sampling techniques (probability vs. nonprobability),
- Sampling frames,
- Sample structures and stratification,
- Sample allocation between strata, and
- Methods for reducing nonsampling errors.

D.1 Sampling techniques

D.1.1 Probability vs. nonprobability sampling

5.16 The statistician, confronted with any measurement problem, must initially consider the possibility of installing a rigorous probability sample. In the context of PPIs, probability sampling means the selection of a sample panel of producers and products (transactions) from a universe of industrial activity in which each producer and product has a known chance of selection.

5.17 Nonprobability sampling is known as judgmental or purposive sampling, or expert choice, and samples are chosen by experts to be representative. In practice, however, different experts would rarely agree on what is representative, and the samples are subject to biases of unknown size. Judgmental sampling may be justified when sample sizes are small, but concern about their biases increases with sample size.

5.18 Using a probability sample comes with two well-known advantages. First, it ensures that the items to be priced are selected in an impartial and objective fashion. In the absence of probability sampling, a danger exists that only items that are easy to price will be selected, resulting in biased estimates (indices). In particular, there is likely to be poor coverage of technologically advanced items, like machine tools, electronic equipment, aircraft, or home electronics in the PPI. These are difficult to price because of rapid changes in specifications. There is also a tendency to place too much emphasis on simpler products, like food items, cement, textiles, or steel bars, for which a comparable series of price quotations can easily be provided.

5.19 The second advantage is that a probability sample permits the measurement of the quality of the survey results through estimates of the variance or sampling error. The quality of results in this context relates to the chance of a difference between the results obtained from the sampled observations and the result that would have been obtained in a complete enumeration of all reporting units in the universe. The use of a probability sample, of course, does not permit the measurement of errors arising from nonresponse, inaccurate reports, obsolete weights, unrepresentativeness of the commodities priced, or any other nonsampling source.

5.20 Probability sampling conceivably could be used at all stages of the selection process. For example, a random sample of products could be selected from a comprehensive list of all goods produced by all mining and manufacturing firms. For each selected commodity, a random sample of producers could be picked using a comprehensive list of producers; for each selected producer, a random sample of specific brands could then be chosen for regular price reporting from a complete list of each producer's output. A less rigorous approach might involve random choice of producers or retailers, followed by a purposive selection of individual products or items; alternatively, the producers or retailers might be selected on a nonprobability basis using cutoff sampling (described next), while a random sample is picked from all items made by the selected producers. This mixture of nonrandom with random selection procedures and cutoff sam-

[2] There are many textbooks that can be consulted on the theory and application of sampling. One text used quite often is Cochran (1977), available worldwide.

pling procedures narrows the interpretation that may be placed on estimated sampling errors but still will retain the advantage that a certain amount of objectivity is imparted to the selection process.

5.21 Optimal sample design requires, for all units in the population, information that will allow effective stratification and increased efficiency due to selection by PPS. Different variants of probability sampling can be used by statistical agencies:

Simple random sampling—every possible unit has an equal chance of being drawn.

Systematic sampling—every k^{th} unit is selected, after a random start. This sampling is affected by any ordering or pattern in the sampling frame. Ordering leads to a form of implicit stratification, and a pattern in the frame can lead to biased samples.

PPS—each unit has a probability of selection in proportion to it size (or some other indicator of importance, but size is commonly used). Once these probabilities of selection are assigned, either simple random or systematic sampling techniques can be used.

5.22 Despite the attractions of probability sampling methods, there will be situations where it is neither necessary nor desirable. Price indices are an area of statistics where the risks in not having a probability sample are relatively low. The potential diversity of the change in prices charged by various producers of a given commodity over many time periods is relatively low. Compare this to the potential diversity for sales or capital expenditures of firms making the same product over the same period of time. The largest firm may become the smallest, and vice versa. Some may even abandon production of the commodity, and new firms may enter. In summary, the measurement of price changes appears to require less rigor with respect to probability sampling than do other areas of statistical measurement. The additional costs that may be involved in probability sampling can be allocated to other areas in the survey, such as price data collection or improvements to source data on weights.

5.23 That said, without probability sampling, statistical agencies will not be able to produce meaningful measures of sampling error to guide users in distinguishing between real changes in prices and those due to statistical noise. They also will experience difficulty in statistical decision making to improve the sample design and allocate resources more efficiently. Good measures of sampling error provide statistical offices with data for reallocating the sample to areas with high variance to reduce statistical error.

5.24 In several countries, the range of domestically produced mining and manufacturing goods is so limited and the number of firms producing them so small that there is no point in making a selection; the survey should try to cover all products and all producers.

5.25 In other cases, there may be no practical way of determining the universe in advance. A basic requirement for probability sampling is to define the universe (or population) and to identify all units in the universe. The universe list must be kept up to date with all units classified by an industry code such as the ISIC or NACE, which in practice is a costly and difficult business.

5.26 The cost of installing and administering a probability sample may be judged too high. There clearly are high costs involved in the design, selection process, control, and administration of a probability sample for collecting price observations.

5.27 Estimates of variability in price movements also are needed. This information is rarely available for all units in the population, certainly not at a detailed product or item level. One way of dealing with this is to use a two-phase sample, where certain information is collected from a sample of units, and then these units are resampled using this information. In the U.K. PPI for example, detailed product data are collected from a sample of producers as part of the EU's PRODCOM survey. These producers then form the sampling frame for the PPI, and the detailed data are used for stratification and PPS.

5.28 Probability selection often will be inappropriate because the survey of producers' prices ideally should form part of an integrated program of price statistics. This means that the choice of items to be priced at the intermediate (that is, producers' prices) stage may depend on the items selected for pricing at an earlier (for example, imports) or at a later (for example, exports or consumption) stage.

5.29 Thus, for most countries a strict probability approach will not be possible, or the costs will greatly outweigh the advantages, so a combination of probability and purposive sampling techniques is employed.

D.1.2 Cutoff sampling

5.30 Cutoff sampling is a strategy frequently used by countries to select samples. In this approach, a predetermined threshold is established with all units at or above the threshold included in the sample (selected with certainty) and units below the threshold level not included (zero probability of selections). Cutoff sampling generally results in a high degree of coverage among a small number of prospective units. This occurs because the distribution of the selection variable (for example, production or sales) is concentrated in a small number of large establishments.[3]

5.31 The problem with such an approach is that the smaller establishments may have different price movements from the larger units and, thus, introduce an element of bias into the price index. The bias would be the difference between the average price change for the noncovered units and the price change for the overall population. If the importance of units excluded is very small or the bias is very small, the effect on the overall error may be very small. Usually the total error is measured by the root mean square error, RMSE,

$$\sqrt{Variance + Bias^2} \ ,$$

and the sample with the lower total error is deemed more efficient. Thus, the approach that produces the lowest total error or RMSE will be preferred. It is possible that a cutoff sample could be more efficient if the bias component of the excluded units is small. For example, if the noncovered units have substantial variation with regard to price change but small bias (that is, the average price change is not much different), the RMSE could be smaller using the cutoff sample, and the survey costs could be much lower.

5.32 Cutoff sampling has a great deal of practicality for selecting the industries and products in a multistage sampling scheme. For example, in selecting the industries in the manufacturing sector that will be included as sample strata, a threshold can be established that only industries that represent 1 percent or more of output will be chosen. Another aspect of sampling where the cutoff approach can be used is in the selection of the representative products within an establishment. If, for example, the selected establishment is assigned four price observations, then the four products with the most sales can be selected.

5.33 Cutoff sampling is not the same as probability sampling. Sampling errors for cutoff samples will not be accurate because the sample is not necessarily representative of the index population. Statistical offices will need to make special efforts to measure bias among smaller firms in order to calculate the RMSE to get a meaningful measure of error.

D.1.3 Multitiered stratification

5.34 Alternatively, it may be useful to use stratified samples in which various classes of establishments are sampled separately. Often it is helpful to identify three or four strata based on their size, such as large, medium-sized, and small establishments, with each stratum having a different sampling rate. For example, large establishments (based on turnover or employment) may be sampled with certainty (that is, all selected in the sample), medium-sized establishments may be sampled at a rate of 25 percent (one out of every four), and small establishments may be sampled at a rate of 2 percent (1 out of every 50).

D.2 Sampling frames

5.35 Whether selecting a sample using probability or nonprobability techniques, we need to define the universe (population) from which we wish to sample, that is, construct a sampling frame. In most countries it is possible to define the population using various lists of enterprises (business registers), compiled for administrative purposes. For the PPI, these business registers probably will be less than ideal for use as sampling frames, however, and will require some manipulation before being used. On the other hand, it is likely that the business registers also will form the sampling frame for any official censuses or surveys of production, in which case some of this manipulation will have been done. The results of the censuses and surveys also will have been used to update and improve the business register.

5.36 The ideal **sampling frame** would

[3]See de Haan, Oppredoes, and Schut (1999) for an analysis of cutoff sampling in the CPI.

- *Be a complete list of all eligible units (producing and exporting) within the geographic and industry or product coverage required.*

5.37 Registers typically are compiled as the by-product of an administrative system such as tax collection or social security schemes. Alternatively, lists can be compiled using records such as bank accounts. Such lists generally contain, at a minimum, information about geographical location and size (turnover or number of employees) but may not indicate the principal activity of an enterprise or identify it as an exporter. Supplementary lists may be needed where certain areas of coverage are known to be inadequate. For example, in the United Kingdom a Builders' Address File is maintained separately from the main business register since construction is recognized as a particular problem. In the United States, population census housing lists are supplemented by new construction information taken from building permit records. Also, information on the location of shops and value of expenditures for the CPI can be collected as part of the Household Budget Survey (HBS) or as a separate Point of Purchase Survey.

- *Be updated instantly with all births and deaths of units and changes in addresses, fax numbers, etc.*

5.38 Maintaining an up-to-date register is resource intensive. It generally is the case that information about the bigger units is more up to date than data on smaller units. This is a particular problem during periods of changing economic structure when some industries or residential areas are expanding, and new units may be starting up in large numbers. If units are not removed from the sampling frame when they no longer exist, they may be selected as part of the sample. This needs to be borne in mind when determining sample sizes. Also, a common error with systematic sampling is to substitute the next unit in the list when a dead unit is sampled, but this should be avoided since the probability of selection of that next unit is enhanced. The sampling interval should be repeated as usual and dead units simply dropped.

- *Hold certain fields for each unit, allowing sorting of the list and stratification as required.*

5.39 For example, industry classification at the ISIC four-digit level and information about value of output would be maintained for PPI purposes (ideally of each product, at the six-digit CPA level, produced by each unit). This information would be updated annually.

5.40 Lists maintained primarily for tax collection purposes are likely to hold information on the values on which taxes are levied, for example, value added, profits, sales. Lists maintained for social security reasons will have information about numbers of employees, wage bills, etc. In countries where production surveys or censuses are performed for national accounts purposes, information on output and intermediate consumption can be held in the business register, too. In the United Kingdom, detailed information on the value of output of products (at the nine-digit level) is collected from a sample of enterprises each year in compliance with EU legislation (PRODCOM), and this information is stored in the register (for sampled enterprises only).

- *Identify each unit uniquely at the correct institutional level.*

5.41 In practice some units may be listed more than once, and others may be grouped under one listing. Ideally, a structure would identify enterprises and their corresponding establishment structure with separate classification and other stratification information for each establishment. If such information is not immediately available from the business register, additional steps or surveys may be needed to collect this information as part of the process of sample frame refinement.

D.3 Sample structure

5.42 The sample structure is likely to depend both on whether industry or area statistics in our price surveys are considered a higher priority than products or population subgroups, or vice versa, and on what information is held in the sampling frame.

5.43 Consider the PPI structure using the following example:

- We require PPIs for industries (four-digit ISIC) and PPIs for products (six-digit CPA);
- Our product classification system is mapped onto our industrial classification system so that each product falls under a single industry;

- There are establishments producing a range of products falling under more than one industry heading.

5.44 The first step in this process may involve selecting the industries and products that will be represented in the PPI. In most countries some industries and products are extremely small in terms of output or sales—for example, industries or products that comprise less than 0.02 percent of total output and sales in a sector such as manufacturing. (If this is not the case, then all industries and products could be included for estimation.) It would be possible to use a cutoff approach where those industries and products below the threshold level (in this example, 0.02 percent of sales) are excluded from the sample of industries or products, but their weight is allocated to another closely related stratum or distributed across a number of other strata. A sampling frame is then built for each industry and product.

5.45 The statistical office should review the industries that fall below the cutoff point and determine if any traditionally important industries or products should be included. Also, newly emerging industries that are expected to grow in importance might be included because they will eventually exceed the threshold. Finally, for the industries not selected, the statistical office should determine if there are logical combinations of industries that can be made to reach the threshold level. For example, ISIC industries 3118 (sugar factories and refineries) and 3119 (manufacture of cocoa, chocolate, and sugar confectionary), may both fall below the threshold level, but by combining the two industries, they would exceed the threshold. Thus a combined industry (3118,9, manufacture of sugar, cocoa, and chocolate) could be derived.[4]

5.46 To construct *industry PPIs,* we would classify each establishment by a four-digit ISIC heading based on its principal activity, draw a sample of establishments within each heading, select products and transactions to be priced from each establishment in the sample, and then weight them accordingly to give industry PPIs.

[4]Another alternative would be for the statistical office to develop a sample at the three-digit level, combining all the lower-level four-digit industries into one group.

5.47 To construct *product PPIs,* we would need output or sales information for each establishment for each six-digit product that it produces, enabling us to form a list of all producers for each six-digit product. From each list we would sample transactions and weight them accordingly to give product PPIs.

5.48 Obviously, running both lists and both samples as described above, in parallel, would be inefficient and burdensome on enterprises, and it would require a large amount of product information at the outset. In practice a compromise usually is made. In some countries, the United Kingdom for example, where detailed product information is available (at least for a subpopulation) and users place importance on product PPIs, establishments are listed under product headings and sampled to give product PPIs, which are then weighted together to give industry PPIs. This approach does not allow for the fact that establishments' behavior does not follow the strict mapping of products onto industries (third bullet in paragraph 5.43); that is, some establishments classified in one industry (A) will produce products (as a result of secondary activities) that are mapped into a different industry (B). Prices for these secondary products **should** be included in the industry PPI where the establishment is classified (A), despite the fact that the product heading appears elsewhere (B).

5.49 A compromise is to employ a two-stage[5] sampling scheme—that is, the frame is stratified first by the four-digit industry, then stratified by size within each industry. Next, samples are selected for each stratum and product samples are drawn from those establishments selected. Each transaction selected must then be classified under a product heading, and product PPIs can be compiled using all prices for each product, regardless of the industry in which the establishments are classified. With two-stage sampling of this sort, some accuracy of the product PPIs will be sacrificed. This is the structure employed in the United States.

[5]A distinction is made between two-stage sampling, where a sample of establishments is selected and then a sample of transactions is selected from each, and two-phase sampling, where a sample of establishments is selected to provide detailed output data, and this sample then is used as a new sampling frame. This new frame can be sorted and stratified much more effectively than the original frame as a result of the information collected in phase one.

D.3.1 Clustering of price-forming units

5.50 It may be useful and more efficient to cluster the basic units in the frame into price-forming units.[6] A price-forming unit is an entity whose price levels and movements are more or less identical (perfectly correlated). For example, several establishments owned by a single enterprise may constitute a profit-maximizing center and operate under the same price-setting regime. These establishments would constitute a cluster or price-forming unit. If a two-stage sample structure is used with industries as the principal strata, then establishments will be classified by industry and then clustered within industries.

D.4 Stratification

5.51 It is a well-known principle of sampling that stratification into segments for which the dispersion of price changes is lower (more homogeneous) than the overall dispersion tends to increase the efficiency of the sample by reducing variance.

5.52 For example, in the two-stage sample described above, the list of price-forming units is first stratified by industry classification, for example, the four-digit ISIC. Each industry stratum then can be further stratified by variables appropriate for that industry. The ideal variant for stratification is the value to be measured in the survey—that is, price change—but in practice we use proxy variables that we assume to be correlated with price change. For example, the size of the production unit may cause differences in production technologies and, thus, different responses to changes in demand or input costs.

5.53 In the U.S. PPI, the sample design ensures that all units (that is, products or producers) above a certain size are included. The remaining units are sampled with probability of selection proportionate to size. The alternate approach of setting broad strata, such as those with value of sales of 1 million to 5 million, 5 million to 10 million, etc., will result in units within each stratum having an equal chance of selection and, when selected, an equal weight. In a PPS sample design, a unit with five million in sales will have roughly a five times greater chance of selection than a unit with one million in sales. Further, the unit falling into the sample on a PPS selection would have a weight inverse to its size, an additional improvement over broad stratum sampling.

5.54 Ideally, stratification should be optimized to minimize sampling errors. For example, the number of strata (L) can be optimized based on a relationship such as

$$(5.1)\ V(\overline{y_{st}}) = \frac{S_y^2}{n}\left[\frac{\rho^2}{L^2} + \left(1-\rho^2\right)\right]$$

where S_y^2 is the variance of the variable being estimated (y), in this case price change; n is sample size; and ρ is the correlation between y and the variable used for stratification, in this case a proxy for price change such as output or sales.

D.5 Sample allocation

5.55 Given that there is always an upper limit on the amount of data that can be collected because of resource constraints, decisions must be made about how to allocate the data collection between the strata—that is, we must decide how many establishments to sample in each stratum and how many prices to collect from each. It is generally more expensive to increase the number of establishments sampled as opposed to increasing the number of prices collected from each establishment, although simply increasing the latter may add little to accuracy when intraestablishment variance is low. So, it is generally the case that the number of establishments to be sampled is the constraint, rather than the total number of prices collected.

5.56 Ideally, the sample allocation would be optimized so that accuracy is maximized within the cost constraint, according to some equation linking sample size with accuracy. For example, the simplest form of optimal allocation is to make the sampling fraction (f_h) in a stratum (h) proportional to the standard deviation S_h in the stratum, and inversely proportional to the square root of the cost

[6] This is not an application of the sampling technique called *cluster sampling*, where units are arranged into clusters, a number of the clusters are selected, and then all units in these clusters are sampled. In cluster sampling the clusters should be internally heterogeneous in the survey variables since those selected should be representative of those not selected. Here, the term *clustering* is being used to describe a method for increasing sample efficiency by grouping together homogeneous units. Strictly speaking, these clusters should be referred to as strata.

(c_h) of including a unit from that stratum in the sample—that is,

$$(5.2)\quad f_h \propto \left(S_h \big/ \sqrt{c_h} \right)$$

Thus more heterogeneous and cheaper strata are sampled at higher rates. Often, costs do not differ between strata, so the optimum allocation reduces to $f_h \; \alpha \; S_h$, the so-called Neyman allocation.

5.57 If probability sampling techniques have been used, it is possible, in theory, to estimate variances at each level. Take the following alternative sample structures as examples:

(i) *Only industry PPIs are needed, so the frame is stratified by the four-digit ISIC and then by size, and two-stage PPS sampling is employed to select establishments within each heading and then transactions from each establishment.*

5.58 The variance of each industry PPI will depend on the variance among (inter) establishments in that industry and the variance within (intra) the establishments in the sample. Since the second stage of the sampling does not stratify each establishment's frame of transactions by product, the intraestablishment variance is likely to be relatively large, particularly if the industry produces a wide range of products. In this case, an optimizing model will allocate the total number of establishments to be sampled across industries and size classes according to interestablishment variance in each stratum. The model is likely to suggest collecting a large number of prices from each establishment, particularly from those showing large internal variance.

(ii) *Only product PPIs are needed, so the frame is stratified by six-digit product codes, and two-stage PPS sampling is employed to select establishments within each code and then transactions from each establishment.*

5.59 Again, the variance of each PPI will depend on the variance among (inter) establishments producing a product, and the variance within (intra) each establishment in the sample. The intra-establishment variance might be because of differences in variety or terms of transaction, but it likely will be relatively small compared with the inter-establishment variance. So, an optimization model will allocate the sample of establishments in proportion to the variance within strata but will suggest collecting a fairly low number of prices for each product from each establishment.

(iii) *Industry* ***and*** *product PPIs are needed, so the frame is stratified by the four-digit ISIC and then by size, and two-stage PPS sampling is employed to select establishments within each heading and then transactions from each establishment. Transactions within each establishment are stratified by product code.*

5.60 Calculation of the variances of the industry and product PPIs is complex, and thus the optimization algorithm also is complex. There are variances among establishments in each industry, and within each product stratum in each establishment in the sample.

5.61 The above examples assume that probability sampling techniques are used and that variances therefore can be estimated. In sample surveying, however, we usually assume very limited information about the frequency distribution followed by sample measurements. This means that in practice, optimization often is done using a variety of pieces of information, applied to more or less formal optimization models. Information that may be available includes the following:

- The total sample size that resources allow;
- The number of units in each industry frame;
- The economics of each industry, that is, the value of output, company and product composition, product dispersion, price-setting mechanisms, etc.;
- Which PPIs need to be published—it may be necessary to allocate larger sample sizes to some strata industries or products than simple empirical methods would indicate in order for PPIs to be published at a detailed level without fear of breaching confidentiality guidelines; and
- Response rates.

5.62 The aim often is simply to produce industry indices with comparable accuracy and to publish a reasonable amount of product detail. As for the number of prices collected from each establishment, it may be necessary to use a general rule, such as the average number of prices should be around 4 or 5 with no single establishment providing more than 15 or 20.

E. An Example of Sample Selection and Recruitment of Establishments

5.63 For sample selection to proceed, all of the earlier steps of sample design must have been completed. Decisions have been made on the sampling techniques to use at each stage of the sampling process. Assume for simplicity that the manufacturing sector has been chosen as the first area to be included in the PPI. (Subsequently, mining, agriculture, public utilities, transport, etc. may be added.) For this purpose, information on establishments such as industry, output, sales, name, and location is available from a recent Census of Manufacturing or a Census of Establishments. Industries at the four-digit ISIC level have been selected using a cutoff sampling strategy. All industries with output (sales) greater than 0.02 percent of total manufacturing output have been chosen. (The cutoff value—0.02 percent—is determined by the amount of economic activity considered significant within the country. If the number of industries is too large given the resources available, a higher cutoff threshold may need to be used.)

5.64 In addition, quite a few industries have production concentrated among a few large enterprises, while others have less-concentrated production. It would be helpful to stratify the industries by size of firm. In those industries where production is highly concentrated among a few large enterprises (for example, three firms represent 90 percent of production), the large enterprises are selected. In those industries with a more disperse concentration, the largest firms could be selected with certainty (that is, chosen with a probability of 1.0), while a sample of smaller firms could be selected using random sampling techniques (for example, PPS sampling as described below). In general, the number of sampling units for the smaller firms should increase as the concentration ratio (percentage of industry output by large firms) becomes smaller. For example, for industries where the concentration ratio is 70 percent, a sample of four units among the smaller establishments might be adequate, but if the concentration ratio is less than 50 percent, the number of units might be twice that size. Using such a process also requires that appropriate weights be assigned to each selected unit. For the certainty units, the weight would be the firm's output (sales), while for other units it would be the sampling interval (see example below).

5.65 At this point the frame is stratified, allocations of sampling units have been made, and the sampling technique has been decided upon. Usually, three phases are left to sample selection:

(i) Select establishments;
(ii) Recruit establishments; and
(iii) Select transactions.

E.1 Selection of establishments

5.66 The sampling frame of establishments has been stratified by four-digit industry and size for probability sampling (purposive sampling could be used instead, and some of the issues involved in this are discussed under "Selecting products and transactions in the establishment"). In this situation, either systematic or PPS sampling could be used, or a combination of the two. A common application of PPS is to assign a probability of 100 percent to units in the largest strata (as discussed above), and then select randomly from each of the other strata, with probability of selection proportionate to size.

5.67 A combination of systematic sampling and PPS is used in the United States, where a stratum frame would be ordered by size and cumulative totals calculated. For example, assume that we know the average cost per establishment for collecting price information, and that the costs will not vary significantly by industry. Based on this information, we determine that the number of establishments in the sample would be 400 (total data collection costs divided by average cost per establishment). If the industry for which we are drawing the sample represents 1.0 percent of the total sector output, then we would allocate four establishments to the industry (400 x .01), and we can proceed to draw the sample from the frame. Assume the information below in Table 5.1 is available from the sampling frame.

Table 5.1. Step 1 for Establishment Sample Selection

Establish-ment Identifier	Size (value of produc-tion in millions)	Cumulative Size	Cumulative Percentage
E	200	200	34
C	100	300	52
D	80	380	66
B	60	440	76
G	50	490	84
F	40	530	91
H	30	560	97
A	20	580	100

The sampling interval is calculated:

$$\text{Sampling interval} = \frac{\text{cumulative grand total}}{\text{number of sample units}} = \frac{580}{4} = 145.$$

5.68 All establishments with production values greater than the sampling interval (145) have 100 percent probability of selection and are known as "certainty units" (Establishment E). These selected units are removed from the frame, we recalculate the cumulative size, and a new sampling interval is calculated using the reduced frame and the remaining number of sample units to be allocated (as shown in Table 5.2).

$$\text{Sampling interval} = \frac{\text{cumulative grand total}}{\text{sample allocation}} = \frac{380}{3} = 127.$$

5.69 If there are new certainty units in the reduced sample, these are removed (not in this case) and the process is repeated until a sampling interval is calculated for which there are no certainty units. This sampling interval is used for systematic sampling. The remaining sample is sorted (largest to smallest as shown in Table 5.3), a random number between 0 and 1 is generated, and the sampling interval is multiplied by this random number to give the starting point for the sampling pattern.

Table 5.2. Step 2 for Establishment Sample Selection

Establishment Identifier	Size (value of production in millions)	Cumulative Size
C	100	100
D	80	180
B	60	240
G	50	290
F	40	330
H	30	360
A	20	380

Table 5.3. Step 3 for Establishment Sample Selection

Establishment Identifier	Size (value of production in millions)	Cumulative Size
C	100	100
D	80	180
B	60	240
G	50	290
F	40	330
H	30	360
A	20	380

Random number = 0.34128

Starting point: $0.34128 \times 127 = 43$

Sampling pattern:

43	(43 + 127)	(43 + 127 + 127)
43	170	297

Thus, Establishments C, D, and F are selected, giving a total sample of C, D, E, and F.

5.70 The weights assigned to each establishment would be as follows. Establishment E will have a weight of 200. It was selected with certainty, and it will maintain the same weight because it is representing itself in the sample. Establishments C, D, and F will each have a weight of 127 because they are representing all the other establishments not selected in the sample. Thus, the total of their weights must be the total of all the noncertainty establishments, which is 380 in this example. Additional detail on the source of weights and methods for proportional allocation of weights within establishments to products is presented in Chapter 4, Sections D and E.

5.71 An alternative approach used in some countries is to use cutoff samples so that a certain level of output or sales is achieved. For example, there may be a desire to have the sample represent 70 percent of the output in each industry in the sample. In such a case, a cutoff sample is used. Establishments in the industry sampling frame are ranked in order of the output (largest to smallest). The percentage of output that each establishment represents to the total for the industry is calculated. The cumulative percentage then is derived. A cutoff of 70 percent is established, so that all establishments below this threshold in the cumulative rankings are dropped and the sample will consist of those remaining. This approach guarantees that the sample consists of large establishments.

5.72 In the previous example if one used the cutoff procedure, establishments E, C, D, and B would have been selected because their cumulative percentage of output is 76.

E.2 Recruiting establishments

5.73 Recruiting an establishment means securing the cooperation of its staff (particularly if the survey is voluntary), so that data will be of a high quality. It is highly recommended that each establishment receive a personal visit during which the purpose and function of the price survey are explained, and the sample of transactions or varieties to be priced is selected. Supplementary data for weighting transactions also can be collected during the visit. All these tasks can be more effectively carried out via personal visits rather than via telephone calls or mailed questionnaires.

E.3 Selecting products and transactions in the establishment

E.3.1 Probability and cutoff sampling procedures

5.74 The probability approach also can be used for selecting products and transactions by soliciting information from establishment records. Once in the establishment, however, the respondent may be reluctant to provide detailed records for selecting products and transactions. One alternative would be to ask the respondent to list the products produced and provide an estimate of the percentage each product represents of total sales. This information can be used to select the sample by ranking the products from highest to lowest and then making the selection using the same techniques discussed above.

5.75 Another alternative, if the respondent is unwilling to provide product percentages, is to ask him or her to rank the products in order of importance. Using the ranking information, estimated percentages can be established. Consider the information in Table 5.4 that is provided by a respondent in an establishment with eight products. The respondent was able to rank the products in order of importance. Each product can then be assigned its importance based on the reverse order of its ranking: Product G is assigned 5, Product H is assigned

Table 5.4. Selection of Products Using the Ranking Method

Product	Ranking	Importance	Estimated Percentage	Cumulative Percentage
G	1	5	33	33
H	2	4	27	60
I	3	3	20	80
J	4	2	13	93
K	5	1	7	100
Total		15	100	

4, etc. Next, an estimated percentage of sales is calculated using each importance as a percentage of the total of the assigned importances. Assume that the sample design indicates that three products are wanted for this establishment. These percentages can then be used to select a sample of products through the probability sampling procedures described above or through cutoff sampling procedures.

5.76 If probability procedures are used, the sampling interval is first calculated:

Sampling interval = 100/3 = 33.

A random number is selected to determine the starting point and the sampling pattern:

Random number = 0.45814
Starting point = 0.45814(33) = 15
Sampling pattern = 15, 48 (15 + 33), and 81 (48 + 33)

The selected sample will be Products G, H, and J. (Note that we do not select Product I because it is below the third interval in the sampling pattern.)

5.77 If the cutoff procedure is used, the first three products (G, H, and I) will be selected. With the cutoff procedure the three most important products are selected.

5.78 In addition, representative transactions for continuous pricing will need to identified. The respondent should be asked to supply information on various transactions that apply to the selected products. Again, the data can be in the form of actual values from company records, estimated percentages, or by ranking. If two transactions per product are required, then the same procedures as those just described would be followed to select the two transactions.

5.79 In the above examples, if the respondent could not provide any information or if he or she says that they are all equally important, then equal probability would be assumed. In such a case, each product or transaction would be assigned the same importance (that is, 100 divided by the number of products), and the selection procedure would continue as explained above.

E.3.2 Purposive sampling

5.80 Since the selection will be based largely on the judgment of the members of establishment staff present at the recruitment meeting (respondents), it is important that these people are knowledgeable and hold senior positions, probably from the marketing, sales, or accounting departments.

5.81 The first step is to stratify by products produced by the establishment selected for the industry sample. As a general guide, it is reasonable to have between 3 and 10 product strata (depending on the size of the establishment) that are deemed representative of the establishment's output. It should be possible to obtain a sales figure or estimate for each stratum, or at least to order the strata by size. In the establishment, if exports make up more than 20 percent of total sales, and export prices are thought to move differently than domestic market prices, then, ideally, the product strata should be further stratified between exports and domestic market. Separate prices should be collected for exports and domestic products, as necessary.

5.82 Then for each stratum, one or two specific transactions should be chosen, bearing in mind the general rule that the average number of prices from establishments should be around 4 or 5, with no single establishment providing more than 15 or 20 (strata may have to be combined if the number is too large). The aim is to choose transactions and terms of sales that account for a significant proportion of sales, are broadly representative of other production, and are expected to be available for sale or stay in production at future price collections.

5.83 Weights for each transaction selected could be determined by proportional allocation of the establishment weight to each product and transaction selected. This procedure is discussed in Chapter 4, Section E.

E.4 Recording product specifications

5.84 After transactions have been selected, the price-determining characteristics must be carefully discussed and recorded on the collection form. (See Chapter 6 for more details on recording product specifications.) Examples of such characteristics are as follows:

Product specifications:

- Type of product;
- Brand name or model number; and
- Main price-determining characteristics—size, weight, power, etc.

Transaction specifications for the PPI:

- Type of buyer—exporter, wholesaler, retailer, manufacturer, government;
- Type of contract—single or multiple deliveries, orders, one-year, agreed volume;
- Unit of measure—per unit, meter, ton;
- Size of shipment—number of units;
- Delivery basis—free on board, sale with or without delivery to customer;
- Type of price—average, list, free on board, net of discount; and
- Type of discount—seasonal, volume, cash, competitive, trade.

F. Sample Maintenance and Rotation

5.85 Price surveys are panel surveys in that data are collected from the same establishments on more than one occasion. The general problems with such surveys are that the panel becomes depleted as establishments stop producing, the panel becomes increasingly unrepresentative as time passes and the universe changes, and some establishments may resent the burden of responding and leave the panel or provide poor-quality data. All these problems cause bias.

5.86 A widely used method to alleviate some of these problems is to limit the length of time that establishments stay on the panel by using some form of panel rotation.[7] Rotation has two main benefits: (i) it ensures that most producers participate in the survey for a limited time and, therefore, the burden is shared among enterprises, and (ii) it helps to alleviate the problems caused by a sample being out of date—that is, sample depletion and not being representative of current trends. Recruiting new establishments helps to ensure that new products are represented in the price surveys.

F.1 Approaches to sample rotation

5.87 Obviously, sample rotation has a cost since new panel members need to be recruited. There are several options regarding how rotation might be done. First, a rotation rate should be fixed. For example, if the whole panel is to be rotated every five years, then the annual rate is 20 percent. This could be implemented by dividing the industry headings into five groups and dealing with one group each year. Or 20 percent of all respondents, across all industries, could be dropped each year and replacements recruited. An establishment's rotation cycle could be related to its size, so that larger establishments stay in the sample for more than five years, and small establishments stay in for fewer than five years.

5.88 If sample rotation is done by industry group, product group, or geographic location, this provides a good opportunity to review the sample design and reallocate and select new establishments as necessary. Rotation and sample revision fit best within a system of annual chain linking in which the product structure and weights can be updated each year.[8]

F.2 Procedures for introducing a new sample of establishments

5.89 The procedures used to introduce a new sample of establishments are similar to the overlap procedure used for linking replacement price observations or introducing a new product structure in a weight update. Assume the rotation strategy calls for replacing 20 percent of all industries. If the PPI sample consists of 100 four-digit industries, then each year the statistical office will replace the samples in 20 industries. For each of the targeted industries, a sampling frame is needed to select a new sample of establishments. The staff must then re-

[7] In many countries, the rotation is limited to the smaller respondents, for whom it is felt that responding to surveys imposes a significant burden. This need not be the general case, and the use of full-panel sample rotation is encouraged.

[8] Annual weight update is not a requirement for sample rotations; it simply makes the process a bit easier because weights already are being updated at most levels of the index. When there is no system for annual weight updates, sample rotation does require a two-tier system of weights—fixed weights at higher levels of aggregation for aggregating to higher-level indices and separate weights for low-level indices that are updated periodically.

cruit the establishments, as discussed in Section E.2.

5.90 The new industry sample will have new weights for the selected establishments, products, and transactions. The new sample and weights will be used directly to replace the old sample. During the same month, the data collection staff will have to collect price observations for both the new and the old sample. The old sample prices are used to calculate the index in the usual way, and the new sample will provide new base-period prices to calculate the index for the next period using the new weights. For example, the old sample for a particular industry may consist of five establishments and 20 price observations, while the new sample may have eight establishments and 32 price observations. Both samples are collected during the overlap month, that is, 13 establishments with 52 price observations (assuming no establishment in the old sample is also in the new). The 20 observations from the old sample are used for the current-period index calculation. The 32 price observations for the new sample provide basic data for setting new base prices in the new sample.

5.91 The index formula used will influence the relationship between the price reference period for the weights and the reference period for the base prices. If the statistical office compiles a Lowe or Laspeyres index, it will use the first set of prices collected in the new sample to set the base prices for the index. The base price reference period and the weight reference period need to align if the Laspeyres price index is used. If the weight reference period for the establishment and product weights are, for example, annual revenue for 2000 and the prices collected for the new sample are for June 2003, then the new prices will have to be estimated backward to the annual average for 2000. This is accomplished by applying the price change for the industry between June 2003 and the annual average for 2000 to the June 2003 price observations. For example, if the prices in the industry rose by 10 percent between the annual average index for 2000 and the June 2003 index, then each price observation would be deflated by the factor 1.10.[9] This calculation adjusts the new price observations for the average price change in the industry between the weight reference period and the current period.

5.92 Consider a similar example for the Lowe index. Again, assume that the weight reference period is for 2000 and that the base price reference period is December 2001. In this case, the statistical office will need to update the weights for price changes between the 2000 annual average and December 2001. The price index for the industry is used to calculate the price change between 2000 and December 2001, and this price change is applied to all the weights. Next, the June 2003 prices will need to be adjusted backward to December 2001. The industry price index is used to measure the price change between December 2001 and June 2003. This price relative then is used to deflate the June 2003 price observations to obtain December 2001 base prices.[10]

5.93 If the statistical office is using a Young index, the process is much simpler because the new weights are used directly in the computation of the index using the new prices without any adjustments. (See Chapter 15, Sections D.2 and D.3, for a discussion of the Lowe and Young indices.)

5.94 These procedures ensure that the new prices and weights are consistent with the index number formula within each four-digit industry selected for sample rotation. For higher-level indices, the weight reference period may not be the same as for the industries going through sample rotation. In practice, the aggregation weights used to combine industry and product often have a different price reference period than for the sample rotation groups. For example, the industry and product group weights used to produce higher-level indices (three-digit, two-digit, etc.) may have a reference date of 2000 because they come from an establishment census conducted in 2000. The index reference period might also be 2000 = 100, because of a statistical agency policy to re-reference index numbers once every five years. On the other hand, the weights from the industry sampling frame used to

[9] The statistical office could also do these calculations using information from the product indices. This would involve deriving more deflation factors for the base prices—one for each product in the industry. Then each observation would be deflated by the price change in its product index, rather than by the industry index.

[10] If product indices are used, then the calculations must be made using the changes in product indices. Again, this will involve calculation of more price changes—one for each product.

draw the rotated sample may be for 2001, because the weights for the rotated industries are taken from an annual industry survey (perhaps with a special supplement for industries scheduled for sample rotation). The price index reference period could be December 2002 because the price information is readily available from sample respondents.

5.95 Thus, there can be a difference between the base price reference period for the new sample at the lowest level (elementary aggregate)—December 2002—and the index reference period for higher-level indices—annual average for 2000. In such cases, the price change from the lower-level indices will be used to move the higher-level indices forward to the current period. For example, in industry 3411 (manufacture of pulp, paper, and paperboard) the index level in December 2002 was 108.0, and in September 2003 it was 110.2 with an index reference period 2000 = 100. The sample of 10 establishments and 40 price observations for this industry was rotated in January 2003 using base prices from December 2002. The elementary indices for the products in this industry have a price reference date of December 2002. To estimate the industry index, the statistical office will have to use the price change from the new sample and link it to the level of the higher-level index. This can be done in two ways, depending on whether the statistical office uses a direct or chained price index formula (see Chapter 9, Section B.3). Assume a direct index is used where the current price for October 2003 is compared with the base price in December 2002, resulting in a price index of 102.96 (December 2002 = 100). The long-term price relative (1.0296), times the industry 3411 price index for December 2002 (108.0), gives the October 2003 index level of 111.2. Alternatively, if the monthly chained index form is used, where the October prices are compared with the September prices, then the lower level index is linked to the September 2003 higher-level index. Assume the one-month price relative was 1.0091 in October 2003. The September 2003 industry 3411 index (110.2, where 2000 = 100) is multiplied by this price relative to derive the October 2003 industry index of 111.2. The results of the formulas should be the same. The advantage to using the monthly chained index form is that it facilitates making quality adjustments, as discussed in Chapter 7, Section C.3.3.

G. Summary of Sampling Strategies for the PPI

5.96 The approach to a sampling strategy in the PPI requires a number of steps to gain enough information and design a survey that will produce reasonable estimates of price change within the level of resources provided. The following points provide a logical sequence to the sampling issues presented in this chapter.

(i) *Determine the survey objectives, uses, coverage, and resources before determining the data to be collected, the periodicity of collection, and the type of sampling that will be employed.*

5.97 It is important to decide at the beginning of the process if price changes for both industry and products will be needed and the degree of accuracy required. It will also be important to decide whether monthly or quarterly indices will be produced. These, in turn, will determine the level of resources allocated to the program. Alternatively, if there is a fixed level of resources available, it is possible to work with cost controls to determine affordable sample sizes and collection frequency at the expense of accuracy.

(ii) *Identify sources to use to develop a sampling frame for selecting the establishments and products for covered sectors and industries.*

5.98 The availability of an up-to-date business register with appropriate selection parameters (for example, industrial codes and measures of size) could serve as a source for developing sampling frames for selected industries. Many of the sources of weight data discussed in Chapter 4 also could be used to develop a sampling frame. These include industrial census, surveys, and administrative records.

(iii) *Use probability sampling techniques to the extent possible.*

5.99 While probability sampling throughout the selection process is a desirable goal, it may not be entirely affordable. An alternative is to use cutoff sampling at certain stages in the process, such as selecting the industries within a sector or the products within major groups. Sampling frames for each industry or product then can be established to conduct sampling using PPS techniques.

(iv) *To make the sample more efficient, use multiple levels of stratification within the sample design.*

5.100 In most cases, three strata will be identified within the sample—industry, product, and establishment. However, the sample could be more efficient and representative if additional strata are used, such as establishment size (large, medium, and small), region or location (if there are price trend differences by location within country), and export versus domestic market production (if there are price trend differences for these markets). Additional strata will be helpful to the design wherever there might exist differing price trends or price variability within the chosen strata.

(v) *The price sample should be based on actual transactions with the characteristics of those transactions fully described.*

5.101 Often there is a tendency to use average prices or unit values (sales value ÷ quantity sold) as the price reported in the PPI. These are not true transaction prices, in that they represent the average of a number of transactions for which there could be differences in quality or pricing characteristics. Therefore, it is important to select a sample of individual transactions with a detailed description of all of the characteristics that determine the price. These transaction prices and their characteristics then will be observed through time.

(vi) *Initial recruitment of establishments should be completed by personal visits.*

5.102 Initial sample recruitment should be conducted through personal interviews with establishment managers in order to accurately select representative products and transactions. The purpose of the survey must be explained, along with the need for the continuous reporting of price data for the selected transactions.

(vii) *Samples of establishments and products must be maintained so the reliability of the PPI remains intact. A program of sample maintenance is needed for this purpose, and sample rotation also may be desirable.*

Products produced by establishments will frequently change in response to market conditions. Also, establishments will cease operations and new ones will begin production. The PPI sample size must be maintained in order for PPI estimates of price change to be accurate. Therefore, it is necessary to have a program targeted toward keeping the sample intact and the products representative of current production, in terms of both the goods being produced and the establishments producing them.

6. Price Collection

A. Introduction

6.1 This chapter gives an overview of price collection issues. It describes a range of options for each aspect of collection, but it is not prescriptive, since different solutions can be used depending on individual country circumstances. Price collection is a vital part of the overall PPI compilation process. Without good quality price collection procedures, it is difficult or impossible to produce accurate and reliable results, regardless of how rigorous the subsequent processing is throughout the remaining steps of producing the PPI. Chapter 12 on organization and management of the PPI also provides guidance on price collection within the framework of the whole PPI system.

B. Timing and Frequency of Price Collection

6.2 Calculating the PPI entails collecting prices from businesses relating to particular products and time periods. Businesses can be both sellers or buyers of products, so that prices may be collected for sales of goods and services for use in an output price index or purchases of goods and services used in the production process for use in an input price index. Both output and input PPIs are often needed, particularly for use as deflators.

6.3 The frequency of collection is often monthly, although a number of countries operate a quarterly collection system. For the purposes of this chapter, it is assumed that price collection is monthly, which is the most common practice. When collecting prices for a particular period, there are two basic choices of collection period: point-in-time or period averages.

B.1 Point-in-time prices

6.4 Point-in-time prices relate to the price of a product on a particular date in the month—for example, first day, first Monday, the nearest trading day to the fifteenth of the month, etc. This approach makes the collection date straightforward, and it should be well understood by the business establishment that prices provided relate to transactions on that date.

6.5 The main advantage of point-in-time pricing is that comparisons from month to month will be consistent, which is particularly important when there are step changes in prices taking place during the month, such as a general price increase or duty changes. One of the disadvantages of a set point in time for producer price indices is that a transaction may not have taken place on the specified date. If this happens, respondents can be asked to provide details of a transaction that occurred as near as possible to the specified date. Another problem is that point-in-time estimates are more susceptible to short-term external influences (for example, extreme weather, labor stoppages) that could affect the price on the particular day of price collection. They may also miss short-term price changes (for example, rise and fall) that occur between pricing dates.

B.2 Period prices

6.6 Period prices are an estimate of the price across the month and so are an average price for the month. A period price should take into account when price changes occurred during the month. For example, if a product was priced at 10 for the first 10 days of a month, and then increased to 15 for the remaining 20 days, then the average price would be 13.33 (that is, $[10 \times 10 + 20 \times 15]/30$). This averaging is usually done by the statistical office and requires the exact date of the price change to be supplied by the respondent.

6.7 This approach usually yields a smoother time series and is less susceptible to the timing of price increases. The method is also easier for respondents since they can select a transaction and specify the relevant transaction date within the period. A key feature (compared with point-in-time estimation) with this method is that when a price changes partly through the month, the full effect of the price change is not included in the index until the following month. This is appropriate for an index used for deflation purposes as well as for an inflation measure.

6.8 Often a single price quotation is taken to represent the average price over the particular reference period. A more accurate measure of an average transaction price is the *unit value price*. In theory, unit value prices, which are total sales divided by the total number of units sold in a period, are the most complete method of pricing.

6.9 If this method is used, the commodity must be either homogeneous or able to be expressed in terms of some common physical unit. A homogeneous commodity can be distinguished by

- Its point of purchase (outlet effect),
- The various competing brands or product lines of the commodity being sold at an outlet (brand effect), or
- The various package sizes at which the commodity is sold (packaging effect).

6.10 The time period in which unit values are calculated should be the "longest period which is short enough so that individual variations in price within the period are regarded as unimportant" (Diewert, 1995a).

6.11 Unfortunately, this method is very problematic and is not generally recommended, since any change in product quality, product mix, or timing can seriously distort the average unit price. In limited circumstances—for example, for a highly volatile but narrowly defined and homogeneous product like petroleum—this method can be used.

6.12 Often the index will be less timely when compared with point-in-time estimates, since the average cannot be calculated until the end of the period. Further, care must be taken to ensure that the average prices relate to a narrowly defined product of constant quality, rather than a broad commodity group.

6.13 Transportation costs should be excluded from the unit value calculation because the pricing basis for output PPIs is the basic price—that is, the amount received by the producer, exclusive of any taxes on products and transport and trade margins. In other words, the pricing point is ex-factory, ex-farm, ex-service provider, and so on.

B.3 Choice of point-in-time or period prices

6.14 The choice of collection period is influenced by a number of issues, such as the frequency of collection, the practicalities of price collection, and the uses of the index. The choice of collection method becomes less important the more frequent the collection; thus, the choice is more important for quarterly collection than monthly collection, although it is still an important consideration for monthly collection. The use of the index is an important consideration. Since PPIs are used to deflate sales data, ideally the index should relate to the time period of the sales flows. Most economic statistics relate to a period rather than a point in time, and so again, in principle, the price index should do the same.

B.4 Frequency

6.15 A distinction can be made between the frequency of collection and timing of observations. Monthly prices can be observed quarterly, for example.

6.16 The choice of collection frequency is determined by issues such as costs and the periodicity needed for deflation of output or sales data. In the EU, members are required to provide monthly data to the Statistical Office of the European Communities (Eurostat) under the short-term indicators regulation. Normally, prices will be collected from every producer in the sample for each time period. This will ensure that all price changes are captured by the PPI.

6.17 While collecting prices for every period is appropriate for most industries, there may be industries where prices are generally stable, products take a long time to produce, or prices change at predetermined times—for example, each January. Collecting a price every period in such a situation may be an unnecessary burden on businesses. For respondents in these industries, a price for each period is still required, although it may be possible to reduce the periodicity of collection; this is a case where using "carryforward" price imputation is desirable. In some exceptional circumstances, respondents may be allowed to give forward prices, but care must be taken to avoid complacency. In these cases, usually involving long-term contracts, respondents can make a commitment that the price will not change in the defined forward period. If this turns out to be incorrect, the frequency of reporting would be changed.

B.5 Definition of a price observation

6.18 Chapter 5 on sampling explains how a representative sample of products should be selected. This section describes how prices for these selected products should be determined and collected. A price observation is defined as the price of a specific product at the point in time or for the period of price collection and its terms of sale. To ensure consistency in the final index, the price observation should compare like with like for each period. The product should be defined as tightly as possible so that the returned price is consistent from period to period and changes in quality can be identified (see Chapter 7, Section B). The price should be one that a customer has paid for the specified product and include all available discounts and special offers—that is, a real transaction price. (See Section D.4.1 below.)

6.19 If the product specification changes from one period to another, the price needs to be adjusted to ensure consistency. For instance, the quantity per order may increase, resulting in a lower unit-selling price. If the new quantity sold was available at the same unit price last period as it is this period, this is *not* a genuine price decrease and should not be reflected in the index. Rather, the comparison should be made between the same quantity purchased in both periods so that the index compares the same specifications (that is, like with like).

6.20 The returned price should be provided in a consistent currency, but even for domestic sales this may not necessarily be the currency of the home country (for example, price could be provided in euros or U.S. dollars). In that case procedures should be in place to convert all returned prices to home currency values. However, it must be clear what coverage is intended—either production for the home market or for the home and export market.

B.5.1 List prices

6.21 The PPI's aim is to measure *actual* prices paid to or received from producers for goods or services. These are commonly referred to as transaction prices. By definition, these prices include all discounts or rebates given.

6.22 The price of goods or services as quoted in a catalogue or advertisement is often referred to as the list price, book price, or recommended retail price. These prices are typically higher than transaction prices, as discounts or rebates apply to transaction prices.

6.23 In most areas of the economy, the prices actually paid or received for goods or services are not the list prices. Typically, negotiations between the producer and purchaser result in some form of discount or rebate, particularly to large purchasers. In most cases they are substantial reductions off the list price and will vary over time. PPI compilers should ensure that actual transaction prices are obtained rather than list prices.

6.24 It is usually easier for a respondent to provide a list price rather than a transaction price. For the reasons already stated, this is not appropriate. Because it is difficult to price a transaction, to achieve constant quality, compilers should ensure that the product priced is the same as that priced in the previous period.

B.6 Issues for high inflation or hyperinflation

6.25 During periods of high inflation or hyperinflation, the timing of price collection takes on significant importance. Prices may well change substantially during the collection period.

6.26 The frequency of collection also becomes more important, so quarterly collection may be inadequate for policymakers in a hyperinflationary period. Even in times of low inflation, it is important that early signals of upstream inflationary pressure are captured. Validation of the data may also prove to be more difficult, because it is likely that every price quote would fail validation checks set during "normal" inflation, and it would be more difficult to spot erroneous returns.

6.27 On a wider scale, there is also a potential problem of feedback or circularity fueling inflation. Some companies may use the PPI to fix their prices (as part of a contract with the customer), which could then feed into the calculation of future PPIs. There is always a risk of this in detailed indices, but the risk would be higher in periods of very high inflation.

C. Product Specification

6.28 The PPI price collection survey is unusual compared with most business surveys, since there is a requirement to get a detailed product specification from respondents before the routine monthly collection can begin; this process is often called initialization or recruitment.

6.29 A separate set of processes and survey forms is required for the initialization procedure. The collection method used can also be different for the initialization period—for example, it may be possible and desirable to make a personal visit to each new respondent at initialization, but subsequent routine price collection would be done via postal collection. The initialization form should put more emphasis on explaining the purpose of the survey and contain more details about the product specification requirements. This form should also contain a product list for the respondent to identify which products they produce from the list. The initialization process may also be conducted by specialist staff such as field officers (see Section D.6).

6.30 The following section on product specification can apply to both the initialization process and routine collection procedures.

C.1 Purpose of product specification

6.31 For each product group or service, prices for a set of specific representative products need to be fully specified for pricing. These products should be typical of the price movements of the range of individual products within the product group or service under consideration. The selection of products from within the product range of each producer would ideally be undertaken from a complete census of the relevant transactions. Obviously, in most cases this information is not available. In some cases there can be a trade-off between having infrequent data on a more complete and detailed basis and more frequent product-updating procedures that rely on the respondents to the price collection to self-select products that are representative of their output and, hence, product group. The sample selection aspects of product selection are covered in Chapter 5.

C.2 Aspects of product specification

6.32 There are a number of different aspects of product specification. For example, simply giving a product name will not be sufficient if the size of the package changes, which would, in turn, affect the price received. The essential purpose of a good product specification is to ensure that a consistent price is collected from period to period, relating to a consistent product with the same terms of sale in each period. Table 6.1 lists the main criteria that could affect the price of a product and could form part of a specification.

6.33 The above details combine to give a tighter specification for the product than just the description alone. Specifying a product in this way also supports the adjustment of the price associated with any changes in the product quality or the terms and conditions of sale. Some respondents object to providing full specification details because of concerns about confidentiality, and in these cases the specification can be held in detail by the statistical office, but a shorter encoded specification can be used on printed material such as forms. If this is done, it is essential to review the specification regularly.

C.3 Other forms of description

6.34 For some industries, a specification for a particular product may not be appropriate. For example, some industries produce goods or services on a made-to-order basis, and the same product is not produced in successive periods. Examples of this could be furniture manufacturers, shipbuilders, and accounting services. In these instances, a generic specification may be more appropriate. This would be a specification, as described previously, but for a standard product, rather than for a specific product. This could be a product that the company has made at some point in the past, or a basic model that it customizes individually for each customer. See Section D.5.2 below for more details on this type of pricing.

D. Collection Procedures

D.1 Survey collection techniques

6.35 The aim of all survey collection techniques is to facilitate the transmission of price data from businesses to the statistical office in a secure and

Table 6.1. Criteria That Affect a Product's Price

Item	Criteria/Reason
Product name	Company's name for the product within the specified product group. This should ideally contain information on the model/variety of the product.
Serial number	For the company's reference. This allows for changes in product name.
Description	In addition to the product name, this gives an opportunity for the company to specify what (if any) enhancements or add-ons are included in the product. For example, with cars, a number of options are usually available (metallic paint, sunroof), all of which could affect the price of the product.
Size of transaction	The amount of the product sold in the transaction and whether volume discounts apply.
Units of sale	Units used in describing the product.
Class of customer	Some companies may have different pricing structures for different customers (for example, retail and trade). A reference number can be used to maintain customer confidentiality.
Discounts	Many companies offer trade, volume, competitive, or preferred customer discounts. All applicable discounts should be described.
Payment terms	Companies may have different prices for different payment or credit terms.
Carriage terms	Whether transport costs are included and what type of transport.
Currency	Currency the price will be provided in.

cost-effective manner, while minimizing the administrative burden on the respondent. A range of approaches to PPI data collection are discussed below—postal survey, automated telephone response, personal interview, telephone interview, and Internet data provision. All of these methods rely on good questionnaire design, good respondent relations, or good interviewing techniques. The highly sensitive and confidential nature of the price data provided by businesses may necessitate extra security requirements in data collection and processing.

D.2 Questionnaire design

6.36 Regardless of which data collection method is used, good questionnaire design is essential for the successful collection of prices. The questionnaire should be designed to make it easy for the producer to use and understand what is required.

6.37 The layout should facilitate the extraction of data and should contain detailed descriptions of the products to be priced. Detailed descriptions not only help the producer but also help in validation and the identification of quality changes. Quality adjustments cannot be made in the absence of detailed product specifications (see Chapter 7 for more details on quality adjustment techniques). Detailed product descriptions also ensure that the same products are priced each month, which gives important continuity and enables the statistical office to validate the data.

6.38 The questionnaire should be designed to help the respondent extract information quickly and to enable speedy and accurate processing in the office. To meet these objectives, the questionnaire should

- Provide clear instructions on what the respondent is required to do;
- Define why the establishment has been chosen, what the survey is, and how the data are collated or published;
- Enable respondents to complete the form quickly and accurately;
- Ensure supporting notes are available for each item of data to be collected;
- Use plain and clear language;
- Clearly identify the organization from which the survey has been sent and give a contact point and telephone number so respondents can get in touch to resolve any problems;
- Request reasons for price changes; and
- Ask whether the products are still representative or sold in volume.

6.39 Different designs can be used to make the questionnaire easier to complete for certain classes of respondents. For example, different designs could be used for production sector and service sector questionnaires. Also, a questionnaire with a checklist design that provides all the important specifications and price-determining characteristics will help respondents and data collectors. They will be able to verify the transaction and provide any new specifications or changes to the price basis that may apply when a previous transaction is no longer available and a replacement is selected.

6.40 One way to make the form easier for businesses to complete is to put the last recorded price on the questionnaire using a "tailored form" with unique product descriptions for each respondent. This will require the statistical agency to have much better form design and printing capabilities. It is controversial in terms of the impact on the results, however; while it is easier for the producer to complete, there is a greater risk that less care is taken in the completion of the survey, and the producer is more likely to repeat last period's price even if a price change has taken place. There is also the risk that confidentiality will be breached if the form goes astray or even to the wrong part of the organization.

6.41 For help in validation and to reduce recontact with the producer, it is useful to provide a comments block to allow the respondents to explain any unusual movements in their prices. It is also important to emphasize to the respondent that any change in specification must be reported. An example of a PPI postal survey price collection form that takes many of these issues into consideration is provided in Figure 6.1 at the end of this chapter.

D.3 Medium of data collection

6.42 The following section outlines a range of survey collection methods. The principles of questionnaire design outlined above apply to each of these methods.

D.3.1 Self-completion: return of data by postal survey

6.43 Key points of good practice that should be followed in questionnaire design are outlined below.

6.44 The form should be clearly addressed to the company in question. It should

- Display on the front page the name of the institution from which it has been dispatched;
- Explain why it has been sent, how the results will be used, and from whom the end product can be obtained;
- Include the name and number of the direct contact in the office should respondents require assistance in completing the return; and
- Include any statutory obligations respondents are under to complete the form and the penalties for not doing so.

6.45 Within the form, sufficient descriptions and explanations should be included for the respondent to follow, including

- Guidance notes for each section requiring data,
- Clear definition of the product requiring data,
- The time period or point in time to be covered by the return,
- Instructions on how to change the description of the product,
- Information on linking the product description to industry tariff codes, and
- A period of back prices for amendment, if necessary.

6.46 Allow for changes to administrative information on the form, including

- Space to record comments,
- The name and contact number of the person completing the form,
- Changes to the mailing address of the company, and
- Notes on how to return the form (prepaid envelope).

6.47 The main advantage of the postal survey approach is that it is inexpensive, particularly when coupled with modern data-handling technology, which reduces the need for operators to physically type data into systems. Large and dispersed geographical areas can be covered for minimum extra cost. This assumes, of course, that the postal system in the country is accurate and dependable with deliveries.

6.48 Disadvantages of the postal survey approach include the difficulty in achieving a high level of response from respondents because the collection mode is not interactive. This can be mitigated if there is legislation in place to penalize for nonresponse. Potential quality problems can arise when respondents do not pay adequate attention to the notes and complete the form incorrectly. For this reason, it is wise to explain the requirement through a meeting or telephone conversation when the respondent is originally selected to participate in the survey; personal contact with the respondent, even by phone, should be encouraged as a general method for improving the quality of data returns.

6.49 Nonresponse follow-up contacts and resolving queries about reported data can add a significant cost to the postal approach.

D.3.2 Automated telephone data submission

6.50 Usually the PPI price survey collects price details for a small number of products from each respondent. The brevity of the questionnaire makes the PPI ideal for telephone-based data entry systems, in which the respondent reports the information directly over the telephone by following voice prompts and entering data using a touch-tone telephone. The prerecorded dialogue in such systems enables the respondents to report their monthly data quickly and accurately. Usually a letter is dispatched asking the respondent to make a telephone return. This approach has the advantage of making it possible to program the dialogue to allow validation of the data to take place during the telephone call. This can be done by asking the respondent to leave a voice message or by switching the call to a data collection analyst. Generally, this method is beneficial to the statistical office by reducing desk processing and hence reducing costs. Some on-line validation can take place that may benefit respondents by saving them from being recontacted by the statistical office.

6.51 The possible disadvantages of this system are confusion to some users caused by the technology, and, because respondents can leave a voice message without discussion, some further clarification contact may be needed. This method is also less useful when there are complex product specifications, which need to be updated frequently. For example, the United Kingdom does not use this collection method for the PPI computer price index.

D.3.3 Personal interviewer

6.52 This involves a face-to-face meeting with each respondent on a regular basis (for example, monthly, quarterly) by a trained interviewer to obtain the data necessary for the survey. The main advantage of this approach is that the data can be validated at the source, and problems and differences of understanding can be resolved during the discussion.

6.53 However, the big disadvantage is the cost of employing interviewers and the high travel costs, particularly where long distances are involved. There is also a disadvantage to the respondent, who would have to spend more time in face-to-face meetings with statistical office representatives. Field collection for the PPI is not as viable as for the CPI because

(i) Outlets are not clustered in population centers and are often in decentralized industrial areas; and

(ii) Inspection of products cannot be carried out, leading to less quality control of specifications.

6.54 A further variation here is to use another collection method (for example, postal questionnaire) on a regular basis and have a less frequent personal interview to clarify details such as the product range and representativity. For example, some statistical offices visit each respondent on a rotating basis over a five-year period. This also gives the statistical office the chance to "train" the respondent to provide good quality data. This approach can be particularly beneficial if used at the point where the business is initially brought into the sample, since many of the problems can be dealt with in a face-to-face meeting.

D.3.4 Telephone interviews

6.55 Each respondent is called during the collection period and asked for the data required for the survey, with the interviewer validating the form when speaking to the respondent. The data collection staff can be assisted with a set dialogue or through computer-aided telephone interviewing. It is important to provide adequate training to deal with questions that arise during the call. The main advantage of this approach is the data validation during the telephone call, but, again, this is costly in terms of staffing, and there can be difficulty getting through to the respondent to get the information. Telephone interviewing is becoming more difficult with technical developments that allow respondents to answer telephone calls only from selected people (via voicemail). Also, the respondent may not have the data immediately at hand, which could lead to guessing rather than the correct data. The main concern with this method is that it is likely to lead to bias caused by respondents repeating previous observations—that is, stating that there has been no change.

D.3.5 Internet data provision

6.56 This method of data collection offers great potential in terms of efficiency and economy. Respondents can be given the questionnaire and reminded to respond through this channel. Systems to validate the data in real time are also possible. This is a benefit to the business respondent since it reduces recontact time (although the benefits of a one-to-one dialogue are lost compared with other methods). Since the returned data are in electronic format, it is efficient for further processing by the statistical agency and the response times are quicker than postal-based collection.

6.57 There are, however, a number of issues related to Internet collection. To be effective, a very large proportion of the businesses in the country must have access to the Internet. Also, Internet security is vital, given the commercially sensitive nature of producer prices.

D.3.6 Electronic capture of data from disc

6.58 This method involves the supply of a floppy disc containing an electronic questionnaire. The respondent loads the disc and completes the information before returning the disc to the statistical office. Data are then transferred to the statistical office database. This method allows on-line validation techniques to be built into the questionnaire to save recontact time. However, the procedures for dealing with floppy discs are onerous, and for short surveys such as the PPI (with few data items to be collected), the benefits are limited.

D.3.7 Electronic data transfer

6.59 This method of collection involves the transfer of data files directly from the establishment's systems and allows a large volume of data to be collected with a minimum ongoing collection burden to the respondent. The initial setup procedures can be quite burdensome, but the regular collection costs are reduced. The statistical office has to clearly define the data format and information system protocols. It is possible that this type of collection could allow full unit value data to be collected, which can be beneficial for tightly defined and homogeneous products.

D.3.8 E-mail collection

6.60 The use of e-mail is another collection method, which allows the survey form to be delivered and returned electronically. This approach is less efficient than some of the other electronic methods outlined above; however, it could be useful where postal services are less reliable. It is also useful as a reminder technique, since it offers speedy contact with respondents. Again, security is a key issue, and, since E-mail can be less secure than some of the other forms of electronic collection, the legal issues should be examined carefully.

D.3.9 Alternative sources

D.3.9.1 Published sources

6.61 Some data items are available from public sources such as trade publications. Examples include prices for metals that are traded on financial markets, which are reported in the financial press and international journals such as *Metal Bulletin*. Published sources provide a high-quality source of price data for these products. Their advantage is that they are readily available and relatively inexpensive; they also reduce the respondent burden. Before using published source data, the statistical office must be sure that the source is reliable and that the prices reported are genuinely independent market prices. It is important to verify that the prices are actually based on business transactions of these products. It is also a good idea to become as familiar as possible about the methodology used by the compiling organization.

6.62 A further subset of published sources that is becoming increasingly important is data collection from company websites. Many companies have created extensive websites allowing customers to search by product specification and in some cases allowing customers to set their own product configuration. It is then possible to buy the product directly from the site. This type of website offers tremendous potential for PPI price collection and also for independent validation of prices received through the more conventional channels. There are a few issues to consider, such as the extent to which the Internet prices are list prices instead of transaction prices and whether large buyers attract lower negotiated prices under long-term contract, for example.

6.63 Another important issue is that the Internet-advertised prices may be retail prices. But in some circumstances retail and producer prices are the same. For example, one of the best sectors to use Internet advertising for price collection and validation is in personal computers (PCs). In this case, major manufacturers have set up websites that enable the public and businesses to buy directly from the manufacturer (again, it is important to be aware that bulk discounts could be offered).

6.64 It is worth noting that even if published sources are not used as direct inputs to the compilation of the index, they can provide valuable information for editing, external verification, and the preparation of analyses of the main index movements.

D.3.9.2 Regulatory data sources

6.65 For some products or services it is possible to get data from government regulators. It can be difficult to get access to this information if confidentiality constraints apply, but where it is possible to get this type of data, it can be of very high quality. Telecommunication and rail fare data are two examples of service prices collected in this way in several countries. The potential overlap between retail or producer price can also be an issue when using this type of data, but in many cases there are different tariffs for business and consumer use.

D.4 Field procedures

6.66 The following section outlines practical field practices adopted by many statistical offices.

D.4.1 Price discounts

6.67 Producer prices should be transaction prices, not list prices. This means that all discounts should be taken into account. Discounts can be given for a variety of reasons, such as prompt payment, volume of the purchase, competitive price-cutting, and so on. It is important that this be made clear during the survey collection process. Discounts arising from high-volume transactions can cause particular difficulty. Problems occur if the volume sold to the representative customer changes from period to period, which could lead to changes in the discount rate in each period. In such a case the price index would be seen to move simply because of changes in the volume mix, rather than as a result of a pure price change. This type of problem commonly occurs in quarry products, such as road stone and railway ballast. A possible approach in such circumstances is to seek prices for the same specific, typical transaction volume each month.

6.68 A common form of discounting is to provide a larger quantity of the product for the same price, sometimes for a limited period. The product specification should include details on quantity to enable an adjustment to be made to include this type of discount. Retrospective discounts based on sales volumes are an important feature of the manufacturing sector but are difficult to collect with normal survey techniques, so these tend not to be included. An example is the bonus paid to car deal-ers by manufacturers based on sales volumes, which is separate from the original sales transaction.

D.4.1.1 Rebates

6.69 Rebates are a form of discount where the discount is (generally) paid after the purchase and are normally based on the cumulative value of purchases over a specified time—for example, a rebate is given at the end of the year based on the customer's total purchases over that year.

6.70 The collection of discounted prices and the identification of discounts are complicated in practice by a number of factors. First, the pricing structure used by the company may be complex and the conditions under which discounts apply may be described in nonstandard terms. Second, differences in pricing and discounting procedures among companies require that data collection be tailored to each company. Third, in a number of areas, the level of discounts and rebates is commercially very sensitive information, and senior company officials may know only the full level of discounts offered to major customers. Taken together, these three factors mean that identifying and keeping track of discounts constitute the major tasks facing PPI compilers.

6.71 Rebates in PPI indices pose major practical problems in that they are often determined by future events—for example, the buyer receives a rebate at the end of the year on the basis of how much he or she purchased during the year. Thus, at the start of the year, although it is known that the buyer will receive a rebate, the precise amount is unknown. The special problem posed by rebates of this sort is that the final price to be paid may not be known until the end of the period concerned, when the total value of purchases will be known and hence the level of rebate can be calculated. This type is often referred to as a retrospective price fall.

6.72 Often a rebate paid to the buyer in the form of a reduction in the cost of the purchase over a year occurs in a particular month. This can lead to reported prices showing dramatic price falls for that particular period. PPI compilers should take care to ensure this does not occur.

D.4.1.2 Treatment of rebates in PPI

6.73 The question arises as to how such rebates should be treated. Should the price paid each month be shown in the index as the price for the item? If so, how should the rebate be treated—as a retrospective price reduction? If so, should the previous prices be revised?

6.74 On balance it is considered that where the rebate is already in existence, the rebate should be treated as a discount and deducted from the monthly price and not treated as a retrospective price reduction. The basis for calculating the rebate should be the buyer's normal volume of purchases (if the buyer is a new customer, then the basis for calculating the rebate should be the average quantity purchased by that category of buyer).

6.75 Changes in the level of rebates should be reflected only when the actual rebate for the same quantity purchased or sold changes. Changes in the rebates paid to a particular customer for changing the volume of purchases should not be reflected as a price change.

6.76 As price indices are designed to measure price changes for a constant quantum of purchases or sales, the rebate collected should be the rebate applicable to that constant quantity and clearly specified in the pricing basis.

6.77 Where rebates are specified in terms of a monetary value of purchases or sales, it is important to realize that because of inflation a monetary value does not represent a constant real quantum. As a consequence, the monetary value should, if possible, be converted to a quantity. If this is not possible, then the dollar value should be updated each year according to the change in the price of the item concerned.

6.78 If the quantity or value of a respondent's purchases or sales changes significantly, the pricing basis should be changed to reflect this. The change in rebate associated with this should not be allowed to affect the index.

6.79 Where a number of levels of rebates are offered, it is necessary to ascertain the importance of each level of rebate and to price those that are significant.

6.80 Caution must be adopted when dealing with retrospective price falls. Revisions to previously published indices can create major problems for users, who use them to negotiate contracts.

D.5 Other variables

D.5.1 Quality or specification changes

6.81 If any variable in the product specification changes, the respondent should be questioned about the change and whether new features have been added. If it is a simple change that has no effect on the price, then the specification should be updated and a marker placed on the product description to indicate that it has been changed. If the quality increases (in terms of producer's cost), then this should be reflected in the index as a price fall. If the quality decreases, then this should be reflected as a price rise.

6.82 If the price of the product changes and it is solely or partly because of the change in the specification of the product, then the data collection form returned to the respondent should contain the new price, but for index purposes a quality adjustment should be made to record no price changes for the product on the price database (since it is not an inflationary pressure causing the price to move). If, for example, the price of the product moves 10 percent and the respondent assesses that only 5 percent of the movement was related to pure price change, with the remaining 5 percent owing to a different product with changed specification being quoted (a quality improvement), then the price relative should move by only 5 percent.

D.5.2 Unique products

6.83 A unique product is a product that is only manufactured once to the specification of an individual customer. Within a group of products, each product will be different from the others—for example, industrial furnaces, ships, or an audit contract. In these cases the price cannot be observed over multiple periods. There are a number of approaches to solving this problem, as follows:

(i) *Model Pricing:* Ask the respondent to provide a notional product with a basic range of characteristics, based on recent orders. For each period the respondent is asked to supply a hypothetical price quote based on this hypothetical product. It is important to update the product specification at regular intervals. Described below are three methods of model selection that can be used. Refer to Chapter 10 for further discussion.

   (a) An actual product sold in some recent period, which is representative of the respondent's output, can be selected and specified in detail as the model to be priced;
   (b) A hypothetical model that is representative of the types of products produced by the respondent can be established. While this model may never have been (or never will be) produced, it must represent an item that could be readily produced; and
   (c) A component model can be established. These are used in those cases where no single model can represent the output of the respondent. In such cases a number of models can be selected or a notional model incorporating the key components from the various items produced can be established—that is, incorporating the different types of materials used and different production techniques. In the latter case the model would be purely hypothetical in that it might never be built, but it would nevertheless be representative for measuring price changes.

(ii) *Repeat Recent Real Sale:* Ask the respondent to provide a price quote for a recent real sale and to provide a hypothetical price for this exact product design for the subsequent months. If the order is not repeated again after a reasonable interval—for example, six to eight months—then a replacement product is sought.

(iii) *Specification Pricing:* A base model of the product or service is agreed upon with the company, and then in each subsequent month the company supplies the price for each individual part of the model—for example, one hour of an accountant's time or a ton of steel and so on. When the data are returned to the office, they are collated using a formula agreed upon with the company to arrive at a price each month.

(iv) *Component Pricing:* This approach entails collecting prices for a selection of component parts and using them as inputs to produce a final output price. It is important to include the prevailing margin achieved by the producer, and it is also important to have a dialogue with the producer to ensure that the components remain representative and constitute a very high proportion of the total inputs. For example, a recent EU task force report on large capital equipment suggested that this approach would be sufficient if the value added for the assembly of the components did not exceed 10 percent of the total product value.

6.84 These approaches are much more burdensome on respondents, since they cannot simply look at recent sales data to provide price quotes. To accurately supply price quotes using these approaches would lead to the respondent incurring substantial costs, so in practice there is an element of estimation in this process.

6.85 In all the above unique product cases, the main difficulty is persuading the respondents of the value of this approach, because they do not produce this specific product as described. To resolve these issues, a field officer visit may be necessary. It is important to include base-level products such as these in the PPI since they are often associated with high-value goods, which would not be included otherwise. It is all too easy to select a related, but simple, product—for example, in shipbuilding to ignore large, unique ships and to concentrate on only small, regularly produced products such as dinghies.

6.86 A further approach to solving the unique product problem could be to build a hedonic model. This would enable the unique model to be valued from its characteristic set. This method is more normally associated with quality adjustment but could be extended where sufficient data are available. See Chapters 7 and 21 on quality adjustment for more details on this approach.

D.5.3 Unit values

6.87 In some circumstances it is possible to use unit value indices to overcome the problems of unique products, but this is recommended only for products with a very narrowly defined product group—for example, for road stone. See Section B.2 above for a discussion of the limitations of unit value indices.

D.5.4 Transfer prices

6.88 Transfer prices are defined in the *1993 SNA* (paragraph 3.79) as

> affiliated enterprise may set the prices of transactions among themselves artificially high or low in order to effect an unspecified income payment or capital transfer.

Transfer prices should be used with caution because they often do not fully reflect the true value of the goods or services being transacted. It is important to aim to collect market prices, or real transaction prices. The best way to get real transaction prices is to ensure that the price recorded is to a third party, not to another part of the same business. In some circumstances it might not be possible to get real market prices without losing too many price quotes to produce a representative index. If the only price available is to another part of the same business, which is potentially but not necessarily a transfer price, careful attention must be paid to market price movements for similar products to ensure that the interenterprise sales reflect market conditions. It is important to avoid recording stable prices for these transactions over the long term if market prices are changing. Obtaining good weight information on the value of output or sales will also be difficult because the revenue weights should reflect market prices.

D.5.5 Sampling issues

6.89 The objective of price indices is to measure pure price change over time—that is, to measure the extent to which the cost of an identical basket of products changes over time, not affected by changes in quality or quantity or the terms of sale. This is often referred to as pricing to constant quality; it is not a simple objective to achieve because the characteristics of products being sold in the marketplace, including their terms of sale, change over time. Frequently, the precise commodity priced in one period is no longer available in the next period either because there has been some change in the characteristics of the commodity or something new has taken its place.

6.90 Sampling issues, such as sample loss relevance, avoidance of making quality adjustments, selecting noncomparable replacements, and inadequate matching procedures, greatly affect the representativeness of the sample over time; ignoring such issues could bias the index. Refer to Chapter 7 for further detail.

6.91 PPI compilers must devise techniques to minimize differences and eliminate their effect on the index.

D.5.6 Price discrimination

6.92 Price discrimination refers to the situation where the same product sells at different prices in different markets. An example would be the same grade of wheat sold at different prices in different markets, or a different volume of wheat sold at the same price to different markets. When such price discrimination occurs, the average price of wheat can change over time because of changes in the proportions of wheat sold to each market.

6.93 How should such price changes be reflected in the PPI? The answer will depend on the reason for the price discrimination and requires an examination of the various forms of price discrimination. There are four main forms of price discrimination (listed below). All four factors (or any combination) could apply to a particular specification.

6.94 In cases of price discrimination, it is extremely difficult to determine the price change when the destination of the purchaser changes. In these instances, PPI compilers should specify only the purchasers of product and the quality or conditions of sale differences between purchasers.

D.5.6.1 Differences in selling terms and provision of credit

6.95 The conditions under which goods are sold often vary between markets (buyers). For instance, prices may be lower in one market because the goods are paid for on delivery, while prices in another market may be higher, reflecting the fact that goods are sold on credit. In these situations it seems reasonable to argue that identical goods are not being sold in each market. What is in fact being sold in the second market is a mixture of the good and credit. It follows that, in these cases, shifts in destination should not be reflected as price changes.

D.5.6.2 Differences owing to timing of contracts

6.96 Where goods are sold on a long-term contract basis, price differences may arise between different markets simply because of differences in the period when the contracts in respect of these markets were signed. In these cases it seems clear that changes in prices owing to changes in destination should be reflected in the index. Failure to do so would run the risk of missing out on long-term price changes for products where the destination is changing over time.

D.5.6.3 Competitive pressures

6.97 In some markets, goods may have to be sold at lower prices because of competition from other countries (for example, dumping of EU agricultural products), while in other markets producers may be able to achieve higher prices because of the absence of such competition. In these cases, shifts between markets represent pure price changes and should be reflected as such.

D.5.6.4 Hidden quality differences

6.98 For some items, such as tinplate, respondents supply prices only for broadly specified products. In these cases, destination may serve as a de facto quality specification—for example, the quality of tinplate shipped to destination A is different from that shipped to B. In this situation, changes in destination should not be reflected as price changes.

D.6 Field officer visits

6.99 Field officer visits serve two broad purposes. First, field officers are often used in the initialization or recruitment process to identify representative products from within the respondents' product range and to discuss the exact reporting requirements for the PPI. (This approach is used by the United States, Australia, and France; the French use qualified engineering staff to visit companies.) In some regions (for example, in Europe, where a detailed survey on products by industry of origin, PRODCOM, is conducted by member states) it is known that a group of products are manufactured by a company in a particular sector. In such a case, there are two options when selecting products: either let the business pick the most representative product (that is, the one that accounts for the largest percentage of the respondent's turnover for the class of product), or let the field officer select the product with the respondent. Each approach has advantages and disadvantages. For example, only the respondents know the best or easiest product for them to supply data for; however, it is also important to get a price quote for several products that are representative of the respondent's range of products. This may require collaboration between the respondent and the field officer. For more information on product selection, see Chapter 5.

6.100 The second main purpose of the field officer visit is to assist respondents with problems in completing returns—for example, in the case of unique products or late responses. The field officer contacts and visits the company to understand its specific concerns and problems in completing the form and to work with the company to overcome them. This is largely a reactive activity used to solve problems, but another alternative is to have a rolling program of visits so that each respondent (or key respondent) is visited over a set period. This helps to keep the respondent educated and ensures that problems do not linger unnoticed for long periods.

D.7 Industry specialists

6.101 The role of the industry specialist is similar to that of the field officer, but the industry specialist concentrates on a narrow range of industries. Since prices are required from very specialized industries such as chemicals and semiconductors, it is difficult for the statistical office to ensure the quality of data returns and have a meaningful dialogue with respondents unless the organization includes analysts with a more detailed knowledge of these complex industries. A small team becomes experts in a certain field—for example, computers. The teams are fully up to speed on changes in the market, their respondents' activities within the market, and specific problems relating to completion of survey data. These experts then analyze returns in line with the industry intelligence and support respondents when they provide data.

D.8 Delinquency follow-up

6.102 It is important to achieve high response rates; to achieve this, procedures should be established to follow up with nonresponders. Problems with maintaining adequate response rates will occur in price surveys that do not have delinquency follow-up procedures, even when the surveys have statutory penalties.

6.103 Delinquency follow-up can be done using any of the data collection methods outlined above. A reminder telephone call is an effective technique, since this enables the respondents to discuss any difficulties they have with the survey at the same time, and it is often possible to take data over the telephone (although a price taken over the telephone should be marked for later verification). This technique has the advantage of generating quick results, but it does require the statistical agency to maintain an up-to-date list of contacts and their telephone numbers. Given the labor-intensive nature of reminder telephone calls, it is possible to target these calls on key responders, which are usually the responders with the largest weights.

6.104 A follow-up letter is often effective, particularly if the country has a legal penalty for nonresponse. In such a case, it is possible to make the wording of the follow-up letter stronger, with more emphasis on the legal penalty. It is usual to follow a set procedure including the use of a recorded delivery letter if formal legal proceedings are to be conducted for nonresponse.

E. Respondent Relations

6.105 Respondents are very important to statistical offices, because without them there are no data. Therefore, developing good relations with and gaining the trust of data providers is an integral part of producing good estimates.

E.1 Dealing with refusals

6.106 On occasion, you will come across a respondent who states, "I've had enough" or "I'm not doing it anymore." In general, providing respondents with relevant information related to their concern will ensure they will continue to provide data. In the examples above, refusals could reflect issues with confidentiality, lack of importance, and too many forms.

6.107 Possible ways of dealing with these concerns are as follows:

(i) *Confidentiality:* When dealing with issues of confidentiality, a statistical agency that is independent of other government agencies has an advantage over those that are not. The IMF (2003) provided a framework for assessing the data quality of the PPI. This framework identified the importance of independence and confidentiality to ensure trust of respondents. Reassurance that the data will not be released to any other agency or person is much easier with legislation backing the statistical office.

(ii) *Objective of the Collection:* Reassure the provider that the prices are aggregated into an index that is published monthly or quarterly, and inform the provider of the importance of contributing and the use of the statistics produced.
(iii) *Respondent Workload:* Talk with the respondent to ensure that the data are easy to obtain and that the current specifications are still relevant; see if the provider can get rotated out of the sample if the company has been contributing data for many years.

E.3 Reducing respondent workload

6.108 Apart from the ongoing use of tailored forms, which significantly simplify the collection process for respondents, PPI compilers can actively reduce provider workload by

(i) Identifying commercially available data that meet the methodological requirements of price indices and using these as a substitute for data collected from respondents. The cost incurred in purchasing these data is compared with expected collection costs, with the benefit of reducing provider load taken into account; and
(ii) Identifying administrative data sources of prices that meet the methodological requirements of price indices and using these as a substitute for data collected from respondents.

F. Verification

F.1 Verification and validation of prices

6.109 Verification aims to identify potentially incorrect prices as early in the process as possible, consult with the respondent, and amend the data if necessary. Three key checks are required:

- Data reported were accurately entered into the processing system,
- All requested data were provided, and
- Data reported were valid (outlier detection).

6.110 This section covers only the first two bullet points above—that is, simple data checks at the point that data enter into the PPI system. Validation assesses whether the data returned by respondents are credible in relation to other data for the same industry or commodity, and the treatment of data that are not credible are covered in Chapter 9.

F.2 Verification tolerance

6.111 The first stage in the verification process is to determine that the data entered into the system for further processing are an accurate reflection of the data returned. This can be achieved through either a manual audit or an automated system. These checks should determine whether

(i) All data fields required have been completed,
(ii) The data entered in the database agree with those reported, and
(iii) All data fields are completed within an expected parameter range.

When the data have been accurately recorded by the statistical office but basic data checks are not passed, the analyst will need to contact the respondent to verify the information or to get the correct data. Returned prices may be compared with those received for the previous period. If the price change is outside a specified range, then the price should be marked for further investigation. Respondents providing dubious prices can then be contacted to check that the large change is correct and to provide a reason for the large change. Large price changes fall into two main categories: those that are erroneous and those that are correct but genuinely unusual. The second category is more difficult to deal with because they could be outliers, which might result in the need for special treatment within the estimation procedure. Outlier treatment is discussed in Chapter 9.

F.3 Setting tolerances

6.112 The tolerances for data verification checks should be set so that any changes outside the boundaries of expectation are flagged for the data reviewer.

6.113 Tolerances may need to be set independently for each product group. For products that have volatile prices, such as oil or seasonal items, it may be appropriate to have quite wide verification tolerances. Other products may have more stable prices, and so narrower tolerances would be more appropriate. To set verification tolerances for a particular product, price changes over a period of time, say two or more years, need to be analyzed. The range of price changes can then be considered, and the top and bottom 10 percent, for example, can be used to set the tolerances for verification.

6.114 In addition to checking for large price movements, another check looks for prices that have not moved for a considerable period. Most companies will review their prices on a regular basis, often annually. If a company's price has not moved for 15 months, for example, they may be returning the same price out of habit. In these cases the company should be contacted to see if the true price is being returned. In periods of low inflation, the periods of stability between price changes might get longer and the number of companies not reporting price changes for considerable periods could increase.

G. Related Price Issues

G.1 Lagged prices

6.115 In some cases it is not possible to get prices in time for the current-period's compilation. To ensure that the PPI is published in a timely manner, the previous available price or lagged price can be used. These lagged prices often come from administrative sources, and the delays result from the time required to collate data by the external supplier. Examples include financial intermediation service charges and insurance premiums.

G.2 Seasonal products

6.116 Some products are available for only part of the year—for example, items associated with religious festivals and certain fresh fruits or vegetables. The practice commonly adopted in such cases is to carry the last reported price forward, until the next season's trading starts and a new price can be collected. This procedure tends to dampen the index movements when the product is out of season and causes upward and downward movements in the product index when the product is back in season. One solution is to impute the missing prices based on the short-term movements of prices for similar products. An alternative solution is to have variable weights for each period, so that the product has a zero weight when not in season. The disadvantage of this is that it makes index point effect analysis less straightforward, since care has to be taken to ensure that the correct weights are applied in each month. See Chapters 10 and 22 on seasonal products and Chapters 7 and 9 on imputation techniques for more details.

Figure 6.1. Example of PPI Collection Form

Statistics Canada Statistique Canada

INDUSTRIAL PRICE REPORT

CONFIDENTIAL (WHEN COMPLETED)

SI VOUS PRÉFÉREZ CE QUESTIONNAIRE EN FRANCAIS, VEUILLEZ COCHER ☐

Authority – Statistics Act, Revised Statutes of Canada, 1985, Chapter S19.

Month Year

This survey is being conducted to collect prices of representative commodity transactions. The prices you report are essential to the production of indexes measuring the movement of prices for important industry and commodity groups in the Canadian economy. The resulting indexes are used in developing estimates for real manufacturing output, real capital inputs and for contract escalation.

INDIVIDUAL PRICE REPORTS ARE KEPT CONFIDENTIAL.

M00000

COMPANY NAME
ATTN: PERSON NAME
ADDRESS
CITY, PROVINCE
POSTAL CODE

The reporting form sets out our request for price information for the period shown. We urge you to read the instructions carefully and fill in the requested information.

Should you require further information with respect to this report, please contact the Prices Division Contact indicated on the reverse side. Please feel free to call collect or call 1-866-230-2248 for general enquiries.

The information and data pre-coded on this form reflects the respondent's preference.

Thank you for your cooperation.

Prices Division
Ottawa, Ontario
K1A 0T6

RESPONDENT R00000	PRICES DIVISION CONTACT: (613) 951-	Si vous préférez ce questionnaire en français veuillez cocher ( )

To complete this INDUSTRIAL PRICE REPORT:
1) **In BOX A enter the TRANSACTION PRICE in effect on the 15th of the month indicated.**
2) **In BOX B enter "NS" if no sales occurred and give an estimate in BOX A for the transaction prices.**
3) **If there is any change in the DESCRIPTION OF PRODUCT and/or TRANSACTION DESCRIPTION please amend.**

PRODUCT ID P0000000	COMMODITY DESCRIPTION:	PRODUCT n of m

DESCRIPTION OF PRODUCT:

TRANSACTION DESCRIPTION:

Date of last reported price change:			C1 - C4: Transaction Description as specified above				Circle reasons for price change	Further explanation of price change (pertinent market information)
DATE	**A**	**B**	**C1**	**C2**	**C3**	**C4**	**D**	
yyyy-mm							**1 2 3 4 5 6**	
yyyy-mm							**1 2 3 4 5 6**	
yyyy-mm							**1 2 3 4 5 6**	
yyyy-mm							**1 2 3 4 5 6**	
yyyy-mm							**1 2 3 4 5 6**	

REASON FOR PRICE CHANGE: **1. Material costs** **2. Labour costs** **3. Competitive factors** **4. Physical content** **5. Terms of sale** **6. Others - describe**

ITEM NUMBER	

SPECIFICATION

If **any of the information** relating to this item has changed, please cross through the existing details, write any amendments alongside and insert an X in the box ☐

UNIT OF SALE

TERMS OF SALE

If the price of this item is no longer representative of the trends in your actual selling prices for this category of goods insert an X in the box ☐

CURRENCY

Please complete the boxes marked with an asterisk below, include future prices if appropriate and insert the date in the month to which the price relates eg. 1st, 14th

Month	Date	Price	Ditto
		.	
		.	☐
		.	☐
		.	☐
		.	☐

Month	Date	Future Price	Ditto
		.	☐
		.	☐
		.	☐

1/99 BP161D

7. Treatment of Quality Change

A. Introduction

A.1 Why quality change is an issue

7.1 When routinely compiling an output or input PPI, specific varieties of goods and services in the index regularly appear and disappear. New goods and services can appear because technical progress makes production of new varieties possible. Even without technical progress in the supplying activity, however, products previously feasible but not produced may emerge because the technology of the using activity or the tastes of the final consumer have shifted. Existing varieties often decrease in importance or disappear from the market altogether as new varieties appear. Moreover, the priced set of products often is a small sample of the full range of products that exist at any given time. The set of priced products is a subset of those available in the sample of establishments, which in turn is a subset of the population of establishments. Products in the sample may appear and disappear, not because they are truly new to, or no longer produced or used by, all establishments, but because they may be only new to, or no longer produced by, the establishments in the sample.

7.2 This chapter covers how to deal with the problem of continuous change in the assortment of transactions whose prices make up a PPI. The overarching principle for designing methods to deal with variety turnover is that, at the most detailed level, the prices of items between any two periods may be directly compared only if the items are essentially the same. Violating this principle would mean that a given monthly price ratio measures not only the change in price, but also the value of the qualitative difference between two items. This contaminates the estimate of relative price change with an element, quality, that measures relative volume rather than price. It degrades the accuracy of the price index formed with the price ratios or relatives for the specific transactions.

7.3 What does "essentially the same" mean in practical terms? In Chapter 9, this *Manual* calls the specific varieties (or item specifications) exchanged in market transactions *products*. A good or service transaction is essentially the same as another good or service transaction if both would be classified as the same product. It follows that products are the most detailed entities on which prices may be compared from period to period. There may be many transactions in a given month for a given product description. Thus, the price of a product is the *unit value* of transactions in the product for the month.

7.4 For measurement purposes, a product equates to a *complete description*. A product description is complete if at a given time there is no variation in the prices of goods or services with that description that might be exchanged between economic agents. Practically speaking, zero variation is rarely possible, in part because price statistics ordinarily aggregate time into monthly periods. Realistically, then, the quality of a description and thus a product specification is in proportion to the price variation at any given time among the transactions fitting that description. In developing products, compilers aim to minimize price variation across the transactions classified by any one description, consistent with maintaining their ability to make successive observations on the average price charged for that description over time.

7.5 The form of this description often is simply text. It also can be highly structured, however. In *structured product descriptions*, the product's characteristics are specific levels of indicators for several dimensions that are known to affect the average transaction price.[1] Each set of these indica-

[1] See Chapter 6 on structured product descriptions, also termed checklists by some statistical agencies.

tors' levels frames a specific product. Examples of these dimensions are the horsepower of a car, the speed of a computer, or the species of a piece of fruit. Examples of product-determining levels or specific settings of these respective dimensions are 325 horsepower, 2 gigahertz, or flame red grape. Another set of products for cars, computers, or fruit would be described by the characteristics levels 110 horsepower, 3 gigahertz, or Thompson green grape.

7.6 For price measurement purposes, the comparative quality of a product comprises its description and price. Distinct descriptions represent different qualities of products, to the extent that they contain different levels of characteristics that affect the average price of transactions of things with that description in a given month. When comparing descriptions, the practice of price statistics thus judges quality by price. If products with two distinct descriptions are transacted at the same time, the description with the higher price must be the higher quality. This corresponds to what is called a higher revealed preference or value in use of the product (demand side), as well as higher content in the input needed to make the product (supply side). For index compilers, then, quality is an ordinal concept, comprising the set of complete product descriptions ordered by price for a given month.

7.7 When a new product appears, a new description manifests itself as well. The new description is different from the descriptions of existing products because the level of at least one characteristic in the description has changed. The difference in the characteristic explains the difference in price compared with varieties already available. For example, a new variety of computer emerges with a processor speed of 3 gigahertz instead of 2, and it has, say, a $325 premium over 2 gigahertz computers already available. Thus, the value of the additional gigahertz of speed is $325 and the new computer is, by implication, of higher quality than the old one.

7.8 All of this appears clear enough. Why, then, is quality change an issue? The issue is related to how much a compiler knows about an emergent variety relative to those continuing to be produced. The computer example above relied on having an overlapping month during which the outgoing product still is sold. An establishment may discontinue a product, immediately replacing it with another item having a different configuration of characteristics and thus a different description. Does a compiler go to another establishment for an overlap price? If so, there may be a number of other characteristics that differ in the comparison besides speed. Are all of these relevant to assessing quality and thus volume change? Does the new product have a characteristic that is itself completely new and not evident at any level in existing products? How should a compiler value a completely new characteristic manifest at some positive level? How important is the new variety in the product group when it is first detected?

7.9 Chapter 9 calls the basic groups of products *elementary aggregates*. Elementary aggregates are the smallest aggregates for which compilers combine price ratios or relatives into index numbers. Those products in an elementary aggregate whose price time series continue are the matched models or matched products. Those products whose price time series ends or begins in a given month comprise the set of unmatched models or unmatched products. There are, therefore, two practical problems a compiler faces when constructing the index for the elementary aggregate: what to do with the matched models and what to do with the unmatched models (missing or new).

7.10 Standard index number theory and methodology handle the part of the product group index for the matched models. This does not eliminate the practical problem for the compiler even for the matched models. According to the statistical evidence, for the matched models there is often significant and rapid change in the shares given products represented in an elementary aggregate. Compilers generally have no current weights at the level of products. How can they know before a product disappears that it is becoming unimportant? How can they prevent declining relevance of their samples by giving new products a chance of selection? Still, these are more or less conventional issues of weighting comparisons of like products. What about the quality problem?

7.11 The compiler's other fundamental problem is how to use last month's prices of items missing in the current month and the prices of items in the current month that are new in the price index of a basic product group, if at all. In this comparison, one confronts the problem of adjusting for differences in *quality*.

7.12 Compilers can address both problems by reselecting product samples frequently and simply using the price index comprising the matched models for the elementary aggregate. This *matched-models method* will ensure that the items in the index more closely represent current transactions and maintain, in all probability, sample relevance. At the same time, it can reduce the importance of unmatched models relative to matched models in any given comparison of adjacent months. Reselecting samples more frequently, however, is more costly and tends to increase the respondent burden of conventional survey methods. Statistical organizations may not be able to afford the staff, travel, and other expenses to support the frequency of reselections needed to maintain sample relevance by this method alone. Further, assuming sampling is undertaken as frequently as necessary, measures also must be taken to ensure some overlap between the successively reselected samples, particularly if the samples are randomly selected. It is, therefore, impractical, if not impossible, to avoid the problem of comparing unmatched models entirely through sample reselections. The quality change issue still comes down to what can be done when comparing sets of unmatched products (missing and new) from two periods with different characteristics and where, moreover, the sets of unmatched models for the two months generally comprise a different number of products.

7.13 As this chapter shows, compilers and researchers have developed a number of methods to address this problem. The method most often used still relies on the matched-models part of the price index for the elementary aggregate. Others, however, use additional information about product characteristics to bring the price information from the unmatched models into the estimator of the elementary aggregate index. A simple inventory of methods will not by itself address a compiler's problem. A number of empirical studies for PPIs and CPIs have found the choice of method can matter substantially (Armknecht and Weyback, 1989; Dulberger, 1989; Lowe, 1995; and Moulton and Moses, 1997). This chapter also is a guide to selecting methods based on the measurement circumstances.

A.2 Why the matched-models method may fail

7.14 The matched-models approach to variety turnover described in Section A.1 is subject to three broad sources of error: (i) missing products, (ii) sample space change (sampling issues), and (iii) new products. The first and third sources of error are the two types of unmatched models in section A.1: disappearing products and new products. The second causes the weights of the matched models or products to change from period to period and, along with the missing and new products, underlies the loss of sample relevance over time.

A.2.1 Missing products

7.15 For each sampled establishment, compilers measure the long-run price change for a product by comparing the price of the product in the current period—usually a month—with the average in the price reference period—usually a specific year. Ideally, price collectors begin recording the price of the products in the index in the first month of the reference period. This would then be the month when the products in the index entered the sample. When a cooperating establishment stops reporting the price of a product, it may be discontinued or it may not be available to the same specification—its quality has changed—and it is effectively missing in the current period. We thus encounter the first potential source of error in the matched-models method. There are several specific contexts for this. It may be a seasonal product, or the product may be a custom-made good or service supplied each time to a customer's specification.[2] There are four main approaches for dealing with missing products:

- Approach 1: The price change of the discontinued product may be *imputed by the aggregate price change of a group of other products* whose price evolution compilers judge to be similar to that of the missing product. Should a replacement product be observed, this amounts

[2] Sometimes compilers know in advance that the price of a product changes only at certain times of the year (electric power, for example). These instances are not missing prices since the compiler knows for certain there is no price change for this product most months of the year, and companies usually announce in advance when the price will change.

to an implicit quality adjustment comparing the price of the replacement product with the imputed price of the discontinued one.

- Approach 2: A *replacement product may be selected, comparable in quality* to the missing product, and its price used directly to form a price relative.

- Approach 3: The replacement may be deemed noncomparable with the missing product, but prices of both the missing and replacement products may be available in an *overlap period* before the product was missing. Compilers use the price difference in this overlap period to quality-adjust the replacement product's price until there are at least two observations on the replacement product.

- Approach 4: The price of a noncomparable replacement may be used with an *explicit adjustment for the quality difference* to extract the pure price change.

7.16 In most instances, compilers make an adjustment to the price (price change) of the replacement product to remove that part because of quality differences from the product it replaces. (This presumes the compiler has a basis for deciding which old product a new product replaces. More often, the change is for the outputs of a given establishment and the choice considered obvious.) The quality adjustment is a coefficient multiplied by the price of the replacement product to make it commensurate, from the producer's point of view, with the price of the original.

7.17 The simplest example of adjusting for quality change is handling the variety of package sizes encountered in all price indices. Suppose that the size of the missing product and its replacement differ, where quantity k of the replacement sells for the same price in the current month as quantity j of the original in the previous month. The conventional matched models approach (approach 1) is equivalent to imputing the price change of the index of matched models in the elementary aggregate to the unmatched models. Approach 2 would amount to finding another instance of the product of the same size with all other characteristics the same and directly comparing the two prices by forming the ratio of the price of the replacement product with the price of the missing product in the previous month. There is no overlap price in this example, precluding application of approach 3.

7.18 Alternatively, the compiler can undertake a range of explicit quality adjustments (approach 4). Suppose one package of the original contains j units of the replacement, while the replacement package contains k units. To make the price of one unit of the replacement commensurate with the price of one unit of the original, it must be multiplied by j/k, the quality adjustment. If $j = 2$ and $k = 3$, the required quality adjustment to be applied to the price of the replacement product is 2/3. Suppose a package of the replacement actually sells in the current month at the same price as a package of the original in the previous month. The price of the replacement, after adjusting for the change in quality, is only 2/3 that of the price of the original. If one unit of the replacement sells for twice the price of the original, then the quality-adjusted price is 4/3 ($2 \times 2/3$) that of the original: the price increase is 33 percent, not 100 percent.

7.19 The critical assumption in this explicit adjustment by the quantity in the package is that there is no difference in inputs between the different package sizes. If packaging and marketing use inputs, for example, or there are other input requirements in providing the different package sizes, the simple proportional adjustment by package size will not be correct. There are two options. If the compiler somehow knows the unit cost of producing the two package sizes of product through interviewing the establishment representative, he or she can divide the price ratio of the new package size to the old by the ratio of the unit cost of the new package size to the old package size. This illustrates the so-called resource-cost adjustment for quality differences.

7.20 In the final type of explicit approach, the compiler collects data on the range of sizes available in the market of an otherwise identical product in the current month and estimates a linear or log linear regression of price on package size.

$$Price = a + b \times Package\ Size$$

7.21 This is the so-called hedonic method. If the intercept or constant a is zero, this would confirm the validity of our first unit-value approach to correcting for package size. If a assumes a value different from 0, however, he or she could impute the value of the old size in the current month by evaluating the estimated regression equation at the old size. The price relative for the old item in the current month would be this estimated current-month price divided by its observed price in the previous month. This also would provide an estimated overlap price of the old size to the new one in the current month. In subsequent months, the monthly price relative would be the current to previous month ratio of the prices of the new-size product.

7.22 This chapter discusses these four approaches to quality adjustment in some detail along with the assumptions they imply. Because the prices of the unavailable products are not measured by definition, the veracity of some of the maintained assumptions about their price changes, had they been available, is difficult to establish. Nevertheless, the objective of each of the methods is to produce matched comparisons of the prices of products: to compare like with like from month to month. When products are replaced with new ones of a different quality, then a quality-adjusted price is required to produce a match. If the adjustment is inappropriate, there is an error, and, if it is inappropriate in a systematic direction, there is a bias. Careful quality adjustment practices are required to avoid error and bias.

A.2.2 Sampling issues

7.23 Sampling issues comprise four main areas of concern. First, samples lose relevance. A given set of matched models or products is likely to become increasingly unrepresentative of the population of transactions over time. It may be that the prices of old products being dropped are relatively low and the prices of new ones relatively high, and their prices are different even after quality adjustment (Silver and Heravi, 2002). For strategic reasons, firms may wish to dump old models, among other reasons to make way for the introduction of new models priced relatively high. Ignoring such unmatched models in PPI measurement will bias the index downward (see Section G.2.3 in this chapter). Ironically, the matched-models method compilers employ to ensure constant quality may itself lead to bias, especially if used with an infrequently updated product sample. (See also Koskimäki and Vartia, 2001, for an example.)

7.24 Second, because of the additional resources required to make quality adjustments to prices, it may be in the interests of the respondents, and indeed fall within their guidelines, to avoid making noncomparable replacements and quality adjustments. They keep with their products until they are no longer produced—that is, continue to monitor old products with limited sales. Such products may exhibit unusual price changes as they near the end of their life cycle. These unusual price changes arise because marketing strategies typically identify gains to be made from different pricing strategies at different times in the life cycle of products, particularly at the introduction and end of the cycle (Parker, 1992). Yet their weight in the index, which is based on their sales share when they were sampled, would remain constant in the index and probably would be too high at the end of the life cycle. Further, new and, therefore, unmatched products with possibly large sales would be ignored. Undue weight would be given to the unusual price changes of matched products at the end of their life cycle. This issue again is resolved by more frequent sample reselection, though not necessarily of the sample of establishments but of products within a given sample of establishments.

7.25 Third, the methodology for selecting replacement products advises compilers to choose a comparable replacement to avoid the need for explicit quality adjustments to prices. Obsolete products are by their nature at the end of their cycle, and replacements, to be comparable, must also be near or at the end of their cycles. Obsolete products with unusual price changes at the end of their cycle are replaced by other obsolete products with unusual price changes. This compounds the problem of unrepresentative samples and continues to bias the index against technically superior products delivering cheaper service flows.

7.26 Finally, the sampling problem with the matching procedure occurs when the respondent continues to report prices of products until replacements are forced, that is, until the products are no longer available, but has instructions to replace them with popular products. This improves the coverage and representativity of the sample. But the wide disparity between the characteristics of the old, obsolete products and new, popular ones makes accurate quality adjustment more difficult. The (quality-adjusted) price changes of very old and very new products may not be similar as required by the imputation methods under approach 1. The differences in quality likely are beyond what can be attributed to price differences in some overlap period under approach 3, since one product is in the last stages of its life cycle and the other in its first. Further, the technical differences between the products are likely to be of an order that makes it more difficult to provide reliable, explicit estimates of the effect of quality differences on prices under approach 4. By implication, many of the methods of dealing with quality adjustment for unavailable products will work better if the switch to a replacement product is made sooner rather than later. Sampling issues thus are closely linked to quality adjustment methods. This will be taken up in Chapter 8, in the section on product selection and the need for an integrated approach to dealing with both representativity and quality-adjusted prices.

A.2.3 New products

7.27 The third potential source of error is distinguishing between new products and quality changes in old ones, also covered in Chapter 8. When a truly new product is introduced, there are at least two reasons why early sales are at high prices that later fall, often precipitously: capacity limitations and market imperfections. Both of these may be present shortly after introduction of a new product because there are few suppliers for it.

7.28 Early in the product life cycle, production processes may have limited capacity; therefore, producers find themselves operating at relatively high and increasing marginal costs of production. Marginal costs of operation tend to decline as more producers enter the market or as existing producers redesign and upgrade production facilities for higher volume. Both of these bring operating levels back from high marginal cost, near full capacity levels.

7.29 With or without early capacity constraints, the small number of suppliers early in the life cycle allows what economists call *market imperfections* to arise. In an imperfectly competitive market, the producer can charge a monopoly price higher than the marginal cost of production. As more competitors enter the market for the new good or service, the monopoly power of early sellers decreases and the price tends to drop toward marginal cost. For example, the introduction of the zipper closure for clothing was a completely new good that led to an initial gain to zipper producers, who could extract an additional surplus from the purchasers (clothing manufacturers). As other zipper suppliers entered the market, the price fell.

7.30 The initially high price at introduction and its full subsequent decline would not be brought into the index fully by the usual methods. Compilers commonly either wait until the index is rebased or until a product in the sample becomes unavailable to seek a replacement product and admit the possibility of detecting a new good. After capacity constraints or monopoly profits diminish, subsequent price changes may show little difference from other broadly similar products. Standard approaches thus wait too long to pick up these early downtrends in the prices of new goods.

7.31 At the extreme, capturing the initial price decline requires a comparison between the first observed price and a hypothetical price for the period before its introduction. The hypothetical price would be the price below which there would be no positive market equilibrium quantity bought and sold.[3] Again, frequent resampling offers the possibility of catching new goods early in the product cycle when their prices are high and market share relatively low, thereby capturing early price declines as producers relieve capacity constraints and new entrants compete market imperfections away.

7.32 Finally, it is important to emphasize that there is not only a price decline but also a market share increase in the stylized product life cycle. Frequent resampling and focused scanning for new products should be at least somewhat effective in capturing the price declines in early product cycles. Compilers face a potentially serious problem, however, if they have no market share information to go with the prices. The stylized facts of the product cycle are that a new product comes in at a

[3]This hypothetical price differs from the *reservation price*, the other conceptual solution to the problem of new goods offered, for example, by Hicks (1940) and Fisher and Shell (1972). For a CPI, this preceding price is the highest notional price at which the quantity demanded would have been zero. The user's reservation price thus will be *higher* than the first observed price. For a PPI, the comparison would be between the price in the period of introduction and the lowest notional price in the preceding period at which the quantity supplied would have been zero. The supplier's reservation price will be *lower* than the first observed price. The product life cycle is based on the typical track of the market equilibrium price and market share, on both the technical possibilities of suppliers and the preferences of users, rather than one to the exclusion of the other.

high price and a low market share. The price then declines and market share increases. Both prices and market share then stabilize for a period, until a successor product emerges at a high price and low market share and then begins to take market share from the now mature existing product. Early and normally large price declines for new products thus should figure into the elementary aggregate price index at relatively low weight, while later and normally smaller price declines figure in at successively higher weight. Without current market share data, early price declines may well be overemphasized and the growth in the price index for the elementary aggregate underestimated.

A.3 Temporarily missing products

7.33 Products that are *temporarily* missing are not available and thus not priced in the month in question but are expected to be priced in subsequent months. The lack of availability may be because, for example, inventories are insufficient to meet demand, or material inputs are seasonal, as is the case with some fruits and vegetables for food canning. There may also be shortages.

7.34 The standard approach for seasonal products is the first of the four alternative methods for missing products: imputing the missing prices until the item reappears based on the price movements of similar products. Standard good survey management practice requires that seasonal products be separately identified by the respondent as "temporarily missing" or "seasonal," so compilers can remain alert to the product's reappearance later in the year. Principles and methods for such imputations and the conceptual difficulties in compiling month-on-month indices for such products are outlined in Armknecht and Maitland-Smith (1999), Feenstra and Diewert (2001), and Chapter 22. Otherwise, there is no difference between items missing temporarily and permanently.

A.4 Outline for remainder of chapter

7.35 Section B.1 first considers further what is meant by quality change and then considers conceptual issues for the valuation of quality differences. The meaning of quality change requires a conceptual and theoretical platform so that adjustments to prices for quality differences are made against a well-considered framework. Section B.2 examines quality-adjustment techniques in a national accounting context. Readers interested only in methods of quality adjustment will find them in Sections C through G. Section C provides an overview of the methods available for dealing with unavailable price observations. Methods for quality-adjusting prices are classified into two types: *implicit* and *explicit* adjustments, covered in greater depth in Sections D and E, respectively. Section F considers how to choose among methods of quality adjustment.

7.36 The implicit and explicit adjustment methods are outlined under a standard long-run Laspeyres framework, whereby prices in a base or reference period are compared with those in each subsequent period. However, where products are experiencing rapid technological change, these methods may be unsuitable. The matching and repricing of like products—and patching in of quality-adjusted replacement prices when the matching fails—is appropriate when failures are the exception. But in high-technology product markets likely to experience rapid turnover of models, they are the rule. Section G considers alternative methods using chained or hedonic frameworks to meet the needs of rapidly changing production portfolios. Section H examines frequent resampling as an intermediary, and for imputation a more appropriate, approach. Chapter 22 discusses issues relating to seasonal products in more detail.

B. What Is Meant by Quality Change

B.1 Nature of quality change

7.37 Bodé and van Dalen (2001) undertook an extensive study of the prices of new automobiles in the Netherlands between 1990 and 1999. The average price increase per car over this period was around 20 percent, but the mix of average quality characteristics changed at the same time. For example, the horsepower (HP) of new cars increased on average from 79 to 92 HP; the average efficiency of fuel consumption improved from 9.3 to 8.4 litres/100km; the share of cars with fuel injection went from 51 percent to 91 percent; the share of cars with power steering went from 27 percent to 94 percent; and the share of cars with airbags went from 6 percent to 91 percent. There were similar increases for central locking, tinted glass, and many more features.

7.38 Standard price index practice matches the prices of a sample of models in, for example, January with the same models in subsequent months. This holds the characteristics mix constant to keep quality differences from contaminating the estimate of price change. However, as considered later in this chapter, the resulting sample of matched models (products) is one that gives less weight (if any) to models subsequently introduced. Yet the later models benefit from more recent technological developments and may have different price changes given the quality of services they provide. One approach to correct for such quality changes using the whole sample of both new and existing models is a dummy variable hedonic regression (see Section G.2.1). Bodé and van Dalen (2001), using a variety of formulations of hedonic regressions, found the quality-corrected prices of these new automobiles to be about constant over this period. In this case, the value of the quality improvements explained the entire nominal price increase.

7.39 Recorded changes in prices are the outcome of shifts in both demand and supply. Chapter 21 explains that these shifts arise from a number of sources, including environmental changes; changes in users' technology, tastes, and preferences; and changes in producers' technology. More formally, the observed data on prices are the loci of the intersections of the demand curves of different final users with varying tastes or intermediate users with possibly varying technologies, and the supply curves of different producers with possibly varying technologies. Separately identifying the effects of changes in environment, technology, and tastes and preferences on the spectrum of product characteristics present in markets at any given time is conceptually and empirically difficult. Fortunately, as Bodé and van Dalen and others demonstrate, compilers do not have to separately identify these effects to produce a good price index in the face of quality change. They need only identify their combined impact.

7.40 Our concern is not just with the changing mix of the observed characteristics of products. There is the practical problem of not always being able to observe or quantify characteristics, such as style, reliability, ease of use, and safety. The *System of National Accounts 1993* (*1993 SNA*, Chapter 16) notes factors other than changes in physical characteristics that improve quality. These include

> Transporting a good to a location in which it is in greater demand is a process of production in its own right in which the good is transformed into a higher quality good. [Paragraph 16.107]

The same good provided at a more convenient location may command a higher price and be of higher quality. Further, different times of the day or periods of the year may also give rise to quality differences:

> For example, electricity or transport provided at peak times must be treated as being of higher quality than the same amount of electricity or transport provided at off-peak times. The fact that peaks exist shows that purchasers or users attach greater utility to the services at these times, while the marginal costs of production are usually higher at peak times…. [Paragraph 16.108]

7.41 Other differences, including the conditions of sale and circumstances or environment in which the goods or services are supplied or delivered, can make an important contribution to differences in quality. A producer, for example, may attract customers by providing better delivery; more credit opportunity; more accessibility; shorter order times; smaller, tailor-made orders; better support and advice; or a more pleasant environment. These sorts of benefits may well be price-determining. If so, they belong among the characteristics in the product's structured definition.

7.42 There is a very strong likelihood some price-determining characteristics will be unmeasured in any quality adjustment situation. Compilers cannot produce timely statistics if they are perpetually seeking more characteristics data to produce a still better quality adjustment. How many characteristics data are enough? Characteristics data are sufficient when products are described completely enough. Products are described completely enough when there is low variability of prices over transactions with that description in any given month. If we use characteristics from a structured product description to estimate a hedonic regression model, as did Bodé and van Dalen, the model will fit well only if the structured descriptions are reasonably complete. The first criterion for sufficiency of structured characteristics data, then, is a good fit to a hedonic model. If there is a good fit using a set of objective characteristics, there may be still other characteristics such as style and reliability not yet included in the structured description and thus unmeasured, but they cannot contribute much more to the fit of the model. A second, qualitative criterion is that the included characteristics be meaningful to the participants in the market for the product.

B.2 Conceptual issues

7.43 Recalling Chapter 2, a PPI is an index designed to measure the average change in the price of goods and services as they leave the place of production (output prices at basic values) or as they enter the production process (input prices at purchaser's values). There are PPIs for total output and intermediate input. There are also PPIs for a range of net output concepts, at different levels of aggregation, representing different stages of production: primary products, intermediate goods, and finished goods. Changes over time in the prices of inputs are an indicator of potential inflation, which will, to some degree, feed through to output prices as output inflation. Section B.2.1 discusses the output price index. It focuses on the general quality adjustment problem for output price indices and the restrictive assumptions that have to be maintained to use the often-favored *resource-cost approach* to quality adjustment. The principles relating to an input price index follow in Section B.2.2. It outlines the quality adjustment problem for input price indices and the restrictive assumptions that have to be maintained to use the often-favored *user-value* approach to quality adjustment. The discussion continues in Section B.2.3 with a brief introduction to two problems associated with resource-cost and user-value approaches. The first, in Section B.2.4, occurs when technology substantially changes and fixed-input *output indices* make little sense for valuing higher-quality products produced at much lower unit cost. The second is the reconciliation problem in national accounts at constant prices referred to above, a problem that leads the *Manual* to recommendations on a unified valuation system in Section B.2.5.

B.2.1 Fixed-input output price index

7.44 In this *Manual*, the principal conceptual basis for the output PPI is the fixed-input *output price index* (FIOPI). The output PPI thus aims to measure an output price index constructed on the assumption that inputs and technology are fixed.[4] Chapter 18 defines the FIOPI as a ratio of revenue functions. The revenue function of an establishment expresses the value of its output as a function of the prices it receives and the quantities of inputs required to produce the output. It recognizes that only a finite number of varieties or products are producible at any given time but also grants that for given inputs and technology, there may be a continuum of designs from which producers select this finite number of products. Hence, in response to changes in preferences or the technologies of producers using a given establishment's output, there may be different sets of products produced from period to period from a given set of inputs and technology.

7.45 Compilers and even price index theorists are used to thinking in the narrower framework comparing the prices of exactly the same things from period to period.[5] For example, they would measure a shirtmaker's price change on the assumption that the cutting, sewing, folding, packaging, and so forth were all undertaken in the same way from the same labor, capital, and material inputs in the two periods being compared. If the revenue increased by 5 percent, given that everything else remained the same, then the output price also increased by 5 percent. If such things do not change, then a measure of a pure price change results.

7.46 Even if technology and inputs remain the same, the way things are produced and sold may change. For example, the shirtmaker may start improving the quality of his or her shirts by using extra cloth and more stitching using the same machinery. The *price basis* or product description underlying this comparison has changed within a given technological framework. A direct comparison of successive months of shirt prices includes, in this case, not only the effects on revenue from price changes but also changes in product characteristics and quality. To include the increase in revenue resulting from improved quality would be to misrepresent price change—to bias the index upward. Prices would not, in fact, be rising as fast as indicated by such an unadjusted index.

[4] See Chapter 17, Section B.1, for more on this conceptual framework.

[5] See, for example, Gerduk, Gousen, and Monk (1986).

7.47 A pure price relative for a product fixes the product description or price basis by definition. For the price basis not to change, the product's observable characteristics and the way the product is sold must remain fixed. The FIOPI for an elementary aggregate may evolve because producers adjust revenue shares in response to changes in the relative prices of products. Further, new products that are feasible with the same inputs and technology but were not previously produced may appear and supplant existing products.

7.48 There also will be different levels of inputs in different months since more or less might be produced. In addition, technology may well change over time. Each monthly comparison implicitly involves a new FIOPI relevant to these changed background conditions. As noted in Section A, these last two sources of change also manifest as changes in index weights and an evolution of the specific set of products on which prices are available. This is akin to demand-induced shifts.

7.49 As further stated in Section A, quality change is present when a change in the price basis occurs for given products. It also is present when new products appear. Compilers would like to incorporate information on the characteristics of the new varieties into a given month's price change by making explicit quality-adjusted comparisons with the prices of continuing products. They usually try to use an overlap method (Section A.2.1, approach 3) as the basis for bringing the new item into the sample. If the new item simply appears alongside the existing varieties, overlap prices are readily available from the continuing products, and the compilers choose the most similar of these to the new product as the donor for the overlap price.

7.50 The overlap price may not be available, however, because the product most similar to the new one disappears in the month the new product appears (for example, if both were produced by the same establishment, and the new replaces the old). In this case, the compiler must estimate an overlap price for the old variety in the current period or an overlap price for the new variety in the previous period. The explicit quality adjustment methods (approach 4 in Section A.2.1) aim to estimate these overlap prices.

7.51 A variant of the FIOPI framework underlies the *resource-cost* approach to explicit quality adjustment for output prices. In the resource-cost approach, when quality changes, the compiler asks the establishment representative how much it cost to produce the new product and how much it would have cost to produce the old product in the current period. He or she then divides the price relative between the new and old products by their relative cost. Resource-cost adjustment relies on keeping input prices relative to total cost fixed rather than keeping input quantities fixed when comparing the prices for a given set of products between two periods. This variant of the FIOPI is based on the concept of a ratio of *indirect* revenue functions, so named because they maximize revenue subject to a cost function constraint rather than a production function constraint.[6] While the direct revenue function of the FIOPI increases with inputs, the indirect revenue function increases with total cost. If product characteristics change along with prices, the resource-cost adjustment for the change in quality is the factor that, when used as a multiplier for observed total cost, would produce the same revenue (given the initial set of product characteristics) as the revenue realized through producing the new products in the current period. Thus, if the new good is higher quality, we would expect this cost multiplier to be positive and the cost of producing the old product in the current period to be less than the cost of producing the new product. The cost relative between the two products, therefore, is greater than one and when divided into the price relative between them, lowers the estimate of price change by the percentage value of the quality increase.

B.2.2 Fixed-output input price index and other indices

7.52 This *Manual*'s principal conceptual basis for the input PPI is the fixed-output *input price index* (FOIPI). It is the relative change in cost—the market value of inputs—required to produce a fixed level of output when input prices change between the current period and a base period. Assuming producers minimize the cost of producing output, the input price index is a ratio of cost functions that relates establishment total production cost to establishment outputs and the input prices

[6] The cost function is itself a derivative of the production function. The indirect revenue function reflects the production function, and thus technology, *indirectly* through the cost function.

the establishment pays.[7] The prices of inputs should include all of the amounts purchasers pay per unit of the products they use, including transportation, insurance, wholesale or retail margins, and indirect taxes. Chapter 14 calls these purchasers' prices, following the *1993 SNA*.

7.53 A variant of the FOIPI framework underlies the *user-value* approach to explicit quality adjustment for input prices. User-value adjustment relies conceptually on a variant of the FOIPI. It holds output prices fixed relative to total revenue, rather than holding output quantities fixed, when comparing the prices for a given set of input products between two periods. The variant is based on the concept of a ratio of *indirect* cost functions, so named because they minimize cost subject to a revenue function constraint rather than a production function constraint.[8] While the direct cost function of the FOIPI increases with outputs, the indirect cost function increases with total revenue. If product characteristics change along with prices, the user-value adjustment for the change in quality is the factor that, when multiplied by observed total revenue, would produce the same cost in the current period (given the initial set of product characteristics) as the cost realized using the new products as inputs. Thus, if the new input has higher quality, we would expect this revenue multiplier to be positive and the revenue possible from using the old product in the current period to be less than the revenue realized from using the new product. The revenue relative between the two products, therefore, is greater than one and when divided into the input price relative between the two input products, lowers the estimate of their price change by the percentage value of the quality increase.

7.54 Triplett (1990, pp. 222–23) summarizes the history of thought on the resource cost and user-value methods of quality adjustment:

> Fisher and Shell (1972) were the first to show that different index number measurements (they considered output price indexes and consumer price indexes) imply alternative treatments of quality change, and that the theoretically appropriate treatments of quality change for these two indexes correspond respectively, to "resource-cost" and "user-value" measures. Triplett (1983) derives this same result for cases where "quality change" is identified with characteristics of goods—and therefore with empirical hedonic methods [discussed later]; the conclusions are that the resource-cost of a characteristic is the appropriate quality adjustment for the output price index, and its user-value is the quality adjustment for the COL [cost of living] index or input index.
>
> Intuitively, these conclusions are appealing. The output index is defined on a fixed value of a transformation function. The position of a transformation function, technology constant, depends on resources employed in production; accordingly, "constant quality" for this index implies holding resources constant, or a resource-cost criterion.
>
> On the other hand, the COL index is defined on a fixed indifference curve, and the analogous input-cost index is defined on a fixed (user) production isoquant. For these two "input" price indexes, "constant-quality" implies holding utility or output constant, or a user-value criterion….

Writers in economic statistics often have associated the term *user-value approach* with the so-called hedonic method introduced in Section A.2 and discussed further in Section G. This *Manual* draws a distinction between the two. Here, the user-value method is the exact input price index analog to resource-cost adjustment of the output price index. The hedonic method is based on a summarized form of supply-demand equilibria in the market, rather than, as shown in this chapter, a set of potentially restrictive assumptions about how technology works.

B.2.3 A problem with these concepts and their use

7.55 The academic literature as outlined above has recognized the FIOPI as the appropriate basis for the output PPI and the FOIPI as the basis for the input PPI. This has led to the adoption of the resource-cost approach as a preferred method for explicit quality adjustment for the output PPI and user value for the input PPI.

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[7] See Chapter 17, Section C, for more on this conceptual framework.

[8] The revenue function is itself a derivative of the production function. The indirect cost function reflects the production function, and thus technology, *indirectly* through the revenue function.

7.56 As shown in Section B.2.1, the resource-cost method has a microeconomic rationale within the indirect revenue framework for quality-adjusted output price measurement. However, the correctness of dividing a price relative by a resource-cost ratio for a given product makes two potentially restrictive assumptions. The production process for the product whose price is adjusted must be *separable* from the process for the rest of the outputs of an establishment, and the *returns to scale* of that process must be constant and equal to one.[9] These assumptions would be difficult to confirm were the data available to empirically test them (these data usually are not available to compilers). We would prefer to use methods not requiring such assumptions, such as the (observed) overlap price and the hedonic methods, if it is feasible.

7.57 As shown in Section B.2.2, the user-value method also has a microeconomic rationale within the indirect cost framework for quality-adjusted input price measurement. However, the correctness of dividing a price relative by a user-value ratio for a given product requires two potentially restrictive assumptions. The input requirements for the item whose price is adjusted must be separable from the requirements for the rest of the inputs an establishment uses, and the returns to scale of that process must be constant and equal to one. These assumptions would be difficult to confirm were the data available to empirically test them (these data usually are not available to compilers). We again would prefer to use methods not requiring such assumptions, such as the (observed) overlap price and the hedonic methods, if it is feasible.

B.2.4 When technology changes

7.58 The problems with traditional resource-cost and user-value approaches to explicit quality adjustment compound in the presence of technical (and taste) change. Throughout the earlier sections, this chapter has noted the similarity of effects on PPIs between changes in relative price, preferences, technology use, and supplying technology. Broadly speaking, all affect the assortment of products available at any given time and the relative importance of the products in the subset of that assortment persisting from period to period. As noted in Chapter 15, however, changes in weights arising from suppliers' and users' responses to relative price changes given fixed technology and preferences have predictable outcomes. They are the foundation for well-known theorems on the downward bias of Laspeyres price indices and the upward bias of Paasche price indices for output price indices, and the upward bias of Laspeyres price indices and the downward bias of the Paasche price indices for input price indices. Normally, considering substitution effects alone leads to the standard expectation that the Laspeyres output price index will lie below the Paasche output price index, and the Laspeyres input price index will lie above the Paasche input price index.

7.59 The price and output or input share data compilers observe from the economy reflect changes in relative prices, technology, and tastes simultaneously. Thus, changes in the relative importance of products, including their emergence and disappearance, can be unpredictable. Technology change can augment the substitution effects from relative price change, or it can more than offset substitution effects. As a result, the Laspeyres output price index may lie above the Paasche output price index and the Laspeyres input price index below the Paasche input price index in any given period-to-period comparison.

7.60 Regarding the resource-cost method, an establishment representative can find it problematic to assess the cost of changes in the price basis of an output good or service arising partly or wholly from a change in production technology. Much of the cost of the improved reliability, efficiency, design, flexibility, durability, and other output characteristics is difficult to measure. Moreover, the changes in technology that generate the improved characteristics include changes in plant and machinery, quality monitoring, inventory control, labor requirements, work organization, material types, packaging, and selling techniques. All of these are difficult to measure in terms of the

[9] See Chapter 21, Section B.6, on the resource-cost decomposition of the relative change in revenue when both prices and product characteristics change. Separability implies, for practical purposes, that any particular product whose quality has changed must have its own production process unaffected by the production of other, more or less similar product varieties. Constant returns to scale reinforce this restriction by implying that the output of a product may be increased by any given proportion by increasing inputs by the same proportion, without regard to the production of other distinct, more or less similar product varieties.

simple costing referred to above. The new technologies in high-technology products require new methods of production. These production technologies may change, possibly more than once during a year. Determining the cost of a previous variety produced under the current production process or the cost of the current variety under the old process may be conceptually appropriate but practically impossible. Yet answering the cost question without assuming that technology is fixed in the current or previous generation can produce wildly inaccurate results. Consider the market for personal computers, where price declines have been accompanied by rapid quality improvements.

7.61 Holdway (1999) illustrated the problem of using a FIOPI for computer microprocessors such as the Intel Pentium III. He considered changes in the speed of new generations of microprocessors and used the example of the transition from a 66 megahertz (MHz) chip that cost $230 when it was discontinued to a 90 MHz chip valued at $247 in the same month. The additional cost of the 24 MHz at that month's technology's resource costs has to be estimated. Suppose the cost of a single unit of MHz was estimated at $2.0833; multiplying this figure by 24 yields $50. So what is the pure price difference between these two chips? To make the new 90 MHz processor equivalent to the old 66 MHz one, the $50 has to be subtracted from its price and compared with the price of the old one; that is, [(247 – 50) / 230] – 1 = –0.143: a 14.3 percent decrease. This is instead of a nominal price increase of [(247 / 230) – 1] = 0.074 or 7.4 percent.

7.62 Suppose, however, the establishment reports the unit cost of the 66 MHz unit at the technology prevailing when the older, slower unit was designed rather than the unit cost of a 66 MHz unit from the newer technology underlying the 90 MHz chip. In this case, it is very easy to misapply the resource-cost method by not comparing costs within a given generation of production technology. The new 90 MHz processors were built using a better technology. They used 0.50 instead of 0.80 micron technology, allowing more features to be packed into a smaller section of a silicon wafer, which improved performance. Also, the technology used to produce them, including an amortization factor for plant and capital equipment, lowered unit costs (see Holdway, 1999, for details). Suppose an estimate was requested as to how much more it would cost to produce a 90 MHz chip versus a 66 MHz one, maintaining that the cost assessment should assume the 66 MHz wafer technology. Suppose unit costs for the higher-performance chip were $100 more because the old technology was less efficient than the new technology, a common occurrence in high-technology industries. Applying the resource-cost method now provides an estimate of (247) / (230 + 100) – 1 = –0.252, a 25.2 percent decrease. The higher unit cost of the faster chip had to be added back to make it equivalent to the new chip because the resource-cost method measures quality by cost.

7.63 In the latter cases, the method breaks down. The unadjusted price increase was 7.4 percent. With a resource-cost adjustment using estimates based on the new technology, there was a decline of 14.3 percent. Adjusting the prices based on estimates using the old technology to produce the new, higher-performing chip results in a decrease of 25.2 percent. In both cases, the cost declines represent different levels of technology, and the resource-cost approach can give widely different answers. In industries such as computers and electronics, where unit prices are falling and technology is rapidly changing, resource-cost quality adjustment procedures can be misleading as major technology shifts occur.

7.64 PPIs cannot hold the price basis constant over long periods. For example, in the 45 years since the introduction of the commercial computer, the price of computing power has decreased to less than one-half of one-tenth of 1 percent (0.0005) of what it was at its introduction. It has decreased by more than two thousandfold (Triplett, 1999). Nordhaus (1997) found substantial increases in the price of light over much longer periods. Yet if these price changes reflected overall changes in producer prices, absurd estimates of output growth at constant prices would result. The tastes and expectations of consumers along with the technology of the producers change over time, and these changes will be shown in Chapter 21 to affect the implicit prices attributed to the quality characteristics of what is bought and sold.

7.65 Because of the effects of changes in relative price, technology, and taste, we again would prefer to use the (observed) overlap price and the hedonic methods—where feasible—rather than the resource-cost and user-value approaches. Further, rapid technological and taste changes must also be met by more frequent sample updates to avoid rapid loss of sample relevance.

B.2.5 Consistency between supply and use price statistics: assessing product quality at supply rather than use values

7.66 From Section A.1, for price index compilers, a product fundamentally is a structured description of goods and services detailed enough so that there is little variation at a point in time in the prices of goods and services with that description. From Section B.1, minimal variation within a product description equates in the hedonic model, first introduced in Section A.2.1 as an explicit quality adjustment method, to a close fit of a regression of prices on measured characteristics. The fit of the hedonic regression thus is a measure of the sufficiency of the structured product description given by the right-hand side variables (product characteristics).

7.67 With the latter observation in hand, the natural conclusion is that a close regression fit for a given set of product descriptions will be achieved more easily with the left-hand side expressed in supply (basic) prices rather than use (purchasers') prices. The prices of domestic production and imports are basic prices, that is, what the supplier receives. They exclude separately invoiced transport and distribution charges, include subsidies on products, and exclude taxes on products. The prices of uses are purchasers' prices, what the user pays. They include the margins and taxes on products excluded by basic prices and exclude subsidies on products. There can be variations in taxes and subsidies on products unrelated to goods and services flows or to the characteristics of goods and services having value to users and a cost of production. There also can be variations in the transport and distribution services included with goods to deliver them to their users that must be accounted for in explaining variations in purchasers' prices, as noted in the quote from the *1993 SNA* in Section B.1. Distance between the producer and the user is an obvious driver of transport costs, for example. It would be most straightforward, therefore, to assess price changes for these services directly when supplied, rather than embedding them in prices of goods delivered to users.

7.68 Further, quality assessments must be consistent throughout the supply and use accounts for goods and services. As discussed at length in Chapter 14, the PPI covers aggregates in the production subaccount of the national accounts. The production account is an important component of the supply and use table balancing the sources of goods and services supply in the current period with the uses of those goods and services. The sources of supply are domestic production—the output PPI value aggregate—and imports, plus adjustments for transport and distribution services to get goods to their users and taxes and subsidies on products. The uses of goods and services are intermediate consumption—the input PPI's value aggregate—as well as final consumption, capital formation, and exports. Each good or service product, therefore, has its own row in the matrix of supply and use, whose columns are the aforementioned components of supply and use. Even at this highest level of detail, the supply of every distinct good or service, adjusted for transport and distribution margins and taxes, must balance its uses. This will be true both in value and volume terms.

7.69 Because every transaction cannot be tracked, however, supply and use tables cannot be produced at the level of elementary items. It is feasible to track supply and use only at the level of elementary aggregates, basic headings, or even higher-level aggregates of goods and services. Thus, each row of such a supply and use table necessarily contains some quality heterogeneity, and we can speak of it only in average terms. Changes in the total supply and total uses of these detailed goods and services aggregates comprise four parts. There are (i) average quality changes, (ii) changes in basic prices, (iii) changes in taxes and subsidies on products, and (iv) average quantity changes of the elementary products comprising the aggregate. Volume change for an aggregate is an amalgam of quality and quantity changes. Clearly, adjusting price change to eliminate the effects of changes in quality is important here, lest volume be understated or overstated by the amount of quality change erroneously ascribed to price change. The context also highlights the need to have a single valuation of quality change, not one from the supply side (output PPI quality adjustments) and one from the uses side (CPI and other uses price index quality adjustments). Thus, basic price valuations should be used both for supply and use quality adjustments if the supply and use accounts are to balance in both value and volume terms.[10]

B.2.6 Summary

7.70 A number of points emerge:

(i) Data availability will dictate which of the four approaches to quality adjustment—imputation, comparable substitution, overlap price, and explicit adjustment—are used in practice.
(ii) The *Manual* distinguishes between user-value and hedonic (Section E.4) methods of explicit quality adjustment. The user-value method of explicit quality adjustment for input price indices is the logical analog to the resource-cost method for output price indices and generally is not equivalent to the hedonic method (see Chapter 21).
(iii) The *Manual* broadly prefers overlap price, hedonic, and, when there are similar price trends for new products as compared with old, imputation methods of quality adjustment; these approaches do not require special assumptions about technology.
(iv) The *Manual* recognizes that statistical offices will still find the traditional resource-cost technique to their first choice among the second-best methods for making quality adjustment to output price indices. This occurs when information is too limited to do overlap price or hedonic quality adjustment, or the quality level is thought to affect the rate of price change, thus excluding the imputation approach. Resource cost nevertheless requires care when applied to industries with falling unit costs and improved quality of output or varying profit margins.
(v) When resource cost (user value) is the best available technique, it should be applied to output (input) price indices, ensuring consistency with the method's microeconomic foundations.
(vi) The *Manual* advises that quality adjustment methods should use basic price valuations, rather than a mixture of basic prices for supply aggregates (output PPIs) and purchasers' prices for uses aggregates (input PPIs and the CPI) to maintain consistency between supply and use volume measures.

C. An Introduction to Methods of Quality Adjustment When Matched Items Are Unavailable

C.1 Introduction

7.71 It may be apparent from the preceding text that quality adjustments to prices are not going to be a simple issue or involve routine mechanical methods whereby given methodologies are applied to prices in specified industries to yield adjustments. A number of alternative approaches will be suggested, and some will be more appropriate than others for specific items regardless of their industrial group. An understanding of the technological features of the producing industry, the product market, and alternative data sources will be required for the successful implementation of a quality adjustment program. Specific attention must be devoted to product areas with relatively high weights and where large proportions of products are turned over. Some of the methods are not straightforward and require some expertise, although methods learned and used on some products may be applicable elsewhere. The issue of quality adjustment is met by developing a gradual approach on an industry-by-industry basis. It is emphasized that such concerns should not be used as reasons to obviate the estimation of quality-adjusted prices. The practice of statistical agencies in dealing with missing products, even if it is to ignore them, implicitly involves a quality adjustment, and the form of the implicit one undertaken may not be the most appropriate one and may even be misleading. The extent of quality changes and the pace of technological change require that appropriate methods be used.

7.72 To measure aggregate price changes, a representative sample of products are selected from a sample of firms along with a host of details that define each *price*, including details on the

[10] Our assertion that supply and use aggregates must balance in volume terms, just like the supply and use of elementary items, abstracts from nonproportional taxes and subsidies on products. Unlike quality differences among goods and services over time, nonproportional changes in taxes and subsidies on products seem to have unequal volume implications for goods and services aggregates between suppliers and users. This is beyond the subject of this *Manual* but deserves further research and elucidation elsewhere in work on price and volume measurement for the national accounts.

conditions of the sale where relevant. This is to establish an insight into the price basis of the product. This is then followed by a periodic survey for which the firms report prices (reprice the product) each month for these selected products. They do so to the same specifications, that is, on the same price basis. The detailed specifications are included on the repricing form each month as a prompt to ensure that the price basis has remained the same. Respondents must be aware of the need to report the details of any change in the price basis; confusion may lead to biased results. It must be borne in mind that firms have no incentive to report such changes since this will invariably involve additional work in costing the change. Attention should also be devoted to ensuring that the description of the price basis contains all pertinent, price-determining elements. If an element is excluded, any change is much less likely to be reported. In both of these cases, the quality change would be invisible to the price measurement process.

C.2 Methods for making quality adjustments

7.73 When a product is missing in a month for reasons other than being off-season or off-cycle, the replacement may be of a different quality—the price basis may have changed, and one may no longer be comparing like with like. A number of approaches exist for dealing with such situations and are well documented for the CPI, as outlined in Turvey and others (1989); Moulton and Moses (1997); Armknecht, Lane, and Stewart (1997); Moulton, LaFleur, and Moses (1998); and Triplett (2002). Though the terms differ among authors and statistical agencies, they include

- *Imputation*—When no information is available to allow reasonable estimates to be made of the effect on price of a quality change. The price change of all products—or of more or less similar products—are assumed to be the same as that for the missing product.

- *Overlap*—Used when no information is available to allow reasonable estimates to be made of the effect on price of a quality change but a replacement product exists in the same period as the old product. The price difference between the old product and its replacement in the same overlap period is then used as a measure of the quality difference.

- *Direct comparison*—If another product is directly comparable, that is, so similar it has more or less the same quality characteristics as the missing one, its price replaces the unavailable price. Any difference in price level between the new and old is assumed to be because of price changes and not quality differences.

- *Explicit quality adjustment*—When there is a substantial difference in the quality of the old and replacement products, estimates of the effect of quality differences on prices are made to enable quality-adjusted price comparisons.

7.74 Before outlining and evaluating these methods, one should say something about the extent of the problem. This situation arises when the product is unavailable. It is not just a problem when *comparable* products are unavailable, for the judgment as to what is and what is not comparable itself requires an estimate of quality differences. Part of the purpose of a statistical meta-information system for statistical offices (outlined in Chapter 8) is to identify and monitor the sectors that are prone to such replacements and determine whether the replacements used really are comparable.

7.75 Quality adjustment methods for prices are generally classified into the implicit or imputed (indirect) methods explained in Section D (the differences in terminology are notorious in this area) and explicit (direct) methods explained in Section E. Both decompose the price change between the old product and its replacement into quality and pure price changes. However, in the latter, an explicit estimate is made of the quality difference, usually on the basis of external information. The pure price effect is identified as a remainder. For implicit adjustments, a measurement technique is used to compare the old product with the replacement, so that the extent of the quality and pure price change is implicitly determined by the assumptions of the method. The accuracy of the method relies on the veracity of the assumptions, not the quality of the explicit estimate. In Sections D and E, the following methods are considered in detail:

Implicit methods:
- Overlap;
- Overall mean/targeted mean imputation;
- Class mean imputation;
- Comparable replacement;
- Linked to show no price change; and
- Carryforward.

Explicit methods:
- Expert judgment;
- Quantity adjustment;
- Differences in production/option costs; and
- Hedonic approach.

C.3 Some points

C.3.1 Additive versus multiplicative

7.76 The quality adjustments to prices may be undertaken by either adding a fixed amount or multiplying by a ratio. For example, where *m* is the old product and *n* its replacement for a comparison over periods *t, t* + 1*, t* + 2, the use of the overlap method in period *t* + 1 required the ratio p_n^{t+1} / p_m^{t+1} to be used as a measure of the relative quality difference between the old item and its replacement. This ratio could then be *multiplied* by the price of the old item in period *t*, p_m^t to obtain the quality-adjusted prices p_m^{*t} shown in Table 7.1. Such multiplicative formulations are generally advised because the adjustment is invariant to the absolute value of the price. It would be otherwise possible for the absolute value of the change in specifications to exceed the value of the product in some earlier or—with technological advances—later period. Yet for some products, the worth of the constituent parts is not in proportion to their price. Instead, they have their own intrinsic, absolute, additive worth, which remains constant over time. Producers selling over the Internet may, for example, include postage, which in some instances may remain the same irrespective of what is happening to price. If postage is subsequently excluded from the price, the fall in quality should be valued as a fixed sum.

Table 7.1. Estimating a Quality-Adjusted Price

	t	$t + 1$	$t + 2$
old item *m*		p_m^{t+1}	
replacement *n*	p_m^{*t}	p_n^{t+1}	p_n^{t+2}

C.3.2 Base- versus current-period adjustment

7.77 Two variants of the approaches to quality adjustment outlined in Section C.2 are to either make the adjustment to the price in the base period or to make the adjustment to the price in the current period. For example, in the overlap method described above, the implicit quality adjustment coefficient was used to adjust p_m^t. An alternative procedure would have been to multiply the ratio p_m^{t+1} / p_n^{t+1} by the prices of the replacement product p_n^{t+2} to obtain the quality-adjusted prices p_n^{*t+2} etc. The first approach is easier since once the base-period price has been adjusted, no subsequent adjustments are required. Each new replacement price can be compared with that of the adjusted base period. For multiplicative adjustments, the end result is the same whichever approach is used. For additive adjustments, the results differ. It is more appropriate to make the adjustment to prices near the overlap period.

C.3.3 Long-run versus short-run comparisons

7.78 Much of the analysis of quality adjustments in this *Manual* has been undertaken by comparing prices between two periods (for example, periods 0 and 1). For long-run comparisons, suppose the base period is taken as period *t* and the index is compiled by comparing prices in period *t* first with *t* + 1, then with *t* + 2, then with *t* + 3, etc. The short-run framework allows long-run comparisons—say, between periods *t* and *t* + 3—to be built as a sequence of links joined by successive multiplication—say, period *t* with *t* + 2 and period *t* + 2 with *t* + 3. This can also be done by chaining period *t* with *t* + 1, *t* + 1 with *t* + 2, and *t* + 2 with *t* + 3. In Section H, the advantages of the short-run framework for imputations are outlined. In Section G.3, chained indices are considered for industries experiencing a rapid turnover in products. These quality adjustment methods are now examined in turn, and in Section F, the choice of method is discussed.

C.3.4 Statistical metadata

7.79 In Sections D and E, implicit and explicit methods of quality adjustments to prices are discussed. In Section F, the choice between these methods is examined. Any consideration of the veracity of these methods, resource implications, and the choice between them needs to be informed by appropriate information on an industry-by-industry basis. Section C of Chapter 8 considers information requirements for a strategy for such quality adjustment in the face of a statistical metadata system.

D. Implicit Methods

D.1 Overlap method

7.80 Consider an example where the items are sampled in January and prices are compared over the remaining months of the year. Matched comparisons are undertaken between the January prices and their counterparts in successive months. Five products are assumed to be sold in January with prices p_1^1, p_2^1, p_5^1, p_6^1, and p_8^1 (Table 7.2, part a). Two types of similar products are produced in the industrial group concerned, A and B. An index of the elementary level is required for the overall price change of these two product types. At this level of aggregation, the weights can be ignored assuming only one quote is taken on each product. A price index for February compared with January = 100.0 is straightforward in that prices of products 1, 2, 5, 6, and 8 are used and compared only by way of the geometric mean of price ratios, known as the Jevons index (which is equivalent to the ratio of the geometric mean in February over the geometric mean in January—see Chapter 20). In March, the prices for products 2 and 6—one of type A and one of type B—are missing.

7.81 In Table 7.2, the lower part (b) is a numerical counterpart of the upper part (a), further illustrating the calculations. The overlap method requires prices of the old and replacement products to be available in the same period. In Table 7.2(a), product 2 has no price quote for March. Its new replacement is, for example, product 4. The overlap method simply measures the ratio of the prices of the old and replacement products in an overlap period. In this example, the period is February, and the old and replacement products are products 2 and 4, respectively. This is taken to be an indicator of their quality differences. The two approaches outlined in Section C.3.2 are apparent: either to insert a quality-adjusted price in January for product 4 and continue to use the replacement product 4 series, or continue the product 2 series by patching in quality-adjusted product 4 prices. Both yield the same answer. Consider the former. For a Jevons geometric mean from January to March *for establishment A only*, assuming equal weights of unity

$$(7.1)\quad P_J(p^1, p^3) = \left[p_1^3 / p_1^1 \times p_4^3 / \left(\left(p_4^2 / p_2^2 \right) \times p_2^1 \right) \right]^{1/2}$$

$$= [\, 6/4 \times 8/ ((7.5 / 6) \times 5)]^{1/2}$$

$$= 1.386.$$

7.82 Note that the comparisons are long-run ones, that is, they are between January and the month in question. The short-run modified Laspeyres framework provides a basis for short-run changes based on data in each current month and the immediately preceding one. In Table 7.2(a) and (b), the comparison for product type A would first be undertaken between January and February using products 1 and 2. The result would be multiplied by the comparison between February and March using items 1 and 4. Still, this implicitly uses the differences in prices in the overlap in February between items 2 and 4 as a measure of this quality difference. It yields the same result as before:

$$\left[\frac{5}{4} \times \frac{6}{5}\right]^{\frac{1}{2}} \times \left[\frac{6}{5} \times \frac{8}{7.5}\right]^{\frac{1}{2}} = 1.386$$

The advantage of recording price changes for January to October in terms of January to September and September to October is that it allows the compiler to compare immediate month-on-month price changes for data editing purposes. Moreover, it has quite specific advantages for the use of imputations as discussed in Sections D.2 and D.3 for which different results arise for the long- and short-run methods. A fuller discussion of the long-run and short-run frameworks is undertaken in Section H.

Table 7.2. Example of Overlap Method of Quality Adjustment

(a) General Illustration

Product Type	Item	January	February	March	April
A	1	p_1^1	p_1^2	p_1^3	p_1^4
	2	p_2^1	p_2^2		
	3			p_3^3	p_3^4
	4		p_4^2	p_4^3	p_4^4
B	5	p_5^1	p_5^2	p_5^3	p_5^4
	6	p_6^1	p_6^2		
	7			p_7^3	p_7^4
	8	p_8^1	p_8^2	p_8^3	p_8^4

(b) Numerical Illustration

Product Type	Item	January	February	March
A	1	4.00	5.00	6.00
	2	5.00	6.00	
	2. overlap			**6.90**
	2. imputation			**6.56**
	2. targeted imputation			**7.20**
	2. comparable replacement			**6.50**
	3			6.50
	4		7.50	8.00
B	5	10.00	11.00	12.00
	6	12.00	12.00	
	6. imputation			**13.13**
	6. targeted imputation			**12.53**
	7			14.00
	8	10.00	10.00	10.00

7.83 The method is only as good as the validity of its underlying assumptions. Consider $i = 1 \ldots m$ products, where p_m^t is the price of product m in period t, p_n^{t+1} is the price of a replacement product n in period $t + 1$, and there are overlap prices for both products in period t. Now item n replaces m but is of a different quality. So let $A(z)$ be the quality adjustment to p_n^{t+1}, which equates its quality to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A\left(z^{t+1}\right) p_n^{t+1}$. Put simply, the index for the product in question over the period $t - 1$ to $t + 1$ is

$$(7.2)\quad I^{t-1,t+1} = \left(p_m^t / p_m^{t-1}\right) \times \left(p_n^{t+1} / p_n^t\right)$$
$$= \frac{p_n^{t+1}}{p_m^{t-1}} \times \frac{p_m^t}{p_n^t}.$$

7.84 The quality adjustment to prices in period $t + 1$ is defined as before, $p_m^{*t+1} = A(z^{t+1})\, p_n^{t+1}$, which is the adjustment to p_n in period $t + 1$, which equates its value to p_m in period $t + 1$ (had it existed then). A desired measure of price changes between periods $t - 1$ and $t + 1$ is thus:

(7.3) $\left(p_m^{*t+1} / p_m^{t-1}\right)$.

The overlap formulation equals this when

$$\frac{p_m^{*t+1}}{p_m^{*t-1}} = A\left(z^{t+1}\right)\frac{p_m^{t+1}}{p_m^{t-1}} = \frac{p_m^{t+1}}{p_m^{t}} \times \frac{p_m^{t}}{p_m^{t-1}}$$

$A\left(z^{t+1}\right) = \dfrac{p_m^t}{p_n^t}$ and similarly for future periods of the series

(7.4) $A\left(z^{t+1}\right) = \dfrac{p_m^t}{p_n^t}$ for $\dfrac{p_m^{*t+i}}{p_m^{t-1}}$ for $i = 2,...T$.

The assumption is that the quality difference in any period equates to the price difference at the *time of the splice*. The *timing* of the switch from *m* to *n* is thus crucial. Unfortunately, respondents usually hang on to a product so that the switch may take place at an unusual period of pricing, near the end of item *m*'s life cycle and the start of item *n*'s life cycle.

7.85 But what if the assumption does not hold? What if the relative prices in period *t*, $R^t = p_m^t \big/ p_n^t$ do not equal *A(z)* in some future period, say $A\left(z^{t+i}\right) = \alpha_i \;\; R^t$? If $\alpha_i = \alpha$, the comparisons of prices between future successive periods—between $t + 3$ and $t + 4$—are unaffected, as would be expected, since product *n* is effectively being compared with itself.

(7.5) $\dfrac{p_m^{*t+4}}{p_m^{*t-1}} \bigg/ \dfrac{p_m^{*t+3}}{p_m^{*t-1}} = \dfrac{\alpha R^t}{\alpha R^t}\,\dfrac{p_n^{*t+4}}{p_n^{*t+3}} = \dfrac{p_m^{*t+1}}{p_m^{*t-1}}$.

However, if differences in the relative prices of the old and replacement products vary over time, then

(7.6) $\dfrac{p_m^{*t+4}}{p_m^{*t-1}} \bigg/ \dfrac{p_m^{*t+3}}{p_m^{*t-1}} = \dfrac{\alpha_4}{\alpha_3}\,\dfrac{p_n^{*t+4}}{p_n^{*t+3}}$.

Note that the quality difference here is not related to the technical specifications or resource costs but to the relative price purchasers pay.

7.86 Relative prices may also reflect unusual pricing policies aimed at minority segments of the market. In the example of pharmaceutical drugs (Berndt, Ling, and Kyle, 2003), the overlapping prices of a generic and a name brand product were argued to be reflective of the needs of two different market segments. The overlap method can be used with a judicious choice of the overlap period. It should be a period before the use of the replacement, since in such periods the pricing may reflect a strategy to dump the old model to make way for the new one.

7.87 The overlap method is implicitly employed when samples of products are rotated, meaning that the old sample of products is used to compute the category index price change between periods $t - 1$ and *t*, and the new sample is used between *t* and $t + 1$. The splicing together of these index movements is justified by the assumption that—on a group-to-group rather than product-to-item level—differences in price levels at a common point in time accurately reflect differences in qualities.

7.88 The overlap method has at its roots a basis in the law of one price. The law states that when a price difference is observed, it must be the result of some difference in physical quality or some such factor for which consumers are willing to pay a premium, such as the timing of the sale, location, convenience, or conditions. Economic theory would dictate that such price difference would not persist given markets made up of rational producers and consumers. However, *1993 SNA* (Chapter 16) notes three reasons why this might fail:

> First, purchasers may not be properly informed about existing price differences and may therefore inadvertently buy at higher prices. While they may be expected to search out for the lowest prices, costs are incurred in the process.
>
> Secondly, purchasers may not be free to choose the price at which they purchase because the seller may be in a position to charge different prices to different categories of purchasers for identical goods and services sold under exactly the same circumstances—in other words, to practise price discrimination.
>
> Thirdly, buyers may be unable to buy as much as they would like at a lower price because there is insufficient supply available at that price. This

situation typically occurs when there are two parallel markets. There may be a primary, or official, market in which the quantities sold, and the prices at which they are sold are subject to government or official control, while there may be a secondary market—a free market or unofficial market—whose existence may or may not be recognized officially.

7.89 There is extensive literature in economics dealing with theory and evidence of price dispersion and its persistence, even when quality differences have been accounted for. The differences can be substantial: Yoskowitz's (2002) study for raw water found one supplier discriminating against a private customer, charging \$500 per acre foot (AF) while a municipality was charged \$20 per AF, though there was some evidence of arbitrage and learning. It is not the role of this *Manual* to examine such theories and evidence, so readers are referred to the following studies: Stigler (1961) and Lach (2002) on search cost theory; Sheshinski and Weiss (1977) and Ball and Mankiw (1994) on menu cost theory; and Friedman (1977) and Silver and Ioannidis (2001) on signal extraction models.

D.2 Overall mean/targeted mean Imputation

7.90 This method uses the price changes of other products as estimates of the price changes of the missing products. Consider a Jevons elementary price index, that is, a geometric mean of price relatives (Chapter 20). The prices of the missing items in the current period, say, $t + 1$, are imputed by multiplying their prices in the immediately preceding period t by the geometric mean of the price relatives of the remaining matched items between these two periods. The comparison is then linked by multiplication to the price changes for previous periods. It is the computationally most straightforward of methods, since the estimate can be undertaken by simply dropping the items that are missing from both periods from the calculation. In practice, the series is continued by including in the database the imputed prices. It is based on the assumption of similar price movements. A targeted form of the method would use similar price movements of a cell or elementary aggregate of similar products, or be based on price changes at a higher level of aggregation if either the lower level had an insufficient sample size or price changes at the higher level were judged to be more representative of the price changes of the missing product.

7.91 In the example in Table 7.2(b), the January to February comparison for both product types is based on products 1, 2, 5, 6, and 8. For March compared with January—weights all equal to unity—the product 2 and product 6 prices are imputed using the short-run price change for February (p^2) compared with March (p^3) based on products 1, 5, and 8. Since different formulas are used for elementary aggregation, the calculation for the three main formulas are illustrated here (see Chapter 20 for choice of formulas). The geometric mean of the price ratios—the Jevons index—is

$$(7.7)\quad P_J\left(p^2,p^3\right)=\left[\prod_{i=1}^{n}p_i^3/p_i^2\right]^{1/3}$$
$$=\left[\left(p_1^3/p_1^2\right)\times\left(p_5^3/p_5^2\right)\times\left(p_8^3/p_8^2\right)\right]^{1/3}$$
$$=\left[\left(6/5\right)\times\left(12/11\right)\times\left(10/10\right)\right]^{1/3}$$
$= 1.0939$, or a 9.39 percent increase.

The ratio of average (mean) prices—the Dutot index—is

$$(7.8)\quad P_D(p^2,p^3)=\sum_{i=1}^{N}p_i^3/N\Big/\sum_{i=1}^{N}p_i^2/N$$
$$=\left(p_1^3+p_5^3+p_8^3\right)/3\ \div\ \left(p_1^2+p_5^2+p_8^2\right)/3$$
$= (6 + 12 + 10) / (5 + 11 + 10) = 1.0769,$

or a 7.69 percent increase.

The average (mean) of price ratios—the Carli index—is:

$$(7.9)\quad P_C(P^3,P^2)=\sum_{n=1}^{N}(p_n^3/p_n^2)/N$$
$$=\left[\left(p_1^3/p_1^2\right)+(p_5^3/p_5^2)+(p_8^3/p_8^2)\right]/3$$
$= [(6/5 + 12/11 + 10/10)] / 3 = 1.09697,$

or a 9.697 percent increase.

7.92 In practice, the imputed figure would be entered onto the data sheet. Table 7.2(b) has the overall mean imputation in March for products 2 and 6, using the Jevons index, as 1.0939 x 6 = 6.563 and 1.0939 x 12 = 13.127, respectively (bold type). It should be noted that the Dutot index is in this instance lower that the Jevons index, a result not expected from the relationships established in Chapter 20. The relationship in Chapter 20 assumed the variance in prices would increase over time, whereas in Table 7.2(b), it decreases for the three products. The arithmetic mean of price relatives—the Carli index—equally weights each price change, but the ratio of arithmetic means—the Dutot index—weights price changes according to the prices of the product in the base period relative to the sum of the base-period prices. Item 1 has a relatively low price, and thus weight, in the base period 1 of 4, but this product has the highest price increase, one of 6/5. Therefore, the Dutot index is lower than the Carli index.

7.93 As noted above, it is also possible to refine the imputation method by targeting the imputation: including the weight for the unavailable products in groupings likely to experience similar price changes—say, by product type, industry, and geographical region. Any stratification system used in the selection of establishments would facilitate this. For example, in Table 7.2(b) assume that the price change of the missing product 2 in March is more likely to follow price changes of product 1, and product 6 is more likely to experience price changes similar to products 5 and 8. For March compared with February, with weights all equal to unity, the geometric mean of price ratios (Jevons) is

$$(7.10)\ \ P_J(p^2,p^3)=\prod_{n=1}^{N}(p_n^3/p_n^2)^{1/N}$$
$$=\left[\left(p_1^3/p_1^2\right)^2\times\left(p_5^3/p_5^2\times p_8^3/p_8^2\right)^{3/2}\right]^{1/5}$$
$$=\left[\left(6/5\right)^2\times\left(12/11\times 10/10\right)^{3/2}\right]^{1/5}$$
$$=1.1041.$$

Note the weights used: for product type A, the single price represents 2 prices; for product type B, the prices represent three or 3/2 = 1.5 each.

7.94 The ratio of average (mean) prices—the Dutot index—is

$$(7.11)\ \ P_D(p^2,p^3)=(\sum_{n=1}^{N}p_n^3/N)/(\sum_{n=1}^{N}p_n^2/N)$$
$$=\left[\frac{\left(2p_1^3+1.5p_5^3+1.5p_8^3\right)/5}{\left(2p_1^2+1.5p_5^2+1.5p_8^2\right)/5}\right]$$
$$=\left[\frac{\left(2\times 6\ +\ 1.5\times 12\ +\ 1.5\times 10\right)/5}{\left(2\times 5\ +\ 1.5\times 11\ +\ 1.5\times 10\right)/5}\right]$$
$$=1.0843.$$

7.95 The average (mean) of price ratios—the Carli index—is:

$$(7.12)\ \ P_C(p^2,p^3)=\sum_{i=1}^{N}(p_i^3/p_i^2)/N$$
$$=\frac{2}{5}\left(p_1^3/p_1^2\right)+\frac{3}{5}\left[\left(p_5^3/p_5^2\right)+\left(p_8^3/p_8^2\right)/2\right]$$
$$=\frac{2}{5}\left(6/5\right)+\frac{3}{5}\left[\left(12/11\right)+\left(10/10\right)/2\right]$$
$$=1.1073$$

Alternatively, and more simply, imputed figures could be entered in Table 7.2(b) for products 2 and 6 in March using just the price movements of A and B for products 2 and 6, respectively, and indices calculated accordingly. Using a Jevons index for product 2, the imputed value in March would be 6/5 × 6 = 7.2, and for product 6 it would be $[(12/11)\times(10/10)]^{1/2}\times$ 12= 12.533. It is thus apparent that not only does the choice of formula matter, as discussed in Chapter 20, but so too may the targeting of the imputation. In practice, the sample of products in a targeted subgroup may be too small. An appropriate stratum is required with a sufficiently large sample size, but there may be a trade-off between the efficiency gains from the larger sample and the representativity of price changes achieved by that sample. Stratification by industry and region may be preferred to industry alone if regional differences in price changes are expected, but the resulting sample size may be too small. In general, the stratum used for the target should be based on the analyst's knowledge of the industry and an understanding of similarities of price changes between and within strata. It also should be based on the reliability of the available sample to be representative of price changes.

7.96 The underlying assumptions of these methods require some analysis since—as discussed by Triplett (1999 and 2002)—they are often misunderstood. Consider $i = 1 \ldots m$ products where, as before, p_m^t is the price of product m in period t, and p_n^{t+1} is the price of a replacement product n in period $t + 1$. Now n replaces m but is of a different quality. As before, let $A(z)$ be the quality adjustment to p_n^{t+1}, which equates its quality services or utility to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A(z)\ p_n^{t+1}$. For the imputation method to work, the average price changes of the $i = 1 \ldots m$ products, including the quality-adjusted price p_m^{*t+1} given on the left-hand side of equation (7.13), must equal the average price change from just using the overall mean of the rest of the $i = 1\ldots.m-1$ products on the right-hand side of equation (7.13). The discrepancy or bias from the method is the balancing term Q. It is the implicit adjustment that allows the method to work. The arithmetic formulation is given here, although a similar geometric one can be readily formulated. The equation for one unavailable product is given by

$$(7.13)\ \frac{1}{m}\left[\frac{p_m^{*t+1}}{p_m^t} + \sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] = \left[\frac{1}{(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] + Q,$$

$$(7.14)\ Q = \frac{1}{m}\frac{p_m^{*t+1}}{p_m^t} - \frac{1}{m(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t},$$

and for x unavailable products by

$$(7.15)\ Q = \frac{1}{m}\sum_{i=1}^{x}\frac{p_m^{*t+1}}{p_m^t} - \frac{x}{m(m-x)}\sum_{i=1}^{m-x}\frac{p_i^{t+1}}{p_i^t}.$$

7.97 The relationships are readily visualized if r_1 is defined as the arithmetic mean of price changes of products that continue to be recorded and r_2 is defined as the mean of quality-adjusted unavailable products, that is, for the arithmetic case where

$$(7.16)\ r_1 = \left[\sum_{i=1}^{m-x} p_i^{t+1}/p_i^t\right] \div (m - x)$$
$$r_2 = \left[\sum_{i=1}^{x} p_i^{*t+1}/p_i^t\right] \div x,$$

then the ratio of arithmetic mean biases from substituting equation (7.16) into equation (7.15) is

$$(7.17)\ Q = \frac{x}{m}(r_2 - r_1),$$

which equals zero when $r_1 = r_2$. The bias depends on the ratio of unavailable values and the difference between the mean of price changes for existing products and the mean of quality-adjusted replacement price changes. The bias decreases as *either* (x/m) *or* the difference between r_1 and r_2 decreases. Furthermore, the method relies on a comparison between price changes for existing products and *quality-adjusted* price changes for the replacement/unavailable comparison. This is more likely to be justified than a comparison without the quality adjustment to prices. For example, let us say there were $m = 3$ products, each with a price of 100 in period t. Let the $t + 1$ prices be 120 for two products, but assume the third is unavailable, that is, $x = 1$, and is replaced by a product with a price of 140, of which 20 is the result of quality differences. Then the arithmetic bias as given in equations (7.16) and (7.17) where $x = 1$ and $m = 3$ is

$$\tfrac{1}{3}\left[(-20 + 140)/\ 100\right]$$
$$-\tfrac{1}{3}\left[\left(\tfrac{120}{100} + \tfrac{120}{100}\right)/2\right]$$
$$= 0$$

Had the bias depended on the *unadjusted price* of 140 compared with 100, the imputation would be prone to serious error. In this calculation, the direction of the bias is given by $(r_2 - r_1)$ and does not depend on whether quality is improving or deteriorating, that is, whether $A(z) > p_n^{t+1}$ or $A(z) < p_n^{t+1}$. If $A(z) > p_n^{t+1}$, a quality improvement, it is still possible that $r_2 < r_1$ and for the bias to be negative, a point stressed by Triplett (2002).

7.98 It is noted that the analysis here is framed in terms of a short-run price change framework. This means that the short-run price changes between two consecutive periods are used for the imputation. This is different from the long-run imputation, where a base-period price is compared with prices in subsequent months and where the implicit assumptions are more restrictive.

7.99 Table 7.3 provides an illustration whereby the (mean) price change of products that continue to exist, r_1, is allowed to vary for values between 1.00 and 1.50: no price change and a 50 percent increase. The (mean) price change of the *quality-adjusted* new products compared with the products they are replacing is assumed to not change, that is, $r_2 = 1.00$. The bias is given for ratios of missing values of 0.01, 0.05, 0.10, 0.25, and 0.50, arithmetic means and geometric means. For example, if 50

Table 7.3. Example of the Bias from Implicit Quality Adjustment for $r_2 = 1.00$

	Geometric mean					**Arithmetic mean**				
	Ratio of missing products, *x*/*m*					**Ratio of missing products, *x*/*m***				
r_1	**0.01**	**0.05**	**0.10**	**0.25**	**0.50**	**0.01**	**0.05**	**0.10**	**0.25**	**0.50**
1.00	1	1	1	1	1	0	0	0	0	0
1.01	0.999901	0.999503	0.999005	0.997516	0.995037	–0.0001	–0.0005	–0.001	–0.0025	–0.005
1.02	0.999802	0.999010	0.998022	0.995062	0.990148	–0.0002	–0.0010	–0.002	–0.0050	–0.010
1.03	0.999704	0.998523	0.997048	0.992638	0.985329	–0.0003	–0.0015	–0.003	–0.0075	–0.015
1.04	0.999608	0.998041	0.996086	0.990243	0.980581	–0.0004	–0.0020	–0.004	–0.0100	–0.020
1.05	0.999512	0.997563	0.995133	0.987877	0.975900	–0.0005	–0.0025	–0.005	–0.0125	–0.025
1.10	0.999047	0.995246	0.990514	0.976454	0.953463	–0.0010	–0.0050	–0.010	–0.0250	–0.050
1.15	0.998603	0.993036	0.986121	0.965663	0.932505	–0.0015	–0.0075	–0.015	–0.0375	–0.075
1.20	0.998178	0.990925	0.981933	0.955443	0.912871	–0.0020	–0.0100	–0.020	–0.0500	–0.100
1.30	0.997380	0.986967	0.974105	0.936514	0.877058	–0.0030	–0.0150	–0.030	–0.0750	–0.150
1.50	0.995954	0.979931	0.960265	0.903602	0.816497	–0.0050	–0.0250	–0.050	–0.1250	–0.250

percent of price quotes are missing and the missing quality-adjusted prices do not change, but the prices of existing products increase by 5 percent ($r_1 = 1.05$), then the bias for the geometric mean is represented by the proportional factor 0.9759; that is, instead of 1.05, the index should be 0.9759 × 1.05 = 1.0247. For an arithmetic mean, the bias is –0.025; instead of 1.05, it should be 1.025.

7.100 Equation (7.17) shows that the ratio *x*/*m* and the difference between r_1 and r_2 determine the bias. Table 7.3 shows that the bias can be quite substantial when *x*/*m* is relatively large. For example, when *x*/*m* = 0.25, an inflation rate of 5 percent for existing products translates to an index change of 3.73 percent and 3.75 percent for the geometric and arithmetic formulations, respectively, when $r_2 = 1.00$, that is, when quality-adjusted prices of unavailable products are constant. Instead of being 1.0373 or 1.0375, ignoring the unavailable products would give a result of 1.05. Even with 10 percent missing (*x*/*m* = 0.1) an inflation rate of 5 percent for existing products translates to 4.45 percent and 4.5 percent for the respective geometric and arithmetic formulations when $r_2 = 1.00$. However, consider a fairly low ratio of *x*/*m,* say, 0.05; then even when $r_2 = 1.00$ and $r_1 = 1.20$, Table 7.3 finds 18.9 percent and 19 percent corrected rates of inflation for the respective geometric and arithmetic formulations. In competitive markets, r_1 and r_2 are unlikely to differ by substantial amounts since r_2 is a price comparison between the new product and the old product *after adjusting for quality differences*. If r_1 and r_2 are the same, then there would be no bias from the method even if *x*/*m* = 0.9. There may, however, be more sampling error. It should be borne in mind that it is not appropriate to compare bias between the arithmetic and geometric means, at least in the form they take in Table 7.3. The latter would have a lower mean, rendering comparisons of bias meaningless.

7.101 An awareness of the market conditions relating to the commodities is instructive to any understanding of likely differences between r_1 and r_2. The concern here is when prices vary over the life cycle of the products. Thus, at the introduction of a new model, the price change may be quite different from price changes of other existing products. Assumptions of similar price changes, even when quality adjusted, might be inappropriate. Greenlees (2000) uses the example of personal computers: new computers enter the market at prices equal to or lower than prices of previous models but with greater speed and capability. An assumption that $r_1 = r_2$ could not be justified.

7.102 Some of this bias relates to the fact that markets are composed of different segments of purchasers. Indeed, the very training of industrial (and consumer) marketers involves consideration of developing different market segments and ascribing to each segment appropriate *pricing*, *product quality*, *promotion*, and *place* (methods of distribution). This is known as the 4 Ps of the marketing mix (Kotler, 1991). In addition, marketers are taught to plan the marketing mix over the product's life cycle. Such planning would allow for different inputs of each of these marketing mix variables at different points in the life cycle. This includes *price skimming* during the period of introduction, whereby higher prices are charged to skim off the surplus from segment(s) of purchasers willing to pay more. The economic theory of price discrimination would also predict such behavior. Thus, the quality-adjusted price change of an old product compared with a new replacement product may be higher than price changes of other products in the product group. After the introduction of the new product, its prices may fall relative to others in the group. There may be no law of one price *change* for differentiated products within a market. Berndt, Ling, and Kyle (2003) clearly showed how after patent expiration, the price of brand name prescription pharmaceuticals can increase with the entry of new generic pharmaceuticals at a lower price, particularly as loyal, less-price-sensitive customers maintain their allegiance to the brand name pharmaceuticals.

7.103 There is little in economic or marketing theory to support any expectation of similar (quality-adjusted) price changes for new and replacement products and other products in the product group. Some knowledge of the realities of the particular market under study would be helpful when considering the suitability of this approach. *Two things matter in any decision to use the imputation approach. The first is the proportion of replacements, and Table 7.3 provides guidance here. The second is the expected difference between* r_1 *and* r_2, *and it is clear from the above discussion that there are markets in which they are unlikely to be similar.* This is not to say the method should not be used. It is a simple and expedient approach. Arguably what should not happen is that the method is used as a default process without any prior evaluation of expected price changes and the timing of the switch. Furthermore, attention should be directed to its targeted use, using products expected to have similar price changes. However, the selection of such products should also be based on the need to include a sufficiently large sample so that the estimate is not subject to undue sampling error.

7.104 Some mention should be made of the way these calculations are undertaken. A pro forma setting for the calculations—say, on a spreadsheet—would have each product description and its prices recorded on a (usually) monthly basis. The imputed prices of the missing products are inserted into the spreadsheet being highlighted as imputed. The reasons for highlighting such prices are (i) because they should not be used in subsequent imputations as if they were actual prices and (ii) the inclusion of imputed values may give the false impression of a larger sample size than actually exists. Care should be taken in any audit of the number of prices used in the compilation of the index to code such observations as imputed. It is stressed that this is an illustration of a *short-run* imputation, and, as will be discussed in Section H, there is a strong case for using *short-run* imputations against *long-run* ones.

D.3 Class mean imputation

7.105 The *class mean* (or *substitution relative*) method of implicit quality adjustment to prices as used in the U.S. CPI is discussed in Schultz (1995); Reinsdorf, Liegey, and Stewart (1996); Armknecht, Lane, and Stewart (1997); and Armknecht and Maitland-Smith (1999). It arose from concerns similar to those considered in Section D.2, namely that unusual price changes were found in the early introductory period when new models were being introduced, particularly for consumer durables. In their study of selected products, Moulton and Moses (1997), using U.S. CPI data for 1995, found the average pure price change to be only 0.12 percent for identical products being repriced (on a monthly or bimonthly basis). This is compared with an average of 2.51 percent for comparable substitutes—items judged equivalent to the products they replaced. The corresponding average price change for directly substituted quality-adjusted price changes was 2.66 percent. Therefore, the price movement of continuing products appears to be a flawed proxy for the pure price component of the difference between old and replacement items.

7.106 The class mean method was adopted in the U.S. CPI for automobiles in 1989 and was phased in for most other nonfood commodities beginning in 1992. It differed from the imputation method only in the source for the imputed rate of price change for the old product in period $t + 1$. Rather than using the category index change obtained using all the nonmissing products in the category, compilers based the imputed rate of price change on constant quality replacement products—those products that were judged comparable or that were quality adjusted directly. The class mean approach was seen as an improvement on the overall mean imputation approach because the imputed price changes were based on items that had not just had a replacement. Instead, these items' replacement prices benefited from a quality adjustment, or the new replacement product had been judged to be directly comparable. However, it may be the case that sufficiently large samples of comparable substitutes or directly quality-adjusted products are unavailable. Or it may be that the quality adjustments and selection of comparable products are not deemed sufficiently reliable. In this case, a targeted imputation might be considered. The targeted mean is less ambitious in that it seeks only to capture price changes of similar products, irrespective of their point in the life cycle. Yet it is an improvement on the overall mean imputation as long as sufficiently large sample sizes are used. Similar issues may arise in the PPI; it is for industry analysts to consider such possibilities.

D.4 Comparable replacement

7.107 This is where the respondent makes a judgment that the replacement is of a similar quality to the old product and any price changes are untainted by quality changes. For product type A in Table 7.2(b), product 3 might be judged to be comparable to product 2 and its prices in subsequent months used to continue the series. In March the price of 6.5 would be used as the price in March for product 2, whose January to March price change would be $6.5/6 \times 100 = 1.0833$ or 8.33 percent. Lowe (1998), in the context of CPI compilation, noted the common practice of television set manufacturers changing model numbers when there is a new production run, though nothing physically has changed, or when small changes take place in specifications, such as the type of remote controls or the number or placement of jacks. The method of comparable replacement relies on the efficacy of the respondents and, in turn, on the adequacy of the specifications used as a description of the price basis. Statistical agencies may be rightly wary of sample sizes being worn down by dropping products using imputation and also of the resource-intensive explicit estimates outlined below. The use of repriced products of a comparable specification has much to commend it. If, however, the quality of products is improving, the preceding product will be inferior to the current ones. In addition, continually ignoring the small changes in the quality of replacements can lead to an upward bias in the index. The extent of the problem will depend on the proportion of such occurrences, the extent to which comparable products are accepted in spite of quality differences, and the weight attached to them. Proposals in Chapter 8 to monitor types of quality adjustment methods by product area will provide a basis for a strategy for applying explicit adjustments where they are most needed.

D.5 Linked to show no price change

7.108 Linking attributes any price change between the replacement product in the *current* period and the old product in the preceding period to the change in quality. A replacement product 7 is selected, for example, in Table 7.2(b) from product type B for the missing March product 6. The replacement product 7 may be of a very different quality compared with product 6, with the price difference being quite large. The change in price is assumed to be due to a change in quality. An estimate is made for p_7^2 by equating it to p_7^3 to show no change, that is, the assumed price of product 7 in February is 14 in Table 7.2(b). There is, therefore, assumed to be no price change over the period February to March for product 7. The January to March result for product 6 is $(12/12) \times (14/14) = 1.00$, or no change. However, for the period March to April, the price of item 7 in March can be compared with the imputed p_7^2 for February and linked to the preceding results. So the January to April comparison is composed of the January to February comparison for product 6 and linked to (multiplied by) the February to April comparison for item 7. This linking is analogous to the procedures used for the chained and short-run framework discussed in Sections G.3 and H.3. The method is born out of circumstances where comparable replacement products are not available, and there are relatively large price differences between the old and replacement products, having significant differences in price base and quality. It is not possible to separate out how much of this difference is due to price changes and how much to quality changes, so the method attributes it all to quality and holds price constant. The method introduces a degree of undue price stability into the index. It may well be the case that the period of replacement is when substantial price changes are taking place, these changes being wrongly assigned to quality changes by this method. For CPIs, Article 5 of the European Commission (EC) Regulation No. 1749/96 requires member states to avoid such automatic linking. Such linking is equivalent to the assumption that the difference in price between two successive models is wholly attributed to a difference in quality (Eurostat, 2001, p. 125).

D.6 Carryforward

7.109 With this method, when a product becomes unavailable—say, in period $t + 2$—the price change calculation uses the old t price, carried forward as if there was no change. Thus, from Table 7.2(a) for product type A for the January to March Jevons and Dutot indices (Chapter 20, Section B)

(7.18) $$P_J(p^1, p^3) = \left[\left(p_1^3 / p_1^1 \times p_2^2 / p_2^1\right)\right]^{1/2} \text{, and}$$

$$P_D(p^1, p^3) = [(p_1^3 + p_2^2)/(p_1^1 + p_2^1)],$$

with p_2^2 filling in for the missing p_2^3. This introduces undue stability into the index, which is aggravated if the old price p_2^2 continues to be used to fill in the unobserved prices in subsequent periods. It introduces an inappropriate amount of stability into the index and may give a misleading impression of the active sample size. The practice of the carryforward method is banned for harmonized CPIs under Article 6 of the EC Regulation No. 1749/96 for Harmonized Indices of Consumer Prices (Eurostat, 2001, p. 126). To use this method, an assumption is made that the price from this product type would not change. This method should be used only if it is fairly certain that there would be no price change.

E. Explicit Methods

7.110 All of the aforementioned methods do not rely on explicit information on the value of the change in quality, $A(z)$. Now methods that rely on obtaining an explicit valuation of the quality difference are discussed.

E.1 Expert judgment

7.111 Hoven (1999) describes comparable replacement as a special case of "subjective quality adjustment," because the determination of product equivalence is based on the judgment of the commodity specialist. It is important to mention this because an objection to subjective methods is the inability to provide results that can be independently replicated. Yet in comparable replacement, and for the selection of representative products, a subjective element is part of normal procedure. This is not, of course, a case for its proliferation.

7.112 The use of experts' views may be appropriate for highly complex products where alternative methods are not feasible. Experts, as noted above, should be directed to the nature of the estimate required as discussed in the conceptual section. More than one expert should be chosen, and, where possible, they should be from different backgrounds. Some indication of the interval in which their estimate should lie is also advisable. The well-used Delphi method (for example, see Czinkota, 1997) may be applicable. In this approach, a panel of experts work separately to avoid any bandwagon effect regarding their estimates. They are asked to provide an estimate of the average and range of likely responses. The median is taken of these estimates, and any estimate that is considered extreme is sent back to the expert concerned. The expert is asked to identify reasons for the difference. It may be that the particular expert has a useful perspective on the problem that the other experts had not considered. If the expert argues a case, the response is fed back to the panel members, who are asked if they wish to change their views. A new median is taken, and there are possible further iterations. It is time consuming and expensive but illustrates the care needed in such matters. However, if the adjustment is needed for a product area with a large weighting in the PPI and no other techniques are available, it is a possible alternative.

E.2 Quantity adjustment

7.113 This is one of the most straightforward explicit adjustments to undertake and is applicable to products for which the replacement is of a different size than the available one. In some situations, there is a readily available quantity metric that can be used to compare the products. Examples are the number of units in a package (for example, paper plates or vitamin pills), the size or weight of a container (for example, kilos of animal feed, liters of industrial lubricant), or the size of sheets or towels. Quality adjustment to prices can be accomplished by scaling the price of the old or new product by the ratio of quantities. The index production system may do this scaling adjustment automatically by converting all prices in the category to a price per unit of size, weight, or number. Such scaling is most important. For example, it should not be the case that because an industrial lubricant is now sold in 5-liter containers instead of 2.5-liter ones, its prices have doubled.

7.114 There is, however, a second issue. It should be kept in mind that a pure price change is concerned with changes in the revenue received from the sale of the exact same products, produced under the exact same circumstances, and sold under the exact same terms. In the pharmaceutical context, for example, prices of bottles of pills of different sizes differ. A bottle of 100 pills, each pill having 50 milligrams of a drug, is not the same as a bottle of 50 pills of 100 milligrams each, even though both bottles contain 5,000 milligrams of the same drug. It may also be reasonable to decide that a bottle of aspirin, for example, containing 500 tablets may not have 10 times the quality of a 50-tablet bottle. If the smaller size is no longer available and there is a change, for example, to a larger size container, and a *unit* price decrease of 2 percent accompanies this change, then it should not be regarded as a price fall if there is a differential in the cost of producing and margin on selling the larger size of 2 percent or more. If, however, the respondent acknowledged that the change in packaging size for this product led to a 1 percent saving in resource costs (and margin) and prices of other such products without any quantity changes were also falling by 1 percent, then the pure price change would be a fall of 1 percent. In practice, the respondent may be able to make some rough estimates of the effect on the unit cost of the change in packaging size. However, it may well be that no such information is available, and the general policy is to not automatically interpret unit price changes arising from packaging size changes as pure price changes if contrary information exists.

7.115 Consider another example: a brand name bag of fertilizer of a specific type, previously available in a 0.5 kg. bag priced at 1.5 is replaced with a 0.75 kg. bag at 2.25. The main concern here is with rescaling the quantities as opposed to differential cost or margin adjustments. The method would use the relative quantities of fertilizer in each bag for the adjustment. The prices may have increased by [(2.25/1.5) × 100 = 150] 50 percent, but the quality (size)-adjusted prices have remained constant [(2.25/1.5) × (0.5/0.75) × 100 = 100].

7.116 The approach can be outlined in a more elaborate manner by referring to Figure 7.1. The concern here is with the part of the unbroken line between the price and quantity coordinates (1.5, 0.5) and (2.25, 0.75), both of which have *unit* prices of 3 (price = 1.5/0.5 and 2.25/0.75). There should be no change in quality-adjusted prices. The delta symbol (Δ) denotes a change. The slope of the line is β, which is ΔPrice/ΔSize = (2.25 – 1.5)/(0.75 – 0.50) = 3, that is, the change in price arising from a unit (kg.) change in size. The quality (size)-adjusted price in period $t-1$ of the old m bag is

$$(7.19)\quad \hat{p}_m^{t-1} = p_m^{t-1} + \beta\Delta size$$

$$= 1.5 + 3\,(0.75 - 0.5) = 2.25.$$

The quality-adjusted price change shows no change as before:

$$p_n^t \Big/ \hat{p}_m^{t-1} = 2.25/2.25 = 1.00.$$

The approach is outlined in this form so that it can be seen as a special case of the hedonic approach discussed later, where price is related to a number of quality characteristics of which size may be one.

7.117 The method can be seen to be successful on intuitive grounds as long as the unit price of different-sized bags remains constant. If the switch was from a 0.5 kg. bag to a 0.25 kg. one priced at 0.75, as shown by the continuation of the unbroken line in Figure 7.1, to coordinate (0.75, 0.25) quality-adjusted prices would again not change. However, assume the *unit* (kg.) prices were 5, 3, and 3 for the 0.25, 0.5, and 0.75 kg. bags, respectively, as shown in the example below and in Figure 7.1 by the *broken* line. Then the measure of quality-adjusted price change would depend on whether the 0.5 kg. bag was replaced by the 0.25 kg. one (a 67 percent increase) or the 0.75 kg. one (no change). This is not satisfactory because the choice of replacement size is arbitrary. The rationale behind the quality adjustment process is to ask: does the difference in unit price in each case arise from differences in unit costs of producing and margins on selling? If so, adjustments should be made to the unit prices to bring them in line; if not, adjustments should be made to the unit price for that proportion due to changes in costs or margins from economies or diseconomies of package size production. It may be obvious from the nature of the product that a product packaged in a very small size with disproportionately high unit price has an unusually high profit margin or will have quite different unit production costs and an appropriate replacement for a large-sized product would not be this very small one.

Example of Quantity Adjustments

Size	First Price	First Unit Price	Second Price	Second Unit Price
0.25	0.75	3.00	1.25	5.00
0.50	1.50	3.00	1.50	3.00
0.75	2.25	3.00	2.25	3.00

E.3 Differences in production and option costs

7.118 A natural approach is to adjust the price of the old product by an amount equal to the costs of the additional features. This approach is associated with resource-cost valuations discussed in Section B.2. Yet Section B.2 advocated a user-value approach, the appropriate valuation being the change in production costs associated with a quality change plus any price-cost margin. This amounts to a comparison of relative prices using

(7.20) $p_n^t / \hat{p}_m^{t-1}$, where $\hat{p}_m^{t-1} = p_m^{t-1} + x$

and x is the cost or contribution to revenue of the additional features in period $t - 1$. The respondent is a natural expert source of such information. Greenlees (2000) provides an example for new trucks and motor vehicles in the United States in 1999. Just before the annual model year introductions, Bureau of Labor Statistics (BLS) staff visit selected manufacturers to collect cost information. The data are used in the PPI and International Price Comparison programs, as well as in the CPI, and the information-gathering activity is a joint operation of the three programs. Allowable product changes for the purpose of quality adjustments include occupant safety enhancements, mechanical and electrical improvements to overall vehicle operation or efficiency, changes that affect length of service or need for repair, and changes affecting comfort or convenience.

7.119 The traditional approach to the producer orientation of the PPI implies that resource cost is the appropriate criterion for quality adjustment to prices (Triplett, 1983). One distinction, then, between the use of producer cost estimates in the CPI and PPI is that only the former program will add retail markups. Another important difference may occur in situations where product improvements are mandated by government. Some of these mandated improvements provide no direct benefit to the purchaser. In those cases, it is appropriate to make a quality adjustment to prices for the associated resource cost in the PPI but not in the CPI, where the appropriate criterion is user value. Yet the discussion in Section B.2 argues for uniformity of treatment via a user-value concept for price index numbers used on the supply-and-use side of national accounts, in the context of this *Manual*, for PPI input and output indices.

7.120 As an example of option cost adjustments, assume the producer prices for a product in periods t and $t + 2$ were 10,000 and 10,500, respectively, but assume the price in period $t + 2$ is for the item with a new feature or option. Also, let the price of the additional feature in period $t + 2$ be 300. Then the price change would be 10,200/10,000 = 1.02, or 2 percent. The adjustment may take a multiplicative form (see Section A); the additional options are worth 300/10,500 = 0.028571 of the period $t + 2$ price. The adjusted price in period t is, therefore, 10,285.71 and the price change 10,500/10,285.71 = 1.020833, or about 2 percent. If in subsequent periods either of these elements change, then so too must $\hat{p}_n^{t-1}$ for those comparisons. Option cost is thus a method for use in stable markets with stable technologies. Alternatively, it may be preferable to estimate a one-off adjustment to the preceding base-period price and then compare all subsequent products with the new option to this estimate; that is, 10,500/10,300 = 1.019417, or approximately 2 percent.

7.121 In the example above, the prices available for the options were sales prices. For resource cost estimates, the sales prices as estimates of user values must be adjusted to cost estimates by removing markups and indirect taxes. Similarly, and more appropriate to the context of Section B.2, production costs of options need to be upgraded to user values by adding price cost markups and indirect taxes. Often such data are available for only one period. If the markups are considered to be in the same proportion in subsequent periods, then there is no problem since the retail price changes would proxy the producer ones after adjustment for proportionate margins. However, if the average age or vintage of the products have changed, then they will be at different stages in their life cycles and may have different margins.

7.122 Consider the addition of a feature to a product. Office chairs, for example, can be produced and sold as standard or with a lever mechanism for height adjustment. The specification may always have been the standard model, but this may no longer be in production. The new spec may be a model with height adjustment. The cost of the option is, therefore, known from before, and a continuing series can be developed by using equation (7.20) and simply adding the option cost back into the base period, old price. Even this process may have its problems. First, the cost (user value) of producing something as standard, since all new chairs now have the height adjuster, may be lower than when it was an option. The option cost method would thus understate a price increase. It may be that the manufacturer has an estimate of the effects of such economies of scale to allow for further adjustments. Triplett (2002) cites a study by Levy and others (1999) in which an automobile antitheft system was installed as standard but disabled when the option was not required. It was seemingly cheaper to produce this way. Second, by including something as standard, the revenue received may be less for some sales than the marginal cost of producing it. The decision to include it as standard precludes buyers from refusing it. It may be that they will turn to other manufacturers who allow them to exclude the option, although it is unlikely that this will be the sole criterion for the purchase. The overall effect would be that the estimate of the option cost, priced for those who choose it, is likely to be higher than the implicit revenue purchasers accord it as standard. Third, the height adjuster may be valued at an additional amount x when sold separately. There is likely to be a segment of the market that particularly values price adjusters and is willing to spend the additional amount. However, when it is sold as standard, many of the purchasers will not value it so highly since these were the very ones who chose the standard chair. The overall user value would be less than x, although it is not immediately apparent how much less. Some statistical offices take one-half x as the adjustment. Some insight into the proportion of the market purchasing the standard products would help generate more precise estimates.

7.123 Option cost adjustments are similar to the quantity adjustments, with the exception that the additional quality feature of the replacement is not limited to size. The comparison is $p_n^t / \hat{p}_m^{t-1}$, where $\hat{p}_m^{t-1} = p_m^{t-1} + \beta\Delta z$ for an individual z characteristic where $\Delta z = \left(z_n^t - z_m^{t-1}\right)$. The characteristic may be the amount of RAM on a personal computer (PC) as a specific model is replaced by one that is identical except for amount of RAM. If the relationship between price and RAM is linear, this formulation is appropriate. On the web pages of many computer manufacturers, the price of additional RAM is independent of other features, and a linear adjustment is appropriate. Bear in mind that a linear formulation values the worth of a fixed additional amount of RAM as the same irrespective of the machine's total amount of RAM.

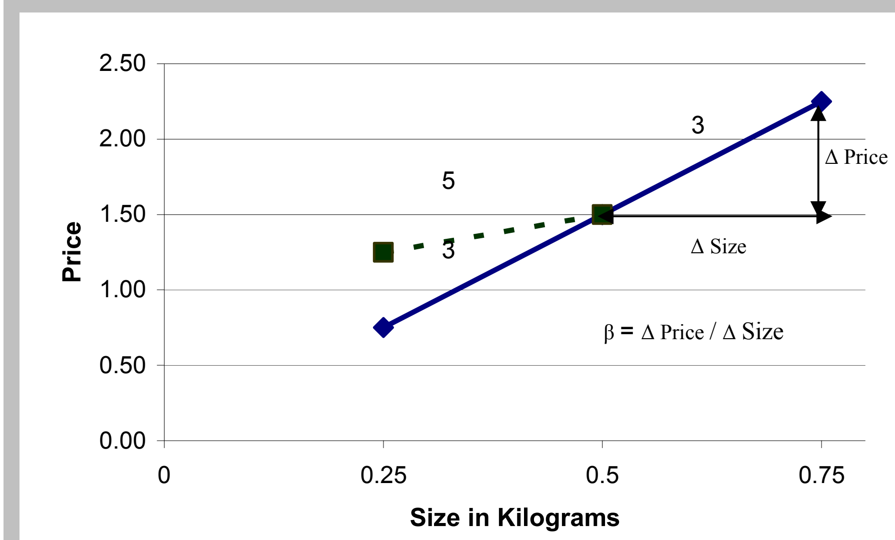
7.124 The relationship may be nonlinear. For example, for every additional 1 percent of x, y increases by 1.5 percent $(\beta = 1.015)$, in this case

$$(7.21)\quad \hat{p}_m^{t-1} = p_m^{t-1}\beta^{z}$$

for $p_n^t / \hat{p}_m^{t-1}$ as a measure of quality-adjusted price changes. Again, the z change may reflect the service flow, but the nonlinearity in the price–z relationship may reflect the increasing or decreasing utility to the scale of the provision. The characteristic may be priced at a higher rate in up-market models of the product versus down-market ones, that is, $\beta \geq 1$ in equation (7.21).

7.125 The similarity between the quantity adjustment and the option cost approach can be identified by simply considering Figure 7.1 with the *z* characteristic being the option horizontal axis. The similarity between the quantity adjustment and the option cost approach is apparent because both relate price to some dimension of quality: the size or the option. The option cost approach can be extended to more than one quality dimension. Both approaches rely on the acquisition of estimates of the change in price resulting from a change in the options or size: the β slope estimates. In the case of the quantity adjustment, this is taken from a product identical to the one being replaced except for the size. The β slope estimate in this case would be perfectly identified from the two pieces of data. It is as if changes in the other factors' quality were accounted for by the nature of the experiment; this is done by comparing prices of what is essentially the same thing except for change in quantity. There may be, for example, two items that are identical except for a single feature. This allows the value of the feature to be determined. Yet sometimes the worth of a feature or option has to be extracted from a much larger data set. This may be because the quality dimension takes a relatively large range of possible numerical values without an immediately obvious consistent valuation. Consider the simple example of one feature varying in a product: processor speed in a PC. It is not a straightforward matter to determine the value of an additional unit of speed. To complicate matters, there may be several quality dimensions to the items, and not all combinations of these may exist as items in the market in any one period. Furthermore, the combinations existing in the second period being compared may be quite different from those in the first. All of this leads to a more general framework.

Figure 7.1. Quality Adjustment for Different-Sized Items



E.4 Hedonic approach

E.4.1 Principles and method

7.126 The hedonic approach is an extension of the two preceding approaches. First, the change in price arising from a unit change in quality—the slope of the line in Figure 7.1—is now estimated from a data set comprising prices and quality characteristic values of a larger number of varieties. Second, the quality characteristic set is extended to cover, in principle, all major characteristics that might influence price, rather than just the quantity or option adjustment. The theoretical basis for hedonic regressions will be covered in Chapter 21 and is briefly reviewed after the following example.

7.127 First, it should be noted that the method requires an extension of the data set to include values for each product of price-determining quality characteristics. Under the matched-models method, each respondent needed to supply sufficient data on each item to allow it to be identified for subsequent repricing. The extension required is that all price-determining characteristics should be available for each item. Checklists for the characteristics of a product have been found by Merkel (2000) to improve the quality of data collected, as well as to serve the needs of hedonic adjustments (see also Chapter 6 on price collection and Liegey, 1994). If a product is missing, any difference in the characteristics of its replacement can be identified, and, as will be shown, a valuation can be ascribed to such differences using the hedonic approach.

7.128 Appendix 7.1 provides data taken from the U.K. Compaq and Dell websites in July 2000 on the prices and characteristics of 64 desktop PCs. Figure 7.2 is a scatter diagram constructed from these data relating the price (£) to the processing speed (MHz). It is apparent that PCs with higher speeds command higher prices—a positive relationship. Under the option cost framework described above, a switch from a 733 MHz PC to a 933 MHz one would involve a measure of the slope of the line between two unique points. The approach requires that there are 733 MHz and 933 MHz PCs that are otherwise identical. From Figure 7.2 and Appendix 7.1, it is apparent that in each instance there are several PCs with the same speed but different prices, owing to differences in other things. To estimate the required value given to additional units of speed, an estimate of the slope of the line that best fits the data is required. In Figure 7.1, the actual slope was used; for the data in Figure 7.2, an estimate of the slope needs to be derived from an estimate of the equation of the line that best fits the data, using ordinary least squares

Figure 7.2. Scatter Diagram of PC Prices

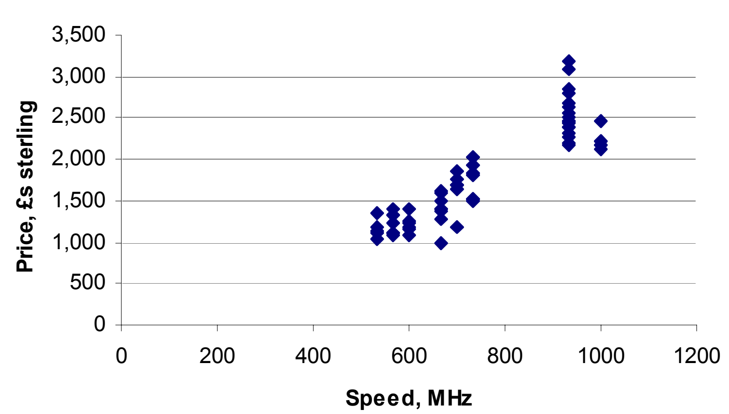


Table 7.4. Hedonic Regression Results for Dell and Compaq PCs

Dependent Variable	Price	Natural Log of Price
Constant	–725.996 (2.71)**	6.213 (41.95)***
Speed (Processor, MHz)	2.731 (9.98)***	0.001364 (9.02)***
RAM (random-access memory, megabytes)	1.213 (5.61) ***	0.000598 (5.00) ***
HD (hard drive capacity, megabytes)	4.517 (1.96)*	0.003524 (2.76)**
Brand (benchmark: Compaq Deskpro)		
Compaq Presario	–199.506 (1.89)*	–0.152 (2.60)**
Compaq Prosignia	–180.512 (1.38)*	–0.167 (2.32)*
Dell	–1,330.784 (3.74)***	–0.691 (3.52)***
Processor (benchmark: AMD Athlon)		
Intel Celeron	393.325 (4.38)***	0.121 (2.43)**
Intel Pentium III	282.783 (4.28)***	0.134 (3.66)***
ROM-drive (benchmark: CD-ROM)†		
CD-RW (compact disk-rewritable)	122.478 (56.07)***	0.08916 (2.88)**
DVD drive (digital video disk)	85.539 (1.54)	0.06092 (1.99)*
Dell × Speed (MHz)	1.714 (4.038)***	0.000820 (3.49)***
N	63	63
$\overline{R}^2$	0.934	0.934

† Read-only memory.
Figures in brackets are *t*-statistics testing a null hypothesis of the coefficient being zero.
***, **, and * denote statistically significant at a 0.1 percent, 1 percent, and 5 percent level, respectively, tests being one-tailed.

(OLS) regression. Facilities for regression are available on standard statistical and econometric software, as well as spreadsheets. The estimated (linear) equation in this instance is

$$(7.22)\quad \hat{P}rice = -658.436 + 3.261\ Speed$$

$$\overline{R}^2 = 0.820 .$$

The coefficient on speed is the estimated slope of the line: the change in price (£3.261) resulting from a 1 MHz change in speed. This can be used to estimate quality-adjusted price changes for PCs of different speeds. The $\overline{R}^2$ finds that 82 percent of price variation is explained by variation in processing speed. A *t*-statistic to test the null hypothesis of the coefficient being zero was found to be 18.83; recourse to standard tables on *t*-statistics found the null hypothesis was rejected at a 1 percent level. The fact that the estimated coefficient differs from zero cannot be attributed to sampling errors at this level of significance. There is a probability of 1 percent that the test has wrongly rejected the null hypothesis. However, the range of prices for a given speed—933 MHz, for example—can be seen from Appendix 7.1 to be substantial. There is a price range of about £1,000, which suggests other quality characteristics may be involved. Table 7.4 provides the results of a regression equation that relates price to a number of quality characteristics using the data in Appendix 7.1. Such estimates can be provided by standard statistical and econometric software, as well as spreadsheets.

7.129 The first column provides the results from a linear regression model, the dependent variable being price. The first variable is processor speed with a coefficient of 2.731; a unit MHz *increase* in processing speed leads to an estimated £2.731 *increase* (positive sign) in price. A change from 733 MHz to 933 MHz would be valued at an estimated 200 (2.731) = £546.20. The coefficient is statistically significant, its difference from zero (no effect) not being due to sampling errors at a 0.1 percent level of significance. This estimated coefficient is based on a multivariate model; the coefficient measures the effect of a unit change in processing speed on price *having controlled for the effect of other variables* in the equation. The result of 3.261 in equation (7.22) was based on just one variable and did not benefit from this. That number is different from this improved result.

7.130 The brand variables are dummy intercepts taking values of 1 if, for example, it is a Dell computer and zero otherwise. While brands are not in themselves quality characteristics, they may be proxy variables for other factors such as after-service reliability. The inclusion of such brand dummies also reflects segmented markets as communities of buyers as discussed in Chapter 21, Appendix 21.1. Similar dummy variables were formed for other makes and models, including the Compaq Presario and Compaq Prosignia. The Compaq Deskpro, however, was omitted to form the benchmark against which other models are compared. The coefficient on Dell is an estimate of the difference between the worth of a Dell and a Compaq Deskpro, other variables being constant (that is, £1,330.78 cheaper). Similarly, an Intel Pentium III commands a premium estimated at £282.78 over an AMD Athlon.

7.131 The estimate for processor speed was based on data for Dell and Compaq PCs. If the adjustment for quality is between two Dell PCs, it might be argued that data on Compaq PCs should be ignored. Separate regressions could be estimated for each make, but this would severely restrict the sample size. Alternatively, an interaction term or slope dummy can be used for variables that are believed to have a distinctive brand-interaction effect. Take Dell × Speed, which takes the value of speed when the PC is a Dell and zero otherwise. The coefficient on this variable is 1.714 (see Table 7.4); it is an estimate of the additional (positive sign) price arising for a Dell PC over and above that already arising from the standard valuation of a 1 MHz increase in speed. For Dell PCs, it is 2.731 + 1.714 = £4.445. Therefore, if the replacement Dell PC is 200 MHz faster than the unavailable PC, the price adjustment to the unavailable PC is to add 200 × £4.445 = £889. Interactive terms for other variables can similarly be defined and used. The estimation of regression equations is easily undertaken using econometric or statistical software, or data analysis functions in spreadsheets. An understanding of the techniques is given in many texts, including Kennedy (2003) and Maddala (1988). In Chapter 21, Appendix 21.1, econometric concerns particular to the estimation of hedonic regressions are discussed.

7.132 The $\bar{R}^2$ is the proportion of variation in price explained by the estimated equation. More formally, it is 1 minus the ratio of the variance of the residuals $\sum_{i=1}^{n}\left(p_i^t - \hat{p}_i^t\right)^2 / n$, of the equation to the variance of prices $\sum_{i=1}^{n}\left(p_i^t - \bar{p}_i^t\right)^2 / n$. The bar on the R^2 denotes that an appropriate adjustment for degrees of freedom is made to this expression, which is necessary when comparing equations with different numbers of explanatory variables. At 0.934, $\bar{R}^2$ is high. However, high $\bar{R}^2$ can be misleading for the purpose of quality adjustment. First, such values inform us that the explanatory variables account for much of price variation. This may be over a relatively large number of varieties of goods in the period concerned. This is not the same as implying a high degree of prediction for an adjustment to a replacement product of a single brand in a subsequent time period. For their accuracy, predicted values depend not just on the fit of the equation but also on how far the characteristics of the product whose price is to be predicted are from the means of the sample. The more unusual the product, the higher the prediction probability interval. Second, $\bar{R}^2$ informs us as to the *proportion* of variation in prices explained by the estimated equation. It may be that 0.90 is explained, while 0.10 is not. If the dispersion in prices is large, this still leaves a large absolute margin of prices unexplained. Nonetheless, a high $\bar{R}^2$ is a necessary condition for the use of hedonic adjustments.

7.133 Hedonic regressions should generally be conducted using a semi-logarithmic formulation

(Chapter 21). The dependent variable is the (natural) logarithm of the price. However, the variables on the right-hand side of the equation are taken in their normal units, thus the semi-logarithmic formulation. A double-logarithmic formulation also takes logarithms of the right-hand side *z* variables. However, if any of these *z* variables are dummy variables—taking the value of zero in some instances—the double logarithmic formulation breaks down. Logarithms of zero cannot be taken (thus the focus on the semi-logarithmic form). This concern with linear and semi-log formulations is equivalent to the consideration of additive and multiplicative formulations discussed in Section A. A linear model would, for example, ascribe an extra £282.78 to a PC with an Intel Pentium III as opposed to an AMD Athlon, irrespective of the price of the PC. This is common in pricing strategies using the World Wide Web. However, more often than not, the same options are valued at a higher price for up-market goods and services. In this case, our equation (7.22) for a multivariate model is

$$(7.23)\quad \text{Price} = \beta_0 \times \beta_1^{z_1} \times \beta_2^{z_2} \times \beta_3^{z_3} \times \ldots \times \beta_n^{z_n} \times \varepsilon \ \text{ or}$$

$$\ln \text{Price} = \ln \beta_0 + z_1 \beta_1 + z_2 \beta_2 + z_3 \beta_3 + \ldots + z_n \beta_n + \ln \varepsilon.$$

Note that this is a semi-logarithmic form; logarithms are taken of only the left-hand side variable, that is, price. Each of the *z* characteristics enter the regression without having logarithms taken. This has the advantage of allowing dummy variables for the possession or otherwise of a feature to be included on the right-hand side. Such dummy variables take the value of 1 if the product possesses the feature and zero otherwise, it not being possible to take a logarithm of the value zero. Issues on choice of functional form are discussed in more detail in Chapter 21.

7.134 The taking of logarithms in the first equation (7.23) allows it to be transformed in the second equation to a linear form. This allows the use of a conventional OLS estimator to yield estimates of the logarithm of the coefficients. These are given in column 3 of Table 7.4 and have a useful direct interpretation: if these coefficients are multiplied by 100, they are the percentage change in price arising from a 1-unit change in the explanatory variable. For processor speed, there is an estimated 0.1364 percent change in price for each additional MHz the replacement product has over and above the unavailable one. When dummy variables are used, the coefficients—when multiplied by 100—are estimates of the percentage change in price given by $\left(e^{\beta} - 1\right) 100$; for example, for a rewritable CD drive (CD-RW) compared with a read-only CD drive (CD-ROM), it is $\left(e^{0.08916} - 1\right)100 = 9.326$ percent. There is some bias in these estimated coefficients on dummy variables for the (semi-) logarithmic equation; one-half of the variance of the regression equation should be added to the coefficient before using it (Teekens and Koerts, 1972). For CD-ROM, the *t*-statistic is 2.88; this is equal to the coefficient divided by its standard error. The standard error is 0.08916/2.88 = 0.03096, and the variance is $0.03096^2 = 0.000958$. To adjust to variance of the regression equation, add 0.000958/2 to 0.08916 = 0.089639 or 8.9639 percent.

7.135 The approach is particularly useful when the market does not reveal the price of the quality characteristics required for the adjustment. Markets reveal prices of products, not quality characteristics, so it is useful to consider products as tied bundles of characteristics. A sufficiently large data set of products with their characteristics and sufficient variability in the mix of characteristics between the products allows the hedonic regression to provide estimates of the implicit prices of the characteristics. The formal theory is provided in Chapter 21. There are a number of ways of implementing the method, which are outlined below. Before doing so, it is useful to note how these coefficients should be interpreted in light of theoretical needs.

E.4.2 On theory

7.136 Some mention should be made of the interpretation of the coefficients from hedonic regressions. The matter will be discussed in further detail in Chapter 21, Section B.5. This section summarizes the conclusion. There used to be an erroneous perception that the coefficients from hedonic methods represented estimates of user value as opposed to resource cost. For CPI construction, the former has generally been accepted as the relevant concept, while for PPI construction, it is the latter (however, see Section B.2). Rosen (1974) found that hedonic coefficients may reflect both user value and resource cost, both supply and demand influences. There is, in econometric terms, an identification problem, in which the observed data do not permit the estimation of the underlying demand-and-supply parameters. However, suppose the *production technology of sellers is the same* but buyers differ. Then the hedonic function describes the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. There are different tastes on the consumer side, so what appears in the market is the result of firms trying to satisfy consumer preferences all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. Now suppose sellers differ but *buyers' tastes are the same*. Here the hedonic function $p(z)$ identifies the structure of demand. Of these possibilities, uniformity of tastes is unlikely while uniformity of technologies is more likely, especially when access to technology is unrestricted in the long run. Griliches (1988, p. 120) has argued in the context of a CPI:

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions *per se*, though it can also be useful for these purposes….what is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

It is thus necessary to take a pragmatic stance. In many cases, the implicit quality adjustment to prices outlined in Section C may be inappropriate because their implicit assumptions are unlikely to be valid. The practical needs of economic statistics require in such instances explicit quality adjustments. To not do anything on the grounds that the measures are not conceptually appropriate would be to ignore the quality change and provide wrong results. Hedonic techniques provide an important tool, making effective use of data on the price-quality relationship derived from other products in the market to adjustment for changes in one or more characteristics.

7.137 The proper use of hedonic regression requires an examination of the coefficients of the estimated equations to see if they make sense. It might be argued that the very multitude of distributions of tastes and technologies and interplay of supply and demand make it unlikely that *reasonable* estimates will arise from such regressions. A firm may apply and cut a profit margin and prices for reasons related to long-run strategic plans, for example, yielding coefficients that prima facie do not look reasonable. This does not negate the usefulness of examining hedonic coefficients as part of a strategy for evaluating estimated hedonic equations. First, there has been extensive empirical work in this field, and the results for individual coefficients are, for the most part, quite reasonable. Even over time, individual coefficients can show quite sensible patterns of decline (van Mulligen, 2003). Second, as shall be seen, it might be argued that the prediction and its error should be our concern and not the values of individual coefficients (Pakes, 2001).

E.4.3 Implementation

7.138 The implementation of hedonic methods to estimate quality adjustments to noncomparable replacements can take a number of forms. The first form is when the repricing is for a product with different characteristics. What is required is to adjust either the price of the old or replacement (new) product for some valuation of the difference in quality between the two products. This patching of missing prices is quite different from the use of hedonic price indices to be discussed in Section 7.G.2 and in Chapter 21. These use hedonic regressions to provide hedonic price indices of overall quality-adjusted prices. The former is a partial application, used on noncomparable replacements when products are no longer produced. The latter, as will be seen in Section 7.G.2, is a general application to a sample from the whole data set. The partial patching is considered here.

7.139 Hedonic imputation: *predicted vs. actual*—In this approach, a hedonic regression of the (natural logarithm of the) price of model i in period t on its characteristics set z_{ki}^{t} is estimated *for each month*, as given by

$$(7.24) \quad \ln\ p_i^t = \beta_0^t + \sum_{k=1}^{K} \beta_k^t z_{ki}^t + \varepsilon_k^t .$$

Let us say the price of a product m available in January (period t) is unavailable in March (period $t + 2$). The price of product m can be predicted for March by inserting the characteristics of the old unavailable product m into the estimated regression equation for March; this process is repeated for successive months. The predicted price for the old product in March and the price comparison with January (period t) are given, respectively, by

$$\text{(7.25a)}\quad \hat{p}_m^{t+2} = \exp\left[\hat{\beta}_k^{\,t+2} + \sum \beta_k^{t+2} \quad z_{k,m}^{t}\right],$$

and $\hat{p}_m^{t+2} / p_m^t$, that is, the *old* model's price is adjusted. In the example in Table 7.2(a), $\hat{p}_2^3$, $\hat{p}_2^4$, etc. and $\hat{p}_6^3$, $\hat{p}_6^4$, etc. would be estimated and compared with p_2^1 and p_6^1, respectively. The blanks for products 2 and 6 in Table 7.2(a) would be effectively filled in by the estimated price from the regression equation.

7.140 An alternative procedure is to select for each unavailable m product a replacement product n. In this case, the price of n in period $t + 2$ is known, and a predicted price for n in period t is required. The predicted price for the new product and required price comparison are

$$\text{(7.25b)}\quad \hat{p}_n^t = \exp\left[\beta_0^t + \sum \beta_k^t \quad z_{k,m}^{t+2}\right],$$

and $p_n^{t+2} / \hat{p}_n^t$, that is, the *new* model's price is adjusted. In this case, the characteristics of product n are inserted into the right-hand side of an estimated regression for period t. The price comparisons of equation (7.25a) may be weighted by w_m^t, as would those of its replaced price comparison in equation (7.25b).

7.141 A final alternative is to take the geometric mean of the formulations in equations (7.25a) and (7.25b) on grounds analogous to those discussed in Chapter 15 and by Diewert (1997) for similar index number issues.

7.142 Hedonic imputation: *predicted vs. predicted*—A further approach is the use of predicted values for the product in *both* periods, for example, $\hat{p}_n^{t+2} / \hat{p}_n^t$, where n represents the product. Consider a misspecification problem in the hedonic equation. For example, there may be an interaction effect between a brand dummy and a characteristic, say, between Dell and speed in the example in Table 7.4. Having both characteristics may be worth more on price (from a semi-logarithmic form) than their separate individual components (for evidence of interaction effects see, Curry, Morgan, and Silver, 2000). The use of $p_n^{t+2} / \hat{p}_n^t$ would be misleading since the actual price in the numerator would incorporate the 5 percent premium while the one predicted from a straightforward semi-logarithmic form would not. It is stressed that in adopting this approach, a recorded, actual price is being replaced by an imputation. Neither this nor the form of bias discussed above are desirable. Diewert (2002e) considers a similar problem and suggests an adjustment to bring the actual price back in line with the hedonic one.

7.143 The comparisons using predicted values in both periods are given as

$\hat{p}_n^{t+2} / \hat{p}_n^t$ for the *new* product,

$\hat{p}_m^{t+2} / \hat{p}_m^t$ for the disappearing or *old* product, or

$$\text{(7.26)}\quad \left[\left(\hat{p}_n^{t+2} / \hat{p}_n^t\right)\left(\hat{p}_m^{t+2} / \hat{p}_m^t\right)\right]^{1/2}$$

as a (geometric) mean of the two.

7.144 Hedonic adjustments using *coefficients*—In this approach, a replacement product is used and any differences between the characteristics of the replacement n in period $t + 2$ and m in period t are ascertained. A predicted price for n in period t, that is, $\hat{p}_n^t$, is compared with the actual price p_n^{t+2}. However, unlike the formulation in equation (7.25b) for example, $\hat{p}_n^t$ may be estimated by applying the subset of the k characteristics that distinguished m from n, to their respective implicit prices in period t estimated from the hedonic regression, and adjusting the price of p_m^t. For example, if the nearest replacement for product 2 was product 3, then the characteristics that differentiated product 3 from product 2 are identified and the price in the base period p_3^1 is estimated by adjusting p_2^1 using the appropriate coefficients from the hedonic regression in that month. For example, for washing machines, if product 2 had an 800 revolutions per minute (rpm) spin speed and product 3 had an 1,100 rpm spin speed, other things being equal, the shadow price of the 300 rpm differential would be estimated from the hedonic regression, and p_2^1 would be adjusted for comparison with p_3^3. Note that if the z variables in the characteristic set are perfectly independent of each other, the results from this approach will be similar to those from equation (7.25b). This is because interdependence among the variables on the right-hand side of the hedonic equation—multicollinearity—leads to imprecise estimates of the coefficients (see Chapter 21, Appendix 21.1).

7.145 Hedonic *indirect adjustment*—An indirect current-period hedonic adjustment may be used, which requires the hedonic regression to be estimated only in the base period t.

$$(7.27)\ \frac{p_n^{t+2}}{p_m^t} \div \frac{\hat{p}_n^t}{\hat{p}_m^t}.$$

The first term is the change in price between the old and replacement items in periods t and $t + 2$, respectively. But the quality of the product has changed, so this price change needs to be divided by a measure of the change in quality. The second term uses the hedonic regression in period t in both the numerator and denominator. So the coefficients—the shadow prices of each characteristic—remain the same. It is not prices that change. The predicted prices differ because different *quantities* of the characteristics are being inserted into the numerator and denominator; the replacement n characteristics in the former and old product m characteristics in the latter. The measure is the change in price after removing (by division) the change in the quantity of characteristics each valued at a constant period t price. Conceptually, the constant valuation by a period $t + 2$ regression would be equally valid and a geometric mean of the two ideal. However, if hedonic regressions cannot be run in real time, equation (7.27) is a compromise. As the spread between the current and base-period results increases, its validity decreases. As such, the regression estimates should be updated regularly using old- and current-period estimates, and results compared retrospectively as a check on the validity of the results.

E.4.4 Need for caution

7.146 The limitations of the hedonic approach should be kept in mind. Some points are summarized below though readers are referred to the Bibliography and to Chapter 21, Appendix 21.1. First, the approach requires statistical expertise for the estimation of the equations. The prevalence of user-friendly software with regression capabilities makes this less problematic. Statistical and econometric software carry a range of diagnostic tests to help judge if the final formulation of the model is satisfactory. These include $\bar{R}^2$ as a measure of the overall explanatory power of the equation; F-test and t-test statistics to enable tests to be conducted to determine whether the differences between the coefficients on the explanatory variables are jointly and individually different from zero at specified levels of statistical significance. Most of these statistics make use of the errors from the estimated equation. The regression equation can be used to predict prices for each product by inserting the values of the characteristics of the products into the explanatory variables. The differences between the actual prices and these predicted results are the residual errors. Biased or imprecise results may arise from a range of factors, including heteroscedasticity (nonconstant variances in the residuals suggesting nonlinearities or omission of relevant explanatory variables), a nonnormal distribution for the errors, and multicollinearity, where two or more explanatory variables are related. The latter, in particular, has been described as the "bane of hedonic regressions..." (Triplett, 1990). Such econometric issues are well discussed in the context of hedonic regressions (Berndt, 1991; Berndt, Griliches, and Rappaport, 1995; Triplett, 1990; Gordon, 1990; Silver, 1999; and Chapter 21, Appendix 21.1) and more generally in introductory econometric texts such as Kennedy (2003) and Maddala (1988). The use of predicted values when multicollinearity is suspected is advised, rather than individual coefficients, for reasons discussed above.

7.147 Second, the estimated coefficients should be updated regularly. However, if the adjustment is to the old model, then the price comparison is between the price of the new model and the quality-adjusted price of the old model. The quality difference between the old and new model is derived using coefficients from a hedonic regression from a previous period as estimates of the value of such differences. There is, at first glance, no need to update the hedonic regression each month. The valuation of a characteristic in the price reference period may, however, be quite out of line with its valuation in the new period. For example, a feature may be worth an additional 5 percent in the reference period instead of 10 percent in the current period because it might have been introduced at a discount at that point in its life cycle to encourage usage. Continuing to use the coefficients from some far-off period to make price adjustments in the current period is similar to using out-of-date base-period weights. The comparison may be well defined but have little meaning. If price adjustments for quality differences are being made to the old item in the price reference period using hedonic estimates from that period, then there is a need to update the estimates if they are considered out of date, for example, due to changing tastes or technology, and splice the new estimated comparisons onto the old. Therefore, regular updating of hedonic estimates when using the adjustments to the old price is recommended, especially when there is evidence of parameter instability over time.

7.148 Third, the sample of prices and characteristics used for the hedonic adjustments should be suitable for the purpose. If they are taken from a particular industry, trade source, or web page and then used to adjust noncomparable prices for products sold by quite different industries, then there must at least be an intuition that the marginal returns for characteristics are similar among the industries. A similar principle applies for the brands of products used in the sample for the hedonic regression. It should be kept in mind that high $\overline{R}^2$ statistics do not alone ensure reliable results. Such high values arise from regressions in periods before their application and inform us of the proportion of variation in prices across many products and brands. They are not in themselves a measure of the prediction error for a particular product, sold by a specific establishment of a given brand in a subsequent period, although they can be an important constituent of this.

7.149 Fourth, there is the issue of functional form and the choice of variables to include in the model. Simple functional forms generally work well. These include linear, semi-logarithmic (logarithm of the left-hand side), and double-log (logarithms of both sides) forms. Such issues are discussed in Chapter 21, Appendix 21.1. The specification of a model should include all price-determining characteristics. Some authors advise quite simple forms with only the minimum number of variables, as long as the predictive capacity is high (Koskimäki and Vartia, 2001). For the CPI, Shepler (2000) included 33 variables in her hedonic regressions of refrigerators, a fairly homogeneous product. These included 9 dummy variables for brand, 4 dummy variables for color, 5 types of outlets, 3 regions as control variables, and 11 characteristics. These characteristics included capacity, type of ice-maker, energy-saving control, number of extra drawers, sound insulation, humidifier, and filtration device. Typically, a study would start with a larger number of explanatory variables and a general econometric model of the relationship; the final model is a more specific, parsimonious one since it has dropped a number of variables. The dropping of variables occurs after experimenting with different formulations and seeing their effects on diagnostic test statistics, including the overall fit of the model and the accordance of signs and magnitudes of coefficients with prior expectations. Reese (2000), for example, started with a hedonic regression for U.S. college textbooks. It included about 50 explanatory variables; subsequently, those variables were reduced to 14 with little loss of explanatory power.

7.150 Finally, Bascher and Lacroix (1999) list several requirements for successful design and use of hedonic quality adjustment in the CPI, noting that these requirements require heavy investments over a long period. They involve (i) intellectual competencies and sufficient time to develop and reestimate the model and employ it when products are replaced; (ii) access to detailed, reliable information on product characteristics; and (iii) a suitable organization of the infrastructure for collecting, checking, and processing information.

7.151 It should be noted that hedonic methods may also improve quality adjustment in the PPI by indicating which product attributes do *not* appear to have material impacts on price. That is, if a replacement product differs from the old product only in characteristics that have been rejected as price-determining variables in a hedonic study, this would support a decision to treat the products as comparable or equivalent and include the entire price difference (if any) as pure price change. Care has to be exercised in such analysis because a feature of multicollinearity in regression estimates is that the imprecision of the parameter estimates may give rise to statistical tests that do not reject null hypotheses that are false, that is, they do not find significant parameter estimates. However, the results from such regressions can nonetheless provide valuable information on the extent to which different characteristics influence price variation. This in turn can help in the selection of replacement products. The enhanced confidence in product substitution and the quality adjustment of prices from the hedonic approach with its parallel reduction in reliance on linking have been cited as significant benefits in the reliability of the measurement of price changes for apparel in the U.S. CPI (Reinsdorf, Liegey, and Stewart, 1996). The results from hedonic regressions have a role to play in identifying price-determining characteristics and may be useful in the design of quality checklists in price collection (Chapter 6).

F. Choosing a Quality Adjustment Method

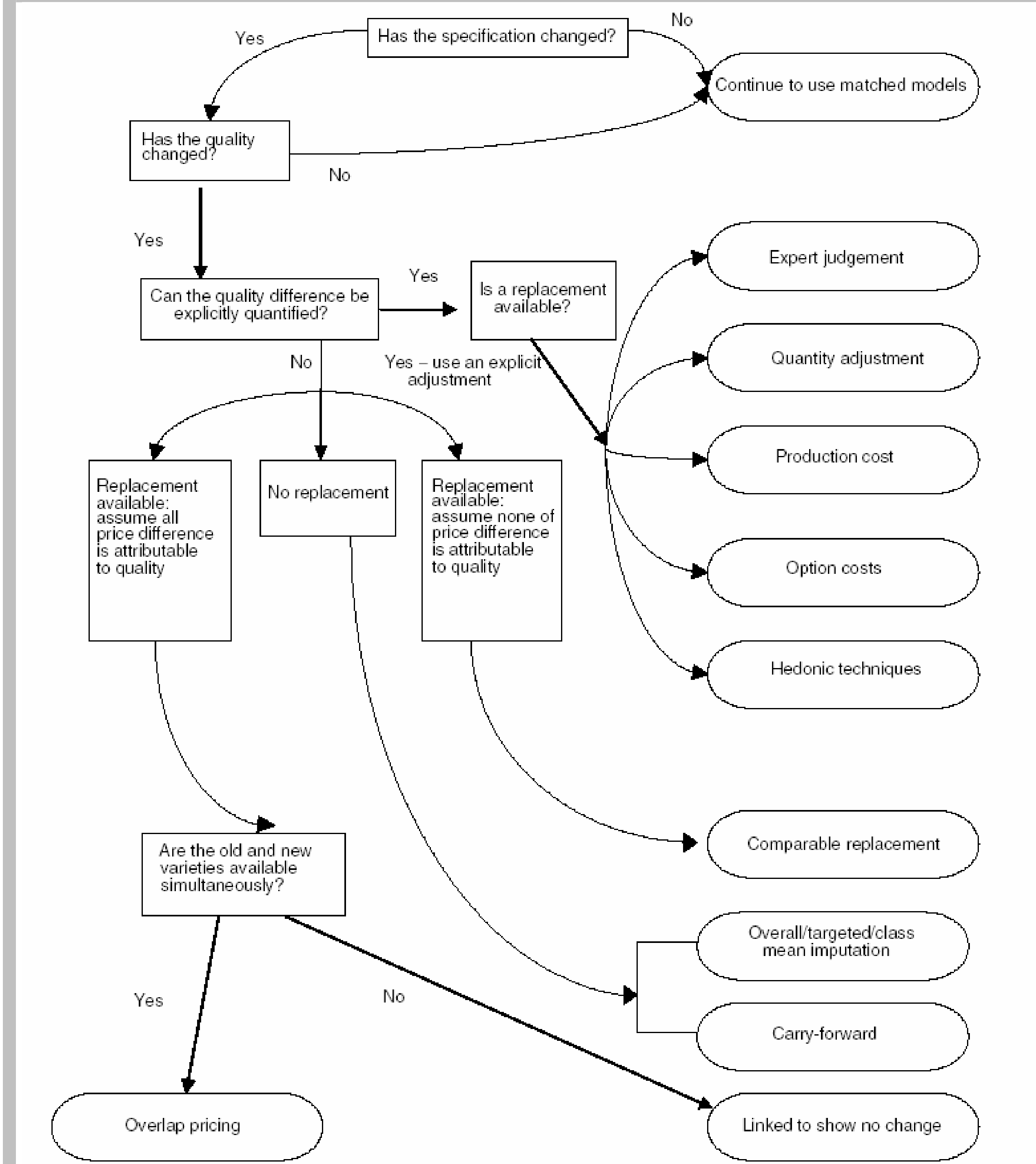
7.152 Choosing a method for quality-adjusting prices is not straightforward. The analyst must consider the technology and market for each commodity and devise appropriate methods. This is not to say the methods selected for one industry will be independent of those selected for other industries. Expertise built up using one method may encourage its use elsewhere, and intensive use of resources for one commodity may lead to less resource-intensive methods in others. The methods adopted for individual industries may vary among countries as access to data, relationships with the respondents, resources, expertise and features of the production, and market for the product vary. Guidelines on choosing a method arise directly from the features of the methods outlined above. A good understanding of the methods and their implicit and explicit assumptions is essential when choosing a method.

7.153 Consider Figure 7.3, which provides a useful guide to the decision-making process. Assume the matched-models method is being used. If the product is matched for repricing—without a change in the specification—no quality adjustment is required. This is the simplest of procedures. However, a caveat applies. If the product belongs to a high-technology industry where model replacement is rapid, the matched sample may become unrepresentative of the universe of transactions. Alternatively, matching may be under a chained framework, where prices of products in a period are matched to those in the preceding period to form a link. A series of successive links of matched comparisons combined by successive multiplication makes up the chained matched index. Alternatively, hedonic indices may be used, which require no matching. The use of such methods is discussed in Section G. At the very least, attention should be directed to more regular product resampling. Continued long-run matching would deplete the sample, and an alternative framework to long-run matching would be required.

7.154 Consider a change in the quality of a product, and assume a replacement product is available. The selection of a comparable product to the same specification and the use of its price as a *comparable replacement* require that none of the price difference is due to quality. They also require confidence that all price-determining factors are included on the specification. The replacement product should also be representative and account for a reasonable proportion of sales. Caution is required when nearly obsolete products at the end of their life cycles are replaced with unusual pricing by similar products that account for relatively low sales, or with products that have substantial sales but are at different points in their cycle. Strategies for ameliorating such effects are discussed below and in Chapter 8, including early substitutions before pricing strategies become dissimilar.

7.155 Figure 7.3 shows where quality differences can be quantified. *Explicit estimates* are generally considered to be more reliable, but they are also more resource intensive (at least initially). Once an appropriate methodology has been developed, explicit estimates can often be easily replicated. General guidelines are more difficult here since the choice depends on the host of factors discussed above, which are likely to make the estimates more reliable in each situation. Central to all of this is the quality of the data on which the estimates are based. If reliable data are unavailable, subjective judgments may be used. Product differences are often quite technical and very difficult to specify and quantify. The reliability of the method depends on the knowledge of the experts and the variance in opinions. Estimates based on objective data are, as a result, preferred. Good *production cost* estimates, along with good data on markups

Figure 7.3. Flowchart for Making Decisions on Quality Change



Source: Chart based on work of Fenella Maitland-Smith and Rachel Bevan, OECD; see also a version in Triplett (2002).

and indirect taxes in industries with stable technologies where differences between the old and replacement products are well specified and exhaustive, are reliable by definition. The *option cost* approach is generally preferable when old and new products differ by easily identifiable characteristics that have once been separately priced as options. The use of *hedonic regressions* for partial patching is most appropriate where data on price and characteristics are available for a range of models and where the characteristics are found to predict and explain price variability well in terms of a priori reasoning and econometrics. Use of hedonic regressions is appropriate where the cost of an option or change in characteristics cannot be separately identified and has to be gleaned from the prices of products sold with different specifications in the market. The estimated regression coefficients are the estimate of the contribution to price of a unit change in a characteristic, having controlled for the effects of variations in the quantities of other characteristics.

7.156 The estimates are particularly useful for valuing changes in the quality of a product when only a given set of characteristics change, and the valuation is required for changes in these characteristics only. The results from hedonic regressions may be used to target the salient characteristics for product selection. The synergy between the selection of prices according to characteristics defined as price determining by the hedonic regression and the subsequent use of hedonics for quality adjustment should reap rewards. The method should be applied where there are high ratios of noncomparable replacements and where the differences between the old and new products can be well defined by a large number of characteristics.

7.157 If explicit estimates of quality are unavailable and no replacement products are deemed appropriate, then *imputations* may be used. The use of imputations has much to commend it in terms of resources. It is relatively easy to employ, although some verification of the validity of the implicit assumptions might be appropriate. It requires no judgment (unless targeted) and is therefore objective. Targeted mean imputation is preferred to overall mean imputation as long as the sample size on which the target is based is adequate. Class mean imputation is preferred when models at the start of their life cycles are replacing those near the end of their life cycles, although the approach requires faith in the adequacy of the explicit and comparable replacements being made.

7.158 Bias from using imputation is directly related to the proportion of missing products and the difference between quality-adjusted prices of available matched products and the quality-adjusted prices of unavailable ones (see Table 7.3). The nature and extent of the bias depends on whether short-run or long-run imputations are being used (the former being preferred) and on market conditions (see Section H). Imputation in practical terms produces the same result as deletion of the product, and the inclusion of imputed prices may give the illusion of larger sample sizes. Imputation is less likely to give bias for products where the proportion of missing prices is low. Table 7.2 can be used to estimate likely error margins arising from its use, and a judgment can be made as to whether they are acceptable. Its use across many industries need not compound the errors since, as noted in the discussion of this method, the direction of bias need not be systematic. It is cost-effective for industries with large numbers of missing products because of its ease of use. But the underlying assumptions required must be carefully considered if widely used. Imputation should by no means be the overall, catchall strategy, and statistical agencies are advised against its use as a default device without due consideration to the nature of the markets, possibility of targeting the imputation, and the viability of estimates from the sample sizes involved if such targeting is employed.

7.159 If the old and replacement products are available simultaneously and the quality difference cannot be quantified, an implicit approach can be used whereby the price difference between the old and replacement product in a period in which they both exist is assumed to be due to quality. This *overlap* method, by replacing the old product with a new one, takes the ratio of prices in a period to be a measure of their quality difference. It is implicitly used when new samples of products are taken. The assumption of relative prices equating to quality differences at the time of the splice is unlikely to hold true if the old and replacement products are at different stages in their life cycles and different pricing strategies are used at these stages. For example, there may be deep discounting of the old product to clear inventories and price skimming of market segments that will purchase new models at relatively high prices. As with comparable replacements, early substitutions are advised so that the overlap is at a time when products are at similar stages in their life cycles.

7.160 The use of the *linked to show no change* method and the *carryforward* method is not generally advised for making quality adjustment imputations for the reasons discussed unless there is deemed to be some validity to the implicit assumptions.

G. High-Technology and Other Sectors with Rapid Turnover of Models

7.161 The measurement of price changes of products unaffected by quality changes is primarily achieved by matching models, the aforementioned techniques being applicable when the matching breaks down. But what about industries where the matching breaks down on a regular basis because of the high turnover in new models of different qualities than the old ones? The matching of prices of identical models over time, by its nature, is likely to lead to a depleted sample. There is both a dynamic universe of all products produced and a static universe of the products selected for repricing (Dalén, 1998). For example, if the sample is initiated in December, by the subsequent May the static universe will be matching prices of those products available in the static universe in both December and May but will omit the unmatched new products introduced in January, February, March, April, and May, and the unmatched old ones available in December but unavailable in May. There are two empirical questions to answer for any significant bias to be detected. First, whether the sample depletion is substantial; such depletion is a necessary condition for bias. Second, whether the unmatched new and unmatched old products are likely to have different quality-adjusted prices versus the matched ones in the current and base period.

7.162 Thus, the matching of prices of identical models over time may lead to the monitoring of a sample of models increasingly unrepresentative of the population of transactions. There are old models that existed when the sample was drawn but are not available in the current period, and there are new ones coming into the current period that are not available in the base period. It may be that the departures have relatively low prices and the entrants relatively high ones and that by ignoring these prices a bias is being introduced. Using old low-priced products and ignoring new high-priced ones has the effect of biasing the index downward. In some industries, the new product may be introduced at a relatively low price and the old one may become obsolete at a relatively high one, serving a minority segment of the market (Berndt, Ling, and Kyle, 2003). In this case, the bias would take the opposite direction; the nature of the bias depends on the pricing strategies of firms for new and old products.

7.163 This sampling bias exists for most products. However, our concern is with product markets where the statistical agencies are finding the frequency of new product introductions and old product obsolescence sufficiently high that they may have little confidence in their results. First, some examples of such product markets will be given. Then, two procedures will be considered: the use of hedonic price indices instead of partial hedonic patching and chaining.

G.1 Some examples

7.164 Koskimäki and Vartia (2001) attempted to match prices of personal computers over three two-month periods (spring, summer, and fall) using a sample of prices collected as part of the standard price collection for the Finnish CPI, which has some similarities to a PPI. Of the 83 spring prices, only 55 matched comparisons could be made with the summer prices, and of those, only 16 continued through to the fall. They noted that the sample of matched pairs became increasingly biased: of the 79 models in the fall, the 16 matched ones had a mean processor speed of 518 MHz compared with 628 MHz for the remaining 63 unmatched ones; the respective hard disk sizes were 10.2 gigabytes (GB) and 15.0 GB; and the percentages of high-end processors (Pentium III and AMD Athlon) were 25 percent and 49.2 percent, respectively. Hardly any change in *matched* prices was found over this six-month period, while a hedonic regression analysis using all of the data found quality-adjusted price falls of around 10 percent. Instructions to respondents to hold on to models until forced replacements are required may lead to a sample increasingly unrepresentative of the population and be biased toward technically inferior variants. In this instance, the hedonic price changes fell faster since the newer models became cheaper for the services supplied.

7.165 Kokoski, Moulton, and Zieschang (1999) used hedonic regressions in an empirical study of interarea price comparisons of food products across U.S. urban areas using U.S. CPI data. They found a negative sign on the coefficients of dummy variables for whether the sample products were from newly rotated samples, where the dummy variable = 1, or samples before rotation, where the dummy variable = 0. This indicated that quality-adjusted prices were lower for the newly included products compared with the quality-adjusted prices of the old products.

7.166 Silver and Heravi (2002) found evidence of sample degradation when matching prices of U.K. washing machines over a year. By December, only 53 percent of the January basket of model varieties was used for the December/January index, although this accounted for 81.6 percent of January expenditure. Models of washing machines with lower sales values dropped out quicker. However, the remaining models in December accounted for only 48.2 percent of the value of transactions *in December*. The active sample relating to the universe of transactions in December had substantially deteriorated. The prices of unmatched and matched models differed, as did their vintage and quality. Even when prices were adjusted for quality using hedonic regressions, prices of unmatched old models were found to be lower than matched ones; there was also evidence of higher prices for unmatched new models. Quality-adjusted prices fell faster for the matched sample than the full sample: about 10 percent for the former compared with about 7 percent for the latter. Residuals from a common hedonic surface and their leverage were also examined. The residuals from unmatched new models were higher than matched ones, while residuals from unmatched old models were much lower. Unmatched observations had nearly twice the (unweighted) leverage than matched ones; their influence in the estimation of the parameters of the regression equation was much greater and their exclusion more serious.

7.167 These studies demonstrate how serious sample degradation can occur and how unmatched excluded products may be quite different from included ones. Two procedures for dealing with such situations will be considered: the use of hedonic price indices instead of the partial hedonic patching discussed above and chaining. Both rely on a data set of a representative sample of products and their characteristics *in each period*. A checklist of structured product characteristics to be completed each reporting period is one way changes in quality characteristics can be prompted and monitored: this is especially useful in high-technology industries (Merkel, 2000). If a new product is introduced and has or is likely to have substantial sales, then it is included as a replacement or even an addition. Its characteristics are marked off against a checklist of salient characteristics. The list will be developed when the sample is initiated and updated as required. Alternatively, web pages and trade associations may be able to provide lists of models and their prices; however, the need for transaction prices as opposed to list prices is stressed.

G.2 Hedonic price indices

7.168 It is important to distinguish between the use of hedonic regressions to make adjustments for quality differences when a noncomparable substitute is used, as in Section E, and their use in their own right as *hedonic price indices,* which are measures of quality-adjusted price changes. Hedonic price indices are suitable when the pace and scale of replacements of products are substantial. There are two reasons for this. First, an extensive use of quality adjustments may lead to errors. Second, the sampling will be from a matched or replacement universe likely to be biased. With new models being continually introduced and old ones dying, the coverage of a matched sample may deteriorate and bias may be introduced as the price changes of the new or old models differ from those of the matched ones. A sample must be drawn in each month, and price indices must be constructed, but, instead of being controlled for quality differences by matching, they will be controlled for, or partialed out, in the hedonic regression. Note that all the indices described below use a fresh sample of the data available in each period. If there is a new product in a period, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old products drop out, they are still included in the data for the indices in the periods in which they exist. In Section E.4.4 of this chapter, the need for caution was stressed in the use of hedonic regressions for quality adjustments due to theoretical and econometric issues, some of which will be considered in the appendix to Chapter 21. This need for caution extends to the use of the results from hedonic indices and is not repeated here for the sake of brevity.

7.169 In Chapter 17, theoretical price indices will be defined and practical index number formulas considered as bounds or estimates of these indices. Theoretical index numbers will also be defined in Chapter 21 to include goods made up of tied characteristics, so that something can be said about how such theoretical indices relate to different forms of hedonic indices. A number of forms will be considered in Chapter 21, and the account is outlined here.

G.2.1 Hedonic functions with dummy variables on time

7.170 The sample covers the two time periods being compared—for example, t and $t + 2$—and does not have to be matched. The hedonic formulation regresses the price of product i, p_i, on the $k = 2 \ldots K$ characteristics of the products z_{ki}. A single regression is estimated on the data in the two time periods compared, the equation also including a dummy variable D^{t+2} being 1 in period $t + 2$, zero otherwise:

$$(7.28)\quad \ln p_i = \beta_0 + \beta_1 D^{t+2} + \sum_{k=2}^{K} \beta_k z_{ki} + \varepsilon_i .$$

The coefficient β_1 is an estimate of the quality-adjusted price change between period t and period $t + 2$. It is an estimate of the change in (the logarithm of) price, having controlled for the effects of variation in quality via $\sum_{k=2}^{K} \beta_k z_{ki}$. Note that an adjustment is required for β_1: the addition of one-half (standard error)^2 of the estimate as discussed in Goldberger (1968) and Teekens and Koerts (1972). Two variants of equation (7.28) are considered. The first is the direct *fixed-base version*, which compares period t with $t + 2$ as outlined: January–February, January–March, etc. The second is a rolling *chained version* evaluated for period t with $t + 1$; then again for $t + 1$ with $t + 2$ and so on, the links in the chain being combined by successive multiplication. A January–March comparison, for example, would be the January–February index multiplied by the February–March one. There is also a *fully constrained version.* This entails a single constrained regression for a period of time—January to December, for example—with dummy variables for each month. However, this is impractical in real time because it requires data on future observations.

7.171 The approach just described uses the dummy variables on time to compare prices in period t with prices in each subsequent period. In doing so, the β parameters are constrained to be constant over the period being compared. A fixed-base, bilateral comparison using equation (7.28) makes use of the constrained parameter estimates over the two periods compared and, given an equal number of observations in each period, is a form of a symmetric average. A *chained* formulation would estimate an index between periods 1 and 4—represented here as $I^{1,4}$—as

$$I^{1,4} = I^{1,2} \times I^{2,3} \times I^{3,4} .$$

7.172 There is no explicit weighting in these formulations; this is a serious disadvantage. In practice, cutoff sampling might be employed to include only the most important products. If sales data are available, a weighted least squares estimator (WLS) should be used, as opposed to an OLS estimator. It is axiomatic in normal index number construction that the same weight should not be given to each price comparison since some products may account for much larger sales revenues than others. The same consideration applies to these hedonic indices. Diewert (2002e) has argued that sales *values* should form the basis of the weights over quantities. Two products may have sales equal to the same quantity, but, if one is priced higher than another, its price changes should be weighted higher accordingly for the result to be meaningful in an economic sense. In addition, Diewert (2002e) has shown that it is value *shares* that should form the weights, since values will increase—over period $t + 2$, for example—with prices, the residuals, and their variance thus being higher in period $t + 2$ than in t. This heteroscedasticity is an undesirable feature of a regression model resulting in increased standard errors. Silver (2002) has further shown that a WLS estimator does not purely weight the observations by their designated weights. The actual influence given is also due to a combination of the residuals and the leverage effect. The latter is higher since the characteristics of the observations diverge from the average characteristics of the data. He suggests that observations with relatively high leverage and low weights be deleted and the regression repeated.

G.2.2 Period-on-period hedonic indices

7.173 An alternative approach for a comparison between periods t and $t + 2$ is to estimate a hedonic regression for period $t + 2$ and insert the values of the characteristics of each model existing in period t into the period $t + 2$ regression to predict, for each item, its price. This would generate predictions of the prices of items existing in period t based on their z_i^t characteristics, at period $t + 2$ shadow prices, $\hat{p}_i^{t+2}(z_i^t)$. These prices (or an average) can be compared with the actual prices (or the average of prices) of models in period t, $p_i^t(z_i^t)$ as a, for example, Jevons hedonic base-period index:

$$(7.29a)\quad P_{JHB} = \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} p_i^t(z_i^t)\right]^{1/N^t}}$$

$$\approx \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} \hat{p}_i^t\right]^{1/N^t}} \approx \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} p_i^t\right]^{1/N^t}}.$$

7.174 Alternatively, the characteristics of models existing in period $t + 2$ can be inserted into a regression for period t. Predicted prices of period $t + 2$ items generated at period t shadow prices, $p_i^t(z_i^{t+2})$, are the prices of items existing in period $t + 2$ estimated at period t prices, and these prices (or an average) can be compared with the actual prices (or the average of prices) in period $t + 2$, $p_i^{t+2}(z_i^{t+2})$; a Jevons hedonic current-period index is

$$(7.29b)\quad P_{JHC} = \frac{\left[\prod_{i=1}^{N^{t+2}} p_i^{t+2}(z_i^{t+2})\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} p_i^t(z_i^{t+2})\right]^{1/N^{t+2}}}$$

$$= \frac{\left[\prod_{i=1}^{N^{t+2}} \hat{p}_i^{t+2}\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} p_i^t(z_i^{t+2})\right]^{1/N^{t+2}}} = \frac{\left[\prod_{i=1}^{N^{+2t}} p_i^{t+2}\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} p_i^t(z_i^{t+2})\right]^{1/N^{t+2}}}.$$

7.175 For a fixed-base, bilateral comparison using either equation (7.29a) or (7.29b), the hedonic equation is estimated for only one period, the current period $t + 2$ in equation (7.29a) and the base period t in equation (7.29b). For reasons analogous to those explained in Chapters 15, 16, and 17, a symmetric average of these indices would have some theoretical support. It would be useful as a retrospective study to compare the results from both approaches (7.29a) and (7.29b). If the discrepancy is large, the results from either should be treated with caution, similar to the way a large Laspeyres and Paasche spread would cast doubt on the use of either of these indices individually. It would be evidence for the need to update the regressions more often.

7.176 Note that a geometric mean of equations (7.29a) and (7.29b) uses all of the data available in each period, as does the hedonic index using a time dummy variable in (7.28). If in (7.28) there is a new product in period $t + 2$, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old products drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure, unlike using matched data and hedonic adjustments on noncomparable replacements when products are no longer available.

7.177 With the dummy variable approach, there is no explicit weighting in its formulation in equations (7.29a) and (7.29b), and this is a serious disadvantage. In practice, cutoff sampling might be employed to include only the most important products or if value of output data are available, a WLS—as opposed to OLS—estimator used with value of output shares as weights, as discussed in Chapter 21, Appendix 21.1.

7.178 The indices ask counterfactual questions. Asking what the price of a model with characteristics z would have been if it had been on the market in a period ignores the likelihood that the appearance of that model would in turn alter the demand for other computers, thus altering the coefficients of the hedonic regression as well. The matter is particularly problematic when *backcasting*, that is, using a current period's specification in some previous period's regression as in equations (7.29a) and (7.29b). If the specifications increase rapidly, it may not be sensible to ask the value of some high-tech model when such technology was in an earlier stage of development. It should be kept in mind that hedonic coefficients may as much reflect production technology as demand (see Chapter 21), and old technologies simply may not have been able to produce goods to the standards of later ones. The question reversed—what would be the value of a previous period's specification in a subsequent period's regression—while subject to similar problems, may be more meaningful. In general, the solution lies in estimating regressions as often as possible, especially in markets subject to rapidly changing technologies.

G.2.3 Superlative and exact hedonic indices (SEHI)

7.179 In Chapter 15, Laspeyres and Paasche bounds will be defined on a theoretical basis, as will superlative indices, which treat both periods' data symmetrically. These superlative formulas, in particular the Fisher index, are also seen in Chapter 14 to have desirable axiomatic properties. The Fisher index is supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds and was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index is shown to be best from the stochastic viewpoint and also does not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices are found to correspond to (be *exact* for) underlying Leontief aggregator functions with no substitution possibilities, while superlative indices are exact for flexible functional forms including the quadratic and translog forms for the Fisher and Törnqvist indices, respectively.

7.180 If data on prices, characteristics, *and quantities* are available, analogous approaches and findings arise for hedonic indices (Fixler and Zieschang, 1992a, and Feenstra, 1995). Exact theoretical bounds on a hedonic index have been defined by Feenstra (1995). Consider a theoretical index now defined only over products defined in terms of their characteristics. The prices are still of products, but they are wholly defined through their characteristics *p(z)*. An arithmetic aggregation for a linear hedonic equation finds a Laspeyres lower bound (as quantities supplied are *increased* with increasing relative prices) is given by

$$(7.30a)\quad \frac{R(p(z)^{t+2},\, S(v)^{t+2})}{R(p(z)^{t},\, S(v)^{t})} \geq \frac{\sum_{i=1}^{N} x_i^t \hat{p}_i^{t+2}}{\sum_{i=1}^{N} x_i^t p_i^t} = \sum_{i=1}^{N} s_i^t \left(\frac{\hat{p}_i^{t+2}}{p_i^t}\right),$$

where R denotes the revenue function at a set of output prices, p, input quantities, x, and technology, $S(v)$, following the fixed-input output price index model. The price comparison is evaluated at a fixed level of period t technology and inputs. s_i^t are the shares in total value of output of product i in period t, $s_i^t = x_i^t p_i^t \Big/ \sum_{i=1}^{N} x_i^t p_i^t$ and

$$(7.30b)\quad \hat{p}_i^{t+2} \equiv p_i^{t+2} - \sum_{i=1}^{N} \beta_k^{t+2} (z_{ik}^{t+2} - z_{ik}^t)$$

are prices in periods $t + 2$ adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. Note that the summation is over the same i in both periods since replacements are included when a product is missing and equation (7.30b) adjusts their prices for quality differences.

7.181 A Paasche upper bound is estimated as

$$(7.31a)\quad \frac{R(p(z)^{t+2},\, S(v)^{t+2})}{R(p(z)^{t},\, S(v)^{t+2})} \geq \frac{\sum_{i=1}^{N} x_i^{t+2} \hat{p}_i^{t+2}}{\sum_{i=1}^{N} x_i^{t+2} p_i^t} = \left[\sum_{i=1}^{N} s_i^{t+2} \left(\frac{\hat{p}_i^{t}}{p_i^{t+2}}\right)\right]^{-1},$$

where $s_i^{t+2} = x_i^{t+2} p_i^{t+2} \Big/ \sum_{i=1}^{N} x_i^{t+2} p_i^{t+2}$ and

$$(7.31b)\quad \hat{p}_i^{t} \equiv p_i^{t} + \sum_{i=1}^{N} \beta_k^{t} (z_{ik}^{t+2} - z_{ik}^t),$$

which are prices in period t adjusted for the sum of the changes in each quality characteristic weighted by its respective coefficients derived from a linear hedonic regression.

7.182 In Chapter 17, it is shown that Laspeyres, P_L, and Paasche, P_P, price indices form bounds on their respective true economic theoretic indexes. Using reasoning similar to that in Chapter 17 applied to equations (7.31a) and (7.31b), it can be shown that under homothetic preferences

$$(7.32)\ P_L \leq P(p^0, p^1, \alpha) \leq P_P .$$

7.183 The approach is similar to that used for adjustments to noncomparable replacement items in equation (7.27). First, the SEHI approach uses all of the data in each period, not just the matched sample and selected replacements. Second, it uses coefficients from hedonic regressions on changes in the characteristics to adjust observed prices for quality changes. Third, it incorporates a weighting system using data on the value of output of each model and their characteristics, rather than treating each model as equally important. Finally, it has a direct correspondence to formulation defined from economic theory.

7.184 Semi-logarithmic hedonic regressions would supply a set of β coefficients suitable for use with these base- and current-period geometric bounds:

$$(7.33a)\ \prod_{i=1}^{N}\left(\frac{p_i^{t+2}}{\hat{p}_i^{t}}\right)^{s_i^{t+2}} \geq \frac{R(p(z)^{t+2}, q, T)}{R(p(z)^{t}, q, T)} \geq \prod_{i=1}^{N}\left(\frac{\hat{p}_i^{t+2}}{p_i^{t}}\right)^{s_i^{t}},$$

where $\hat{p}_i^t \equiv p_i^t \exp[\sum_{i=1}^{N} \beta_k^t (z_{ik}^{t+2} - z_{ik}^t)]$ and

$$(7.33b)\ \hat{p}_i^{t+2} \equiv p_i^{t+2} \exp[-\sum_{i=1}^{N} \beta_k^{t+2} (z_{ik}^{t+2} - z_{ik}^t)] .$$

7.185 In equation (7.33a), the two bounds on their respective theoretical indices have been shown to be brought together under an assumption of homothetic preference (see Chapter 17). The calculation of such indices is no small task. For examples of its application see Silver and Heravi (2001a and 2003) for comparisons over time and Kokoski, Moulton, and Zieschang (1999) for price comparisons across areas of a country.

7.186 Note that unlike the hedonic indices in Sections G.2.1 and G.2.2, the indices in equations (7.30b), (7.31b), and (7.33b) need not be based on matched data. Kokoski, Moulton, and Zieschang (1999) used a sample from a replacement universe of otherwise matched data from the U.S. Bureau of Labor Statistics CPI, although the sample benefited from rotation. Silver and Heravi (2001a and 2003) used scanner data for the universe of transactions via a two-stage procedure. First, cells were defined according to major price-determining features much like strata; such features included all combinations of brand, outlet type, and screen size (for television sets). There may be a gain in the efficiency of the final estimate, since the adjustment is for within-strata variation, much in the way that stratified random sampling improves on simple random sampling. The average price in each matched cell could then be used for the price comparisons using equations (7.30a), (7.31a), or (7.33a), except that to ensure that the quality differences in each cell coming from characteristics other than these major ones did not influence the price comparison, adjustments were made for quality changes using equations (7.30b), (7.31b), or (7.33b). This allowed all matched, old unmatched, and new unmatched data to be included. If the average price in a cell of equation (7.30a) was increased because of the inclusion of a new improved product, equation (7.30b) would be used to remove such improvements, on average. For example, consider a brand *X*, 14-inch television set without stereo sound assembled by establishments in a given elementary aggregate industrial group. In the next period, there may be matched cells: 14-inch television set for brand *X*, which also includes stereo. The new model may have to be grouped in the same cell with the brand *X*, 14-inch television sets with and without stereo and the average price of the cells compared in equations (7.30a), (7.31a), or (7.33a), with a quality adjustment for the stereo of the form undertaken by equations (7.30b), (7.31b), or (7.33b). There may be a gain in the efficiency of the final estimate, since the adjustment is for within-strata variation, much in the way that stratified random sampling improves on simple random sampling. The estimated coefficient for stereo would be derived from a hedonic equation estimated from data of other television sets, some of which possess stereo.

7.187 The description above illustrates how weighted index number formulas such as

Laspeyres, Paasche, Fisher, and Törnqvist might be constructed using data on prices, quantities, and characteristics for a product. Silver and Heravi (2003) show that as the number of characteristics over which the summation takes place in equations (7.30a), (7.31a), or (7.33a) increases, the more redundant becomes the adjustment in equations (7.30b), (7.31b,) or (7.33b). When all characteristic combinations are used (equations [7.30a], [7.31a], or [7.33a]) as strata, the calculation extends to a matched-models problem, in which each cell uniquely identifies a product. For matched data, equations (7.30b), (7.31b, or (7.33b) serve no purpose and the aggregation in equations (7.30a), (7.31a), or (7.33a) would be over all products and reduce to the usual index number problem. Diewert (2003), commenting on the method, explains that when matching is relatively large, the results given are similar to those from superlative hedonic index numbers. Note that the theoretical indices in Chapter 21 are concerned with both goods that are hedonic tied bundles of characteristics *and* goods that are nonhedonic commodities. The framework of equations (7.30), (7.31), or (7.33) allows both types of goods to be included, and there are no adjustments necessary in equations (7.30b), (7.31b), or (7.33b) for the latter nonhedonic ones.

7.188 The above has illustrated how weighted index number formulas might be constructed using data on prices, quantities, and characteristics for a product when the data are not matched. This is because continuing with matched data may lead to errors from (i) multiple quality adjustments from products no longer produced and their noncomparable replacements and (ii) sample selectivity bias from sampling from a replacement universe as opposed to a double universe.

G.2.4 Difference between hedonic indices and matched indices

7.189 In previous sections, the advantages of hedonic indices over matched comparisons were referred to in terms of the inclusion by the former of unmatched data. This relationship is developed more formally here. Triplett (2002) argued and Diewert (2003) showed that an unweighted geometric mean (Jevons) index for matched data gives the same result as a logarithmic hedonic index run on the same data. Consider the matched sample m and z^{t+2} and z^t as overall quality adjustments to the dummy variables for time in equation (7.28), that is, $\sum_{k=2}^{K} \beta_k z_{ki}$. The very first line in equation (7.34) is shown by Aizcorbe, Corrado, and Doms (2001) to equal the difference between two geometric means of quality-adjusted prices. The sample space $m = M^t = M^{t+2}$ is the same model in each period. Consider the introduction of a new model n introduced in period $t + 2$ with no counterpart in t and the demise of an old model o so it has no counterpart in $t + 2$. So M^{t+2} is composed of m and n, and M^t is composed of m and o, and M consists only of the matched models m. Silver and Heravi (2002) have shown the dummy variable hedonic comparison to now be

$$(7.34) \ln p^{t+2}/p^t$$

$$= [m/(m+n)\sum_m (\ln p_m^{t+2} - Z_m)/m$$

$$+ n/(m+n)\sum_n (\ln p_n^{t+2} - Z_n)/n]$$

$$- [m/(m+o)\sum_m (\ln p_m^{t} - Z_m)/m$$

$$+ o/(m+o)\sum_o (\ln p_o^{t} - Z_o)/o]$$

$$= [m/(m+n)\sum_m (\ln p_m^{t+2} - Z_m)/m$$

$$- m/(m+o)\sum_m (\ln p_m^{t} - Z_m)/m]$$

$$+ [n/(m+n\sum_n (\ln p_n^{t+2} - Z_n)/n$$

$$- o/(m+o)\sum_o (\ln p_o^{t} - Z_o)/o].$$

7.190 Consider the *second* expression in equation (7.34). First, there is the change for the m matched observations, the quality adjustment being redundant. This is the change in mean prices of matched models m in period $t + 2$ and t adjusted for quality. Note that the weight in period $t + 2$ for this matched component is the proportion of matched to all observations in period $t + 2$. Similarly, for period t the matched weight depends on how many unmatched old observations are in the sample in this period. In the last line of equation (7.34), the change is between the unmatched new and the unmatched old mean (quality-adjusted) prices in periods $t + 2$ and t. Thus, matched methods can be seen to ignore the last line in equation

(7.34) and will differ from the hedonic dummy variable approach in at least this respect. The hedonic dummy variable approach, in its inclusion of unmatched old and new observations, can be seen from equation (7.34) to possibly differ from a geometric mean of matched price change. The extent of any difference depends, in this unweighted formulation, on the proportions of old and new products leaving and entering the sample and on the price changes of old and new ones relative to those of matched ones. If the market for products is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes (see Silver and Heravi, 2002, and Berndt, Ling, and Kyle, 2003, for examples). Different market behavior and changes in technology will lead to different forms of bias.

7.191 If sales weights replace the number of observations in equation (7.34), then different forms of weighted hedonic indices can be derived as explained in Chapter 21, Section A.5. Silver (2002) has also shown that the hedonic approach will differ from a corresponding weighted or unweighted hedonic regression in respect to the leverage and influence the hedonic regression gives to observations.

G.3 Chaining

7.192 An alternative approach for dealing with products with a high turnover is to use a chained index instead of the long-term fixed-base comparison. A chained index compares prices of items in period t with period t + 1 ($\text{Index}_{t,t+1}$) and then as a new exercise, studies the universe of products in period t + 1 and matches them with items in period t + 2. These links, $\text{Index}_{t,t+1}$ and $\text{Index}_{t+1,\ t+2,}$ are combined by successive multiplication continuing to, say, $\text{Index}_{t+5,t+6}$ to form $\text{Index}_{t,t+6.}$ Only items available in both period t and period t + 6 would be used in a fixed-base PPI. Consider the five products 1, 2, 5, 6, and 8 over the four months January to April as shown in Table 7.2. The price index for January compared with February (J:F) involves price comparisons for all five products. For (F:M), it involves products 1, 4, 5, and 8; for (M:A), it involves products 1, 3, 4, 5, 7, and 8. The sample composition changes for each comparison as products die and are born. Price indices can be calculated for each of these successive price comparisons using any of the unweighted formulas described in Chapter 21. The sample will grow when new products appear and shrink when old products disappear, changing in composition through time (Turvey, 1999).

7.193 Sample depletion may be reduced in long-run comparisons by the judicious use of replacement items. However, as discussed in the next chapter, the replacement sample would include a new product only when a replacement was needed, irrespective of the number of new products entering the market. Furthermore, the replacement product is likely to be either of a similar quality, to facilitate quality adjustment and thus have relatively low sales, or be of a different quality with relatively high sales but requiring an extensive quality adjustment. In either case, this is unsatisfactory.

7.194 Chaining, unlike hedonic indices, does not use all the price information in the comparison for each link. Products 2 and 6, for example, may be missing in March. The index makes use of the price information on products 2 and 6, when they exist, for the January–February comparison but does not allow their absence to disrupt the index for the February–March comparison. It may be that product 4 is a replacement for product 2. Note how easily it is included as soon as two price quotes become available. There is no need to wait for rebasing or sample rotation. It may be that product 7 is a replacement for product 6. A quality adjustment to prices may be required for the February–March comparison between products 6 and 7, but this is a short-run, one-off adjustment, The compilation of the index continues for March–April using product 7 instead of product 6. *SNA* (1993, Chapter 16, paragraph 16.54) picks up on the point in its sections on price and volume measurement:

> In a time series context, the overlap between the products available in the two periods is almost bound to be greatest for consecutive time periods (except for sub-annual data subject to seasonal fluctuations). The amount of price and quantity information that can be utilized directly for the construction of the price or volume indices is, therefore, likely to be maximized by compiling chain indices linking adjacent time periods. Conversely, the further apart the two time periods are, the smaller the overlap between the ranges of products available in the two periods is likely to be, and the more necessary it becomes to resort to implicit methods of price compari-

sons based on assumptions. Thus, the difficulties created by the large spread between the direct Laspeyres and Paasche indices for time periods that are far apart are compounded by the practical difficulties created by the poor overlap between the sets of products available in the two periods.

7.195 The chained approach has been justified as the natural discrete approximation to a theoretical Divisia index (Forsyth and Fowler, 1981, and Chapter 16). Reinsdorf (1998b) has formally determined the theoretical underpinnings of the index, concluding that in general, chained indices will be good approximations of the theoretical ideal. However, they are prone to bias when price changes "swerve and loop," as Szulc (1983) has demonstrated (see also Forsyth and Fowler, 1981, and de Haan and Opperdoes, 1997).

7.196 The dummy variable hedonic index uses all of the data in January and March for a price comparison between the two months. Yet the chained index ignores unmatched successive pairs as outlined above; nevertheless, this is preferable to its fixed-base equivalent. The hedonic approach, by predicting from a regression equation, naturally has a confidence interval attached to such predictions. The width of the interval is dictated by the fit of the equation, the distance of the characteristics from their mean, and the number of observations. Matching, chained or otherwise, does not suffer from any prediction error. Aizcorbe, Corrado, and Doms (2001) undertook an extensive and meticulous study of high-technology goods (personal computers and semiconductors) using quarterly data for the period 1993–1999. The results from comparable hedonic and chained indices were remarkably similar over the seven years of the study. For example, for desktop central processing units, the index between the seven years of 1993: Q1 and 1999: Q4 fell by 60.0 percent (dummy variable hedonic), 59.9 percent (chained Fisher), and 57.8 percent (chained geometric mean). The results differed only in quarters when there was a high turnover of products, and, in these cases, such differences could be substantial. For example, for desktop central processing units in 1996: Q4, the 38.2 percent annual fall measured by the dummy variable hedonic method differed from the chained geometric mean index by 17 percentage points. Thus, with little model turnover, there is little discrepancy between hedonic and chained matched-models methods and, for that matter, fixed-base matched indices. It is only when binary comparisons or links have a high model turnover that differences arise (see also Silver and Heravi, 2001a and 2003).

7.197 There is a possibility that the introduction of new models and exits of old ones instantaneously affects the prices of all existing models. In such a case, the price changes of existing models will suffice. They will reflect the price changes of new entrants and old departures not part of the sample. This argument is used for the case that direct matched-models comparisons, chained matched-models comparisons, and hedonic indices should give the same results. It is an empirical matter, and its plausibility will vary among industries. It is more likely to apply to fast-moving goods with little to no development costs or barriers to entry.

7.198 It is possible to make up for missing prices by using a partial, patched hedonic estimate as discussed above. Dulberger (1989) computed hedonic indices for computer processors and compared the results with those from a matched-models approach. The hedonic dummy variable index fell by about 90 percent from 1972–1984, about the same as for the matched-models approach where missing prices for new or discontinued products were derived from a hedonic regression. However, when using a chained matched-models approach with no estimates or imputations for missing prices, the index fell by 67 percent. It is also possible to combine methods; de Haan (2003) used matched data when available and the time dummy only for unmatched data—his double-imputation method.

H. Long-Run and Short-Run Comparisons

7.199 This section outlines a formula to help quality adjustment. The procedure can be used with all of the methods outlined in Sections D and E. Its innovation arises from a possible concern with the long-run nature of the quality-adjusted price comparisons being undertaken. In the example in Table 7.2, prices in March were compared with those in January. Assumptions of similar price changes are required by the imputation method to hold over this period for long-run imputations. This gives rise to increasing concern when

Table 7.5. Example of Long-Run and Short-Run Comparisons

Item	January	February	March	April	May	June
Comparable replacement						
A	2	2	2	2	2	2
B	3	3	4	n/a	n/a	n/a
C	n/a	n/a	n/a	6	7	8
Total	5	5	6	8	9	10
Explicit adjustment						
A	2	2	2	2	2	2
B	3	3	4	**5/6 x 6=5**	**5/6 x 7=5.8**	**5/6x8= 6.67**
C	**6/5 x 3=3.60**	n/a	n/a	6	7	8
Total	5	5	6	8	9	10
Overlap						
A	2	2	2	2	2	2
B	3	3	4	**6 x 4/5=4.8**	n/a	n/a
C	n/a	n/a	5	6	7	8
Total	5	5	6	6.8	9	10
Imputation						
A	2	2	2.5	3.5	4	5
B	3	3	4	**3.5/2.5 x 4= 5.6**	**4/3.5 x 5.6=6.4**	**5/4 x 6.4=8**
Total	5	5	6.5	9.1	8.4	13

Figures in bold are estimated quality-adjusted prices described in the text.
Note: n/a = not available.

price comparisons continue over longer periods, such as between January and October, January and November, and January and December, and even subsequently. In this section, a *short-run* formulation outlined in Sections C.3.3 and D.2 is more formally considered to help alleviate such concerns. Consider Table 7.5, which, for simplicity, has a single product *A* that exists throughout the period, a product *B* that is permanently missing in April, and a possible replacement *C* in April.

H.1 Short-run comparisons: illustration of some quality adjustment methods

7.200 A *comparable replacement C* may be found. In the previous example, the focus was on the use of the Jevons index at the elementary level since it is shown in Chapter 20 that this has much to commend it. The example here uses the Dutot index, the ratio of arithmetic means. This is not to advocate it but only to provide an example using a different formulation. The Dutot index also has much to commend it on axiomatic grounds but fails the commensurability (units of measurement) test and should be used only for relatively homogeneous items. The long-run Dutot index for April compared with January is

$$P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{\text{Apr}} / N}{\sum_{i=1}^{N} p_i^{\text{Jan}} / N} \right],$$

which is 8/5 = 1.30, a 30 percent increase. The *short-run* equivalent is the product of a long-run index up to the immediately preceding period and an index for the preceding to the current period, that is, for period $t + 4$ compared with period t:

$$(7.35)\ \ P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{t+3}/N}{\sum_{i=1}^{N} p_i^{t}/N}\right] \times \left[\frac{\sum_{i=1}^{N} p_i^{t+4}/N}{\sum_{i=1}^{N} p_i^{t+3}/N}\right],$$

or, for example, using a comparison of January with April:

$$P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{Mar}\ /\ N}{\sum_{i=1}^{N} p_i^{Jan}\ /\ N}\right] \times \left[\frac{\sum_{i=1}^{N} p_i^{Apr}\ /\ N}{\sum_{i=1}^{N} p_i^{Mar}\ /\ N}\right],$$

which is of course $\frac{6}{5} \times \frac{8}{6} = 1.30$ as before.

7.201 Consider a *noncomparable replacement with an explicit quality adjustment*: say *C*'s value of 6 in April is quality-adjusted to be considered to be worth only 5 when compared with the quality of *B*. The quality adjustment to prices may have arisen from an option cost estimate, a quantity adjustment, a subjective estimate, or a hedonic coefficient as outlined above. Suppose the long-run comparison uses an adjusted January price for *C*, which is *B*'s price of 3 multiplied by 6/5 to upgrade it to the quality of *C*, that is, 6/5 × 3 = 3.6. From April onward, the prices of the replacement product *C* can be readily compared with its January reference period price. Alternatively, the prices of *C* in April onward might have been adjusted by multiplying them by 5/6 to downgrade them to the quality of *B* and enable comparisons to take place with product *B*'s price in January: for April, the adjusted price is 5/6 × 6 = 5; for May, the adjusted price is 5.8; and for June, it is 6.67 (see Table 7.5). Both procedures yield the same results for long-run price comparisons. The results from both methods (rounding errors aside) are the same for product *B*.

7.202 However, for the overall Dutot index, the results will differ because the Dutot index weights price changes by their price in the initial period as a proportion of total price (Chapter 20, equation [20.1]). The two quality adjustment methods will have the same price changes but different implicit weights. The Dutot index in May is 9/5.6 = 1.607 using an adjustment to the initial period, January's price, and 7.8/5 = 1.56 using an adjustment to the current period, May's price. The short-run indices give the same results for each adjustment:

$\frac{8}{5.6} \times \frac{9}{8} = 1.607$ using an adjustment to the initial period, January's price, and

$\frac{7}{5} \times \frac{7.8}{7} = 1.56$ using an adjustment to the current period, May's price.

7.203 The *overlap method* may also take the short-run form. In Table 7.5, there is a price for *C* in March of 5 that overlaps with *B* in March. The ratio of these prices is an estimate of their quality difference. A long-run comparison between January and April would be $\left(6 \times \frac{4}{5} + 2\right) \Big/ 5$ = 1.36. The short-run comparison would be based on the product of the January to March and March to April link: $\frac{6.8}{6} \times \frac{6}{5} = 1.36$.

7.204 At this unweighted level of aggregation, it can be seen that there is no difference between the long-run and short-run results when products are not missing, comparable replacements are available, explicit adjustments are made for quality, or the overlap method is used. The separation of short-run (most recent month-on-month) and long-run changes may have advantages for quality assurance to help spot unusual short-run price changes. But this is not the concern of this chapter. The short-run approach does, however, have advantages when imputations are made.

H.2 Implicit short-run comparisons using imputations

7.205 The use of the short-run framework has been considered mainly for temporarily missing values, as outlined by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001). However, similar issues arise in the context of quality adjustment. Consider again Table 7.5, but this time there is no replacement product *C* and product *A*'s prices have been changed to trend upward. Product *B* is again missing in April. A long-run imputation for product *B* in April is given by $\frac{3.5}{2} \times 3 = 5.25$. The price change is thus $(5.25 + 3.5)/5 = 1.75$, or 75 percent. One gets the same result by simply using product *A* (3.5/2 = 1.75), since the implicit assumption is that price movements of product *B*, had it continued to exist, would have followed those of *A*. However, the assumption of similar long-run price movements may in some instances be difficult to support over long periods. An alternative approach would be to use a short-run framework whereby the imputed price for April is based on the (overall) mean price change between the preceding and current period, that is, $\frac{3.5}{2.5}\times 4 = 5.6$ in the above example. In this case, the price change between March and April is (5.6 + 3.5)/(2.5 + 4) = 1.40. This is combined with the price change between January and March: (6.5/5) = 1.30, making the price change between January and April $1.30 \times 1.40 = 1.82$, an 82 percent increase.

7.206 Consider why the short-run result of 82 percent is larger than the long-run result of 75 percent. The price change for *A* between March and April of 40 percent, on which the short-run imputation is based, is larger than the average *annual* change of *A*, which is just over 20 percent. The extent of any bias from this approach was found in the previous section to depend on the ratio of missing values and the difference between the average price changes of the matched sample and the quality-adjusted price change of the product that was missing, had it continued to exist. The short-run comparison is to be favored if the assumption of similar price changes is considered more likely to hold than the long-run one.

7.207 There are data on price changes of the product that is no longer available—product *B* in Table 7.5—up to the period preceding the period in which it is missing. In Table 7.5, product *B* has price data for January, February, and March. The long-run imputation makes no use of such data by simply assuming that price changes from January to April are the same for *B* as for *A*. Let the data for *B*'s prices in Table 7.5 (second to last row) now be 3, 4, and 6 in January, February, and March, respectively, instead of 3, 3, and 4. The long-run estimate for *B* in April is 5.25 as before. The estimated price change between March and April for *B* is now a *fall* from 6 to 5.25. A short-run imputation based on the price movements of *A* between March and April would more correctly show an increase from 6 to (3.5/2.5) × 6 = 8.4.

7.208 There may, however, be a problem with the continued use of short-run imputations. Returning to the data for *A* and *B* in Table 7.5, consider what happens in May. Adopting the same short-run procedure, the imputed price change is given in Table 7.5 as 4/3.5 × 5.6 = 6.4 and for June as (5/4) × 6.4 = 8. In the former case, the price change from January to May is

$$\left[\frac{(6.4+4)}{(5.6+3.5)}\right]\times\left[\frac{(5.6+3.5)}{(3+2)}\right]=2.08$$

and in the case of June

$$\left[\frac{(8+5)}{(6.4+4)}\right]\times\left[\frac{(6.4+4)}{(3+2)}\right]=2.60$$

against long-run comparisons for May:

$$\left[\frac{((4/2)\times 3+4)}{(3+2)}\right]=2.00$$

and long-run comparisons for June:

$$\left[\frac{((5/2)\times 3+5)}{(3+2)}\right]=2.50\,.$$

7.209 A note of caution is required here. The comparisons use an imputed value for product *B* in April and also an imputed one for May. The price comparison for the second term in equation (7.35), for the current versus immediately preceding period, uses imputed values for product *B*. Similarly, for the January to June results, the May to June comparison uses imputed values for product *B* for both May and June. The pragmatic needs of quality adjustment may demand this. If comparable replacements, overlap links, and resources for explicit quality adjustment are unavailable, an imputation must be considered. However, using imputed values as lagged values in short-run comparisons introduces a level of error into the index that will be compounded with their continued use. Long-run imputations are likely to be preferable to short-run changes based on lagged imputed values unless there is something in the nature of the industry that cautions against such long-run imputations. There are circumstances when the respondent may believe the missing product is missing temporarily, and the imputation is conducted under the expectation that production will subsequently continue. A wait-and-see policy is adopted under some rules—three months, for example—after which it is deemed to be permanently missing. These are the pragmatic situations that require imputations to extend over consecutive periods. These circumstance promote lagged imputed values to compare against current imputed values. This is cautioned against, especially over a period of several months. There is an intuition that the period in question should not be extensive. First, the effective sample size is being eaten up as the use of imputation increases. Second, the implicit assumptions of similar price movements inherent in imputations are less likely to hold over the longer run. Finally, there is some empirical evidence, albeit from a different context, against using imputed values as lagged actual values. (See Feenstra and Diewert's 2001 study using data from the U.S. Bureau of Labor Statistics for their International Price Program.)

7.210 The short-run approach described above will be developed in the next section, where weighted indices are considered. The practice of estimating quality-adjusted prices is usually at the elementary product level. At this lower level, the prices of products may subsequently be missing and replacements with or without adjustments and imputations are used to allow the series to continue. New products and varieties are also being introduced; the switching of sales between sections of the index becomes prevalent. The turmoil of changing quality is not just about the maintaining of similar price comparisons but also about the accurate reweighting of the mix of what is produced. Under a Laspeyres framework, the bundle is held constant in the base period, so any change in the relative importance of products produced is held to be of no concern until the next rebasing of the index. Yet capturing some of the very real changes in the mix of what is produced requires procedures for updating the weights. This was considered in Chapter 5. The concern here is with a higher-level procedure equivalent to the short-run adjustments discussed above. It is one particularly suited to countries where resource constraints prohibit the regular updating of weights through regular household surveys.

H.3 Single-stage and two-stage indices

7.211 Consider aggregation at the elementary level (Chapter 6). This is the level at which prices are collected from a representative selection of establishments across regions in a period and compared with the matched prices of the same products in a subsequent period to form an index for a good. Lamb is an example of a good in an index. Each price comparison is equally weighted unless the sample design gave proportionately more chance of selection to products with more sales. The elementary price index for lamb is then weighted and combined with the weighted elementary indices for other products to form the PPI. A Jevons elementary aggregate index, for example, for period $t + 6$ compared with period t is given as

$$(7.36)\quad P_J \equiv \prod_{i \in N(t+6) \cap N(t)}^{N} \left(p_i^{t+6} / p_i^{T} \right).$$

Compare this with a two-stage procedure:

$$(7.37)\quad P_J \equiv \prod_{i \in N(t+5) \cap N(t)}^{N} \left(p_i^{t+5} / p_i^{T} \right) \times \prod_{i \in N(t+6) \cap N(t+5)}^{N} \left(p_i^{t+6} / p_i^{T+5} \right).$$

7.212 If a product is missing in period $t + 6$, an imputation may be undertaken. If equation (7.36) is used, the requisite assumption is that the price change of the missing product, had it continued, is equal to that of the average of the remaining products *over the period* t *to* t + *6*. In equation (7.37), the missing product in period $t + 6$ may be included in the first stage of the calculation, between periods t and $t + 5$, but excluded in the second stage, between periods $t + 5$ and $t + 6$. The requisite assumption is that price changes *between* t – *1 and* t are similar. Assumptions of short-run price changes are generally considered to be more valid than their long-run counterparts. The two-stage framework also has the advantage of including in the worksheet prices for the current period and the immediately preceding one, which, as will be shown in Chapter 9, promotes good data validity checks.

7.213 Feenstra and Diewert (2001) applied a number of mainly short-run imputation procedures to price comparisons for the U.S. Bureau of Labor Statistics International Price Program (IPP). Although such price indices are not the direct interest of this *Manual*, the fact that about one-quarter of the individual products tracked did not have price quotations in any given month makes it an interesting area to explore the results from different imputation procedures. When using the two-stage procedure, they advise against carrying forward imputed prices as if they were actual values for the subsequent price comparison. The resulting price relatives for the subsequent period based on prior imputations had a standard deviation about twice that of price relatives where no imputation was required, leading them to conclude that such a practice introduced a significant amount of “noise” into the calculation. Feenstra and Diewert (2001) found more variance in price changes in the long-run imputation method than the short-run method. They also found from both theory and empirical work that when actual prices are available in a future data set and they are used to interpolate back on a linear basis the missing prices, such estimates lead to much lower variances than the short-run imputation approach. However, such linear interpolations require the statistical agency to store past information until a price quote becomes available, interpolate back the missing price, and then publish a revised PPI.

Appendix 7.1: Data for Hedonic Regression Illustration

Price (£)	Speed (MHz)	RAM	HD	Dell	Presario	Prosignia	Celeron	Pentium III	CD-RW	DVD	Dell × Speed
2,123	1,000	128	40	0	1	0	0	0	0	0	0
1,642	700	128	40	0	1	0	0	0	0	0	0
2,473	1,000	384	40	0	1	0	0	0	0	0	0
2,170	1,000	128	60	0	1	0	0	0	0	0	0
2,182	1,000	128	40	0	1	0	0	0	0	1	0
2,232	1,000	128	40	0	1	0	0	0	1	0	0
2,232	1,000	128	40	0	1	0	0	0	0	0	0
1,192	700	384	40	0	1	0	0	0	0	0	0
1,689	700	384	60	0	1	0	0	0	0	0	0
1,701	700	384	40	0	1	0	0	0	0	1	0
1,751	700	384	40	0	1	0	0	0	1	0	0
1,851	700	384	40	0	1	0	0	0	0	0	0
2,319	933	128	15	0	0	0	0	1	0	0	0
2,512	933	256	15	0	0	0	0	1	0	0	0
2,451	933	128	30	0	0	0	0	1	0	0	0
2,270	933	128	10	0	0	0	0	1	0	0	0
2,463	933	256	10	0	0	0	0	1	0	0	0
2,183	933	64	10	0	0	0	0	1	0	0	0
1,039	533	64	8	0	0	1	1	0	0	0	0
1,139	533	128	8	0	0	1	1	0	0	0	0
1,109	533	64	17	0	0	1	1	0	0	0	0
1,180	533	64	8	0	0	1	1	0	1	0	0
1,350	533	128	17	0	0	1	1	0	1	0	0
1,089	600	64	8	0	0	1	0	1	0	0	0
1,189	600	128	8	0	0	1	0	1	0	0	0
1,159	600	64	17	0	0	1	0	1	0	0	0
1,230	600	64	8	0	0	1	0	1	1	0	0
1,259	600	128	17	0	0	1	0	1	0	0	0
1,400	600	128	17	0	0	1	0	1	1	0	0
2,389	933	256	40	0	1	0	0	1	0	0	0
1,833	733	256	40	0	1	0	0	1	0	0	0

Appendix 7.1 (concluded)

Price (£)	Speed (MHz)	RAM	HD	Dell	Presario	Prosignia	Celeron	Pentium III	CD-RW	DVD	Dell× Speed
2,189	933	128	40	0	1	0	0	1	0	0	0
2,436	933	256	60	0	1	0	0	1	0	0	0
2,397	933	256	40	0	1	0	0	1	0	1	0
2,447	933	256	40	0	1	0	0	1	1	0	0
2,547	933	256	40	0	1	0	0	1	0	0	0
2,845	933	384	60	0	1	0	0	1	0	0	0
2,636	933	384	60	0	1	0	0	1	0	0	0
1,507	733	64	30	0	1	0	0	1	0	0	0
1,279	667	64	10	1	0	0	0	1	0	0	667
1,379	667	128	10	1	0	0	0	1	0	0	667
1,399	667	64	30	1	0	0	0	1	0	0	667
1,499	667	128	30	1	0	0	0	1	0	0	667
1,598	667	128	30	1	0	0	0	1	1	0	667
1,609	667	128	30	1	0	0	0	1	0	1	667
1,389	667	64	10	1	0	0	0	1	0	1	667
999	667	64	10	1	0	0	1	0	0	0	667
1,119	566	64	30	1	0	0	1	0	0	0	566
1,099	566	128	10	1	0	0	1	0	0	0	566
1,097	566	64	10	1	0	0	1	0	1	0	566
1,108	566	64	10	1	0	0	1	0	0	1	566
1,219	566	128	30	1	0	0	1	0	0	0	566
1,318	566	128	30	1	0	0	1	0	1	0	566
1,328	566	128	30	1	0	0	1	0	0	1	566
1,409	566	128	10	1	0	0	0	1	0	0	733
1,809	733	384	10	1	0	0	0	1	0	0	733
1,529	733	128	30	1	0	0	0	1	0	0	733
1,519	733	128	10	1	0	0	0	1	0	1	733
1,929	733	384	30	1	0	0	0	1	0	0	733
2,039	733	384	30	1	0	0	0	1	0	1	933
2,679	933	128	30	1	0	0	0	1	0	0	933
3,079	933	384	10	1	0	0	0	1	0	0	933
2,789	933	128	10	1	0	0	0	1	0	1	933
3,189	933	384	10	1	0	0	0	1	0	1	933

8. Item Substitution, Sample Space, and New Goods

A. Introduction

8.1 In the introduction to Chapter 7, the use of the matched-models method was recognized as the accepted approach to ensure that the measurement of price changes was untainted by changes in their quality. However, it was noted that the approach might fail in three respects: missing items, sampling issues, and new goods and services (hereafter "goods" includes services). Missing items are the subject of Chapter 7, in which several implicit and explicit methods of quality adjustment to prices, and the choice between them, were discussed. In this chapter, attention is turned to two other reasons why the matched-models method may fail: sampling issues and new goods. The three sources of potential error are briefly outlined.

- **Missing items.** A problem arises when an item is no longer produced. An implicit quality adjustment may be made using the overlap or imputation method, or the respondent may choose a replacement item of a comparable quality, and its price may be directly compared with the missing item's price. If the replacement is of a noncomparable quality, an explicit price adjustment is required. This was the subject of Chapter 7, Sections C through F. In Section G of Chapter 7 a caveat was added. Items in industries where model replacements were rapid, continued long-run matching would deplete the sample, and quality adjustment becomes unfeasible on the scale required. Chained matching or hedonic indices were deemed preferable.

- **Sampling issues.** The matching of prices of identical items over time, by its nature, is likely to lead to monitoring of a sample of items increasingly unrepresentative of the population of transactions. Respondents may keep with their selected items until they are no longer produced—that is, continue to monitor items with unusual price changes and limited sales. Yet on item replacement, respondents may select unpopular comparable items to avoid explicit quality adjustments; obsolete items with unusual price changes are replaced by near-obsolete items that also have unusual price changes, compounding the problem of unrepresentative samples. The substitution of an item with relatively high sales for an obsolete one has its own problems, since the difference in quality is likely to be substantial and substantive, beyond what can be attributed to, say, the price difference in some overlap period. One would be in the last stage of its life cycle and the other in its first. The issue has implications for sample rotation and item substitution.

- **New products.** A third potential difficulty arises when something "new" is produced. There is a difficulty in distinguishing between new items and quality changes in old ones, which will be discussed below. When a new good is produced, there is a need for it to be included in the index as soon as possible, especially if the product is expected to be responsible for relatively high sales. New goods might have quite different price changes than existing ones, especially at the start of their life cycle. In the initial period of introduction, producers often gain more from their ability to receive higher prices from their recently introduced product than might be attainable once the market settles into a competitive equilibrium. But by definition, there is no price in the period preceding the introduction of the *new* product. So even if prices of new products were obtained and included in the index from the initial introduction date, there would still be something missing—the initial high price producers can reap by exploiting any monopoly power in the period of launch.

8.2 The problem of missing items was the subject of Chapter 7. In this chapter, sampling issues arising out of the matched-models approach and the problem of introducing new goods into the index are considered.

B. Sampling Issues and Matching

B.1 Introduction

8.3 The matching procedure has at its roots a conundrum. Matching is designed to avoid price changes being contaminated by quality changes. Yet its adoption constrains the sampling to a static universe of items that exist in *both* the reference and base periods. Outside of this there is, of course, something more: items that exist in the reference period but not in the current period, and are therefore not matched; and similarly those new items existing in the current period but not in the reference one—the dynamic universe (Dalén, 1998, and Sellwood, 2001). The conundrum is that the items not in the matched universe, the new items appearing after the reference period and the old items that disappeared from the current period, may be the ones whose price changes differ substantially from existing matched ones. They will embody different technologies and be subject to different (quality-adjusted) strategic price changes. The very device used to maintain a constant quality sample may itself give rise to a sample biased away from technological developments. Furthermore, when this sample is used to make imputations (Chapter 7, Sections D.1 and D.2) as to the price changes of replacement items, it reflects the technology of a sample not representative of current technological changes.

8.4 A formal consideration of matching and the dynamic universe is provided in Appendix 8.1. Three universes are considered:

- An *intersection* universe, which includes only matched items;
- A dynamic *double universe*, which includes all items in the base comparison period and all in the current period, although they may be of different qualities; and
- A *replacement* universe, which starts with the base-period universe but also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

8.5 It is, of course, difficult to ascertain the extent to which matching from the intersection universe constrains the penetration of the sample into the dynamic double universe, since statistical agencies generally do not collect data for the latter. Its extent will, in any event, vary between commodities. Sellwood (2001) advocated simulations using the universe of scanner data. Silver and Heravi (2002) undertook such an experiment using scanner data on the consumer prices of washing machines in the United Kingdom in 1998. A matched Laspeyres index—based on price comparisons with matched models existing in both January and December—covered only 48 percent of the December expenditure on washing machines, as a result of new models that were introduced after January not being included in the matched index. Furthermore, the January to December matched comparison covered only just over 80 percent of the January expenditure, because of the exclusion of models available in January but not in December. A biannual sample rotation (rebasing) increased the December expenditure coverage to just over 70 percent and a monthly (chained) rotation to about 98 percent (see also Chapter 7, Section G.1, for further examples). Two implications arise from this. First, selection of item substitutes (replacements) puts coverage of the sample to some extent under the control of the respondents. Guidelines on directed replacements in particular product areas have some merit. Second, chaining, hedonic indices (as considered in Chapter 7, Section G) and regular sample rotation also have merit in some commodity areas as devices to refresh the sample.

B.2 Sample space and item replacement or substitution

8.6 The respondents often are best placed to select replacement items for repricing. They are aware of not only the technological basis of the items being produced but also their terms of sale. The selection of the replacement for repricing might be quite obvious to the respondent. There may be only a slight, nominal improvement to the item. For example, the "improved" item is simply a replacement variety sold instead of the previous one. The replacement could have a different code or model number and will be known to the respondent as simply a different color or packaging. The specification list given to the respondent is a critical prompt as to when a repriced item is different, and it is important that this include all price-determining factors.

8.7 The respondent prompted by the specification list takes on the role of identifying whether an item is of comparable quality or otherwise. If it is judged to be comparable when it is not, the quality difference is taken to be a price difference, and a bias will result if the unrecognized quality changes are in a consistent direction. Informed comparable substitution requires general guidelines on what makes a good substitute as well as product-specific information on likely price-determining characteristics. It also requires timely substitution to maximize the probability of an appropriate substitute being available.

8.8 Liegey (1994), for CPI purposes, notes how useful the results from hedonic regressions are in the selection of items. The results provide an indication of the major quality factors that make up the product or service, in terms of explaining price variation. Not only would the selection of items be more representative, but the coefficients from hedonic regressions, for their subsequent use to estimate quality-adjusted prices, would be more tailored to the sample in hand.

8.9 On repricing, respondents traditionally are required to find substitute items that are as similar as possible to the items being replaced. This maximizes the likelihood that the old and replacement item will be judged equivalent and so minimizes the need to employ some method of quality adjustment. Yet, replacement items should be chosen so that they intrude into the sampled items in a substantial and representative manner so as to make the sampled items more representative of the dynamic universe. The inclusion of a popular replacement item to refresh the sample—one at the same point in its life cycle as the original popular one selected in the base period—allows for a useful and accurate price comparison and increases the chance of an appropriate quality adjustment being undertaken. It is of little merit to substitute a new item with limited sales for a missing item with limited sales, just because they have similar features of both being "old." The index would become more unrepresentative. Yet if replacements are made for items at the end of their life with popular replacements items at the start of their life, the quality adjustment will be substantial and substantive. More frequent sample rotation or directed replacements will be warranted for some commodity areas.

- Replacements offer an opportunity to cut back on and possibly remove sample bias in the period of replacement, though not prior to it;
- The more frequent the replacement, the less the bias;
- If there is more than one new (replacement) item in the market, there may still be bias since only the most popular one will be selected, and it may be at a different stage in its life cycle than others and priced differently;
- The analysis assumes that perfect quality adjustments are undertaken on replacements. The less frequent the replacement, the more difficult this might be, because the very latest replacement item on the market may have more substantial differences in quality than earlier ones;
- If the replacement item has relatively high sales, is of comparable quality, and is at the same stage in its life cycle as the existing one, then its selection will minimize bias;
- If there is more than one new (replacement) item and the most comparable one is selected at the old technology, it will have low market share and unusual price changes;
- Given advance market or production information, replacements undertaken before obsolescence are likely to increase the sample's share of the market, include items more representtative of the market, and facilitate quality adjustment.

8.10 The problem of item substitution is analogous to the problems that arise when an establishment closes. It may be possible to find a comparable establishment not already in the sample, or a noncomparable one for which, in principle, an adjustment can be made for the better quality of service of the new one. It is not unusual for an establishment to close following the introduction of a new factory. Thus, here is an obvious replacement factory. However, if the new establishment has comparable prices but a better range of items, delivery, and service quality, there is a gain to purchasers from substituting one factory's output for the other. Yet, since such facilities have no direct price, it is difficult to provide estimates of the value of such services in order for an adjustment to be made for the better quality of service of the new one. The index thus would have an upward bias, which would be lost on rebasing. In such cases, substituting the old establishment for a new one that provides a similar standard of service may be preferable.

B.3 Sample rotation, chaining, and hedonic indices

8.11 It is important also to recognize the interrelationships among the methods for handling item rotation, item replacement, and quality adjustment. When PPI item samples are rotated, this is a form of item substitution, except that it is not "forced" by a missing item but is undertaken for a general group of items to update the sample. Rotation has the effect of making future forced replacements less likely. Yet the assumptions implicit in its use are equivalent to those for the overlap adjustment technique: price differences are an adequate proxy for the change in price per unit of quality between items disappearing from the sample and replacement items. Consider the initiation of a new sample of items by probability or judgmental methods or a combination of the two. Prices for the old and new sample are returned in the same month and the new index is compiled on the basis of the new sample, with the results being linked to the old. This is an implicit use of the overlap method, in which all price differences between the new and old items are taken to be quality changes. Assume the initiation is in January. The prices of an old item in December and January are 10 and 11, respectively, a 10 percent increase, and those for the replacement item in January and February are 16 and 18, respectively, an increase of 12.5 percent. The new item in January is of a better quality than the old, and this difference in quality may be worth 16 – 11 = 5; that is, the price difference is assumed to be equal to the quality difference, which is the assumption implicit in the overlap method. Had the price of the old item in December been compared with the quality-adjusted price of the new item in January under this assumption, the price change would be the same: 10 percent (that is, (16 – 5)/10 = 1.10). If, however, the quality difference in January between the two prices was more than the revenue difference to the producer, the result would be wrong. In practice, the need to simultaneously replace and update a large number of items requires the assumptions of the overlap method. This process should not be regarded as error-free, and in cases where the assumptions are likely to be particularly untenable (discussed in Chapter 7, Section D.2), explicit adjustments of the form discussed in Chapter 7, Section E, should, resources permitting, be used.

8.12 It was noted above that when samples are updated, any difference in average quality between samples is dealt with in a way that is equivalent to the overlap adjustment technique. Sample rotations to freshen the sample between rebasing are an expensive exercise. However, if rebasing is infrequent and there is a substantial loss of items in particular industries, then this might be appropriate for those industries. In the next section the need for a metadata system to facilitate such decisions will be outlined. The use of a more frequent sample rotation aids the process of quality adjustment in two ways. First, the updated sample will include newer varieties, comparable replacements with substantial sales will be more likely to be available, and noncomparable ones will be of a more similar quality, which will aid good explicit adjustments. Second, because the sample has been rotated, there will be fewer missing items than otherwise and thus less need for quality adjustments.

8.13 A natural extension of more frequent sample rotation is to use a chained formulation in which the sample is reselected each period. In Chapter 7, Section G.3, the principles and methods are outlined in the context of sectors in which there is a rapid turnover of items, and such principles are echoed here. Similarly, the use of hedonic indices as outlined in Chapter 7, Section G.2, or the use of short-run comparisons discussed in Chapter 7, Section H, might be useful in this context.

8.14 Chaining, as discussed in Chapter 7, Section G.3, allows the price changes of a new commodity to be included as soon as the commodity can be priced for two successive periods. The new price's effect on the index in the initial period of introduction is ignored. Fisher and Shell (1972) suggest that the preceding price is imputed as the reservation price given the current-period technology, where the reservation price is defined as the maximum price at which zero production of the good is forthcoming, given current-period inputs and prices of other outputs in the preceding period (see Hicks, 1940, and Hausman, 1997, for equivalent considerations for the consumer price index). Similar concerns arise for disappearing commodities. A disappearing good's price has to be imputed in the current period, which is imputed as the reservation price given the preceding period technology, defined as the maximum price in the current period at which no production of the good is forthcoming, given inputs in the preceding period and prices of other outputs in the current period. The estimation of such reservation prices is not practical, though

Hausman (1997) provides an example in the context of the CPI. If the new commodity is not entirely new, in the sense that it is providing more services than those of the old product, a hedonic estimate of the reservation price can be used to estimate the cost of the base situation characteristics for the missing price of the disappearing good or the cost of the current situation characteristics for the missing reference price of the new variety (Zieschang, 1988). However, this applies only when the good is not entirely new, so that the price can be determined in terms of a different combination of the existing character set.

C. Information Requirements for a Strategy for Quality Adjustment

8.15 It should be apparent from the above that a strategy for quality adjustment must not only be linked to sample representativity, but it also requires building a statistical metadata system. The approach for the index as a whole cannot be described simply. It requires the continual development of market information and the recording and evaluation of methods on a commodity-by-commodity basis. The rationale for such a metadata system relates to the variety of procedures for quality adjustments to prices discussed in Chapter 7, Section C.3.4, and how their suitability might vary on a case-by-case basis, all of which require documentation.

C.1 Statistical metadata system

8.16 The methods used for estimating quality-adjusted prices should be well documented as part of a statistical metadata system. Metadata is systematic, descriptive information about data content and organization that helps those who operate the statistics production systems to remember what tasks they should perform and how they should perform them. A related purpose is to introduce new staff to and train them in the production routines (Sundgren, 1993). The metadata, as proposed in this context, are also to help identify where current methods of quality adjustment require reconsideration and will prompt the use of alternative methods.[1] The dramatic increase in the volume of statistical data in machine-readable form has some arguing for keeping metadata in such a form. This is to encourage transparency in the methods used and help ensure that they are understood and continued as staff members leave and others join. Changes in quality adjustment methodology can in themselves lead to changes in the index. Indices for products using new procedures should be spliced onto existing indices. The metadata system also should be used as a tool to help with quality adjustment. Because so much of the rationale for the employment of different methods is specific to the features of the industries concerned, data should be kept on such features. The metadata system should help in the following ways:

- Statistical agencies should monitor the incidence of missing items against each three-digit ISIC code, and if the incidence is high, then at the four- or more digit level or by elementary aggregate to the most detailed level of the system. Where the incidence is high, the ratios of temporary missing prices, comparable replacements, and noncomparable replacements to the overall number of prices, and the methods for dealing with each of these three circumstances, also should be monitored to provide the basis of a statistical metadata system. The advantage of a top-down approach is that resources are saved by monitoring at the detailed level only the product areas that are problematic.
- Product-specific information—such as the timing of the introduction of new models, pricing policies, especially in months when no changes were made, and popularity of models and brands according to different data sources—should be included as the system develops.
- An estimate, if available, of the weight of the product concerned should be given so that a disproportionate effort is not given to relatively low-weighted items. All of this will lead to increased transparency in the procedures used and allow effort to be directed where it is most needed.
- The statistical metadata system will benefit from contacts among market research organizations, retailers, manufacturers, and trade associations for items for which replacement levels are high. The development of such links may lead, for example, to option cost estimates, which can be easily introduced. Where possible, staff should be encouraged to learn more about specific industries whose weights are relatively high and where item replacement is

[1] Metadata may also serve user needs, the oldest and most extensive form being footnotes (Silver, 1993).

common. Such links to these organizations will allow staff to better judge the validity of the assumptions underlying implicit quality adjustments.

- Industries likely to be undergoing regular technological change should be identified. The system should attempt to ascertain the pace at which models change and, where possible, the timing.
- Price-determining characteristics for product areas using hedonic regressions, information from market research, store managers, trade and other such bodies, and the experience of price collectors should be identified. This should contribute to the statistical metadata system and be particularly useful in providing subsequent guidelines on item selection.
- The system should undertake an analysis of what have in the past been judged to be "comparable" replacements in terms of the factors that distinguish the replacement and old item. The analysis should identify whether different respondents are making similar judgments and whether such judgments are reasonable.
- When hedonic regressions are used either for partial patching of missing prices or as indices in their own right, information on the specification, estimated parameters, and diagnostic tests of the regression equations should be kept along with notes as to why the final formulation was chosen and used along with the data. This will allow the methodology for subsequent updated equations to be benchmarked and tested against the previous versions.
- Price statisticians may have more faith in some quality adjustment procedures than others. When such procedures are used extensively, it might be useful to note, as part of the metadata system, the degree of faith the statistician has in the procedures. Following Shapiro and Wilcox (1997b) this may be envisaged as a traditional confidence interval: the statistician believes at a 90 percent level of confidence that the quality-adjusted price change is 2 percent (0.02) with an overall width of 0.005, for example. There may be an indication as to whether the interval is symmetric or positively or negatively one-sided. Alternatively, statisticians may use a simple subjective coding on a scale of one to five.

D. Incorporating New Goods

D.1 What are new goods and how do they differ from quality changes?

8.17 A new model of a good may provide more of a currently available set of service flows. For example, a new model of an automobile is different from an existing one in that it may have a bigger engine. There is a continuation of a service and production flow, and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of a new good's quality changes against an updated existing model is that, first, the former cannot be easily linked to an existing item as a continuation of an existing resource base and service flow because of the very nature of its newness. Some forms of genetically modified seeds, frozen foods, microwave ovens, and mobile phones, while extensions of existing services, have a dimension of service that is quite new. Second, new goods can generate a welfare gain to consumers and surplus to producers by their very introduction, and the simple introduction of the new good into the index, once two successive price quotes are available, misses this gain.

8.18 Oi (1997) directs the problem of defining new goods to that of defining a monopoly. If there is no close substitute, the good is new. He argues that some individual new videos may have quite small cross-elasticities with other videos; their shared service is to provide movie entertainment and they are similar only in this respect. The same argument may apply to some new books and new breakfast cereals. However, Hausman (1997) found cross-elasticities for substitution to be quite substantial for new breakfast cereals. There are many new forms of existing commodities, such as fashionable toys, which are not easily substitutable for similar items, and thus manufacturers could generate a substantial surplus over and above what might be expected from their production costs. The ability of manufacturers to generate monopoly surpluses is one way of considering whether items are new.

8.19 However, Bresnahan (1997, p. 237) notes that for the United States, *Brandweek* counted more than 22,000 new-product introductions in 1994—the purpose of their introduction being, as differentiated products, to be distinct and not exact substitutes for existing ones. Their distinctiveness is in many cases the rationale behind their launch. However, the extent of differentiated markets makes impractical the definition and treatment of such things as new. Oi (1997, p. 110) provides the pragmatic case: "Our theory and statistics would be unduly cluttered if separate product codes had to be set aside for Clear Coke and Special K." Furthermore, the techniques for their inclusion are not readily applicable, and the sound practical advice given by Oi (1997) to keep matters uncluttered is therefore not unreasonable.

8.20 Merkel (2000, p. 6) is more practical in devising a classification scheme that will meet the needs of PPI compilation (see also Armknecht, Lane, and Stewart, 1997, for CPIs). Merkel considers *evolutionary* and *revolutionary* goods. The former are defined as

> …extensions of existing goods. From a production inputs standpoint, evolutionary goods are similar to pre-existing goods. They are typically produced on the same production line and/or use largely the same production inputs and processes as pre-existing goods. Consequently, in theory at least, it should be possible to quality adjust for any differences between a pre-existing good and an evolutionary good.
>
> In contrast, revolutionary goods are goods that are substantially different from pre-existing goods. They are generally produced on entirely new production lines and/or with substantially new production inputs and processes than those used to produce pre-existing goods. These differences make it virtually impossible, both from a theoretical and practical standpoint, to quality adjust between a revolutionary good and any pre-existing good.

8.21 Quality adjustments to prices are therefore suitable for evolutionary goods under the FIOPI framework but unsuitable for revolutionary goods. The definitions are designed to distinguish between the two types of goods not in terms of what is analytically appropriate, but by what is practically meaningful for the needs of PPI construction. It is quite possible for a new item made from the same inputs and processes as the old one to have a high cross-elasticity of substitution and, thus, command revenue for each item beyond what might be expected from a normal markup. Yet practical needs are important in this context, especially because the methods for estimating the producers' surplus are not practically possible given their substantial resource needs of data and econometric expertise.

D.2 The issues

8.22 There are two major concerns regarding the incorporation of new goods into the PPI. First is their identification and detection; second is the related decision on the need and timing for their inclusion. This refers to both the weight and price changes of the new goods. Consider some examples.

8.23 First, the production of cellular phones, for example, was in some countries at such a significant level that their early inclusion in the PPI became a matter of priority. They simply rose from nothing to quite a large proportion of output in their industry. Furthermore, their price changes were atypical of other goods in their industry. Being new, they may be produced using inputs and technologies quite different from those used for existing ones.

8.24 Many new goods can command substantial sales and be the subject of distinct pricing strategies at introduction because of substantial marketing campaigns. Dulberger (1993) provided some estimates for U.S. PPIs for dynamic random-access memory (DRAM) computer memory chips. She calculated price indices for the period from 1982 to 1988 with varying amounts of delay in introducing new chips into the index. The indices were chained so that new chips could be introduced, or not, as soon as they were available for two successive years. Using a Laspeyres chained index, the fall of 27 percent, if there is no delay in introducing new goods, was compared with falls of 26.2 percent, 24.7 percent, 19.9 percent, 7.1 percent, and 1.8 percent, if the introductions were delayed by one year, two years,…, five years, respectively. In all cases, the index is biased downward because of the delay. The longer the delay, the more the price changes of new products are estimated by products whose market shares may be quite small. Berndt and others (1997) provided a detailed study of the new anti-ulcer drug Tagamet and found the effects of preintroduction marketing on its price and market share at introduction to be quite substantial. Not unexpectedly, price falls were found for the generic form of a pharmaceutical on the expiration of the patent, but *increases* were found for the branded form because loyal customers were willing to pay a premium over the price prior to the patent expiration (Berndt, Ling, and Kyle, 2003).

8.25 Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new products may lead to errors in the measurement of price changes if the unusual price movements cycles are ignored at critical stages in the product life. Strategies are required for the early identification of new products and mechanisms for their incorporation either at launch, if preceded by major marketing strategies, or soon after, if there is evidence of market acceptance. This should form part of the metadata system. Waiting for a new product to achieve market maturity may result in an implicit policy of ignoring the quite disparate price movements that accompany their introduction (Tellis, 1988, and Parker, 1992). This is not to say that new goods will always have different price changes. Merkel (2000) gives the example of "lite" varieties of foods and beverages, similar to the original ones but with less fat. They have prices very close to the original ones and serve to expand the market. While there is a need to capture such expansion when the weights are revised, the price changes for the existing items can be used to capture those of the lite ones.

D.3 Methods

8.26 The methods outlined here include those that fall under what should be normal PPI procedures and those that would require exceptional treatment. In the former case, consideration will be given in Section D.3.1 to the rebasing of the index, rotating of items, introduction of new goods as replacements for discontinued ones, and a strategy for dealing with new-item bias. In the latter, techniques that require different sets of data will be outlined. The use of chained matched models and hedonic indices were outlined and discussed in Chapter 7, Section G, "High-Technology and Other Sectors with Rapid Turnover of Models."

D.3.1 Sample rebasing, rotation, directed replacements, and sample augmentation

D.3.1.1 Sample rebasing and rotation

8.27 The concern here is mainly with *evolutionary goods*. A new good may be readily incorporated in the index at the time of rebasing the index or when the sample is rotated. If the new good has, or is likely to have, substantial sales and is not a replacement for a preexisting one, or is likely to command a much higher or lower market share than the preexisting one it is replacing, then new weights are necessary to reflect this. New weights are fully available only at rebasing, not on sample rotation. There will be a delay in the new item's full inclusion, and the extent of the delay will depend on how close its introduction is to the next rebasing and, more generally, the frequency with which the index is rebased. The term rebasing here is effectively concerned with the use of new weights for the index. Even if the index was rebased annually and chained, it would take until the annual rebasing before weights could be assigned, and even then there might be a further six-month delay in the sampling and collating of the survey results for the weights. More frequent rebasing allows for an earlier introduction of the new good and is advised when the weights are not keeping pace with product innovations.

8.28 At the elementary level of aggregation, equal (implicit) weight is given by the Jevons index—for example, to each price relative. The Dutot index gives each price change the weight of its price relative to the sum of the prices in the initial base period of the comparison (Chapter 20, Section B). If an industry is expected to be subject to dynamic innovations, then the sample may be increased without any changes to the weight for the group. There simply would be more items selected to form the arithmetic or geometric average price change. As new varieties become available, they can be substituted for some of the existing ones, with a wider range from which to draw a comparable one or with less effort involved in the quality adjustment procedure for a noncomparable one.

8.29 Some statistical agencies rotate (resample) items within industry groups. Opportunities exist to introduce new items within a weighted group under such circumstances. The resource practicalities of such schemes require items to be rotated on a staggered basis for different industries, with industries experiencing rapid change being rotated more frequently. For example, DVDs could replace VCR tapes using the overlap method, with the difference in prices in the overlap period assumed to be equal to their quality difference. The assumptions implicit in such procedures have been outlined above, and their likely veracity needs to be considered. Since evolutionary items are defined as continuations of the service flow of exiting ones, the hedonic framework may be more suitable; further methods and their choice were discussed in Chapter 7, Sections D through F. However, the principle remains for including new goods in an index within a weighting system as a substitute for old goods.

8.30 Yet in many countries rebasing is infrequent and sample rotation not undertaken. Furthermore, rotating samples on a frequent basis should not be considered as a panacea. Sample rotation is an arduous task, especially when performed over a range of industries experiencing rapid change. Even frequent rotation, say, every four years, may miss many new goods. Experience in the United States has found that frequent rotation (resampling) has had a negative impact on participation rates, since respondents shy away from incurring the indirect costs associated with being interviewed about their product range and technology (Merkel, 2000). Yet it is not necessary for statistical agencies to wait until an item is obsolete before the new one is introduced. It is quite feasible for statistical agencies to preempt the obsolescence of the old item and direct an early substitution of the new. In some industries, the arrival of a new good is well advertised in advance of the launch, while in others it is feasible for a statistical agency to have more general procedures for directed substitutions, as will be outlined below. Without such a strategy and infrequent rotation and rebasing, a country would be open to serious new product bias. In summary,

- The treatment of a new good as a replacement for an existing one can be undertaken if the old item's weights suitably reflect the new good's sales and if suitable quality adjustments can be made to its price to link it to the existing old price series.
- If the new good does not fit into the preexisting weighting structure, it can be included on rebasing, though this may be infrequent in some countries.
- Regular sample rotation provides a means by which the inclusion of such items can be formally reconsidered. Since this is undertaken on a staggered basis, only the weights within the industry are reallocated, not those between industries.
- Directed sample substitution, as opposed to waiting for sample rotation, may be used to preempt the arrival of new goods.
- Revolutionary items, tectonic shifts, and entirely new products will not fit into existing weighting structures, and alternative means are required.
- Directed replacements for evolutionary goods as replacement items and for revolutionary goods to augment the sample are considered below.
- The chained framework outlined in Chapter 15, Section F, may be more appropriate for product areas with high turnover of items.

D.3.1.2 Directed replacements and sample augmentation

8.31 For *evolutionary goods* in industries with a rapid replacement and introduction of such goods, a policy of directed substitution might be adopted. Judgment, experience, and a statistical metadata system should help identify such industries. The existing items should be coded into well-defined product lines. The respondents then are contacted on a regular (say, annual) basis to establish whether a new version has been introduced and, if so, what percentage of the product line's revenue is represented by the new version. Replacement could be decided by a number of criteria. If the new version is designed as a replacement for an existing one, then substitution might be automatic. Once a substitute has been made, the prices require adjustment for the quality differences using the overlap method, imputation, or an explicit estimate based on production or option costs or a hedonic regression. Examples of forms to help guide this process of directed substitution are given in Merkel (2000).

8.32 It is important to emphasize that, on the introduction of new versions of these evolutionary goods, a price may be charged over and above that which can be ascribed to the resource costs behind its difference from the old one. A new version of, for example, electrical cable may have stronger and more flexible plastic coating and the resource cost behind its production may be quite small. Yet it may be sold at a much higher price than the old version because it's seen to be superior to other products in the market. This price increase is a real one that should, after subtraction of the difference in resource costs, be captured by the PPI. After a while prices may be reduced as the novelty of the item wears off or as competitors bring out improved products. The directed substitution becomes important so that the unusual price increases at the introduction are captured by the PPI. It is also necessary so that the coverage of items becomes more representative. Directed substitution allows both.

8.33 However, for *revolutionary goods* substitution may not be appropriate. First, they may not be able to be defined within the existing classification systems. Second, they may be primarily produced by a new establishment, which will require extending the sample to include such establishments. Third, there will be no previous items to match it against and make a quality adjustment to prices since, by definition, they are substantially different from preexisting goods. Finally, there is no weight to attach to the new establishment or item.

8.34 The first need is to identify new goods and the proposal for contacts with market research companies, outlet managers, and manufacturers discussed in Section C.1 on producing a supporting metadata system. *Once identified, sample augmentation is appropriate for the introduction of revolutionary goods, as opposed to sample substitution for evolutionary goods*. It is necessary to bring the new revolutionary good into the sample in addition to what exists. This may involve extending the classification, the sample of outlets, and the item list within new or existing outlets. The means by which the new goods are introduced is more problematic.

8.35 Once two price quotes are available, it should be possible to splice the new good onto an existing or obsolete one. This of course misses the impact of the new item in its initial period, but as discussed below, including such effects is not a trivial exercise. Consider the linking of a good that is likely to be replaced in the market by the new good. For example, a quite new electrical kitchen appliance may use the price index for existing kitchen appliances up to the period of the link, and then the price changes for the new good in subsequent periods. This would create a separate and additional price series for the new good, which augments the sample, as illustrated in Table 8.1. Product C is new in period 2 and has no base-period weight. Its price change between periods 1 and 2, had it existed, is assumed to follow the overall index for products A and B. For period 3 onward a new, linked price series is formed for product C, which for period 3 is $101.40 \times 0.985 = 99.88$, and for period 4 is $101.40 \times 0.98 = 99.37$. New revised weights in period 2 show product C's weight to be 20 percent of all products. The new index for period 3 is

$$101.40\,[(0.8\,(101.9/101.4) + 0.2\,(99.88/101.4))]$$

$$= 0.8\,(101.9) + 0.2\,(99.88)$$

$$= 101.50$$

and for period 4,

$$101.40\,[(0.8\,(102.7/101.4) + 0.2\,(99.37/101.4))]$$

$$= 0.8\,(102.7) + 0.2\,(99.37)$$

$$= 102.05.$$

8.36 If product C was an evolutionary good replacing product B, there would be no need to introduce new weights and no need to augment the sample, as undertaken above. However, since the revolutionary product C has no weight in the base period, the splicing requires a revision of the weights at the same time. The selection of the series onto which the new item is spliced, and, in turn, the product groups selected for the weight revision, requires some judgment. Items whose market share is likely to be affected by the introduction of the new good should be selected. If the new good is likely to be responsible for a significant share of revenue, such that it will affect the weights of a broad class of product groups, then there may be a case for a realignment of the overall weighting procedure. Such seismic shifts can of course occur, especially in the communications industry, and for a wide range of industries when regulations are removed or trade barriers are relaxed in less developed economies. In some countries, a new industry or plant can, in itself, amount to sizable proportions of a sector's weights. The change in weights also may be required for *disappearing* goods no longer produced in an economy. As noted in Chapter 15, Section F, chaining and hedonic indices may be appropriate when there is a rapid turnover in such new and obsolete goods. Chaining is an extension of the above procedure and can be used to introduce a new good as soon as it is available for two successive periods.

Table 8.1. Sample Augmentation Example

Products	Base Weight	Revised Weight	Period 1	Period 2	Period 3	Period 4
A	0.6	0.5	100.00	101.00	101.50	102.50
B	0.4	0.3	100.00	102.00	102.50	103.00
All items		*0.8*	*100.00*	*101.40*	*101.90*	*102.70*
C				100.00	98.50	98.00
Spliced *C*		0.2	100.00	101.40	99.88	99.37
Revised all items			*100.00*	*101.40*	*101.50*	*102.05*

8.37 Item augmentation also may be used for evolutionary goods that are likely to be responsible for a substantial share of the market, while not displacing the existing goods. For example, a country with a local brewery and a licensing agreement with a foreign brewery will have joint production of the two beers. The revenue share for beer from the brewery remains the same, but one segment of the market now drinks foreign as opposed to domestic beer. Respondents may be directed to a forced substitution of some of the sample of domestic beers for foreign ones, with the weight remaining the same. This would be similar to a quality adjustment using a noncomparable replacement as discussed in Chapter 7, Section F. Alternatively, the sample may be augmented since there is concern that a smaller sample of domestic beers may not be sufficiently representative. The augmentation process may be similar to that outlined in Table 8.1, with the new foreign beer *C* accounting for 20 percent of the market. Had the advent of foreign beers displaced some of the alcoholic spirits market, then the revision of weights would extend into this product group. As noted in Chapter 7, Section G, chaining and hedonic indices may be appropriate when there is a rapid turnover in new and obsolete goods. With chaining, the good needs to be available only for two successive periods to allow for its introduction.

E. Summary

8.38 The concern in this chapter with sample space and new goods arises out of a very real concern with the dynamic nature of modern markets. New goods and quality changes are far from new issues, and as Triplett (1999) has argued, it has not been demonstrated that the *rate* of new product developments and introductions is much higher now than in the past. However, it is certainly accepted that the *number* of new products and varieties is substantially greater than before. Computer technology provides cost-effective means for collecting and analyzing much larger sets of data. In Chapter 6, the use of handheld computers for data capture was considered, as was the availability of bar-code scanner data. Yet the proper handling of such data requires consideration of issues and methods that go beyond those normally considered for the static intersection universe, which underscores matched samples. In the appendix to this chapter a formal outline of such sampling issues is provided. In this section some of the more important issues are reiterated.

- Where nothing much in the quality and range of available goods changes, there is much that is advantageous to the use of the matched-models methods. It compares like with like from like establishments.
- Statistical metadata systems are needed for quality adjustment issues to help identify the industries in which matching provides few problems. This focuses attention on those that are problematic by collecting and providing information that will facilitate quality adjustment. It also allows for transparency in methods and facilitates retraining.
- Where there is a very rapid turnover in items, such that serious sample depletion takes place quickly, replacements cannot be relied on to replete the sample. Alternative mechanisms, which sample from or use the double universe

of items in each period, are required. These include chained formulations and hedonic indices as discussed in Chapter 7, Section G.

- Some new goods can be treated as evolutionary and incorporated using noncomparable replacements with an associated quality adjustment. The timing of the replacement is critical for both the efficacy of the quality adjustment and the representativity of the index.
- Instructions to respondents on the selection of replacement items are important because they also have a bearing on the representativity of the index. The replacement of obsolete items with newly introduced items leads to difficulties in undertaking quality adjustments, while their replacement with similar items leads to problems of representativity.
- Sample rotation is an extreme form of the use of replacements and is one mechanism for refreshing the sample and increasing its representativity. However, a disadvantage is the possible bias arising from the implicit assumptions underlying the quality adjustment overlap procedure not being met.
- Revolutionary goods may require the augmentation of the sample to make room for new price series and new weighting procedures. The classification of new goods into evolutionary goods and revolutionary goods has a bearing on the strategy for their introduction, directed replacement (substitution), and sample augmentation.

Appendix 8.1: Appearance and Disappearance of Products and Establishments

8.39 In earlier chapters, especially Chapter 5 on sampling, it was generally assumed that the target quantity for estimation could be defined on a fixed set of products. In this appendix the important complications arising from the products and establishments continually changing are considered. The rate of change is rapid in many industries. With this in mind, sampling for price change estimation is a dynamic rather than static problem. Somehow, the prices of new products and in new establishments have to be compared with old ones. *It is important to realize that whatever methods and procedures are used in a price index to handle these dynamic changes, the effects of these procedures will always amount to an explicit or implicit estimation approach for this* ***dynamic universe***.

Representation of change in a price index[2]

8.40 From a sample selection perspective, there are three ways of handling dynamic changes in an elementary aggregate universe, where varieties and establishments move in and out: (i) by *resampling* the whole elementary aggregate at certain points in time, (ii) by a one-to-one *replacement* of one variety or establishment for another one, and (iii) by *adding and deleting* single observation points (items in establishments) within an index link.

Resampling

8.41 In *resampling*, the old sample is reconsidered as a whole so as to make it representative of the universe in a later period. This does not necessarily mean that all, or even most, sampling units have to be changed, only that a fresh look is taken at the representativity of the whole sample and changes undertaken as appropriate. The methods used for *resampling* could be any of those used for the initial sampling. In the case of probability sampling, it means that every unit belonging to the universe in the later period needs to have a nonzero probability equal to its relative market share of being included in the sample.

8.42 *Resampling* or *sample rotation* is traditionally combined with the overlap method outlined in Chapter 7, Section D. It is similar to the procedure used when combining two links in chained indices. The first period for which the new sample is used is also the last period for which the old sample is used. Thereby, price change estimation is always based on one sample only—the old sample up to the overlap period and the new sample from the overlap period onward, as discussed in further detail below. *Resampling* is the only method that is fully able to maintain the representativity of the sample and, resources permitting, should be undertaken frequently. The necessary frequency depends on the rate of change in a particular product group. It relies, however, on the assumption that the price differences between the old and new items are appropriate estimates of quality differences. At its ex-

[2]A fuller version of this appendix can be found in Dálen (1998).

treme, *resampling* amounts to drawing a new sample in each period and comparing the average price between the samples, instead of the usual procedure of averaging price changes for matched samples. Although being the logical end-point from a representativity point of view, resampling each period would aggravate the quality adjustment problem by its implicit quality adjustment procedure, and, thus, it is not recommended.

Replacement

8.43 A replacement can be defined as an individual successor to a sampled product that either disappeared completely from the market or lost market share in the market as a whole or a specific establishment. Criteria for selecting replacements may differ considerably. There is first the question of when to replace. Usual practices are to do it either when an item disappears completely or when its share of the sales is reduced significantly. Another possible, but less-used, rule would be to replace an item when another variety within the same group, or representative item definition, has become larger with regard to sales, even if the old variety still is sold in significant quantities.

8.44 Second is the question of how to select the replacement item. If the rule for initial selection was most sold or with probability proportionate to (sales) size, then the replacement rule could follow the same selection rule. Alternatively, the replacement could be that item that is most like the old one. The advantage of the former rule is better representativity. The advantage of the most-like rule is, at least superficially, that it might result in a smaller quality adjustment problem.

8.45 It is important to realize that, at least with today's practices, replacements cannot adequately represent new items coming into the market. This is because what triggers a replacement is not the appearance of something new but the disappearance or reduced importance of something old. If the range of varieties in a certain group is increasing, sampling can represent this increase only directly from the set of new varieties, such as in the case of *resampling*.

Adding and deleting

8.46 It is possible to add a new observation point into an elementary aggregate within an index link. If, for example, a new brand or model of a durable was introduced without replacing any particular old model, it would be desirable to add it to the sample starting from the time of its introduction. In order to accommodate this new observation into the index system, its reference price needs to be imputed. A practical way to do this is to divide its price in the month of introduction by the price index of all other items in the elementary aggregate from the reference period to the month of introduction. In this way, its effect on the index for months up to the introduction month will be neutral.

8.47 Similarly, an item that disappears could just be deleted from the sample without replacement. Price change can then be computed over the remaining items. If no further action is taken, this means that the price change for the deleted item that was measured up to the month prior to deletion will be disregarded from the month of deletion. This may or may not be desirable, depending on the circumstances in the particular product group.

Formulating an operational target in a dynamic universe

8.48 A rigorous approach to the problem of statistical estimation requires an *index estimation strategy* that includes both the *operational target of measurement* and the *sampling strategy* (design and estimator) needed for estimating this target. This strategy would have to consist of the following components:

(i) A definition of the universe of *transactions* or *observation points* (usually a product variety in an establishment) in each of the two time periods between which we want to estimate price change;
(ii) A list of all *variables* defined on these units. These variables should include prices and quantities (number of units sold at each price), but also all relevant price-determining characteristics of the products (and possibly also of the establishments)—the price basis;
(iii) The *target algorithm* (index formula) that combines the variable values defined in (ii) for the observation points in the universe defined in (i) into a single value;
(iv) Procedures used for *initial sampling* of items and establishments from the universe defined in (i);

(v) Procedures within the time span for *replacing, resampling and adding or deleting* observations; and
(vi) The *estimation algorithm* (index formula) applied to the sample with the purpose of minimizing the expected error of the sample estimate compared with the target algorithm under (iii). This algorithm, in principle, needs to consider all the procedures taken in replacement and *resampling* situations, including procedures for quality adjustment.

8.49 The kind of rigorous strategy outlined above is generally not used in practical index construction because of its complexity, though its required information system was discussed in Section C.1. A few comments on such possible strategies are made below.

A two-level aggregation system

8.50 A starting point for discussing this objective is a two-level structuring of the universe of commodities and establishments considered in the scope of a price index. These levels are

- The *aggregate* level. At this level there is a fixed structure of item groups $h = 1,\ldots, H$ (or perhaps a fixed cross-structure of item groups by regions or establishment types) within an index link. New goods and services for updating the universe of commodities would be defined in terms of new groups at this level and moved into the index only in connection with a new index link.
- The *elementary* level. Within this level the aim is to capture the properties of a changing universe in the index by comparing new and old items. The micro comparison from s to t must be defined so that new products and establishments enter into the market and old products and establishments disappear from the market.

The common starting point for three alternative approaches at the elementary level is a pure price formulation of price change from period s to period t at the aggregate level:

$$\text{(A8.1)}\quad I^{st} = \frac{\sum_h Q_h P_h^t}{\sum_h Q_h P_h^s} = \sum_h W_h^s I_h^{st} \,,$$

where $W_h^s = \dfrac{Q_h P_h^s}{\sum_h Q_h P_h^s}$ and $I_h^{st} = \dfrac{P_h^t}{P_h^s}$.

The quantities, Q_h, are for $h = 1\ldots H$ item groups from any period or functions of quantities from several periods, for example, a symmetric average of the base and current periods s and t. Special cases of such a pure price index are the Laspeyres ($Q_h = Q_h^s$), Paasche ($Q_h = Q_h^t$), Edgeworth ($Q_h = Q_h^s + Q_h^t$), and Walsh ($Q_h = [\,Q_h^t\, Q_h^t\,]^{1/2}$) price indices outlined in Chapters 15 through 17. Alternative formulations for an elementary-level estimation strategy now enter in the definition of I_h^{st}. As a further common starting point the set of items or establishments belonging to h in period u *(=* s or t*) are defined* as Ω_h^u. The concept of an *observation point* is introduced, usually a tightly specified item in a specific establishment, such that, say, $\Omega_h^u = \{1,\ldots,j,\ldots, N_h^u\}$. For each observation point $j \in \Omega_h^u$, there is a price p_j^u and a quantity sold q_j^u. There are now three possibilities for defining the operational target.

The intersection universe

8.51 The elementary index is defined over the intersection universe, that is, only over observation points existing in both s and t. This index may also be called the *identical units index*. It is equivalent to starting out with the observation points existing in s and then dropping (deleting) missing or disappearing points. An example of such an index is:

$$\text{(A8.2)}\quad I_h^{st} = \frac{\sum_{j\in\Omega_h^s\cap\Omega_h^t} q_j p_j^t}{\sum_{j\in\Omega_h^s\cap\Omega_h^t} q_j p_j^s} \quad .$$

The intersection universe decreases successively over time as fewer matches are found for each long-run comparison between s and t, s and $t + 1$, s and $t + 2$, etc., until it eventually becomes empty. An attraction of the intersection universe is that there are, by definition, no replacements involved in this target and, thus, normally no quality adjustments. If the identical units index is combined with a short index link, followed by *resampling* from the universe in a later period, sampling from this universe is a perfectly reasonable strategy, as long as the assumptions implicit in the overlap procedure, that the price differences at that point in time reflect the quality differences, are valid.

The double universe

8.52 The polar opposite approach to the intersection universe is to consider P_h^s and P_h^t as average prices defined over two separately defined universes in the two periods. A double universe target could then be considered: one universe in period *s* and another in period *t*. This seems to be a natural way of defining the target, since both time periods should be of equal status, and all products existing in any of these should be taken into account. The difficulty with this approach is that the two universes are rarely comparable in terms of quality. Some kind of adjustment for average quality change would need to be brought into the index. The natural definition of the average prices involved in this approach is as unit values. This would lead to the following definition of a *quality-adjusted unit value index*:

$$\text{(A8.3)} \quad I_h^{st} = \bar{P}_h^t \Big/ \bar{P}_h^s g_h^{st} \quad ,$$

where $\bar{P}_h^t = \dfrac{\sum_{j \in \Omega_h^t} q_j^t p_j^t}{\sum_{j \in \Omega_h^t} q_j^t}$ and $\bar{P}_h^s = \dfrac{\sum_{j \in \Omega_h^s} q_j^s p_j^s}{\sum_{j \in \Omega_h^s} q_j^s}$.

In equation (A8.3), g_h^{st} is the average quality change in *h* (also interpretable as a *quality index*), which needs further definition. For example, g_h^{st} could be thought of as a hedonic adjustment procedure, where characteristics are held constant. Equation (A8.3) was outlined in Chapter 7, Section E, in forms that include explicit hedonic quality adjustments, g_h^{st}, but as part of Laspeyres, Paasche, Fisher, and Törnqvist indices. This operational target is attractive for commodities where the rate of turnover of varieties is very fast, but average quality changes slowly, or reliable estimates of quality changes can be made. The commonly used representative item method is not really compatible with a double-universe target. It implicitly focuses on preselected primary sampling units that are used for both periods *s* and *t.*

The replacement universe

8.53 Neither sampling from the intersection nor sampling from the double universe bears a close resemblance to usual practices for constructing price indices. In particular, the representative item method combined with one-to-one replacements, which is the most common sampling method used in practice, needs a rationalization in terms of operational targets, which differs from these alternatives. Such a rationalization of sampling from a *replacement universe* is considered below.

Definition 1a: For each $j \in \Omega_h^s$ and $j \notin \Omega_h^t$ we define replacement items $a_j \in \Omega_h^t$ whose price enters into *j*'s place in the formula. (For $j \in \Omega_h^s$ and $j \in \Omega_h^t$, $a_j = j$.) In addition to a replacement, a quality change from *j* to a_j is included, which gives rise to a quality adjustment factor g_j, interpreted as the factor with which p_j^s must be multiplied for the producer to be indifferent between producing items *j* and a_j at prices p_j^s and $p_{a_j}^t$.

$$\text{(A8.4)} \quad I_h^{st} = \frac{\sum_{j \in \Omega_h^s} q_j p_{a_j}^t}{\sum_{j \in \Omega_h^s} q_j p_j^s g_j} \quad .$$

However, this first step toward an operational use of the formula requires, first, a need to define g_j, possibly arising from a hedonic regression as described in Chapter 7, Section G.2. Second, there is a need to define a_j. A natural procedure is to use a *dissimilarity function* from *j* to a_j. The notation $d(j, a_j)$ is introduced for this function. The common procedure of choosing the most similar item in cases of replacement now corresponds to minimizing the dissimilarity function. However, some further specifications need to be made. *When* is the replacement defined to take place? In practice, this ought to be done when the first chosen variety is no longer representative. Mathematically, this could be defined as Definition 1b.

Definition 1b: Observation point *j* should be replaced in the first period in which $q_j^t < c q_j^s$, where *c* is a suitably chosen constant between 0 and 1 (a modification would be required for seasonal items).

The *choice* of replacement point would then be governed by a rule such as Definition 1c.

Definition 1c: a_j should be chosen so that $d(j, a_j)$ is minimized for *j*.

However, since some priority should be given to observation points that are important in terms of quantities or values, Definition 1c can then be modified to become Definition 1d.

Definition 1d: a_j should be chosen so that $d(j, a_j)/q^t_{a_j}$ is minimized for *j*. (Some other function of *d(.)* and $q^t_{a_j}$ could be chosen in its place.)

8.54 The dissimilarity function needs to be specified; it may depend on the item group *h*. In general this must be some kind of metric defined on the set of characteristics of the product and establishment in question. For example, priority could be given to its dissimilarity either to "same establishment" or "same product," which could easily be worked into such a metric. A more troublesome concern is the inclusion of as many new points in Ω^t_h as possible in the index definition, to make the sample representative. As Definitions 1a–d now stand, the same new point could replace many predecessors, whereas there may be many new points that will not be sampled unless there is a need for a replacement. This shortcoming of the replacement universe is an inherent trait in the replacement method as such. The replacement method is designed only to maintain the representativity of the old sample, not that of the new sample.

9. PPI Calculation in Practice

A. Introduction

9.1 This chapter provides a general description of the ways PPIs are calculated in practice. The methods used in different countries are not exactly the same, but they have much in common. Both compilers and users of PPIs are interested in knowing how most statistical offices actually set about calculating their PPIs.

9.2 As a result of the greater insights into the properties and behavior of price indices that have been achieved in recent years, it is now recognized that some traditional methods may not necessarily be optimal from a conceptual and theoretical viewpoint. Concerns have also been voiced in a number of countries about possible biases that may be affecting PPIs. These issues and concerns need to be addressed in the *Manual*. Of course, the methods used to compile PPIs are inevitably constrained by the resources available, not merely for collecting and processing prices but also the revenue data needed for weighting purposes. In some countries, the methods used may be severely constrained by a lack of resources.

9.3 The calculation of PPIs usually proceeds in two stages. First, price indices are estimated for the *elementary aggregates,* and then these *elementary price indices* are averaged to obtain *higher-level indices* using the relative values of the revenue weights for elementary aggregates as weights. Section B starts by explaining how the elementary aggregates are constructed and which economic and statistical criteria need to be taken into consideration in defining the aggregates. The index number formulas most commonly used to calculate the elementary indices are then presented and their properties and behavior illustrated using numerical examples. The pros and cons of the various formulas are considered together with some alternative formulas that might be used. The problems created by disappearing and new products are also explained, as are the different ways of imputing for missing prices.

9.4 Section C of the chapter is concerned with the calculation of higher-level indices. The focus is on the ongoing production of a monthly price index in which the elementary price indices are averaged, or aggregated, to obtain higher-level indices. Price updating of weights, chain linking, and reweighting are discussed, with examples provided. The problems associated with introduction of new elementary price indices and new higher-level indices into the PPI are also covered. The section explains how it is possible to decompose the change in the overall index into its component parts. Finally, the possibility of using some alternative and rather more complex index formulas is considered.

9.5 Section D concludes with data editing procedures, since these are an integral part of the process of compiling PPIs. It is essential to ensure that the right data are entered into the various formulas. There may be errors resulting from the inclusion of incorrect data or from entering correct data inappropriately and errors resulting from the exclusion of correct data that are mistakenly believed to be wrong. The section examines data editing procedures that try to minimize both types of errors.

B. Calculation of Price Indices for Elementary Aggregates

9.6 PPIs typically are calculated in two steps. In the first step, the elementary price indices for the elementary aggregates are calculated. In the second step, higher-level indices are calculated by averaging the elementary price indices. The elementary aggregates and their price indices are the basic building blocks of the PPI.

B.1 Composition of elementary aggregates

9.7 Elementary aggregates are constructed by grouping individual goods and individual services into relatively homogeneous products and transactions. They may be formed for products in various regions of the country or for the country as a whole. Likewise, elementary aggregates may be formed for different types of establishments or for various subgroups of products. The actual formation of elementary aggregates thus depends on the circumstances and the availability of information, and they may therefore be defined differently in different countries. However, some key points should be observed:

- Elementary aggregates should consist of groups of goods or services that are as similar as possible, and preferably fairly homogeneous.
- They should also consist of products that may be expected to have similar price movements. The objective should be to try to minimize the dispersion of price movements within the aggregate.
- The elementary aggregates should be appropriate to serve as strata for sampling purposes in light of the sampling regime planned for the data collection.

9.8 Each elementary aggregate, whether relating to the whole country, an individual region, or a group of establishments, will typically contain a very large number of individual goods, services, or products. In practice, only a small number can be selected for pricing. When selecting the products, the following considerations need to be taken into account:

(i) The transactions selected should be ones with price movements believed to be representative of all the products within the elementary aggregate.
(ii) The number of transactions within each elementary aggregate for which prices are collected should be large enough for the estimated price index to be statistically reliable. The minimum number required will vary between elementary aggregates, depending on the nature of the products and their price behavior.
(iii) The object is to try to track the price of the same product over time for as long as possible, or for as long as the product continues to be representative. The products selected should therefore be ones that are expected to remain on the market for some time so that like can be compared with like.

B.1.1 Aggregation structure

9.9 The aggregation structure for a PPI is discussed in Chapter 4, Section C.4, and in Figure 4.1. Using a classification of business products such as PRODCOM, CPC, or CPA, the entire set of produced goods and services covered by the overall PPI can be divided into broad *sections, divisions,* and *groups,* then further refined into smaller *classes* and *subclasses.* Each elementary aggregate is assigned a product code. This enables statistical offices to aggregate elementary indices at the lowest level to higher product classes, groups, divisions, etc. In addition, each elementary aggregate is assigned an industry (activity) code from a standard industrial classification such as ISIC or NACE and thus can be aggregated by industry from the four-digit to the three-digit and higher levels. The overall PPI should be the same whether aggregated by industry or product as long as each elementary aggregate has the same weight in the industry and product aggregations.

9.10 The methods used to calculate the elementary indices from the individual price observations are discussed below. Working from the elementary price indices, all indices above the elementary aggregate level are higher-level indices that can be calculated from the elementary price indices using the elementary revenue aggregates as weights. The aggregation structure is consistent so that the weight at each level above the elementary aggregate is always equal to the sum of its components. The price index at each higher level of aggregation can be calculated on the basis of the weights and price indices for its components—that is, the lower-level or elementary indices. The individual elementary price indices are not necessarily sufficiently reliable to be published separately, but they remain the basic building blocks of all higher-level indices.

B.1.2 Weights within elementary aggregates

9.11 In many cases, the explicit revenue weights are not available to calculate the price indices for elementary aggregates. Whenever possible, however, weights should be used that reflect the relative importance of the sampled products, even if the weights are only approximate. Often, the elementary aggregate is simply the lowest level at which reliable weighting information is available. In this case, the elementary index has to be calculated as an unweighted average of the prices of which it consists. However, even in this case it should be noted that when the products are selected with probabilities proportional to the size of some relevant variable such as sales, for example, weights are implicitly introduced by the sample selection procedure. In addition, statistical offices can work with establishment respondents to obtain estimated weight data, as discussed in Chapter 4.

9.12 For certain elementary aggregates, information about output of particular products and market shares from trade and industry sources may be used as explicit weights within an elementary aggregate. Weights within elementary aggregates may be updated independently and possibly more often than the elementary aggregates themselves (which serve as weights for the higher-level indices).

9.13 For example, assume that the number of suppliers of a certain product such as car fuel supplied to garages is limited. The market shares of the suppliers may be known from business survey statistics and can be used as weights in the calculation of an elementary aggregate price index for car fuel. Alternatively, prices for water may be collected from a number of local water supply services where the population in each local region is known. The relative size of the population in each region may then be used as a proxy for the relative revenues to weight the price in each region to obtain the elementary aggregate price index for water.

9.14 A special situation occurs in the case of tariff prices. A tariff is a list of prices for the provision of a particular kind of good or service under different terms and conditions. One example is electricity, for which one price is charged during daytime and a lower price is charged at night. Similarly, a telephone company may charge a lower price for a call on the weekend than a weekday. Another example may be bus tickets sold at one price to regular passengers and at lower prices to children or seniors. In such cases, it is appropriate to assign weights to the different tariffs or prices to calculate the price index for the elementary aggregate.

9.15 The increasing use of electronic recording for transactions in many countries, in which both prices and quantities are maintained as products are sold, means that valuable new sources of information may become increasingly available to statistical offices. This could lead to significant changes in the ways in which price data are collected and processed for PPI purposes. The treatment of electronic data transfer is examined in Chapters 6, 7, and 21.

B.2 Compilation of elementary price indices

9.16 An elementary price index is the price index for an elementary aggregate. Various methods and formulas may be used to calculate elementary price indices. This section provides a summary of pros and cons that statistical offices must evaluate when choosing a formula at the elementary level; Chapter 20 provides a more detailed discussion.

9.17 The methods statistical offices most commonly use are illustrated by means of a numerical example in Table 9.1. In the example, assume that prices are collected for four representative products within an elementary aggregate. The quality of each product remains unchanged over time so that the month-to-month changes compare like with like. No weights can be applied. Assume initially that prices are collected for all four products in every month covered so that there is a complete set of prices. There are no disappearing products, no missing prices, and no replacement products. These are quite strong assumptions because many of the problems encountered in practice are attributable to breaks in the continuity of the price series for the individual transactions for one reason or another. The treatment of disappearing and replacement products is taken up later.

9.18 Three widely used formulas that have been, or still are, in use by statistical offices to calculate elementary price indices are illustrated in Table 9.1. It should be noted, however, that these are not the only possibilities, and some alternative formulas are considered later.

- The first is the *Carli* index for $i = 1,\ldots, n$ products. It is defined as the simple, or unweighted, arithmetic mean of the price relatives, or price ratios, for the two periods, 0 and t, to be compared.

Table 9.1. Calculation of Price Indices for an Elementary Aggregate[1]

	January	February	March	April	May	June	July
				Prices			
Product *A*	6.00	6.00	7.00	6.00	6.00	6.00	6.60
Product *B*	7.00	7.00	6.00	7.00	7.00	7.20	7.70
Product *C*	2.00	3.00	4.00	5.00	2.00	3.00	2.20
Product *D*	5.00	5.00	5.00	4.00	5.00	5.00	5.50
Arithmetic mean prices	5.00	5.25	5.50	5.50	5.00	5.30	5.50
Geometric mean prices	4.53	5.01	5.38	5.38	4.53	5.05	4.98
				Month-to-month price relatives			
Product *A*	1.00	1.00	1.17	0.86	1.00	1.00	1.10
Product *B*	1.00	1.00	0.86	1.17	1.00	1.03	1.07
Product *C*	1.00	1.50	1.33	1.25	0.40	1.50	0.73
Product *D*	1.00	1.00	1.00	0.80	1.25	1.00	1.10
				Current to reference month (January) price relatives			
Product *A*	1.00	1.00	1.17	1.00	1.00	1.00	1.10
Product *B*	1.00	1.00	0.86	1.00	1.00	1.03	1.10
Product *C*	1.00	1.50	2.00	2.50	1.00	1.50	1.10
Product *D*	1.00	1.00	1.00	0.80	1.00	1.00	1.10
Carli index—Arithmetic mean of price relatives							
Month-to-month index	100.00	112.50	108.93	101.85	91.25	113.21	100.07
Chained month-to-month index	100.00	112.50	122.54	124.81	113.89	128.93	129.02
Direct index on January	100.00	112.50	125.60	132.50	100.00	113.21	110.00
Dutot index—Ratio of arithmetic mean prices							
Month-to-month index	100.00	105.00	104.76	100.00	90.91	106.00	103.77
Chained month-to-month index	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Direct index on January	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Jevons index—Geometric mean of price relatives or ratio of geometric mean prices							
Month-to-month index	100.00	110.67	107.46	100.00	84.09	111.45	98.70
Chained month-to-month index	100.00	110.67	118.92	118.92	100.00	111.45	110.00
Direct index on January	100.00	110.67	118.92	118.92	100.00	111.45	110.00

[1]All price indices have been calculated using unrounded figures.

$$(9.1)\quad P_C^{0:t} = \frac{1}{n}\sum\left(\frac{p_i^t}{p_i^0}\right)$$

- The second is the *Dutot* index, which is defined as the ratio of the unweighted arithmetic mean prices.

$$(9.2)\quad P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^t}{\frac{1}{n}\sum p_i^0}$$

- The third is the *Jevons* index, which is defined as the unweighted geometric mean of the price relative or relatives, which is identical to the ratio of the unweighted geometric mean prices.

$$(9.3)\quad P_J^{0:t} = \prod\left(\frac{p_i^t}{p_i^0}\right)^{1/n} = \frac{\prod\left(p_i^t\right)^{1/n}}{\prod\left(p_i^0\right)^{1/n}}$$

The properties of the three indices are examined and explained in some detail in Chapter 20. Here, the purpose is to illustrate how they perform in practice, to compare the results obtained by using the different formulas, and to summarize their strengths and weaknesses.

9.19 Each *month-to-month* index shows the change in the index from one month to the next. The *chained month-to-month* index links together these month-to-month changes by successive multiplication. The *direct* index compares the prices in each successive month directly with those of the reference month, January. By simple inspection of the various indices, it is clear that the choice of formula and method can make a substantial difference in the results obtained. Some results are striking—in particular, the large difference between the chained Carli index for July and each of the direct indices for July, including the direct Carli.

9.20 The properties and behavior of the different indices are summarized in the following paragraphs and explained in more detail in Chapter 20. First, the differences between the results obtained by using the different formulas tend to increase as the variance of the price relatives, or ratios, increases. The greater the dispersion of the price movements, the more critical the choice of index formula and method becomes. If the elementary aggregates are defined so that the price movements within the aggregate are minimized, the results obtained become less sensitive to the choice of formula and method.

9.21 Certain features displayed by the data in Table 9.1 are systematic and predictable and follow from the mathematical properties of the indices. For example, it is well known that an arithmetic mean is always greater than, or equal to, the corresponding geometric mean—the equality holding only in the trivial case in which the numbers being averaged are all the same. The direct Carli indices are therefore all greater than the Jevons indices, except in May and July when the four price relatives based on January are all equal. In general, the Dutot index may be greater or less than the Jevons index but tends to be less than the Carli index.

9.22 One general property of geometric means should be noted when using the Jevons formula. If any one observation out of a set of observations is zero, its geometric mean is zero, whatever the values of the other observations. The Jevons index is sensitive to extreme falls in prices, and it may be necessary to impose upper and lower bounds on the individual price relatives of, say, 10 and 0.1, respectively, when using the Jevons. Of course, extreme observations are often the results of errors of one kind or another, and so extreme price movements should be carefully checked in any case.

9.23 Another important property of the indices illustrated in Table 9.1 is that the Dutot and the Jevons indices are transitive, whereas the Carli index is not. Transitivity means that the chained monthly indices are identical to the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices that link together the month-to-month-indices. The intransitivity of the Carli index is illustrated dramatically in Table 9.1, in which each of the four individual prices in May returns to the same level as it was in January, but the chained Carli index registers an increase of almost 14 percent over January. Similarly, in July, although each individual price is exactly 10 percent higher than in January, the chained Carli index registers an increase of 29 percent. These results would be regarded as perverse and unacceptable in the case of a direct index, but even in the case of the chained index, the results seems so intuitively unreasonable as to undermine the credibility of the chained Carli index. The price changes between March and April illustrate the effects of "price bouncing," in which the same four prices are observed in both periods, but they are switched between the different products. The monthly Carli index from March to April increases, whereas both the Dutot and the Jevons indices are unchanged.

9.24 The message emerging from this brief illustration of the behavior of just three possible formulas is that different index numbers and methods can deliver very different results. Index compilers have to familiarize themselves with the interrelationships between the various formulas at their disposal for the calculation of the elementary price indices so that they are aware of the implications of choosing one formula rather than another. However, knowledge of these interrelationships is not sufficient to determine which formula should be used, even though it makes it possible to make a more informed and reasoned choice. It is necessary to appeal to additional criteria to settle the choice of formula. Two main approaches may be used, the axiomatic and the economic approaches.

B.2.1 Axiomatic approach to elementary price indices

9.25 As explained in Chapters 16 and 20, one way to decide on an appropriate index formula is to require it to satisfy certain specified axioms or tests. The tests throw light on the properties possessed by different kinds of indices, some of which may not be obvious. Four basic tests illustrate the axiomatic approach.

Proportionality Test: If all prices are λ times the prices in the price reference period (January in the example), the index should equal λ. The data for July, when every price is 10 percent higher than in January, show that all three direct indices satisfy this test. A special case of this test is the *identity test,* which requires that if the price of every product is the same as in the reference period, the index should be equal to unity (as in May in the example).

Changes in the Units of Measurement Test (or Commensurability Test): The price index should not change if the quantity units in which the products are measured are changed—for example, if the prices are expressed per liter rather than per pint. The Dutot index fails this test, as explained below, but the Carli and Jevons indices satisfy the test.

Time Reversal Test: If all the data for the two periods are interchanged, then the resulting price index should equal the reciprocal of the original price index. The Carli index fails this test, but the Dutot and the Jevons both satisfy the test. The failure of the Carli index to satisfy the test is not immediately obvious from the example but can easily be verified by interchanging the prices in January and April, for example, in which case the backward Carli index for January based on April is equal to 91.3, whereas the reciprocal of the forward Carli index is 1/132.5, or 75.5.

Transitivity Test: The chained index between two periods should equal the direct index between the same two periods. The example shows that the Jevons and the Dutot indices both satisfy this test, whereas the Carli index does not. For example, although the prices in May have returned to the same levels as in January, the chained Carli index registers 113.9. This illustrates the fact that the Carli index may have a significant built-in upward bias.

9.26 Many other axioms or tests can be devised, as presented in Chapter 16, but the above (summarized in Table 9.2) are sufficient to illustrate the approach and also to throw light on some important features of the elementary indices under consideration here.

9.27 The sets of products covered by elementary aggregates are meant to be as homogeneous as possible. If they are not fairly homogeneous, the failure of the Dutot index to satisfy the units of measurement, or commensurability, test can be a serious disadvantage. Although defined as the ratio of the unweighted arithmetic average prices, the Dutot index may also be interpreted as a weighted arithmetic average of the price relatives in which each ratio is weighted by its price in the base period.[1] However, if the products are not homogeneous, the relative prices of the different products may depend quite arbitrarily on the quantity units in which they are measured.

9.28 Consider, for example, salt and pepper, which are found within the same CPC subclass. Suppose the unit of measurement for pepper is changed from grams to ounces, while leaving the units in which salt is measured (say, kilos) unchanged. Because an ounce of pepper is equal to 28.35 grams, the "price" of pepper increases by more than 28 times, which effectively increases the

[1]This can be seen by rewriting equation (9.1) as

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^0 \left(p_i^t / p_i^0\right)}{\frac{1}{n}\sum p_i^0}.$$

Table 9.2. Properties of Main Elementary Aggregate Index Formulas

	Formula		
Formula properties	Carli—Arithmetic mean of price relatives	Dutot—Relative of arithmetic mean prices	Jevons—Geometric mean of price relatives
Proportionality	yes	yes	yes
Change-of-units of measurement	yes	no	yes
Time reversal	no	yes	yes
Transitivity	no	yes	yes
Allows for substitution	no	no	yes

weight given to pepper in the Dutot index by more than 28 times. The price of pepper relative to salt is inherently arbitrary, depending entirely on the choice of units in which to measure the two goods. In general, when there are different kinds of products within the elementary aggregate, the Dutot index is unacceptable conceptually.

9.29 The Dutot index is acceptable only when the set of products covered is homogeneous, or at least nearly homogeneous. For example, the Dutot index may be acceptable for a set of apple prices, even though the apples may be of different varieties, but not for the prices of different kinds of fruits, such as apples, pineapples, and bananas, some of which may be much more expensive per item or per kilo than others. Even when the products are fairly homogeneous and measured in the same units, the Dutot index's implicit weights may still not be satisfactory. More weight is given to the price changes for the more expensive products, but they may well account for only small shares of the total revenue within the aggregate, in practice. Purchasers are unlikely to buy products at high prices if the same products are available at lower prices.

9.30 It may be concluded that from an axiomatic viewpoint, both the Carli and the Dutot indices, although they have been and still are widely used by statistical offices, have serious disadvantages. The Carli index fails the time reversal and transitivity tests. In principle, it should not matter whether we choose to measure price changes forward or backward in time. We would expect the same answer, but this is not the case for the Carli index. Chained Carli indices may be subject to a significant upward bias. The Dutot index is meaningful for a set of homogeneous products but becomes increasingly arbitrary as the set of products becomes more diverse. On the other hand, the Jevons index satisfies all the tests listed above and also emerges as the preferred index when the set of test is enlarged, as shown in Chapter 20. From an axiomatic point of view, the Jevons index is clearly the index with the best properties, even though it may not have been used much until recently. The Jevons index also allows for some substitution effects consistent with a unitary elasticity of substitution. There seems to be an increasing tendency for statistical offices to switch from using Carli or Dutot indices to Jevons.

B.2.2 Economic approach to elementary price indices

9.31 The objective of the economic approach is to estimate for the elementary aggregates an "ideal" (or "true") economic index—that is, one consistent with the economic theory of revenue-maximizing producers explained in Section F of Chapter 20. The products for which respondents provide prices are treated as a basket of goods and services produced by establishments to provide revenue, and producers are assumed to arrive at their decision about the quantities of outputs to produce on the basis of revenue-maximizing behavior. As explained in Chapters 1, 15, and 17, an ideal theoretical economic index measures the ratio of revenues between two periods that an establishment can attain when faced with fixed technologies and inputs. Changes in the index arise only from changes in prices. The technology is assumed to be held fixed, although the revenue-maximizing producer can make substitutions between the products produced in response to changes in their relative prices. In the absence of information about quantities or revenues within an elementary aggregate, an ideal index can be estimated only when certain special conditions are assumed to prevail.

9.32 There are two special cases of some interest. The first case is when producers continue to produce the same *relative* quantities whatever the relative prices. Producers prefer not to make any substitutions in response to changes in relative prices. The cross-elasticities of supply are zero. The technology by which inputs are translated into outputs in economic theory is described by a production function, and a production function with such a restrictive reaction to relative price changes is described in the economics literature as Leontief. With such a production function, a Laspeyres index would provide an exact measure of the ideal economic index. In this case, the Carli index calculated for a random sample of products would provide an estimate of the ideal economic index that the products are selected with probabilities proportional to the population revenue shares.[2]

9.33 The second case occurs when producers are assumed to vary the quantities they produce in inverse proportion to the changes in relative prices. The cross-elasticities of supply between the different products they produce are all unity, the revenue shares remaining the same in both periods. Such an underlying production function is described as Cobb-Douglas. With this production function, the *geometric Laspeyres*[3] index would provide an exact measure of the ideal index. In this case, the Jevons index calculated for a random sample of products would provide an unbiased estimate of the ideal economic index provided that the products are selected with probabilities proportional to the population expenditure shares.

9.34 From the economic approach, the choice between the sample Jevons index and the sample Carli index rests on which is likely to approximate more closely the underlying ideal economic index—in other words, whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the cross-elasticities could take on any value ranging up to $+\infty$ for an elementary aggregate consisting of a set of strictly homogeneous products—that is, perfect substitutes.[4] It may be conjectured that for demand-led industries where producers produce less of a commodity whose relative price has increased to meet the reduced quantity demanded, the average cross-elasticity is likely to be closer to unity. Thus, the Jevons index is likely to provide a closer approximation to the ideal economic index than the Carli index. In this case, the Carli index must be viewed as having an upward bias. However, there are some establishments in industries, including utilities, in which supply is relatively unresponsive to demand changes, and the Carli index would be more appropriate, given that sampling is with probability proportional to base-period revenue shares. And, yet again, there would be establishments in industries in which quantities produced increase as prices increase, and, with probability sampling proportional to base-period revenues, neither the Carli nor the Jevons index would be appropriate from the economic approach.

9.35 The insight provided by the economic approach is that the Jevons and Carli indices can be justified from the economic approach depending on whether a significant amount of substitution is more likely than no substitution, especially as elementary aggregates should be deliberately constructed to group together similar products that are close substitutes for each other.

9.36 The Jevons index does not imply, or assume, that revenue shares remain constant. Obviously, the Jevons can be calculated whether changes do or do not occur in the revenue shares in practice. What the economic approach shows is that *if* the revenue shares remain constant (or roughly constant), *then* the Jevons index can be expected to provide a good estimate of the underlying ideal

[2]It might appear that if the products were selected with probabilities proportional to the population quantity shares, the sample Dutot would provide an estimate of the population Laspeyres. However, if the basket for the Laspeyres index contains different kinds of products whose quantities are not additive, the quantity shares, and hence the probabilities, are undefined.

[3]The geometric Laspeyres is a weighted geometric average of the price relatives, using the revenue shares in the earlier period as weights. (The revenue shares in the second period would be the same in the particular case under consideration.)

[4]It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price "index" is given by the ratio of the unit values in the two periods, as explained below.

economic index. Similarly, *if* the relative quantities remain constant, *then* the Carli index can be expected to provide a good estimate, but the Carli index does not actually imply that quantities remain fixed. Reference should be made to Section F of Chapter 20 for a more rigorous statement of the economic approach.

9.37 It may be concluded that on the economic approach, as well as the axiomatic approach, the Jevons emerges as the preferred index in general, although there may be cases in which little or no substitution takes place within the elementary aggregate, and the Carli might be preferred. The index compiler must make a judgment on the basis of the nature of the products actually included in the elementary aggregate.

9.38 Before leaving this topic, it should be noted that it has thrown light on some of the sampling properties of the elementary indices. If the products in the sample are selected with probabilities proportional to expenditures in the price reference period,

- The sample (unweighted) Carli index provides an unbiased estimate of the population Laspeyres, and
- The sample (unweighted) Jevons index provides an unbiased estimate of the population geometric Laspeyres.

9.39 These results hold, regardless of what the underlying economic index may be.

B.3 Chained versus direct indices for elementary aggregates

9.40 In a direct elementary index, the prices of the current period are compared directly with those of the price reference period—in a chained index, prices in each period are compared with those in the previous period, the resulting short-term indices being chained together to obtain the long-term index, as illustrated in Table 9.1.

9.41 Provided that prices are recorded for the same set of products in every period, as in Table 9.1, any index formula defined as the ratio of the average prices will be transitive—that is, the same result is obtained whether the index is calculated as a direct index or as a chained index. In a chained index, successive numerators and denominators will cancel out, leaving only the average price in the last period divided by the average price in the price reference period, which is the same as the direct index. Both the Dutot and the Jevons indices are therefore transitive. As already noted, however, a chained Carli index is not transitive and should not be used because of its upward bias. Nevertheless, the direct Carli remains an option.

9.42 Although the chained and direct versions of the Dutot and Jevons indices are identical when there are no breaks in the series for the individual products, they offer different ways of dealing with new and disappearing products, missing prices, and quality adjustments. In practice, products continually have to be dropped from the index and new ones included, in which case the direct and the chained indices may differ if the imputations for missing prices are made differently.

9.43 When a replacement product has to be included in a direct index, it often will be necessary to estimate the price of the new product in the price reference period, which may be some time in the past. The same happens if, as a result of an update of the sample, new products have to be linked into the index. Assuming that no information exists on the price of the replacement product in the price reference period, it will be necessary to estimate it using price relatives calculated for the products that remain in the elementary aggregate, a subset of these products, or some other indicator. However, the direct approach should be used only for a limited period. Otherwise, most of the reference prices would end up being imputed, which would be an undesirable outcome. This effectively rules out the use of the Carli index over a long period, because the Carli index can be used only in its direct form anyway, being unacceptable when chained. This implies that, in practice, the direct Carli index may be used only if the overall index is chained annually, or at intervals of two or three years.

9.44 In a chained index, if a product becomes permanently missing, a replacement product can be linked into the index as part of the ongoing index calculation by including the product in the monthly index as soon as prices for two successive months are obtained. Similarly, if the sample is updated and new products have to be linked into the index, this will require successive old and new prices for the present and the preceding month. However, for a chained index, the missing observation will affect the index for two months, since the missing observation is part of two links in the chain. This is not the case for a direct index, where a single, nonestimated missing observation will affect only the index in the current period. For example, when comparing periods 0 and 3, a missing price of a product in period 2 means that the chained index excludes the product for the last link of the index in periods 2 and 3, while the direct index includes it in period 3 (since a direct index will be based on products with prices available in periods 0 and 3). However, in general, the use of a chained index can make the estimation of missing prices and the introduction of replacements easier from a computational point of view, whereas it may be inferred that a direct index will limit the usefulness of overlap methods for dealing with missing observations. This is discussed further in Section B.5.

9.45 The direct and the chained approaches also produce different by-products that may be used for monitoring price data. For each elementary aggregate, a chained index approach gives the latest monthly price change, which can be useful for both editing data and imputing missing prices. By the same token, however, a direct index derives average price levels for each elementary aggregate in each period, and this information may be a useful by-product. However, the availability of cheap computing power and spreadsheets allows such by-products to be calculated whether a direct or a chained approach is applied, so that the choice of formula should not be dictated by considerations regarding by-products.

B.4 Consistency in aggregation

9.46 Consistency in aggregation means that if an index is calculated stepwise by aggregating lower-level indices to obtain indices at progressively higher levels of aggregation, the same overall result should be obtained as if the calculation had been made in one step. For presentational purposes, this is an advantage. If the elementary aggregates are calculated using one formula, and the elementary aggregates are averaged to obtain the higher-level indices using another formula, the resulting PPI is not consistent in aggregation. However, it may be argued that consistency in aggregation is not necessarily an important or even appropriate criterion. Also it may be unachievable, particularly when the amount of information available on quantities and revenues is not the same at the different levels of aggregation. In addition, there may be different degrees of substitution within elementary aggregates compared with the degree of substitution between products in different elementary aggregates.

9.47 As noted in Section B.2.2 above, the Carli index would be consistent in aggregation with the Laspeyres index if the products were to be selected with probabilities proportional to revenues in the price reference period. However, this is typically not the case. The Dutot and the Jevons indices are also not consistent in aggregation with a higher-level Laspeyres. However, as explained below, the PPIs actually calculated by statistical offices are usually not true Laspeyres indices anyway, even though they may be based on fixed baskets of goods and services. As also noted earlier, if the higher-level index were to be defined as a geometric Laspeyres index, consistency in aggregation could be achieved by using the Jevons index for the elementary indices at the lower level, provided that the individual products are sampled with probabilities proportional to revenues. Although unfamiliar, a geometric Laspeyres index has desirable properties from an economic point of view and is considered again later.

B.5 Missing price observations

9.48 The price of a product may not be collected in a particular period, either because the product is missing temporarily or because it has permanently disappeared. The two classes of missing prices require different treatments. Temporary unavailability may occur for seasonal products (particularly for fruit, vegetables, and clothing) because of supply shortages or possibly because of some collection difficulty (for example, an establishment was closed or a respondent was on vacation). The treatment of seasonal products raises a number of particular problems. These are dealt with in Chapter 22 and will not be discussed here.

B.5.1 Treatment of temporarily missing prices

9.49 In the case of temporarily missing observations for products, one of four actions may be taken:

- Omit the product for which the price is missing so that a matched sample is maintained (like is compared with like), even though the sample is depleted.

- Carry forward the last observed price.
- Impute the missing price by the average price change for the prices that are available in the elementary aggregate.
- Impute the missing price by the price change for a particular comparable product from a similar establishment.

Omitting an observation from the calculation of an elementary index is equivalent to assuming that the price would have moved in the same way as the average of the prices of the products that remain included in the index. Omitting an observation changes the implicit weights attached to the other prices in the elementary aggregate.

9.50 Carrying forward the last observed price should be avoided wherever possible and is acceptable only for a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index toward zero change. In addition, there is likely to be a compensating step-change in the index when the price of the missing product is recorded again, which will be wrongly missed by a chained index but will be included in a direct index to return the index to its proper value. The adverse effect on the index will be increasingly severe if the product remains unpriced for some length of time. In general, carryforward is not an acceptable procedure or solution to the problem unless it is certain the price has not changed.

9.51 Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates when the prices can be expected to move in the same direction. The imputation can be made using all the remaining prices in the elementary aggregate. As already noted, this is numerically equivalent to omitting the product for the immediate period, but it is useful to make the imputation so that if the price becomes available again in a later period, the sample size is not reduced in that period. In some cases, depending on the homogeneity of the elementary aggregate, it may be preferable to use only a subset of products from the elementary aggregate to estimate the missing price. In some instances, this may even be a single comparable product from a similar type of establishment whose price change can be expected to be similar to the missing one.

9.52 Table 9.3 illustrates the calculation of the price index for an elementary aggregate consisting of three products, where one of the prices is missing in March. The upper part of Table 9.3 shows the indices where the missing price has been omitted from the calculation. The direct indices are therefore calculated on the basis of *A*, *B*, and *C* for all months except March, where it is calculated on basis of *B* and *C* only. The chained indices are calculated on the basis of all three prices from January to February and from April to May. From February to March and from March to April, the monthly indices are calculated on the basis of *B* and *C* only.

9.53 For both the Dutot and the Jevons, the direct and chain indices now differ from March onward. The first link in the chained index (January to February) is the same as the direct index, so that the two indices are identical numerically. The direct index for March ignores the price decrease of product *A* between January and February, while this is taken into account in the chained index. As a result, the direct index is higher than the chained index for March. On the other hand, in April and May, where all prices again are available, the direct index catches the price development, whereas the chained index fails to track the development in the prices.

9.54 In the lower half of Table 9.3, the missing price for product *A* in March is imputed by the average price change of the remaining products from February to March. While the index may be calculated as a direct index comparing the prices of the present period with the reference period prices, the imputation of missing prices should be made on basis of the average price change from the preceding to the present period, as shown in the table. Imputation on the basis of the average price change from the price reference period to the present period should not be used, since it ignores the information about the price change of the missing product that has already been included in the index. The treatment of imputations is discussed in more detail in Chapter 7.

Table 9.3. Imputation of Temporarily Missing Prices

	January	February	March	April	May
			Prices		
Product *A*	6.00	5.00		7.00	6.60
Product *B*	7.00	8.00	9.00	8.00	7.70
Product *C*	2.00	3.00	4.00	3.00	2.20
Omit missing product from the index calculation					
Carli index—Arithmetic mean of price relatives					
Direct index	100.00	115.87	164.29	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.67	118.18	84.62	91.67
Chained month-to-month index	100.00	106.67	126.06	106.67	97.78
Direct index	100.00	106.67	144.44	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Month-to-month index	100.00	112.62	122.47	81.65	87.31
Chained month-to-month index	100.00	112.62	137.94	112.62	98.33
Direct index	100.00	112.62	160.36	125.99	110.00
Imputation					
Carli index—Arithmetic mean of price relatives					
Impute price for A *in March as 5 (9/8 + 4/3)/2 = 6.15*					
Direct index	100.00	115.87	143.67	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Impute price for A *in March as 5 [(9 + 4)/(8 + 3)] = 5.91*					
Month-to-month index	100.00	106.67	118.18	95.19	91.67
Chained month-to-month index	100.00	106.67	126.06	120.00	110.00
Direct index	100.00	106.67	126.06	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for A *in March as* $5\,(9/8 \times 4/3)^{0.5} = 6.12$					
Month-to-month index	100.00	112.62	122.47	91.34	87.31
Chained month-to-month index	100.00	112.62	137.94	125.99	110.00
Direct index	100.00	112.62	137.94	125.99	110.00

B.5.2 Treatment of products that have permanently disappeared and their replacements

9.55 Products may disappear permanently for various reasons. The product may disappear from the market because new products have been introduced or the establishments from which the price has been collected have stopped selling the product. When products disappear permanently, a replacement product has to be sampled and included in the index. The replacement product should ideally be one that accounts for a significant proportion of sales, is likely to continue to be sold for some time, and is likely to be representative of the sampled price changes of the market that the old product covered.

9.56 The timing of the introduction of replacement products is important. Many new products are initially sold at high prices that then gradually drop over time, especially as the volume of sales increases. Alternatively, some products may be introduced at artificially low prices to stimulate demand. In such cases, delaying the introduction of a new or replacement product until a large volume of sales is achieved may miss some systematic price changes that ought to be captured by PPIs. It may be desirable to try to avoid forced replacements caused when products disappear completely from the market and to try to introduce replacements when sales of the products they replace are decreasing and before they cease altogether.

9.57 Table 9.4 shows an example where product *A* disappears after March and product *D* is included as a replacement from April onward. Products *A* and *D* are not available on the market at the same time, and their price series do not overlap. To in-

Table 9.4. Disappearing Products and Their Replacements with No Overlap

	January	February	March	April	May
			Prices		
Product *A*	6.00	7.00	5.00		
Product *B*	3.00	2.00	4.00	5.00	6.00
Product *C*	7.00	8.00	9.00	10.00	9.00
Product *D*				9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 9 /[(5/3 + 10/7) 0.5] = 5.82*					
Direct index	100.00	99.21	115.08	154.76	155.38
Dutot index—Ratio of arithmetic mean prices					
Impute price for D *in March as 9 /[(5 + 10)/(4 + 9)] = 7.80*					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Impute price for D *in January as 9 /[(5 + 10)/(3 + 7)] = 6.00*					
Direct index	100.00	106.25	112.50	150.00	143.75
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for D *in March as* $9/[(5/4 \times 10/9)^{0.5}] = 7.64$					
Month-to-month index	100.00	96.15	117.13	117.85	98.65
Chained month-to-month index	100.00	96.15	112.62	132.73	130.94
Impute price for D *in January as* $9/[(5/3 \times 10/7)^{0.5}] = 5.83$					
Direct index	100.00	96.15	112.62	154.30	152.22
Omit the price					
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Monthly index	100.00	96.15	117.13	117.85	98.65
Chain month-to-month index	100.00	96.15	112.62	132.73	130.94

clude the new product in the index from April onward, an imputed price needs to be calculated either for the base period (January) if a direct index is being calculated, or for the preceding period (March) if a chained index is calculated. In both cases, the imputation method ensures that the inclusion of the new product does not, in itself, affect the index.

9.58 In the case of a chained index, imputing the missing price by the average change of the available prices gives the same result as if the product is simply omitted from the index calculation until it has been priced in two successive periods. This allows the chained index to be compiled by simply chaining the month-to-month index between periods $t-1$ and t, based on the matched set of prices in those two periods, on to the value of the chained index for period $t-1$. In the example, no further imputation is required after April, and the subsequent movement of the index is unaffected by the imputed price change between March and April.

9.59 In the case of a direct index, however, an imputed price is always required for the reference period to include a new product. In the example, the price of the new product in each month after April still has to be compared with the imputed price for January. As already noted, to prevent a situation in

Table 9.5. Disappearing and Replacement Products with Overlapping Prices

	January	February	March	April	May
			Prices		
Product *A*	6.00	7.00	5.00		
Product *B*	3.00	2.00	4.00	5.00	6.00
Product *C*	7.00	8.00	9.00	10.00	9.00
Product *D*			10.00	9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	99.21	115.08	128.17	131.75
Dutot index—Ratio of arithmetic mean prices					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	106.25	105.88	104.35	95.83
Chained month-to-month index	100.00	106.25	112.50	117.39	112.50
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	106.25	112.50	121.88	118.75
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	106.25	112.50	109.09	104.55
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	96.15	117.13	107.72	98.65
Chained month-to-month index	100.00	96.15	112.62	121.32	119.68
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	96.15	112.62	121.32	119.68
Impute price for D *in January as 6 /(5/10) = 12.00*					
Direct index	100.00	96.15	112.62	121.32	119.68

which most of the reference period prices end up being imputed, the direct approach should be used only for a limited period of time.

9.60 The situation is somewhat simpler when there is an overlap month in which prices are collected for both the disappearing and the replacement product. In this case, it is possible to link the price series for the new product to the price series for the old product that it replaces. Linking with overlapping prices involves making an implicit adjustment for the difference in quality between the two products, since it assumes that the relative prices of the new and old product reflect their relative qualities. For perfect or nearly perfect markets, this may be a valid assumption, but for certain markets and products it may not be so reasonable. The question of when to use overlapping prices is dealt with in detail in Chapter 7. The overlap method is illustrated in Table 9.5.

9.61 In the example, overlapping prices are obtained for products *A* and *D* in March. Their relative prices suggest that one unit of product *A* is worth two units of product *D*. If the index is calculated as a direct Carli index, the January base-period price for product *D* can be imputed by dividing the price of product *A* in January by the price ratio of *A* and *D* in March.

9.62 A monthly chained index of arithmetic mean prices will be based on the prices of products *A*, *B*, and *C* until March, and from April onward by *B*, *C*, and *D*. The replacement product is not included until prices for two successive periods are obtained. Thus, the monthly chained index has the advantage that it is not necessary to carry out any explicit imputation of a reference price for the new product.

9.63 If a direct index is calculated as the ratio of the arithmetic mean prices, the price of the new product needs to be adjusted by the price ratio of *A* and *D* in March in every subsequent month, which complicates computation. Alternatively, a reference period price of product *D* for January may be imputed. However, this results in a different index because the price relatives are implicitly weighted by the relative reference period prices in the Dutot index, which is not the case for the Carli or the Jevons index. For the Jevons index, all three methods give the same result, which is an additional advantage of this approach.

B.6 Other formulas for elementary price indices

9.64 A number of other formulas have been suggested for the price indices for elementary aggregates. The most important are presented below and discussed further in Chapter 20.

B.6.1 Laspeyres and geometric Laspeyres indices

9.65 The Carli, Dutot, and Jevons indices are all calculated without the use of explicit weights. However, as already mentioned, in certain cases there may be weighting information that could be exploited or developed in the calculation of the elementary price indices. If the reference period revenues for all the individual products within an elementary aggregate, or estimates thereof, were available, the elementary price index could itself be calculated as a Laspeyres price index, or as a geometric Laspeyres. The Laspeyres price index is defined as

$$(9.4)\quad P_L^{0:t} = \sum w_i^0 \left(\frac{p_i^t}{p_i^0} \right), \quad \sum w_i^0 = 1,$$

where the weights, w_i^0, are the revenue shares for the individual products in the reference period. If all the weights were equal, equation (9.4) would reduce to the Carli index. If the weights were proportional to the prices in the reference period, equation (9.4) would reduce to the Dutot index.

9.66 The geometric Laspeyres index is defined as

$$(9.5)\quad P_{JW}^{0:t} = \prod \left(\frac{p_i^t}{p_i^0} \right)^{W_0^i} = \frac{\prod \left(p_i^t \right)^{W_i^0}}{\prod \left(p_i^0 \right)^{W_i^0}}, \quad \sum w_i^0 = 1,$$

where the weights, w_i^0, are again the revenue shares in the reference period. When the weights are all equal, equation (9.5) reduces to the Jevons index. If the revenue shares do not change much between the weight reference period and the current period, then the geometric Laspeyres index approximates a Törnqvist index.

B.6.2 Some alternative index formulas

9.67 Another widely used type of average is the harmonic mean. In the present context, there are two possible versions: either the harmonic mean of price relatives or the ratio of harmonic mean of prices.

9.68 The harmonic mean of price relatives is defined as

$$(9.6)\ \ P_{HR}^{0:t} = \frac{1}{\frac{1}{n}\sum \frac{p_i^0}{p_i^t}}.$$

The ratio of harmonic mean prices is defined as

$$(9.7)\ \ P_{RH}^{0:t} = \frac{\sum n/p_i^0}{\sum n/p_i^t}.$$

Equation (9.7), like the Dutot index, fails the commensurability test and would be an acceptable possibility only when the products are all fairly homogeneous. Neither formula appears to be used much in practice, perhaps because the harmonic mean is not a familiar concept and would not be easy to explain to users. However, at an aggregate level, the widely used Paasche index is a weighted harmonic average.

9.69 The ranking of the three common types of mean is always

arithmetic mean ≥ geometric mean ≥ harmonic mean.

It is shown in Chapter 20 that, in practice, the Carli index, the arithmetic mean of the relatives, is likely to exceed the Jevons index, the geometric mean, by roughly the same amount that the Jevons exceeds the harmonic mean, equation (9.6). The harmonic mean of the price relatives has the same kinds of axiomatic properties as the Carli but with opposite tendencies and biases. It fails the transitivity and time reversal tests discussed earlier. In addition it is very sensitive to "price bouncing," as is the Carli index. As it can be viewed conceptually as the complement, or rough mirror image, of the Carli index, it has been argued that a suitable elementary index would be provided by a geometric mean of the two, in the same way that, at an aggregate level, a geometric mean is taken of the Laspeyres and Paasche indices to obtain the Fisher index. Such an index has been proposed by Carruthers, Sellwood, Ward, and Dalén—namely,

$$(9.8)\ \ P_{CSWD}^{0:t} = \sqrt{I_C^{0:t} \cdot I_{HR}^{0:t}}.$$

P_{CSWD} is shown in Chapter 20 to have very good axiomatic properties but not quite as good as Jevons index, which is transitive, whereas the P_{CSWD} is not. However, it can be shown to be approximately transitive and, empirically, it has been observed to be very close to the Jevons index.

9.70 More recently, as attention has focused on the economic characteristics of elementary aggregate formulas, consideration has been given to formulas that allow for substitution between products within an elementary aggregate. The increasing use of the geometric mean is an example of this. However, the Jevons index is limited to a functional form that reflects an elasticity of demand equal to one that, while clearly allowing for some substitution, is unlikely to be applicable to all elementary aggregates. A logical step is to consider formulas that allow for different degrees of substitution in different elementary aggregates. One such formula is the unweighted Lloyd-Moulton formula:

$$(9.9)\ \ P_{LM}^{0:t} = \left[\sum \frac{1}{n}\left(\frac{P_i^t}{P_i^0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the elasticity of substitution. The Carli and the Jevons indices can be viewed as special cases of the P_{LM} in which σ = 0 and σ = 1. The advantage of the P_{LM} formula is that σ is unrestricted. Provided a satisfactory estimate can be made of σ, the resulting elementary price index is likely to approximate the Fisher and other superlative indices. It reduces substitution bias when the objective is to estimate an economic index. The difficulty is in the need to estimate elasticities of substitution, a task that will require substantial development and maintenance work. The formula is described in more detail in Chapter 20.

B.7 Unit-value indices

9.71 The unit-value index is simple in form. The unit value in each period is calculated by dividing total revenue on some product by the related total quantity. It is clear that the quantities must be strictly additive in an economic sense, which implies that they should relate to a single homogeneous product. The unit-value index is then defined as the ratio of unit values in the current period to those in the reference period. It is not a price index as normally understood, since it is essentially a measure of the change in the average price of a *single* product when that product is sold at different prices to different purchasers, perhaps at different times within the same period. Unit values, and unit-value indices, should not be calculated for sets of heterogeneous products.

9.72 However, unit values do play an important part in the process of calculating an elementary price index, because they are the appropriate *average* prices that need to be entered into an elementary price index. Usually, prices are sampled at a particular time or period each month, and each price is assumed to be representative of the average price of that product in that period. In practice, this assumption may not hold. In this case, it is necessary to estimate the unit value for each product, even though this will inevitably be more costly. Thus, having specified the product to be priced in a particular establishment, data should be collected on both the value of the total sales in a particular month and the total quantities sold in order to derive a unit value to be used as the price input into an elementary aggregate formula. It is particularly important to do this if the product is sold at a discount price for part of the period and at the "regular" price in the rest of the period. Under these conditions, neither the discount price nor the regular price is likely to be representative of the average price at which the product has been sold or the price change between periods. The unit value over the whole month should be used. With the possibility of collecting more and more data from electronic records, such procedures may be increasingly used. However, it should be stressed that the product specifications must remain constant through time. Changes in the product specifications could lead to unit-value changes that reflect quantity, or quality, changes and should not be part of price changes.

B.8 Formulas applicable to electronic data

9.73 Respondents may well have computerized management accounting systems that include highly detailed data on sales both in terms of prices and quantities. Their primary advantages are that the number of price observations can be significantly larger and that both price and quantity information are available in real time. Much work has been undertaken on the use of scanner data as an emerging data source for CPI compilation, and there are parallels for the PPI. There are a large number of practical considerations, which are discussed and referenced in the *CPI Manual* (ILO and others, 2004) and also in Chapter 6, Section D, of this *Manual*, but it is relevant to discuss briefly here possible index number formulas that may be applicable if electronic data are collected and used in PPI compilation.

9.74 The existence of quantity and revenue information increases the ability to estimate price changes accurately. It means that traditional index number approaches such as Laspeyres and Paasche can be used, and that superlative formulas such as the Fisher and Törnqvist-Theil indices can also be derived in real time. The main observation made here is that since price and quantity information are available for each period, it may be tempting to produce monthly or quarterly chained indices using one of the ideal formulas mentioned above. However, the compilation of subannual chained indices has been found in some studies to be problematic because it often results in an upward bias referred to as "chain drift."

C. Calculation of Higher-Level Indices

C.1 Target indices

9.75 A statistical office must have some target index at which to aim. Statistical offices have to consider what kind of index they would choose to calculate in the ideal hypothetical situation in which they had complete information about prices and quantities in both time periods compared. If the PPI is meant to be an economic index, then a superlative index such as a Fisher, Walsh, or Törnqvist-Theil would have to serve as the theoretical target, since a superlative index may be expected to approximate the underlying economic index.

9.76 Many countries do not aim to calculate an economic index and prefer the concept of a *basket index*. A basket index is one that measures the change in the total value of a given basket of goods and services between two time periods. This general category of index is described here as a *Lowe* index after the early 19th-century index number pioneer who first proposed this kind of index (see Chapter 15, Section D). The meaning of a Lowe index is clear and can be easily explained to users, important considerations for many statistical offices. It should be noted that, in general, there is no necessity for the basket to be the actual basket in one or other of the two periods compared. If the theoretical target index is to be a basket or Lowe index, the preferred basket might be one that attaches equal importance to the baskets in both periods—for example, the Walsh index.[5] Thus, the same kind of index may emerge as the theoretical target on both the basket and the economic index approaches. In practice, however, a statistical office may prefer to designate the basket index that uses the actual basket in the earlier of the two periods as its target index on grounds of simplicity and practicality. In other words, the Laspeyres index may be a target index.

9.77 The theoretical target index is a matter of choice. In practice, it is likely to be either a Laspeyres or some superlative index. However, even when the target index is the Laspeyres, there may a considerable gap between what is actually calculated and what the statistical office considers to be its target. It is now necessary to consider what statistical offices tend to do in practice.

C.2 PPIs as weighted averages of elementary indices

9.78 Section B discussed alternative formulas for combining individual price observations to calculate the first level of indices, called elementary aggregates. The next steps in compiling the PPI involves taking the elementary indices and combining them, using weights, to calculate successively higher levels of indices as shown in Chapter 4, Figure 4.1.

9.79 A higher-level index is an index for some revenue aggregate above the level of an elementary aggregate, including the overall PPI itself. The inputs into the calculation of the higher-level indices are

- The elementary price indices, and
- Weights derived from the values of elementary aggregates in some earlier year, or years.

The higher-level indices are calculated simply as weighted arithmetic averages of the elementary price indices. This general category of index is described here as a *Young* index after another one of the 19th-century index number pioneers who advocated this type of index (see Chapter 15, Section D).

9.80 The weights typically remain fixed for a sequence of at least 12 months. Some countries revise their weights at the beginning of each year to try to approximate as closely as possible to current production patterns. However, many countries continue to use the same weights for several years. The weights may be changed only every five years or so.

9.81 The use of fixed weights has the considerable practical advantage that the index can make repeated use of the same weights. This saves both time and money. Revising the weights can be both time consuming and costly, especially if it requires new establishment production surveys to be carried out.

9.82 The second stage of calculating the PPI does not involve individual prices or quantities. Instead, a higher-level index is a Young index in which the elementary price indices are averaged using a set of predetermined weights. The formula can be written as follows:

$$(9.10)\quad P_Y^{0:t} = \sum w_i^b I_i^{0:t}\ , \quad \sum w_i^b = 1,$$

where $P_Y^{0:t}$ denotes the overall PPI, or any higher-level index, from period 0 to t; w_i^b is the weight attached to each of the elementary price indices; and $I_i^{0:t}$ is the corresponding elementary price index. The elementary indices are identified by the subscript i, whereas the higher-level index carries no subscript. As already noted, a higher-level index is any index, including the overall PPI, above the elementary aggregate level. The weights are derived from revenue in period b, which in practice has to precede period 0, the price reference period.

9.83 It is useful to recall that three kinds of reference periods may be distinguished for PPI purposes:

[5]The quantities that make up the basket in the Walsh index are the geometric means of the quantities in the two periods.

- *Weight Reference Period*: The period covered by the revenue statistics used to calculate the weights. Usually, the weight reference period is a year.
- *Price Reference Period*: The period for which prices are used as denominators in the index calculation.
- *Index Reference Period*: The period for which the index is set to 100.

9.84 The three periods are generally different. For example, a PPI might have 1998 as the weight reference *year*, December 2002 as the price reference *month*, and the *year* 2000 as the index reference period. The weights typically refer to a whole year, or even two or three years, whereas the periods whose prices are compared are typically months or quarters. The weights are usually estimated on the basis of an establishment survey that was conducted some time before the price reference period. For these reasons, the weight and the price reference periods are invariably separate periods in practice.

9.85 The index reference period is often a year, but it could be a month or some other period. An index series may also be re-referenced to another period by simply dividing the series by the value of the index in that period, without changing the rate of change of the index. The expression "base period" can mean any of the three reference periods and can sometimes be quite ambiguous. "Base period" should be used only when it is absolutely clear in context exactly which period is referred to.

9.86 Provided the elementary aggregate indices are calculated using a transitive formula such as the Jevons or Dutot, but not the Carli, and provided that there are no new or disappearing products from period 0 to *t*, equation (9.10) is equivalent to

$$(9.11)\quad P^{0:t} = \sum w_i^b I_i^{0:t-1} I_i^{t-1:t}\ ,\quad \sum w_i^b = 1\ ,$$

where $P^{0:t}$ is a higher-level PPI.

Table 9.6. Aggregation of Elementary Price Indices

Index	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
A	0.20	100.00	102.50	104.88	101.16	101.15	100.00
B	0.25	100.00	100.00	91.67	109.09	101.67	108.20
C	0.15	100.00	104.00	96.15	104.00	101.92	103.77
D	0.10	100.00	92.86	107.69	107.14	100.00	102.67
E	0.30	100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
B	0.25	100.00	100.00	91.67	100.00	101.67	110.00
C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00
Higher-level indices							
G = *A*+*B*+*C*	0.60	100.00	101.83	99.03	103.92	105.53	110.00
H = *D*+*E*	0.40	100.00	99.46	101.25	101.79	104.29	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00

The advantage of this version of the index is that it allows the sampled products within the elementary price index from $t - 1$ to t to differ from the sampled products in the periods from 0 to $t - 1$. Hence, it allows replacement products and new products to be linked into the index from period $t - 1$ without the need to estimate a price for period 0, as explained in Section B.5. For example, if one of the sampled products in periods 0 and $t - 1$ is no longer available in period t, and the price of a replacement product is available for $t - 1$ at t, the new replacement product can be included in the index using the overlap method.

C.3 A numerical example

9.87 Equation (9.10) applies at each level of aggregation above the elementary index. The index is additive—that is, the overall index is the same whether calculated on the basis of the original elementary price indices or on the basis of the intermediate higher-level indices. This facilitates the presentation of the index.

9.88 Table 9.6 (on the previous page) illustrates the calculation of higher-level indices in the special case where the weight and the price reference period are identical, that is, $b = 0$. The index consists of five elementary aggregate indices (*A–E*), which are calculated using one of the formulas presented in Section 9.B.2, and two intermediate higher-level indices, *G* and *H*. The overall index (Total) and the higher-level indices (*G* and *H*) are all calculated using equation (9.10). Thus, for example, the overall index for April can be calculated from the two intermediate higher-level indices of April as

$$P^{Jan:apr} = 0.6 \times 103.92 + 0.4 \times 101.79 = 103.06$$

or directly from the five elementary indices

$$P^{Jan:apr} = 0.2 \times 108.75 + 0.25 \times 100 + 0.15 \times 104 + 0.1 \times 107.14 + 0.3 \times 100 = 103.06 .$$

Note from equation (9.11) that

$$(9.12)\quad P^{0:t} = \sum w_i^b P_i^{0:t-1} P_i^{t-1:t} \neq P^{0:t-1} \sum w_i^b \; P_i^{t-1:t} \Rightarrow \frac{P^{0:t}}{P^{0:t-1}} \neq \sum w_i^b P_i^{t-1:t} .$$

This shows that if the month-to-month indices are averaged using the fixed weights w_i^b, the resulting index is *not* equal to the month-to-month higher-level index. As explained below, to be able to obtain the month-to-month higher-level index, the weights applied to the month-to-month indices need to be updated to reflect the effects of the price changes that have taken place since January.

C.4 Young and Lowe indices

9.89 It is useful to clarify the relationship between the Lowe and Young indices. As already noted, when statistical offices explain their PPIs to users, they often describe them as Lowe indices, which measure the change over time in the value of a fixed basket of goods and services. When they calculate their PPIs, however, the formula they actually use is that of a Young index. The relationship between the two indices is given in equation (9.13), where P_{Lo} is the Lowe index and P_Y is the Young index:

$$(9.13)\quad P_{Lo} = \frac{\sum p_j^t q_j^b}{\sum p_j^0 q_j^b} = \frac{\sum p_j^t q_j^b}{\sum p_j^b q_j^b} \Bigg/ \frac{\sum p_j^0 q_j^b}{\sum p_j^b q_j^b} = \sum w_j \left(\frac{p_j^t}{p_j^0} \right) = P_Y ,$$

$$\text{where } w_j = \frac{p_j^0 q_j^b}{\sum p_j^0 q_j^b} .$$

The individual quantities (q_j^b) in the weight reference period *b* make up the basket. Assume initially that the weight reference period *b* has the same duration as that of the two periods 0 and *t* that are being compared. It can be seen from equation (9.13) that

(i) The Lowe index is equal to a Young index in which the weights are *hybrid* value shares obtained by revaluing the quantities in the weight reference period *b* (q_j^b), at the prices of the price reference month 0;[6]

(ii) The Lowe index can be expressed as the ratio of the two Laspeyres indices for periods *t* and 0, respectively, based on month *b*; and

[6]Since the weights are usually revenues, this is often referred to as price updating the weights to the price reference period and will be discussed further in Section C.6.

(iii) The Lowe index reduces to the Laspeyres index when $b = 0$ and to the Paasche index when $b = t$.

9.90 In practice, the situation is more complicated for actual PPIs because the duration of the reference period b is typically much longer than periods 0 and t. The weights w_j usually refer to the revenues over a year, or longer, while the price reference period is usually a month in some later year. For example, a monthly index may be compiled from January 2003 onward with December 2002 as the price reference month, but the latest available weights during the year 2003 may refer to the year 2000, or even some earlier year.

9.91 Conceptually, a typical PPI may be viewed as a Lowe index that measures the change from month to month in the total revenue of an annual basket of goods and services that may date back several years before the price reference period. Because it uses the fixed basket of an earlier period, it is sometimes loosely described as a "Laspeyres-type" index, but this description is unwarranted. A true Laspeyres index would require the basket to be that purchased in the price reference month, whereas in most PPIs the basket not only refers to a different period from the price reference month but to a period of a year or more. When the weights are annual and the prices are monthly, it is not possible, even retrospectively, to calculate a monthly Laspeyres price index.

9.92 A Lowe index that uses quantities derived from an earlier period than the price reference period is likely to exceed the Laspeyres (see Section D.1 of Chapter 15), and by a progressively larger amount, the further back in time the weight reference period is. The Lowe index is likely to have an even greater upward bias than the Laspeyres index as compared with some target superlative index and the underlying economic index. Inevitably, the quantities in any basket index become increasingly out of date and irrelevant the further back in time the period to which they relate. To minimize the resulting bias, the weights should be updated more frequently, preferably annually.

9.93 A statistical office may not wish to estimate an economic index and may prefer to choose some basket index as its target index. In that case, if the theoretically attractive Walsh index were to be selected as the target index, a Lowe index would have the same bias, as just described, given that the Walsh index is also a superlative index.

C.5 Factoring the Young index

9.94 It is possible to calculate the change in a higher-level Young index between two consecutive periods, such as $t-1$ and t, as a weighted average of the individual price indices between $t-1$ and t, *provided* that the weights are updated to take into account the price changes between the price reference period 0 and the previous period, $t-1$. This makes it possible to factor equation (9.10) into the product of two component indices in the following way:

$$(9.14)\quad P^{0:t} = P^{0:t-1} \sum w_i^{b(t-1)} P_i^{t-1:t} ,$$

where $w_i^{b(t-1)} = w_i^b P_i^{0:t-1} \Big/ \sum w_i^b P_i^{0:t-1}$.

$I^{0:t-1}$ is the Young index for period $t-1$. The weight $w_i^{b(t-1)}$ is the original weight for elementary aggregate i price updated by multiplying it by the elementary price index for i between 0 and $t-1$, the adjusted weights being rescaled to sum to unity. The price-updated weights are hybrid weights because they implicitly revalue the quantities of b at the prices of $t-1$ instead of at the average prices of b. Such hybrid weights do not measure the actual revenue shares of any period.

9.95 The index for period t can thus be calculated by multiplying the already calculated index for $t-1$ by a separate Young index between $t-1$ and t with hybrid price-updated weights. In effect, the higher-level index is calculated as a chained index in which the index is moved forward period by period. This method gives more flexibility to introduce replacement products and makes it easier to monitor the movements of the recorded prices for errors, since month-to-month movements are smaller and less variable than the total changes since the price reference period.

9.96 Price updating may also occur between the weight reference period to the price reference period, as explained in the next section.

C.6 Updating from weight reference period to price reference period

9.97 When the weight reference period b and the price reference period 0 are different, as is normally the case, the statistical office has to decide whether or not to price update the weights from b to 0. In practice, the price-updated weights can be calculated by multiplying the original weights for period b by elementary indices measuring the price changes between periods b and 0 and rescaling to sum to unity.

9.98 The issues involved are best explained with the help of a numerical example. In Table 9.7, the base period b is assumed to be the year 2000 so that the weights are the revenue shares in 2000. In the upper half of the table, 2000 is also used as the price reference period. However, in practice, weights based on 2000 cannot be introduced until after 2000 because of the time needed to collect and process the revenue data. In the lower half of the table, it is assumed that the 2000 weights are introduced in December 2002, and that this is also chosen as the new price reference base.

9.99 Notice that it would be possible in December 2002 to calculate the indices based on 2000 shown in the upper half of the table, but it is de-

Table 9.7. Price Updating of Weights Between Weight and Price Reference Periods

Index	Weight	2000	Nov 02	Dec 02	Jan 03	Feb 03	Mar 03
Index with 2000 as weight and price reference period							
Elementary price indices							
	w_{00}						
A	0.20	100.00	98.00	99.00	102.00	101.00	104.00
B	0.25	100.00	106.00	108.00	107.00	109.00	110.00
C	0.15	100.00	104.00	106.00	98.00	100.00	97.00
D	0.10	100.00	101.00	104.00	108.00	112.00	114.00
E	0.30	100.00	102.00	103.00	106.00	105.00	106.00
Higher-level indices							
G = A+B+C	0.60	100.00	102.83	104.50	103.08	104.08	104.75
H = D+E	0.40	100.00	101.75	103.25	106.50	106.75	108.00
Total		100.00	102.40	104.00	104.45	105.15	106.05
Index re-referenced to December 2002 and weights price-updated to December 2002							
Elementary price indices							
	$w_{00(Dec02)}$						
A	0.190	101.01	98.99	100.00	103.03	102.02	105.05
B	0.260	92.59	98.15	100.00	99.07	100.93	101.85
C	0.153	94.34	98.11	100.00	92.45	94.34	91.51
D	0.100	96.15	97.12	100.00	103.85	107.69	109.62
E	0.297	97.09	99.03	100.00	102.91	101.94	102.91
Higher-level indices							
G = A+B+C	0.603	95.69	98.41	100.00	98.64	99.60	100.24
H = D+E	0.397	96.85	98.55	100.00	103.15	103.39	104.60
Total		96.15	98.46	100.00	100.43	101.11	101.97
Rescaled to 2000 = 100		100.00	102.40	104.00	104.45	105.15	106.05

cided to make December 2002 the price reference base. This does not prevent the index with the December 2002 price reference period from being calculated backward a few months into 2002, if desired.

9.100 The statistical office compiling the index has two options at the time the new index is introduced. It has to decide whether the weights in the new index should preserve the quantities in 2000 or the revenues in 2000. It cannot do both.

9.101 If it decides to preserve the quantities, the resulting index is a basket, or Lowe, index in which the quantities are those of the year 2000. This implies that the *movements* of the index must be identical with those of the index based on 2000 shown in the upper part of the table. In this case, if the index is to be presented as a weighted average of the elementary price indices with December 2002 as price reference period, the revenue weights for 2000 have to be price updated to December 2002. This is illustrated in the lower half of Table 9.7, where the updated weights are obtained by multiplying the original weights for 2000 in the upper part of the table by the price indices for the elementary aggregates between 2000 and December 2002 and then rescaling the results to sum to unity. These are the weights labeled $w_{00(Dec02)}$ in the table.

9.102 The indices with price-updated weights in the lower part of Table 9.7 are Lowe indices in which b = 2000 and 0 = December 2002. These indices can be expressed as relatives of the indices in the upper part of the table. For example, the overall Lowe index for March 2003 with December 2002 as its price reference base, namely 101.97, is the ratio of the index for March 2003 based on 2000 shown in the upper part of the table, namely 106.05, divided by the index for December 2002 based on 2000, namely 104.00. Thus, the price updating preserves the movements of the indices in the upper part of the table while shifting the price reference period to December 2002.

9.103 On the other hand, it could be decided to calculate a series of Young indices using the revenue weights from 2000 as they stand without updating. If the revenue shares were actually to remain constant, the quantities would have had to move inversely with the prices between 2000 and December 2002. The quantities that make up the basket for the new Young index could not be the same as those of 2000. The movements of this index would have to be slightly different from those of the price-updated Lowe index.

9.104 The issue is whether to use the known quantities of the weight reference period 2000, which are the latest for which firm data have been collected, or to use the known revenue shares of the weight reference period. If the official objective is to measure a Lowe index that uses a fixed basket, the issue is decided and the statistical office is obliged to price update. On the other hand, some statistical offices may have to decide for themselves which option to adopt.

9.105 Updating the prices without updating the quantities does not imply that the resulting revenue weights are necessarily more up to date. When there is a strong inverse relation between movements of price and quantities, price updating on its own could produce perverse results. For example, the price of computers has been declining rapidly in recent years. If the quantities are held fixed while the price is updated, the resulting revenue on computers would also decline rapidly. In practice, however, the share of revenue on computers might actually be rising because of a very rapid increase in quantities of computers purchased.

9.106 When rapid changes take place in relative quantities as well as relative prices, statistical offices are effectively obliged to change their revenue weights more frequently, even if this means conducting more frequent establishment surveys. Price updating on its own cannot cope with this situation. The revenue weights have to be updated with respect to their quantities as well as their prices, which, in effect, implies collecting new revenue data.

C.7 Introduction of new weights and chain linking

9.107 From time to time, the weights for the elementary aggregates have to be revised to ensure that they reflect current revenue shares and business activity. When new weights are introduced, the price reference period for the new index can be the last period of the old index, the old and the new indices being linked together at this point. The old and the new indices make a chained index.

9.108 The introduction of new weights is often a complex operation because it provides the opportunity to introduce new products, new samples, new data sources, new compilation practices, new elementary aggregates, new higher-level indices, or new classifications. These tasks are often undertaken simultaneously at the time of reweighting to minimize overall disruption to the time series and any resulting inconvenience to users of the indices.

9.109 In many countries reweighting and chaining is carried out about every five years, but some countries introduce new weights each year. However, chained indices do not have to be linked annually, and the linking may be done less frequently. The real issue is not whether to chain, but how frequently to chain. Reweighting is inevitable sooner or later, because the same weights cannot continue to be used forever. Whatever the time frame, statistical offices have to address the issue of chain linking sooner or later. It is an inevitable and major task for index compilers.

C.7.1 Frequency of reweighting

9.110 It is reasonable to continue to use the same set of elementary aggregate weights as long as production patterns at the elementary aggregate level remain fairly stable. However, over time purchasers will tend to move away from products whose prices have increased relatively so that, in general, movements in prices and quantities tend to be inversely related. This kind of substitution behavior implies that a Lowe index based on the fixed basket of an earlier period will tend to have an upward bias compared with a basket index using up-to-date weights.

9.111 Another reason why purchasing patterns change is that new products are continually being introduced on the market while others drop out. Over the longer term, purchasing patterns are also influenced by several other factors. These include rising incomes and standards of living, demographic changes in the structure of the population, changes in technology, and changes in tastes and preferences.

9.112 There is wide consensus that regular updating of weights—at least every five years, and more often if there is evidence of rapid changes in production patterns—is a sensible and necessary practice. However, the question of how often to change the weights and chain link the index is not straightforward, because frequent linking can also have some disadvantages. It can be costly to obtain new weights, especially if they require more frequent establishment surveys. On the other hand, annual chaining has the advantage that changes such as the inclusion of new goods can be introduced on a regular basis, although every index needs some ongoing maintenance, whether annually chained or not.

9.113 Purchasers of certain types of products are strongly influenced by short-term fluctuations in the economy. For example, purchases of cars, major durables, expensive luxuries, etc., may change drastically from year to year. In such cases, it may be preferable to base the weight on an average of two or more years' revenue.

C.7.2 Calculation of a chained index

9.114 Assume that a series of fixed-weight Young indices have been calculated with period 0 as the price reference period, and that in a subsequent period, k, a new set of weights has to be introduced in the index. (The new set of weights may, or may not, have been price updated from the new weight reference period to period k.) A chained index is then calculated as

$$(9.15)\quad P^{0:t} = P^{0:k}\,\Sigma w_i^k P_i^{k:t-1} P_i^{t-1:t}$$
$$= P^{0:k}\,\Sigma w_i^k P_i^{k:t}$$
$$= P^{0:k} P^{k:t}\,.$$

There are several important features of a chained index.

(i) The chained index formula allows weights to be updated and facilitates the introduction of new products and subindices and removal of obsolete ones.
(ii) To link the old and the new series, an overlapping period (k) is needed in which the index has to be calculated using both the old and the new set of weights.
(iii) A chained index may have two or more links. Between each link period, the index may be calculated as a fixed-weight index using equation (9.10) or any other index formula. The link period may be a month or a year, provided the weights and indices refer to the same period.

Table 9.8. Calculation of a Chained Index

Index	Weight 1998	1998	Nov 02	Dec 02	Weight 2000	Dec 02	Jan 03	Feb 03	Mar 03
		1998 = 100				Dec 2002 = 100			
Elementary price indices									
A	0.20	100.00	120.00	121.00	0.25	100.00	100.00	100.00	102.00
B	0.25	100.00	115.00	117.00	0.20	100.00	102.00	103.00	104.00
C	0.15	100.00	132.00	133.00	0.10	100.00	98.00	98.00	97.00
D	0.10	100.00	142.00	143.00	0.18	100.00	101.00	104.00	104.00
E	0.30	100.00	110.00	124.00	0.27	100.00	103.00	105.00	106.00
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Higher-level indices									
G = A+B+C	0.60	100.00	120.92	122.33	0.55	100.00	100.36	100.73	101.82
H = D+E	0.40	100.00	118.00	128.75	0.45	100.00	102.20	104.60	105.20
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Chaining of higher-level indices to 1998 = 100									
G = A+B+C	0.60	100.00	120.92	122.33	0.55	122.33	122.78	123.22	124.56
H = D+E	0.40	100.00	118.00	128.75	0.45	128.75	131.58	134.67	135.45
Total		100.00	119.75	124.90		124.90	126.39	127.99	129.07

(iv) Chaining is intended to ensure that the individual indices on all levels show the correct development through time.

(v) Chaining leads to nonadditivity. When the new series is chained onto the old as in equation (9.15), the higher-level indices after the link cannot be obtained as weighted arithmetic averages of individual indices using the new weights.[7] Such results need to be carefully explained and presented.

9.115 An example of the calculation of a chained index is presented in Table 9.8. From 1998 to December 2002, the index is calculated with the year 1998 as weight and price reference period. From December 2002 onward, a new set of weights is introduced. The weights may refer to the year 2000, for example, and may or may not have been price updated to December 2002. A new fixed-weight index series is then calculated with December 2002 as price reference month. Finally, the new index series is linked onto the old index with 1998 = 100 by multiplication to get a continuous index from 1998 to March 2003.

9.116 The chained higher-level indices in Table 9.8 are calculated as

$$(9.16)\quad P^{00:t} = P^{98:Dec02} \sum w_i^{00(Dec02)} P_i^{Dec02:t} .$$

Because of the lack of additivity, the overall chained index for March 2003 (129.07), for example, cannot be calculated as the weighted arithmetic mean of the chained higher-level indices *G* and *H* using the weights from December 2002.

[7]If, on the other hand, the index reference period is changed and the index series before the link period are rescaled to the new index reference period, these series cannot be aggregated to higher-level indices by use of the new weights.

Table 9.9. Calculation of a Chained Index Using Linking Coefficients

Index	Weight 1998	1998	Nov 02	Dec 02	Linking coefficient	Jan 03	Feb 03	Mar 03
Elementary price indices (1998 = 100)								
A	0.20	100.00	120.00	121.00	1.2100	121.00	121.00	123.42
B	0.25	100.00	115.00	117.00	1.1700	119.34	120.51	121.68
C	0.15	100.00	132.00	133.00	1.3300	130.34	130.34	129.01
D	0.10	100.00	142.00	143.00	1.4300	144.43	148.72	148.72
E	0.30	100.00	110.00	124.00	1.2400	127.72	130.20	131.44
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07
Higher-level indices (1998 = 100)								
G = A+B+C	0.60	100.00	120.92	122.33	1.2233	122.78	123.22	124.56
H = D+E	0.40	100.00	118.00	128.75	1.2875	131.58	134.67	135.45
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07

Index	Linking coefficient	1998	Nov 02	Dec 02	Weight 2000	Jan 03	Feb 03	Mar 03
Elementary price indices (December 2002 = 100)								
A	0.82645	82.65	99.17	100.00	0.25	100.00	100.00	102.00
B	0.85470	85.47	98.29	100.00	0.20	102.00	103.00	104.00
C	0.75188	75.12	99.25	100.00	0.10	98.00	98.00	97.00
D	0.69993	69.99	99.39	100.00	0.18	101.00	104.00	104.00
E	0.80645	80.65	88.71	100.00	0.27	103.00	105.00	106.00
Total	0.80064	80.06	95.88	100.00		101.19	102.47	103.34
Higher-level indices (2000 = 100)								
G = A+B+C	0.81746	81.75	98.85	122.33	0.55	100.36	100.73	101.82
H = D+E	0.77670	77.67	91.65	128.75	0.45	102.20	104.60	105.20
Total	0.80064	80.06	95.88	124.90		101.19	102.47	103.34

C.7.3 Chaining indices using linking coefficients

9.117 Table 9.9 presents an example of chaining indices on new weights to the old reference period (1998 = 100). The linking can be done several ways. As described above, one can take the current index on the new weights and multiply it by the old index level in the overlap month (December 2002). Alternatively, a linking coefficient can be calculated between the old and new series during the overlap period and this coefficient applied to the new index series to bring the index up to the level of the old series. The linking coefficient for keeping the old price reference period is the ratio of the old index in the overlap period to the new index for the same period. For example, the coefficient for the Total index is (124.90 ÷ 100.00) = 1.2490. This coefficient is then applied to the Total index each month to convert it from a December 2002 reference period to the 1998 reference period.[8]

9.118 Another option is to change the index reference period at the time the new weights are introduced. In the current example, the statistical office can shift to a December 2002 reference period and link the old index to the new reference period. This is done by calculating the linking coefficient for each index as the ratio of the new index in the overlap period to the old index. For example, the coefficient for the Total index is (100.00 ÷ 124.90) = 0.80064. This coefficient is applied to the old Total index series to bring it down to the level of the new index. Table 9.9 presents the linking coefficients and the resulting re-reference price indices using the two alternative index reference periods—1998 or December 2002.

C.7.4 Introduction of new elementary aggregates

9.119 First, consider the situation in which new weights are introduced and the index is chain linked in December 2002. The overall coverage of the PPI is assumed to remain the same, but certain products have increased sufficiently in importance to merit recognition as new elementary aggregates. Possible examples are the introduction of new elementary aggregates for mobile telephones or a new multinational company setting up a car factory.

9.120 Consider the calculation of the new index from December 2002 onward, the new price reference period. The calculation of the new index presents no special problems and can be carried out using equation (9.10). However, if the weights are price updated from, say, 2000 to December 2002, difficulties may arise because the elementary aggregate for mobile telephones did not exist before December 2002, so there is no price index with which to price update the weight for mobile telephones. Prices for mobile telephones may have been recorded before December 2002, possibly within another elementary aggregate (communications equipment) so that it may be possible to construct a price series that can be used for price updating. Otherwise, price information from other sources such as business surveys, trade statistics, or industry sources may have to be used. If no information is available, then movements in the price indices for similar elementary aggregates may be used as a proxies for price updating.

9.121 The inclusion of a new elementary aggregate means that the next higher-level index contains a different number of elementary aggregates before and after the linking. Therefore, the rate of change of the higher-level index whose composition has changed may be difficult to interpret. However, failing to introduce new goods or services for this reason would result in an index that does not reflect the actual dynamic changes taking place in the economy. If it is customary to revise the PPI backward, then the prices of the new product and their weights might be introduced retrospectively. If the PPI is not revised backward, however, which is usually the case, little can be done to improve the quality of the chained index. In many cases, the addition of a single elementary aggregate is unlikely to have a significant effect on the next higher-level index into which it enters. If the addition of an elementary aggregate is believed to have a significant impact on the time series of the higher-level index, it may be necessary to discontinue the old series and commence a new higher-level index. These decisions can be made only on a case-by-case basis.

C.7.5 Introduction of new, higher-level indices

9.122 It may be necessary to introduce a new, higher-level index in the overall PPI. This situation may occur if the coverage of the PPI is enlarged or the grouping of elementary aggregates is changed. It then needs to be decided what the initial value the new higher-level index should be when it is included in the calculation of the overall PPI. Take as an example the situation in Table 9.8 and assume that a new higher-level index from January 2003 has to be included in the index. The question is what should be the December 2002 value to which the new higher-level index is linked. There are two options.

- Estimate the value in December 2002 that the new higher-level index would have had with 1998 as price reference period, and link the new series from January 2003 onward onto this value. This procedure will prevent any break in the index series.
- Use 100 in December 2002 as starting point for the new higher-level index. This simplifies the

[8]A linking coefficient is needed for each index series that is being chained.

problem from a calculation perspective, although there remains the problem of explaining the index break to users.

In any case, major changes such as those just described should, so far as possible, be made in connection with the regular reweighting and chaining to minimize disruptions to the index series.

9.123 A final case to consider concerns classification change. For example, a country may decide to change from a national classification to an international one, such as ISIC. The changes in the composition of the aggregates within the PPI may then be so large that it is not meaningful to link them. In such cases, it is recommended that the PPI with the new classification should be calculated backward for at least one year so that consistent annual rates of change can be calculated.

C.7.6 Partial reweighting and introducing new goods

9.124 The weights for the elementary aggregates may be obtained from a variety of sources over a number of different periods. Consequently, it may not be possible to introduce all the new weighting information at the same time. In some cases, it may be preferable to introduce new weights for some elementary aggregates as soon as possible after the information is received. This would be the case for introducing new goods (for example, revolutionary goods, discussed in Chapter 8) into the index when these goods fall within the existing product structure of the index. The introduction of new weights for a subset of the overall index is known as partial reweighting.

9.125 As an example, assume there is a four-digit industry with three major products (*A*, *B*, and *C*) that were selected for the sample in 2000. From the revenue data for 2000, *A* had 50 percent of revenues, *B* had 35 percent, and *C* had 15 percent. From a special industry survey conducted for 2002, the statistical office discovers that *C* now has 60 percent of the revenue and *A* and *B* each have 20 percent. When the new weights are introduced into the index, the procedures discussed in Section C.7.2 for chaining the new index onto the old index can be used. For example, the new product weights for 2002 are used to calculate the index in an overlap month such as April 2003 with a base price reference period of December 2002. For May 2003, the index using the new product weights is again calculated and the price change using the new index is then applied (linked) to the old industry level index for April 2003 (with 2000 = 100) to derive the industry index for May 2003 (2000 = 100). The formula for this calculation is the following:

$$(9.17)\quad P^{00:May03} = P^{00:Apr03}\left[\frac{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:May03}}{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:Apr03}}\right].$$

9.126 Continuing with this example, assume the special survey was conducted because producers are making a new, important product in this industry. The survey finds the new product (*D*) has a significant share of production (perhaps 15 or 20 percent), and it is expected to continue gaining market share. The statistical office would use the same procedure for introducing the new product. In this case, the calculations for the new industry index in April and May would use all four products instead of the original three. The price change in the new sample is linked to the old index as in equation (9.17). The only difference will be that the summations are over *m* (four products) instead of *n* (three) products.

9.127 One could also make the same calculations using the linking coefficient approach discussed in Section C.7.3. The linking coefficient is derived by taking the ratio of the old industry index (2000 = 100) to the new industry index (December 2002 = 100) in the overlap period (April 2003):

$$(9.18)\ \text{Linking coefficient} = \frac{\sum_{i=1}^{n} w_i^{02} P_i^{Dec02:Apr03}}{\sum_{i=1}^{n} w_i^{00} P_i^{00:Apr03}}.$$

The linking coefficient, computed for the overlap period only, is then applied each month to the new index to adjust it to the level of the old index with an index reference period of 2000.

9.128 Another issue is the weights to use for compiling the index for the product groups represented by *A*, *B*, *C*, and *D*. For example, if indices for products *A* and *B* are combined with products *X* and *Y* to calculate a product group index, the new weights for *A* and *B* present a problem because they represent revenues in a more current period than the weights for *X* and *Y*. Also, the indices have different price reference periods. If we had weights for products *X* and *Y* for the same period as *A* and *B*, then we could use the same approach as just described for compiling the industry index. Lacking new product weights for *X* and *Y* means the statistical office will have to take additional steps. One approach to resolve this problem is to price update the weights for products *X* and *Y* from 2000 to 2002 using the change in the respective price indices. Thus, the original weight for product *X* is multiplied by the change in prices between 2002 and 2000 (that is, the ratio of the average price index of *X* in 2002 to the average price index of *X* in 2000). Then use the same base price reference period as for *A* and *B* so that the indices for products *X* and *Y* are each re-referenced to December 2002. The product group index can then be compiled for April 2003 using the new weights for all four products and their indices with December 2002 = 100. Once the April 2003 index is compiled on the December 2002 price reference period, then the linking coefficient using equation (9.18) can be calculated to adjust the new index level to that of the old index. Alternatively, the price change in the new product group index (December 2002 = 100) can be applied to the old index level each month as shown in equation (9.17).

9.129 As this example demonstrates, partial reweighting has particular implications for the practice of price updating the weights. Weighting information may not be available for some elementary aggregates at the time of reweighting. Thus, it may be necessary to consider price updating the old weights for those elementary aggregates for which no new weights are available. The weights for the latter may have to be price updated over a long period, which, for reasons given earlier, may give rise to some index bias if relative quantities have changed inversely with the relative price changes. Data on both quantity and price changes for the old index weights should be sought before undertaking price updating alone. The disadvantage of partial reweighting is that the implicit quantities belong to different periods than other components of the index, so that the composition of the basket is obscure and not well defined.

9.130 One may conclude that the introduction of new weights and the linking of a new series to the old series is not difficult in principle. The difficulties arise in practice when trying to align weight and price reference periods and when deciding whether higher-level indices comprising different elementary aggregates should be chained over time. It is not possible for this *Manual* to provide specific guidance on decisions such as these, but compilers should consider carefully the economic logic and statistical reliability of the resulting chained series and also the needs of users. To facilitate the decision process, careful thought should be given to these issues in advance during the planning of a reweighting exercise, paying particular attention to which indices are to be published.

C.7.7 Long- and short-term links

9.131 Consider a long-term chained index in which the weights are changed annually. In any given year, the current monthly indices when they are first calculated have to use the latest set of available weights, which cannot be those of the current year. However, when the weights for the year in question become available subsequently, the monthly indices can then be recalculated on basis of the weights for the same year. The resulting series can then be used in the long-term chained index rather than the original indices first published. Thus, the movements of the long-term chained index from, say, any one December to the following December, are based on weights of that same year, the weights being changed each December.[9]

9.132 Assume that each link runs from December to December. The long-term index for month *m* of year *Y* with December of year 0 as index reference period is then calculated by the formula[10]

$$(9.19)\quad P^{Dec0:mY} = \left(\prod_{Y=1}^{Y-1} P^{DecY-1:DecY}\right) P^{DecY-1:mY}$$

$$= P^{Dec0:Dec1} \times P^{Dec1:Dec2} \times \ldots \times P^{DecY-2:DecY-1} \times P^{DecY-1:mY}$$

The long-term movement of the index depends on the long-term links only as the short-term links are successively replaced by their long-term counterparts. For example, let the short-term indices for January to December 2001 be calculated as

[9]This method has been developed by the Central Statistical Office of Sweden, where it is applied in the calculation of the CPI. See Statistics Sweden (2001).

[10]In the actual Swedish practice, a factor scaling the index from December year 0 to the average of year 0 is multiplied onto the right-hand side of equation (9.19) to have a full year as reference period.

$$(9.20)\ \ P^{Dec00:m01} = \sum w_i^{00(Dec00)} P_i^{Dec00:m01},$$

where $W_i^{00(Dec00)}$ are the weights from 2000 price updated to December 2000. At the time when weights for 2001 are available, this is replaced by the long-term link

$$(9.21)\ \ P^{Dec00:Dec01} = \sum w_i^{01(Dec00)} P_i^{Dec00:Dec01},$$

where $W_i^{01(Dec00)}$ are the weights from 2001 price backdated to December 2000. The same set of weights from 2001 price updated to December 2001 are used in the new short-term link for 2002,

$$(9.22)\ \ P^{Dec01:m02} = \sum w_i^{01(Dec01)} P_i^{Dec01:m02}.$$

9.133 Using this method, the movement of the long-term index is determined by contemporaneous weights that refer to the same period. The method is conceptually attractive because the weights that are most relevant for most users are those based on production patterns at the time the price changes actually take place. The method takes the process of chaining to its logical conclusion, at least assuming the indices are not chained more frequently than once a year. Since the method uses weights that are continually revised to ensure that they are representative of current production patterns, the resulting index also largely avoids the substitution bias that occurs when the weights are based on the production patterns of some period in the past. The method may therefore appeal to statistical offices whose objective is to estimate an economic index.

9.134 Finally, it may be noted that the method involves some revision of the index first published. In some countries, there is opposition to revising a PPI once it has been first published, but it is standard practice for other economic statistics, including the national accounts, to be revised as more up-to-date information becomes available. This point is considered below.

C.8 Decomposition of index changes

9.135 Users of the index are often interested in how much of the change in the overall index is attributable to the change in the price of some particular product or group of products, such as petroleum or food. Alternatively, there may be interest in what the index would be if food or energy were left out. Questions of this kind can be answered by decomposing the change in the overall index into its constituent parts.

9.136 Assume that the index is calculated as in equation (9.10) or equation (9.11). The relative change of the index from $t - m$ to t can then be written as

$$(9.23)\ \ \frac{P^{0:t}}{P^{0:t-m}} - 1 = \frac{\sum w_i^b P_i^{0:t-m} P_i^{t-m:t}}{\sum w_i^b P_i^{0:t-m}} - 1.$$

Hence, a subindex from $t - m$ to 0 enters the higher-level index with a weight of

$$(9.24)\ \ \frac{w_i^b P_i^{0:t-m}}{\sum w_i^b P_i^{0:t-m}} = \frac{w_i^b P_i^{0:t-m}}{P^{0:t-m}}.$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$(9.25)\ \ \text{Effect} = \frac{w_i^b I_i^{0:t-m}}{I^{0:t-m}}\left(\frac{I_i^{0:t}}{I_i^{0:t-m}} - 1\right) = \frac{w_i^b}{P_i^{0:t-m}}\left(P_i^{t:0} - P_i^{0:t-m}\right).$$

With $m = 1$, it gives the effect of a monthly change; with $m = 12$, it gives the effect of the change over the past 12 months.

9.137 If the index is calculated as a chained index, as in equation (9.15), then a subindex from $t - m$ enters the higher-level index with a weight of

$$(9.26)\ \ \frac{w_i^0 P_i^{k:t-m}}{P^{k:t-m}} = \frac{w_i^0\left(P_i^{0:t-m}/P_i^{0:k}\right)}{\left(P^{0:t-m}/P^{0:k}\right)}.$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$(9.27)\ \ \text{Effect} = \frac{w_i^0}{P^{k:t-m}}\left(P_i^{k:t} - P_i^{k:t-m}\right) = \frac{w_i^0}{\left(I^{0:t-m}/I^{0:k}\right)}\left(\frac{I_i^{0:t} - I_i^{0:t-m}}{I_i^{0:k}}\right).$$

It is assumed that $t - m$ lies in the same link (that is, $t - m$ refers to a period later than k). If the effect of a subindex on a higher-level index is to be calculated across a chain, the calculation needs to be carried out in two steps, one with the old series up to the link period and one from the link period to period t.

9.138 Table 9.10 illustrates an analysis using both the index point effect and contribution of each component index to the overall 12-month change. The next-to-last column in Table 9.10 is calculated using equation (9.25) to derive the effect each component index contributes to the total percentage change. For example, for agriculture the index weight (w_i^b) is 38.73, which is divided by the previous period index ($P_i^{0:t-m}$), or 118.8, and then multiplied by the index point change ($P_i^{t:0} - P_i^{0:t-m}$) between January 2003 and January 2002, 10.5. The result shows that agriculture's effect on the 9.1 percent overall change was 3.4 percent. The change in agriculture contributed 37.3 percent (3.4 ÷ 9.1 × 100) to the total 12-month change.

C.9 Some alternatives to fixed-weight indices

9.139 Monthly PPIs are typically arithmetic weighted averages of the price indices for the elementary aggregates in which the weights are kept fixed over a number of periods, which may range from 12 months to many years. The repeated use of the same weights relating to some past period b simplifies calculation procedures and reduces data collection requirements. It is also cheaper to keep using the results from an old production survey than conduct an expensive new one. Moreover, when the weights are known in advance of the price collection, the index can be calculated immediately after the prices have been collected and processed.

9.140 However, the longer the same weights are used, the less representative they become of current production patterns, especially in periods of rapid technical change when new kinds of goods and services are continually appearing on the market and old ones disappearing. This may undermine the credibility of an index that purports to measure the rate of change in the production value of goods and services typically produced by businesses. Such a basket needs to be representative not only of the producers covered by the index but also of the revenue patterns at the time the price changes occur.

9.141 Similarly, if the objective is to compile an economic index, the continuing use of the same

Table 9.10. Decomposition of Index Change from January 2002 to January 2003

	Industry Sector	2000 weights (w_i^b)	Index (I) 2000	Index (I) Jan 02	Index (I) Jan 03	Percent change from Jan 02 to Jan 03	Effect (Contribution) Percentage points of total price change	Effect (Contribution) Percent of total price change
1	Agriculture	38.73	100	118.8	129.3	8.8	3.4	37.3
2	Mining	6.40	100	132.8	145.2	9.3	0.7	7.3
3	Manufacturing	18.64	100	109.6	120.6	10.0	1.7	18.8
4	Transport and Communication	19.89	100	126.3	131.3	4.0	0.8	9.1
5	Services	16.34	100	123.4	141.3	14.5	2.4	26.8
	Total	100.00	100	120.2	131.1	9.1	9.1	100.0

fixed basket is likely to become increasingly unsatisfactory the longer the same basket is used. The longer the same basket is used, the greater the bias in the index is likely to become. It is well known that the Laspeyres index has a substitution bias compared with an economic index. However, a Lowe index between periods 0 and t with weights from an earlier period b will tend to exceed the Laspeyres substitution bias between 0 and t, becoming larger the further back in time period b is (see Chapter 15, Section D).

9.142 There are several possible ways of minimizing, or avoiding, the potential biases from the use of fixed-weight indices. These are outlined below.

9.143 *Annual chaining*. One way to minimize the potential biases from the use of fixed-weight indices is to keep the weights and the base period as up to date as possible by frequent weight updates and chaining. A number of countries have adopted this strategy and revise their weights annually. In any case, as noted earlier, it would be impossible to deal with the changing universe of products without some chaining of the price series within the elementary aggregates, even if the weights attached to the elementary aggregates remain fixed. Annual chaining eliminates the need to choose a base period, because the weight reference period is always the previous year, or possibly the preceding year.

9.144 *Annual chaining with current weights*. When the weights are changed annually, it is possible to replace the original weights based on the previous year, or years, by those of the current year if the index is revised retrospectively as soon as information on current-year revenue becomes available. The long-term movements in the PPI are then based on the revised series. This is the method adopted by the Swedish Statistical Office as explained in Section C.7.7 above. This method could provide unbiased results.

9.145 *Other index formulas*. When the weights are revised less frequently, say, every five years, another possibility would be to use a different index formula for the higher-level indices instead of an arithmetic average of the elementary price indices. One possibility would be a weighted geometric average. This is not subject to the same potential upward bias as the arithmetic average. More generally, a weighted version of the Lloyd-Moulton formula, given in Section B.6 above, might be considered. This formula takes account of the substitutions that purchasers make in response to changes in relative prices and should be less subject to bias for this reason. It reduces to the geometric average when the elasticity of substitution is unity, on average. It is unlikely that such a formula could replace the arithmetic average in the foreseeable future and gain general acceptance, if only because it cannot be interpreted as measuring changes in the value of a fixed basket. However, it could be compiled on an experimental basis and might well provide a useful supplement to the main index. It could at least flag the extent to which the main index is liable to be biased and throw light on its properties.

9.146 *Retrospective superlative indices*. Finally, it is possible to calculate a superlative index retrospectively. Superlative indices such as Fisher and Törnqvist-Theil treat both periods compared symmetrically and require revenue data for both periods. Although the PPI may have to be some kind of Lowe index when it is first published, it may be possible to estimate a superlative index later when much more information becomes available about producers' revenues period by period. At least one office, the U.S. Bureau of Labor Statistics, is publishing such an index for its CPI. The publication of revised or supplementary indices raises matters of statistical policy, but users readily accept revisions in other fields of economic statistics. Moreover, users are already confronted with more than one CPI in the EU where the harmonized index for EU purposes may differ from the national CPI. Thus, the publication of supplementary indices that throw light on the properties of the main index and that may be of considerable interest to some users seems justified and acceptable.

D. Data Editing

9.147 This chapter has been concerned with the methods used by statistical offices to calculate their PPIs. This concluding section considers the data editing carried out by statistical offices, a process closely linked to the calculation of the price indices for the elementary aggregates. Data collection, recording, and coding—the data-capture processes—are dealt with in Chapters 5 through 7. The next step in the production of price indices is the data editing. Data editing is here meant to comprise two steps:

- Detecting possible errors and outliers, and
- Verifying and correcting data.

9.148 Logically, the purpose of detecting errors and outliers is to exclude errors or the effects of outliers from the index calculation. Errors may be falsely reported prices, or they may be caused by recording or coding mistakes. Also, missing prices because of nonresponse may be dealt with as errors. Possible errors and outliers are usually identified as observations that fall outside some prespecified acceptance interval or are judged to be unrealistic by the analyst on some other ground. It may also be the case, however, that even if an observation is not identified as a potential error, it may actually show up to be false. Such observations are sometimes referred to as inliers. On the other hand, the sampling may have captured an exceptional price change, which falls outside the acceptance interval but has been verified as correct. In some discussions of survey data, any extreme value is described as an outlier. The term is reserved here for extreme values that have been verified as being correct.

9.149 When a possible error has been identified, it needs to be verified whether it is in fact an error or not. This can usually be accomplished by asking the respondent to verify the price, or by comparison with the price change of similar products. If it is an error, it needs to be corrected. This can be done easily if the respondent can provide the correct price or, where this is not possible, by imputation or omitting the price from the index calculation. If it proves to be correct, it should be included in the index. If it proves to be an outlier, it can be accepted or corrected according to a predefined practice—for example, omitting or imputation.

9.150 Although the power of computers provides obvious benefits, not all of these activities have to be computerized. However, there should be a complete set of procedures and records that controls the processing of data, even though some or all of it may be undertaken without the use of computers. It is not always necessary for all of one step to be completed before the next is started. If the process uses spreadsheets, for example, with default imputations predefined for any missing data, the index can be estimated and reestimated whenever a new observation is added or modified. The ability to examine the impact of individual price observations on elementary aggregate indices and the impact of elementary indices on various higher-level aggregates is useful in all aspects of the computation and analytical processes.

9.151 It is neither necessary nor desirable to apply the same degree of scrutiny to all reported prices. The price changes recorded by some respondents carry more weight than others, and statistical analysts should be aware of this. For example, one elementary aggregate with a weight of 2 percent, say, may contain 10 prices, while another elementary aggregate of equal weight may contain 100 prices. Obviously, an error in a reported price will have a much smaller effect in the latter, where it may be negligible, while in the former it may cause a significant error in the elementary aggregate index and even influence higher-level indices.

9.152 However, there may be an interest in the individual elementary indices as well as in the aggregates built from them. Since the sample sizes used at the elementary level may often be small, any price collected, and error in it, may have a significant impact on the results for individual products or industries. The verification of reported data usually has to be done on an index-by-index basis, using statistical analysts' experience. Also, for support, analysts will need the cooperation of the survey respondents to help explain unusual price movements.

9.153 Obviously, the design of the survey and questionnaires influences the occurrence of errors. Hence, price reports and questionnaires should be as clear and unambiguous as possible to prevent misunderstandings and errors. Whatever the design of the survey, it is important to verify that the data collected are those that were requested initially. The survey questionnaire should prompt the respondent to indicate if the requested data could not be provided. If, for example, a product is not produced anymore and thus is not priced in the current month, a possible replacement would be requested along with details of the extent of its comparability with the old one. If the respondent cannot supply a replacement, there are a number of procedures for dealing with missing data (see Chapter 7).

D.1 Identifying possible errors and outliers

9.154 One of the ways price surveys are different from other economic surveys is that, although prices are recorded, the measurement concern is with price *changes*. As the index calculations consist of comparing the prices of matching observations from one period to another, editing checks should focus on the price changes calculated from pairs of observations, rather than on the reported prices themselves.

9.155 Identification of unusual price changes can be accomplished by

- Nonstatistical checking of input data,
- Statistical checking of input data, and
- Output checking.

These will be described in turn.

D.1.1 Nonstatistical checking of input data

9.156 Nonstatistical checking can be undertaken by manually checking the input data, by inspecting the data presented in comparable tables, or by setting filters.

9.157 When the price reports or questionnaires are received in the statistical office, the reported prices can be checked manually by comparing these with the previously reported prices of the same products or by comparing them with prices of similar products from other establishments. While this procedure may detect obvious unusual price changes, it is far from sure that all possible errors are detected. It is also extremely time consuming, and it does not identify coding errors.

9.158 After the price data have been coded, the statistical system can be programmed to present the data in a comparable form in tables. For example, a table showing the percentage change for all reported prices from the previous to the current month may be produced and used for detection of possible errors. Such tables may also include the percentage changes of previous periods for comparison and 12-month changes. Most computer programs and spreadsheets can easily sort the observations according to, say, the size of the latest monthly rate of change so that extreme values can easily be identified. It is also possible to group the observations by elementary aggregates.

9.159 The advantage of grouping observations is that it highlights potential errors so that the analyst does not have to look through all observations. A hierarchical strategy whereby all extreme price changes are first identified and then examined in context may save time, although the price changes underlying elementary aggregate indices, which have relatively high weights, should also be examined in context.

9.160 Filtering is a method by which possible errors or outliers are identified according to whether the price changes fall outside some predefined limits, such as ±20 percent or even 50 percent. This test should capture any serious data coding errors, as well as some of the cases where a respondent has erroneously reported on a different product. It is usually possible to identify these errors without reference to any other observations in the survey, so this check can be carried out at the data-capture stage. The advantage of filtering is that the analyst need not look through numerous individual observations.

9.161 These upper and lower limits may be set for the latest monthly change, or change over some other period. Note that the set limits should take account of the context of the price change. They may be specified differently at various levels in the hierarchy of the indices—for example, at the product level, at the elementary aggregate level, or at higher levels. Larger changes for products with prices known to be volatile might be accepted without question. For example, for monthly changes, limits of ±10 percent might be set for petroleum prices, while for professional services the limits might be 0 percent to +5 percent (as any price that falls is suspect), and for computers it might be –5 percent to zero, as any price that rises is suspect. One can also change the limits over time. If it is known that petroleum prices are rising, the limits could be 10 percent to 20 percent, while if they are falling, they might be –10 percent to –20 percent. The count of failures should be monitored regularly to examine the limits. If too many observations are being identified for review, the limits will need to be adjusted, or the customization refined.

9.162 The use of automatic deletion systems is not advised, however. It is a well-recorded phenomenon in pricing that price changes for many products, especially durables, are not undertaken smoothly over time but saved up to avoid what are termed "menu costs" associated with making a price change. These relatively substantial increases may take place at different times for different models of products and have the appearance of extreme, incorrect values. To delete a price change for each model of the product as being "extreme" at the time it occurs is to ignore all price changes for the industry.

D.1.2 Statistical checking of input data

9.163 Statistical checking of input data compares, for some time period, each price change with the change in prices in the same or a similar sample. Two examples of such filtering are given here, the first based on nonparametric summary measures and the second on the log-normal distribution of price changes.

9.164 The first method involves tests based on the median and quartiles of price changes, so they are unaffected by the impact of any single extreme observation. Define the median, first quartile, and third quartile price relatives as R_M, R_{Q1}, and R_{Q3}, respectively. Then, any observation with a price ratio more than a certain multiple C of the distance between the median and the quartile is identified as a potential error. The basic approach assumes price changes are normally distributed. Under this assumption, it is possible to estimate the proportion of price changes that are likely to fall outside given bounds expressed as multiples of C. Under a normal distribution, R_{Q1}, and R_{Q3} are equidistant from R_M; thus, if C is measured as $R_M - (R_{Q1} + R_{Q3})/2$, 50 percent of observations would be expected to lie within $\pm C$ from the median. From the tables of the standardized normal distribution, this is equivalent to about 0.7 times the standard deviation (σ). If, for example, C was set to 6, the distance implied is about 4σ of the sample, so about 0.17 percent of observations would be identified this way. With $C = 4$, the corresponding figures are 2.7σ, or about 0.7 percent of observations. If $C = 3$, the distance is 2.02σ, so about 4 percent of observations would be identified.

9.165 In practice, most prices may not change each month, and the share of observations identified as possible errors as a percentage of all changes would be unduly high. Some experimentation with alternative values of C for different industries or sectors may be appropriate. If this test is to be used to identify possible errors for further investigation, a relatively low value of C should be used.

9.166 To use this approach in practice, three modifications should be made. First, to make the calculation of the distance from the center the same for extreme changes on the low side as well as on the high side, a transformation of the relatives should be made. The transformed distance for the ratio of one price observation i, S_i, should be

$$S_i = 1 - R_M/R_i \text{ if } 0 < R_i < R_M \text{ and}$$
$$= R_i/R_M - 1 \text{ if } R_i \geq R_M.$$

Second, if the price changes are grouped closely together, the distances between the median and quartiles may be very small, so that many observations would be identified that had quite small price changes. To avoid this, some minimum distance, say, 5 percent for monthly changes, should be also set. Third, with small samples, the impact of one observation on the distances between the median and quartiles may be too great. Because sample sizes for some elementary indices are small, samples for similar elementary indices may need to be grouped together.[11]

9.167 An alternative method can be used if it is thought that the price changes may be distributed log-normally. To apply this method, the standard deviation of the log of all price changes in the sample (excluding unchanged observations) is calculated and a goodness of fit test (χ^2) is undertaken to identify whether the distribution is log-normal. If the distribution satisfies the test, all price changes outside two times the exponential of the standard deviation are highlighted for further checking. If the test rejects the log-normal hypothesis, all the price changes outside three times the exponential of the standard deviation are highlighted. The same caveats mentioned before about clustered changes and small samples apply.

[11]For a detailed presentation of this method, the reader is referred to Hidiroglou and Berthelot (1986). The method can be expanded also to take into account the level of the prices, so that, for example, a price increase from 100 to 110 is attributed a different weight than a price increase from 10 to 11.

9.168 The second example is based on the Tukey algorithm. The set of price relatives are sorted and the highest and lowest 5 percent flagged for further attention. In addition, having excluded the top and bottom 5 percent, exclude the price relatives that are equal to 1 (no change). The arithmetic (trimmed) mean (*AM*) of the remaining price relatives is calculated. This mean is used to separate the price relatives into two sets, an upper and a lower one. The upper and lower "mid-means"—that is, the means of each of these sets (AM_L, AM_U)—are then calculated. Upper and lower Tukey limits (T_L, T_U) are then established as the mean ±2.5 times the difference between the mean and the mid-means:

$$T_U = AM + 2.5\ (AM_U - AM),$$
$$T_L = AM - 2.5\ (AM - AM_L).$$

Then, all those observations that fall above T_U and below T_L are flagged for attention.

9.169 This is a similar but simpler method than that based on the normal distribution. Since it excludes all cases of no change from the calculation of the mean, it is unlikely to produce limits that are very close to the mean, so there is no need to set a minimum difference. However, its success will also depend on there being a large number of observations on the set of changes being analyzed. Again, it will often be necessary to group observations from similar elementary indices. For any of these algorithms, the comparisons can be made for any time period, including the latest month's changes, but also longer periods, in particular, 12-month changes.

9.170 The advantage of these two models of filtering compared with the simple method of filtering is that for each period the upper and lower limits are determined by the data itself and hence are allowed to vary over the year, given that the analyst has decided the value of the parameters entering the models. A disadvantage is that, unless one is prepared to use approximations from earlier experience, all the data have to be collected before the filtering can be undertaken. Filters should be set tightly enough so that the percentage of potential errors that turn out to be real errors is high. As with all automatic methods, the flagging of an unusual observation is for further investigation, as opposed to automatic deletion.

D.1.3 Checking by impact, or data output checking

9.171 Filtering by impact, or output editing, is based on calculating the impact an individual price change has on an index to which it contributes. This index can be an elementary aggregate index, the total index, or some other aggregate index. The impact a price change has on an index is its percentage change times its effective weight. In the absence of sample changes, the calculation is straightforward: it is the nominal (reference period) weight, multiplied by the price relative, and divided by the level of the index to which it is contributing. So the impact on the index *I* of the change of the price of product *i* from time *t* to *t* + 1 is $\pm w_i \left(p_{t+1} / p_t \right) / I_t$, where w_i is the nominal weight in the price reference period. A minimum value for this impact can be set, so that all price changes that cause an impact greater than this change can be flagged for review. If index *I* is an elementary index, then all elementary indices may be reviewed, but if *I* is an aggregative index, prices that change by a given percentage will be flagged or not depending on how important the elementary index to which they contribute is in the aggregate.

9.172 However, at the lowest level, births and deaths of products in the sample cause the effective weight of an individual price to change quite substantially. The effective weight is also affected if a price observation is used as an imputation for other missing observations. The evaluation of effective weights in each period is possible, though complicated. However, as an aid to highlighting potential errors, the nominal weights, as a percentage of their sum, will usually provide a reasonable approximation. If the impact of 12-month changes is required to highlight potential errors, approximations are the only feasible filters to use, since the effective weights will vary over the period.

9.173 One advantage of identifying potential errors this way is that it focuses on the results. Another advantage is that this form of filtering also helps the analyst to describe the contributions to change in the price indices. In fact, much of this kind of analysis is done after the indices have been calculated, as the analyst often wishes to highlight those indices that have contributed the most to overall index changes. Sometimes the analysis results in a finding that particular industries have a relatively high contribution to the overall price change, and that is considered unrealistic. The change is traced back to an error, but it may be late in the production cycle and jeopardize the schedule release date. There is thus a case for identifying such unusual contributions as part of the data editing procedures. The disadvantage of this method is that an elementary index's change may be rejected at that stage. It may be necessary to override the calculated index, though this should be a stopgap measure only until the index sample is redesigned.

D.2 Verifying and correcting data

9.174 Some errors, such as data coding errors, can be identified and corrected easily. Ideally, these errors are caught at the first stage of checking, before they need to be viewed in the context of other price changes. Dealing with other potential errors is more difficult. Many results that fail a data check may be judged by the analyst to be quite plausible, especially if the data checking limits are broad. Some editing failures may be resolved only by checking the data with the respondent.

9.175 If a satisfactory explanation can be obtained from the respondent, the data can be verified or corrected. If not, procedures may differ. Rules may be established that if a satisfactory explanation is not obtained, then the reported price is omitted from the index calculation. On the other hand, it may be left to the analyst to make the best judgment as to the price change. However, if an analyst makes a correction to some reported data, without verifying it with the respondent, it may subsequently cause problems with the respondent. If the respondent is not told of the correction, the same error may persist in the future. The correct action depends on a combination of the confidence in the analysts, the revision policy in the survey, and the degree of communication with respondents. Most statistical offices do not want to unduly burden respondents.

9.176 In many organizations, a disproportionate share of activity is devoted to identifying and following up potential errors. If the practice leads to little change in the results, as a result of most reports ending up as being accepted, then the bounds on what are considered to be extreme values should be relaxed. More errors are likely introduced by respondents failing to report changes that occur than from wrongly reporting changes, and the goodwill of respondents should not be unduly undermined.

9.177 Generally, the effort spent on identifying potential errors should not be excessive. Obvious mistakes should be caught at the data-capture stage. The time spent identifying observations to query, unless they are highly weighted and excessive, is often better spent treating those cases in the production cycle where things have changed—quality changes or unavailable prices—and reorganizing activities toward maintaining the relevance of the sample and checking for errors of omission.

9.178 If the price observations are collected in a way that prompts the respondent with the previously reported price, the respondent may report the same price as a matter of convenience. This can happen even though the price may have changed, or even when the particular product being surveyed is no longer available. Because prices for many products do not change frequently, this kind of error is unlikely to be spotted by normal checks. Often the situation comes to light when the contact at the responding outlet changes and the new contact has difficulty in finding something that corresponds to the price previously reported. It is advisable, therefore, to keep a record of the last time a particular respondent reported a price change. When that time has become suspiciously long, the analyst should verify with the respondent that the price observation is still valid. What constitutes too long will vary from product to product and the level of overall price inflation, but, in general, any price that has remained constant for more than a year is suspect.

D.2.1 Treatment of outliers

9.179 Detection and treatment of outliers (extreme values that have been verified as being correct) is an insurance policy. It is based on the fear that a particular data point collected is exceptional by chance, and that if there were a larger survey, or even a different one, the results would be less extreme. The treatment, therefore, is to reduce the impact of the exceptional observation, though not to ignore it, since, after all, it did occur. The methods to test for outliers are the same as those used to identify potential errors by statistical filtering, described above. For example, upper and lower bounds of distances from the median price change are determined. In this case, however, when observations are found outside those bounds, they may be changed to be at the bounds or imputed by the rate of change of a comparable set of prices. This outlier adjustment is sometimes made automatically, on the grounds that the analyst by definition has no additional information on which to base a better estimate. While such automatic adjustment methods are employed, the *Manual* proposes caution in their use. If an elementary aggregate is relatively highly weighted and has a relatively small sample, an adjustment may be made. The general prescription should be to include verified prices and the exception to dampen them.

D.2.2 Treatment of missing price observations

9.180 It is likely that not all the requested data will have been received by the time the index needs to be calculated. It is generally the case that missing data turns out to be delayed. In other cases, the respondent may report that a price cannot be reported because neither the product, nor any similar substitute, is being made anymore. Sometimes, of course, what started as an apparent late report becomes a permanent loss to the sample. Different actions need to be taken depending on whether the situation is temporary or permanent.

9.181 For temporarily missing prices, the most appropriate strategy is to minimize the occurrence of missing observations. Survey reports are likely to come in over a period of time before the indices need to be calculated. In many cases, they follow a steady routine—some respondents will tend to file quickly, others typically will be later in the processing cycle. An analyst should become familiar with these patterns. If there is a good computerized data-capture system, it can flag reports that appear to be later than usual, well before the processing deadline. Also, some data are more important than others. Depending on the weighting system, some respondents may be particularly important, and such products should be flagged as requiring particular scrutiny.

9.182 For those reports for which no estimate can be made, two basic alternatives are considered here (see Chapter 7 for a full range of approaches): imputation, preferably targeted, in which the missing price change is assumed to be the same as some other set of price changes, or an assumption of no change, as the preceding period's price is used (the carryforward method discussed in Chapter 7). However, this latter procedure ignores the fact that some prices will prove to have changed, and if prices are generally moving in one direction, this will mean that the change in the indices will be understated. It is not advised. However, if the index is periodically revised, the carryforward method will lead to less subsequent revisions than making an imputation, since for most products, prices do not generally change in any given period. The standard approach to imputation is to base the estimate of the missing price observation on the change of some similar group of observations.

9.183 There will be situations where the price is permanently missing because the product no longer exists. Since there is no replacement for the missing price, an imputation will have to be made each period until either the sample is redesigned or until a replacement can be found. Imputing prices for permanently missing sample observations is, therefore, more important than in the case of temporarily missing reports and requires closer attention.

9.184 The missing price can be imputed by the change of the remaining price observations in the elementary aggregate, which has the same effect as removing the missing observation from the sample, or by the change of a subset of other price observations for comparable products. The series should be flagged as being based on imputed values.

9.185 Samples are designed on the basis that the products chosen to observe are representative of a wider range of products. Imputations for permanently missing prices are indications of weakness in the sample, and their accumulation is a signal that the sample should be redesigned. For indices where there are known to be a large number of deaths in the sample, the need for replacements should be anticipated.

10. Treatment of Specific Products

A. Introduction

10.1 This chapter provides examples of how different national statistical agencies handle different industries. The emphasis is on those industries in which price measurement generally is regarded as difficult; however, examples of routine industries are included. *It should be kept in mind that the presentation of these methods is not intended to convey them as "best practices." In fact, it is recognized that in some cases a country's circumstances likely will necessitate deviations from these methodologies.* To underscore this point at the end of each section, a list of outstanding issues is provided—issues that point to problems in the described procedures.

10.2 A general problem in constructing PPIs is formulating a precise characterization of the good or service to be priced. To some extent, that characterization hinges on the definition of the industry to which the producing firm is assigned. For the purposes of this chapter, the ISIC Rev. 3 will be used as a reference. The linkage between the selection of the products to be priced and their industry assignment is independent of whether there is probability or judgmental sampling.

10.3 After selecting a product or output to be priced, the difficult problem is characterizing the good in a way that not only facilitates repricing but also distinguishes between changes in quality and changes in price. The last aspect is extremely important for an accurate measure of price change. Previous chapters in the *Manual* have provided discussion of the conceptual framework underlying many aspects of constructing PPIs. This chapter will provide some examples of different statistical agency practices.

10.4 Within the context of any economy, there will be some industries for which a relatively straightforward application of these methods and concepts is possible and industries for which that is not the case. In this chapter, both types of industries will be addressed.

10.5 Generally, the industries that allow for a straightforward application of the methods and concepts are ones for which the establishment output is countable. That is to say, a member establishment's output is physically measurable or has physical indicators of output that can be used. In either case, the definition of the output's price is clear. Examples of industries falling into this category and discussed below are agriculture (ISIC 01), steel (ISIC 27), and petroleum refining (ISIC 23).

10.6 However, some industries that produce physical output present difficulties for index number compilers. The construction industry (ISIC 45) and the shipbuilding industry (ISIC 35) are two examples. Though the output is easy to count, the creation of the companion price index is difficult for two main reasons: the output is produced over a long time, and it is an outcome of a contract—the output is usually a custom product. Accordingly, it is difficult to price the output on, say, a monthly basis. Such cases of custom capital goods are discussed below.

10.7 Industries producing goods that experience frequent technological change also present some special problems. Though the output of the computer industry (ISIC 30) may be measurable, constructing price indices for computers is difficult when trying to capture quality change that arises from the technological change. The computer industry and motor vehicles (ISIC 34) are examples provided in this chapter.

10.8 The clothing industry (ISIC 18) presents a similar problem. The output is measurable, but the measurement of price change is complicated by the change in quality of the clothing and the influence of seasons. The case of the clothing industry is specifically considered.

10.9 Because service industries generally do not have easily measurable output, it is difficult to apply the concepts set out in the *Manual* to them. Accordingly, this chapter will consider service industries such as retail trade (ISIC 52), wireless telecommunications (ISIC 642), commercial banking (ISIC 65), insurance (ISIC 66), software consultancy (ISIC 7220), legal services (ISIC 7411), and general medical hospitals (ISIC 8511). It will show how different statistical agencies overcome these difficulties to compile service sector producer price indices.

10.10 The discussion below will not fully address issues concerning sample design or sampling methodology. These features will be discussed only to the extent that they affect the establishment of a pricing strategy for the product.

B. Agriculture, ISIC 01[1]

10.11 The construction of a price index for agricultural products generally, and crops in particular, is more difficult because of two circumstances that sometimes combine. First, marked seasonal patterns in some commodities' production make prices unobservable during part of the year. Second, volatility in price and production from year to year, and sometimes within a year, is caused by external forces such as the weather or economic influences.

10.12 These two problems have to be accommodated by building into the indices a method for dealing with gaps in the supply of prices and for smoothing volatile elements while reflecting, as quickly as possible, changes in the trend of agricultural production.

10.13 The following description is drawn from the recently redesigned Canadian farm product price index (FPPI) and the procedures introduced to meet these problems, which are representative of the techniques used by other countries.

10.14 The index follows a seasonal basket concept in which the volume shares of the various commodities are different for each month in the year. Thus there are 12 different baskets used in calculating the months of a calendar year in the FPPI.

10.15 The annual index number for a given year is a weighted average of the corresponding monthly indices, rather than a simple average, as is common in other indices.

10.16 The index is an annually reweighted chain price index, so the annual weighting pattern is updated every year. Each annual weighting pattern, or basket, is based on marketing data for the five most recent years available.

10.17 The linking of new baskets each year is done at the annual index number level, not for any one month.

B.1 Seasonal baskets

10.18 The formula for constructing the seasonal baskets in the Canadian FPPI is a variant of what usually is called the Rothwell formula, after Doris Rothwell, an economist with the U.S. Bureau of Labor Statistics, who proposed it in a 1958 paper for the U.S. CPI. However, the formula was originally proposed in 1924 by two economists with the U.S. Department of Agriculture, Louis H. Bean and O. C. Stine, as an index number for farm prices. Thus the formula adopted for constructing seasonal baskets was originally designed as an indicator of farm price movements.

10.19 The Rothwell formula must be used to calculate indices of fresh produce in the harmonized indices of farm product prices of the European Community, so statisticians of those countries are familiar with it. The formula also is used to calculate series for seasonal commodity groups in the CPIs of several countries, including Japan, France, and the United Kingdom.

10.20 The Rothwell formula is

(10.1) $$P^{(c)}_{y,m/0} = \sum_j p^j_{y,m} q^j_{c,m} / \sum_j p^j_0 q^j_{c,m} \ ,$$

where $$p^j_0 = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} \Bigg/ \sum_{m=1}^{12} q^j_{c,m} = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} \Bigg/ q^j_c \ .$$

In the above formula, $p^j_{y,m}$ is the price of the j^{th} commodity for the *m*th of year *y*, p^j_0 is its price in base year 0, and $q^j_{c,m}$ is its quantity sold in the *m*th month of the basket reference period *c*. Note that in the special case where the basket reference period *c* is the same as the base year 0, the formula becomes

[1] The following description is based on Baldwin (2002).

$$(10.2)\ P^{(0)}_{y,m/0} = \sum_j p^j_{y,m} q^j_{0,m} \Big/ \sum_j p^j_0 q^j_{0,m}\ ,$$

where $p^j_0 = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} \Big/ \sum_{m=1}^{12} q^j_{c,m} = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} / q^j_c$.

Also note that the average base-year price of each commodity is its base-year unit value. In the special case where $q^j_{0,m} = q_0 / 12; m = 1,2,...12$ for every commodity (that is, quantities sold were the same in every month of the base year for every commodity), this variant reduces to the familiar Laspeyres formula.

10.21 The Rothwell formula for the annual index would be

$$(10.3)\ P^{(c)}_{y/0} = \sum_j p^j_y q^j_c \Big/ \sum_j p^j_0 q^j_c\ ,$$

where $p^j_y = \sum_{m=1}^{12} p^j_{y,m} q^j_{c,m} \Big/ \sum_{m=1}^{12} q^j_{c,m} = \sum_{m=1}^{12} p^j_{y,m} q^j_{c,m} \Big/ q^j_c$.

In the special case where the basket reference period *c* is identical with the base year 0, the formula becomes

$$(10.4)\ P^{(0)}_{y/0} = \sum_j p^j_y q^j_c / \sum_j p^j_0 q^j_c\ ,$$

where $p^j_y = \sum_{m=1}^{12} p^j_{y,m} q^j_{0,m} \Big/ \sum_{m=1}^{12} q^j_{0,m} = \sum_{m=1}^{12} p^j_{y,m} q^j_{0,m} \Big/ q^j_0$.

Note that even when the base-year prices are unit values, prices for other years are not, since they are weighted according to another period's monthly sales pattern.

10.22 In the Canadian FPPI, the monthly weighting patterns are calculated as follows: For each product, the average quantities sold for the five years from 1994 through 1998 were calculated for each month of the year. The quantities sold of most agricultural products can be measured directly: The availability of measures such as bushels or head obviate the need for deflation. The 12 monthly shares are then calculated. To obtain the monthly revenue weight for a given product, the annual revenue weight for a particular year is multiplied by the relevant monthly share. The sum of these monthly weights yields the annual weight. As described below, the annual weights change every year, but the monthly share patterns are held constant until the next major review, in about five years. This approach allows the relative importance of commodities in the 12 monthly baskets to change from year to year, reflecting the changes in the relative prices of the different commodities.

10.23 A major strength of this approach is calculating highly seasonal products available for only a few months in the year. Using the previous annual basket approach, such commodities had the same basket share in every month of the year. One had to impute prices in months when no quantities were sold. Using a monthly basket approach, if there were no sales for a commodity in a given month from 1994 to 1998, then it simply fell out of the index basket. There was no need to impute a price for it.

10.24 Problems with changing seasonal patterns may remain. If a seasonal commodity had no sales in a given month from 1994 to 1998, but some thereafter, the prices for that month would be ignored. For example, if the season for corn lengthened, perhaps because of global warming, to include sales in November, where before no sales occurred after October. This shift in the overall seasonal pattern of production of an agricultural commodity would not be reflected until the next update of the seasonal patterns. Changes in the length of a season do not occur very often, and it is the *beginning or end* of the season that is being ignored. Ignoring that is much less serious than assuming all months would have about an 8 percent (one-twelfth) share of the annual sales.

10.25 Imputations cannot always be avoided. If you typically have a weight for a product in a certain month, but for some reason, such as early frost in October, no sales of that product occurred that year, an imputed price would have to be assigned to it. This kind of scenario is more likely to occur than the one discussed above. In such situations, the imputed price would be the weighted average price for the in-season months through September. Although one could argue for other solutions, such an imputation is simple, does not depend on price information external to the stratum or the commodity in question, and gives the same annual price as if one simply ignored October in calculating the annual price.

10.26 The problems of imputation, as well as the formation of the seasonal basket, are ones faced by seasonal commodities, such as clothing.

B.2 Annual price index

10.27 The annual price indices are weighted averages of the monthly index numbers. The weights are the monthly expenditure weights. In this, they differ from the simple means of the monthly index numbers. A weighted average is used because the monthly shares of sales of many farm products are highly unequal. Most occur in only two or three months of the year, and in the same two or three months of the year, year after year. One cannot have much confidence in an annual index based on equal weighting of the monthly indices if the different months have such unequal contributions to annual output. This is more so the case since product prices are highly and negatively correlated with sales, being much lower in the months with the largest shares than in other months of the year.

10.28 Although they are close, the annual prices at the most detailed level are not unit values of the commodities. The annual unit value for a commodity is calculated as the total annual revenue divided by the total annual quantity sold. This amounts to a weighted average of monthly prices, weighted by *same-year* quantities. The annual prices in the FPPI are weighted averages of monthly quantities for the seasonal profile reference period, currently from 1994 to 1998.

B.3 Annual chaining

10.29 The index is updated every year, from the receipts for a five-year period. The basket for 1999, for example, is based on the sales from 1993 to 1997, revalued to 1998 average prices.

10.30 Consider the updating done for the January 1999 index. The quantities sold from 1993 to 1997 are evaluated at prices for 1998 to provide a new basket. Using this basket, indices are recalculated for each month from January 1998 onward; the index will automatically be on a 1998 time reference, so the ratio of this index to the previously calculated 1998 index gives the link factor. Indices for the months of 1999 are multiplied by this link factor. In January 2000, the same procedure is followed, instead using quantities sold for the period 1994 to 1998.

B.4 Linking at the annual index

10.31 Linking series that are computed with both monthly and annual measures can be a problem because it is not possible to preserve continuity for both. Most series get linked at the monthly level so that the monthly index changes are not distorted by shifts between the baskets. This can be done by linking in December, so that December and January prices are compared in terms of the new basket.

10.32 For this index, the monthly baskets change anyway, so there is no advantage in linking by the month. Linking at the year preserves the year-to-year movement as a measure of pure price change.

B.5 Analysis of monthly price changes

10.33 Monthly baskets have the disadvantage of no measure of pure price change between months. Even if there is no change in prices from one month to the next, a change in the index is possible because of the change in the basket. However, it is possible to decompose the monthly change in the FPPI into a pure price change component and a residual component for all months except January. The pure price change component measures what the change in the FPPI would be if there were no change in the monthly basket. This calculation may require the calculation of imputed prices for some commodities that may have gone out of season by the next month.

10.34 The decomposition is as follows:

$$(10.5)\quad P^{(c)}_{y,m/y-1} - P^{(c)}_{y,m-1/y-1} = \frac{\sum (p_{y,m} - p_{y,m-1}) q_{c,m-1}}{\sum p_{y-1} q_{c,m-1}} + \frac{\sum p_{y,m}(q_{c,m} - q_{c,m-1})}{\sum p_{y-1} q_{c,m}},$$

where summation is over commodities. Therefore, the monthly percentage change in the Rothwell index can be decomposed between a pure price change component,

$$(10.6)\quad \left(\frac{\sum (p_{y,m} - p_{y,m-1}) q_{c,m-1}}{\sum p_{y,m-1} q_{c,m-1}} \right) \times 100,$$

and a residual component,

$$\left(\frac{\sum p_{y,m}(q_{c,m} - q_{c,m-1})}{\sum p_{y,m-1} q_{c,m-1}} \right) \times 100.$$

(As can be seen, the residual component is not a pure quantity change component since there are different prices in the numerator and the denominator.)

10.35 Where very large basket shifts exist from one month to the next, it may not be acceptable to take the previous month's basket as appropriate for comparing prices between the previous and current month. An Edgeworth-Marshall-type cross should then be calculated

$$(10.7)\quad \left(\frac{\sum (p_{y,m} - p_{y,m-1})\overline{q}_{c,m-1\&m}}{\sum p_{y,m-1}\overline{q}_{c,m-1\&m}}\right) \times 100,$$

where $\overline{q}_{c,m-1\&m} = (q_{c,m-1} + q_{c,m})/2$.

10.36 Equation (10.6) answers the question of what the monthly percentage change in the FPPI would have been if there had been no change in the monthly basket from the previous month, with the previous month's FPPI remaining as published. Equation (10.7) answers the question of what the monthly percentage change in the FPPI would have been if both previous and current month estimates had been calculated using a common monthly basket representing sales in both months. Equation (10.6) thus is more closely connected to the published FPPI than is equation (10.7). Yet the latter may be a better measure of month-to-month price change because it uses quantity weights from two time periods.

10.37 An Edgeworth-Marshall cross has the advantage of being consistent in aggregation and satisfying the property of transactions equality. (If the volume of sales in month *m* is five times larger in month *m* than in month *m* – 1, month *m* will be about five times more important in determining the basket shares of the price comparisons.)

10.38 A Fisher cross is another way to incorporate information from two time periods. However, such an index does not satisfy transactions equality. In a Fisher cross, the price comparisons are weighted using each basket, and then their geometric mean is taken; the two baskets are treated as being of about equal importance, which may be contrary to reality as in the example where sales in month *m* is 5 times the sale in month *m* – 1.

10.39 An Edgeworth-Marshall cross also has an advantage over a Walsh cross, another index that combines information from two time periods, in that it does not remove seasonally disappearing commodities from the comparison. For a Walsh cross

$$(10.8)\quad \left(\frac{\sum (p_{y,m} - p_{y,m-1})\overline{\overline{q}}_{c,m-1\&m}}{\sum p_{y,m-1}\overline{\overline{q}}_{c,m-1\&m}}\right) \times 100,$$

where the average $\overline{\overline{q}}_{c,m-1\&m} = \sqrt{q_{c,m-1} \times q_{c,m}}$.

If a commodity were missing in either month, its mean quantity sold would be zero, and it would have no impact on the measured price change; in the Edgeworth-Marshall cross, all commodities with sales in at least one of the two months would have an influence on the estimated price change.

10.40 In calculating an Edgeworth-Marshall cross using equation (10.7), one must impute prices for commodities unavailable in either month *m* – 1 or *m* (but not both) and not, as with equation (10.6), only for those unavailable in month *m*.

10.41 The December–January change is distorted not only by the switch from one monthly basket to another, but also by the switch from one annual basket to another. As the annual basket changes every year, comparisons of 12-month changes between the same months of successive years do not provide a measure of pure price change. This problem is met by calculating each new index for 24 months, as previously described. Although the monthly index numbers are not used for the first 12 months, comparisons between them and the 12 months that follow can be used as measures of pure price change for 12 periods. In other words, the 1998 indices, on a 1998 base, are not used in the index, only those indices for 1999 are. Since they use the same basket, comparisons of the May 1998 indices (1998 = 100) and May 1999 indices give a pure price change measure.

B.6 Other issues

10.42 *Use of receipts in the absence of quantities sold*. For some products, such as maple products, quantities are not provided, though there are cash receipts. In this index, the price movements are taken from the movement of the total crops index.

This ensures that each kind of product is represented in the index with an appropriate weight.

10.43 *Choice of time reference*. The FPPI is referred to 1997 = 100. As the index is a chained fixed-basket index with the basket changing every year, the choice of time reference has nothing to do with the estimated price movements over time. The base was chosen to correspond with Canada's choice of 1997 as the reference for most of its economic series, including the *System of National Accounts*.

10.44 The Canadian approach raises issues that some countries may want to avoid or come up with alternative methods. As described earlier, the seasonal basket combines information from annual and monthly data, creating issues about (i) how to select the market basket and (ii) how to interpret the switch between annual and monthly quantity data. Relatedly, one also must choose the appropriate base year. A fuller discussion of seasonal adjustment is provided in Chapter 22.

C. Clothing, ISIC 18

10.45 As measured by the ABS, the output of the Australian clothing industry covers the production of a wide range of garments, from basics to high-fashion items. The output can be categorized in a numbers of ways, but industry and commodity classifications generally adopt the traditional split of

- Women's and girls' clothing,
- Men's and boys' clothing,
- Infants' clothing, and
- Clothing not elsewhere classified.

10.46 A further dissection by functional type of clothing can be made within these categories. For example, women's and girls' clothing could be divided into women's dresses, girls' dresses, women's skirts, girls' skirts, women's sleepwear, girls' sleepwear, and so on.

10.47 Alternative classifications may focus on aspects such as formal or fashion wear, business wear, casual wear, or sporting wear, or on the type of material used, including cotton or polyester.

10.48 Having selected the items to be covered by the index (for example, women's dresses), the respondents to be included in the index and the specific items to be priced need to be selected. The selection of respondents will normally be based on data from surveys or censuses of manufacturers.

10.49 Selection of the actual specifications to be priced will require contact with the manufacturers and may be complex. Key principles in selecting the actual specifications from any particular manufacturer are

- Specifications should provide adequate coverage of the types of garments produced by the manufacturer within that item category. In particular, they should represent the pricing practices adopted by the manufacturer. That is, the factors that cause prices to move differently across specifications should be taken into account. These may include type of material used (for example, cotton fabric shirts may move differently in price compared with polyester fabric shirts) and type of customer to whom items were sold (for example, if the manufacturer produces its own brand of underwear sold to up-market retailers and a "house brand" of underwear sold to major discount chains, then specifications for both categories will need to be selected since their prices are likely to move differently).
- One should be able to price the specifications on an ongoing basis to maintain constant quality. To do so on an ongoing basis, full details of the specification need to be obtained (see below).

10.50 A general problem in pricing clothing is the distinct seasonal variations in the clothing produced in most countries as manufacturers switch from summer to winter clothing. Since some garments are produced for only part of the year, some technique is required to handle the period when these seasonal items are not produced. The most common technique is to simply repeat prices for the out-of-season items.

10.51 As was mentioned regarding agricultural products, the problem of missing items is common when dealing with seasonal commodities. Imputations are therefore necessary. Section B.5 of Chapter 9 discusses imputations.

10.52 Another problem is finding the same items to price in the new season (for example, this winter) as were priced in that season of the previous year (that is, last winter). Items will often change because of fashion and style changes and the changing relative costs of different fabrics (for example, wool versus synthetics). Where the same item cannot be repriced and a different item is priced instead, it will be necessary to assess what price movement should be shown.

10.53 Quality change can be identified by any changes in the characteristics that incur costs. For a type of clothing, the quality change associated with a substitution of polyester for cotton can be handled by valuing the different cost. A wide range of factors can affect the quality of these garments. Major factors include

- Type of fabric used, for example, pure cotton, cotton blend, polyester;
- Quality of fabric, for example, weight, thread count, type of dyeing;
- Quality of make, for example, type of seams, buttonholes, collar, pleats.

10.54 With clothing, a natural question is what to do about fashion changes that are generally tied to seasonal variation. Opinions differ on whether a specific quality change should be made for fashion. Some might argue that a quality adjustment should be performed because the fashion element is the key price-determining characteristic. Others might argue that fashion changes manifest themselves in changes in other characteristics, such as fabric, and therefore do not require additional adjustments. If there are no changes in any of the measurable characteristics of the article of clothing, then some imputation for the cost of design, which would be quite difficult, may be necessary. Furthermore, no such adjustments typically are made for other products traditionally redesigned every year, such as automobiles. (The quality adjustment procedures for automobiles are discussed below.) Finally, although manufacturers devote considerable efforts to establish their designs as the fashion of the season, there is no certainty of success. Accordingly, the validity of computing a quality adjustment for fashion rests to some extent on whether the fashion can be deemed successful.

10.55 The practical problems for the price statistician are, first, to detect these changes and, second, to place a value on them. To detect quality changes, it is necessary to list on the prices questionnaire the actual specifications being priced from particular respondents, for example: "Brand X, Men's dress shirt, style No. xxxx, 100% cotton, size 38–43, long sleeves, single cuffs, etc., sold to major retail customer."

10.56 In addition to the detailed specifications respondents specifically should be asked on the questionnaire whether there have been any changes in the quality of the specifications being priced.

10.57 Seasonal dimensions can be handled by creating checklists that are seasonally based. Thus, an item selected would be Women's summer dresses, fall dresses, and so on.

10.58 There are three prominent issues that arise with measuring price change for clothing. First, as mentioned above, is how to impute missing prices and quantities. Second, there is an issue concerning the price reductions that result from seasonal clearances. Since such clearances sometimes result in drastic reductions in price, and they occur after the commodity is out of season, it is not clear that they should be considered. Third, there is a question of whether changes in fashion should be considered as a quality change. Earlier it was argued that such changes should not be considered quality change.

D. Petroleum Refining, ISIC 23[2]

10.59 Petroleum refining is a manufacturing activity that converts crude oil into various petroleum products. The primary outputs of petroleum refineries are refined petroleum products, including fuels, lubricants, and petrochemical products.

10.60 Crude oil is a complex mixture of hydrocarbons, water, salts, sulfur, metals, dirt, and other impurities. The crude oil must be cleaned and separated into various products. Often the molecular structure must be altered to improve its properties. Different products must be blended to produce usable mixtures. The major steps in a refinery include

- Desalting—removing salt, water, dirt, and other impurities;
- Crude and vacuum distillation—separating the crude oil into separate products;
- Conversion—modifying the composition of the products; and

[2]The following description was provided by Suzanna Lee of Statistics Singapore.

- Blending—putting together measured amounts of products.

These steps are called process units because they process the crude oil directly.

10.61 To compile a PPI for oil refining in Singapore, the price movements of commodities produced for sale by oil refineries are measured. Motor gasoline, jet fuel, kerosene, diesel fuel, fuel oil, lubricating oil, naphtha, liquefied propane gas (LPG), and asphalt are the primary outputs of oil refineries. Oil companies had been successful in selling motor gasoline to consumers under unique brand names like Shell, BP, Exxon, Mobil, and so on. Despite the differentiation, the products produced by refiners are essentially the same. It is simple in terms of data requirements and respondents' burden to construct a price index using sales revenue.

10.62 The weighting pattern in the PPI for petroleum refining is based on the relative importance of the commodities' production value in the base year. The production value is the sale value received by manufacturers for the sales of their output.

10.63 Before conducting a regular price survey, it is necessary to conduct a preliminary survey to identify the actual product, specification, brand, and grade of items produced by oil refiners. The survey frame for conducting the preliminary survey is usually obtained from the Census of Industrial Production register. A list of establishments classified by manufacturing activities, the products manufactured (at seven-digit Standard International Trade Classifications, SITC), and the corresponding sales values of these products were extracted from the Census. In most countries, oil refining is concentrated mainly in a few very large petroleum refineries. If the number of companies is few, no sampling is required, and the coverage of the establishments that engaged in oil refining can be exhaustive. For the preliminary survey, customized forms with the major commodity items (at seven-digit SITC) preprinted were sent out to respondents requesting them to provide the top-three-selling product brands under each of the commodity items listed. In addition they were requested to delete items that were no longer manufactured and to add new items not listed in the customized survey forms. They were requested to rank the various brands in order of importance and to provide a detailed specification or description pertaining to brand, grade, size measurements, frequency of production, unit of measurement, and actual ex-factory prices at which they were sold.

10.64 The returns from respondents were reviewed and verified through telephone inquiries to ensure that the products or brands reported fell within the commodity items requested. The final selection of products for the monthly price survey was based on how often and how regularly the product was produced, how specific and detailed the specifications were for pricing, and how significant was the product's share of the establishment's production. Based on these selection conditions, items that were made to order or were produced on project or ad hoc basis were excluded. This helps reduce problems posed by the substitution of such items in the future for price monitoring.

10.65 In Singapore, customized survey forms preprinted with the description of each selected brand item and unit of measurement are then sent to the refiners for monthly prices collection. Respondents are directed to quote ex-factory prices that exclude outward transport charges and excise duties, if any. The prices on the 15th of the month or the closest date are to be provided. Respondents are encouraged to give transaction prices, not list prices that can be substantially different. If ex-factory prices were not available, respondents were requested to state the basis of the quotation. The prices reported must be consistent and comparable from one reporting period to the next. Although spot prices of oil products are published on a daily basis in the newspaper, spot prices often reflect only a minute proportion of total production. It is more accurate, as well as quite cost-effective, to obtain actual transacted prices directly from the small number of oil refiners involved.

10.66 In many countries, the government is concerned with protecting air quality in the environment and has enacted laws to reduce the level of air pollution caused by the emission of pollutants from motor vehicles. The production of cleaner motor gasoline (unleaded petrol), as mandated by the government, raised the production cost for refiners manufacturing the clean gasoline. Manufacturers faced with additional production costs increased the price for the clean gasoline. Cleaner gasoline represents a quality improvement because the characteristics of the gasoline have changed. Therefore, the price increase in the unleaded gasoline mandated by the government should be subjected to an appropriate quality adjustment, since price change for motor gasoline should track only pure price changes.

10.67 A preferred way to handle quality adjustments is to estimate the value of the quality change directly. The quality adjustment for cleaner gasoline could be estimated directly if information could be obtained on the additional production cost incurred by manufacturers having to produce cleaner gasoline. Besides requiring data on the additional production cost, the proportion of clean gasoline produced in relation to total gasoline production is another consideration. Refineries could manufacture gasoline both for local consumption and for exports to other countries. The vehicular emissions standards of other countries may not necessarily be the same as those required within the refiners' country. Furthermore, government control of fuel emission from vehicles also may be gradually introduced over a long time frame, say, over 5 to 10 years, implying that the proportion of the unleaded to leaded gasoline manufactured could vary substantially from year to year, depending on both internal and external demand. While keeping the more aggregated index weight constant, the more detailed weights of leaded and unleaded gasoline may need to be adjusted on the basis of information on the volume of leaded and unleaded gasoline produced by refiners.

10.68 Often, information on additional production costs and volume changes is not available from refiners or product knowledge experts. In such cases, price compilers may have little choice but to resort to using indirect quality adjustment methods such as overlap pricing or class mean imputation. The decision on which method to adopt for quality adjustment depends on the information available to price compilers.

10.69 Essential fuel products such as petrol may be subjected to price controls that prohibit any price increases. In such a situation, it may be necessary to accept the official controlled prices reported by respondents. Nevertheless, a black market for the commodity might exist, and if it is possible to measure the black market price increases of the controlled items, such prices should be included in the index. Price controls greatly complicate the task of measuring price changes, and price statisticians must be aware of the problem so as to alert users of the more detailed category index that includes the price control items.

10.70 An issue that arises in the case of petroleum products is the potential irrelevancy of a fixed-weight index. As the discussion above makes clear, it is relatively easy for a refinery to switch from one product to another in anticipation of changes in demand—the largest and most common source of which is seasonal changes. In such a situation a fixed-weight index may have an incorrect set of weights for the period being considered. This problem can be solved by frequently updating the weights or using one of the superlative indices discussed in Chapters 1 and 15.

E. Steel Mills, ISIC 27[3]

10.71 The primary output of the steel mill industry is the production of steel shapes, such as sheet, strip, plate, bar, rod, pipe, and tube, from molten metal. Steel is generally produced by either the blast furnace/basic oxygen mill process or by the electric arc furnace process. The first process involves converting a charge of iron ore, coke, and other components into molten metal. The metal may be poured into any of several semifinished products, such as ingot, billet, and slab. The primary output of the industry includes semifinished products that leave the mill. Most semifinished steel is converted internally into higher-value forms such as sheet, plate, and bar. Blast furnace/basic oxygen mills also may use a continuous casting process, in which the molten metal instead is converted directly into more finished shapes. Electric mills convert steel scrap, pig iron, and other charge components into molten metal, which is then usually converted directly into sheet, plate, rod, or bar.

10.72 The primary output of the steel mill industry also includes various products such as forgings, nails, and wire when they are made at the steel mill for shipment to other establishments. The following describes the methods underlying the U.S. PPI for steel mills.

10.73 Items should be selected at the company level for the desired product aggregation, which are the publication cells. Item selection within publication cells is based entirely on probability proportionate to size sampling. Such sampling within companies has been based, whenever possible, on value of shipments or sales data. Value-based figures are preferred and are usually available for

[3]For more information, see Bestock and Gerduk (1993).

higher levels of aggregation. Quantity measures, usually tonnage, are viewed as acceptable at the more detailed levels. For example, there is little difference in value between several diameters of pipe of the same carbon steel grade. Yet there is a significant difference in value between carbon steel and stainless steel grades.

10.74 Prices are collected from each reporter based on basic shapes—sheet, plate, bar, rod—and on the type of steel—carbon, stainless, or alloy steel. Providing such specificity helps to align the products with revenue statistics. Additional specifications also are collected to further identify the product. These might include some sort of item identification, order or part number, the specific grade (there are hundreds of them), dimensional information, and other features, such as shipments to various types of buyers. The types of transactions include contract sales, multiyear sales to manufacturers, and distributor sales. Discounts, if applicable, are reflected in the price. Surcharges also may be included in the price. Steel companies have at times imposed temporary surcharges to cover sudden increases in the cost of scrap metal, and nickel, molybdenum, and other alloys.

10.75 The collected prices should be transaction prices. Reporters sometimes prefer to provide list prices to shield their pricing strategy. However, list prices in this industry can be especially nonrepresentative of price movements because of the potential impact of imports on market prices. One way to overcome this concern, as practiced in the United States, is to define the transaction price as an average and have a one-month lag. Because the industry firms typically set the transaction price at a discount from list prices, which yields widely varying prices of similar products based on different transaction characteristics, price averaging across all customers for a specific product each month provided the most representative transaction price obtainable. In addition, average pricing proved popular with reporters since it required no additional formatting of price records and avoided revealing any buyer-specific details. The major drawback was having to accept the one-month lag.

10.76 Quality adjustment in this industry is a less critical issue than for many others because the nature of the products rarely changes. However, if a product made of one alloy of steel is altered to be made with another alloy of steel that enhances strength, then a quality change would be made based on the cost differential for the alloy. This procedure is described in Chapter 7.

10.77 New products are introduced slowly by the industry, and therefore there is little problem with new and disappearing goods. The sheet metal that covers a new car is substantially lighter and stronger than 40 years ago, but it probably did not change much from that used several years ago. Given this pace of product change, the regular resampling process is sufficient. However, one possible issue with product change in the steel industry is that, when a certain grade or type of steel starts to become obsolete, output may be reduced as buyers slowly switch to a better product. The type of buyer may change as well. As with production cost data, diligent and reliable reporters provide the best means to maintaining index quality. Fortunately, the very limited and gradual rate of product change in the industry probably minimizes this problem.

10.78 The most common quality improvements in the steel industry usually are associated with the production process and often embodied in the building of a new mill, since it is hard to substantially redesign a mill once it is built. Changes in the production process, however, are not the type of change for which quality adjustments are made. The introduction of a new, lower-cost way to produce an existing good without changing any of its characteristics can result in lower price that should properly be treated as a price change. However, care must be taken to ensure that the new process does not alter the characteristics of the good. In that case, some adjustment in the observed change in price should be made. Ideally, reporters who are in the sales or accounting divisions will check with engineers.

10.79 Since steel prices collected are an average from the previous month, the published index is the measure of price change from two months previous. Although this inconsistency in timing was accepted to overcome reluctance to report transaction prices, some statistical agencies may have an alternative means of addressing reporter concerns.

F. Electronic Computers, ISIC 30[4]

10.80 The U.S. PPI program developed a computer price index that has served as a model for many countries. The methodology of that index is described below.

10.81 The primary output of the computer industry is the assembly of components into general-purpose computer systems that process data according to a stored set of instructions. These instructions are contained in the computer software (operating and application) and are often included in the computer system by the manufacturer. Establishments that primarily manufacture machinery or equipment that incorporate computers for the purpose of performing functions such as measuring, displaying, or controlling process variables are classified based on the manufactured end product.

10.82 The output of the computer industry can be disaggregated into several product categories. These categories should be broadly defined because the rapid pace of industry technological change can render narrowly defined categories obsolete. The PPI publication structure for computers is based on product detail collected by the U.S. Census Bureau in their Current Industrial Report (CIR) survey, which is described below:

- Large-scale;
- Midrange, excluding PCs and workstations;
- Personal computers and workstations, excluding portable computers;
- Portable computers with attached displays; and
- Other computers.

10.83 Note that the large-scale and mid-range product designations are problematic. These two Census product categories were originally intended to include computer host or server systems that were differentiated by memory size. Systems with 64 megabytes (MB) or more of memory were considered large, and systems with fewer than 64 MB were considered midrange. As mentioned above, any technologically based characteristic used to define product categories can quickly be rendered obsolete by rapid changes in computer output. Because PPI sample intervals average six to seven years, if 64 MB were maintained as the dividing point, then the advances in memory and corresponding fall in price quickly would make the midrange category obsolete. It would force all computer servers, including PC servers, into the large-scale category. When the PPI resamples the computer industry, it will avoid descriptors such as large-scale or midrange and use higher-level and more stable classifications such as host or multiuser computers. The U.S. Census Bureau recently adjusted its classification of computer servers and describes them as "Host Computers (multiusers)."

10.84 Rapid changes in computer output can create the classification problem of new product classes that do not fit neatly into an existing product classification structure. For instance, handheld devices such as the Palm Pilots are the fastest growing product segment in the computer industry. This product category did not exist when the PPI completed its last sample of the computer industry. The best fit under the current publication structure for handhelds is portable computers. However, index users, including producers, have come to view the portable computer designation as including only laptops or notebooks. If handheld devices were to be introduced into the PPI through a targeted sample augmentation, then the publication structure should be flexible enough to adapt. User needs and agency resources hinder the degree of flexibility. At a minimum, the PPI would revise the product title "portable computers" to "portable computers, including handhelds." If revising the title of an existing product classification does not satisfy analytical requirements, then a more aggressive adaptation could include the introduction of a new, more specific product category into the publication structure, such as "handheld computers, including personal digital assistants (PDAs)."

10.85 Both of the product classification issues described above are related to rapid post-sample changes in output. Similar adjustments at the disaggregate level may be required for the output of other high-tech industries such as semiconductors and telecommunications.

10.86 In the U.S. PPI program, computer producers were selected with a probability proportionate to size, and then individual products representing current output were selected based on their relative importance to a producer's value of shipments. Establishments that report to PPI provided detailed product specifications for each of the items (com-

[4]For more information, see Holdway (2001).

puters) that were sampled for which the producers provide monthly price updates. (The average number of computers sampled per producer was 4 but ranged from 2 to 12, based on the producer's size.) Because of rapid technological change, producers generally are unable to maintain a matched model for more than three or four months. Therefore, new computers or updated versions of predecessor computers are continually introduced into the PPI as sampled products become obsolete. Product substitution caused by rapid product displacement, in effect, provides an automatic sample update mechanism. However, new technologies or changes in characteristic quantities embodied in computer replacements challenge a statistical agency's ability to publish constant quality indices.

10.87 The PPI is based on a Laspeyres formula that is designed to approximate a FIOPI. The FIOPI defines a theoretical framework that is approximated by measuring changes in industry revenue-holding inputs, including technology fixed. The assumption of fixed inputs in a dynamic economy is problematic but generally can be addressed through adjustments for product attribute changes that are valued by the attending changes in the marginal cost of the product.

10.88 This resource-cost approach to valuing quality change is often difficult to apply to high-tech products directly because of insufficient information.

10.89 The PPI program has developed alternative valuations of quality change when resource-cost data are not available from computer producers or when technology enables higher quality at a lower unit cost.

10.90 Hedonic methods have been used to estimate quality change valuations for computers in the PPI since 1990. The hedonic function is based on the premise that the characteristics that make up a complex product can be unbundled and their influences on price measured.

10.91 The correct specification for a hedonic model is often a technical issue that is more dependent on product- and market-specific knowledge than econometrics. If appropriate data, including transaction costs, are available to support a model, then regressions can provide estimated coefficient values (implicit prices) for each of the independent variables described in a specification. Discussion of hedonic models was provided in Chapter 7, Section E.4.

10.92 When cost data are unavailable, then the implicit prices from a hedonic model can be used to value changes in the quantities of characteristics reported to the PPI.

10.93 The mechanics of quality-adjusting price relatives when computer characteristics change are described below:

ICP = Implicit characteristic price from hedonic model,

P_0 = Price of predecessor computer in reference period,

P_c = Price of replacement computer in comparison period,

PR = Price relative, and

$$PR = \frac{P_c - ICP}{P_0}.$$

10.94 The above example is based on an increase in the quantity of computer characteristics such as system memory or hard drive capacity. If the quantity of computer characteristics declines in period c, then the value of ICP is added to rather than subtracted from P_c.

10.95 Many of the primary inputs to computer production such as microprocessors, memory, and disk drives exhibit extraordinarily rapid price declines. For example, the PPI's index for microprocessors has declined at an average annual rate of about 20 percent. Information from trade journals indicates that disk drive prices, on a price per unit of storage capacity, have dropped at a rate at least as fast as microprocessors.

10.96 The independent variables specified in the PPI's hedonic models include all of the inputs mentioned above and many others. Because the costs of these components change rapidly, the PPI has opted for frequently updated cross-sectional models rather than less-frequent updates of pooled data.

10.97 Ideally, the PPI would update its cross-sectional computer models on a monthly basis, but resource constraints limit the PPI to quarterly updates. Nevertheless, the PPI has greater confidence in the constant quality measures provided by quarterly cross-sectional updates relative to a pooled model. Frequent updates of cross-sectional models also help the PPI estimate implicit prices for new characteristics shortly after they are introduced. The availability of a large amount of computer-related data on the Internet has aided the updating of the hedonic regressions.

10.98 Regularly updated cross-sectional models provide implicit prices that are based on market conditions at or close to the point at which a product replacement actually occurs, thereby enabling an improved approximation of constant quality indices in the PPI's real-time monthly production environment.

10.99 Because a longitudinal analysis of the relationship between prices and characteristics is the preferred way of basing quality adjustments, an issue that some agencies may want to address concerns the manner in which a sequence of updated cross-sectional regressions approximates a longitudinal regression.

G. Motor Vehicles, ISIC 34

10.100 The primary output of the broad motor vehicle-building industry is the manufacture of motor vehicles and the manufacture of engines and parts for motor vehicles. The discussion below describes the PPI for the Australian automobile industry and the methods of the ABS.

10.101 The output can be defined by the main activities of the industry, such as

- Motor vehicle manufacture,
- Motor vehicle engines and parts,
- Motor vehicle body manufacturing,
- Automotive electrical and instrument manufacturing, and
- Other automotive component manufacturing.

10.102 The first requirement in attempting to measure price change for this sector is to establish a clear understanding of the industry. In particular, one must determine the major categories of motor vehicles.

10.103 The following discussion focuses on complete motor vehicle manufacture. The concepts discussed also will be of assistance in considering issues involved with pricing other motor vehicle activities.

10.104 The next stage is to select respondents representative of these activities. In the case of motor vehicle manufacturers, this normally will be relatively straightforward, since there are usually only a few motor vehicle manufacturers in most countries. This may result in the sample actually including 100 percent coverage of manufacturers.

10.105 According to the *1993 SNA* the correct pricing basis for output producer price indices is basic prices, that is, to capture prices at the factory gate, where possible, excluding any taxes, delivery, or wholesaler margins. The prices also should be transaction prices that allow for discounts or incentive adjustments.

10.106 The prices also should reflect market values in cases where the manufacturers are vertically integrated with sellers. In such cases, cross-subsidization may make it difficult to obtain the proper price. However, this situation has become increasingly rare, since vertically integrated enterprises accounting systems usually require market valuations on transactions to manage monitoring effectively and to satisfy stricter taxation requirements.

10.107 In the case of motor vehicles, large fleet operators may be able to bypass the normal distribution chain. If this type of purchase is significant, it may be necessary to separately price such transactions. In some cases, government may be an important customer, as well as large taxi chains, or large motor vehicle hire chains. These customers may be able to deal directly with the manufacturer and could attract particular discounts.

10.108 Usually match pricing on a particular day of the month (such as on the 15th) will be adequate for monthly indices, because motor vehicle prices tend not to be as volatile as some commodities.

10.109 A major issue for producing an index for any technologically advanced commodity, such as motor vehicles, is quality change. While vehicle manufacturing tends to follow models that will be on the production run for at least a year (giving some opportunity to assess more fundamental technological change), motor vehicle suppliers are constantly offering packaged deals on these models. Given the array of options available for automobiles, price statisticians have the challenge of pricing to constant quality. However, because either manufacturers or distributors (wholesalers and retailers) can add options, it is important to consider only those offered by the manufacturer for a PPI for the auto manufacturing industry.

10.110 Examples of motor vehicle features that may be relevant for item selection and assessment of quality change include

- Make and model,
- General type of vehicle (for example, sports, 4-wheel drive, limousine, sedan, wagon, etc.),
- Engine size,
- Exterior dimensions,
- Interior dimensions,
- Torque,
- Antilock brake system (ABS)
- All-wheel drive,
- Fuel consumption (high consumption regarded as a negative attribute, while the type of fuel used has differing assessments depending on relative fuel costs and efficiencies),
- Air bags,
- Traction-control systems,
- Safety rating,
- Acceleration,
- Brake horsepower,
- Curbside weight,
- Air conditioning,
- Cruise control,
- Compact disk (CD) player and stacker,
- Global positioning system (GPS),
- Keyless entry,
- Security system,
- Power windows,
- Electric sunroof,
- Electric mirrors, and
- Metallic paint.

10.111 One method commonly employed for change of specification is the overlap method of pricing, discussed in Chapter 7. To undertake this method, prices must be available for the old and new model at the same time, which often may not be possible. The price comparison uses the old specification price in the earlier period and the replacement specification in the next period. Implicitly, the price difference is said to represent the market's evaluation of the quality difference between the two items.

10.112 An adjustment for changes in quality also can be made by valuing the difference in production cost attributable to the change in characteristics. This method has conceptual appeal in the case of PPIs, because assessments of quality change are best made on production cost estimates of differences in models. This method is frequently employed in the quality assessment of motor vehicles. A great deal of costing information that can be used for this purpose often is available from manufacturers. Similar sources of information may include motoring magazines or assessments made by motoring clubs or insurance companies.

10.113 Another approach is to use hedonic methods for quality adjustment purposes (see Chapters 7 and 21 for an in-depth explanation of hedonic methods). This will require an extensive data set of motor vehicles' prices with the quantities of all characteristics influencing price, preferably on the correct pricing basis (that is, basic prices), from which to calculate the hedonic function. The implicit prices of the motor vehicle characteristics from the hedonic function are used to value the differences in new and replacement motor vehicles within the ongoing sample. Alternatively, if complete time-series data sets of prices and characteristics are available, then the time dummy method could be used to directly estimate a price index from the hedonic function. It is important that the hedonic function on which these implicit characteristic prices are based should be updated at least annually. An excellent reference on the use of hedonic methods for constructing constant quality price indices for motor vehicles is that of Bodé and van Dalen (2001).

10.114 A number of private companies collect and collate pricing data on motor vehicles. Such sources often are used for detailed hedonic analysis of quality change. Whatever the quality assessment technique used, price statisticians may find it useful to refer to websites that provide reliable and free comparisons between different models and makes. An example of such a site is www.autobytel.com.

10.115 It should be noted that the set of characteristic changes also should include those mandated by governments. Some typical examples include

- Catalytic converters to limit pollution,
- Seatbelts or airbags,
- Systems that prevent ignition without the use of seatbelts, and
- Speed-limiting or warning mechanisms.

Legally mandated features should be seen as a quality improvement because they cost extra to produce and reflect a greater volume of production. Manufacturers usually can supply estimates of the extra production costs imposed by the addition of these features.

10.116 The price statistician needs to be concerned with some issues in implementing quality adjustments for automobiles. For example, automobile purchasers often order models with options—that is, the purchased model differs from the standard model. If such options are popular in a time period, then a high percentage of the cars purchased may have those options. If, on realizing the option's popularity, the manufacturer decides to make the option standard, then care must be taken in estimating the quality adjustment. To illustrate, suppose that all of the automobiles purchased in a given time period were ordered with the option and that in the next time period the option becomes standard. In this case, no quality adjustment should be conducted in the month that the option becomes standard, because in the previous month the value of the option should have been accounted for. When dealing with options, care must be taken to recognize the market penetration of the option before performing a quality adjustment, should the option become standard. Another caveat applies when performing quality adjustments for changes in features that can return to the original level. For example, suppose that because of relatively stable fuel prices, engine horsepower starts increasing and quality adjustments are performed for the increase. If fuel prices rise sharply and induce reductions in horsepower to the level of the reference model, then a decision must be made on how to treat horsepower change. One the one hand, a quality erosion could be recorded (relative to the last model), but, on the other hand, there is no quality change relative to the reference model.

H. Shipbuilding, ISIC 35

10.117 Many industries produce what can be described as custom capital goods. These are goods for which the buyer contracts with the producing firm to provide a capital good made to specific requirements. The goods would not be produced otherwise. Two examples are shipbuilding (discussed below) and construction (discussed in the next section). These examples describe the methods of the Australian Bureau of Statistics.

10.118 The primary output of the shipbuilding industry is the construction and repair of ships (vessels 50 tons and more in displacement), manufacture of submarines, and manufacture of major components for ships and submarines.

10.119 The output can be defined further by the main activities of the industry:

- Dry dock operations,
- Hull cleaning,
- Ship repairing,
- Shipbuilding, and
- Submarine construction.

10.120 The prices statistician is faced with many problems when attempting to measure changes in prices for the output of the shipbuilding industry. Ships take a long time to build, and, as a result, there may be no sales in a particular period. A significant proportion of the output of the industry relates to ships that are "unique," in that the same ship (for example, specialized naval ships) will not be produced again in the near to medium term, if ever. However, in some sectors of the industry the same general type of ship may be produced on an ongoing basis (for example, high-speed passenger ferries), although the actual specifications of each vessel are different, reflecting the needs or preferences of the particular buyers. That is, while the basic hull design may be the same, the fitout of each ship is markedly different: engines, propulsion systems, different layout of passenger space (cabins versus sit-up lounges), amount of cargo space, and type of navigation equipment fitted.

10.121 These features mean that it is difficult, if not impossible, to price the same specifications over time.

10.122 Possible approaches to measuring price changes are

- Escalated contract prices,
- Input or component prices, and
- Model pricing (also referred to as quoted prices).

10.123 In the case of escalated contract prices, the contract for the sale of the ship specifies a base price, which is subject to escalation over time according to movements in costs (that is, of labor and material inputs). This may be a common industry practice for very large contracts, such as naval contracts for a number of ships of the same design (for example, a class of destroyers) where the contract may have a long life (for example, 10 years or more). Under this method, the use of escalated input costs provides a simulated output price for the base-period item each month (or quarter) for the life of the contract.

10.124 This method requires a reliable technique for escalating the input costs. This may require the use of other proxy indices (for example, for labor and material inputs), which must match those used in the actual contract for the pricing method to be representative. The major problem with this pricing method is that it provides only a basis for measuring prices for the life of the contract. For example, if shipyard A finishes building naval patrol boats, and the next navy contract is for frigates to be built at shipyard B, how can the two series be reliably linked? Forming a link could be a problem, especially if the base contract price for the frigates implicitly includes an allowance for price changes in the intervening period (because of productivity in the industry, for example) not fully reflected in the escalation provisions of the patrol boat contract.

10.125 The input or component price approach is commonly used because of its relative ease. The basis of this approach is the concept that the price of an item can be viewed as a function of

- Cost of direct inputs, that is, materials, major components, labor, energy, and so on;
- Cost of indirect inputs and overheads, that is, depreciation, administrative expenses, and so on;
- Productivity—efficiency with which inputs are put together; and
- Profit margins.

10.126 At its simplest, the input prices approach uses movements in the cost of the major direct inputs as a proxy for output prices. For example, using a breakdown of the major materials and types of labor used in building the ship, the major inputs and their relative weighting can be determined. Using this approach, a ship is viewed as a bundle of standardized components, for example, main engine(s), gearboxes, navigation equipment, hull, and so on, which are combined together using various amounts and types of labor. Actual specifications (for example, specific make and model of engine, aluminum plates) can then be selected and priced over time.

10.127 Such a simplistic approach is unlikely to be satisfactory over the long run, because it assumes that all other factors remain constant. In particular, it does not consider the profit margin and may not capture substitution toward more productive inputs.

10.128 The solution most widely adopted for handling the problem of unique products such as ships has been model pricing.

10.129 The model pricing approach requires the respondent to quote a price each period for a standard product with specifications that are held constant. For example, a shipbuilder is asked to select a representative ship that was constructed in the past, and to quote each period what the price would be to undertake that project if it were up for contract.

10.130 The obvious problems with this approach are

- The workload imposed on the respondent. To accurately reprice a ship on an ongoing basis represents a major task. Most respondents will be reluctant to do so.
- Getting the respondents to take the exercise seriously and to reflect market conditions. This is especially a problem when market conditions change dramatically, and, as a result, margins change. In periods of recession, respondents may not have undertaken any work in that area in the recent past and are not tendering for any such work. This adds to the hypothetical nature of the exercise. As a consequence, it is crucial to pay particular attention to the prices obtained and to maintain regular contact with the respondent to ensure that the prices being obtained are representative of actual market transactions.

10.131 For a particular respondent, there are two main methods of model selection that can be used:

- An actual ship sold in some recent period, which is representative of the respondent's output, can be selected and specified in detail as the model to be priced.
- A hypothetical model that is representative of the types of products produced by the respondent can be established. While this model may never have been (or never will be) produced, it

must represent an item that could be produced readily.

10.132 Whatever type of model is selected, it is essential that the model be specified in sufficient detail so the respondent reports prices for that defined model, and no variation from the model occurs over time without notification to the statistical agency.

10.133 The model should be broken down into the individual material and labor components. The following example illustrates the level of detail required:

(i) Materials used (with types of materials and quantities used listed)
  a. Hull construction
    i. Aluminum plates—type(s) × number
    ii. Aluminum beams—type(s) × number
(ii) Major components
  a. Main engines
    i. Makes and models
  b. Gearboxes
    i. Makes and models
  c. Propulsion system(s)
    i. Water jets—make and model
(iii) Fabrication labor (with type of labor, for example, skilled, semiskilled, and numbers of hours listed)
(iv) Design and drafting costs
(v) Overhead
(vi) Profit margin (the representative margin that would apply if contract were signed today, in current competitive climate).

10.134 For each pricing period, the respondent will need to recost each component. The respondent must understand that the profit margin quoted should reflect actual business conditions in the pricing period, and, therefore, this component is expected to fluctuate with market conditions (that is, be higher in boom periods and lower—even negative on occasion—in recessionary periods).

10.135 The model pricing procedure is thus equivalent to the respondent preparing a competitive tender (bid) each month to supply the model. Using model prices amounts to an attempt to reflect, each month, the real conditions prevailing in the marketplace, that is, the conditions that the respondent would take into account if submitting a competitive tender for a real project.

10.136 Model pricing is prone to error, particularly where market conditions are changing dramatically or the contact officer completing the form changes.

10.137 To minimize the potential for error the following steps need to be taken:

- Respondents supplying model prices should be subject to an annual interview. At such interviews the representativeness of the model should be reviewed and the prices supplied should be checked to ensure they reflect market conditions.
- Whenever a contact changes, the new contact should be visited personally and the basis of the model pricing explained.

10.138 For the shipbuilding industry (and the construction industry, examined next) the validity of the method used to derive a monthly price is a fundamental issue. More specifically, care must be taken to ensure that the derived monthly price is a good estimate of a transaction price—in other words, it responds to market conditions that the producer would have faced in the month if a project were being directly priced.

I. Construction, ISIC 45[5]

10.139 As will be described below, many of the aspects of the approach to the shipbuilding industry are applicable to the construction industry.

10.140 This industry includes only activities concerned with the actual construction of a building—it does not include the value of land or the development of land. The latter activities would be included in ISIC 701, Real Estate Activities with Owned or Leased Land.

10.141 The output of the building construction industry is the construction of buildings and the alteration, addition, renovation, or general repair of buildings. Building outputs are diverse, not reproduced over time, and often unique. Even with the same type of building (for example, an office build-

[5]For further information on this price index, refer to Australian Bureau of Statistics (2004).

ing) outputs can differ according to design, floor area, building materials, and construction method.

10.142 Furthermore, projects differ according to the underlying tasks. Projects entailing mostly renovations and repair will be quite different from those concerning new buildings. The scope of projects may vary in terms of the elements of construction required. Some services that are project-dependent include demolition and design.

10.143 The location of a project also can have a major impact on the price. In many countries, building prices may reflect urban, suburban, and regional factors.

10.144 Although the building price may reflect all the characteristics of the construction service, the fundamental problem is comparing building output prices from one period to the next.

10.145 There are many concepts of building price. The basic price, the preferred valuation of output in the *1993 SNA,* is the amount receivable by the producer from the purchaser for a good or service produced, calculated as output minus any tax payable, plus any subsidy receivable.[6]

10.146 The ABS uses the basic price of building output, which covers the amount receivable by the prime contractor from the client for the building, excluding taxes, subsidies, the value of land, design, and other professional fees.

I.1 House building

10.147 The ABS compiles a price index for house building using the matched-model method. More information on the matched-model method can be found in Chapter 1, Sections I, K, and L, and in Chapters 7 and 8. This approach is used for houses because of the great degree of regularity in their design, size, construction materials, and construction methods, and the fact that most home-building companies in Australia specialize in the construction of a range of well-specified models. A representative sample of home models is selected in each city, prices obtained for each period, and the price movements for each model weighted together.

10.148 Constant quality is preserved by calculating price movements on a matched-sample basis, that is, the price movements between adjacent periods are based on the same models in each period. If the specification of an individual model changes substantially or a price cannot be obtained, then that model is excluded from the calculation of price movement. Adjustments are made to raw prices to compensate for any minor changes in specifications. For example, if a particular model is essentially the same as in the previous period but the current price reflects a new addition or special feature, then the price of the model is adjusted using standard quality adjustment principles so that the model can be matched directly with that in the previous quarter.

I.2 Residential building other than houses and nonresidential building

10.149 The building output can be defined as a whole final structure or as a collection of particular elements that constitute the construction process. These elements should be narrowed down to include only those that would generally be covered in a standard construction contract between client and builder. Examples of excludable elements are any site works (such as demolition, land clearance, roads), external services (such as drainage, water and electricity connection), and design and other professional services.

10.150 Of the several compilation methods available, the ABS has chosen a method based on a breakdown of building construction into a set of common components. This so-called component-cost method treats building output as a set of standardized homogeneous components representing subcontracted work-in-place. This and other methods for compiling building price indices are described in OECD and Eurostat (1997).

10.151 Typical projects were selected to represent construction activity in a range of functional categories, such as office buildings, shops, and factories. Each project was broken down into a set of

[6]Other concepts of building price include the producer's price, the purchaser's price, and the seller's price. The producer's price is the amount receivable by the producer minus any value-added tax, or similar deductible tax invoiced to the purchaser. The purchaser's price is the producer's price plus the value of any nondeductible value-added tax payable by the purchaser. The seller's price reflects all the elements that contribute to the price paid by the final owner (Organization for Economic Co-operation and Development and Eurostat, 1997, pp. 14–22). It would include the purchaser's price plus the value of land, design and other professional fees, the client's profit margin, and other costs.

standard well-defined components, each component consisting of a quantity, a unit rate, and a value (quantity multiplied by rate). The selection and analysis of projects was undertaken by a firm of quantity surveyors.

10.152 Projects are priced each period by updating the unit rate of each component while holding the quantity constant. The resultant component values are aggregated to produce a current-period project value. Project indices are weighted together to produce index numbers for strata, such as building function, region, and total industry.

10.153 Not all components need to be directly priced each period. First, pricing can focus on a subset of components that contribute to the bulk of the building cost. Second, it may be possible to use one item to represent several components that fall under the same building trade or that exhibit similar price behavior. The unit rate (price) collected, for example, for one specification of the formwork to a suspended slab could represent several formwork components (say, formwork to slabs, columns, and beams). Finally, because the components are standardized, it will be possible for one specification to be applicable to several building types. Thus, instead of collecting a distinct set of prices for every component of each project, it may be possible to greatly reduce the number of prices collected by arriving at a group of representative items. For example, an office building, a shopping center, a hospital, a hotel, and an apartment building will share a set of common components (the components will have different quantities and values for each project but share the same definition). For some of these common components, just one price may suffice for all of the projects. The items will need to be specified in considerable detail to enable consistent pricing from quarter to quarter. These element prices will then have to be aggregated to form the price of the building.

10.154 The ABS has contracted with a consultant to provide the unit rates for the building indices. The consultant, a large national quantity surveyor or cost consultant has access to up-to-date market prices for work-in-place and inputs. The consultant first builds up the unit rates from scratch, using prices and ratios for labor, materials, and plant. Profit margins and overheads are added. The rate is then modified according to up-to-date information on prices tendered for current projects. Every quarter the consultant provides 62 unit rates for each of nine geographic regions.

10.155 The ABS decided to obtain rates from a consultant, rather than collect rates through a survey of builders. This decision was based on economic considerations. The establishment of a collection for more than 550 items, with at least three different respondents per item, would have involved setting up more than 1,650 specifications, most of which would have been complex models. This would have required significant time and resources. In addition, the maintenance of such a collection would be relatively complex and costly, ideally requiring the services of in-house building industry experts.

10.156 The advantages of the component cost method are (i) the index captures changes in productivity and subcontractor profit margins (unlike the simpler factor inputs method) by pricing work-in-place; (ii) it can be used for a range of structures, selected to best represent building activity; (iii) pricing is relatively simple and less resource intensive than the quoted price method (in which respondents provide prices for whole hypothetical structures), and consequently promises more plausible results; and (iv) it requires less information than hedonic methods.

10.157 The component cost method used by the ABS does not measure changes in the prime contractor’s profit margin—it is fixed at 5 percent. On the basis of information gained from interviews with contractors, the ABS determined 5 percent to be representative of a reasonable margin under normal conditions. This course of action was taken because the options for estimating the margin were judged to be highly subjective, or they involved the collection of very sensitive figures from respondents who are traditionally quite guarded about divulging such information. Work on developing a reliable measure for prime contractors' margin is continuing, and if a reliable measure is developed it will be incorporated in the index.

I.3 Construction of roads and bridges

10.158 Roads and bridges are simpler structures, compared with buildings, in terms of the number of elements in their construction and the variability of their design. The price of roads, however, still can vary considerably because of considerations such as pavement type, pavement area, location, terrain, and soil quality.

10.159 The ABS has adopted a method that breaks down road and bridge construction into a set of common components. The components and their weights were derived from an analysis of cost and tender documentation relating to a selection of representative main road and highway projects in most states of Australia. The broad-level road construction components are preliminaries, drainage, earthworks, pavement and surface, and furniture and landscaping. The bridge construction components are piling and columns, substructure, bearings, girders, and completion of superstructure. Representative specifications are chosen for these components and prices are collected on a quarterly basis, mostly through survey methods.

10.160 The earthworks component has been the most troublesome because it is a major contributor to the cost of road construction, but its cost can be highly variable (and unpredictable) since it depends on location, terrain, and soil quality. Since the cost of earthworks is dependent mainly on the use of operated equipment, hire rates for earthmoving equipment (including machine, operator, fuel, maintenance, overheads, and profit) are being used as a proxy measure.

I.4 Other nonbuilding construction

10.161 Apart from buildings, roads, and bridges, a diverse range of construction output includes those associated with telecommunications; electricity generation, transmission, and distribution; railways; harbors; pipelines; recreation; and heavy industry. Although the output in these categories can be unique, like buildings, roads, and bridges, the output also can be composed of standard components or processes, suggesting that an index compilation method that focuses on the pricing of the components rather than the entire structure can be used.

10.162 The ABS is experimenting with an index for telecommunications construction in which the output consists of about two hundred well-defined work activities, grouped into broad categories such as cable laying, optical fiber cable laying, aerial works, installation of new telephone services, general civil works, broadband services, and mobile networks.

10.163 For electricity-related construction, research conducted to date has shown that the construction of transmission and distribution networks is the most promising in allowing pricing to constant quality. The construction of transmission and distribution networks is frequently an ongoing activity, or at least regular activity, rather than infrequent or irregular as in the case of generation plants. As with other areas, more frequent transactions provide a better opportunity for repeat measurement. Further, transmission and distribution networks have some standard elements, such as towers, lines, and cabling, for which it is more likely that prices will be collected over time to a constant quality.

10.164 The ABS has done little work to date on compiling producer price indices for other components of nonbuilding construction.

10.165 Currently, in the ABS Stage of Production PPIs, the price movements in road and bridge construction are used to represent those occurring in other nonbuilding construction. As indices for the various components of other nonbuilding construction are developed they will be incorporated. Statistics Canada currently produces indices for electricity utilities construction, covering distribution systems and transmission lines. The indices are compiled from a combination of output and input prices.

10.166 Aside from the issues mentioned above in the case of shipbuilding, an issue regarding the pricing of construction projects is whether to include the price of land. As mentioned at the outset, activities that include land development are included in another ISIC category. One might argue that the price of a construction project is dependent on the land, or that the building and the land are bundled together.

J. Retail Trade, ISIC 52[7]

10.167 The primary output of retail trade industries is the provision of the marketing functions necessary to allow consumers access to various goods. In other words, the retailer acts as an intermediary between goods producers and consumers. As opposed to wholesale trade industries, customers

[7]The details presented in this section were provided by staff in the Producer Price Program at the U.S. Bureau of Labor Statistics.

are able to make unit purchases of items (that is, not required to buy in bulk) that are generally packaged in some manner. Basic functions involved in retailing include standardization or grading of goods, storage and transportation, buying, risk bearing, financing, selling, and product planning. The selling function is probably the most obvious one seen by the consumer. Selling includes the pricing of the product and the presentation, which includes activities such as tagging, packaging, display, space allocation, advertising, and promotion.

10.168 The U.S. PPI program has developed a method of constructing the retail trade services provided by grocery and department stores. Major service lines are defined by type of store within each industry, as shown below in the examples for grocery stores and department stores:

(i) Grocery stores
  (a) Supermarkets
  (b) Convenience stores
(ii) Department stores
  (a) Discount or mass-merchandise department stores
  (b) National chain department stores
  (c) Conventional department stores

Within each of these, there may be further disaggregation according to the services provided. For example, in the case of supermarkets, disaggregation is by department: meat, bakery, fresh produce, and so on. Such disaggregation of activities is determined by the organizational unit and is taken to refer to homogeneous units of activity.

10.169 Once the products are selected, price-determining characteristics associated with the product are identified. These include the type of product, size or weight, and, often, material composition. In addition, store characteristics associated with providing the service also are collected or assigned based on secondary source data. Store characteristics include area, number of available product choices, hours of operation, and the existence and age of scanners and software for processing customers.

10.170 In most cases the price collected is a margin price (with the exception noted below). The margin price captures the intermediary nature of the retail service and is calculated by taking the selling price and subtracting the purchase price of the last shipment received (less all rebates and allowances) for a specific good. Furthermore, it is consistent with national income accounting conventions that define the output of retail trade as the margin. However, the U.S. Bureau of Economic Analysis (BEA) defines the margin as the selling price of a good in the retail market less the cost of replacing the good in the store's stock. This definition is difficult to implement because it requires collecting the replacement cost of the item. It is far easier to use a last in, first out (LIFO) accounting methodology, that is, using the last shipment received for pricing the acquisition cost of the sold item.

10.171 For a limited number of cases, the selling price of the good is used instead of the margin price. These cases occur when the value added by the retailer in preparation of the product for sale is large, or when there is a fee for a service where the customer is clearly paying for something incidental to the sale of goods. Examples include sales from in-store restaurants run by the retail establishment, alterations of purchased goods, and delivery charges incidental to the purchase of a good.

10.172 Two approaches have been used in implementing the margin pricing methodology. The first looks at the margin price of a unique product. This sample of goods approach is used to represent the output of the entire store. One concern with this approach is that the marketing of the selected sample of goods may not always be representative of the marketing of other goods sold in the store. Using this approach, changes in store characteristics may not explain changes in margin for the selected sample of products. The second approach looks at the average margin value of a relatively homogeneous grouping of products. Though this approach may allow store characteristics to better explain the margin, the average margin might be overly affected by differences among the products in the group.

10.173 There are three potential problems with using a margin price: setting the base price, negative margins, and weighting.

10.174 Base prices are established as the price received in the first month before index calculation. Sale prices are common in retail trade industries, and if a sale price is used as a base price then a permanent bias is introduced in the indices. However, if sale prices are not used in the base period, the price movement from the base month to the next month is incorrect because the index methodology requires that the base be the first month’s price. To solve the sale price problem, the most recent nonsale price before the month of the base price is used as the base price. To solve the price movement problem created by the use of nonsale prices as the base price, the first six months of calculation are not published, and the index is reset to begin calculation in the seventh month. This sixth-month interval is necessary because new indices are introduced only every six months in the U.S. PPI program. Although six months of pricing data are lost, the bias is eliminated from the index. The base price problem is related to the Laspeyres formula bias problem in the sense that it is actually a weighting problem that derives from the price movement of items based on sale prices being overrepresented (having too large of a weight) in the index.

10.175 The negative price problem derives from the fact that some retail trade establishments, for example, supermarkets, drug stores, and gasoline stations, will, on occasion, sell individual products at a loss. This is done to draw customers with the expectation that they will purchase not only the product being sold at a loss but also products with positive margins. Since the index calculation system used in the U.S. PPI program does not allow negative or zero prices to be used, a procedure was implemented that uses a Dutot index (the ratio of average prices) to calculate price indices in the retail trade industries. This procedure can be briefly described as an unweighted summation of margin prices for three relatively homogeneous products. The monthly percentage change for these three products is used in index calculation instead of the price for a single product. The Dutot methodology also decreases the variability of index movement that is often a characteristic of margin-priced indices.

10.176 The weighting problem is concerned with the aggregation of margin and nonmargin goods. Industry definitions in retail trade industries include not only selling merchandise but also, in certain cases, the manufacture and sale of products. Consider a bakery that manufactures bread for resale or in-store consumption and carries other prepackaged bakery products for resale. Since all production is given a chance of selection, prices for some of the manufactured goods and some of the prepackaged bakery goods for resale may be collected. The prices for the manufactured goods are the retail price, while the price of the prepackaged bakery goods is the margin price. Combining retail prices and margin prices in the same product category creates item weighting problems. Suppose the bakery sells two loaves, one manufactured on-premise and the other a prepackaged loaf bought from another bakery. One might suppose that the selling of the two loaves should have equal weighting. However, the retail-priced loaf should have a larger weight than the margin-priced loaf because the output associated with the former is manufacturing and selling, while with the latter it is just selling.

10.177 The easiest solution to this problem is to create two properly weighted product categories separating the margin-priced goods from the nonmargin-priced goods. However, if budget allocations are not large enough to support a separate sampling of the two transaction types, an alternative solution needs to be developed. Separate data on manufactured and resold goods must be acquired and used to derive or reapportion the sample unit weight among transactions that are margin priced versus those that are not. It should be noted that for the United States, Canada, and Mexico, this problem will disappear when the NAICS is fully implemented. Under NAICS, all combined manufacturing and selling transactions are classified in the manufacturing sector and not in the retail trade sector.

10.178 A fundamental issue in pricing retail trade industries is adjusting for changes in the quality of the service, which is assumed to be dependent on a store's characteristics. If store characteristics change, all items being priced from that location have to be adjusted to account for changes in the service, provided they are related to a change in store characteristics. A retail store is considered to be providing the same service activity when slight modifications are made to the products being sold. Major changes to products could require changes in the retail service—different displays or different inventory requirements.

10.179 Hedonic models should quantify the correlation that exists between store margins and store characteristics. Store characteristics include

- Total store area,
- Selling area,
- Checkout scanners,
- Age of scanner software,
- Number of stock-keeping units,
- Number of full-time-equivalent employees,

- Type and location of store,
- Hours of operation,
- Total sales volume, and
- Time since last renovation.

10.180 These characteristics are readily available, and, instead of using them directly, it may prove more effective to transform them into ratios. For example, forming a ratio of checkouts to store traffic yields a measure of checkouts per customer that may indicate better or faster service with all else remaining equal. Other possible ratios to include are checkouts to sales, employees to sales, stock-keeping units to store area, employees to store area, and checkouts to store area.

10.181 A key aspect of retail trade services is the services that can be linked to the maintenance of an inventory. However, the valuation of inventories is difficult to incorporate in price indices.[8]

K. Telecommunication, ISIC 642[9]

10.182 This section examines the compilation of the U.S. PPI for a relatively new and difficult component of the telecommunication industry—wireless communication.

10.183 The primary output of wireless telecommunications is that of placing parties in communication through a radio network, parallel to the traditional wire line network of the telephone system. The U.S. PPI program has developed an index for this industry. Cellular telephone services include Traditional Cellular Service, Personal Communications Services (PCSs), and Enhanced Specialized Mobile Radio (ESMR) and are defined as being "voice-grade" and interactive. Paging is defined as "less than voice-grade" because it involves only the sending of characters or numbers. Paging services, however, can include customer notification of voicemail messages waiting at a voice mailbox provided by the paging company.

10.184 There are three types of voice-grade wireless services: Traditional Cellular Service, ESMR, and PCSs. The primary distinction among these is in how they are licensed and in the frequency and power level used in transmitting and receiving. To the buyer, the technical differences among the three are not noticeable. The services can be augmented with add-on features, including services such as voicemail.

10.185 In the United States, licenses for Traditional Cellular Service are granted by the federal government according to geography. In the 1980s, about 300 licenses were distributed for urban areas and about 400 for rural areas. There were only two licenses available for each Metropolitan Statistical Area (MSA) or Residential Service Area (RSA).

10.186 ESMR systems operate at a lower frequency and higher power setting than cellular service. ESMR uses an improved "push-to-talk" technology previously used only by dispatch services for taxicabs and mobile repair operations.

10.187 PCSs are provided in the same manner as conventional cellular systems, except that license areas are much larger. These systems work at a higher frequency and lower power setting, which require more cell stations in a given area. There were about five licenses auctioned per MSA, which greatly increased competition in each area.

10.188 Paging services allow messages to be sent to a subscriber. Messages can be delivered or stored for later delivery. The messages can be just numbers or characters and numbers.

10.189 In the United States, the item classification is associated with the regulatory designations that are, in turn, related to the various technologies used to provide the service.

10.190 The fundamental issue in developing a pricing methodology was to define a net transaction price that would reflect discounts, new service plans, and new service features.

10.191 The price measure for all wireless telecommunications services was chosen to be the unit value and is defined in terms of minutes for each applicable rate for all similar services in the reference period. All minutes for a particular service are included in the measurement each month. A unit value is calculated for that service and represents the average cost per minute of that service for all customers. The average cost per minute is multiplied by the reference period weight of that service and then aggregated with other services to yield the final average billed price for the overall service.

[8]See, for example, Diewert and Smith (1994).
[9]See Deuchars, Moriya, and Junko Kunihiro (2001).

10.192 The unit-value method captures the universe of transactions within the entire service line (publication category) each month. There may be many (hundreds) of different service plans in existence for a particular type of service offered by a company. The unit-value approach provides an easy and inexpensive way (both in terms of cost to the agency and respondent burden) to capture new plans and thereby includes yet another source of price variation. This method also captures price change resulting from bundling and unbundling service features in much the same manner as it captures the introduction of new rates. The unit-value approach is operationally easier than trying to price a few specific plans and then trying to substitute to new plans each month.

10.193 The shifting of the weights inherent in the unit-value method is another of its advantages because it provides a straightforward way to obtain an accurate price through the automatic adjustment for the changing popularity of various service characteristics. Furthermore, because no single product is priced—in effect a series of product varieties is priced—the need and subsequent use of quality adjustment is greatly reduced.

10.194 However, the unit-value approach does not remove the concern about new-item bias. Such a bias arises with the use of the Laspeyres index because of the fixed-quantity assumption. New services in this industry are expected to be introduced frequently and to become popular quickly. The unit-value approach can handle the introduction of new service characteristics that attach to existing services—for example, the provision of a voicemail option to standard cellular service. When *completely new* services are introduced, there is concern about when to include them and how to adjust for their weight.

10.195 Cellular, PCSs, and ESMR are tracked similarly. Differences are based on the price of the total service package, including the monthly access, usage charges, and features. The reporter begins participation in the index by filling out the attached worksheet (see Figure 10.1). Part I of the worksheet includes three steps: (i) determine all of the different types of charges possible, including all optional features, whether they are billed or free; (ii) enter the total number of units used of each type of service; (iii) enter the total number of access lines or subscribers and divide the total units by the number of subscribers. This gives the average number of charged units per subscriber. Part II of the worksheet calculates the average revenue per unit for each service. First, the reporter enters the total billed revenue for each type of charge. The billed revenue is divided by the total number of units to give the average revenue per unit. Part III of the worksheet computes the average revenue bill. The average number of units per subscriber is multiplied by the average revenue per unit to produce a total weighted average.

10.196 Only six basic services for paging are offered. These are the combination of the two types of paging (numeric and alphanumeric) and the three types of service areas (local, regional, and national). The first step is to ascertain the ratio of revenue represented by the six categories of service for each company. The next step is to determine the ratio, in billed units, of all the billed components to each of the six services.

10.197 The Sample Worksheet (Figure 10.1) at the end of this chapter illustrates the implementation of the methodology.

10.198 The telecommunications industry is a very dynamic industry, which raises the issue of how best to capture the effects of new goods and changes in the quality of the service. Many approaches can be undertaken to address these issues. For example, in some countries it may be preferable to rely on a comprehensive set of bills from providers; this would be true if there is rigidity in plans and the ability of consumers to switch among plans. Such an approach also would require frequent updating of the sample to ensure the capture of both new plans and new services.

L. Commercial Banking, ISIC 65

10.199 Financial institutions explicitly and implicitly charge their customers for financial services, as recognized in the recommended methodology for compiling their aggregate value in the *1993 SNA*. The challenge for price index compilers is to account for these and other components of the price of financial services in the construction of producer and consumer price indices. This section briefly considers valuation and price concepts and then turns to the anatomy of a financial services price index and the attending compilation issues. In addition, a presentation of the U.S. PPI for the banking industry is provided, which implements the framework discussed.

L.1 Output of the banking industry

10.200 The primary output of the banking industry is the provision of financial services, including financial intermediation. For this industry, financial intermediation can be defined as the provision of services associated with the matching of savers and investors. In the course of providing these services, banks provide various transaction and credit services. Because the focus is on the financial service transactions, these are the basis for the identification of a bank's output.

10.201 One of the primary challenges in this industry is to measure financial intermediation services implicitly measured, or FISIM, as defined in the *1993 SNA*. Banks often provide services for which they do not explicitly charge. Paying or charging different rates of interest to lenders and borrowers covers the cost of providing these services and provides an operating surplus. As pointed out in the *1993 SNA*, this scheme of interest rates avoids the need to charge customers individually for services provided and leads to the pattern of interest rates observed in practice. Thus, the price measure must capture both direct and indirect charges for the provided services.

10.202 According to the financial firm model of a financial institution, the price of services is given by the user cost of money. The user cost of money is analogous to the more familiar user cost of capital in the sense that it provides a way for valuing the flow of services emanating from a stock of a nonfinancial asset; either one may be viewed as a rental rate. Interested readers can reference Barnett (1978, 1980), Diewert (1974c), Donovan (1978), Fixler (1993), Fixler and Zieschang (1992b), and Hancock (1985) for an in-depth treatment of the user cost of money concept. The sign of the user cost indicates the role of the product in the financial operations of the firm; a positive user cost indicates a financial input and a negative user cost indicates a financial output. Given that the indicators of account activity are positive, the convention is to take the negative of the user cost, which hereafter is called the user cost price.

10.203 For financial assets such as loans, the user cost price is the margin between accrued payments to the owner of the asset, *including expected holding gains*, and the opportunity cost of money. As its name implies, the opportunity cost of money is a concept akin to the reference rate discussed in the *1993 SNA*. For a depositor or lender to a financial institution, the user cost of money is the difference between the opportunity cost rate and the rate payable by the institution to the lender.

10.204 The user cost price captures both the implicit and explicit charges for services associated with an account and thus is an appropriate measure of the price of those services. The explicit charges would comprise all overt service levies in monetary units against the customer's account for transaction processing and the like, and the implicit charges would be based on the user cost.[10]

10.205 In this formulation, the amount of (deflated) dollars in each account is an indicator of the financial service activity. Using deflated dollars would be necessary if the price index is based on a constant quantity assumption.

10.206 The price of services attached to an account i is given by the following sum of implicit and explicit components, based on our previous discussion:

$$p_i^t = \left[\left(r_i^t + h_i^t - \rho_i^t\right) + s_i^t\right],$$

where

p_i^t = the price of services at time t on account I,

r_i^t = the average rate of return payable on account i during period t,

h_i^t = the average rate of gain (loss) on holding account i during period t,

ρ_i^t = the reference rate, and

s_i^t = the rate charged for explicit service charges payable on account i during period t.

10.207 It should be noted that the user cost price can be discounted by the factor $1 + \rho$; this may be important in volatile countries where the risk-free rate is relatively high. In addition, the user cost

[10] In December 2003, as part of its comprehensive revision to the national accounts, the U.S. Bureau of Economic Analysis implemented a user cost method to value the implicit financial services provided by commercial banks. The change and its impact on GDP are discussed in Moulton and Seakin (2003) and Fisher, Reinsdorf, and Smith (2003).

prices can be deflated by a general price index; this would be necessary to adjust nominal interest rates and service fees for changes in price movements, and again might be important in countries experiencing relatively high rates of inflation.

10.208 The computation of *h* does not include the value of any "write down" arising from a reassessment of the credit risk of the borrower. Unlike a holding gain or loss generated as a result of exposure to exchange risk, accumulated write downs appear on the institution's balance sheet as a liability counter-entry to the contract value of the loan asset, rather than a direct "mark (down) to market" of the value of the compromised asset. The write down increases the liability recorded against booked loan assets and is shown as an "other change in the volume of assets" rather than a revaluation of holding gain or loss.

10.209 Using the above, the price relative for an individual financial product that is an asset to the issuing or creating financial institution (such as a loan) is to have the following form:

$$R_i^{t,t-1} = \frac{p_i^t}{p_i^{t-1}} = \left[\frac{\left(r_i^t + h_i^t - \rho_i^t\right) + s_i^t}{\left(r_i^{t-1} + h_i^{t-1} - \rho_i^{t-1}\right) + s_i^{t-1}} \right].$$

Thus, the price relative for services attaching to an asset account represents the relative change in the total service charge rate.

10.210 The computation of the various interest rates is done by dividing income data by corresponding balance sheet items. For example, the interest rate received on loans would be computed as interest received on loan divided by the amount of outstanding loans on the balance sheet. Instead of using the balance sheet entry at a point in time, one could use an average over two time periods—this would be an estimate of opening and closing values and allow for some loans to be paid off while new loans are made.

10.211 All of the other components of the user cost price would be computed in a similar manner.

10.212 For liability products, the price of financial services again comprises an implicit and explicit component, as

$$p_i^t = \left[\left(\rho_i^t - r_i^t + h_i^t\right) + s_i^t\right],$$

which yields a relative of the form

$$R_i^{t,t-1} = \frac{p_i^t}{p_i^{t-1}} = \left[\frac{\left(\rho_i^t - r_i^t + h_i^t\right) + s_i^t}{\left(\rho_i^{t-1} - r_i^{t-1} + h_i^{t-1}\right) + s_i^{t-1}} \right].$$

10.213 The holding gain *h* is interpreted in the liability case to be an increase (decrease) in the size of liability through time, possibly because of a feature of the contract forming the liability.

10.214 In principle, the reference rate should be some risk-free rate. However, the selection of an actual rate is more complicated. For example, should only short-term government securities be used, or should the rate represent a weighted average of government security rates, where the rates reflect the holdings of such securities by banks? An additional question concerns the time period to be used. As indicated, the computation of interest rates is based on historical data. If one were to use current-period market rates for the reference rate, then one may find that the differences between the reference rate and the computed interest rate are volatile. One solution would be to compute the reference rate as the other interest rates; the reference rate would be the interest income from government securities divided by the balance sheet entry for such securities.

10.215 For both asset and liability products, it also may be useful to collect indicators of activity, such as number of accounts, number of automated teller machines (ATMs), or indicators of the average utilization of specific service dimensions on each account, such as transaction processing, statement generation, assessment of creditworthiness via loan applications, and applications for letters of credit, as applicable to the type of account. Variations in these other indicators of service would indicate variations in the quality or nature of service across accounts and institutions, to the extent that these variations are correlated with the explicit and implicit service charges on accounts to adjust service fees and interest rates to enable adjustments for quality of services.

10.216 In fact, one could consider the account as the primary unit of output for a financial institution; the output would be expressed in terms of the numbers of accounts, and the user cost prices above would be multiplied by the average balance in each type of financial product.

10.217 Because the financial services industry generally is regulated, the data needed to construct weights in a PPI should be available, often from administrative reports such as those required of depository institutions by central banks. Other financial intermediaries, such as insurance firms, in general also must complete regulation forms. Since most countries monitor their financial institutions, and there are international agreements regarding reporting requirements and accounting methods, the data elements from financial institution regulatory sources will be at least somewhat comparable internationally.

L.2 U.S. PPI commercial bank index (not yet in production)

10.218 The U.S. PPI program has developed an implementation of the above-described approach to measuring price change in banking. The details of that index are described below.

10.219 The specific types of services provided by banks can further define the output. For example, in the United States, the major service lines are

- Loans,
- Deposits,
- Trust services, and
- Other banking services.

10.220 In many countries, universal (one-stop) banking is allowed, so additional services such as insurance, brokerage, and travel may be added.

10.221 Loans are assets of a bank defined as funds advanced to a borrower to be repaid at a later date, usually with interest. Included in the loan category are residential real estate, nonresidential real estate, home equity, commercial and industrial, agricultural, new and used auto, and credit card loans.

10.222 Deposits are liabilities of a bank defined as funds placed with a bank in an account subject to withdrawal. Because the focus is on financial services, the services associated with deposit products are viewed as outputs of the bank. Included in the deposit category are demand, time, and savings accounts.

10.223 Trust activities involve the bank's acting in a fiduciary capacity for an individual or a legal entity, such as a corporation or an individual's estate. This typically involves holding and managing trust assets for the benefit of a third party.

10.224 Other banking services include standby letters of credit, correspondent banking, sale of securities, and cash management.

10.225 Account selections can be made within account classes defined by the institution or from the reports required by the regulatory authorities. A model list of account classes would include

- Mortgage loans,
- Agricultural loans,
- Commercial loans,
- Consumer and other loan services,
- Retail (deposits),
- Trust services, and
- Other banking services.

10.226 Once an account is selected, the next step is to specify the services to be priced. In some instances, account classes are sufficiently homogeneous, and it is unnecessary to select a sample of individual account numbers within the class. For example, for loans and deposits, the unique item to be priced is represented by a homogeneous group of accounts (for example, all 15-year fixed-rate residential mortgages or all 1-year certificates of deposit). On the other hand, trusts and other banking services could be priced by selecting an individual transaction and tracking the cost of the profiles through time.

10.227 Once the actual service is selected, its price-determining characteristics are identified to permit monthly repricing of the same unique item. The following characteristics are common for most services:

- Type of service; for example, mortgage loans, money market savings account, corporate trust;
- Term of service; for example, 15-year loan, 5-year certificate of deposit; and
- Type of fees; for example, late payment, automatic teller machine, early withdrawal penalty.

10.228 The U.S. PPI program has implemented the user cost framework described above. A key feature of valuing the implicit component of the financial service price is the reference rate, or the opportunity cost rate of money, which does not include any intermediation services. As shown above, the user cost prices for assets and liabilities differ. The price of an asset (for example, loan) is equal to the asset holding rate less a reference rate. The asset holding rate is the interest received plus service charges. For liabilities (deposits), the price is equal to a reference rate less the liability holding cost rate. The liability holding cost rate is the interest paid to depositors less service charges.

10.229 In measuring the prices for both loans and deposits, the same reference rate is used. Possible reference rates include the central bank lending rate (discount rate), interbank lending rate (federal funds rate), or a weighted average of the interest rates on all banks' securities holdings where the weights are shares of the different securities in a bank's security portfolio.

10.230 In practice, the price of these services can be expressed as shown below. Again, both services are priced at the portfolio level.

$$\text{Loan Price} = \left[\left(\frac{\text{Earned interest income} + \text{Fees}}{\text{Average loan balance}}\right) - \text{Reference rate}\right] \times \$1,000 .$$

10.231 Earned interest income includes all interest actually received in a given month for the portfolio of loans being priced. This includes interest earned on both old and new loans. The average loan balance is calculated by averaging the ending daily balances of the loans in the portfolio over the month.

$$\text{Deposit Price} = \left[\text{Reference rate} - \left(\frac{\text{Interest payments} - \text{Earned fees}}{\text{Average deposit balance}}\right)\right] \times \$1,000.$$

10.232 Interest payments include all interest actually paid to depositors on the funds held in the portfolio in a given month. Earned fees should include all fees, such as those for ATM withdrawals or insufficient funds, that are actually collected by the bank. Again, the deposit balance is calculated by taking the average of the ending daily balances of the portfolio.

10.233 For both equations, the calculation within the outer brackets results in a rate. This rate is multiplied by $1,000 to yield a service price used in the index calculation. When the price is positive, the service will be considered on output. However, whenever the price is negative, the service will be considered a financial input, and the price will be excluded from index calculation until it becomes positive again. In other words, the influence of that particular good is excluded; in effect its price is imputed to be that of other members of its group until it becomes positive again.

10.234 For trust and all other banking services, the price is equal to the actual fee charged for performing the service. These fees can be a percentage of assets or a flat fee.

10.235 The PPI program uses bank revenue data collected by the U.S. Bureau of the Census and the Federal Deposit Insurance Corporation, the U.S. agency that takes the lead in compiling income and balance sheet data for U.S. depository institutions.

10.236 Net interest revenue will be allocated between loan and deposit products by using the reference rate. Intuitively, the net interest rate can be decomposed into borrower and depositor components using the reference rate

$$\begin{aligned}\text{Loan rate} - \text{Deposit rate} &= (\text{Loan rate} - \text{Reference rate}) \\ &\quad + (\text{Reference rate} - \text{Deposit rate}).\end{aligned}$$

10.237 For individual products, there will be an adjustment for changes in the purchasing power of money in the volumes of assets and liabilities. In the case of asset products, the PPI will deflate by a price index that corresponds to the asset. For example, for a portfolio of car loans, the PPI Auto index will be the deflator. For deposits and any asset not associated with a particular price index, the GDP chain-linked price index will be the deflator.

10.238 Another fundamental issue in pricing banking services is the ability to maintain constant quality. One can view the deflation described above as maintaining the constant quality of the money volumes. However, just as in any other service or product, there are observable service characteristics that can be monitored, such as access to ATMs, ability to use Internet banking, debit cards, and so on. When such changes are observed, quality adjustments should be performed.

M. Insurance, ISIC 66[11]

10.239 The provision of insurance is another financial service that presents conceptual problems in compiling a producer price index. In this section, the construction of an index for the property and casualty insurance industry is examined.

10.240 The primary output of the property and casualty insurance industry is the assumption of risk (transfer of risk from the policyholder) and financial intermediation.

10.241 The U.S. PPI program has developed a producer price index for the property and casualty insurance industry. The service output is measured by the policies underwritten by the insurer.

10.242 A given policy lists the events for which restitution would be made to the policyholder and the attending payment levels. These can be viewed as the amount of risk being transferred to the insurer.

10.243 The output can be further defined by the specific types of property and casualty insurance coverage. The major service lines in the United States are

- Private passenger auto insurance,
- Homeowner's insurance,
- Commercial auto insurance,
- Commercial multiple peril insurance,
- Worker's compensation insurance,
- Medical malpractice insurance,
- Product liability insurance,
- Inland marine insurance,
- Surety insurance, and
- Fidelity insurance.

10.244 A policy is selected by sampling from a frame of previously selected (via sampling) lines of service for an insurer.

10.245 The following policy characteristics are common in most property and casualty insurance lines:

- Type of property or casualty description—characteristics of the insured property;
- Type of coverage—including physical damage coverage and liability coverage;
- Dollar limit of coverage—the maximum amount of money the insurer is legally obligated to pay in the event of a claim;
- Coinsurance clause—percentage of the value of the property to be reimbursed by the insurer;
- Deductible—the insured bears the first part of any loss covered by the policy up to a specified amount;
- Length of policy period—time frame for which the policy is in effect;
- Perils covered—specific risks that the insurer assumes;
- Location of the insured property—risks vary by geographic location;
- Past loss experience—premiums generally are lower if the insured has a past record of making fewer claims;
- Valuation of insured property—either the actual cash value of the property, which adjusts for depreciation, or the replacement cost; and
- Valuation of risk exposure—a valuation for liability coverage.

10.246 In addition to assuming risk, insurers act as financial intermediaries. They receive a flow of premiums on the policies they sell, a liability, and transform those premiums into earning assets by investing them, chiefly through the purchase of safe investments such as government bonds. This investment income is crucially important to the industry and greatly affects their pricing decisions. Companies may well reduce premiums when the rate of return increases and raise premiums when the rate of return is lower.

10.247 The price of assuming risk and providing financial intermediation is defined to be the sum of premium plus the rate of return on investment. Or,

$$\text{Price} = \text{Premium}\ (1 + r),$$

where r is the annual return on the invested portion of the premium for the particular line of insurance that is being priced. This rate is stated as a percentage of all premiums paid.

10.248 In the case of mutual companies where policyholders also are the stockholders of the company, there is an additional consideration. Because

[11]The U.S. approach to constructing an insurance price index is described by Dohm and Eggleston (1998).

these companies typically pay out a dividend rebate to the policyholders on an annual basis, such a dividend would be subtracted from the premium to obtain a net transaction price. Accordingly the price is expressed as

$$\text{Price} = \text{Premium}\ (1 + r) - \text{Dividend}.$$

10.249 To track premium movement in the property and casualty industry, companies provide estimated premiums for frozen policies. In other words, the premium's price determining characteristics are held constant while the policy is repriced on a monthly basis.

10.250 The insurance company estimates the current premium for this frozen policy by applying current charges to its characteristics. This premium remains unchanged until the policy is priced again the following year.

10.251 The cost of freezing the policy is that it does not capture modifications in the policy over time. Policyholders can change the level of liability, reduce the deductibles, or change the nature of the risk—for example, the addition of a teenage driver dramatically changes the risk associated with an automobile policy.

10.252 To hold inflation-sensitive characteristics constant, periodic adjustments are made to account for inflation. For homeowner's insurance, the dollar limit of coverage is adjusted annually to account for construction price inflation. The assumption is that the policyholder is insuring to secure a constant flow of services from the insured property. If price inflation affects the cost of repair or replacement of the damaged property, the coverage limit should be escalated to reflect this increase. This adjustment is made on the anniversary date of the policy. This procedure reflects the actual coverage adjustments made by insurers at the time of policy renewal.

10.253 Because the index tracks several thousand policies selected on a probability basis, policy anniversary dates are spread throughout the year. This yields a smoother-behaving index than making this adjustment for all repriced items at one time.

10.254 The source for the inflation adjustment depends on the insurance company. If the company cannot make a recommendation as to how the inflation-sensitive policy characteristics should be adjusted, the analyst decides the appropriate index to use. For example, the E. H. Boeckh Building Cost Index is used to escalate the coverage limits for homeowner's insurance. For adjustments to worker's compensation insurance, the workforce in the group is held constant (same number of people in the same jobs), and the wage rates are adjusted to account for general wage inflation by using the U.S. Bureau of Labor Statistics' Employment Cost Index.

10.255 The investment rate of return is calculated by all insurance companies as a percentage of the premium. An annual report is prepared by all companies that includes this calculation. The report provides the investment rate of return by insurance line calculated as a percentage of premium. As with the inflation-sensitive policy characteristics, the rate of return is updated annually for each priced item on the policy anniversary date.

10.256 The fundamental issue in pricing insurance services over time is the ability to identify and adjust for changes in risk. For changes in explicitly endogenous risk factors such as changes in coverage or deductibles, companies have suitable cost data to allow for meaningful cost-based quality adjustment.

10.257 However, for changes in exogenous risk factors that go beyond the scope of policy negotiations, such as an increased incidence of theft or a severe hurricane season, company-specific data would be not be sufficient to definitively quantify risk. Only outside data sources will be able to identify short-term versus long-term changes in risk.

10.258 Such an outside data source is used in the quality adjustment of private passenger auto insurance, where risk changes occur even though the age of the insured auto remains the same. To keep the age constant, the model year of the auto is updated once a year to the next model year. For example, in a policy that insured a three-year-old car in year t, the model year of the car would be upgraded by one year in year $t + 1$ to maintain the three-year-old car status. However, changing the model year also can move the auto into a different risk category known as a symbol group. This can occur because the characteristics of the auto may have changed, or it may occur because the risk associated with the car has changed without any characteristics change, such as the car is now more popular among thieves. Insurance companies are unable to assess this risk change on their own, but a valuation can be obtained from outside sources. In the United States, there are third-party firms (Insurance Services Office) that assemble and evaluate risk information and provide risk ratings used industrywide. Changes in this risk rating are used to explicitly adjust the premium.

10.259 Although both frozen policy and the annual resampling of policies approaches to repricing are susceptible to new-item bias, the problem is greater when using a frozen policy. Over time, a frozen policy may no longer be representative. Mandated coverage may change, or new insurance products may be introduced.

10.260 Although bias may not be as prevalent when following an actual policy, it can occur if the general population has changed its preference for the type of insurance product it purchases or if the policy represents a smaller portion of the company's business.

10.261 The U.S. PPI program has developed a directed substitution procedure to reduce new item bias. This procedure captures evolutionary changes to a current product or a service that did not exist when the sample was selected. Periodically, each company will be contacted in order to review the insurance products included in the sample. Evolutionary changes in the industry will be identified and disaggregation will be performed to determine whether a substitution should be made from the current product to an evolutionary product or whether to add the new feature to the description of the current product. Producer cost-based quality adjustment will then be attempted to adjust for these changes.

10.262 The measure of insurance output in national accounts (see, for example, *1993 SNA,* Annex II) is based on a premiums-less-claims concept. Accordingly, some might view that the proper measure of the price of insurance services should be based on such a net premium concept. Others might argue that because insurance services are a type of financial services, the price of insurance services should be analogous to the price of banking services described above—that is, a user cost price approach should be used.

10.263 Regardless of the measure of price, any approach must address the issue of how to address changes in the risk being assumed by the insurer. There is both an identification dimension and a measurement dimension to this issue. The first concerns that ability of the price statistician to identify the changes in risk that are properly assigned to the insurance service, and the second concerns the ability to measure the change in risk in a way that a quality adjustment can be performed.

N. Software Consultancy and Supply, ISIC 7220[12]

10.264 The compilation of PPIs for the output of the software consultancy and supply industry is challenging because of the diversity of the output and factors such as rapid obsolescence, frequent quality improvements, and increasing productivity.

10.265 Software output includes custom software produced on order from specific users, computer programming services provided on a fee or contract basis, and ready-made or prepackaged software sold on license to a number of users. There also is interest in developing price indices for deflation of expenditure on own-account software.

10.266 The output of the industry is highly diverse. Prepackaged software covers a large heterogeneous range of software, including systems software, applications software, and other types of software such as games. Output also includes documentation, maintenance, and training services. Custom software and contract programming are client specific and will vary depending on the requirements of the client.

10.267 When compiling a price index for prepackaged software one must take account of the fact that products change fairly frequently and then make adjustments for changes in quality. Matched-model methods (see Chapters 7 and 8) have been used for this output; however, they do not capture quality improvements such as enhanced power and performance, and as a result they "understate quality-adjusted price declines" (Bureau of Economic Analysis, 2000). The difficulties of making adjustments for changes in quality are identifying the quality change and then estimating its monetary value. Hedonic analysis is seen as the most promising method for producing a constant quality price index for prepackaged software (see Chapters 7 and 21).

[12]See O'Rourke and McKenzie (2002).

10.268 Function point analysis has been identified as a potential means of pricing comparable units of custom software and own-account software. By breaking up software into components that can be measured according to functionality provided to users, function point analysis can be used to analyze the unit cost of software, making possible the comparison of heterogeneous output. This approach can be problematic since these metrics themselves can be both subjective and difficult to estimate (Gartner Group, 1999).

10.269 Because of difficulties in pricing comparable software over time, input cost indices have been used for own-account software and charge-out rates have been used for computer programming services. These methods are comparatively easy to develop; however, they are problematic in practice. Input cost indices for own-account software do not take into account improvements over time in the productivity of information technology (IT) professionals, arising from substantial improvements in other IT products used as inputs in providing the service (for example, computer hardware, software applications for debugging code in creating new softwares) (O'Rourke and McKenzie, 2002).

10.270 Custom software is produced to meet a diversity of clients' needs and consequently is not a standard output for which constant quality price changes can easily be measured over time. As custom software "consists of a mixture of new programming and existing programs or program modules, including prepackaged software, that are incorporated into new systems," PPIs for this output have been made by weighting indices for prepackaged software and own-account software (BEA, 2000).

10.271 As noted above, a major issue with the construction of software price indices is selecting a method of quality adjustment. It also may be necessary to set up a rule that allows for a distinction between a quality change that can be said to create a new good and a product change that would fit into the modification of existing products. Such a distinction enables the use of different methods—in the case of a new product it may not be necessary to perform any price change for the existing product. Setting the criteria for such distinctions is a task that must be confronted by all goods and services that experience rapid technological change. Software has an additional problem in that much of the work is performed on a contract basis, and in such cases the ability to measure the extent of quality change is likely to be very difficult.

O. Legal Services, ISIC 7411[13]

10.272 The legal services industry is a difficult area in which to apply conventional price index techniques, and at present only a small number of countries have established indices. The methods described here have been discussed by the Voorburg Group on Services Prices and have been implemented by the ABS.

10.273 There are many ways to list the activities that might be included as output of legal services firms. One way would be according to products as classified by the CPC system of the UN. Accordingly, legal services products are

CPC	Description
821	Legal services
8211	Legal advisory and representation services in the different fields of law
82111	Legal advisory and representation services concerning criminal law
82119	Legal advisory and representation services in judicial procedures concerning other fields of law
8212	Legal advisory and representation services in statutory procedures of quasi-judicial tribunals, boards, etc.
8213	Legal documentation and certification services
8219	Other legal services

10.274 In most countries, this industry is generally composed of large firms primarily serving large corporate clients and small firms primarily serving households and small businesses. It is important to seek assistance from industry bodies in the sample design process. These associations provide splits of the main revenue-earning activities of the industry (for example, corporate and personal law, activities relating to patents, real estate) to aid in deciding which services to price and what may be an appropriate stratification. Industry associations also may be able to provide lists of organizations within the industry from which to sample, or at least have information to supplement a statistical organization's business register. Probability proportional to size sampling (should a measure of size variable exist)

[13]See McKenzie (2001).

or judgment sampling to best represent the spread of activities undertaken in the industry and the largest revenue-earning firms are key aims of the sample design. Because of the large number of small firms in the industry, stratification by size of firm may also be required with a small sample of small firms chosen to represent this sector.

10.275 The most common forms of charging within the legal services industry in Australia are summarized below. Note that combinations of these charging methods often may be used, depending on negotiations with the client and the range of services required.

10.276 ***Hourly rates.*** The basis for most forms of charging within the legal services industry is derived from an analysis of the time involved for the staff (for example, partners, associates, juniors) to provide the service required. The hourly charge-out rate is a common form of billing in most countries. Lawyers, whether on their own or in large firms, tend to keep precise time sheets to determine billable hours.

10.277 ***Fixed fees.*** Fixed fees are common for more routine legal matters, such as drawing up a simple will, closing a title on a house, or patent registration. A fixed rate is typically employed when both the time and staffing level needed to complete the project are known in advance, and thus an analysis of relevant charge-out rates would be the basis for the fixed-fee schedule.

10.278 ***Ad valorem pricing.*** Ad valorem pricing is the term used when the price is a proportion of the value of the subject of the legal work; for example, the value of a property being conveyed or the amount recovered in court action. For the second example, the actual fee is subject to risk and may have little relation to hourly charge-out rates, since an amount is payable only if the case is won, and this may not bear a strong relationship to the time spent working on the case.

10.279 Determining product specifications for the price of legal services is difficult because each legal case involves a different mix of professional staff levels and can involve different combinations of component services. This absence of product standardization complicates price statisticians' ability to track the price for the same service provided over time. For lawsuits, especially when ad valorem pricing is used, the concept of a constant quality service is particularly difficult to define because every case is different, and the price for the service depends on the outcome of the case. Despite these problems, legal service organizations tend to keep detailed records on each client, in terms of the type of work performed and time spent by level of staff member. Obtaining access to such records can greatly assist in arriving at a suitable pricing method for surveyed businesses.

10.280 Three main methodologies are used by countries currently pricing legal services: repricing of fixed fees, charge-out rates, and model pricing.

10.281 ***Repricing of fixed fees.*** There are a number of services within the legal industry for which fixed fees, or fees based on some form of scale, apply. Identifying major revenue-earning items for which this form of pricing applies, and tracking a sample of fixed fees charged by respondents, can be an effective way of representing price change for a portion of the legal services industry. Examples of the types of services that can be priced using this approach are

- Preparation of simple wills,
- Settling simple divorces,
- Patent registration (for a range of patent services such as standard patents, trademarks, designs, etc.),
- Registration fees for property, and
- Transfer of real estate.

10.282 In the latter examples, the price may be dependent on the value of the item being registered or transferred. In such cases, it is important to keep this value constant when repricing the fees associated with these activities, for example, fees applying to the transfer of a property worth a certain fixed amount (for example, $200,000), depending on the most representative value(s) for the relevant transaction(s) applicable to the country. These representative values should be subject to some form of indexation (or possibly a moving average) over time to represent changing prices in the value of items subject to the legal services, since this has a major influence on the actual price change for the associated legal services.

10.283 ***Charge-out rates.*** Respondents could be asked to report a selection of hourly charge-out rates by level of staff for a specific type of project (for example, in servicing a major corporate client, criminal representation), depending on the firm's major revenue-earning activities. The assumption is that changes in hourly charge-out rates will approximate changes in the final charges paid by clients for the various services the firm provides.

10.284 ***Model pricing.*** Specifications are developed (in consultation with legal professionals) for a range of legal services provided by the industry (see examples 10.1–10.5) for the type of the detail required in specifying certain types of legal services for model pricing). These specifications are then sent to respondents each sample period for repricing.

10.285 The fixed-fee method is appealing in that it is relatively low cost and should be effective in pricing to constant quality. It is, of course, important to ensure that the respondent is reporting for the identical service in each period. However, the fixed-fee method may cover only a small proportion of the revenue-earning activities of the legal services industry in most countries and therefore may be of little use if this percentage is very low for the country concerned.

10.286 Proper implementation of a model pricing strategy is likely to result in reliable price indices. However, this method is particularly costly to the statistical organization, because considerable liaison with industry associations and potential respondents is required to set up appropriate models. These also must be maintained over time as the nature of services provided within the industry evolves. Of equal importance is the burden placed on respondents in using this approach. Repricing the precise model each period will be time consuming, and there is the danger that a respondent will refuse to cooperate or not take the exercise seriously (that is, not provide prices that relate to current market conditions).

10.287 Charge-out rates by classification of lawyer tend to be readily available from law firms; they are adjusted to reflect market conditions and form the basis for the prices charged in a large range of legal services. Therefore, charge-out rates can be a relatively cost-effective way (for both the statistical organization and respondents) of measuring price change in the industry. However, the collection of a charge-out rate schedule may not adequately account for the impact on prices resulting from changes in labor productivity within the industry (for example, when the amount of labor inputs required for legal services in general may decline because of more effective use of technology).

10.288 It should be noted that a similar approach could be adapted to many types of business services, such as accountancy and advertising.

10.289 Although the method presented above does provide a means for monthly repricing, the method does not overcome the difficulty of maintaining constant quality. This difficulty in part derives from the custom nature of the services being provided.

Example specifications for model pricing of legal services from Australia

Example 10.1 Standard specification for obtaining an injunction

1. You are contacted by the managing director of a client company. He advises that the company has been served with a Section 218 notice and requests an urgent appointment to see you.

2. Preliminary interview with managing director in which he
   a) Gives you a brief outline of the circumstances giving rise to the issue of the notice,
   b) Hands you relevant documents, and
   c) Inquires what steps should be taken.

3. You first check the Section 218 notice to see when the time expires after which the creditor can issue a statement of claim.

4. Having ascertained the date, you then advise the managing director that, with the short time available, you will study the documents and give him your advice shortly.

5. Over the ensuing 48 hours you spend time studying the documents and then write to the solicitors who issued the Section 218 notice. The purpose of this letter is
   a) To point out that the debt is disputed and therefore a Section 218 notice is inappropriate;
   b) To advise them that, in any event, your client has a claim that exceeds the amount of the disputed debt;

c) To ask them whether, in view of the above, they still intend proceeding; and
d) To advise them that unless you hear within seven days that they do not intend proceeding, you will apply to the court for an injunction.

6. Within seven days you receive a reply advising that they do not consider your client's position to be financially strong, that they do not consider that the debt is genuinely disputed, and that they do intend proceeding if the money is not paid or security given.

7. Following this, there is a lengthy consultation with the managing director to prepare the documentation necessary to obtain an injunction. As it is necessary to persuade the court that the dispute over the debt is a matter of substance and not simply a smokescreen designed to put off the evil day, full details of the dispute must be given.

8. The following documents are then prepared for filing with the court:
   a) Statement of claim and notice to defendant asking for
      i) A declaration that the alleged debt is not owing,
      ii) Damages for breach of contract, and
      iii) Costs.
   b) An ex parte application for injunction restraining the defendant company from issuing or advertising a winding-up petition on the grounds that
      i) The debt is disputed,
      ii) There is a claim that exceeds the amount of the disputed debt, and
      iii) The presentation and advertising of the petition will do irreparable harm to the plaintiff company.
   c) An affidavit from the managing director of the plaintiff company setting out full details of the disputed debt and its background.

9. These documents are then filed with the court. The motion is ex parte because of the urgency of the situation and the fact that when the proceedings are filed, the time for the expiration of the notice almost has arrived. If there were ample time, then the motion would inevitably have to be served on the other party.

10. In the first instance, the papers are placed before the judge, who makes an order after perusing the papers. The judge does not call counsel for further clarification of the issues.

11. You arrange to have the order sealed in the court and served on the defendant company forthwith to prevent the statement of claim for winding-up proceeding.

Example 10.2 Standard specification for registration of debenture

Assume: Your client company is asked to give a debenture to its trading bank to see fluctuating overdraft and other banking accommodation up to $25,000. The bank's debenture is to take priority over an existing debenture to a finance company securing a $10,000 fixed item of plant.

1. Receive letter of instruction from lending institution. Check
   a) Level of accommodation and
   b) Interest and finance rate (servicing ability).

2. Search company, including Memorandum and Articles. Check that company has power to give such a security. Obtain full details of prior charge.

3. Confirm lending institution's instructions and conditions with client.

4. Peruse trading bank's debenture (standard form).

5. Complete details in debenture form.

6. Prepare Declaration of Due Execution and company resolutions, including Disclosures of Interest. Ensure that the same conform with the company's Articles of Association.

7. Prepare Deed of Modification of Priority and obtain confirmation form on debenture holders to the terms thereof.

8. Attendances on execution of the following documents:
   a) Resolutions incorporating appropriate Disclosures of Interest,
   b) Debenture,

c) Declaration of Due Execution, and
d) Deed of Modification of Priority.

9. Attendances on Disclosure.

10. Arrange noting of appropriate insurance policy.

11. Forward priority document to other debenture holder for execution and return.

12. Register a copy of debenture with the Companies Office.

13. Forward certificates to lending institutions to confirm compliance with requirement and request drawdown of funds.

14. Report to client.

15. Uplift funds and disburse.

16. Attend to stamping of Deed of Priority.

17. Forward final solicitor's certificate to lending institutions together with debentures, security documents, including Section 105 certificate, Deed of Priority, and insurance policy.

18. Final report to client.

Example 10.3 Standard specification for estate administration

Work involved in approximate chronological order

1. Preliminary interview with executor, discussion of terms of the will, the nature and approximate length of time of the administration of the estate, and likely cost.

2. Written report to executor, supplying copy of will and summary of administration.

3. Drafting affidavit to Lead Grant of Probate, Affidavit of Death where necessary, Notice of Application to the court for Grant of Probate.

4. Attendance on executor and having affidavits sworn.

5. Filing application and affidavit(s) at High Court Registry.

6. Preparing and sending letters to post bank, life insurance company, and trading bank seeking particulars of derived and accrued interest for tax purposes and amount payable to estate.

7. Receiving responses form them.

8. Searching certificate of title to house property.

9. Obtaining two death certificates (and birth certificate if age is not admitted by the insurance company).

10. Receiving minute of court's order on application for probate, preparing formal Grant of Probate, and declaration in value of estate.

11. Sealing Grant of Probate and filing declaration at High Court Registry.

12. Preparing Transmissions by Survivorship of jointly owned house property.

13. Preparing taxation returns to date of death.

14. Reporting to executor, supplying Schedule of Assets and Liabilities.

15. Attending on widow/er for execution of transmission.

16. Advising rates authority/Valuation Department, insurance company, electrical supply authority, and Telecom about telephone of transfer of joint house property to widow/er.

17. Attending at Land Transfer Office with transmission (death certificate annexed) and certificate of title for registration.

18. Receiving release of probate from High Court.

19. Completing life insurance discharge, specimen signature, and withdrawal forms for post bank and trading bank accounts.

20. Arranging execution of discharge and withdrawal forms by the executor.

21. Forwarding life insurance policy, death certificate (and birth certificate) discharge, and probate to life insurance company requesting payment.

22. Receiving return of probate from life insurance company together with check in settlement. Arranging for payment into trust account.

23. Forwarding specimen signature and withdrawal forms and probate to trading bank and arranging for closing of account, final bank statements, and payment.

24. Receiving return of probate from trading bank and check in settlement. Arranging for payment of check into trust account.

25. Forwarding specimen signature and withdrawal forms and probate to post bank and requesting payment to the estate.

26. Receiving return of probate from post bank together with check in settlement. Arranging for payment into trust account.

27. Payment of debts, including funeral expenses.

28. Reporting to executor and beneficiary and arranging payment of interim distribution to beneficiary if required.

29. Receiving, checking, and paying assessment from Inland Revenue Department in respect of taxation return to the date of death.

30. Preparing final estate accounts.

31. Preparing a trustee's tax returns from date of death to date of distribution.

32. Final report to executor and beneficiary supplying final statements.

33. Attendance on executor to discuss final accounts and make payment of the balance held in trust.

34. Sundry telephone attendances (say, five) during administration.

Example 10.4 Standard specification for incorporation

Assume: You are consulted by a husband and wife who have purchased a suburban bookstore. They wish to operate the business as a limited liability company with a nominal capital of $10,000. The husband and wife wish to be the shareholders and directors, and they wish their accountant to be the secretary.

1. Preliminary discussion canvassing
   a) Reason for incorporation,
   b) Concept of limited liability,
   c) Level of paid-up capital and reasons why,
   d) Selection of proposed name,
   e) Shareholders, Directors, and Secretary,
   f) Type of business,
   g) Registered office, and
   h) Bankers.

2. Name approval. Forward application for name approval to Registrar of Companies, Wellington, including disbursement.

3. Receive name approval.

4. Draft company documents, including
   a) Articles of Association,
   b) Memorandum of Association,
   c) Notice of Situation of Registered office,
   d) Particulars of Directors and Secretary,
   e) Consents to Act as Director,
   f) Consents to Act as Secretary, and
   g) Minutes of first meeting of Directors.

5. Order Common Seal.

6. Collect funds from clients to cover disbursements, including approval fee, registration costs, and Common Seal.

7. Attendances pertaining to explanation of Articles, Memorandum, powers and rights pursuant thereto, obligations of officers, accounting method, opening of banking accounts, and payment of capital.

8. Attendances on execution of documents.

9. Submit documents to Companies Office for registration.

10. Receive advice as to incorporation.

11. Report and account to clients and advise of incorporation and forward incorporation documents, including
    a) Certificate of Incorporation,
    b) Articles of Association,
    c) Memorandum of Association, and
    d) Minutes of first meeting of Directors.

12. Forward minutes of first meeting to Directors.

13. Forward copies of Articles and Memorandum of Association to nominees.

Example 10.5 Standard specification for a traffic offense

1. Preliminary interview—charge of dangerous driving against stock agent involving a noninjury collection. Instructed to defend the charge.

2. Attending at court when information adjourned for a defended hearing.

3. Interviewing defense witness (passerby) and defendant to prepare a hearing.

4. Speaking to Ministry of Transport to obtain details of prosecution evidence. If necessary, making a request under the Official Information Act 1983.

5. Attending at court to conduct defended hearing in which traffic officer in charge of prosecution and other driver give evidence for prosecution, as well as defendant and passerby for defendant. Total time involved at court: one and a half hours.

6. Preparing application for limited license involving affidavits from defendant and employer representative.

7. Attending at court to obtain limited license against opposition from Ministry of Transport.

P. General Medical Hospitals, ISIC 8511[14]

10.290 Establishing a price index for health services is challenging because of the complexity of measuring service output.

10.291 Many countries do not encounter the problem because hospital expenditures are part of government expenditures and usually valued at factor cost.

10.292 However, for countries like the United States, which have privately and publicly provided health services, it becomes necessary to price them. The U.S. PPI program has developed a price index for hospital services, described below.

10.293 The primary output of the hospital industry is the complete service patients receive during their stay or visit to the hospital. The hospital's output is represented by the full content of the patient bill. Any items or services included on the patient bill were treated as part of the output and were included in our repricing effort. This output is classified in one of two ways:

- Inpatient treatments and
- Outpatient treatments.

10.294 For an inpatient, the output is obtained using all items or services rendered during the patient's length of stay (that is, admission to discharge). These items or services may include room, board, medical supplies, drug treatments, medical and surgical procedures, or ancillary services.

10.295 For an outpatient, there is not an actual admission to the hospital (that is, length of stay is zero), therefore, the services an outpatient receives will occur on a single visit to the hospital. Outpatient services may include treatments for minor injuries, minor surgical procedures, or ancillary services.

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[14]For further information regarding the U.S. PPI for General Medical and Surgical Hospitals, see Catron and Murphy (1996). This article describes the original survey sample and design, analyzes hospital price inflation as measured by the PPI, summarizes the results, and briefly compares the hospital industry measures of the PPI with those of the CPI.

10.296 There is no distinction between the inpatient and outpatient services below the industry level.

10.297 After the hospitals were selected, the hospital services needed to be identified for collection. Because of the endless combinations of hospital services, a method was devised to eliminate a time-consuming process of item selection at the hospital. Instead, the services to be priced were preselected using data from the U.S. Agency for Healthcare Research and Quality.

10.298 The following characteristics were used in preselecting each service:

- Type of Patient (inpatient or outpatient),
- Type of Payer (Medicare, Medicaid, Commercial Insurance, etc.), and
- Assigned Diagnosis-Related Group (DRG) (for inpatients only).

10.299 DRG is a coding system in which patient categories are defined by diagnoses or procedures and modified by age, complications, coexisting conditions, or discharge status. Each DRG groups like patients with like ailments and anticipates the level of care required during hospitalization. DRGs are prospective in nature, in that they are based on expected costs rather than actual costs.

10.300 For each inpatient stay at a hospital, patients are assigned one of 497 DRGs. For example, a patient may be assigned DRG 127 *Heart Failure and Shock,* depending on what the principal diagnosis and procedures are. Other factors such as complications, comorbidities, age, and discharge status also play a role in DRG assignment. The DRG, along with diagnosis and procedures, will be listed on the patient bill. However, the payment a hospital receives may or may not be based on the assigned DRG. The payment will depend on the payer and the type of reimbursement. Both are covered in the next section.

10.301 This output is represented by the full content of the patient bill. Each hospital sampled for the PPI was asked to provide a representative patient bill for each of the preselected services. For each patient bill selected, the information necessary for pricing purposes was recorded (payer information, diagnosis information, reimbursement, etc.).

10.302 Because of the importance of third-party payers, the public and private insurers, it is important to distinguish between the price and reimbursement within the hospital industry. The term price usually refers to the total charges that appear on the patient bill. Reimbursement would be the actual amount that the hospital receives as payment.

10.303 What a hospital charges and what it receives are usually two very different amounts. The PPI program is interested in what a hospital actually receives (reimbursement) for its services, not what it charges (price). Differences arise from many sources, but chiefly, discounted prices for various services are frequently negotiated with third-party payers. Thus, the PPI's primary purpose is to capture reimbursement as the net transaction price.

10.304 The most common types of reimbursement for hospitals are per diem rates, DRG/case rates, and percentage of total billed charges. These are not all inclusive, and many methods may be used. However, these three, or variants of them, are seen in the majority of cases.

10.305 The simplest reimbursement method for a hospital is total billed charges. However, it is rarely used. In most cases, a percentage of total billed charges is paid. This percentage is negotiated before services are rendered and is often in effect for a year or more for a given covered population.

10.306 Per diem rates also are very common. This type of reimbursement involves a per day payment for each day of stay in the hospital, regardless of actual charges or costs incurred. This per day rate depends on a number of factors, the two main ones being the number and mix of cases. Many times, multiple sets of per diem rates will be negotiated on the basis of service type (for example, medical-surgical, obstetrics, intensive care, neonatal intensive care, rehabilitation). The per diem rate is multiplied by the length of stay to calculate the total reimbursement. As with DRGs, the hospital keeps any overpayment but has to absorb any underpayment.

10.307 The fundamental difficulty in measuring price changes in the hospital industry is that no identical transactions occur for each repricing period. A patient generally does not repeatedly visit the hospital for the same episode of an illness or ailment. As such, each patient stay or visit to a hospital can be defined as a custom service.

10.308 Each patient represents a unique combination of age, gender, lifestyle, sensitivity to drugs, allergies, medical history, genetics, mental attitude, and so forth.

10.309 Actual treatment paths, as represented by the randomly selected patient bills, formed the basis of the repricing effort. These treatment paths cannot be directly observed in subsequent months (as stated above); however, the hospitals are able to report reimbursement based on identical inputs (payer, diagnosis, length of stay, etc.). This procedure removes any price variability resulting from a direct comparison of different patient bills.

10.310 Another issue is new-item bias. This occurs when repricing is based on inputs that are not current. Over time, treatment guidelines and protocols change. In addition, some hospital services change from being treated as an inpatient to an outpatient or vice versa.

10.311 The PPI program will try to overcome this problem by periodically evaluating the most current, widely accepted treatment protocols for a select set of DRGs. By utilizing data from an outside source, if it is determined that a new or alternate treatment has become prevalent and has begun to replace an old method, then particular items with the old treatment method will be replaced with the newer method. The proportion of new or alternate procedures introduced in our sample will reflect that of the population as a whole. This procedure should allow the index to reflect the most current treatment problems.

10.312 An obvious issue with the repricing of complex services such as health care is quality adjustment. Because objective measures of quality change are difficult to construct, it may be tempting to use changes in resource costs as an estimate (see the discussion in Chapter 7). However, it is much more difficult to draw a relationship between resource costs and quality change for services in which quality has a significant subjective component.

Figure 10.1. Sample Worksheet

WIRELESS TELECOMMUNICATIONS (EXCEPT PAGING) WORKSHEET

PART I: AVERAGE UNIT PER ACCESS LINE

List all types of charges assessed by company for the selected area in column 1. Enter the total number of units for each type of charge in column 2. Enter the total number of access lines in column 3. Divide column 2 by column 3 and enter in column 4. The reporter may be reluctant to provide data for columns 2 and 3. If the reporter will calculate the percentages, it is only necessary to fill out columns 1 and 4.

Column 1	Column 2	Column 3	Column 4 (Column 2/ Column 3)
Type of Charge	**Total Units: Billed and Free**	**Total Number of Access Lines**	**Average Number per Access Line**
ACCESS LINE			
1.0000*	*BY DEFINITION		
USAGE CHARGE BASED ON TIME			
Peak minutes	32,400,000		162
Off-peak minutes	26,600,000	200,000	133
Roaming minutes	2,000,000	200,000	10
Landline minutes			
Other charges			
USAGE CHARGES OTHER THAN TIME			
Landline, per call	400,000	200,000	2
Other charges, Daily rate	200,000	200,000	1
FEATURES/OPTIONS AND FEATURE PACKAGES			
Custom calling package	130,000	200,000	0.65
Call waiting	40,000	200,000	0.20
Call forwarding	20,000	200,000	0.10
3-way conference	10,000	200,000	0.05

No answer transfer	20,000	200,000	0.10
Voice messaging	40,000	200,000	0.20

WIRELESS TELECOMMUNICATIONS (EXCEPT PAGING) WORKSHEET

PART II: AVERAGE REVENUE PER UNIT
Copy all the charges in Part I, column 1, to Part II, column 1. Obtain the net billed revenues for each type of charge and divide by the total quantity used of each charge. **OR**
The reporter may be reluctant to provide data for columns 2 and 3. If the reporter will calculate the average revenue, it is only necessary to fill columns 1 and 4.

Column 1	**Column 2**	**Column 3**	**Column 4 (Column 2/ Column 3)**
Type of Charge	**Total Net: Billed Revenue**	**Total Units: Billed and Free**	**Average Revenue per Unit**
ACCESS LINE	5,350,600	200,000	26.7530
USAGE CHARGE BASED ON TIME			
Peak minutes	8,388,360	32,400,000	0.2589
Off-peak minutes	2,191,840	26,600,000	0.0824
Roaming minutes	1,944,400	2,000,000	0.9722
Landline minutes			
Other charges			
USAGE CHARGES OTHER THAN TIME			
Landline, per call	60,000	400,000	0.1500
Other charges,			

Daily rate	300,000	200,000	1.5000

FEATURES/OPTIONS AND FEATURE PACKAGES

Custom call package	449,800	130,000	3.4600
Call waiting	194,000	40,000	4.8500
Call forwarding	103,000	20,000	5.1500
3-way conference	57,500	10,000	5.7500
No answer transfer	85,000	20,000	4.2500
Voice messaging	192,000	40,000	4.8000

WIRELESS TELECOMMUNICATIONS (EXCEPT PAGING) WORKSHEET

PART III: COMPUTE AVERAGE REVENUE BILL

Copy all the types of charges in Part I, column 1 to Part III, column 1. Copy average number per access line from Part I, column 4, to column 2. Copy average revenue per unit from Part II, column 4. Multiply column 2 by column 3 and enter in column 4. Sum column 4 to base period total or "price."

Column 1	Column 2	Column 3	Column 4 (Column 2 X Column 3)
Type of Charge	Average Number per Access Line (Part I, Col. 4)	Average Revenue per Unit (Part II, Col.4)	Weighted Revenue
ACCESS LINE	1.000	26.7530	26.7530
USAGE CHARGE BASED ON TIME			
Peak minutes	162	0.2589	41.9418
Off-peak minutes	133	0.0824	10.9592
Roaming minutes	10	0.9722	9.7220
Landline minutes			

Other charges			
USAGE CHARGES OTHER THAN TIME			
Landline, per call	2	0.1500	0.3000
Other charges, Daily rate	1	1.5000	1.5000
FEATURES/OPTIONS AND FEATURE PACKAGES			
Custom calling package	0.65	3.4600	2.2490
Call waiting	0.20	4.8500	0.9700
Call forwarding	0.10	5.1500	0.5150
3-way conference	0.05	5.7500	0.2875
No answer transfer	0.10	4.2500	0.4250
Voice messaging	0.20	4.8000	0.9600
BASE-PERIOD TOTAL			97.0686

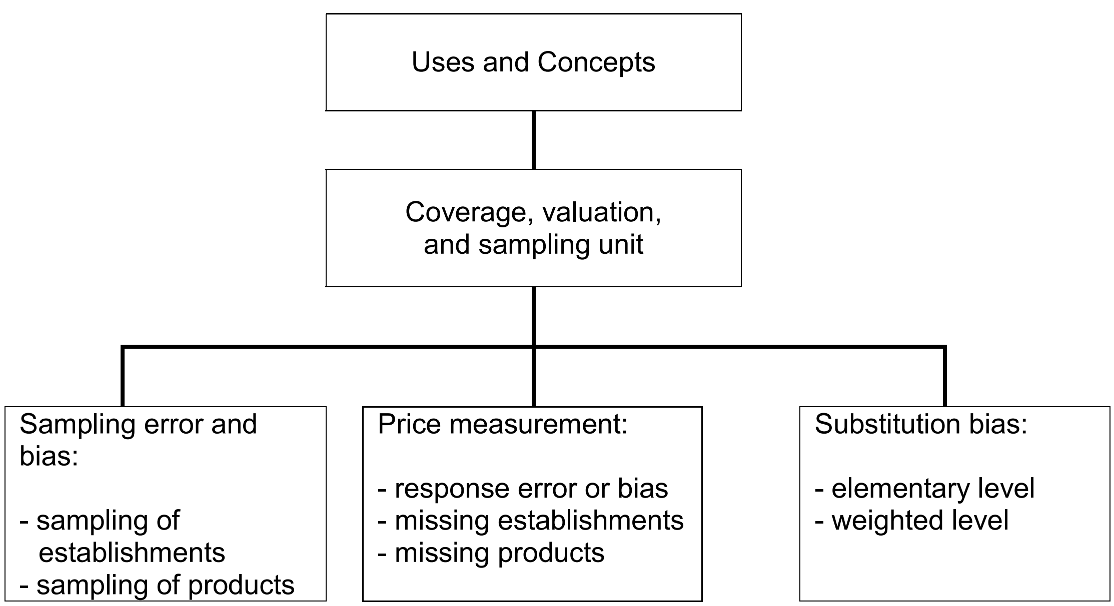
11. Errors and Bias in the PPI

A. Introduction

11.1 A number of sources of error and bias have been discussed in the preceding chapters and will be discussed again in subsequent chapters. The purpose of this chapter is to briefly summarize such sources to provide a readily accessible overview. Both conceptual and practical issues will be covered. To be aware of the limitations of any PPI, it is necessary to consider what data are required, how they are to be collected, and how they are to be used to obtain overall summary measures of price changes. The production of PPIs is not a trivial task, and any program of improvement must match the estimated cost of a potential improvement in accuracy against the likely gain. In some instances, one may have to take into account the user requirements necessary to meet specific needs or engender more faith in the index, in spite of the relatively limited gains in accuracy matched against their cost.

11.2 Figure 11.1 outlines the potential sources of error and bias in PPIs. The distinction between errors and bias is, however, first considered in Section B. In sampling, for example, the nature of the sample design (for example, the use of cutoff sampling—see Chapter 5) may *bias* the sample toward larger establishments whose average item price changes are below the average of all establishments. In contrast, an unrepresentative sample with

Figure 11.1. Outline of Sources of Error and Bias



disproportionate larger establishments may be selected by chance and similarly include item prices that are, on average, below those of all establishments. This is *error* since it is equally likely that a sample might have been selected whose average price change was, on average, above those of all establishments

11.3 The discussion of bias and errors first requires consideration of the conceptual framework on which the PPI is to be based and the PPI's related use(s). This will govern a number of issues, including the decision as to the coverage or domain of the index and choice of formula. Errors and bias may arise if the coverage, valuation, and choice of the sampling unit fail to meet a conceptual need; this is discussed in Section C. Section D examines the sources of errors and bias in the sampling of transactions. The sampling of item prices for a PPI is undertaken in two stages: sampling of establishments and the subsequent sampling of items produced (or purchased) by those establishments. Bias may arise if establishments or items are selected with, on average, unusual price changes, possibly due to omissions in the sampling frame or a biased selection from the frame. Sampling error, as discussed previously and in Chapter 5, can arise even if the selection is random from an unbiased sampling frame and will increase as the sample size decreases and as the variance of prices increases. Sampling error arises simply because an estimated PPI is based on samples and not a complete enumeration of the populations involved. The errors and biases discussed in Section D are for the sample on initiation. Section E is concerned with what happens to sampling errors and bias in subsequent matched price comparisons.

11.4 Once the sample of establishments and their items has been selected, the sample will become increasingly out of date and unrepresentative as time progresses. The extent and nature of any such bias will vary from industry to industry. The effect of these dynamic changes in the universe of establishments and the items produced on the static, fixed sample are the subject of Section E. Sample rotation will act to refresh the sample of items, while rebasing may serve to initiate a new sample of items *and* establishments. Establishments will close, and items will no longer be produced on a temporary or permanent basis. Sample augmentation and replacement aid the sampling of establishments, although replacement occurs only when an establishment is missing. Sample augmentation tries to bring into the sample a new major establishment. It is a more complicated process because the weighting structure of the industry or index has to be changed (Chapter 8). When item prices are missing, the sampling of items may become unrepresentative. Imputations can be used, but they do nothing to replace the sample. In fact, they lower the effective sample size, thereby increasing sampling error. Alternatively, comparable replacement items or replacements with appropriate quality adjustments may be introduced. As for new goods providing a substantively different service, the aforementioned difficulties of including new establishments extend to new goods, which are often neglected until rebasing. Even then, their inclusion is quite problematic (Chapter 8).

11.5 The discussion above has been concerned with how missing establishments and items may bias or increase the error in sampling. But the normal price collection procedure based on the matched-models method may have errors and bias as a result of the prices collected and recorded being different from those transacted. Such response errors and biases, along with those arising from the methods of treating temporarily and permanently missing items and goods, are outlined in Section F as errors and bias in price measurement. Section F is concerned with deficiencies in methods of replacing missing establishments and items so that the matched-models method can continue, while Section E is concerned with the effect of such missing establishments and items on the efficacy of the sampling procedure.

11.6 The final source of bias is substitution bias. Different formulas, as shown in Chapters 15 through 17, have different properties and replicate different effects depending on the weighting system used and the method of aggregation. At the higher level, where weights are used, substitution effects were shown to be included in superlative formulas but excluded in the traditional Laspeyres formula (Chapter 15). Similar considerations were discussed at the lower level. Whether it is desirable to include such effects depends on the concepts of the index adopted. A pure fixed-base period concept would exclude such effects, while an economic cost-of-living approach (Chapters 17 and 20) would include them. The concepts in Figure 11.1 can be used to address definitional issues such as coverage, valuation, and sampling, as well as price measurement issues such as quality adjustment and the inclusion of new goods and establishments.

11.7 It is worthwhile to list the main sources of errors and bias:

(i) Inappropriate coverage and valuation (Section C);
(ii) Sampling error and bias, including
  a) Sample design on initiation (Section D), and
  b) Effect of missing items and establishments on sampling error (Section E);
(iii) Matched price measurement (Section F), including
  a) Response error/bias,
  b) Quality adjustment bias,
  c) New goods bias, and
  d) New establishments bias; and
(iv) Formula (substitution) bias (Section G), including
  a) Upper-level item and establishment substitution, and
  b) Lower-level item and establishment substitution.

11.8 It is not possible to judge which sources are the most serious. In some countries and industries, the increasing differentiation of items and rate of technological change make it difficult to maintain a sizable, representative matched sample, and issues of quality adjustment and the use of chained or hedonic indices might be appropriate. In other countries, a limited coverage of economic sectors where the PPI is used might be the major concern. Inadequacies in the sampling frame of establishments might also be a concern.

11.9 There is no extensive literature on the nature and extent of errors and bias in PPI measurement, Berndt, Griliches, and Rosett (1993) being a notable exception. However, there is substantial literature on errors and bias in CPI measurement, and Diewert (1998a and 2002c) and Obst (2000) provide a review and extensive reference list. Much of this literature includes problem areas that apply to PPIs as well as CPIs.

B. Errors and Bias

11.10 In this section, a distinction is made between *error* and *bias*. The distinction is most appropriate to the discussion of sampling, although the same framework will be shown to apply to nonsampling errors and bias. Yet an error or bias can also be discussed in terms of how an existing measure corresponds to some true concept of a PPI and will vary depending on the concept advocated, which in turn will depend on the use(s) required of the measure. These issues are discussed in turn.

B.1 Sampling error and bias

11.11 Consider the collection of a random *sample* of prices whose overall population average (arithmetic mean) is μ.[1] The *estimator* is the method used for estimating μ from sample data. An appropriate estimator for μ is the mean of a *sample* drawn using a random design. An *estimate* is the value obtained using a specific sample and method of estimation, let us say $\overline{x_1}$, the sample mean. The population mean μ, for example, may be 20, but the arithmetic mean from a sample of a given size drawn in a specific way may be 19. This *error* may not be *bias*, it may simply be that by chance a random sample was drawn with, on average, below-average prices. If an infinite number of samples were drawn using sufficiently large samples, the average of the $\overline{x_1}$, $\overline{x_2}$, $\overline{x_3}$, ... sample means would in principle equal μ. The estimator is said to be *unbiased*; if it is not, it is called *biased*. The error caused by $\overline{x_1}$ being different from μ = 20 did not arise from any systematic under- or over-estimation in the way the sample was drawn and the average calculated. If an infinite number of such estimates were drawn and summarized, no error would be found, the estimator not being biased and the discrepancy being part of the usual expected sampling error.[2]

[1]The discussion is in terms of prices and not price *changes* for simplicity.

[2]This is *sampling error,* which can be estimated as the differences between upper and lower bounds of a given probability, more usually known as *confidence intervals*. Methods and principles for calculating such bounds are explained in Cochran (1963), Singh and Mangat (1996), and most introductory statistical texts. Moser and Kalton (1981) (continued)

11.12 It should be stressed that any one sample may give an inaccurate result, even though the method used to draw the sample and calculate the estimate is, *on average*, unbiased. Improvements in the design of the sample, increases in the sample size, and less variability in the prices (more detailed price specifications for the price basis) will lead to less error, and the extent of such improvements in terms of the sample's probable accuracy is measurable. Note that the accuracy of such estimates is measured in principle by confidence intervals, that is, probabilistic bounds in which μ is likely to fall. Smaller bounds at a given probability are considered to be more *precise* estimates. It is in the interest of statistical agencies to design their sample and use estimators in a way that leads to more precise estimates.

11.13 The calculation of such intervals requires a measure of the variance of a PPI in which all sources of sampling error are caught. However, the sampling of prices involves sampling of establishments and items, and probabilistic methods generally are not used at each stage. Judgmental and cutoff methods are often considered to be more feasible and less resource intensive. Estimates of the variance, however, require probabilistic sample designs at all stages. Yet it is feasible to develop partial (conditional) measures in which only a single source of variability is quantified (see Balk and Kerston, 1986, for a CPI example). Alternative methods for nonprobability samples are discussed in Särndal, Swensson, and Wretman (1992).

11.14 *Efficiency* gains (smaller sampling errors) may be achieved for a given sample size and population variance by using better *sample designs* (methods of selecting the sample) as outlined in Chapter 5. Yet it may be that the actual selection probabilities deviate from those specified in the sample design. Errors arising from such deviations are called *selection errors*.

11.15 While an unbiased estimator may give imprecise results, especially if small samples are used, a biased estimator may give quite precise results. Consider the sampling from only large establishments. Suppose such prices were, on average, less than μ, but assume these major establishments covered a substantial share of the revenue of the industry concerned, then the mean of the estimates from all such possible samples $\underline{m}$ may be quite close to μ, even if smaller establishments had different prices. However, the difference between m and μ would be of a systematic and generally predictable nature. On average, m would exceed μ, the bias[3] being $(\mu - m)$.

provide a good account of the different types of errors and their distinction.

B.2 Nonsampling error and bias

11.16 The above framework for distinguishing between errors and biases is also pertinent to nonsampling error. If, for example, the prices of items are incorrectly recorded, a response error results. If such errors are *unsystematic*, then prices are overrecorded in some instances but, counterbalancing this, underrecorded in others. Overall, errors in one direction should cancel out those in the other, and the net error, on average, will be expected to be small. If, however, the establishments selected and kept in the sample are older and produce at higher (quality-adjusted) prices than their newer, high-technology equivalent establishments, then there is a systematic bias. The results are biased in the sense that if an infinite number of similar random samples of older establishments were taken from the population of establishments, the average or expected value of the results would differ from the true population average, and this difference would be the bias. The distinction is important. Increasing the sample size of a biased sample, of older establishments for example, when samples are rebased reduces the error but not the bias.

11.17 This distinction between errors and bias is for the purpose of *estimation*. When using the results from a sample to estimate a population parameter, both error and bias affect the accuracy of the results. Yet there is also a distinction in the statistical literature between types of errors according to their *source*: sampling versus nonsampling (response, nonresponse, processing, etc.) error. Although they are both described as errors, the distinction remains that if their magnitude cannot be estimated from the sample itself, they are biases, and some estimate of μ is required to measure them. If they can be estimated from the sample, they are errors.

[3]Since μ is not known, estimates of sampling error are usually made; they are but one component of the variability of prices around μ.

B.3 Concepts of a true or good index

11.18 The discussion of errors and bias so far has been in terms of estimating μ as if it were the required measure. This has served the purpose of distinguishing between errors and bias. However, much of the *Manual* has been concerned with the choice of an appropriate index number formula. It is now necessary to consider bias in terms of the difference between the index number formula and methods used to calculate the PPI and some concept of a true index. In Chapter 17, *true* theoretical indices were defined from economic theory. The question is, if producers behave as optimizers and switch production toward products with relatively high price increases, which would be the appropriate formula to use? The result was a number of superlative index number formulas. They did not include the Laspeyres index or the commonly used Young index (Chapter 15), which give unduly low weights to products with relatively high price increases because no account is taken of substitution effects (see Chapter 17). For industries whose establishments behave this way, Laspeyres is biased downward. An understanding of bias thus requires a concept of a true index. According to economic theory, a true index makes assumptions about the nature of economic behavior of industries. These presuppositions dictate which formulas are appropriate and, given these constructs, determine if there is any bias.

11.19 A good index number formula can be defined by axiomatic criteria as outlined in Chapter 16. The Young and Carli indices, for example, were argued to be biased upward since they failed the time reversal test; the product of the indices between periods 0 and 1 and periods 1 and 0 *exceeded* unity.

11.20 In PPI number theory and practice there are quite different conceptual approaches. On the one hand, there is the revenue-maximizing concept defined in economic theory mentioned above. On the other hand, there is the fixed-basket approach.[4] An index based on the latter approach would not suffer, in the strictest sense of the concept, from the biases of substitution (formula) or new goods because the concept is one of measuring the prices of a *fixed* basket of goods. However, it may be argued on the grounds of representativeness that the baskets should be updated and substitution effects incorporated.

[4]A discussion of the debate is in Triplett (2001).

C. Use, Coverage, and Valuation

11.21 Errors and biases can arise from the inappropriate use of a PPI, regardless of the methodology used to compile it. Since price changes can vary considerably from product to product, the value of the price index will depend partly on which products or items are included in the index and how the item prices are determined (Chapter 15, Section B.1). In Chapter 2, different uses of the PPI were mentioned and aligned with different domains and valuation principles. Thus, the discussion of errors and biases starts with a need to decide whether the coverage and valuation practices are appropriate for the purposes required.

11.22 In general terms, a PPI can be described as an index designed to measure either the average change in the price of goods and services as they leave the place of production or as they enter the production process. Thus, producer price indices fall into two clear categories: input prices (at purchasers' prices) and output prices (at basic prices). In Chapter 15, a value-added deflator was described as a further PPI. This is used to deflate the value of a sector or economy, with outputs less the value of the intermediate inputs used to produce the output. First, some major uses are noted, and the domain or coverage of the index is considered. Second, the principles of valuation are reiterated.

C.1 Uses and coverage

11.23 The input PPI is a short-term indicator of inflation. It tracks potential inflation as price pressure builds up and goods and services enter the factory gate. Output PPIs or PPIs at different stages of production show how price pressure evolves up to the wholesaler and retailer. They are *indicators* of *producer* price inflation excluding the effect of price pressure from imports and including that which goes into exports. Separate import and export PPIs should form part of the family of PPIs. There may be a deficiency in the coverage of a PPI. If, for example, an output PPI is restricted to the industrial sector, this is a source of error when examining overall inflation if price changes for other sectors that differ from the industrial sector.

11.24 The PPI indices may be biased when used for national accounts deflation. First, their coverage may be inadequate yet still be used by national accountants. For example, if only a manufacturing PPI is used to deflate industrial output, and price changes from the missing, quarrying, and construction sectors differ in the aggregate from those of manufacturing, there is a bias. The *undercoverage* bias is in the *use* of the index, not necessarily in its construction, although statistical agencies should be sensitive to the needs of users. Second, overcoverage bias means some elements are included in the survey that do not belong to the target population. The bias surfaces if their price changes differ on aggregation from the included ones. Third, the classification of activities for the PPI should be at an appropriately low level of disaggregation, and the system of classification should be the same as that required for the production accounts under the *1993 SNA*. Finally, the use of the Laspeyres PPI formula as a deflator induces a bias, since a Paasche formula is theoretically appropriate (see Chapter 18) for the measurement of changes in output at constant prices. The extent of the bias will also increase as the weights become more out of date.

11.25 Highly aggregated PPIs are used for the macroeconomic analysis of inflation. Certain industries or products with volatile price changes may be excluded. Such indices may be excluded because they introduce substantial sampling error into the aggregate indices, and their exclusion helps with the identification of any underlying trend.

11.26 The preceding discussion has considered the coverage or domain of the index in terms of the activities included. However, such issues also may extend to the geographic scope. The exclusion of establishments in rural areas, for example, may lead to bias if their price changes differ from those in urban areas. Such issues are considered in Sections D and E under *sampling*.

C.2 Valuation

11.27 The valuation of an output PPI is to value output at basic prices with any VAT or similar deductible tax, invoiced to the purchaser, excluded. Such tax revenues go to the government and should be excluded because they are not part of the establishment's receipts. Transport charges and trade margins invoiced separately by the producer should also be excluded. An input PPI should value intermediate inputs with nondeductible taxes included, since they are part of the actual costs paid by the establishment. For input PPIs, changes in the tax procedures—due to a switch to import duties on intermediate inputs, for example—can lead to bias. In such instances, ex-tax or ex-duty indices might be produced. In any event, it is necessary to ensure that establishments treat indirect taxes in a consistently appropriate way, especially when such tax rates fluctuate.

D. Sampling Error and Bias on Initiation

11.28 In Chapter 5, appropriate approaches to sample design were outlined. The starting point for potential bias in sample design is an inadequate sampling frame. It is one of the most pernicious sources of error because the inadequacies of a sampling frame are not immediately apparent to users. Yet a sampling frame biased to particular sizes of establishments or industrial sectors will yield a biased sample irrespective of the probity of the sample selection. Since sampling is generally in two stages—the sampling of establishments and the items within establishments—a sampling frame is required for establishments and for items within establishments. The latter relies on the establishment producing data on the revenues, quantities, and prices (or revenue per unit of output) for the items produced. Any bias here, perhaps because some components produced are priced and recorded at the head office, may lead to bias. It should be kept in mind that even when purposive sampling is used, there is an implicit frame from which the respondent selects items. It should be clear to the respondent what the frame should be.

11.29 The selection of the sample of establishments from the sampling frame should be random or, failing that, purposive. In the latter case, the aim should be to include major items whose price changes are likely to represent overall price changes. Chapter 5 provided a fairly detailed account of the principles and practice of sample selection and the biases that may ensue. The distinction has already been drawn between *bias* and sampling *error,* and the possibility has been raised that unbiased selection will be accompanied by estimates with substantial error, due to high variability in the price (change) data and relatively low sample sizes.

E. Sampling Error and Bias: The Dynamic Universe

11.30 Chapters 7 and 8 also considered sampling issues. Under the matched-models method, prices will be missing in a period if the item is temporarily or permanently out of production. If overall imputations are used to replace the missing prices, the sample size is being effectively reduced and the sampling error increased. In a comparison between prices in period 0 and period *t,* imputation procedures (Chapter 7) ignore the prices in period 0 of items whose prices are missing in period *t*. If such old prices of items no longer produced differ from other prices in period 0, there is a bias due to their exclusion. Similarly, new items produced after period 0, and thus not part of the matched sample, are ignored; if their prices in period *t* differ, on average, from the prices of matched items in period *t*, there is a bias. Sampling error and bias, therefore, may arise due to the exclusion of prices introduced after initiation and dropped when they go missing. This is over and above any errors and bias in the sample design on initiation. Its concern is ensuring that the sample is representative of the dynamic universe.

11.31 As the sample of establishments and items deteriorates, the need for rebasing the index—to update the weights and sample of establishments and items or the rotation of sample items—becomes increasingly desirable. However, these are costly and irregular procedures, and, for some industries, more immediate steps are required. Rebasing and sample rotation are used to improve the sampling of establishments and items. Strategies for dealing with missing establishments and missing prices also have an effect on the sampling of establishments and items. Such strategies involve introducing *replacement* establishments and items that replenish the sample in a more limited way than rebasing and sample rotation. Quality adjustments to prices are required if the replacement establishment or item differs from the missing ones, although this is the concern of price measurement bias in Section F. New establishments and goods may also need to be incorporated into the sample to avoid sampling bias. There is a need in such instances to *augment* the sample. Such augmentation may require a change to the weighting system and, as discussed in Chapter 8, should be undertaken only when the incorporation of major new establishments or goods is considered necessary. Thus, bias in sampling due to differences between the dynamic universe and the static one on initiation may, to some extent, be militated by sample replacement and augmentation (Chapter 8).

11.32 Circumstances may arise in which there is a serious sample deterioration due to missing items as differentiated items rapidly turn over. In such cases, hedonic indices or chaining based on resampling the universe each month was advised in Chapter 7, Section G.

F. Price Measurement: Response Error and Bias, Quality Change, and New Goods

F.1 Response error and bias

11.33 Errors may happen if the reporting or recording of prices is inaccurate. If the errors occur in a systematic manner, there will be bias. The item descriptions that define the price basis should be as tightly specified as is reasonably possible, so that the prices of like items are compared with like. Allowing newer models to be automatically considered comparable in quality introduces an upward bias if quality is improving. Similar considerations apply to improvements in the service quality that accompanies an item. The period to which the prices relate should be clearly indicated, especially where prices vary over the month in question and some average price is required (Chapter 6). Errors in valuation can be reduced by clear statements of the basis of valuation and discussions with respondents if the valuation principles of their accounting systems differ from the valuation required. This is of particular importance when there are changes in tax rates or systems. Diagnostic checks for extremely unusual price changes should be part of an automated quality assurance system, and extreme values should be checked with the respondent and not automatically deleted. Price collectors should visit establishments on initiation and then periodically as part of a quality assurance auditing program (see Chapter 12).

F.2 Quality change bias

11.34 Bias can arise, as discussed in Section E, because newly introduced items do not form part of the matched sample, and their (quality-adjusted) prices may differ from those in the matched sample. This *sampling* bias from items of improved quality and new goods was the subject of Section E. It was also noted that statistical agencies may deplete the sample by using imputation or use replacements to replenish the sample. The concern here is with the validity of such approaches for price measurement, not their effects on sampling bias.

11.35 In Chapter 7, a host of explicit and implicit quality adjustment methods were outlined. From a practical perspective, the quality change problem involves trying to measure price changes for a product that exhibited a quality change. The old item is no longer produced, but a replacement one or alternative is there. If the effect of quality on price is, on average, either improving or deteriorating, then a bias will result if the prices are compared as if they were comparable when they are not. An explicit quality adjustment may be made to the price of either of the items to make them comparable. A number of methods for such explicit adjustments were outlined in Chapter 7, including expert judgment, quantity adjustment, option and production costs, and hedonic price adjustments. If the adjustment is inappropriate, there will be an error, and, if the adjustments are inappropriate in a systematic direction, there will be a bias. For example, using quantity adjustments to price very small lots of output, for which customers pay more per unit for their convenience, would yield a biased estimate of the price adjustment due to quality change (Chapter 7, Section E.2).

11.36 There are also implicit approaches to quality adjustment. These include the overlap approach overall and targeted mean imputation; class mean imputation; comparable substitution, spliced to show no price change; and the carryforward approach. Imputations are widely used, whereby the price changes of missing items are assumed to be the same as those of the overall sample or some targeted group of items. Yet such approaches increase error through the drop in sample size and may lead to bias if the items being dropped are at stages in their life cycle where their pricing differs from that of other items. Such bias is usually taken to overestimate price changes (Chapter 7, Section D).

11.37 The choice of appropriate quality adjustment procedure was argued in Chapter 7 to vary among industries to meet their particular features. There are some products, such as consumer durables, materials, and high-technology electronic products, in which the quality change is believed to be significant. If such products have a significant weight in the index, overall bias may arise if such changes are ignored or the effects of quality change on price is mismeasured. Whichever of the methods are used, an assumption is being made about the extent to which any price change taking place is due to quality; bias will ensue if the assumption is not valid.

F.3 New-goods bias

11.38 Over time, new goods (and services) will appear. These may be quite different from what is currently produced. An index that does not adequately allow for the effect on prices of new goods may be biased. Introducing new goods into an index is problematic. First, there will be no data on weights. Second, there is no base-period price to compare the new price with. Even if the new good is linked into the index, there is no (reservation) price in the period preceding its introduction to compare with its price on introduction. Including the new good on rebasing will miss the price changes in the product's initial period of introduction, and it is in such periods that the unusual price changes are expected if the new good delivers something better for a given or lower price. Similar considerations apply to new establishments (Section G.4). New-goods and new-establishment bias is assumed to overstate price changes, on average.

F.4 Temporarily missing bias

11.39 The availability of some items fluctuates with the seasons, such as fruits and vegetables. A number of methods are available to impute such prices during their missing periods. Bias has been shown to arise if inappropriate imputation approaches are used. Indeed, if seasonal items constitute a large proportion of revenue, it is difficult to give meaning to month-on-month indices, although comparisons between a month and its counterpart in the next year will generally be meaningful (see Chapter 22).

G. Substitution Bias

11.40 Given the domain of an index and the valuation principles, the value of the revenue accruing to the establishment can be compared over two periods, let us say, 0 and 1. It is shown in Chapter 15 that the change in such values between periods 0 and 1 can be broken down into two components: the overall price and overall quantity change. An index number formula is required to provide an overall, summary measure of the price change. In practice, this may be undertaken in two stages. At the higher level, a weighted average of price changes (or change in the weighted average of prices) is compiled with information on revenues (quantities) serving as weights. At the lower level, the summary index number formulas do not use revenue or quantity weight, and use only price information to measure the elementary aggregate indices of average price changes (or changes in average prices). It is recognized that in many cases, only weighted calculations are undertaken. Five approaches were used in Chapters 15 through 17 to consider an appropriate formula at the higher level, a similar analysis being undertaken for lower-level elementary aggregate indices in Chapter 20.

G.1 Upper-level substitution bias

11.41 Different formulas for aggregation have different properties. At the upper-weighted level, substantial research from the axiomatic, stochastic, Divisia, fixed-base, and economic approach has led to an understanding of the bias implicit in particular formulas. Chapters 15 through 17 discuss such bias in some detail. The Laspeyres formula is generally considered to be used for PPI construction for the practical reason of not requiring any current-period quantity information. It is also recognized that the appropriate deflator that generates estimates of output at constant prices is a Paasche one (Chapter 18). Thus, if estimates of a series of output at constant prices is required, the use of Laspeyres deflator will result in bias. In practice, for a price comparison between periods 0 and *t*, period 0 revenue weights are not available, and a Young index is used, which weights period 0 to *t* price changes by an earlier period *b* revenue shares. Chapter 15 finds this index to be biased. Superlative index number formulas, in particular the Fisher and Törnqvist indices, have good axiomatic properties and can also be justified using the fixed-base, stochastic, and economic approaches. Indeed, Laspeyres can be shown to suffer from *substitution* bias if particular patterns of economic behavior are assumed. For example, producers may seek to maximize revenue from a given technology, and inputs may shift production to items with above-average relative price increases. The Laspeyres formula, in holding quantities constant in the base period, does not incorporate such effects in its weighting, giving unduly low weights to items with above-average price increases. Therefore, it suffers from a downward bias. It can be similarly argued that the fixed, current-period weighted Paasche index suffers from an upward bias, while the Fisher index is a symmetric mean of the two, falling within these bounds. Calculating the Fisher index retrospectively on a trailing basis will give insights into upper-level substitution bias.

11.42 The extent of the bias depends on the extent of the substitution effect. The Laspeyres index is appropriate if there is no substitution. However, the economic model assumes that the technology of production is the same for the two periods being compared. If, for example, the factory changes its technology to produce the same item at a lower cost, the assumptions that dictate the nature and extent of the bias break down.

G.2 Lower-level substitution bias

11.43 In some countries or industries, elementary aggregate indices at the lower level of aggregation are constructed that use only price information. The prices are aggregated over what should be the same item. In practice, however, item specifications may be quite loose and the price variation between items being aggregated quite substantial.

11.44 The axiomatic (test), stochastic, and economic approaches can also be applied to the choice of formula on this lower level (Chapter 20). The Carli index, as an arithmetic mean of price changes, performed badly on axiomatic grounds and is not recommended. The Dutot index, as a ratio of arithmetic means, was shown to be influenced by the units of measurements used for price changes and is not advised when items do not meet tight quality specifications. The Jevons index, as the geometric mean of price changes (and equivalently, the ratio of geometric means of prices), performed well when tested by the axiomatic approach but incorporates a substitution effect that goes the opposite way to that predicted by the aforementioned economic model. It has an implicit unitary elasticity, which requires revenues to remain constant over the periods compared. For a consumer price index, the economic model is one of consumers substituting *away* from items with above-average price increases so more of the relatively cheaper items are purchased. Constant revenue shares is an appropriate assumption in these circumstances. However, producer theory requires producers to substitute *toward* items with above-average price increases, and assumptions of equal revenues are not tenable. Chapter 20 details a number of formulas with quite different properties. However, it concludes that since the axiomatic, stochastic, fixed-base, and economic approaches, as noted in Section G.1, find superlative index numbers to be superior (Chapters 15 to 17), a more appropriate course of action is to attempt to use such formulas at the lower level, rather than replicate their effects using only price data, a task to which they are unsuited. Respondents should be asked to provide revenue or quantity data as well as price data. Failing that, an appropriate index number formulas are advocated depending on the expected nature of the substitution bias.

G.3 Unit-value bias

11.45 Even if quantity or revenue data were available at a detailed item level, there is still potential for bias due to the formula used to define prices. If an establishment produces thousands of an item each day, the price may not be fixed. Minor variations in the nature of what is produced may affect the price if it is estimated as the total revenue divided by the quantity produced. If production moves to higher-priced items, then average prices will increase simply because of a change in the mix of what is produced; there will be an upward bias.

G.4 New-establishment (substitution) bias

11.46 The need to include new establishments in the sample has already been referred to in Section E under *sampling bias*. Products produced by new establishments may not only have different (usually lower) prices, arguing for their inclusion in the sample, but gain increasing acceptance as purchasers substitute goods from new establishments for goods from old establishments. Their exclusion may overstate price changes. When an establishment in the sample closes, an opportunity exists to replace it with a new establishment, thus militating against sampling bias as discussed in Section E. However, the quality of not only the item being replaced, but also the level of service, geographical convenience, and any other factors surrounding the terms of sale, must be considered in any price comparison to ensure that the pricing is for a consistently defined price basis.

11.47 The sections above are merely an overview of the sources of error and bias and are intended to be neither exhaustive nor detailed accounts. The detail is to be found in the individual chapters concerned. The multiplicity of such sources argues for statistical agencies undertaking audits of their strengths and weaknesses and formulating strategies to counter such errors and bias in a cost-effective manner.

PART III

Operational Issues

12. Organization and Management

A. Introduction

12.1 The producer price index is used for many purposes by government, business, labor, universities, and other kinds of organizations, as well as by members of the general public. Accuracy and reliability are paramount for a statistic as important as the PPI. Whether the PPI is used as a deflator of national account values, an indicator of inflation, in escalation of contracts, in revaluation of fixed assets or stocks, or in other economic analyses, the process of producing the PPI needs to be carefully planned.

12.2 Individual circumstances vary to such an extent that this *Manual* cannot be too prescriptive about timetables or critical path analysis of all the steps involved. However, the description in this chapter provides an outline of the kinds of activities that should result from a detailed examination of the logistics of the whole periodic operation of compiling the index.

12.3 The following guidance seeks to present some options in the organization of data collection. The examples given are based on experience and provide an indication of goals a country may seek. In recognizing these options, this chapter, which talks about organization and management of PPI procedures, covers the relationships between the price collectors (who may be stationed at regional offices in large countries) and PPI staff at the central office (covering the work carried out in the central office, the flow of information among each part of the organization, and related activities for coordinating collection and processing data). Because of the size, frequency, cost, or complexity of the collection of prices as the basis of the index, in some countries not all these operations and relationships will be appropriate.

B. Initiation of the Price Collection Process

12.4 This process involves PPI staff visiting individual businesses drawn from a sample to establish cooperation, stress the importance of the index, and receive basic information, such as the exact goods and services produced by the business, relative importance of transactions with various clients, individuals to contact on a recurrent basis, and so on. The range and number of businesses visited and the types of goods and services priced will vary among countries. In some countries these operations may be conducted by telephone. Once this process has commenced, a questionnaire for the price collection may be designed. (See Chapter 6, Section D, for information on questionnaire design.)

12.5 Although the precise method of current price collection will vary, each price collector will usually be responsible for collecting from a certain business or from certain types of businesses. This may enable the collector to specialize in certain subject areas of the index. Collectors will contact the same businesses in each collection period to attempt to price the same transactions of goods and services. Price collection is usually done monthly or quarterly, but the frequency can change if the prices for certain transactions change at known intervals. For example, goods or services with prices usually subsidized or regulated by government will change prices when government action is taken. These prices may be collected directly by PPI staff in the main office based on external information such as contact with other government offices or through the media. In any case, checks must be in place to ensure all price data are reported. (See Chapter 6, Section B, for information on timing and frequency of price collection.)

B.1 Mode of price collection

12.6 One of the decisions facing any statistical agency carrying out a price collection program is whether to use in-house staff or tender the collection to an external organization. For example, another part of the agency, another government department that specializes in surveys, or a private market research company could perform this function.

12.7 The nature of the price collection and the distribution and profile of statistical staff may help determine whether the collection is suitable for contracting out to another agency or even the private sector. Where price collection is continuous, involves complicated decision making (such as quality adjustment), or is collected from a small number of businesses, it may be advantageous to keep the collection in-house. However, if the collection takes place over just a few days per month from a large number of businesses, is relatively straightforward, and involves only routine or simple decision making (perhaps selecting from a list of codes), then contracting out to another agency can be considered. For example, if the statistical office does not have a dedicated data collection staff, it could contract with other agencies such as commerce, industry, and agriculture to collect PPI data. Another possibility could be a private research company, if there are market research companies with suitable skills existing in the country. The statistical office must also take confidentiality requirements into consideration when contracting with another agency to guarantee that there are no breaches in confidentiality. This may involve national statistical laws that address the issue of data collection by contractors and enforcement of penalties for breaking confidentiality requirements.

12.8 Contracting out price collection can lead to lower costs, because the statistical office is no longer responsible for overheads such as collector pensions. When price collection is carried out using electronic methods such as computer-assisted telephone interviews (CATI) or computer-assisted personal interviews (CAPI), the responsibility for purchasing and maintaining data-capture devices may also be transferred to the contractor.

12.9 Contracting out may also allow statistical office staff to spend more time analyzing data rather than collecting it. By separating the role of data collector and data checker, statistical staff can feel more comfortable questioning the validity of price data. The accuracy of collected data can be linked to the performance of the contractor through performance measures, which drive incentives payments (and penalties if targets are not achieved).

12.10 The same considerations may be used when deciding whether the survey division or PPI staff should conduct the price collection. Usually, some mixed mode of operation will be in place. Staff from the survey division may handle straightforward and routine price collection; more complicated and specialized industries such as chemicals and semiconductors will require price collection by specialists, whether from PPI staff or consultants of a statistical office.

C. Quality in Field Data Collection

12.11 Quality is an important part of price collection; a high-quality price collection enables a statistical agency to have confidence in the index it produces and ensure that observed price changes are genuine and not the result of collector error. Procedures must be developed to ensure that a high standard of collection is maintained for every collection period. These procedures will form the basis of collector training and should be included in any training material developed for price collectors. Guidance should cover price index principles, organizational issues, and validation procedures. For a discussion of the components that should be in a training regimen, statistical offices should review Finkel and Givol (1999), which includes both technology and price collection methods. Additional training requirements for statistical offices appear in Section F.1 of this chapter.

C.1 Training

12.12 The statistical office should have a general training program for staff working on the price programs. There are four basic components of such a program.

12.13 First, fundamental (basic) training must provide information on how to collect data, code data elements, review and edit basic price data, and compile collected data to produce indices. In addition, the training should impart to staff information on the purposes and uses of the collected prices.

12.14 Second, the program should highlight the need for continuous training of staff at all levels. Staff should provide feedback at all levels—from respondents to data collectors and from supervisors to staff. There should be regularly scheduled meetings between staff and supervisors at all levels to assess the program and identify current and potential problems.

12.15 Third, statistical offices also need to provide professional training for staff in computer technology, economics, statistics, and even psychology (for dealing effectively with respondents).

12.16 Fourth, annual seminars or retreats for staff can be effective in discussing the strengths and weakness of the program during the previous year and in planning for the upcoming year. This is particularly true when the program undertakes major changes in index methods, new weights, new sample designs, and so on.

C.2 Transaction descriptions

12.17 Accurate price transaction descriptions are critical in ensuring price transaction continuity. Descriptions should be comprehensive to ensure that collectors or reporters can price the same transaction in each collection period. Collectors must record all information that uniquely defines the price transaction selected. So, for example, in price collection for production of clothes, color, size, and fabric composition must be specified to ensure that the same price transaction is priced each month.

12.18 Accurate price transaction descriptions will assist the price collector, respondent, and PPI staff in choosing a replacement for a price transaction that has been terminated and will also help to identify changes in quality. PPI staff should be encouraged to spend some time, each collection period, going through reported descriptions to ensure that the correct price transactions are being priced. Collectors or respondents should also be encouraged to review their descriptions to ensure that they contain all the relevant information, and it may be useful to ask collectors occasionally to switch collections with another collector so that they understand the importance of comprehensive descriptions.

C.3 Continuity

12.19 Continuity is one of the most important principles of price collection. Because a price index measures price changes, the same price transaction must be priced every month so that a true picture of price changes is established. It is not possible to be prescriptive because the concept of equivalence will vary among countries, but for practical purposes a detailed description of the price transactions must be kept. Some guidelines may be drawn up by the statistical office's PPI Head Office staff to cover different price transactions. All transaction specifications, such as the same purchaser, similar delivery terms, valuation of currencies, changes in subsidies, tax laws, and so on must be met.

12.20 Collectors or respondents should report prices at similar times within each collection period. This is particularly important when pricing volatile price transactions with sharp fluctuations.

C.4 Data entry queries

12.21 Once the price data are correct and complete, a series of validation checks may be run. In deciding which checks should be carried out, take into account the validation checks carried out in the field, whether by price collectors in the regional office, survey division officials in the main office, or by PPI analysts. For example, CATI will increase the potential for validation at the time of price collection and reduce the need for detailed scrutiny by PPI Head Office staff. It would not be productive or cost effective to repeat tests.

12.22 The range of tests carried out for all collection methods may include the following:

(i) ***Price Change***: The price entered may be compared with the price for the same defined transaction in the same business in the previous month and queries raised where this is outside preset percentage limits. The queries may vary depending on the price transaction or group of price transactions and may be determined by looking at historical evidence of price variation. If there is no valid price for the previous month, for example, because the produced good was out of stock and no transaction could be made, the check can be made against the price two or three months ago. The price may also be compared with other transactions conducted by the same business in the current month.

(ii) ***Maximum/Minimum Prices***: A query may be raised if the price entered exceeds a maximum or is below a minimum price for group of goods or services of which the particular product is representative. The range may be derived from the validated maximum and minimum values observed for that price transaction in the previous month expanded by a standard scaling factor. This factor may vary among price transactions.

12.23 If computer-assisted techniques are used, these tests can be easily implemented to take place at the time of collection; otherwise, they will need to be conducted by PPI Head Office staff as soon as possible after collection and before prices are processed on the main system. A failure in the CATI or CAPI should not result in collectors being unable to price the price transaction, but it should prompt them to check and confirm their entries and prompt for an explanatory comment.

12.24 Queries raised may be dealt with either by staff analysts at the PPI Head Office or by the price collector or respondent contacted for resolution. For example, scrutiny of a form might show that a big price difference has arisen because the transaction priced was a new product replacing one that had been discontinued. In this case, there may be no need to raise a query with the price collector unless there is evidence to suggest that labeling the transaction as a "new product" is incorrect.

12.25 When an error is discovered too late in the process to resolve, PPI Head Office staff will need to reject it and exclude that price transaction from that month's index. The price transaction must be excluded from the base month so that the basket is kept constant. (See Chapter 6, Section F, for more details on the verification process.)

C.5 Feedback

12.26 When price collectors are used, they should be encouraged to give feedback to PPI Head Office staff on their experiences. Collectors are a valuable source of information and often give good early feedback on changes in the different industries. Collectors can often warn of size or product changes before the PPI Head Office staff can derive this information from other sources, such as trade magazines or the business press. Collector's feedback can form the basis of a collector newsletter and can support observed price movements and provide supplementary briefing material. Significant changes in price transactions within a business may require an additional visit by PPI analysts to the business to update the price transaction descriptions.

D. Quality Checks in Price Collection

D.1 Role of auditors

12.27 The routine of collecting prices in the field (where the "field" represents an array of collection methods; see Chapter 6, Section D.3) needs to be carefully planned and monitored with arrangements to reflect local conditions. Circumstances vary, and it is not appropriate to be too prescriptive. Some of the measures mentioned below may be irrelevant if PPI analysts in the Head Office collect the prices centrally. However, when data are collected locally and sent into the Head Office or reported directly by the businesses in the sample, price collectors and contacts must send in data on time. If data are not timely, it is necessary to find out the reason and take appropriate action. It is also important to check that the information sent in is accurate and complete.

12.28 One way to monitor the work of price collectors is to employ auditors to occasionally accompany collectors during field collection—whether data are collected by phone or personal visits—or to carry out a retrospective check on data that have been collected.

D.1.1 Monitoring data collection

12.29 If an auditor intends to accompany a price collector during a personal or telephone interview, he or she must inform the collector in advance to arrange meeting details. In general, the auditor will not monitor the entire price collection process but will spend a few hours observing the price collection in a specific location. For example, it may be necessary to observe the collection of certain price transactions or in particular businesses where collection might be problematic, based on analysis of the price trends in the past. Special workstations in a regional or main office may be set up for auditors who listen and track a price collector using a computer-assisted method such as CATI.

12.30 Before monitoring, the auditor will need to carry out preparation work—a premonitoring check. Such a check could involve looking at descriptions, prices, price history, and indicator codes of the price transactions collected in the particular business or section of the index. This type of check will enable the auditor to understand the standard of collection before going into the field and may suggest in which areas of the collection the auditors should concentrate their efforts.

12.31 An auditor's main duty is to ensure that the price collector is following proper procedures and instruction and performing the collection competently. While the auditor may not have the role of a trainer, he or she may give some coaching to correct any errors. The collector should use this opportunity to ask the auditor relevant questions.

12.32 Following a monitoring visit, auditors should compile a report detailing their observations. This report should include a summary report of findings, issues for action, and a recommended course of action. Auditors may advise that a collector receive extra training on certain aspects of the price collection, and PPI Head Office staff (or the contractor, if the collection has been outsourced) should act on this. The auditor's report will be used as a starting point on the auditor's next visit. In other instances, general problems may arise when solutions need to be disseminated to all price collectors, perhaps by issuing revised instructions or through a newsletter or other written or electronic means.

D.1.2 Backchecking and process auditing

12.33 Another approach to monitoring the standard of price collection is to carry out a "backcheck"—a retrospective check of a proportion of the prices recorded during the collection. Backchecks can be used to

(i) Assess the competence of overall individual price collectors;
(ii) Audit the general standard of price collection;
(iii) Identify general training needs or the specific needs of an individual;
(iv) Highlight any key issues, such as problems with documentation and instructions issued by PPI Head Office staff; and
(v) Identify areas where collection is problematic—for example, if all collectors have problems in certain types of businesses—demonstrating the need for more detailed PPI Head Office staff instructions.

12.34 Backchecking could be done by an expert independent of the process (employed by the statistical office when price collection is outsourced). It is carried out by contacting the business(es) selected and recollecting the prices and other relevant information. This activity should be carried out at about the same time as the original collection period to avoid problems of price changes occurring in the interim. Backcheckers must explain to the contact person at the business the reason for the check and stress the importance of his or her participation. The response by the business will usually be favorable.

12.35 For a backcheck to be useful, the results must be compared with preestablished performance criteria. These criteria should set, for example, the acceptable number of price errors per number of price transactions checked. Well-defined criteria will enable easy identification of a poorly performing collector or section of the index.

12.36 The need for a backcheck may be triggered by

(i) Price differences—if different, the auditor should see if there has been a price change since the original collection took place;
(ii) Insufficient price transaction description—the auditor should determine if it is uniquely defined so that another collector can replicate the process;
(iii) Wrong price transaction priced, such as incorrect details of the transaction being chosen; and
(iv) Price transactions wrongly recorded as missing or temporarily out of stock.

12.37 A report should be sent to the PPI Head Office staff for scrutiny once the backcheck has been completed. The Head Office will then need to take appropriate action, which may include, for example, retraining or sending out supplementary instructions.

12.38 Auditing and backchecking are important ways to improve quality, but there is a trade-off between this and the burden on business imposed by the audit process. For example, businesses are likely to object to being asked for the same information twice (once as a check). There are, of course, ways to audit collection without imposing extra burdens on business—for example, by monitoring telephone conversations with businesses and examining the quality of the data and supporting information received by different analysts.

D.1.3 Other auditor functions

12.39 The range of tasks auditors carry out will vary from one statistical agency to another, and monitoring the standard of price collection will always be their main focus. However, there are a number of other areas to which auditors can be called to contribute.

12.40 Auditors may be required to help with initiating the price collection process and price transaction sampling. Auditors can also carry out other work. For example, if a particular price transaction is causing difficulty for price collectors, auditors can speak to collectors and businesses and find out why. Auditors who work on CPI and PPI can also lead to more consistency among the indices, and they can advise both CPI and PPI Head Office staff on availability of goods and services and other economic activities that may be of importance.

12.41 Auditors who learn that a particular business is participating in numerous surveys of the statistical agency can inform the Head Office of this information to help the agency find ways to reduce respondent burden or better coordinate collection at the business.

D.2 Quality checks by PPI Head Office staff

12.42 Regular quality checking is recommended, as necessary, to

- Ensure that the price collectors' reports are sent in when they are due. If not, it is necessary to find out the reason and take appropriate action;
- Confirm that the reports contain what they are supposed to contain—that is, that required fields have not been left blank, that numeric fields contain numbers, and that nonnumeric fields do not;
- Review and edit each return. Substitutions may have to be made centrally, or those made by the collectors may have to be approved. Unusual (or simply large) price changes may need to be queried. Transactions priced in multiple units or varying weights may have to be converted to price per standard unit. Missing prices must be dealt with according to standard rules relating to the cause; and
- Identify and correct errors introduced when keying the numbers into the computer or transcribing them onto worksheets.

12.43 As stated above, logical checks conducted in the field by an automated process can reduce the number of checks and errors handled by PPI Head Office staff.

12.44 Note that the way the data are organized in worksheets or in the computer may differ from the way they are collected in the field. Their origin should, however, be recorded so that reference to them can be made should processing disclose any problems. Even if codes provided to the collectors to list price transactions and to describe or qualify the prices are unchanged in processing, other codes may have to be used for information that comes in from the collectors in noncoded form. How the checking is organized will vary from country to country. In some cases, local or regional supervisors will do some of it; in other cases, it will be more appropriate for it all to be done centrally.

12.45 Some of these tasks can be done by computer; others, manually. Therefore, no general suggestion can be made about the sequence of the work or about its division into different parts. Procedures should be in place to check that all documents, messages, or files are returned from the field so that price collectors can be contacted about missing returns. Initial checks should then be carried out to ensure that data are complete and correct. If any prices fail these checks, a query should be raised with the price collector for clarification. Since some of the checking may require reference back to the price collectors (or to their supervisors or respondents when direct mail questionnaires are used), the timetable for producing the index must allow for this communication to take place.

12.46 Following the price data checks, a series of validation checks may be run. In deciding which checks should be carried out, account should be taken of the validation checks carried out in the field. For example, computers will increase the potential for validation at the time of price collection and reduce the need for detailed scrutiny at the PPI Head Office. In addition, it would not be productive or cost-effective to repeat the tests already carried out, except as a secondary audit or random check.

D.2.1 Reports

12.47 Reports (on paper or computer) should be generated routinely for most representative price transactions. Reports help the analyst pick out particular prices that are different from those reported for similar firms elsewhere or that lie outside certain specified limits. A computer printout can list all cases that either fall well outside the range of prices obtained earlier for that representative price transaction or that show a marked percentage change from last time for the same price transaction in the same business. The limits used will vary from price transaction to price transaction and can be amended. The analyst can study the printout, first ascertaining whether there has been a keying error, then examining whether the explanation furnished by the collector adequately explains the divergent price behavior, and finally determining whether a query should be sent back to the supervisor or collector. The timetable should allow for this step, and anomalous observations should be discarded when an acceptable explanation or correction cannot be obtained in time. (Also see Chapter 9, Section D, on editing data.)

12.48 Other reports may be regularly produced on the basis of several periods' (or months') reporting (to detect accumulated patterns) that will enable broader problems to be detected. For example,

- One collector's reports might show many more "not available" remarks than those of other collectors, perhaps indicating either a motivational or training need on the part of that collector, or a change in retail trade patterns in a particular area;
- Substitution for a particular representative price transaction might become more numerous than before, suggesting a possible need for revision of the specification or the choice of another representative price transaction; or
- The dispersion of price changes for a particular representative price transaction might be much larger than it used to be, raising the question of whether it has been appropriately specified.

12.49 The routine computer-generated reports should enable those in charge of the index to detect all such problems.

D.2.2 Dispersion index report

12.50 This dispersion index report is a list of price transactions with the current *index* for each price transaction, number of valid quotes for each price transaction, and the range of price relatives. The index dispersion prints can identify situations with price relatives that fall outside the range of the main bulk of quotes. These quotes can be identified and investigated and appropriate action taken if necessary.

D.2.3 Quote report

12.51 The quote report consists of a range of information on a price transaction that the index dispersion report has highlighted as warranting further investigation. Information listed may include current price, recent previous prices, and base price, together with similar quotes from other reporting businesses. The report can be used to identify the quotes that require further investigation and also to investigate rejected prices.

D.2.4 Algorithms

12.52 Algorithms can be created that identify and invalidate price movements that differ significantly from the norm for a price transaction. For some seasonal price transactions for which price movements are erratic, it may be more appropriate to construct an algorithm to look at price level rather than price change.

12.53 Although algorithms can be an efficient way to highlight problematic data, a word of caution should be expressed about using them. Analysts will want to assure themselves that their use does not result in systematic bias in the index. This issue may also need to be addressed in any editing routines (as presented in Chapter 9, Section D), although it is less likely to be problematic in the context of manual editing.

E. PPI Production and Quality Assurance

E.1 Organizational structure and responsibilities

12.54 Statistical offices could adopt a number of organizational models for effective work. In deciding on the appropriate organizational structure, statistical offices should take into account the following:

- The need for clarity of reporting lines;
- The need for a clear division of responsibilities;
- Centralized or decentralized management of fieldwork;
- Production management versus technical development; and
- Compatibility with corporate structures in the National Statistical Institute, for example, in relation to quality management, methodological research, and dissemination.

12.55 In some cases—for instance, when little in-house expertise in fieldwork practices exists—it may be advantageous for fieldwork to be conducted by a different organization in either the public or private sector. In these circumstances, an effective contractual relationship must exist with the data collection agency. There should also be agreed delivery targets and performance measures to cover such items as data delivery timetables, response rates, and levels of accuracy. Consideration should also be given to the independent auditing of the contractor's work on a sample basis. It is worth noting that even if fieldwork is conducted in a different division within the organization, a contractual relationship between fieldwork and main office functions can alleviate tension and improve the quality of a PPI.

E.2 Monthly compilation

12.56 The system used for the regular computation of the index must be sufficiently flexible to allow for changes in the kind of data obtained. A modular or mix-mode approach may be seen as an advantage.

12.57 Analytical computations provide comparisons between the published index (or indices) and what an alternative index would have produced using different methods or data. They help explain the relationship with subindices, characterize the movements in the index over time, and allow methodological experimentation. The following examples of such investigations make clear some of the computational capabilities for analytical indices:

- Alternative aggregations of subindices;
- Alternative computations of indices for seasonal goods;
- Effects of different weights; the effects of introducing newly significant product categories;
- Price updating of weights;
- Number and duration of missing observations; how a different method of estimating them would affect the index;
- Comparison of indices computed with various subsamples of the data as a means of estimating variance; variances of price ratios;
- Computation of a Standard Reference Index (one with no explicit quality adjustments) so that an Implicit Quality Index is obtained;
- Numbers of sampled products; rates of forced replacements; lengths of time products remain in sample; and
- Frequency distributions of quality adjustments.

12.58 To examine such matters, the database must contain not only prices but price transaction descriptions, details of product replacements, explanatory remarks attached to observed prices, information on the data suppliers, and so on. Generally, historical databases are too large to be stored live on the system and need to be archived. However, analysis of seasonal trends requires 12 months of data on current computer systems. Detailed documentation relating to the archived material will need to be kept to guard against loss of vital information caused by changes in computing staff or computers. Consideration should also be given to appointing a data custodian with responsibility for all archived records.

E.3 Spreadsheets

12.59 Spreadsheets may be used for compiling subindices that require special procedures or when data are collected by different methods than the main method or technique. A spreadsheet has the advantage of additional flexibility and scope for combining responsibility for data collection, data input, and computation. The compilers' specialized knowledge (they usually are PPI analysts in the Head Office) about the markets or businesses where these prices may be observed, combined with analytical tools applied to the spreadsheet, will help them detect any irregularities in the data, facilitate investigation of whether these reflect reporting or input errors, and allow rapid rectification. A compiler can jump between numerical data entry and a chart—displaying, for example, current-month and previous-month entries—and this helps the rapid and simple detection of anomalies, and he or she can then follow-up with the data supplier. As time passes, the resolution of problems that have arisen and adaptation to new circumstances will result in changes to the spreadsheet. Unless *quality management controls* are put in place, the spreadsheet may be unclear and improperly documented. If so, two unfortunate consequences can arise.

- If that developer is absent, retires, or moves to another job, the successor will find it very difficult to maintain the continuity and quality of the spreadsheet.
- New procedures introduced to deal with new circumstances may be inconsistent with procedures used for other subindices for which other people are responsible.

12.60 Good documentation and communication with colleagues will diminish these risks. At a minimum, the spreadsheets and changes to them must be understandable with adequately explicative row and column headings or notes attached to headings. Furthermore, changes in procedures or formulas, rebasing, and application of new weights should always be introduced by moving computation to a new sheet within the workbook, not by modifying the old sheet. The new sheet and the old sheet will then exist side by side so that they can be compared. Passwords can prevent inadvertent changes to cells containing formulas and can lock cells containing input data once editing is completed. Only people with authority to edit the spreadsheets should know the passwords. Regular backup by copying the whole workbook to another disk is also essential. Working experience with spreadsheets in a PPI has revealed some other important quality management controls:

- Spreadsheet designs should be as similar as possible,
- PPI analysts should present these to each other on a regular basis, and
- At least two analysts should verify the results in the spreadsheets.

E.4 Monthly consultations

12.61 PPI managers in the main headquarters may find it useful to convene monthly consultations with their analysts before publication of the index. Although various checks have been implemented throughout the process (fieldwork, editing, compilation of subindices, etc.) added value is obtained by comparing results of different economic branches in the current month. Analysts could be asked to present areas of significant change in the index (whether by percentage points or relative importance) and explain the reasons that led to these changes, such as changes in the local economy, global prices, exchange rates, institutional or government intervention, and so on. Countries may find it useful to convene all the analysts of price indices on a monthly basis to compare changes in the different markets.

E.5 Introducing changes

12.62 Various checks should be carried out when introducing changes in a PPI. These may include a comparison of the old and new basis using data from parallel running of collections—for example, when handing over to a new collection contractor, division, or individual within the organization. Checks may include reestimating backward—for example, when new base prices are being imputed for a complete range of goods or services. Any anomalies can then be investigated further.

E.6 Disaster recovery

12.63 Price indices are important and high-profile statistics produced by a statistical office and can affect a wide range of users. There may be a legal obligation for the index to be published within a short time period after the end of the relevant month. Many contracts within an economy may be linked to the indices, whether consumer prices or producer prices. Any delay in publication can have significant effects on subsequent months, threatening future publications. With significant delays, it could take months to return to the existing tight publication timetables. It is critical, therefore, that statistical offices develop a robust and tested Disaster Recovery Plan (DRP), however unlikely the need to implement it may appear. There are a number of possible causes of disaster:

(i) Failure of an external contractor to fulfill obligations to supply information when the data collection is contracted out to a private company,
(ii) Failure of the computer system, and
(iii) Major natural disaster or other event (for example, terrorist activity) affecting the operations center or PPI Head Office staff.

12.64 When the whole operation of the PPI (including fieldwork) is conducted in-house by the statistical office, the DRP for the organization will already include special procedures to ensure the continuance of PPI production in times of disaster. If price collection is contracted out, the first cause of disaster mentioned above may become relevant. Therefore, the DRP may include arrangements for alternative fieldwork operations—whether by outsourcing the procedures to a third party or utilizing in-house capability (if such exists).

F. Performance Management, Development, and Training

12.65 Equally important to organizational structure in production of a PPI is the ability to ascertain an effective performance management system for individuals. Performance management can be seen as a continuous process designed to improve work outputs by focusing on what people actually achieve rather than the amount of effort put into the work. It should provide the link between the objectives of the individual and those of the team and the wider organization so that work plans are coherent across the organization and everybody knows what they are doing and why they are doing it. The performance management system should provide clear objectives for monitoring and evaluation to enable feedback on performance and also to assist with the identification of the development needs of individuals. Performance management should be continuous.

F.1 Training requirements

F.1.1 Introduction

12.66 Effective training will motivate staff and equip them to deliver a good-quality index. At its simplest, training will give a background understanding of the nature and uses of the index and its compilation. Training and development take many different forms and may include

- Tutoring by the line manager or supervisor,
- Attending an induction course or reading a manual, and
- Accompanying an experienced price collector.

12.67 A written training plan can be useful in identifying training and development needs against the organization's goals and targets. It can also identify the resources required to deliver these needs and evaluate whether training has been delivered effectively and objectives have been met.

F.1.2 Compilers and collectors

12.68 Further training will be required for specific skills depending on the roles of the individuals and their jobs. Training should continue beyond the induction stage to cover changed procedures and include retraining when performance is unsatisfactory.

(i) Price collectors will need to be trained specifically in field procedures, including relations with businesses, selection and definition of a valid price, special rules for certain individual price transactions (including seasonal price transactions), how to complete forms, and, where appropriate, how to use computers.
(ii) Compilers of the index will need to be trained specifically on the validation, consistency checking, and calculation of centrally collected indices; weighting procedures and how to aggregate prices; and treatment of seasonal price transactions and special procedures relating to some sections of the index.
(iii) It can also be beneficial to provide training in local or national trading or statistical regulations, culture, and commodity information.

12.69 Significant benefits can result from the interaction between price collectors and index compilers. Benefits will also be gained from a liaison between statistical offices and experts from industry, who can advise on issues such as how to identify quality features, and so on, on particular price transactions such as electrical goods, personal computers, or clothing and footwear.

12.70 It can be beneficial if statisticians from headquarters are personally responsible for supervising price collection (at least for certain parts of the index) so that they have firsthand experience of the problems involved and provide assistance when difficulties arise. It is also a good idea to arrange for regular visits to headquarters by groups of collectors and their supervisors. It is good for morale, and price collectors will, arguably, do a better job if they feel that they belong to a team. They can see that their work is appreciated and their problems are understood. It will help convey that the accuracy and conscientiousness of their contribution is recognized as being crucial to the quality of the index.

Visits to PPI Head Office staff by price collectors will help the statisticians keep in touch with conditions in the field and, for example, obtain more information about new goods and aspects of quality change.

12.71 Compilers of the index also may wish to visit the field occasionally and participate in or observe the price collection. This will provide them with a better appreciation of the practical problems associated with price collection, a better feel for data and index quality, and a greater understanding of the skills required to help with price collection in the event of an emergency. In a PPI, this may be of great importance, especially when collecting price transaction data in more complicated economic branches.

F.1.3 Documentation

12.72 Manuals and other documents such as desk instructions may serve for initial training and later on should enable the collectors and compilers to remind themselves of all the relevant PPI rules and procedures. Documents should be well organized and well indexed so that answers to problems can quickly be found.

12.73 All concerned should check the documentation and update it regularly; the pile of paper containing amendments should never grow large and should be replaced by a new consolidated version. One way of achieving this is to have a loose-leaf manual so that individual pages can be replaced whenever necessary, or to keep an electronic version that can be updated by designated individuals. It is important that the updating be done in a systematic and controlled way. A variety of available software programs can help the statistical office. The benefits of using standard electronic software for documentation are threefold:

- More efficient production of documentation, because software helps with initial compilation and reduces the need to print and circulate paper copies;
- Better-informed staff, because they have immediate electronic access to the latest documentation, including desk instructions with search facility by subject and author; and
- Better quality control, since authors can readily amend and date-stamp updates, and access is restricted to "read only" nonauthors.

F.2 Reviews

12.74 Training can be an essential part of continuous quality improvement. Staff may be invited to operational reviews where all team members have the opportunity to raise concerns and, where appropriate, tackle specific issues through individual or group training.

G. Quality Management and Quality Management Systems

12.75 Statistical offices are faced with the continuous challenge of providing a wide range of outputs and services to meet user, that is, customer, needs. Thus, a key element of quality is customer focus and the effective dissemination of relevant, accurate, and timely statistics. In addition, a quality program should include effective customer education on the use of such statistics. In these terms, success can be measured by the achievement of a high level of satisfaction among well-informed users. The IMF has developed the Dissemination Standards Bulletin Board (dsbb@imf.org), which provides dissemination standards and a data quality reference site. The data quality reference site also includes a framework for assessing price statistics systems that is contained in Reports on the Observance of Standards and Codes (ROSC) Data Module for a number of countries. Statistical offices can use the ROSC Data Module to assess their price statistics programs.

12.76 For the quality management of a PPI, it can be argued that the priority area is quality control of the production process itself. For most statistical offices, this will be an area that represents a high risk, given the complexity of the process and the financial implications of an error in the index. If the principles of organizing and managing the collection of data, and subsequent processing of information to produce a PPI, are to be adopted, then it is vital that a quality management system be in place. This will ensure that the data obtained, the processes involved in achieving the specified outputs, and the formulation of policies and strategies that drive them are managed in an effective, consistent manner. The data systems should, wherever possible, be open to verification and mechanisms put in place to ensure outputs meet requirements—in other words, customer satisfaction.

12.77 Taken together, the above-listed elements form the basis of a *quality management system*. Varying perceptions about the meaning of quality exist, but an important common thread is the requirement to react to and serve user needs and to ensure continuous improvement in them. Thus, the implementation of an effective quality management system requires a high level understanding of what customers need and the translation of this need into a coherent statistical and quality framework. Such a framework is also necessary for putting together criteria for judging success. User needs can be canvassed either formally through negotiation of contractual obligations or less formally through talking to customers on a one-on-one basis or through customer surveys. In many countries, issues relating to the governance of the statistical office are set down in a framework document or similar document. This defines the functions and responsibilities of the statistical office and generally guides and directs the work of the office. For instance, an objective stated in the framework document to "improve the quality and relevance of service to customers—both in government and the wider user community" provides a powerful statement for determining workplans. This recognition of the importance of quality can be further endorsed by a published vision of the national statistical institute as a key supplier of authoritative, timely, and high-quality information. This can be encapsulated in published objectives in an annual business plan. These objectives can include improving quality and relevance, thereby increasing public confidence in the integrity and validity of outputs. Performance can be measured against a combination of a number of factors, including accuracy, timeliness, efficiency, and relevance.

12.78 There are a number of examples and case studies of quality systems in practice that illustrate how different models may be applied. Some models may be more suitable than others, depending on the exact mode of PPI operations in different countries.

G.1 Quality management systems

12.79 A number of best-practice standards can be exploited to help organizations improve quality management, some of which have the added advantage of being internationally recognized.

G.1.1 Total quality management

12.80 Total quality management, or TQM, is most closely identified with a management philosophy rather than a highly specified and structured system. The characteristics associated with TQM and an effective quality culture in an organization include

- Clearly defined organizational goals,
- Strong customer focus,
- Strategic quality planning,
- Process orientation,
- Employee empowerment,
- Information sharing, and
- Continuous quality improvement.

G.1.2 Benchmarking

12.81 Benchmarking is a process of comparing with, and learning from, others about what you do and how well you do it, with the aim of creating improvements. The Australian Bureau of Statistics has been particularly active in this area and undertook an exercise in 1998–2000 in partnership with the United Kingdom. Benchmarking projects have also been undertaken in New Zealand, the United States, and Scandinavian countries. Areas that can be considered when benchmarking a PPI collection may include

- Timelines, accuracy, and coverage of collection;
- Benefits of index methodologies for various price transactions, for example, geometric mean versus average of relatives;
- Frequency of collection and publication; and
- Cost of collection per unit of commodity, staff, and so on.

G.1.3 European Foundation for Quality Management Excellence Model

12.82 The European Foundation for Quality Management (EFQM) Excellence Model is a self-assessment diagnostic tool that is becoming widely used by government organizations across Europe to improve quality and performance. It may be described as a tool that drives the philosophy of TQM. It focuses on general business areas and assesses performance against five criteria covering what the business area does (the enablers: leadership, people, policy/strategy, partnership/resources, process) and four criteria on what the business area achieves (the results: people results, customer results, society results, key performance results). Evidence based on feedback from focus groups, questionnaires, and personal interviews is used to score performance, and a resulting action plan for improvement is introduced, which is then included in the business plan.

12.83 Underlying the EFQM Excellence Model is the realization that business excellence measured through customer satisfaction is achieved through an effective leadership, which drives policy and strategy, allocates resources compatible with that policy, and manages employees to manage the processes.

12.84 In the case of statistical offices, where some procedures are governed by statute or regulation, the use of the EFQM Excellence Model enables continuous improvement to go forward across a range of processes and functions. To work effectively, it needs the commitment of senior managers who must be responsible for leading any self-assessment. Unlike the international standard ISO 9000 (see below), however, where assessment is carried out by qualified auditors often from outside the work area, the EFQM Excellence Model relies on input from all staff.

G.1.4 ISO 9000

12.85 The International Standards Organization (ISO) established an international quality standard for management systems—ISO 9000. The quality system is a commonsense, well-documented business management system that is applicable to all business sectors and that helps to ensure consistency and improvement of working practices, including the products and services produced. The ISO standards have been fully revised to match current philosophies of quality management and to provide the structures needed to ensure continuous improvement is maintained.

12.86 The revision of these standards (as of the year 2000) gives users the opportunity to add value to their activities and to improve their performance continually by focusing on the major processes within the organization. ISO standards will result in a closer alignment of the quality management system with the needs of the organization and reflect the way those organizations run their business activities. It will therefore come more into line with TQM and the EFQM Excellence Model.

G.2 Need for quality management in statistics

12.87 Both ISO 9000 and the EFQM Excellence Model have received a great deal of international recognition over recent years. Benchmarking networks also have gained prominence. It is therefore pertinent to ask whether more coordinated use should be made of these and other quality management techniques at a strategic level in fields of statistics where the focus is on international comparability. This is particularly so with statistics such as those in the Harmonized Index of Consumer Prices, which are compiled for treaty purposes by member states of the European Union following detailed methodological guidelines laid down in European law.

12.88 The arguments are fivefold:

(i) Such important nonoptional statistics with production and uses enshrined in legislation must have the full trust of users across the European Union;
(ii) The quality of international comparisons is dependent on the weakest link; thus, good-quality statistics from one country may be of little value if not matched by statistics of equally good quality from other countries;
(iii) The potential for misleading analysis and conclusions arising from differences in the application of standard methodology;
(iv) The reduction of empowerment in ensuring the establishment of adequate control processes when production is delegated to member states; and
(v) The limited scope for centralized validation and quality management of decentralized production.

G.3 Specific quality management models in a PPI

12.89 PPI operations may differ from those of a CPI in several areas. The distinction between fieldwork procedures and complexity of continuity in the samples of goods and services can lead to implementation of different models in a PPI and CPI. For example, the PPI involves telephone interviewing and self-administered postal questionnaires, while the CPI largely uses personal interviewing. Also, the PPI basket has goods and services produced by specific firms according to economic branch, while the CPI basket contains many goods and services that are standard and can be purchased at many outlets. Therefore, several countries have adopted quality management models that may enhance representativity of the index. For example, the ABS undertakes an ongoing Sample Review and Maintenance Program process that is concerned with

- Adequacy of the sample of respondents,
- Adequacy of the specifications priced,
- Appropriateness of the pricing basis underlying the reported prices, and
- Accuracy of reported prices.

12.90 The U.S. Bureau of Labor Statistics conducts a Structured Schedule Review (SSR) that serves as a cornerstone of the quality control program in its PPI. The assumptions that underlie the development of this system are

(i) Survey quality is largely determined at the data initiation stage; and
(ii) Quality-related problems are associated with various causes, such as faulty procedures, inadequate training, and imprecise collection forms or uncontrolled operator errors, and they require an SSR system to assist in diagnosing the source of error.

13. Publication, Dissemination, and User Relations

A. Introduction

13.1 As discussed in Chapter 2, the PPI is one of the most important statistical series for monitoring inflation and assisting in the measurement of GDP at constant prices. It follows, therefore, that the PPI must be published, and otherwise disseminated, according to the policies, codes of practice, and standards set for such data.

13.2 The PPI, therefore, should be

- Released as soon as possible (noting the trade-off between timelines and quality),
- Made available to all users at the same time,
- Released according to preannounced timetables,
- Released separately from ministerial comment,
- Made available in convenient form for users and include analysis of the main contributors to overall change,
- Accompanied by methodological explanation and advice as to where more detailed metadata can be found, and
- Backed up by professional statisticians or economists who can answer questions and provide further information.

13.3 Above all, the PPI should meet the UN's *Fundamental Principles of Official Statistics*. It is published in several languages on the website of the UN (www.un.org). The *Principles* refer to dissemination and to all aspects of statistical work. In addition, the data dissemination standards developed by the IMF should be reviewed and followed by statistical offices. These and other standards are discussed in this chapter.

B. Types of Presentation

B.1 Time-series presentation of level and change

13.4 It is common to give prominence to indices that show changes in aggregate prices between the month for which the most up-to-date data are available, the change from the same month one year earlier, and the one-month change. It is also usual to compare the annual change with the annual change shown one month previously. The model presentation in Section E provides examples of these.

13.5 The arguments for the first presentation shown in the example are as follows. The 12-month comparison provides an indication of price changes in a reasonably long time frame by referring to periods that are unlikely to be influenced by seasonal factors. Also, prices that are often set centrally, such as the prices of or tariffs on utilities, and changes in indirect taxes (which directly affect prices) are usually on an annual timetable and occur in the same month or months each year. However, there may be one-off changes in either of the two months that can have an influence on the index.

13.6 Data on the one-month change, especially for some components of the PPI, need to be treated with caution to avoid, for example, suggesting that a 2 percent change in one month is similar to a 24 percent change over a year. (See Chart 2 in Section E.)

13.7 It is normal practice to set a reference period (usually a year, though a shorter period, such as a month, may be used) for which the price index is set at one hundred. Index numbers for all subsequent periods are percentages of the value for the reference period. Indeed, that is the index that is used as the basic figure from which the other changes are calculated.

13.8 These indices are usually shown to only one decimal place, as are the other changes mentioned here, so figures have to be rounded. Rounding in these circumstances can, however, give a false impression of comparative change and must, therefore, be explained, especially where price changes are small.

13.9 Care also has to be taken to differentiate between changes in index points and percentage changes between one month and the next. If in one month the index is, for example, 200, and the following month it is 201, then the change can be described as one index point (above the previous level of 200) or as an increase of half a percent. Both measures are valid, but they require careful specification.

13.10 The reference period that is set at one hundred is often referred to as the *base period* or the *reference base period*. It is often an arbitrarily chosen date, changed periodically, and not necessarily related to a point in time when methodologies may have changed or when a new basket of goods and services was introduced. The status of the reference period should be made clear in the methodological explanation. For technical reasons, a reference period that is abnormal (for example, in terms of absolute or relative price levels, industry structure, etc.) should be avoided.

13.11 The PPI is, by definition, an index; it is, therefore, not a level or a series of absolute changes in prices. Nevertheless, in the process of presenting the indices, average prices are sometimes calculated for categories of goods and services. It is thus possible to publish some average prices for groups of goods or services and also to show the upper and lower bands of the prices from which the averages have been calculated. Some users of the index find average price levels useful, and they should be made available to researchers who may want them.[1] It must be noted, however, that price-level data may be less reliable than the price change indices for any given group of goods or services because of the sampling strategies used. Further, quality changes can distort comparisons over time.

13.12 So far this chapter has referred to only the broadest aggregates without reference to subgroups of prices or to variants of the PPI that may include or exclude certain items. Nor does it refer to price indices with underlying concepts that may differ from those underlying the PPI. Some of these considerations are discussed later in this chapter.

13.13 All of the above can refer to the most common form of the PPI, which is usually intended to refer to the average price change in a specific country and to include high coverage of producer prices in that country. But it can equally refer to regions of a country or to subcomponents (such as raw materials versus intermediate goods), different product groups or industries of origin, or related or alternative measures of price change. Related or alternative measures and subaggregate indices are discussed in Section B.5.

B.2 Seasonal adjustment and smoothing of index

13.14 The treatment of seasonal products and the estimation of seasonal effects are discussed in Chapter 22. In this chapter, the dissemination of such adjusted or smoothed series is discussed.

13.15 Many economic statistical series are shown seasonally adjusted as well as unadjusted. Normally, however, PPIs are not seasonally adjusted. In cases where there are seasonal factors, statistical series are frequently recalculated using the latest data. As a result, seasonally adjusted series can be retrospectively revised. Unadjusted PPIs are not usually revised, although in a few countries, there is an explicit revision policy to publish a preliminary PPI and then revise that index after some fixed period (usually one–three months). This occurs because not all of the sample is received by the index cutoff date, so the index is released on a preliminary basis; but, after a few months, practically all of the sample is received and a revised index is published.

13.16 In comparing one month with the same month a year earlier, it is implicitly assumed that seasonal patterns are much the same from one year to the next. However, there may be exceptional months when the usual seasonal change is advanced or delayed, in which case the advance or delay should be identified as one of the likely contributors to a change in the PPI or one of its components.

13.17 Changes over periods of less than a year are subject to seasonal influence. To differentiate them from other factors, it is necessary to try to quantify seasonal effects and identify them as contributing to changes in the index.

13.18 Although the PPI itself is not seasonally adjusted normally, some variants of the PPI may be seasonally adjusted (such as the PPI for raw materials or agricultural products) because they are more subject to seasonality and can be revised retrospectively if necessary. If such variants are seasonally adjusted, it is important to explain why.

[1]When releasing data on average prices, confidentiality requirements must be maintained. See Section C.4.

13.19 Seasonal adjustment usually leads to a smoother series than the original unadjusted one. But there are other ways of smoothing a monthly series, such as using three-month moving averages. However, statistical offices do not usually smooth the PPI series in their published presentations. Producer price changes are not usually so erratic from month to month that they disguise price trends. To the extent that there might be an erratic change, the compilers of the index can usually explain the reasons for any sharp fluctuation.

13.20 In cases where any seasonally adjusted or smoothed PPI series is published, it is important to publish the unadjusted series as well, so that the effects of the adjustment process are clear to users who may wish to know what has happened to the actual transaction prices, whether the changes can be put down to seasonal or other factors. Similarly, full explanation should be given for the reasons why a particular seasonal adjustment procedure has been followed.

B.3 Analysis of contributions to change

13.21 The PPI is an aggregate of many different goods and services, whose prices are changing at different rates and possibly in different directions. Many users of the index want to know which goods or services have contributed most to changes in the aggregate index and which prices may be out of step with general price trends. The index compilers are well placed to provide analyses of the contributions to the price change in the current press release and current issue of the PPI publication.

13.22 Sufficient detail should be made available to the users of the index, so they can see for themselves what has happened to various groups of prices. However, because of the time constraints facing many users, the statistician should indicate which prices are the main contributors to the aggregate PPI and which ones may be most different from the aggregate. They can be presented in the forms of tables and charts, so that trends may be compared.

13.23 Similarly, statisticians should indicate any reasons for price changes that may not be immediately obvious but are nevertheless discernable from the published figures. For example, if there had been a sharp price rise or fall one year earlier, then it will affect the current year-on-year change, regardless of what happens to the current-period prices.

13.24 Analysis of contributions to change should also refer to any preannounced price changes, or major changes since the last price reporting date, that will affect the outlook for the index over the following months.

B.4 Economic commentary and interpretation of index

13.25 In undertaking analysis such as that described above, statisticians must be objective, so that users of the data may differentiate clearly between the figures themselves and the interpretation of them. It is, therefore, essential to avoid expressing any judgment of the policy causes or possible implications for future policies. Whether the figures should be seen as good news or bad news is for the users to decide. The statistician's role is to make it as easy as possible for users to form their own judgments from their own particular economic or political perspective.

13.26 There are several ways of avoiding any apparent or real lapses in objectivity in the analysis. The first and perhaps most important way is to publish the figures independently of any ministerial or other kind of political comment. Another is to be consistent in the way the analysis is presented. That is to say, the data should be presented in much the same format every month (see Section B.6). For example, tables and charts should cover the same periods every month and use the same baselines.

B.5 Presentation of related or alternative measures

B.5.1 Core inflation

13.27 For the purposes of economic analysis, it is sometimes desirable to construct measures of *core* or *underlying* inflation that exclude movements in the price index that are attributable to transient factors. Examples of such factors include the impact of monetary and fiscal policy decisions, regular seasonal influences, and inherent volatility. In other words, measures of core or underlying inflation seek to measure the persistent or generalized trend of inflation. For example, central banks require measures of the general trend of inflation when setting monetary policy. For this reason, there is increasing interest by economists and statisticians in developing measures of underlying inflation.

13.28 Several methods can be used to derive a measure of underlying inflation. Most measures focus on reducing or eliminating the influence of exceptionally volatile prices, or focus on exceptionally large individual price changes. The most traditional approach is to exclude particular components of the PPI on a discretionary basis. The items to be excluded are based on the statistician's knowledge of the volatility of particular items in the domestic economy. Items commonly excluded under this approach are fresh meat, fruit and vegetables, and petroleum. Many countries also exclude imported goods, government charges, and government-controlled prices. Care must be taken so as not to exclude so many items that the remainder becomes only a small and unrepresentative component of the total.

13.29 Other methods include smoothing techniques. An example would be annualizing three-month moving averages and abstracting from the effects of government fiscal policy decisions (for example, developing *net* price indices that have indirect taxes held constant or removed from the transaction price). A more difficult method is to exclude or give relatively smaller weight to *outliers*, that is, those items with the highest or lowest increases. This approach is gaining more interest as a method for identifying the inflation signal from price index measures.[2]

B.5.2 Alternative indices

13.30 An example of an alternative index is the PPI by stage of processing. The PPI can be viewed from the perspective that it is composed of the various stages at which price changes occur. The first stage is for primary inputs of raw materials such as iron, bauxite, or agricultural products. The second stage is for intermediate inputs, including such semifinished goods as steel and aluminum products. The last stage is for goods and services that are provided for final sale at the end of the production process. A variant on this traditional processing stage model groups PPI products according to their economic sequence in the chain of production and distribution. The approach requires a detailed analysis of national supply and use tables.

[2]See Roger (2000).

13.31 Another example is the net output PPI. In most PPIs, the price index for each industry would be aggregated by the gross output of that industry. This gives rise to concerns that there is a form of double-weighting in industries that produce significant intermediate products within the industry (for example, steel ingots used as an intermediate input to processed steel products). An index that uses net output by industry (excluding the value of intermediate products used from within the industry) avoids this perceived double-weighting problem.

13.32 Both of these examples involve different analytical weighting structures for basic components in an aggregate PPI. They are considerably more complex than the basic PPI itself but have the intuitive attraction of indices that aim to track the change in prices of different components that contribute differentially to overall price movements. As such, they can be presented as interesting and enlightening constructs derived from the basic PPI data.

13.33 Further examples are PPIs for industrial activities and PPIs for services. No country, at this time, has complete coverage of all goods and services in the PPI. Many countries started by developing PPIs for industrial activities (manufacturing, mining, and energy supply) and then progressively added economic activities over time (for example, agriculture, transport services, construction). This results in the availability of a range of PPIs for different sectors of the economy. However, not of all sectors of the national economy have their own PPI.

13.34 Another area of development in PPI indices is business services. In expanding their PPI into services activities, a number of countries have found high user demand for services to businesses (such as advertising, professional services, insurance, etc.). Because of this demand, several countries have developed corporate services PPIs.

B.5.3 Subaggregate indices

13.35 Countries commonly calculate price indices for hundreds of products (for example, bread or footwear) based on thousands of individual price records. Therefore, the number of possible subaggregates is quite large.

13.36 One kind of subaggregation is by groups of products that, when aggregated, comprise the whole of the PPI. An important consideration here is the relationship between different products within the subgroups. For example, an index may be presented for food; under the “food” heading, indices may be presented for subgroups such as breads, cereals, vegetables, and so on.

13.37 Another type of aggregation is by industry. Indices for each 4-digit industry aggregate to 3-digit, 2-digit, and 1-digit groupings. For each aggregate grouping, there are subgroups that represent the industries within the grouping. Another important consideration is that the PPI by industry and the PPI by product produce the same aggregate price change in the overall PPI, so that the weighting structure used in the product and industry aggregations is consistent (see Chapter 4).

13.38 One of the first considerations in presenting such subaggregate data for related products or by industry is consistency over time. That is, there should be a set of subaggregates for which indices are calculated and presented each month. Users commonly attach great importance to being able to continue their analysis from month to month.

13.39 Another consideration is international standardization of the division of the index into groups of goods and services, which enables comparison among countries. Some countries also have their own subaggregate groupings that may predate the current international standard. The generally accepted international standards for the presentation of subaggregates is the ISIC, Revision 3.1; the CPA; and the CPC. These classifications are important because they define groups of industries or products by the technology used for production or the purpose for which they are produced (for example, manufactured products or transport services). Many national classifications are derived from these international standards by adapting them to local circumstances. Locally, it is important to identify and include certain modifications that make the classification more useful and better understood within the country.

13.40 A further type of subaggregate index is an index that is essentially the same as the PPI except that it excludes certain items. The underlying inflation index discussed earlier is an example. Some countries publish, in addition to their *all items* output PPI (at basic prices), an index or indices that can be derived from PPI sources. An example is an input price index that is measured at purchasers' prices and thus includes transport and trade margins paid by producers when purchasing inputs. In the presentation of all related or alternative measures, the concepts and definitions should be made clear, and it is advisable to give the reasons for the alternative presentation. Most importantly, it should not be suggested that the subaggregate index is more meaningful than or superior to the PPI itself.

B.6 Model press release, bulletin, and methodological statement

13.41 An example of a press release for a fictitious country appears in Section E at the end of this chapter. The example provides only text and charts. It does not include data tables that would normally be attached to support the analysis in the text. Other formats are possible; for example, it might include a seasonally adjusted index.

13.42 Note that the example press release contains the following information:

(i) Details of issuing office,
(ii) Date and time of release,
(iii) Percentage change in new month versus a year ago,
(iv) Comparison of percentage change in a new month with that of previous month,
(v) Information on product groups that contributed to change and any significant component price, and
(vi) Reference to where more information can be found.

13.43 Note also that

- No judgments are offered on policy or economic reasons for the price change, and
- No judgment is given on whether the change is good or bad.

13.44 What is not obvious from just one example is that the format should be the same in all releases from month to month. Using a consistent format is important to avoid appearing to indicate a preference. A format with a selected starting date, for example, might indicate a preferred trend.

13.45 Other pages of the press release should give the monthly indices (base period equals one hundred) from which the percentage changes are calculated. Similar indices should also be given for major groups of goods and services. Charts may also be used to illustrate, for example, which prices have contributed most or least to the overall PPI.

13.46 If any other producer price variant is also being published, then the differences between the indices should be briefly explained, including any methodological differences. Variants that require explanation include stage-of-processing indices and any regional indices or PPI variants that include particular components of producers' expenditure, such as the purchase of inputs, including margins. More detailed explanation can be found in handbooks.

13.47 In addition, the press release should include a short note on methodology similar to the following:

> **What Is the Producer Price Index Measuring and How Is It Done?**
>
> The all-items PPI is an overall measure of the change in prices received by producers for their output, valued at basic prices. The PPI is a key indicator of price movements that contribute to inflation. It measures the average change in prices, from month to month, of the goods and services sold by producers.
>
> Prices are collected each month from establishments that produce goods and services. The amount of revenue received by producers for these goods and services is derived from a regular census of establishments. The prices and revenue received are then combined to calculate the price indices for divisions and groups of industries, and for the all- items index.
>
> The overall index, with all of its component indices, is published each month in our *PPI Bulletin*. The *Bulletin* also contains more information on the methodology used in calculating the PPI. A small booklet is also available. For a detailed account of the methodology used in calculating the PPI, the National Statistical Office has published the *PPI Technical Manual*. For more information on these publications and how they may be obtained, please refer to our website at www.nso.gov.cy or call the numbers listed on the front of this press notice.

B.7 UN's *Fundamental Principles of Official Statistics*, IMF data standards, and ILO standards

13.48 Many international standards apply, in general terms or specifically, to the PPI. One very general but fundamental standard is the UN's *Fundamental Principles of Official Statistics* (1994). It is available in several languages on the UN's website. It refers not just to dissemination but to all aspects of statistical work.

13.49 The introduction to this chapter lists some of the broad principles that are reflected in many of the international standards in some form.

13.50 IMF standards are particularly pertinent in this context because they are specifically aimed at dissemination issues. There are two that refer to statistics, including producer price indices. One is the GDDS and the other is the SDDS. The GDDS provides a general framework with some specific indicators defined as *core* and others defined as *encouraged*. The SDDS is based on the GDDS framework but is more demanding and applies only to those countries that choose to subscribe to it by writing to the IMF Board. Both are available on the IMF Dissemination Standards Bulletin Board (www.dsbb.org).

13.51 The GDDS has several dimensions for dissemination standards. Under the heading of *quality*, the GDDS refers to the necessity to provide information on sources, methods, component detail, and checking procedures. Under *integrity* it refers to declared standards of confidentiality, internal government access before data release, identification of ministerial commentary, information on revision, and advance notice of changes in methodology. Under *access by the public*, it refers to the need for preannounced release dates and simultaneous access for all users. In the tables of data categories, it refers to the PPI as a *core indicator* that should be issued monthly, within one to two months of the data collection date. All of these standards are reflected in the present *Manual*. The ILO also has guidelines on the dissemination of labor statistics on its website (www.ilo.org).

C. Dissemination Issues

C.1 Timing of release

13.52 The PPI should be released as soon as possible (see the discussion in the following section), but it is equally important to release the index according to a strict timetable with an unambiguous embargo time to ensure simultaneous access. It is also important to publish the timetable of release dates as far in advance as possible. Having a fixed release date, published well in advance, is important for two main reasons. First, it reduces the scope for the manipulation of the release date for political expediency. Second, it gives confidence to users that the release date is as early as possible and has not been delayed (or brought forward) for purely political reasons. A third advantage is that users know when to expect the figures and can be prepared to use them.

C.2 Timeliness of release versus data accuracy

13.53 The IMF's GDDS, discussed in Section B.7 above, recommends that the PPI be released within one to two months after the data collection month. It is customary for most countries to release the PPI in the middle of the month after the month covered by the index. This is possible because, in many cases, the data are collected mainly over a limited period in the middle of the month to which the latest data refer. Thus, the statisticians have some time to check and analyze the data, and to prepare the many tables and charts in which the data will be disseminated.

13.54 The accuracy of the index is particularly important because so much depends on the PPI. In addition to the economic policy implications of the index, its components are used in many countries as deflators in the national accounts to derive constant price GDP; they are also used in a variety of commercial contracts. Perhaps the best-known contractual use is the indexing of material inputs.

13.55 The PPI may be subject to revision, depending on the data collection method used and the timeliness of source price data. When PPI data are collected though personal visits, the source prices are practically all available by the end of the month. In such cases, it is rare for the PPI to be revised after first publication. This represents a major difference between the PPI and other economic or socioeconomic aggregates, which are often subject to revision at a later date. In other instances, such as when the PPI source data are collected by a mail survey, the returns arrive more slowly and may not all be available at the time of first publication. In such instances, the statistical office may institute a revision policy in which the monthly PPI is first published on a preliminary basis; then a final estimate is published one to three months later when practically all sample returns have been received.

13.56 It follows that although timeliness is important, the timetable must allow time for the data to be properly prepared and thoroughly checked. In most cases, a revision to the nonseasonally adjusted PPI is not permitted after the release date. If a revision policy is in effect or the PPI series is revised on an ad hoc basis, then the policy or the changes must be fully described and explained when the new data are released. If there is any methodological change, then users should be advised several months before the change occurs.

C.3 Access to data

13.57 With the PPI, as with other statistics, users should be allowed access to as much data as possible for two main reasons. First, some users find the detailed data very useful in their analysis. Second, access to the details increases the understanding of and confidence in the data.

13.58 There are, however, limits on the amount of data that can be made available to users. One constraint is confidentiality, which is addressed in Section C.4. Another is the limited volume of data that most users can absorb. Still another reason is the cost of publishing large amounts of data that few users need.

13.59 In general, the PPI and its major components are deemed to be of such wide importance that they are freely available through press releases and statistical office websites. More detailed data are often published only in statistical office bulletins and other media, and users are charged fees so the statistical office can recover some of the dissemination costs. Similarly, particular users requesting special analyses are usually charged a rate commensurate with the work involved.

13.60 The volume of data to which users should be given access through the various media is also discussed in Sections C.4 and C.5.

C.4 Confidentiality

13.61 Although as much data as possible should generally be made available to users as explained above, there are reasons why confidentiality is important in most instances.

13.62 First, most data supplied by establishments are provided on the understanding that the data will be used only for the purpose of aggregation with other data and will not be released in any other form. This can be especially important when the data are supplied voluntarily, as they often are. Most statistical offices make a pledge that the price data are strictly confidential, or confidentiality requirements may be included in statistical legislation. In such cases, the statistical office will not provide the information to any other organizations or publish the data in a manner that would reveal the individual respondent's information. Many agencies have rules about the minimum number of establishments (for example, three or more) that must report before data can be published or released. In addition, many statistical offices have rules about dominant enterprises within an industry (for example, 75 percent of production), so that data for large firms are not divulged without the firm's consent.

13.63 Second, only a sample of particular product transactions are priced as representative of a much larger group of products. If it were known which varieties are included in the index and which are not, then it might be possible to bias components of the index by manipulating a small number of prices.

13.64 Even the knowledge that price data are or might be collected on one particular day in the month could enable some component price indices to be biased by respondents choosing to change prices on a particular day. However, this provides only a short-run advantage and cannot be sustained.

C.5 Issues of electronic or Internet release

13.65 The World Wide Web has several advantages as a dissemination medium. For the data producer, distribution costs are relatively small, involving no printing or mailing. As soon as the data are on the Web, they are available to all Web users at the same time. Putting a large amount of data on the Web costs little more than doing the same with a much smaller amount. Web users can download the data without rekeying, thus increasing speed and reducing transmission or transposition errors.

13.66 One disadvantage is that all data users do not have equal access to the Web. Another is that users may go straight to the data without reading the metadata that may be crucial to understanding the data. Also, it may be as easy for a user to disseminate the PPI widely by electronic means as it is for the statistical office, thus enabling users to preempt the statistical agency by providing statistics without the metadata that may prevent a misunderstanding of the figures.

13.67 Ideally the PPI—complete with any essential metadata—is released simultaneously to the press and other users. Some statistical offices achieve this by bringing the journalists together, perhaps half an hour before the official release time, to provide them with the printed press release, explain the data, and answer any questions. Then, at release time, the journalists are permitted to transmit the figures to their offices for wider distribution.

13.68 In essence, care must be taken to ensure that the PPI is available at the same time to all users regardless of the dissemination medium used.

D. User Consultation

D.1 Explanation of different uses of PPIs

13.69 The different uses of PPIs are discussed in some detail in Chapter 2. It is important to explain to potential users of the PPI what are suitable uses and what are not. To this end, it is important to explain how the PPI is constructed, in terms of its sources and methods (see Section D.2).

13.70 It is also important to make readily available explanations of alternative or subindices such as stage-of-processing indices, indicating how their uses are different from or supplement the uses of the PPI itself.

D.2 Presentation of methodology

13.71 When the PPI is published each month, users are anxious to see the main figures and to use them. They do not generally want to be burdened with explanations of the methodology underlying the data. Nevertheless, methodological explanations must be accessible to those who may want them and in forms that are comprehensible to users with different levels of expertise and interest.

13.72 Any significant changes in methodology must be fully explained, and users must be notified as far in advance as possible of the change being made.

13.73 In addition to a brief statement in press releases (see Section B.6), methodological explanations should be available on at least two levels. Nonexperts should be able to refer to a booklet that explains the history, principles, and practice underlying the PPI and any alternative measures that may be available. A more thorough explanation of sources and methods should be readily available to those users who are sufficiently interested to use it; an example would be statisticians in training who may be new to working in the production of the PPI. The information must also be kept up to date despite the pressures to devote time to the output at the expense of documentation.

13.74 As noted earlier, the ready availability of a full explanation of sources and methods is essential to confidence and trust in the PPI.

D.3 Role of advisory committees

13.75 For a statistical series as important as the PPI, it is essential that there is an advisory committee or set of committees that is representative of users and producers. There are many contentious issues in the construction of the PPI. In many countries, there has been debate about, for example, which components to include and exclude, particularly when the industrial scope of the PPI is being expanded. The role of an advisory committee is to consider and advise on issues contentious and otherwise. But perhaps its equally important role is *presentational*, in that it provides evidence that the PPI can be trusted and is not a tool of government propaganda.

13.76 In those countries where advisory committees have not been the norm, there may be a fear on the part of statisticians that by including nongovernment participants, their expectations may be raised beyond what the statisticians can deliver, thus increasing their dissatisfaction. On the other hand, the inclusion of nongovernment users can lead to a greater understanding of the realities and practical constraints of meeting theoretical needs. This is the usual experience of statistical offices that already have advisory bodies that include representatives of all the major constituencies inside and outside government.

13.77 It is, therefore, important that the advisory committee should be composed of academics, employers, trade unions, and others who have an interest in the index from differing points of view. It is also important that its reports are made available to the public fully and without undue delay.

D.4 Presentation of issues concerning index quality

13.78 The PPI may be regarded with suspicion at many different levels. It usually refers to producers in the industrial sector (mining, manufacturing, and energy supply), but this sector is becoming a smaller segment of the economy. Therefore, unless the PPI is expanded to cover more economic activities, it will be criticized for being less relevant than it was in the past. Also, there may be criticism of the index because of suspicion that it does not keep track of newer types of goods and services, changes in the quality of products, or newer marketing and sales methods. In transition economies, there is also the concern about the ability of the PPI to measure the newly emerging private economy with many small-sized producers.

13.79 In the light of such suspicion, it is important for the producers of the index to be willing to discuss these issues and explain how they are being dealt with. As with other issues discussed here, the producers of the index must be open about their methods and the extent to which they can overcome the theoretical and actual problems that have been identified.

13.80 It follows that the statisticians who produce the index should publish explanations of quality issues, whether or not the quality of the index is being questioned currently.

E. Press Release Example

National Statistical Office of [name of country]

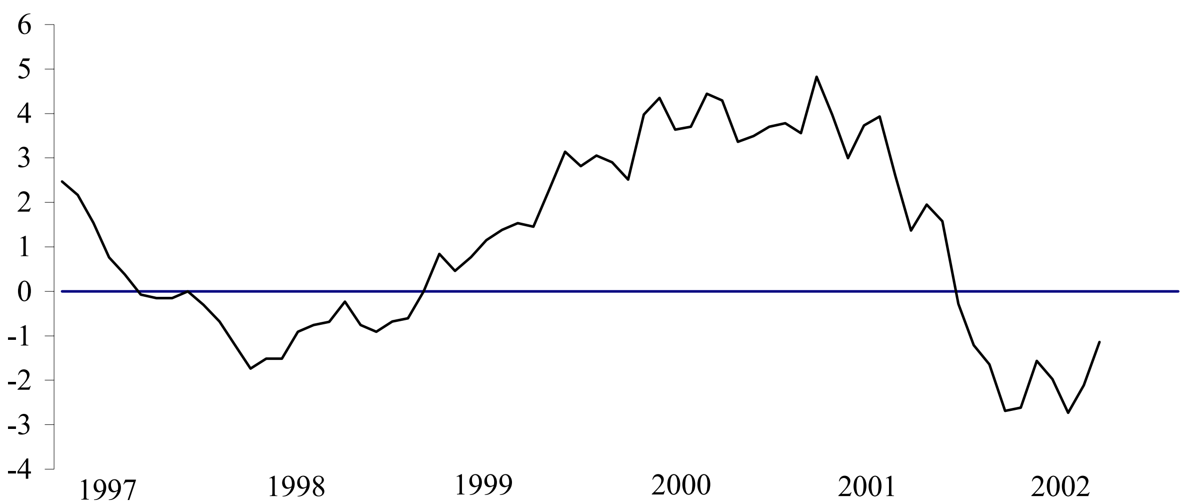
Friday August16, 2002
for release at 11:00 a.m.

PRODUCER PRICE INDEX (PPI)

JULY 2002: PRESS RELEASE

In July 2002, producer prices were 1.5 percent lower than in July 2001 for finished products in the PPI product structure. This 12-month change was less than the 12-month changes recorded in June (–2.7 percent) and November (–3.1 percent).

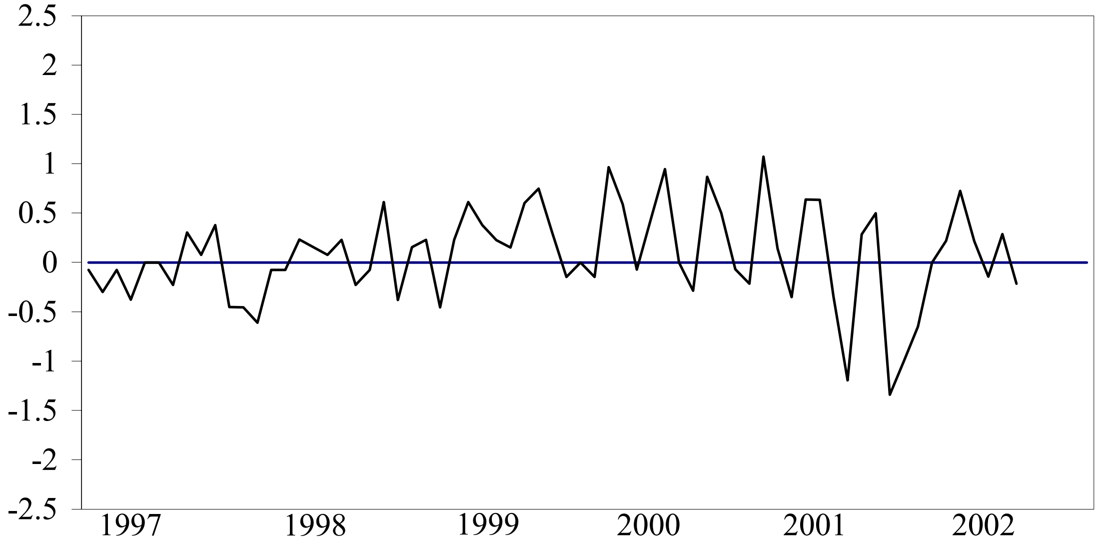
Chart 1. Percentage Change in the Producer Price Index: 1997–2002
Relative to the Same Month of the Previous Year *[line chart]*



Main contributors to the overall 1.5 percent decrease
Over the year the index for finished energy goods dipped 5.2 percent, prices for finished consumer foods declined 1.3 percent, and the index for finished goods other than foods and energy edged down 0.2 percent. At the earlier stages of processing, prices received by producers of intermediate goods decreased 1.5 percent for the 12 months ended July 2002, and the crude goods index fell 6.2 percent during the same period.

Current-period changes
The PPI for finished goods decreased 0.2 percent in July from the June level. Prices for finished consumer goods other than foods and energy declined 0.4 percent in July, compared with a 0.3 percent advance in June. The capital equipment index decreased 0.4 percent in July, compared with a 0.1 percent increase in June. The index for finished consumer foods edged down 0.1 percent, following a 0.1 percent increase in June. The index for finished energy goods increased 0.1 percent in July, after showing no change in the prior month.

Chart 2. Percentage Change in the Producer Price Index: 1997–2002
Relative to the previous month *[line chart]*



Issued by the National Statistical Office of (Country), address of NSO.
Press inquiries 1 111 1111, public inquiries 2 222 2222 (name of a contact is helpful), fax number, and e-mail address.
Background notes on the PPI are given in the annex to this note.
More notes and details are given on our website at www.nso.gov.cy.

PART IV

Conceptual and Theoretical Issues

14. The System of Price Statistics

A. Introduction

14.1 This chapter is about value aggregates and their associated price indices in an integrated system of economic statistics. To understand why value aggregates are important, we foreshadow the next chapter, which addresses concepts for decomposing value aggregates into price and volume components. Chapter 15 begins with defining a value aggregate in equation (15.1) as the sum of the products of the prices and quantities of goods and services. Equations (15.2) and (15.3) characterize a price index as the factor giving the relative change in the value aggregate arising from changes in prices. Not surprisingly then, to define a price index, it is first necessary to define precisely the associated value aggregate.

14.2 Four of the principal price indices in the system of economic statistics—the PPI, the CPI, and the export and import price indices (XPI and MPI)—are well-known and closely watched indicators of macroeconomic performance. They are direct indicators of the purchasing power of money in various types of transactions and other flows involving goods and services. As such, they also are used as deflators in providing summary measures of the volume of goods and services produced and consumed. Consequently, these indices are important tools in the design and conduct of the monetary and fiscal policy of governments, and they are also useful in informing economic decisions throughout the private sector. They do not, or should not, comprise merely a collection of unrelated price indicators but provide instead an integrated and consistent view of price developments pertaining to production, consumption, and international transactions in goods and services. By implication, the meaningfulness of all of these indices derives in no small measure from the significance of the value aggregates to which each refers.

14.3 Section B of this chapter establishes the relationships among the four major price series, as well as their relationships with a number of supporting or derivative price indices. It does this by associating them with certain aspects of the interlocking aggregates defined in the *1993 SNA*. Section C briefly considers purchasing power parities in the system of economic statistics.

14.4 The reader interested in a survey of the goods and services accounts of the *1993 SNA* and how it interrelates to the full range of price indices in the economy will find the entire chapter of interest. Users engaged principally in compiling the PPI should focus on Sections B.1.1, B.1.2, B.1.3.1, B.1.3.2, B.1.3.6, and B.2, since these deal directly with the PPI. This sequence of sections skips over explanations of how the *1993 SNA* builds up the consumption, capital formation, and external trade flows in the supply and use table (SUT) of the *1993 SNA* from the accounts of individual economic agents. Also skipped in this sequence are the total economy price indices for total supply, final uses, GDP, and the price index for labor services.

14.5 Section B.5 also may be of interest to compilers, because it focuses on how the PPI relates to other major price indices. Chapter 4 of this *Manual* on weights and their sources cross-references the current chapter, which defines the institutional unit and transactions scope of the PPI. It also lays out the conceptual framework for the weights of the PPI and its net production and stage-of-processing variants. These sources comprise the Output submatrix of the Supply matrix, and the Intermediate Consumption and Value-Added submatrices of the Uses matrix. Chapter 6 on price collection discusses the practical dimensions of defining the price to be collected, cross-referencing the current chapter regarding the basic price valuation basis for the PPI output aggregates.

B. Major Goods and Services Price Statistics and National Accounts

B.1 National accounts as a framework for the system of price statistics

14.6 The significance of a price index derives from its referent value aggregate.[1] This chapter considers a core system of value aggregates for transactions in goods and services that is clearly of broad economic interest: the system of national accounts. The major price and quantity indices should, in principle, cover those value aggregates in the national accounts system representing major flows of goods and services and levels of tangible and intangible stocks. If the coverage of the major indices is not complete relative to the national accounts aggregates, then it should be compatible with and clearly related to the components of these aggregates. This chapter shows how the national accounts positions headline price indices such as the PPI and CPI, and how we can logically link these indices.

14.7 The *1993 SNA*, paragraph 1.1, describes itself as follows:

> 1.1 The System of National Accounts (SNA) consists of a coherent, consistent and integrated set of macroeconomic accounts, balance sheets and tables based on a set of internationally agreed concepts, definitions, classifications and accounting rules. It provides a comprehensive framework within which economic data can be compiled and presented for purposes of economic analysis, decision taking and policy making.

14.8 The accounts cover the major activities taking place within an economy, such as production, consumption, financing, and the accumulation of capital goods. Some of the flows involved, such as income, saving, lending, and borrowing, do not relate to goods and services, so not all of them can be factored into price and quantity components. However, the *1993 SNA* also contains a comprehensive framework, the supply and use table, discussed in more detail below, within which the relationships among the main flows of goods and services in the economy are established and displayed. The coverage and contents of these flows are defined, classified, and measured in a conceptually consistent manner. The table clearly shows the linkages among major flows of goods and services associated with activities such as production, consumption, distribution, importing, and exporting. It provides an ideal framework for designing and organizing a system of internally consistent price statistics that relate to a set of economically interdependent flows of goods and services. Not only are the relationships among consumer, producer, import, and export prices established within such a table, but so are their linkages with price indices for major macroeconomic aggregates such as GDP.

14.9 This overview of price indices first takes a top-level view of the major national accounts aggregates. It then reviews the underlying construction of these aggregates. It first considers the types of economic agents that the national accounting system recognizes. It then considers the economic accounts kept on transactions that build up to the main aggregates. As these accounts are built from their foundations, precise relationships emerge between the well-known headline price indicators—the PPI, CPI, XPI, and MPI—and the closely watched national accounts aggregates.

B.1.1 Supply and use of goods and services in the aggregate

14.10 At the most aggregate level, the supply and use of goods and services in the national accounts is the simple textbook macroeconomic identity equating total supply with total uses. Total supply is the sum of output Y, imports M, and taxes less subsidies on products T. Total uses is the sum of intermediate consumption Z, the final consumption of households C and government G, capital formation I, and exports X:[2]

[1]As noted in Chapter 2, price indices may be used as deflators and general economic indicators. They also may be used in the calculation of escalators for adjusting contract, government pension, and transfer payments. A distinction may be drawn between a price index, which is defined in this chapter as the price component of relative change in a value aggregate, and an escalator, which is one of the uses of a price index. While an escalator may be chosen as equal to a selected price index, the optimal determination of escalators can lead to more complex functions of price indices than a simple identity relationship.

[2]This chapter uses the standard terminology of the *1993 SNA*, in which net accumulation of current output to enable (continued)

(14.1) $Y + M + T = Z + C + G + I + X$.

14.11 Rearranging this identity by subtracting intermediate consumption and imports from both sides reveals the familiar alternative expressions for GDP from the production (*value-added*) and expenditure approaches:

$$\begin{aligned}(14.2)\ (Y - Z) + T \\ &= \text{value added } + T \\ &\equiv C + G + I + X - M \\ &= \text{ GDP.}\end{aligned}$$

GDP is internationally recognized as the central national accounts aggregate for measuring national economic performance. It is essentially a measure of production, distinct from final demand. More precisely, it measures the value added of the productive activity carried out by all the units resident in an economy. Since imports are not included in GDP, a price index for GDP tracks internally generated inflation. Compiling indices for tracking the parts of relative change in GDP and its components that can be attributed to price and volume change is perhaps the primary objective for developing price statistics in modern statistical systems.

14.12 As explained in more detail later, the SUT in the *1993 SNA* is a comprehensive matrix covering the economy that exploits the identities, equations (14.1) and (14.2), at a disaggregated level. Each row of the matrix shows the total uses of a commodity, or group of commodities, while each column shows the total supplies from domestic industries and imports. The SUT provides an accounting framework that imposes the discipline of both conceptual and numerical consistency on data on flows of goods and services drawn from different sources. The flows have to be defined, classified, and valued in the same way, and any errors have to be reconciled. The SUT provides a good basis for compiling a set of interdependent price and quantity indices. In the following sections, the various elements or building blocks that make up the SUT are considered.

future production is called Capital formation rather than Investment.

B.1.2 Institutional units and establishments: economic agents and units of analysis in the national accounts

14.13 In building the accounting system and the major aggregates *Y, M, T, Z, C, G, I,* and *X* of equations (14.1) and (14.2), the *1993 SNA* first organizes the economy of a country into the kinds of entities or agents that undertake economic activity. These agents are called *institutional units* and comprise five types of units that are resident in the economy, as well as a single nonresident category—the rest of the world. An institutional unit is *resident* in an economy if its primary center of economic interest is located there.[3] The five types of resident institutional sectors are nonfinancial corporations, financial corporations, general government, households, and nonprofit institutions serving households (NPISHs). Generally speaking, the *1993 SNA* associates with institutional units the ability to hold title to productive assets, and thus they represent the smallest units on which complete balance sheets can be compiled.[4]

14.14 For analyzing production, the *1993 SNA* identifies a unit or agent smaller than an institutional unit, called an *establishment* or *local kind of activity unit* (LKAU). Within an institutional unit, the establishment is the smallest unit organized for production whose costs and output can be identified separately. Generally, establishments specialize in the production of only a few types of output at a single location.

14.15 In addition to production activity, institutional units may engage in final consumption of goods and services and in capital formation, represented by the accumulation of goods and services as productive assets. The *1993 SNA* classification of institutional units into sectors is shown in Box 14.1. Notice that the *1993 SNA* institutional sectors repre-

[3]For example, this is determined by physical domicile for households, according to whether the household has been living within the geographic boundaries of a country for a year or more.

[4]The *1993 SNA* classification or sectoring of institutional units does not strictly follow the legal status of institutional units, but rather their function. Hence, a government-owned nonfinancial enterprise producing output sold at prices substantially covering its costs and for which a balance sheet can be compiled would be classified as a nonfinancial corporation, along with nonfinancial legal corporations. For further details, see *1993 SNA*, Chapter IV.

Box 14.1. Institutional Sectors in the *System of National Accounts 1993*

S.1 Total economy

- **S.11 Nonfinancial corporations**
  Ultimate subdivisions public, national private, and foreign-controlled
- **S.12 Financial corporations**
  Ultimate subdivisions public, national private, and foreign-controlled
  - S.121 Central bank
  - S.122 Other depository corporations
    - S.1221 Deposit money corporations
    - S.1222 Other depository corporations, except deposit money corporations
  - S.123 Other financial intermediaries, except insurance corporations and pension funds
  - S.124 Financial auxiliaries
  - S.125 Insurance corporations and pension funds
- **S.13 General government**
  Alternate scheme n = 1, social security funds shown as a separate branch of government S.1314
  Alternate scheme n = 2, social security funds included as components of central, state, and local branches, and S.1314 deleted
  - S.13*n*1 Central government
  - S.13*n*2 State government
  - S.13*n*3 Local government
  - S.1314 Social security funds
- **S.14 Households**
  Classified according to the largest source of income received
  - S.141 Employers *(Mixed income,*[1] *owning an unincorporated enterprise with paid employees)*
  - S.142 Own-account workers *(Mixed income, owning an unincorporated enterprise without paid employees)*
  - S.143 Employees *(Compensation of employees)*[2]
  - S.144 Recipients of property and transfer income[3]
    - S.1441 Recipients of property income
    - S.1442 Recipients of pensions
    - S.1443 Recipients of other transfers
- **S.15 Nonprofit institutions serving households (NPISHs)**

S.2 Rest of the world

[1]To understand how subsectors S.141 and S.142 of households are formed, an explanation of the term *mixed income* is in order. This, in turn, requires the national accounts income concept of operating surplus. The operating surplus of an enterprise is the residual of the value of output less purchases of goods and services, inputs, wages and salaries, employers' social contributions (social security and pension payments), and taxes net of subsidies payable on production that are unrelated to products. The mixed income of household unincorporated enterprises is algebraically defined identically with the operating surplus of other enterprises. However, for unincorporated household enterprises, the compensation of the owners or proprietors of the enterprise may not be included in the recorded compensation of employees item, and thus the difference between output and operating cost will include compensation for the owners' labor. The distinct terminology merely recognizes that the owners' wages are often inextricably mixed with the operating surplus for these units.

[2]Compensation of employees comprises wages and salaries and the employer-provided benefits comprising employers' social contributions.

[3]Property income comprises interest, dividends, and rent.

sent the units typically covered in economic and household censuses and surveys. The *1993 SNA*, as indicated by its name, focuses on the activities of institutional units that are resident in a nation. A provision for the rest of the world (S.2 in Box 14.1) is made to capture the transactions of resident institutional units with nonresidents. Transactions of nonresidents with other nonresidents are out of scope for the national or regional accounts of a given country or region.

B.1.3 Constructing the system of supply and use flows from accounting data on institutional units

14.16 Equations (14.1) and (14.2) identified the basic aggregates comprising the total supply and uses of goods and services in the economy, and derived GDP in terms of these aggregates. To separate the price and volume components of supply and use, it is necessary to build these basic aggregates from the institutional sector accounts of the economy's economic agents. One must detail the production and consumption activities of these agents, as well as the types of goods and services they produce and consume. The framework within which this information is organized is the SUT in the national accounts. As it is built, the SUT also effectively begins to accumulate data on the price (or quantity or volume) index weights considered in Chapter 4. The basic accounts of the *1993 SNA* in which all of these aggregates are recorded at the level of institutional units are the *production*, *use of income*, *capital,* and *external goods and services* accounts. These accounts organize the information for the following top-level aggregates:

- Production account: Output Y, intermediate consumption Z, and value added $Y - Z$;
- Use of income account: Household consumption C and government consumption G;
- Capital account: Capital formation K; and
- External goods and services account: Exports X and imports M.

B.1.3.1 Recording transactions in goods and services

14.17 Before further elaborating on these four goods and services accounts, it is important to specify how each entry in the value aggregates comprising them is to be recorded. The items in the value aggregate equation (14.1) represent detailed goods and services flows classified into categories of transactions. There are two defining aspects of recording transactions: timing and valuation.

B.1.3.1.1 Timing of transactions covered

14.18 To associate each transaction with a date, the national accounts consider a transaction to have been consummated when the event takes place that creates the liability to pay. In the case of flows of goods and services, this occurs when the ownership of the good is exchanged or when the service is delivered. When change of ownership occurs or the service delivered, a transaction is said to have *accrued*. In general, this time need not be the same as the moment at which the payment actually takes place.

B.1.3.1.2 Valuation

14.19 There are two valuation principles in the national accounts, one for suppliers and one for users. For suppliers, transactions in goods and services are to be valued at *basic prices*. The basic price is the price per unit of a good or service *receivable* by the producer.[5] Because the producer does not receive taxes (if any) on products but does receive subsidies (if any) on products, taxes on products are excluded from the basic price, while subsidies on products are included.[6] The producer also does not receive invoiced transportation and insurance charges provided by other suppliers, or any distribution margins added by other retail or wholesale service producers, and these also are excluded from the basic price. On the other hand, the user, as purchaser, pays all of these charges. Users' purchases are therefore valued at *purchasers' prices*, which add taxes net of subsidies on products and margins for included transportation, insurance, and distribution services to the basic price.

[5]The term *receivable* is used to indicate that the price refers to an *accrued* transaction for the seller, and the term *payable* is used to indicate a transaction that has *accrued* to the purchaser.

[6]The *1993 SNA* distinguishes between *taxes on products* and *other taxes on production*. Taxes net of subsidies on products T include all taxes payable per unit or as a fraction of the value of goods or services transacted. Included in T are excise, sales, and the nonrefundable portion of value-added taxes, duties on imports, and taxes on exports. Subsidies on products include all subsidies receivable per unit or as a fraction of the value of goods or services produced, including in particular subsidies paid on imports and exports. *Other taxes on production* comprise, for example, taxes on real property and taxes on profits. *Other subsidies on production* include, for example, regular payments by the government to cover the difference between the costs and revenues of loss-making enterprises. Of total taxes and subsidies on production, only taxes and subsidies on *products* are considered in defining basic and purchasers' prices. By implication, there are no taxes payable on products included in either of the aggregates Y or M, while subsidies receivable on products are included in these aggregates.

Table 14.1. Production Account for an Establishment, Institutional Unit, or Institutional Sector
(1993 SNA *items in bold refer to flows in goods and services*)

Uses		Resources	
P.2	**Intermediate consumption (purchasers' prices)**	**P.1**	**Output (basic prices)**
B.1	*Gross value added* (balances the account; that is, it is the difference between output [P.1] and intermediate consumption [P.2])		
			Of which, memorandum items breaking down total output for classifying the market/nonmarket status of the producer unit:
		P.11	**Market output**
		P.12	**Output for own final use**
		P.13	**Other nonmarket output**

14.20 Accordingly, output Y and imports M in equations (14.1) and (14.2) are valued at *basic prices*, to which are added taxes less subsidies on products T to arrive at total supply.[7] The components of total uses are valued at purchasers' prices. This is clearly interpreted for the final consumption of households and government. For capital formation expenditures, the notion of purchasers' prices also includes the costs of setting up fixed capital equipment. For exports, purchasers' prices include export taxes net of subsidies, according to the "free on board" (f.o.b.) value at the national frontier. Now each of the four major goods and services accounts are discussed in turn.

B.1.3.2 Production

14.21 An institutional unit engaged in production is said to be an *enterprise*. By implication, any of the five types of resident institutional units can be an enterprise. The *production account* for enterprises in the *1993 SNA* appears, with minor reordering of elements, essentially as shown in Table 14.1. An identical presentation also applies to the establishments or LKAUs owned by enterprises, and, in fact, an establishment can be defined operationally as the smallest unit for which a production account can be constructed. There are cases in which an establishment or LKAU is synonymous with, or at least inseparable from, the institutional unit that owns it. This is true of single-establishment corporations and of household unincorporated enterprises, for example. In other cases, an enterprise may own multiple establishments. The production account also can be produced for various establishment and enterprise groupings, including institutional sectors, but also for establishment industry or activity groups. In the production account and throughout the *1993 SNA*, the transaction codes beginning with "P." refer to entries for transactions in goods and services. The codes beginning with "B." refer to so-called "balancing items," which are defined residually as the difference between a re-

[7]The reader may have noted that transportation, insurance, and distribution margins have somehow disappeared after having been introduced. Whether these services are included with the good or invoiced separately does not affect the total expenditure on goods and services by the purchaser. For the economy as a whole, these transactions cancel out, but when industry activity and product detail are considered, they will have redistributive effects among goods and services products. This point is revisited in the discussion of the supply and use table.

sources total and the sum of itemized uses of those resources.

14.22 For classifying an establishment or LKAU, output is broken down into market output and two types of nonmarket output. Market output (P.11) is sold at economically significant prices substantially covering the cost of production. Nonmarket output is provided without charge or at prices so low they bear no relationship to production cost. The two types of nonmarket output are output for own final use (P.12) and other nonmarket output (P.13). Output for own final use includes the production of, for example, machine tools and structures (fixed capital formation items) by an establishment for the sole use of the establishment itself or other establishments in the same enterprise; the imputed rental value of certain productive assets owned by households, such as (and currently limited to) owner-occupied dwellings; and the production of certain other unincorporated household enterprises, such as agricultural products produced by a farmer for consumption by his own family or his employees. Other nonmarket output comprises the output of general government and NPISHs distributed free or sold at prices that are not economically significant. For price index construction, only those transactions of establishment units that involve economically significant prices, and thus market output (P.11), are relevant. However, the prices collected for market output items also may be used to value the own final use portion of nonmarket output (P.12). Our scope of coverage for price indices thus extends to cover this component of nonmarket output as well.

14.23 A production unit's resources derive from the value of its output, and its uses of resources are the costs it incurs in carrying out production. The production account therefore uses both the basic price and purchasers' price methods of valuation, as appropriate to a production unit in its roles as a supplier and a user of products. For the *supply (resources)* of goods and services, products are valued at *basic prices*, the national currency value *receivable* by the producer for each unit of a product. They *include* subsidies and *exclude* the taxes on products and additional charges or margins on products to pay for included retail and wholesale trade services, and for included transportation and insurance. For *uses* of goods and services, products are valued at *purchasers' prices*, the national currency value *payable* by the user for each unit of a product, *including* taxes on products, trade, and transport margins, and *excluding* subsidies on products.

14.24 *Product detail in the production account.* In addition to breaking down output into its market and nonmarket components, output and intermediate consumption also can be broken down by type of product. Classifying product types using, for example, the international standard CPC, version 1.0, the production account for each establishment appears as in Table 14.2.

14.25 *Industry detail in the production account.* The entries in Table 14.3 of total output by product and total market and nonmarket outputs for each establishment allow us to classify establishments by their principal activity or industry *and* market/nonmarket status. To reflect the information required for this classification, positions for the activity and market or nonmarket classification codes of the establishment are shown in the first line of Table 14.2.[8] The activity classification involves principally, if not exclusively, sorting establishments according to the types of product (CPC code *ccccc* or other product code, such as the CPA) for which the total output is greatest. The major categories of the ISIC, Revision 3, are shown in Box 14.2.

[8]As indicated in Table 14.3, The *1993 SNA* recommends use of the ISIC for activities or industries, the CPC for domestic products, and the closely related HS for exported and imported products. Each country may adapt the international standard to its specific circumstances. If the adaptation amounts to adding further detail, the classification is said to be derived from the international standard. NACE is an industrial classification derived from the ISIC. If the adaptation reorganizes the way in which detailed categories are grouped compared with the international standard but provides for a cross-classification at some level of detail, it is said to be related. NAICS, which is being implemented by Canada, Mexico, and the United States, is an industrial classification related to the ISIC. The European Union's PRODCOM classification of industrial products is derived from its CPA, which, in turn, is related to the international standard CPC through a cross-classification defined at a high level of product detail.

Table 14.2. Production Account with Product Detail for an Establishment/LKAU
(1993 SNA *items in bold refer to flows in goods and services*)

Uses	Resources
P.2 Intermediate consumption (purchasers' prices), ***of which***	**P.1 Output (basic prices),** ***of which***
CPC 0 Agriculture, forestry and fishery products	CPC 0 Agriculture, forestry and fishery products
CPC 1 Ores and mineral; electricity, gas, and water	CPC 1 Ores and mineral; electricity, gas, and water
CPC 2 Food products, beverages and tobacco; textiles, apparel and leather products	CPC 2 Food products, beverages and tobacco; textiles, apparel and leather products
CPC 3 Other transportable goods, except metal products, machinery and equipment	CPC 3 Other transportable goods, except metal products, machinery and equipment
CPC 4 Metal products, machinery and equipment	CPC 4 Metal products, machinery and equipment
CPC 5 Intangible assets; land; constructions; construction services	CPC 5 Intangible assets; land; constructions; construction services
CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services
CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social and personal services	CPC 9 Community, social and personal services
B.1 *Gross value added*	
	Memorandum items breaking down total output for classifying the market/nonmarket status of the producer:
	P.11 Market output
	P.12 Output for own final use
	P.13 Other nonmarket output

Note: Establishment ID: eeeeeeee. Activity/industry code (ISIC): aaaa. Institutional unit ID: uuuuuuuu.
Institutional sector code: S.nnnnn. Market status: P.1n.

14.26 The associated products are grouped in the production accounts by activity and output transaction status, and each entry of the accounts is summed across all establishments within each industry *and* output transaction status category. Table 14.3 shows a model production account for an industry (identified by activity code *aaaa*). This account is an aggregate of the production accounts of establishments classified into that industry and according to whether they are principally market, own final use, or other nonmarket producers. In most cases, both the establishment and industry production accounts would show higher product detail than has been shown here, preferably at the four- or five-digit CPC level, or higher with country-specific extensions.

14.27 *Output aggregate for the PPI in the production account.* The PPI is an index of the prices of the outputs of establishments. The position of the PPI in the *1993 SNA* is defined by the relationship of its output value aggregate to those defined in the national accounts. Box 14.2 considers the formation of the PPI value aggregate according to its industry coverage, arguing that the PPI's industry coverage should be complete. The coverage of the PPI across the type of output by market status is shown under the column of Table 14.3 labeled P.11 Output (basic prices), market. For most establishments, output for own final use, P.12, comprises only capital formation, such as acquisition of machine tools or construction. Household establishments also may pro-

Table 14.3. Industry/Activity Production Account with Detail for Products and Market/Nonmarket

(1993 SNA *items in bold refer to flows in goods and services*)

Uses						Resources					
P.2 Intermediate consumption (purchasers' prices), market, *of which*		**P.2 Intermediate consumption (purchasers' prices), own final use, *of which***		**P.2 Intermediate consumption (purchasers' prices), other nonmarket, *of which***		**P.11 Output (basic prices), market, *of which***		**P.12 Output (basic prices), own final use, *of which***		**P.13 Output (basic prices), other nonmarket, *of which***	
						PPI output aggregate					
CPC 0	Agriculture, forestry, and fishery products	CPC 0	Agriculture, forestry, and fishery products	CPC 0	Agriculture, forestry, and fishery products	CPC 0	Agriculture, forestry, and fishery products	CPC 0	Agriculture, forestry, and fishery products	CPC 0	Agriculture, forestry, and fishery products
CPC 1	Ores and mineral; electricity, gas, and water	CPC 1	Ores and minerals; electricity, gas, and water	CPC 1	Ores and minerals; electricity, gas, and water	CPC 1	Ores and minerals; electricity, gas, and water	CPC 1	Ores and minerals; electricity, gas, and water	CPC 1	Ores and minerals; electricity, gas, and water
CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2	Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3	Other transportable goods, except metal products, machinery and equipment	CPC 3	Other transportable goods, except metal products, machinery, and equipment	CPC 3	Other transportable goods, except metal products, machinery, and equipment	CPC 3	Other transportable goods, except metal products, machinery, and equipment	CPC 3	Other transportable goods, except metal products, machinery, and equipment	CPC 3	Other transportable goods, except metal products, machinery, and equipment
CPC 4	Metal products, machinery and equipment	CPC 4	Metal products, machinery and equipment	CPC 4	Metal products, machinery, and equipment	CPC 4	Metal products, machinery and equipment	CPC 4	Metal products, machinery and equipment	CPC 4	Metal products, machinery, and equipment
CPC 5	Intangible assets; land; constructions; construction services	CPC 5	Intangible assets; land; constructions; construction services	CPC 5	Intangible assets; land; constructions; construction services	CPC 5	Intangible assets; land; constructions; construction services	CPC 5	Intangible assets; land; constructions; construction services	CPC 5	Intangible assets; land; constructions; construction services
CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services
CPC 7	Financial and related services; real estate services; and rental and leasing services	CPC 7	Financial and related services; real estate services; and rental and leasing services	CPC 7	Financial and related services; real estate services; and rental and leasing services	CPC 7	Financial and related services; real estate services; and rental and leasing services	CPC 7	Financial and related services; real estate services; and rental and leasing services	CPC 7	Financial and related services; real estate services; and rental and leasing services
CPC 8	Business and production services	CPC 8	Business and production services	CPC 8	Business and production services	CPC 8	Business and production services	CPC 8	Business and production services	CPC 8	Business and production services
CPC 9	Community, social, and personal services	CPC 9	Community, social, and personal services	CPC 9	Community, social, and personal services	CPC 9	Community, social, and personal services	CPC 9	Community, social, and personal services	CPC 9	Community, social, and personal services
B.1	*Gross value added, market output establishments*	B.1	*Gross value added, output for own final consumption establishments*	B.1	*Gross value added, other nonmarket output establishments*						

Note: Activity/industry code (ISIC): aaaa.

Box 14.2. Industry/Activity Coverage of the Producer Price Index Output Value Aggregate

The principal economic activities of the International Standard Industrial Classification of All Economic Activities (ISIC), Revision 3, are

- A Agriculture, hunting, and forestry
- B Fishing
- C Mining and quarrying
- D Manufacturing
- E Electricity, gas, and water supply
- F Construction
- G Wholesale and retail trade; repair of motor vehicles, motorcycles, and personal and household goods
- H Hotels and restaurants
- I Transport, storage, and communications
- J Financial intermediation
- K Real estate, renting, and business activities
- L Public administration and defense; compulsory social security
- M Education
- N Health and social work
- O Other community, social, and personal service activities
- P Private households with employed persons
- Q Extra-territorial organizations and bodies

These are characteristic of the activities identified in most national industrial classifications. In assembling data on the supply and use flows in the economy, a detailed industry production account such as that in Table 14.3 is effectively constructed for each type of activity in the economy. The major activity categories are shown in the ISIC list above. (More is said about the comprehensive presentation of supply and use for the total economy in Section B.1.3.) With the product output and expenditure, detail Table 14.3 shows more explicitly the typical goods and services coverage of the PPI within the output aggregate P.1 of the production account for each industry. In most countries, PPIs cover goods-producing industries, such as the "mining and manufacturing" activities C and D and sometimes also agriculture (A), fishing (B), and construction (F). Most PPIs also cover the two "industrial" service activities—electricity, gas, and water supply (E) and transport, storage, and communications (I). In principle, the PPI should cover the market output of all activities, and a number of countries are working on rounding out PPI coverage to other service-producing activities beyond transportation and utilities.

duce goods for households' own consumption, such as food, and this activity is included within the *1993 SNA* production boundary. Large portions of P.12, output for own final use, may be valued at market prices if close market substitutes are available but otherwise at the cost of production (*1993 SNA*, Paragraph 6.85). In principle, the weighting of items in the PPI could be extended to cover the market-valued portion of P.12. The scope of the PPI would not extend to P.13, other nonmarket output, since this is almost without exception valued at production cost because rarely are market equivalents available, and thus no basis for constructing an explicit price index exists.

B.1.3.3 Consumption

14.28 Final consumption of goods and services in the *1993 SNA* is shown in the *use of income account*, which appears essentially as in Table 14.4 for each institutional unit. Recall that the *1993 SNA* designates goods and services items with the codes "P.*n*." These goods and services flows can be decomposed into price and volume components and thus would draw our interest as price index compil-

Table 14.4. Use of Income Account for Institutional Units and Sectors
(1993 SNA *items in bold refer to flows in goods and services*)

Uses	Resources
P.3 Final consumption expenditure (purchasers' prices)[1]	B.6 *Disposable income*[2]
P.31 Individual consumption expenditure *P.311 Monetary consumption expenditure* *P.312 Imputed expenditure on owner-occupied housing services* *P.313 Financial Intermediation Services Implicitly Measured (FISIM)* *P.314 Other individual consumption expenditure* **P.32 Collective consumption expenditure (general government sector S.13 only)** D.8 Adjustment for the change in the net equity of households in pension funds[3]	
B.8 *Saving* (balances the account; the difference between disposable income [B.6] and the sum of expenditures [P.3] and adjustment [D.8])	

Note: Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn.
[1]By definition, corporations have no final consumption in the *1993 SNA*. Thus, item P.3 and its subdivisions appear with non-zero entries only for household, government, and NPISH units.
[2]The *1993 SNA* derives disposable income in a sequence of accounts producing the balancing items *Value added* B.1 (production account), *Operating surplus* B.2 and *Mixed income* B.3 (generation of income account), *Balance of primary incomes* B.5 (allocation of primary income account), and *Disposable income* B.6 (secondary distribution of income account). Collapsing all of these steps, *Disposable income* B.6 is *Value added* B.1 *less* (net) taxes on production and imports (payable) D.2 *plus* (net) subsidies D.3 (receivable), *plus* compensation of employees receivable, *plus* (net) property income (receivable) D.4, *less* (net) taxes on income and wealth (payable) D.5, *less* (net) social contributions (payable) D.61, *plus* (net) social benefits (receivable) D.62, *less* (net) other transfers (payable) D.7.
[3]This adjustment reflects the treatment by the *1993 SNA* of privately funded pensions as owned by the household beneficiaries of such plans. It maintains consistency between the income and accumulation accounts in the system. It is not relevant to price and volume measurement, and the reader is referred to the *1993 SNA*, Chapter IX, Section A.4, for further details.

ers. Items of final consumption are designated by P.3 with extensions. P.3 comprises individual consumption expenditure (P.31) and collective consumption expenditure (P.32).[9]

[9]Final consumption expenditure (P.3) is made by institutional units classified in the general government (S.13), household (S.14), and NPISH (S.15) institutional sectors only. Corporations (S.11) and (S.12) do not have final consumption expenditure, and thus for these units operating surplus (B.2) is equal to saving (B.8) in the use of income account (Table 14.4).

14.29 *Individual consumption, actual consumption and household consumption expenditures.* The *1993 SNA* distinguishes individual from collective goods and services, a distinction equivalent to that between private and public goods in economic theory. It is mainly relevant to services. Individual services are provided to individual households and benefit those particular households, whereas collective services are provided to the community: public order, administration, security, and defense. Many individual services, such as education, health, hous-

Box 14.3. The Treatment of Housing and Consumer Durables in the *1993 SNA* and CPIs

Dwellings are fixed assets. Purchases of dwellings by households therefore constitute household gross fixed capital formation and are not part of household consumption. They cannot enter into a price index for household consumption. Fixed assets are used for purposes of production, not consumption. Dwellings therefore have to be treated as fixed assets that are used by their owners to produce housing services. The *1993 SNA* actually sets up a production account in which this production is recorded. The services are consumed by the owners. The expenditures on the services are imputed, the services being valued by the estimated rentals payable on the market for equivalent accommodation. The rentals have to cover both the depreciation on the dwellings and the associated interest charges or capital costs.

The existence of these imputed expenditures on owner-occupied housing services has always been recognized in national accounts, and most countries also have included them in their CPIs, even though other imputed expenditures are not included.

Consumer durables, such as automobiles, cookers, freezers, etc., also are assets used by their owners over long periods of time. In principle, they could be treated in the same way as dwellings and be reclassified as fixed assets that produce flows of services consumed by their owners. For certain analytic purposes, it may be desirable to treat them this way. However, to do so in the *1993 SNA* would not simply be a matter of estimating the market rentals that would be payable for hiring the assets. It also would be necessary to set up production accounts in which the durables are used as fixed assets. This has traditionally been regarded as too difficult and artificial. There also are objections to extending further the range of imputed flows included in the *1993 SNA* and GDP. In practice, therefore, expenditures on durables are classified in the *1993 SNA* as consumption expenditures and not as gross fixed capital formation, a practice carried over into CPIs.

ing, and transportation, may be financed and paid for by government or nonprofit institutions and provided free or at a nominal prices to individual households. A large part of government consumption expenditure is not on public goods but on goods or services supplied to individual households. These individual consumption expenditures by governments and NPISHs are described as *social transfers in kind* in the *1993 SNA*.

14.30 Household consumption can have three distinct meanings. First, it can mean the total set of individual consumption goods and services *acquired* by households, including those received as social transfers in kind. Second, it can mean the subset that households actually *pay for* themselves. To distinguish between these two sets, the *1993 SNA* describes the first as the *actual final consumption* of households and the second as *household final consumption expenditures*. A third possible interpretation of household consumption is the actual physical process of consuming the goods and services. It is this process from which utility is derived and that determines households' standard of living. The process of consuming or using the goods or services can take place some time after the goods or services are acquired, since most consumer goods can be stored. The distinction between acquisition and use is most pronounced in the case of consumer durables that may be used over a long time. The treatment of durables is discussed further in Box 14.3.

14.31 The existence of social transfers in kind is not recognized in CPIs, although one should take account of them, especially when considering changes in the cost of living. Moreover, governments may start to charge for services that previously were provided free, a practice that has become increasingly common in many countries. The goods and services provided free as social transfers could, in principle, be regarded also as being part of household consumption expenditures but having a zero price. The shift from a zero to positive price is then a price increase that could be captured by a consumer price index.

14.32 *Monetary and imputed expenditures.* Not all household expenditures are monetary. A monetary expenditure is one in which the counterpart to the good or service acquired is the creation of some kind of financial liability. This may be immediately extinguished by a cash payment, but many monetary expenditures are made on credit. Household consumption expenditures also include certain imputed expenditures on goods or services that households produce for themselves. These are treated as expenditures because households incur the costs of producing them (in contrast to social transfers in kind, which are paid for by government or nonprofit institutions).

14.33 The imputed household expenditures recognized in the *1993 SNA* include all of those on goods that households produce for themselves (mainly agricultural goods in practice) but exclude all household services produced for own consumption *except* for housing services produced by owner occupiers. The imputed prices at which the included goods and services are valued are their estimated prices on the market. In the case of housing services, these are imputed market rentals. In practice, most countries follow the *1993 SNA* by including owner-occupied housing in the CPI. It is not customary, however, to include other imputed prices, such as the prices of vegetables, fruit, dairy, or meat products produced for own consumption.

14.34 *A hierarchy of household consumption aggregates.* The following hierarchy of household consumption aggregates that are relevant to CPIs may be distinguished in the *1993 SNA*. It is worth noting that all household consumption expenditures are individual expenditures, by definition.

P.41 Actual individual consumption, *of which*
- D.63 Social transfers in kind (individual consumption expenditure P.31 of general government S.13 and NPISHs S.15).
- P.31 Individual consumption expenditure, *of which*
  - P.311 Monetary consumption expenditure;
  - P.312 Imputed expenditure on owner-occupied housing services;
  - P.313 Financial Intermediation Services Implicitly Measured (FISIM);
  - P.314 Other individual consumption expenditure:
    - Expenditures on nonhousing production for own consumption;
    - Expenditures on goods and services received by employees as income in-kind.

14.35 The codes P.311, P.312, P.313, and P.314 do not exist in the *1993 SNA* but are introduced for convenience here. These four subcategories of household consumption expenditures are separately specified in Tables 14.4, 14.5, and 14.6. As already noted, D.63 and P.314 are usually excluded from the calculation of CPIs.

14.36 It is worth noting that the treatment of financial services in the *1993 SNA* would imply an augmented treatment of financial services consumption expenditures to include expenditures on bank services not separately distinguished from interest charge, as well as the explicit expenditures on service charges charged directly. This is indicated in the footnote to the CPC 7 item in Table 14.5.

14.37 *Product detail in the use of income account.* As with the production accounts of establishments owned by institutional units, the product detail of goods and services consumption can be expanded in the use of income account according to the type of product consumed.[10] To maintain the integration of the system of price and volume statistics on consumption with those that have just been covered on production, products would be classified according to the same system as in the production

[10]Although the discussion in this chapter maintains a consistent product classification of expenditure across all goods and services accounts, alternative, functional classifications of expenditure have been developed for each institutional sector for specific purposes. The international standard versions of these classifications included in the *1993 SNA* comprise the Classification of Individual Consumption by Purpose (COICOP), the Classification of the Purposes of Nonprofit Institutions Serving Households (COPNI), the Classification of the Functions of Government (COFOG), and the Classification of the Purposes of Producers (COPP). The first column of Tables 14.5 and 14.6 is often compiled from household expenditure survey data, which are collected using functional classifications such as COICOP rather than product classifications. To facilitate constructing the cross-economy framework of the *1993 SNA* considered in this chapter, there is a concordance between the CPC and the COICOP.

Table 14.5. Use of Income Account with Product Detail for Institutional Units and Sectors

(*Left columns show detail of far right column;* 1993 SNA *items in bold refer to flows in goods and service, sector titles in italics indicate whether the column appears in the use of income account for that sector*)

Uses					Resources
P.31 Individual consumption expenditure			**P.32 Collective consumption expenditure**	**P.3 Final consumption expenditure (total, purchasers' prices)**	B.6 *Disposable income*
Monetary individual consumption expenditure (P.311)	*Consumption of owner-occupied housing services (P.312) and FISIM (P.313)*	*Other individual consumption expenditure (P.314)*	*Entries for general government sector (S.13) only*		
CPC 0 Agriculture, forestry, and fishery products		CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	
CPC 1 Ores and minerals; electricity, gas, and water		CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products		CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	
CPC 3 Other transportable goods, except metal products, machinery, and equipment		CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	
CPC 4 Metal products, machinery, and equipment		CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	
CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services		CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services[1]	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	
CPC 8 Business and production services		CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	
CPC 9 Community, social, and personal services		CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	
				D.8 Adjustment for the change in the net equity of households in pension funds	
				B.8 *Saving*	

Note: Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn.

[1]In addition to the real estate, rental, and leasing services of homeowners, the *1993 SNA* treats financial services consumption expenditure as the sum of measured and imputed components. Measured expenditures comprise explicit service charges levied by financial institutions for deposit, loan, advisory services, and the like, while imputed expenditures reflect the income foregone because the household does not lend (keep deposits with a financial institution) or borrow at a reference rate. See Chapter 10. In principle, these imputed expenditures, as well as those for other imputed consumption, are of the same market-equivalent valued type as for owner-occupied housing services and could be covered in the CPI.

Table 14.6. Use of Income Account with Product Detail for the Total Economy
(*Left columns show detail of far right column*; 1993 SNA *items in bold refer to flows in goods and services*)

					B.6 *Disposable income*, total economy S.1, *with uses comprising*
P.31 Individual consumption expenditure, total economy S.1 (purchasers' prices), *comprising*				**P.32 Collective consumption expenditure, total economy S.1 (purchasers' prices)**, *comprising*	**P.3 Final consumption expenditure, total economy S.1,** ***of which***
P.31 Individual consumption expenditure, household sector S.14			**P.31 Individual consumption expenditure, general government S.13 and NPISH S.15 sectors**	**P.32 Collective consumption expenditure, general government sector S.13**	
Consumer price index reference aggregate #1					
CPI reference aggregate #2[1]					
Monetary individual consumption expenditure (P.311)	*Consumption of owner-occupied housing services (P.312) and FISIM (P.313)*	*Other individual consumption expenditure (P.314)*			
CPC 0 Agriculture, forestry, and fishery products		CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products
CPC 1 Ores and minerals; electricity, gas, and water		CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products		CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment		CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment
CPC 4 Metal products, machinery, and equipment		CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment
CPC6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services		CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services
CPC 8 Business and production services		CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social, and personal services		CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services
					D.8 Adjustment for the change in the net equity of households *S.14* in pension funds
					B.8 *Saving, total economy S.1*

Note: Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn.
[1]See also Table 14.7, Capital Account.

Table 14.7. Capital Account
(*Items in bold refer to flows of goods and services*)

Uses	Resources
P.5 Gross capital formation, *of which*	B.10.1 *Changes in net worth due to saving and capital transfers, of which*
P.51 Gross fixed capital formation	
	B.8n *Saving, net*
P.511 Acquisitions less disposals of tangible fixed assets	B.8 *Saving (from use of income account)*
P.5111 Acquisitions of new tangible fixed assets	K.1 Consumption of fixed capital (–)
Of which, residential dwellings	
CPI reference aggregate #2	
P.5112 Acquisitions of existing tangible fixed assets	
Of which, residential dwellings	
CPI reference aggregate #2	
P.5113 Disposals of existing tangible fixed assets	
Of which, residential dwellings	
CPI reference aggregate #2	
P.512 Acquisitions less disposals of intangible fixed assets	
P.5121 Acquisitions of new intangible fixed assets	D.9 Capital transfers receivable (+)
P.5122 Acquisitions of existing intangible fixed assets	D.91 Investment grants
P.5123 Disposals of existing intangible fixed assets	
	D.9 Capital transfers payable (–)
P.513 Additions to the value of nonproduced nonfinancial assets	D.91 Capital taxes payable
P.5131 Major improvements to nonproduced nonfinancial assets	D.91 Other capital transfers payable
P.5132 Costs of ownership transfer on nonproduced nonfinancial assets	D.92 Other capital transfers receivable
P.52 Changes in inventories	
P.53 Acquisitions less disposals of valuables	
K.1 Consumption of fixed capital (–)	
K.2 Acquisitions less disposals of nonproduced nonfinancial assets	
K.21 Acquisitions less disposals of land and other tangible nonproduced assets	
K.22 Acquisitions less disposals of intangible nonproduced assets	
B.9 *Net lending (+)/net borrowing (–)*	
Note: Institutional unit ID: uuuuuuu. Institutional sector code: S.nnnnn.	

account. Table 14.5 shows the major categories of the CPC, version 1.0, within the components of final consumption expenditure.

14.38 *The expenditure aggregate of CPI in the use of income account.* The detailed use of income accounts for institutional sectors can be assembled into a consolidated framework by choosing columns from Table 14.5 for each sector and displaying them together as in Table 14.6. Table 14.6 shows an economywide presentation of final consumption and saving. It also shows that total economy individual consumption comprises the individual consumption entries (P.31) of the household, NPISH, and general government sector use of income accounts. Table 14.6 separately shows the final collective consumption of government (P.32) and consolidates the disposable income (B.6) of all three. The account in Table 14.6 has been arranged specifically to show the consumption coverage of the typical CPI, which comprises the first and second columns.

B.1.3.4 Capital formation

14.39 Capital formation comprises the accumulation of fixed tangible and intangible assets, such as equipment, structures, and software; changes in inventories and works in progress; and acquisitions less disposals of valuables, such as works of art. These items are accounted for in the *1993 SNA capital account*, which appears, with minor resorting, essentially as in Table 14.7 for each institutional unit.

14.40 B.9 *Net lending (+)/net borrowing (–)* is the balancing item of the capital account. It makes the uses on the left, comprising net acquisitions of stocks of various tangible and intangible items, add up to the resources on the right, comprising the sources of income financing them. From the section on institutional units and establishments, it would be easy to conclude that the smallest economic unit to which the capital account can apply is the institutional unit. It was asserted earlier that only institutional units maintain balance sheets and can monitor stock variables that are the focus of this account. However, the physical capital assets whose changes are tracked in the capital account should be compiled, if possible, at the establishment or LKAU level to allow production of data on capital formation by industry. Such data are particularly useful for productivity analysis, even though complete capital accounts cannot be compiled at the establishment level. As with the other goods- and services-related accounts in the *1993 SNA*, the capital account's goods and services items, designated by the code P.5 with extensions, can be exploded by product type.[11] The account, therefore, can be rearranged to show this goods and services detail as in Table 14.8, which, as for Table 14.7, may pertain to an institutional unit, an institutional sector aggregate, or the total economy.

B.1.3.5 External trade

14.41 The external account of goods and services is shown in Table 14.9. It contains the transactions of nonresident institutional units sector—S.2 rest of the world—with the five types of resident units taken together. The external goods and services account is generally taken from the Balance of Payments, which uses adjusted merchandise trade information from the customs records for goods on P.61 and P.71, and assembles services data on P.62 and P.72 from various sources (IMF, 1993).[12] Note, however, that the *1993 SNA* differs from the IMF's *Balance of Payments Manual, Fifth Edition* (*BPM5*), in compiling the external accounts from the nonresident's point of view rather than the resident's point of view (a *BPM5* resident's credit or debit is a *1993 SNA* nonresident's debit or credit). As with the other accounts, the external goods and services account can be exploded to show product detail, as in Table 14.10.

14.42 As noted, the *1993 SNA* treats external trade from the point of view of the nonresident buyer in the case of exports and the nonresident seller in the case of imports. Free on board prices for exports are the purchasers' price valuations relevant for nonresident users of goods and services

[11]In addition to the CPC version 1.0 shown here, the *1993 SNA*, Annex V, contains a nonfinancial assets classification identifying the specific tangible, intangible, produced, and nonproduced fixed assets, as well as inventory and valuables items, recognized by the *1993 SNA*.

[12]Services are valued and recorded when performed. Regarding goods, the *BPM5*, Paragraph 114, states that "... goods for export are generally considered to change ownership when the exporter ceases to carry the goods on his books as real assets and makes a corresponding change in his financial items. Goods for import are considered to change ownership when the importer enters the goods as a real asset and makes a corresponding change in his financial items."

Table 14.8. Capital Account with Product Detail
(1993 SNA *items in bold refer to flows in goods and services)*

					B.10.1 *Change in net worth due to saving and capital transfers, with uses comprising*
P. 51 Gross fixed capital formation			**P.52 Change in inventories**[1]	**P.53 Acquisitions less disposals of valuables**[2]	**P.5 Gross capital formation**
P.511 Acquisitions less disposals of tangible fixed assets, *of which*[3]	**P.512 Acquisitions less disposals of intangible fixed assets, *of which***[4]	**P.513 Additions to the value of non-produced non-financial assets, *of which***[5]			
CPC 0 Agriculture, forestry, and fishery products CPC 4 Metal products, machinery, and equipment CPC 5 Intangible assets; land; constructions; construction services *Of which* *Residential dwellings* *CPI reference aggregate #2*	CPC 5 Intangible assets; land; constructions; construction services	CPC 5 Intangible assets; land; constructions; construction services	CPC 0 Agriculture, forestry, and fishery products CPC 1 Ores and minerals; electricity, gas, and water CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products CPC 3 Other transportable goods, except metal products, machinery, and equipment CPC 4 Metal products, machinery, and equipment CPC 5 Intangible assets; land; constructions; construction services	CPC 0 Agriculture, forestry, and fishery products CPC 1 Ores and minerals; electricity, gas, and water CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products CPC 3 Other transportable goods, except metal products, machinery, and equipment CPC 4 Metal products, machinery, and equipment	CPC 0 Agriculture, forestry, and fishery products CPC 1 Ores and minerals; electricity, gas, and water CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products CPC 3 Other transportable goods, except metal products, machinery, and equipment CPC 4 Metal products, machinery, and equipment CPC 5 Intangible assets; land; constructions; construction services *Of which* *Residential dwellings* *CPI reference aggregate #2*
					K.1 Consumption of fixed capital K.2 Acquisitions less disposals of nonproduced nonfinancial assets
					B.9 *Net borrowing/net lending*

Note: Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn.

[1] *1993 SNA* asset code AN.12 Inventories. Excludes intangible assets, land, and constructions.

[2] *1993 SNA* asset code AN.13 Valuables. Excludes intangible assets, land, constructions, and construction services.

[3] *1993 SNA* asset code AN.111 Tangible fixed assets. Excludes intangible assets, land, and construction services.

[4] *1993 SNA* asset code AN.112 Intangible fixed assets. Excludes, land, constructions, and construction services.

[5] *1993 SNA* asset code AN.2 Nonproduced assets. Excludes intangible assets, constructions, and construction services.

Table 14.9. External Account of Goods and Services
(*All resident institutional units S.1.nnnn with nonresident institutional units S.2;* 1993 SNA *goods and services items shown in bold*)

Uses	Resources
P.6 Exports of goods and services	**P.7 Imports of goods and services**
P.61 Exports of goods	**P.71 Imports of goods**
P.62 Exports of services	**P.72 Imports of services**
B.11 *External balance of goods and services*	

supplied by resident providers, and f.o.b. prices for imports are the basic price valuations for nonresident suppliers of imports to resident users.[13] Regarding Table 14.10, the *1993 SNA*, Paragraph 15.68, states that imported goods should be valued at cost-insurance-freight (c.i.f.) at the level of detailed products. On the other hand, like the *BPM5*, the *1993 SNA* requires that, in total, imports of goods be valued f.o.b. at the border of the exporting country.[14] This is managed by excluding insurance and transportation in a single adjustment to total imports c.i.f. (*1993 SNA*, Paragraphs 14.36–14.41). That part of freight services on imports provided by nonresidents is included in imports of transport services, and that part of insurance services provided on imports by nonresidents is added to imports of insurance services. Transportation and insurance services provided by residents on imports are included in exports of transportation and insurance services.[15]

B.1.3.6 The supply and use table

14.43 The SUT arrays the industries side by side first for market producers, then for own-account producers, and then for other nonmarket producers under *Resources* and *Uses*. An SUT is shown in Table 14.11. It arrays various accounts relevant to monitoring developments in production and consumption within a country according to the *supply* of goods and services (with reference to the *1993 SNA* codes labeling the regions of Table 14.11)

- From resident establishments (arranged in industries) in the form of domestic output (P.1), given by Y in equations (14.1) and (14.2);
- From the rest of the world as imports (P.7), given by M in equations (14.1) and (14.2);

[13]Referring to Chapter 17, taking the nonresident's view implies that the export price index is an input price index and the import price index is an output price index. The opposite would be true from the point of view of the residents—the export price index would be an output price index, and the import price index would be an input price index. As shown in Chapter 17, the point of view taken by the *1993 SNA* has implications for the direction of bias in Laspeyres and Paasche export and import price indices relative to the underlying economic index numbers.

[14]Regarding the point in time and space at which the value of a goods transaction is to be assessed, Paragraph 222 of the *BPM5* states that

"The standard, or rule, is that goods shall cover, in principle, the value of goods and related distributive services at the same time the goods reach the customs frontier of the country from which the goods are to be exported. The value of the goods includes the value of any loading of the goods on board the carrier at that frontier. That is, exports and imports of goods are valued f.o.b. at the customs frontier of the exporting economy. ... The customs frontier need not coincide physically with the national boundary and could be located in the interior of the economy."

[15]This rather roundabout approach is taken to imports by product because, as a practical matter, it may be difficult to obtain insurance and freight charges on imports from customs administrative data systems at the product level of detail. (See *1993 SNA*, Paragraphs 14.40–14.41.) Recent developments in computerized customs documentation have made the itemization of insurance and freight more straightforward, and the *1993 SNA* does allow also for the possibility of determining imports by product at their f.o.b. values, consistent with the aggregate valuation of imports. Were this the case, insurance and freight on imports could be shown as trade and transport margins analogously with such margins on domestically produced goods.

Table 14.10. External Account of Goods and Services with Product Detail

(*All resident institutional units S.1 with nonresident institutional units S.2;* 1993 SNA *goods and services items shown in bold*)

Uses	Resources
P.6 Exports of goods and services	**P.7 Imports of goods and services**
Export Price Index uses aggregate	***Import Price Index supply aggregate***
P.61 Exports of goods	**P.71 Imports of goods**
At f.o.b. values	*At f.o.b. values, of which*
	At c.i.f. values:[1,2]
CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products
CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment
CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment
	Less: *Adjustment to total imports of goods c.i.f. for insurance and freight provided by both residents and nonresidents for delivery to the first resident recipient.*
P.62 Exports of services	**P.72 Imports of services**
CPC 5 Intangible assets; land; constructions; construction services[3]	CPC 5 Intangible assets; land; constructions; construction services[4]
CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services, *of which*	CPC 6 Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services, *of which*
• Distributive trade services; lodging; food and beverage serving services; transport services; and utilities-distribution services; *except* transport services on imports and exports rendered by residents	• Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services; *except* transport services on imports rendered by nonresidents
• Transport services on imports and exports rendered by residents	• Transport services on imports and exports rendered by nonresidents
CPC 7 Financial and related services; real estate services; and rental and leasing services, *of which*	CPC 7 Financial and related services; real estate services; and rental and leasing services, *of which*
• Financial and related services; real estate services; and rental and leasing services; *except* insurance services on imports rendered by residents	• Financial and related services; real estate services; and rental and leasing services; *except* insurance services on imports rendered by nonresidents
• Insurance services on imports rendered by residents	• Insurance services on imports rendered by nonresidents
CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services
B.11 *External balance of goods and services*	

[1]The *1993 SNA* values imports f.o.b. However, it allows for the fact that while f.o.b. valuation by product would be consistent and preferred, compiling such data may be problematic at the product level of detail. Imports of goods c.i.f. by product may be all that is available because the insurance and freight data often are not separately compiled by product in customs systems. See *1993 SNA*, Paragraph 15.68. Totals for these data may be obtained instead from resident and nonresident shippers in the process of compiling the Balance of Payments. Insurance and freight services provided by residents on imports are a services export.

[2]Regarding goods and services valuations in the import price and volume indices, see MPI in Tables 14.11 and 14.12, where it is explained that *both* f.o.b. and purchasers' price valuations are important in constructing the MPI as a deflator for imports f.o.b.. Imports at purchasers' prices would be imports c.i.f. *plus* import tariffs as well as domestic insurance and freight for delivery to the first domestic owner.

[3]Construction services only.

[4]Construction services only.

Table 14.11. The Supply and Use Table (SUT)

(*Production account: double outlines and no fill; use of income account: single outlines and no fill; capital account: diagonal fill; external account of goods and services: vertical fill*)

Supply							**Use**						
P.1 Output at basic prices *of establishments producing for:*			Transport and distribution margin adjustment[1]	D.21–D.31 Taxes less subsidies products[2]	Imports c.i.f.[3]		P.2 Intermediate consumption at purchasers' prices *of establishments producing for:*			P.31 Individual consumption	P.32 Collective consumption	P.5 Gross capital formation	P.6 Exports f.o.b.
P.11 Market	P.12 Own use	P.13 Other nonmarket			+ P.7 Imports f.o.b.	+ c.i.f./f.o.b. adjustment[4]	P.11 Market	P.12 Own use	P.13 Other nonmarket				
Product × Industry	*Product × Industry*	*Product × Industry*	+ ***Goods*** (+) ***Services*** (–)	+ ***Product × 1***	***Goods f.o.b.*** ***Services***[6]	***Goods*** (+)[5] ***Services*** (–)	= *Product × Industry*	*Product × Industry*	*Product × Industry*	+ *Product ×* 1	*Product ×* 1	***Product ×* 1**	+ ***Product × 1***
							+ B.1 Value added 1 × *Industry*	+ B.1 Value added 1 × *Industry*	+ B.1 Value added 1 × *Industry*				
= P.11 Output at basic prices 1 × *Industry*	= P.12 Output at basic prices 1 × *Industry*	= P.13 Output at basic prices 1 × *Industry*					= P.11 Output at basic prices 1 × *Industry*	= P.12 Output at basic prices 1 × *Industry*	= P.13 Output at basic prices 1 × *Industry*				

[1]The sum of items in this column is zero. It appears in the SUT but does not appear in any *1993 SNA* account. Included in this adjustment are transportation, insurance, and distribution charges on goods within the geographical boundaries of the national economy only.

[2]Taxes and subsidies on products are shown in the *1993 SNA* allocation of primary income account for the general government institutional sector S.13, which derives B.5 *Balance of primary incomes* as the balancing item (*1993 SNA*, Annex V, Table A.V.5). B.5 is the sum of B.2 operating surplus, D.2 taxes on production and imports, less D.3 subsidies, plus D.4 property income (net). This account is the source of data for construction of this column in the SUT when exploded to show product detail for the items D.21 and D.31, taxes and subsidies on products.

[3]As noted elsewhere, the *1993 SNA* values goods imports c.i.f. at the product level of detail but f.o.b. in total. Thus, the *1993 SNA* presentation of goods imports in the supply matrix is the sum of P.7 imports f.o.b. and the c.i.f./f.o.b. adjustment on goods imports. To simplify the presentation of this diagram of the SUT and clarify the nature of the negative adjustment to services, assume that insurance and freight provided on imports can be compiled by product, and thus imports f.o.b. can be compiled by product. Insurance and freight provided on imports by residents already are included in the insurance and transportation rows of the P.1 matrix.

[4]The sum of items in this column is zero. It appears in the SUT but does not appear in any *1993 SNA* account.

[5]Insurance and freight on imports of goods by product provided by both residents and nonresidents.

[6]Including insurance and freight on imports provided by nonresidents. Insurance and freight provided on imports by residents are included in the insurance and transportation rows of the P.1 matrix.

- Adjusted for trade and transport margins[16] and taxes less subsidies on products (D.21 through D.31), given by *T* in equations (14.1) and (14.2);

and the *uses* of goods and services

- For current inputs into production by resident producers (arranged in industries) in the form of intermediate consumption (P.2), given by *Z* in equations (14.1) and (14.2);
- For final domestic consumption, including individual consumption by resident households, resident NPISHs, and the government (P.31), and collective consumption by the government (P.32), given by, respectively, *C* and *G* in equations (14.1) and (14.2);
- Capital formation by resident enterprises (P.5) (comprising fixed capital formation (P.51), inventory change (P.52), and acquisitions less disposals of valuables (P.53)), given by *I* in equations (14.1) and (14.2); and
- For export (P.6) and use by the rest of the world, given by *X* in equations (14.1) and (14.2).

14.44 The SUT primarily is a matrix of flows of goods and services designed to highlight the relationship between the production and consumption of institutional units and institutional sectors. For example, households may undertake production in unincorporated enterprises whose activity appears in the production for own final use part of the SUT, but they also may consume goods and services, as represented in individual consumption. The current production transactions of the establishments of all institutional units are grouped together and summarized in one part of the SUT, and the remaining transactions are summarized and organized in another part. The SUT deals principally with flows of transactions in goods and services. Associated with these monetary flows are price and volume components. It is of central interest in monitoring the economy with national accounts statistics to be able to assess the price and volume components of flows of goods and services exchanged for money or credit in market transactions in the SUT. Movements in the price components are of interest in assessing changes in the purchasing power of incomes, as well as in influencing the rate of general price change through monetary policy. Finally, price movements in the various national accounts aggregates are used in private sector decision making and in the escalation of contracts. Movements in the price components of national accounts aggregates are, as discussed at the beginning of this section, measured with price indices.

[16]Trade and transport margins do not appear in the standard sequence of accounts in the *1993 SNA* because these accounts are not shown with product detail. Although these margins are nonzero for individual products, they sum to zero, because the amount added to the domestic supply of *goods* comes from the domestic supply of distribution, insurance, and transport *services*. Margins are thus shown in Table 14.11, separately for margins on domestic production and imports (c.i.f./f.o.b. adjustment), because the SUT displays product detail down the columns. In the aggregate, these adjustments for trade and transport margins on domestic production and the c.i.f./f.o.b. adjustment for imports cancel each other out.

B.2 Variants of the PPI and their relationship to other major price series

B.2.1 PPI variants

B.2.1.1 Price indices for intermediate consumption

14.45 In considering total economy and industry intermediate consumption price indices (ICPIs), the weights correspond to a column-wise reading of the intermediate consumption part of the SUT's use matrix. The intermediate consumption matrix derives from the production account in Table 14.3. It is shown in Tables 14.11 and 14.12 as the region labeled P.2. Because the various margins on basic prices inherent in prevailing purchasers' prices may vary from industry to industry, the ideal sources for purchasers' prices for ICPIs would be enterprise surveys. Such surveys are generally burdensome and expensive. Instead, as noted in the discussion on price indices for total supply, the price index of intermediate consumption by industry can be derived from detailed product components of the supply price index (SPI). This index will be acceptably accurate if the variation in the total tax, subsidy, transport, and distribution margin is not too great from industry to industry within product class. For the total economy, the price index of intermediate consumption is obtained as a weighted average of industries' intermediate input price indices. The weights are the share of each industry's intermediate consumption in the total intermediate consumption in the economy.

Table 14.12. Location and Coverage of the Major Price Indices in the Supply and Use Table

(*The effective coverage of the major indices is shown by areas with gray shading*)

Supply												**Use**								
P.1 Output at basic prices *of establishments producing for:*				*Transport and distribution margin adjustment*		D.21–D.31 Taxes less subsidies on products		Imports c.i.f.				P.2 Intermediate consumption at purchasers' prices *of establishments producing for:*				P.31 Individual consumption	P.32 Collective consumption	P.5 Gross capital formation		P.6 Exports f.o.b.
P.11 Market	P.12 Own use	P.13 Other nonmarket						P.7 Imports f.o.b.		c.i.f./f.o.b. adj.		P.11 Market	P.12 Own use	P.13 Other nonmarket						
PPI		*Product × Industry*	+	*Goods (+)* *Services (–)*	+	*Product × 1*	+	***MPI***	+	*Goods (+)* *Serv. (–)*	=	*Product × Industry*	*Product × Industry*	*Product × Industry*	+	***CPI***[1]	*Product × 1*	***PPI***[2]	+	***XPI***
												+	+	+						
												PPI net output/ Value-added deflator		B.1 Value added 1 × *Industry*						
=	=	=										=	=	=						
PPI		P.13 Output at basic prices 1 × *Industry*									=	***PPI***		P.13 Output at basic prices 1 × *Industry*						

[1]Covering the individual consumption expenditure (P.31) of the household sector (S.13) only and excluding consumption of goods produced by households for own final use, with the exception of imputed rental of dwellings of owner-occupants.
[2]Covering the finished goods component of change in inventories (P.52).

B.2.1.2 Net output PPIs and value-added deflators

14.46 The PPI has been defined in terms of the total market or market-valued output aggregates of the *1993 SNA*, but PPIs sometimes are produced for net output as well as total output. The argument for net output PPIs is that, for a given aggregate of establishments, total output PPIs overweight or "double count" the output of goods used in intermediate consumption within the aggregate. Net output PPIs may be produced for various narrow or broad aggregations of establishments, from detailed industries to the entire population of establishments resident in the economy. The value aggregate of net output PPIs subtracts from total output the value of goods and services used within the aggregate *and* of the *same types* as produced for output by establishments in the aggregate. With one exception, net output is not value added, because it does *not* exclude the intermediate consumption of goods and services used by establishments of the aggregate that are *not* of the same types as produced for output. The exception is when the aggregate is all resident establishments.

14.47 By implication, the net output PPI for all establishments resident in the economy must be closely related to the value-added price index or deflator discussed in Chapter 17. The value aggregate for the all items net output PPI would be value added (Section B.1) as defined in the *1993 SNA* and shown in the production account (Tables 14.1, 14.2, and 14.3). In fact, if the PPI has complete product coverage, including all service products, then net output and value added are the same thing for the total economy.[17] They may be the same even at the industry level under an alternative definition of "net output." See this issue in a stage-of-processing context in Section B.2.1.3.

14.48 The principal issue in interpreting net output PPIs is the definition of the intermediate consumption prices netted from output to arrive at the net output aggregate. These prices should be defined with a view toward the valuation principle inherent in the value aggregates to which they refer. Recall that output (P.1) is valued at *basic prices* while intermediate consumption is valued at *purchasers' prices*. Ideally, the net output PPI would be a type of double-deflation price index, similar in principle to the value-added deflator described in Chapter 17. In such an index, the prices of the goods and services in intermediate consumption would be defined inclusive of taxes on products and charges for included transportation and distribution services, and exclusive of subsidies on products. The prices of goods and services in output would be defined as exclusive of taxes on products and separately invoiced charges for transportation and distribution, and inclusive of subsidies on products. Net output PPIs generally do not attempt the *purchasers' price* valuation of the intermediate consumption of output-type goods and services within the industry aggregate in question. They compromise the concept of the net output index, should there be a change in any component of the purchasers' prices of intermediate consumption goods and services other than the underlying basic prices of products. See Section B.2.1.3 regarding the scope of intermediate consumption in the net output PPI and its alignment with intermediate consumption in value added.

B.2.1.3 Stage-of-processing PPIs

14.49 *Product-based stage-of-processing indices.* The simplest method of forming a set of stage-of-processing PPIs is first to determine an ordering of products a priori, on the basis of judgment, from primary to finished goods. The second step is to produce PPIs for goods grouped by this intrinsic stage of processing classification. Such indices are referred to as product- or commodity-based stage-of-processing indices. They may employ the so-called "end-use" product classifications associated with the commodity flow methods often used in compiling the national accounts.

14.50 *Industry-based stage-of-processing indices.* Industry net output PPIs are associated with industry stage-of-processing PPIs. They are produced in an effort to measure the contribution of the basic prices of goods and services to the change in value added for the economy. They also provide an analytical tool to measure the transmission of inflation through stages of processing, from primary goods and services to those sold for final uses. Industry stage-of-processing PPIs involve a sorting of the product rows and industry columns of the use ma-

[17]Note, however, the equivalence between net output and value-added price indices for the total economy presumes variations in taxes on products, and charges for included (not separately invoiced) transportation and distribution charges on outputs used as inputs are part of the prices of those inputs. The practice of compiling net output PPIs should, but sometimes does not, take this into account.

trix so the matrix is roughly triangular. In other words, any given *product row* in the stage-of-processing-sorted use matrix comprises all zero uses to the left of a particular industry sorted by industry stage of processing. It would have mostly positive uses for that industry and other industries to the right of (and thus at higher stages of processing relative to) that industry. Further, within any given industry column, products earlier in the stage-of-processing sort (above the product in question in the industry column) would tend to have positive uses. There would be zero use of products later in the stage-of-processing sort (below the product in question) in the industry column. Stages of processing are meaningful in this context for goods, but triangularizing the uses matrix tends to classify business services in the primary production category because all industries use them in varying degrees. In this definition of stage of processing, they are primary output because they are produced mainly with labor and capital primary inputs, rather than the outputs of other industries.

14.51 The PPIs constructed for such stage-of-processing-sorted use matrices are compiled as net output indices, exclusive of uses of output-type goods and services within the industry aggregate in question. Hence, net output PPIs generally are associated with industry stage-of-processing PPIs. For most aggregates, total economy industry net output is equivalent to value added. Unfortunately, when the coverage of services is incomplete, output and intermediate consumption prices cannot be fully characterized except for goods, the largest industry for which the net output aggregation is feasible. Here, only the price index for the net output of goods can be characterized, which differs from the value added of the goods industry because intermediate consumption of services is still not netted from the net output of goods.

14.52 There is a second interpretation of stage-of-processing PPIs. In this view, they are conceptually the same as value-added price indices or deflators for industries that have been sorted by stage of processing according to the above diagonalization process. By implication, a PPI for an industry at a late stage of processing would expressly exclude the price change of primary products from the price change of the tertiary or finished products of the late-stage industry. Again, universal product coverage, including services, is needed for output and intermediate consumption, and many countries are lacking particularly in the coverage of the prices of service products. When there are no service price indices, value-added deflators cannot be computed even for goods-producing industries, because the services component of intermediate consumption is missing.

B.2.2 Relationship of the PPI to other major price indices

14.53 It is instructive at this point to associate the four major headline price indices compiled by most countries with the component aggregates and matrices of the SUT. The four main price indices and their associated national accounts aggregates and matrices in the SUT are

(i) Output of resident producers (P.1): producer price index[18]
(ii) Individual consumption expenditure on goods and services (P.31), except consumption from own production but including the imputed rent of owner-occupied dwellings, of the household sector (S.13) only: consumer price index;
(iii) Exports (P.6): export price index; and
(iv) Imports (P.7): import price index.

14.54 The location and coverage of these major price indicators as they directly apply to goods and services value aggregates in the national accounts is diagrammed in Table 14.12. Recall that Section A of this chapter characterized a price index as a function of price relatives and weights, noting that, other than the formula for the index itself, the requisite features of the relatives and weights would be determined by the value aggregate. These factors were

- What items to include in the index,
- How to determine the item prices,
- Which transactions that involve these items to include in the index, and

[18]This chapter has also described net output PPIs, whose associated value aggregate is value added (B.1) for the economy as a whole, as well as for individual industries, under the assumption that all products including services are covered in the PPI. As noted earlier, if product coverage (for example, of services) is incomplete, then the net output concept deviates from value added because the intermediate consumption of noncovered goods is not subtracted from output.

Table 14.13. Definition of Scope, Price Relatives, Coverage, and Weights for Major Price Indices

Index	Items to Include	Price Determination for Relatives	Transactions Coverage	Sources of Weights
PPI	All types of domestically produced or processed goods and services that are valued at market prices.	Basic prices, determined for goods as the date when available for sale (available for change of ownership) or service price when service rendered.	Output of resident enterprises, comprising sales plus change in finished goods inventories for goods, and sales for services.	The product by industry matrices of market output (P.11) and output for own final use (P.12) in the expanded industry production account and the SUT.
CPI	All types of goods and services purchased by households for individual consumption.	Purchasers' prices, determined for goods and services on the date when used, including taxes on products, excluding subsidies on products, and including transportation and distribution margins.	Consumption expenditures of the households sector (S.13) of institutional units, excluding consumption from own production, except for imputed expenditures for rental of owner-occupied dwellings.	The product column of the CPI consumption subaggregate of individual consumption (P.31) of the household sector (S.13) in the expanded use of income account and in the SUT.
XPI	All types of transportable goods and services purchased by nonresidents from residents. Goods exported without change of ownership for significant processing by nonresidents and subsequent reimport are included.	*From the point of view of the nonresident purchaser,* purchasers' prices at the national frontier of the exporting country (f.o.b.), including export taxes and excluding export subsidies, and including transport and distribution margins from the production location to the national frontier.	All transportable goods and services produced or processed by residents and purchased by nonresidents, except goods in transit or goods exported and minimally processed by nonresidents for reimport.	The product column of exports (P.6) in the expanded external account of goods and services and the SUT.
MPI	All types of transportable goods and services purchased by residents from nonresidents. Goods imported without change of ownership for significant processing by residents and subsequent reexport are included.	*From the point of view of the nonresident seller,* basic prices at the national frontier of the exporting country (f.o.b.), excluding import taxes and including import subsidies, and excluding transport and distribution margins from the production location to the national frontier.	All transportable goods and services produced or processed by nonresidents and purchased by residents, except goods in transit or goods imported and minimally processed by residents for reexport.	The product column of imports (P.7) in the expanded external account of goods and services and the SUT.

- From what source to draw the weights used in the selected index formula.

Based on our survey of the goods and services accounts of the *1993 SNA* culminating in the SUT, these particulars for each of the four major indices can be summarized as in Table 14.13.

B.2.3 CPI versus PPI as a measure of inflation in market transactions

14.55 Central banks take an interest in the major price indices, particularly if they implement an "inflation-targeting" monetary policy. The CPI is the most widely available macroeconomic price statistic, and in many countries it may be the only available option for inflation measurement. When available, the PPI ordinarily is produced monthly, on a timetable similar to that of the CPI. It is useful, therefore, to compare the two indices as candidates for inflation measurement.

14.56 Both reference aggregates for the CPI (consumption plus capital formation) are important components of total final expenditure and GDP in virtually all countries. Indeed, reference aggregate 2 (consumption plus capital formation) has been promoted by some analysts as a better measure of change in the prices of actual transactions in goods and services than CPIs based on reference aggregate 1 (consumption), which gives substantial weight to the imputed rent of owner-occupied housing. On the other hand, the total value of transactions in goods and services also includes intermediate consumption and acquisitions or disposals of tangible and intangible capital assets, so as an inflation index for total goods and services transactions, the CPI's coverage is rather limited under either definition 1 or 2. The CPI's purchasers' price valuation principle also includes taxes less subsidies on products, which may not be desired in an inflation indicator for underlying price change.

14.57 In contrast, the PPI covers, in principle, total output, which by definition implicitly includes intermediate consumption as well as value added.[19] A second desirable feature of the PPI is that it provides some information on the transmission of inflation through the economy by stage of processing. As noted earlier, product-based stage-of-processing PPIs may be used to provide information on transmission of inflation through the economy from primary products to finished products. If industry value-added indices are compiled, then industry-based stage-of-processing net output price indices can be used to inform on the transmission of inflation from primary activity to tertiary activity. As noted earlier, the latter indices require price indices for intermediate consumption, which most often are estimated using available information on basic prices, trade and transport margins, and taxes and subsidies on products, rather than from direct surveys, although the latter may be used and are preferable if the survey resources are available.[20]

B.3 Other goods and services price indicators in national accounts

B.3.1 Price indices for total supply

14.58 Consistent with our earlier discussion of the PPI coverage, total market-valued output is defined as the sum of market output (P.11) and output for own final use (P.12). Total output (P.1) is the sum of market-valued output and other nonmarket output (P.13). Total supply at basic prices is the sum of output and imports (P.7). Markup adjustments at the product level for trade and transport margins on domestic production, insurance and freight on imports, and taxes (D.21) less subsidies (D.31) on products would be added to total supply at basic prices to produce total supply at purchasers' prices.

14.59 In decomposing total supply into price and volume components, the total SPI at basic prices can be seen to be a weighted mean of the total out-

[19]However, progress in extending the industry coverage of the PPI to cover all output producing activities, services in particular, has proceeded slowly owing to the technical difficulty of specifying service products and measuring the associated prices.

[20]Although it is possible to produce something similar to industry-based stage-of-processing indices with information only on basic prices deriving from the output-based PPI in conjunction with a product by industry intermediate consumption matrix, such indices do not capture changes in trade and transportation margins or taxes less subsidies on production. To the extent that such changes are occurring, such indices measure the value-added deflators with an error. However, for inflation measurement, particularly with a view toward an inflation targeting monetary policy, it may be desirable to remove the contribution to change in such industry-based stage-of-processing indices that arises from changes in taxes net of subsidies on products.

put price index (YPI) and the MPI. The YPI comprises in turn the PPI and an implicit deflator index (IDI) for other nonmarket output. To obtain the deflator for total supply at purchasers' prices, the SPI would be multiplied by an index of the total markup for trade, insurance, and transport margins,[21] and taxes net of subsidies on products.

14.60 Total supply price indices at product levels of detail are useful in compiling and reconciling discrepancies in supply and use tables expressed in volume terms. In addition, SPIs are employed in producing industry price indices for intermediate consumption (P.2), which are useful for compiling GDP volume measures from the production approach. Although principally used as a compilation aid and in deflation of value added at basic prices via the double-deflation approach (see Section B.4.2), SPIs could also serve as analytical indicators in their own right because of their coverage of all goods and services transactions in the economy relating to production and external trade. As such, they may be useful as indicators for economic policy analysis and evaluation requiring broad transaction coverage, in monetary policy formulation, for example.

B.3.2 Price indices for final uses

14.61 The price indices for final uses comprise deflators for individual consumption (P.31), collective consumption (P.32), gross fixed capital formation (P.51), change in inventories (P.52), acquisitions less disposals of valuables (P.53), and exports (P.6). Of the major price indices discussed above, the CPI is the principal source of detailed (product-level) information for P.31, and the PPI is a significant source of detailed information for P.51 and the principal source for the finished goods component of P.52. The SPI may be the principal source for the input inventories component of P.52 in the absence of a detailed intermediate inputs purchase price survey, and the XPI is the deflator for P.6. The SPI can serve as a source of detailed product information for P.32, P.51, and P.53. The deflator for total final uses is designated as the final uses price index, or the FPI. It would be computed as a weighted mean (formula to be determined) of the component indices just discussed.

B.3.3 GDP deflator

14.62 As noted above in the discussion of the SPI and the IPI, the GDP price deflator[22] can be compiled in two ways, corresponding to the two goods and services methods of compiling GDP: the production approach and the expenditure approach. Recall that the production approach derives from the definition of value added, which is the difference between output (P.1) (at basic prices) and intermediate consumption (P.2) (at purchasers' prices). The *1993 SNA* recommends the use of double deflation for value added, by which output at basic prices Y is deflated by the all items YPI to obtain output volume, and intermediate purchases are deflated by an intermediate purchases price index to obtain intermediate input volume. Real value added is then computed as the difference between output volume and intermediate input volume.[23] This operation is equivalent to deflating value added in current prices with a double-deflation-type price index having a positive weight on the YPI and a negative weight on the IPI.[24] The total value added at current basic prices divided by real value added obtained via double deflation yields the implicit deflator for value added at basic prices. Finally, the GDP deflator at purchasers' prices is the value-added price index (at basic prices for output and purchasers' prices for intermediate input) multiplied by the in-

[21]These margins matter only when developing supply price indices at purchasers' prices for individual products and product subaggregates. For all products they cancel out, leaving only taxes less subsidies on products contributing to the total markup on total supply at basic prices.

[22]The terminology "GDP price index" could be used here with no confusion of meaning, but we follow conventional usage as set out in Chapter 17. This does not imply that a price index that declines with increases in some prices is in fact not a price index—this *Manual* considers a price index to be that part of the relative change in a value aggregate that can be attributed to the associated change in prices, whether such a change increases or decreases the aggregate. See Chapter 15.

[23]See *1993 SNA*, Chapter XVI.

[24]In the usual case just described, the value-added deflator is as a Paasche index (Chapter 15, equation [15.6]) of the output price index $\mathrm{YPI}^{s,t}$ and the intermediate input price index $\mathrm{IPI}^{s,t}$, where the weight on the $\mathrm{IPI}^{s,t}$ is

$$w_I^t = \frac{-\mathrm{P.2}^t}{\mathrm{P.1}^t - \mathrm{P.2}^t} \ .$$

As noted in Chapter 15, equation (15.11), the corresponding volume index has the Laspeyres or "constant price" form, which is equivalent to the double-deflation real value-added volume measure described in the text divided by base-period value added.

Table 14.14. Generation of Income Account for Establishment, Institutional Unit, or Institutional Sector
(1993 SNA *goods and services items shown in bold*)

Uses	Resources
D.1 Compensation of employees	B.1 *Value added*[1]
D.11 Wages and salaries	
D.12 Employers' social contributions	
D.121 Employers' actual social contributions	
D.122 Employers' imputed social contributions	
D.2 Taxes on production and imports	
D.29 Other taxes on production[2]	
D.3 Subsidies	
D.39 Other subsidies on production (–)[3]	
B.2 *Operating surplus*[4]	

[1]From the production account.
[2]Taxes on production unrelated to products.
[3]Subsidies on production unrelated to products.
[4]Balancing item of the generation of income account.

dex of the markup on value added of output taxes less output subsidies on products.

14.63 Alternatively, the final expenditure deflator FPI may be combined with the MPI using a double-deflation-type approach. GDP volume is calculated from expenditure data by deflating imports (P.7) by the MPI and subtracting it from the volume of final uses, calculated by deflating final uses by the FPI. The implicit GDP deflator would be the ratio of GDP at current prices with GDP volume so calculated.

B.3.4 Labor services price indices

14.64 The *1993 SNA* provides for the income components comprising value added in the *generation of income account*, shown in Table 14.14. The largest of the income components itemized in this account is compensation of employees (D.1), comprising wages and salaries (D.11) and employers' social contributions (D.12). D.1 represents a value aggregate for a flow of labor services and thus is susceptible to decomposition into price and volume components. Table 14.15 shows the same account exploded by type of labor service (occupation) for an establishment or industry. The price index of labor services or employment cost index (ECI) measures developments in total compensation by occupation within industry. The price of labor services in total compensation terms is of particular interest when compared with the GDP deflator, which indicates the relative purchasing power of labor compensation in terms of production for final consumption. This comparison is useful in assessing cost-push pressures on output prices and as an input into compiling measures of the productivity of labor. A second useful comparison is between the wages and salaries subindex of the ECI[25] with the CPI. The ratio of the ECI with the CPI indicates the purchasing power of wages in terms of consumption goods and services, and tracks the material welfare particularly of the employees subsector (S.143) of the household institutional subsector (S.14) (see Box 14.1).

[25]In the ECI, the price of labor services comprises all of the components of compensation of employees, including employers' social contributions (benefits) as well as wages and salaries. The wages and salaries subindex of the ECI would be another example of a price index adjusted by a markup index. Analogously with the price index for total supply at purchasers' prices or for GDP by production in Table 14.12, the ECI would be adjusted in this case by a "markdown index" taking off employers' social contributions.

Table 14.15. Generation of Income Account for Establishment and Industry with Labor Services (Occupational)[1] Detail
(1993 SNA *goods and services items shown in bold*)

Uses			Resources
D.11 Wages and salaries	**D.12 Employers' social contributions**	**D.1 Compensation of employees**	B.1 *Value added*[2]
1: Legislators, senior officials, and managers	1: Legislators, senior officials, and managers	1: Legislators, senior officials, and managers	
2: Professionals	2: Professionals	2: Professionals	
3: Technicians and associate professionals	3: Technicians and associate professionals	3: Technicians and associate professionals	
4: Clerks	4: Clerks	4: Clerks	
5: Service workers and shop and market sales workers	5: Service workers and shop and market sales workers	5: Service workers and shop and market sales workers	
6: Skilled agricultural and fishery workers	6: Skilled agricultural and fishery workers	6: Skilled agricultural and fishery workers	
7: Craft and related trades workers	7: Craft and related trades workers	7: Craft and related trades workers	
8: Plant and machine operators and assemblers	8: Plant and machine operators and assemblers	8: Plant and machine operators and assemblers	
9: Elementary occupations	9: Elementary occupations	9: Elementary occupations	
0: Armed forces	0: Armed forces	0: Armed forces	
		D.2 Taxes on production and imports	
		D.29 Other taxes on production	
		D.3 Subsidies (–)	
		D.39 Other subsidies on production	
		B.2 *Operating surplus*[3]	

Note: Establishment ID: eeeeeeee. Activity/industry code (ISIC): aaaa. Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn. Market status: P.1n.
[1]Shown are major groups of the International Standard Classification of Occupations 1988 (ISCO-88), ILO.
[2]From the production account.
[3]Balancing item of the generation of income account.

B.4 A framework for a system of price statistics

14.65 To summarize this section's overview of the main price indicators and the national accounts, Table 14.16 shows the price indices needed for the value aggregates in the national accounts and their relation to the four main price indicators. Indices that are functions of two other indices are shown with the general notation $f\left(I_1, I_2; w\right)$, where *f* is an index formula, I_1 and I_2 are price indices (for example, MPI and YPI), *w* is the weight of the second index, with the weight of the first argument in *f* understood to be 1 – *w*. For example, if *f* is the Laspeyres formula, then the output price index *YPI* would be calculated by making the following substitutions: $P_L^{s,t} = YPI^{s,t}$ $r_1^{s,t} = PPI^{s,t}$, $w_1^s = 1 - w_X^s$, $r_2^{s,t} = XPI^{s,t} \times \Delta^{s,t}$, $w_2^s = w_X^s$. The variable *f* also could be chosen as a Paasche formula (with the same substitutions except for change in the time superscript on the weights $w_1^t = 1 - w_X^t$ and $w_2^t = w_D^t$), Fisher Ideal formula, or other index formula.

C. International Comparisons of Expenditure on Goods and Services

14.66 The main price statistics discussed thus far trace price developments of goods and services through time. Purchasing power parities compare price levels expressed in a numeraire currency, such as the U.S. dollar or the euro, of detailed goods and services among different countries or geographical areas for a given accounting period. They eliminate the effect of prices when comparing the levels of GDP between two countries or areas. The price relatives in bilateral PPPs comprise the ratios of the local prices, converted to a numeraire currency, of identical goods and services between the two countries or areas. The weights are proportional to the shares of these items in expenditure on GDP, expressed in a numeraire currency, between the two countries or areas. PPPs thus follow the same scope and valuation concepts as GDP in Table 14.16, with the superscript *t* referring to an area or country rather than month, quarter, or year.

14.67 The sources of price relatives are the same as those for the final uses GDP deflator, and the weights are simply the total final uses, net of imports f.o.b., by product. To ensure the PPP between area *A* and area *B* is the reciprocal of the PPP between *B* and *A*, bilateral PPPs need to be computed using symmetric index numbers such as the Fisher or Törnqvist indices.[26]

14.68 A matrix of bilateral PPPs provides a means of making not only direct bilateral comparisons, but also bilateral comparisons between any two areas as the product of a sequence of bilateral PPPs through any set of intervening areas, beginning with the first area and ending with the second. To ensure the consistency of such comparisons (for example, that a chain beginning with a given area and ending with the same area produces a PPP of unity), bilateral PPPs are adjusted to produce a transitive set of comparisons. The methods for imposing transitivity on a system of bilateral parities compare each area or country's goods and services prices and shares in GDP to a regional set of reference prices and reference shares.

[26]Note that in the international comparisons case the superscripts *s* and *t* of the price and volume decompositions in Section A of this chapter refer to two countries rather than two time periods.

Table 14.16. A Framework for Price Statistics

1993 SNA Aggregate	*1993 SNA* Transaction Codes[1]	Valuation and Needed Detail	*1993 SNA* Source Account	Price Index[2]	Derivation from Other Price Indices
			Supply		
Market-valued output	**P.11 + P.12**	**Basic prices, product by industry**	**Production account with industry and product detail, total economy (S.1)**	***Producer price index***	
Other nonmarket output[3]	P.13	Basic prices (cost of production), product by industry	Production account with industry and product detail, total economy (S.1)	*Implicit deflator index for other nonmarket output*	Derived from volume indicator
Total output	P.1 = P.11 + P.12 + P.13	Basic prices, by product	Production account with industry and product detail, total economy (S.1)	*Output price index*	$YPI = f\left(PPI, IDI; w_m\right),\ w_m = \frac{\text{P.13}}{\text{P.1}}$
Imports	**P.7**	**Basic prices (goods f.o.b. frontier of exporting country, plus separately identified freight and insurance on imports provided by nonresidents, as well as other services provided by nonresidents), by product**	**External transactions in goods and services account with product detail, total economy (S.1)**	***Import Price Index, comprising an import purchasers' price index, multiplied by an f.o.b./purchasers' price markdown index***	
Total supply, basic prices	P.1 + P.7	Basic prices, by product	Supply and use table, total economy (S.1)	*Supply price index*	$SPI = f\left(MPI, YPI; w_Y\right),\ w_y = \frac{\text{P.1}}{\text{P.1}+\text{P.7}}$
Domestic trade, insurance, and transport margin adjustment		Basic prices, for services provided for transportation and distribution within national frontiers, by product	Supply and use table, total economy (S.1)	*Supply markup index (SMI)*	$SMI = \frac{\frac{\text{P.1}^t + \text{P.7}^t + \text{D.21}^t - \text{D.31}^t}{\text{P.1}^t + \text{P.7}^t}}{\frac{\text{P.1}^s + \text{P.7}^s + \text{D.21}^s - \text{D.31}^s}{\text{P.1}^s + \text{P.7}^s}}$ *(in the aggregate)*. Product-level total output markup indices also would include trade and transport margins in the numerator of the above expression.
Freight and insurance on imports adjustment		Basic prices (for services provided from exporter frontier to domestic frontier, regardless of residency of provider), by product	Supply and use table, total economy (S.1)		
Taxes less subsidies on products	D.21 – D.31	Payable, by product	Allocation of primary income account, general government sector (S.13)		
Total supply, purchasers' prices	P.11 + P.12 + P.7 + D.21 – D.31	Purchasers' prices			*SPI* × *SMI*

			Uses		
Intermediate consumption	P.2	Purchasers' prices, products by industries	Production account with product and industry detail, total economy (S.1)	*Intermediate consumption price index*	Usually incorporates product-level information from the total supply price index at purchasers' prices.
Individual consumption	P.31	Purchasers' prices, by product	Use of income account with product detail, total economy (S.1)	*Household consumption price index (HPI)*	Incorporates the CPI and may incorporate product-level information from the CPI and PPI regarding goods and services produced from own consumption and provided to individuals by NPISHs and general government.
Household sector S.14	**CPI reference aggregate #1: P.31, except employers' social contributions and consumption from production for own final use, but including imputed rent of homeowners** **CPI reference aggregate #2: P.31, except employers' social contributions and consumption from production for own final use (and excluding by implication imputed rent of homeowners)**	**Purchasers' prices, by product**	**Reference aggregate #1: Use of income account with product detail, household sector (S.14), with special subclassification of P.31** **CPI reference aggregate #2: Use of income account with product detail, household sector (S.14), with special subclassification of P.31**	***Consumer price index, consumption basis*** *Consumer price index, transactions or inflation basis*	
Collective consumption	P.32	Purchasers' prices, by product	Use of income account with product detail, general government sector (S.13)	*Government price index (GPI)*	*May incorporate product indices from the CPI and PPI.*
Gross fixed capital formation	P.51	Purchasers' prices, by product	Capital account with product detail, total economy (S.1)	*Fixed capital formation price index (KPI)*	*May incorporate product indices from the PPI.*
Household sector S.14	**CPI reference aggregate #2: Gross capital formation in residential dwellings (P.51)**	**Purchasers' prices, by product**	**CPI reference aggregate #2: Capital account regarding acquisitions (P.5111) less disposals (P.5112) of residential dwellings**	*Consumer price index, transactions or inflation basis*	
Change in inventories	P.52	Purchasers' prices, by product	Capital account with product detail, total economy (S.1)	*Inventory price index (NPI)*	*Price index of inventory stocks*
Acquisitions less disposals of valuables	P.53	Purchasers' prices, by product	Capital account with product detail, total economy (S.1)	*Valuables price index (VPI)*	*Price index of valuables stocks*

Table 14.16. (concluded)

1993 SNA Aggregate	*1993 SNA* Transaction Codes[1]	Valuation and Needed Detail	*1993 SNA* Source Account	Price Index[2]	Derivation from Other Price Indices
			Uses		
Exports	**P.6**	**Purchasers' prices (f.o.b. domestic frontier), by product**	**External transactions in goods and services account with product detail, total economy (S.1)**	***Export price index (XPI)***	
Total final uses	P.3 + P.5 + P.6	Purchasers' prices, by product	Supply and use table, total economy (S.1)	*Final uses price index (FPI)*	$FPI = f\left(HPI, GPI, KPI, NPI, VPI, XPI, \vec{w}\right)$ *where* $\vec{w} = \left[w_G, w_K, w_N, w_V, w_X\right]$[4] *and* $w_G = \frac{\text{P.32}}{\text{P.3}+\text{P.4}+\text{P.5}+\text{P.6}}$, $w_K = \frac{\text{P.51}}{\text{P.3}+\text{P.4}+\text{P.5}+\text{P.6}}$, $w_G = \frac{\text{P.32}}{\text{P.3}+\text{P.4}+\text{P.5}+\text{P.6}}$, $w_V = \frac{\text{P.53}}{\text{P.3}+\text{P.4}+\text{P.5}+\text{P.6}}$, $w_X = \frac{\text{P.6}}{\text{P.3}+\text{P.4}+\text{P.5}+\text{P.6}}$.
			Gross Domestic Product		
Value added (net output PPI)	V = P.1 – P.2 + D.21 – D.31	By industry, product and institutional sector, with industry and total value-added price indices adjusted by a markup factor for taxes net of subsidies on products. Net output PPI may exclude from intermediate consumption products, particularly services, on which there may be no price information.	Supply and use table, total economy (S.1)	*Value-added deflator*	*Value-added deflator* $= f\left(SPI, IPI; w_I\right)$, *where* $w_M = \frac{-\text{P.7}}{\text{GDP}}$ $w_I = \frac{-\text{P.2}}{\text{GDP}}$[5]

Gross domestic product	GDP = P.3 + P.5 + P.6 – P.7	By product and institutional sector.	Supply and use table, total economy (S.1)	*GDP deflator*	$GDP\ deflator = f(FPI, MPI; w_M)$ $= SMI^* \times f(SPI, IPI; w_I)$, *where* $SMI^* = \dfrac{\dfrac{P.1^t - P.2^t + D.21^t - D.31^t}{P.1^t - P.2^t}}{\dfrac{P.1^S - P.2^S + D.21^S - D.31^S}{P.1^S - P.2^S}}$ *(in the aggregate).* Industry-level value-added markup indices *SMI** would include the total trade and transport margins on output in the numerator.
Other Price Indices Related to Net Output					
Compensation of employees	D.1	By occupation, industry, and institutional sector	Generation of income account, total economy (S.1)	*Employment cost index*	

[1]P.11 = Market output, P.12 = Output for own final use, D.21 = Taxes on products, and D.31 = Subsidies on products.

[2]The four major price indices are shown in bold.

[3]This category comprises public services output provided free of charge or at economically insignificant prices by general government and NPISHs. This output is valued at cost because it has no market comparator. A price index cannot be directly constructed for this aggregate because there are no economically significant prices for other nonmarket output. The implicit deflator for other nonmarket output (P.13) is derived by dividing a directly compiled volume indicator into the value of other nonmarket output.

[4]Unlike the other aggregations of indices that involve the combination of two component indices, it is shown that the FPI is a simultaneous aggregation of six price indices for the components of final uses. Again, *f* can be any of the indices introduced in this chapter, and with the weight of the first item (here of individual consumption [P.31]) determined as one minus the rest of the weights, and the price relatives given by the list of index arguments.

[5]The negative weights of the second index arguments of both of these formulas for GDP is an indication that they represent a double-deflation-type price index. See *1993 SNA*, Chapter XVI, Section E.

15. Basic Index Number Theory

A. Introduction

> The answer to the question what is the Mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as there are purposes; and we may almost say, in the matter of prices as many purposes as writers. Hence much vain controversy between persons who are literally at cross purposes. (F.Y. Edgeworth, 1888, p. 347)

15.1 The number of physically distinct goods and unique types of services that consumers can purchase is in the millions. On the business or production side of the economy, there are even more products that are actively traded. The reason is that firms not only produce products for final consumption, they also produce exports and intermediate products that are demanded by other producers. Firms collectively also use millions of imported goods and services, thousands of different types of labor services, and hundreds of thousands of specific types of capital. If we further distinguish physical products by their geographic location or by the season or time of day that they are produced or consumed, then there are *billions* of products that are traded within each year in any advanced economy. For many purposes, it is necessary to summarize this vast amount of price and quantity information into a much smaller set of numbers. The question that this chapter addresses is the following: *How exactly should the microeconomic information involving possibly millions of prices and quantities be aggregated into a smaller number of price and quantity variables?* This is the basic *index number problem.*

15.2 It is possible to pose the index number problem in the context of microeconomic theory; that is, given that we wish to implement some economic model based on producer or consumer theory, what is the best method for constructing a set of aggregates for the model? However, when constructing aggregate prices or quantities, other points of view (that do not rely on economics) are possible. Some of these alternative points of view will be considered in this chapter and the next chapter. Economic approaches will be pursued in Chapters 17 and 18.

15.3 The index number problem can be framed as the problem of decomposing the value of a well-defined set of transactions in a period of time into an aggregate price multiplied by an aggregate quantity term. It turns out that this approach to the index number problem does not lead to any useful solutions. Therefore, in Section B, the problem of decomposing a *value ratio* pertaining to two periods of time into a component that measures the overall *change in prices* between the two periods (this is the *price index*) multiplied by a term that measures the overall *change in quantities* between the two periods (this is the *quantity index*) is considered. The simplest price index is *a fixed-basket index.* In this index, fixed amounts of the n quantities in the value aggregate are chosen, and then this fixed basket of quantities at the prices of period 0 and period 1 are calculated. The fixed-basket price index is simply the ratio of these two values, where the prices vary but the quantities are held fixed. Two natural choices for the fixed basket are the quantities transacted in the base period, period 0, or the quantities transacted in the current period, period 1. These two choices lead to the Laspeyres (1871) and Paasche (1874) price indices, respectively.

15.4 Unfortunately, the Paasche and Laspeyres measures of aggregate price change can differ, sometimes substantially. Thus, Section C considers taking an average of these two indices to come up with a single measure of price change. Section C.1 argues that the best average to take is the geometric mean, which is Irving Fisher's (1922) ideal price index. In Section C.2, instead of averaging the Paasche and Laspeyres measures of price change, taking an average of the two baskets is considered. This fixed-basket approach to index number theory leads to a price index advocated by Walsh (1901, 1921a). However, other fixed-basket approaches are also possible. Instead of choosing the basket of period 0 or 1 (or an average of these two baskets), it is possible to choose a basket that pertains to an entirely different period, say, period *b*. In fact, it is typical statistical agency practice to pick a basket that pertains to an entire year (or even two years) of transactions in a year before period 0, which is usually a month. Indices of this type, where the weight reference period differs from the price reference period, were originally proposed by Joseph Lowe (1823), and in Section D indices of this type will be studied. They will also be evaluated from the axiomatic perspective in Chapter 16 and from the economic perspective in Chapter 17.[1]

15.5 In Section E, another approach to the determination of the *functional form* or the *formula* for the price index is considered. This approach, devised by the French economist Divisia (1926), is based on the assumption that price and quantity data are available as continuous functions of time. The theory of differentiation is used to decompose the rate of change of a continuous time value aggregate into two components that reflect aggregate price and quantity change. Although Divisia's approach offers some insights,[2] it does not offer much guidance to statistical agencies in terms of leading to a *definite* choice of index number formula.

15.6 In Section F, the advantages and disadvantages of using a *fixed-base* period in the bilateral index number comparison are considered versus always comparing the current period with the previous period, which is called the *chain system*. In the chain system, a *link* is an index number comparison of one period with the previous period. These links are multiplied to make comparisons over many periods.

[1]Indices of this type will not appear in Chapter 19, where most of the index number formulas exhibited in Chapters 15–18 will be illustrated using an artificial data set. However, indices where the weight reference period differs from the price reference period will be illustrated numerically in Chapter 22, where the problem of seasonal products will be discussed.

[2]In particular, it can be used to justify the chain system of index numbers, which will be discussed in Section E.2.

B. Decomposition of Value Aggregates into Price and Quantity Components

B.1 Decomposition of value aggregates and the product test

15.7 A *price index* is a measure or function that summarizes the *change* in the prices of many products from one situation 0 (a time period or place) to another situation 1. More specifically, for most practical purposes, a price index can be regarded as a weighted mean of the change in the relative prices of the products under consideration in the two situations. To determine a price index, it is necessary to know

(i) Which products or items to include in the index,
(ii) How to determine the item prices,
(iii) Which transactions that involve these items to include in the index,
(iv) How to determine the weights and from which sources these weights should be drawn, and
(v) Which formula or mean should be used to average the selected item relative prices.

All the above price index definition questions except the last can be answered by appealing to the definition of the *value aggregate* to which the price index refers. A *value aggregate V* for a given collection of items and transactions is computed as

$$(15.1)\quad V = \sum_{i=1}^{n} p_i q_i \ ,$$

where p_i represents the price of the ith item in national currency units, q_i represents the corresponding quantity transacted in the time period under consideration, and the subscript i identifies the ith elementary item in the group of n items that make up the chosen value aggregate V. Included in this definition of a value aggregate is the specification of the group of included products (which items to include) and of the economic agents engaging in transactions involving those products (which transactions to include), as well as the valuation and time of recording principles motivating the behavior of the economic agents undertaking the transactions (determination of prices). The included elementary items, their valuation (the p_i), the eligibility of the transactions, and the item weights (the q_i) are all within the domain of definition of the value aggregate. The precise determination of the p_i and q_i was discussed in more detail in Chapter 5 and other chapters.[3]

15.8 The value aggregate V defined by equation (15.1) referred to a certain set of transactions pertaining to a single (unspecified) time period. Now, consider the same value aggregate for two places or time periods, periods 0 and 1. For the sake of definiteness, period 0 is called the *base period* and period 1 is called the *current period.* Assume that observations on the base-period price and quantity vectors, $p^0 \equiv [p_1^0,\ldots,p_n^0]$ and $q^0 \equiv [q_1^0,\ldots,q_n^0]$, respectively, have been collected.[4] The value aggregates in the two periods are defined in the obvious way as

$$(15.2) \quad V^0 \equiv \sum_{i=1}^{n} p_i^0 q_i^0; \quad V^1 \equiv \sum_{i=1}^{n} p_i^1 q_i^1.$$

15.9 In the previous paragraph, a price index was defined as a function or measure that summarizes the *change* in the prices of the n products in the value aggregate from situation 0 to situation 1. In this paragraph, a *price index* $P(p^0,p^1,q^0,q^1)$ along with the corresponding *quantity index* (or *volume index*) $Q(p^0,p^1,q^0,q^1)$ is defined as two functions of the 4n variables p^0,p^1,q^0,q^1 (these variables describe the prices and quantities pertaining to the value aggregate for periods 0 and 1), where these two functions satisfy the following equation:[5]

$$(15.3) \quad V^1/V^0 = P(p^0,p^1,q^0,q^1) \times Q(p^0,p^1,q^0,q^1).$$

If there is only one item in the value aggregate, then the price index P should collapse to the single-price ratio p_1^1/p_1^0, and the quantity index Q should collapse to the single-quantity ratio q_1^1/q_1^0. In the case of many items, the price index P is to be interpreted as some sort of weighted average of the individual price ratios, $p_1^1/p_1^0,\ldots, p_n^1/p_n^0$.

15.10 Thus, the first approach to index number theory can be regarded as the problem of *decomposing* the change in a value aggregate, V^1/V^0, into the product of a part that is due to *price change*, $P(p^0,p^1,q^0,q^1)$, and a part that is due to *quantity change*, $Q(p^0,p^1,q^0,q^1)$. This approach to the determination of the price index is the approach taken in the national accounts, where a price index is used to *deflate* a value ratio to obtain an estimate of quantity change. Thus, in this approach to index number theory, the primary use for the price index is as a *deflator*. Note that once the functional form for the price index $P(p^0,p^1,q^0,q^1)$ is known, then the corresponding quantity or volume index $Q(p^0,p^1,q^0,q^1)$ is completely determined by P; that is, by rearranging equation (15.3):

$$(15.4) \quad Q(p^0,p^1,q^0,q^1) = \left(V^1/V^0\right) / P(p^0,p^1,q^0,q^1).$$

Conversely, if the functional form for the quantity index $Q(p^0,p^1,q^0,q^1)$ is known, then the corresponding price index $P(p^0,p^1,q^0,q^1)$ is completely determined by Q. Thus, using this deflation approach to index number theory, separate theories for the determination of the price and quantity indices are not required: if either P or Q is determined, then the other function is implicitly determined by the product test, equation (15.4).

15.11 In the next subsection, two concrete choices for the price index $P(p^0,p^1,q^0,q^1)$ are considered, and the corresponding quantity indices $Q(p^0,p^1,q^0,q^1)$ that result from using equation (15.4) are also calculated. These are the two choices used most frequently by national income accountants.

B.2 Laspeyres and Paasche indices

15.12 One of the simplest approaches determining the price index formula was described in great detail by Joseph Lowe (1823). His approach to

[3]Ralph Turvey and others (1989) have noted that some values may be difficult to decompose into unambiguous price and quantity components. Some examples of values difficult to decompose are bank charges, gambling expenditures, and life insurance payments.

[4]Note that it is assumed that there are no new or disappearing products in the value aggregates. Approaches to the "new goods problem" and the problem of accounting for quality change are discussed in Chapters 7, 8, and 21.

[5]The first person to suggest that the price and quantity indices should be jointly determined to satisfy equation (15.3) was Irving Fisher (1911, p. 418). Frisch (1930, p. 399) called equation (15.3) the *product test*.

measuring the price change between periods 0 and 1 was to specify an approximate *representative product basket,*[6] which is a quantity vector $q \equiv [q_1,\ldots,q_n]$ that is representative of purchases made during the two periods under consideration, and then to calculate the level of prices in period 1 relative to period 0 as the ratio of the period 1 cost of the basket, $\sum_{i=1}^{n} p_i^1 q_i$, to the period 0 cost of the basket, $\sum_{i=1}^{n} p_i^0 q_i$. This *fixed-basket approach* to the determination of the price index leaves open the following question: How exactly is the fixed-basket vector q to be chosen?

15.13 As time passed, economists and price statisticians demanded a bit more precision with respect to the specification of the basket vector q. There are two natural choices for the reference basket: the base period 0 product vector q^0 or the current period 1 product vector q^1. These two choices led to the *Laspeyres* (1871) price index[7] P_L defined by equation (15.5) and the *Paasche* (1874) price index[8] P_P defined by equation (15.6):[9]

$$(15.5)\quad P_L(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0};$$

$$(15.6)\quad P_P(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^1}.$$

15.14 The above formulas can be rewritten in a manner that is more useful for statistical agencies. Define the period t revenue share on product i as follows:

$$(15.7)\quad s_i^t \equiv p_i^t q_i^t \;/\; \sum_{j=1}^{n} p_j^t q_j^t \quad \text{for } i = 1,\ldots,n$$

and $t = 0,1$.

Then, the Laspeyres index, equation (15.5), can be rewritten as follows:[10]

$$\begin{aligned}(15.8)\quad P_L(p^0,p^1,q^0,q^1) &= \sum_{i=1}^{n} p_i^1 q_i^0 \;/\; \sum_{j=1}^{n} p_j^0 q_j^0 \\ &= \sum_{i=1}^{n} (p_i^1/p_i^0)\, p_i^0 q_i^0 \,/\, \sum_{j=1}^{n} p_j^0 q_j^0 \\ &= \sum_{i=1}^{n} (p_i^1/p_i^0)\;\; s_i^0,\end{aligned}$$

using definitions in equation (15.7).

Thus, the Laspeyres price index, P_L can be written as a base-period revenue share-weighted arithmetic average of the n price ratios, p_i^1/p_i^0. The Laspeyres formula (until the very recent past) has been widely used as the intellectual base for PPIs around the world. To implement it, a statistical agency needs only to collect information on revenue shares s_n^0 for the index domain of definition for the base period 0 and then collect information on item *prices* alone on an ongoing basis. *Thus, the Laspeyres PPI can be produced on a timely basis without current-period quantity information.*

[6]Joseph Lowe (1823, Appendix, p. 95) suggested that the product basket vector q should be updated every five years. Lowe indices will be studied in more detail in Section D.

[7]This index was actually introduced and justified by Drobisch (1871a, p. 147) slightly earlier than Laspeyres. Laspeyres (1871, p. 305) in fact explicitly acknowledged that Drobisch showed him the way forward. However, the contributions of Drobisch have been forgotten for the most part by later writers because Drobisch aggressively pushed for the ratio of two unit values as being the best index number formula. While this formula has some excellent properties, if all the n products being compared have the same unit of measurement, the formula is useless when, say, both goods and services are in the index basket.

[8]Again, Drobisch (1871b, p. 424) appears to have been the first to explicitly define and justify this formula. However, he rejected this formula in favor of his preferred formula, the ratio of unit values, and so again he did not get any credit for his early suggestion of the Paasche formula.

[9]Note that $P_L(p^0,p^1,q^0,q^1)$ does not actually depend on q^1, and $P_P(p^0,p^1,q^0,q^1)$ does not actually depend on q^0. However, it does no harm to include these vectors, and the notation indicates that the reader is in the realm of bilateral index number theory; that is, the prices and quantities for a value aggregate pertaining to two periods are being compared.

[10]This method of rewriting the Laspeyres index (or any fixed-basket index) as a share-weighted arithmetic average of price ratios is due to Irving Fisher (1897, p. 517; 1911, p. 397; 1922, p. 51) and Walsh (1901, p. 506; 1921a, p. 92).

15.15 The Paasche index can also be written in revenue share and price ratio form as follows:[11]

$$(15.9)\ \ P_P(p^0,p^1,q^0,q^1) = 1\Big/\left\{\sum_{i=1}^{n} p_i^0 q_i^1 \Big/ \sum_{j=1}^{n} p_j^1 q_j^1\right\}$$
$$= 1\Big/\left\{\sum_{i=1}^{n} \left(p_i^0/p_i^1\right) p_i^1 q_i^1 \Big/ \sum_{j=1}^{n} p_j^1 q_j^1\right\}$$
$$= 1\Big/\left\{\sum_{i=1}^{n} \left(p_i^1/p_i^0\right)^{-1} s_i^1\right\}$$
$$= \left\{\sum_{i=1}^{n} \left(p_i^1/p_i^0\right)^{-1} s_i^1\right\}^{-1},$$

using definitions in equation (15.7).

Thus, the Paasche price index P_P can be written as a period 1 (or current-period) revenue share-weighted *harmonic* average of the n item price ratios p_i^1/p_i^0.[12] The lack of information on current-period quantities prevents statistical agencies from producing Paasche indices on a timely basis.

15.16 The quantity index that corresponds to the Laspeyres price index using the product test, equation (15.3), is the Paasche quantity index; that is, if P in equation (15.4) is replaced by P_L defined by equation (15.5), then the following quantity index is obtained:

$$(15.10)\quad Q_P(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^0}.$$

Note that Q_P is the value of the period 1 quantity vector valued at the period 1 prices, $\sum_{i=1}^{n} p_i^1 q_i^1$, divided by the (hypothetical) value of the period 0 quantity vector valued at the period 1 prices, $\sum_{i=1}^{n} p_i^1 q_i^0$. Thus, the period 0 and 1 quantity vectors are valued at the same set of prices, the current-period prices, p^1.

15.17 The quantity index that corresponds to the Paasche price index using the product test, equation (15.3), is the Laspeyres quantity index; that is, if P in equation (15.4) is replaced by P_P defined by equation (15.6), then the following quantity index is obtained:

$$(15.11)\quad Q_L(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}.$$

Note that Q_L is the (hypothetical) value of the period 1 quantity vector valued at the period 0 prices, $\sum_{i=1}^{n} p_i^0 q_i^1$, divided by the value of the period 0 quantity vector valued at the period 0 prices, $\sum_{i=1}^{n} p_i^0 q_i^0$. Thus, the period 0 and 1 quantity vectors are valued at the same set of prices, the base-period prices, p^0.

15.18 The problem with the Laspeyres and Paasche index number formulas is that they are equally plausible, but, in general, they will give *different* answers. For most purposes, it is not satisfactory for the statistical agency to provide *two* answers to this question:[13] what is the best overall summary measure of price change for the value aggregate over the two periods in question? Thus, in the following section, it is considered how best averages of these two estimates of price change can be constructed. Before doing this, we ask what is the normal relationship between the Paasche and Laspeyres indices? Under normal economic conditions, when the price ratios pertaining to the two situations under consideration are negatively correlated with the corresponding quantity ratios, it can be shown that the Laspeyres price index will be

[11]This method of rewriting the Paasche index (or any fixed-basket index) as a share-weighted harmonic average of the price ratios is due to Walsh (1901, p. 511; 1921a, p. 93) and Irving Fisher (1911, pp. 397–98).

[12]Note that the derivation in equation (15.9) shows how harmonic averages arise in index number theory in a very natural way.

[13]In principle, instead of averaging the Paasche and Laspeyres indices, the statistical agency could think of providing both (the Paasche index on a delayed basis). This suggestion would lead to a matrix of price comparisons between every pair of periods instead of a time series of comparisons. Walsh (1901, p. 425) noted this possibility: “In fact, if we use such direct comparisons at all, we ought to use all possible ones.”

larger than the corresponding Paasche index.[14] In Appendix 15.1, a precise statement of this result is presented.[15] This divergence between P_L and P_P suggests that if a *single estimate* for the price change between the two periods is required, then some sort of evenly weighted average of the two indices should be taken as the final estimate of price change between periods 0 and 1. This strategy will be pursued in the following section. However, it should be kept in mind that, usually, statistical agencies will not have information on current revenue weights and, hence, averages of Paasche and Laspeyres indices can be produced only on a delayed basis (perhaps using national accounts information) or not at all.

C. Symmetric Averages of Fixed-Basket Price Indices

C.1 Fisher index as an average of the Paasche and Laspeyres indices

15.19 As was mentioned in the previous paragraph, since the Paasche and Laspeyres price indices are equally plausible but often give different estimates of the amount of aggregate price change between periods 0 and 1, it is useful to consider taking an evenly weighted average of these fixed-basket price indices as a single estimator of price change between the two periods. Examples of such *symmetric averages*[16] are the arithmetic mean, which leads to the Drobisch (1871b, p. 425) Sidgwick (1883, p. 68) Bowley (1901, p. 227)[17] index, $P_{DR} \equiv (1/2)P_L + (1/2)P_P$, and the geometric mean, which leads to the Irving Fisher[18] (1922) ideal index, P_F, defined as

$$(15.12)\ P_F(p^0,p^1,q^0,q^1) \equiv \left[P_L(p^0,p^1,q^0,q^1)\right]^{1/2} \times \left[\ P_P(p^0,p^1,q^0,q^1)\right]^{1/2}.$$

At this point, the fixed-basket approach to index number theory is transformed into the *test approach* to index number theory; that is, to determine which of these fixed-basket indices or which averages of them might be best, desirable *criteria* or *tests* or *properties* are needed for the price index. This topic will be pursued in more detail in the next chapter, but an introduction to the test approach is provided in the present section because a test is used to determine which average of the Paasche and Laspeyres indices might be best.

15.20 What is the best symmetric average of P_L and P_P to use as a point estimate for the theoretical cost-of-living index? It is very desirable for a price index formula that depends on the price and quantity vectors pertaining to the two periods under consideration to satisfy the *time reversal test.*[19] An

[14]Peter Hill (1993, p. 383) summarized this inequality as follows: "It can be shown that relationship (13) [that is, that P_L is greater than P_P] holds whenever the price and quantity relatives (weighted by values) are negatively correlated. Such negative correlation is to be expected for price takers who react to changes in relative prices by substituting goods and services that have become relatively less expensive for those that have become relatively more expensive. In the vast majority of situations covered by index numbers, the price and quantity relatives turn out to be negatively correlated so that Laspeyres indices tend systematically to record greater increases than Paasche with the gap between them tending to widen with time."

[15]There is another way to see why P_P will often be less than P_L. If the period 0 revenue shares s_i^0 are exactly equal to the corresponding period 1 revenue shares s_i^1, then by Schlömilch's (1858) Inequality (see Hardy, Littlewood, and Polyá, 1934, p. 26), it can be shown that a weighted harmonic mean of n numbers is equal to or less than the corresponding arithmetic mean of the n numbers and the inequality is strict if the n numbers are not all equal. If revenue shares are approximately constant across periods, then it follows that P_P will usually be less than P_L under these conditions; see Section D.3.

[16]For a discussion of the properties of symmetric averages, see Diewert (1993c). Formally, an average $m(a,b)$ of two numbers a and b is symmetric if $m(a,b) = m(b,a)$. In other words, the numbers a and b are treated in the same manner in the average. An example of a nonsymmetric average of a and b is $(1/4)a + (3/4)b$. In general, Walsh (1901, p. 105) argued for a symmetric treatment if the two periods (or countries) under consideration were to be given equal importance.

[17]Walsh (1901, p. 99) also suggested this index. See Diewert (1993a, p. 36) for additional references to the early history of index number theory.

[18]Bowley (1899, p. 641) appears to have been the first to suggest the use of this index. Walsh (1901, pp. 428–29) also suggested this index while commenting on the big differences between the Laspeyres and Paasche indices in one of his numerical examples: "The figures in columns (2) [Laspeyres] and (3) [Paasche] are, singly, extravagant and absurd. But there is order in their extravagance; for the nearness of their means to the more truthful results shows that they straddle the true course, the one varying on the one side about as the other does on the other."

[19]See Diewert (1992a, p. 218) for early references to this test. If we want the price index to have the same property as a single-price ratio, then it is important to satisfy the time reversal test. However, other points of view are possi-
(continued)

index number formula $P(p^0,p^1,q^0,q^1)$ satisfies this test if

$$(15.13)\quad P(p^1,p^0,q^1,q^0) = 1/P(p^0,p^1,q^0,q^1)\ ;$$

that is, if the period 0 and period 1 price and quantity data are interchanged and the index number formula is evaluated, then this new index $P(p^1,p^0,q^1,q^0)$ is equal to the reciprocal of the original index $P(p^0,p^1,q^0,q^1)$. This is a property that is satisfied by a single price ratio, and it seems desirable that the measure of aggregate price change should also satisfy this property so that it does not matter which period is chosen as the base period. Put another way, the index number comparison between any two points of time should not depend on the choice of which period we regard as the base period: if the other period is chosen as the base period, then the new index number should simply equal the reciprocal of the original index. It should be noted that the Laspeyres and Paasche price indices *do not* satisfy this time reversal property.

15.21 Having defined what it means for a price index P to satisfy the time reversal test, then it is possible to establish the following result:[20] the Fisher ideal price index defined by equation (15.12) above is the *only* index that is a homogeneous[21] symmetric average of the Laspeyres and Paasche price indices, P_L and P_P, and satisfies the time reversal test in equation (15.13) above. Thus, the Fisher ideal price index emerges as perhaps the best evenly weighted average of the Paasche and Laspeyres price indices.

15.22 It is interesting to note that this *symmetric basket approach* to index number theory dates back to one of the early pioneers of index number theory, Arthur L. Bowley, as the following quotations indicate:

> If [the Paasche index] and [the Laspeyres index] lie close together there is no further difficulty; if they differ by much they may be regarded as inferior and superior limits of the index number, which may be estimated as their arithmetic mean … as a first approximation. (Arthur L. Bowley, 1901, p. 227)

> When estimating the factor necessary for the correction of a change found in money wages to obtain the change in real wages, statisticians have not been content to follow Method II only [to calculate a Laspeyres price index], but have worked the problem backwards [to calculate a Paasche price index] as well as forwards. … They have then taken the arithmetic, geometric or harmonic mean of the two numbers so found. (Arthur L. Bowley, 1919, p. 348)[22]

15.23 The quantity index that corresponds to the Fisher price index using the product test, equation (15.3), is the Fisher quantity index; that is, if P in equation (15.4) is replaced by P_F defined by equation (15.12), the following quantity index is obtained:

$$(15.14)\quad Q_F(p^0,p^1,q^0,q^1) \equiv \left[Q_L(p^0,p^1,q^0,q^1)\right]^{1/2} \times\left[Q_P(p^0,p^1,q^0,q^1)\right]^{1/2}.$$

Thus, the Fisher quantity index is equal to the square root of the product of the Laspeyres and Paasche quantity indices. It should also be noted that $Q_F(p^0,p^1,q^0,q^1) = P_F(q^0,q^1,p^0,p^1)$; that is, if the role of prices and quantities is interchanged in the Fisher price index formula, then the Fisher quantity index is obtained.[23]

15.24 Rather than take a symmetric average of the two basic fixed-basket price indices pertaining to two situations, P_L and P_P, it is also possible to return to Lowe's basic formulation and choose the basket vector q to be a symmetric average of the base- and current-period basket vectors, q^0 and q^1. The following subsection pursues this approach to index number theory.

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ble. For example, we may want to use our price index for compensation purposes, in which case satisfaction of the time reversal test may not be so important.

[20]See Diewert (1997, p. 138).

[21]An average or mean of two numbers a and b, $m(a,b)$, is *homogeneous* if when both numbers a and b are multiplied by a positive number λ, then the mean is also multiplied by λ; that is, m satisfies the following property: $m(\lambda a,\lambda b) = \lambda m(a,b)$.

[22]Irving Fisher (1911, pp. 417–18; 1922) also considered the arithmetic, geometric, and harmonic averages of the Paasche and Laspeyres indices.

[23]Irving Fisher (1922, p. 72) said that P and Q satisfied the *factor reversal test* if $Q(p^0,p^1,q^0,q^1) = P(q^0,q^1,p^0,p^1)$ and P and Q satisfied the product test in equation (15.3) as well.

C.2 Walsh index and theory of "pure" price index

15.25 Price statisticians tend to be very comfortable with a concept of the price index based on pricing out a constant representative basket of products, $q \equiv (q_1,q_2,\ldots,q_n)$, at the prices of period 0 and 1, $p^0 \equiv (p_1^0,p_2^0,\ldots,p_n^0)$ and $p^1 \equiv (p_1^1,p_2^1,\ldots,p_n^1)$, respectively. Price statisticians refer to this type of index as a *fixed-basket index* or a *pure price index*,[24] and it corresponds to Knibbs's (1924, p. 43) *unequivocal price index*.[25] Since Joseph Lowe (1823) was the first person to describe systematically this type of index, it is referred to as a *Lowe index*. Thus, the general functional form for the *Lowe price index* is

$$(15.15) \quad P_{Lo}(p^0,p^1,q) \equiv \sum_{i=1}^{n} p_i^1 q_i \Big/ \sum_{i=1}^{n} p_i^0 q_i = \sum_{i=1}^{n} s_i (p_i^1 / p_i^0),$$

where the (hypothetical) *hybrid revenue shares* s_i[26] corresponding to the quantity weights vector q are defined by

$$(15.16) \quad s_i \equiv p_i^0 q_i \Big/ \sum_{j=1}^{n} p_j^0 q_j \quad \text{for } i = 1,2,\ldots,n.$$

15.26 The main reason why price statisticians might prefer a member of the family of Lowe or fixed-basket price indices defined by equation (15.15) is *that the fixed-basket concept is easy to explain to the public*. Note that the Laspeyres and Paasche indices are special cases of the pure price concept if we choose $q = q^0$ (which leads to the Laspeyres index) or if we choose $q = q^1$ (which leads to the Paasche index).[27] The practical problem of picking q remains to be resolved, and that is the problem addressed in this section.

15.27 It should be noted that Walsh (1901, p. 105; 1921a) also saw the price index number problem in the above framework:

> Commodities are to be weighted according to their importance, or their full values. But the problem of axiometry always involves at least two periods. There is a first period, and there is a second period which is compared with it. Price variations have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period? Or those of the second? Or should there be a combination of the two sets? There is no reason for preferring either the first or the second. Then the combination of both would seem to be the proper answer. And this combination itself involves an averaging of the weights of the two periods. (Correa Moylan Walsh, 1921a, p. 90)

Walsh's suggestion will be followed, and thus the ith quantity weight, q_i, is restricted to be an average or *mean* of the base-period quantity q_i^0 and the current-period quantity for product i q_i^1, say, $m(q_i^0,q_i^1)$, for $i = 1,2,\ldots,n$.[28] Under this assump-

[24]See Section 7 in Diewert (2001).

[25]"Suppose, however, that for each commodity, $Q' = Q$, the fraction, $\sum(P'Q) / \sum(PQ)$, viz., the ratio of aggregate value for the second unit-period to the aggregate value for the first unit-period is no longer merely a ratio of totals, it also shows unequivocally the effect of the change in price. Thus, it is an unequivocal price index for the quantitatively unchanged complex of commodities, *A, B, C,* etc.

> "It is obvious that if the quantities were different on the two occasions, and if at the same time the prices had been unchanged, the preceding formula would become $\sum(PQ') / \sum(PQ)$. It would still be the ratio of the aggregate value for the second unit-period to the aggregate value for the first unit-period. But it would be also more than this. It would show in a generalized way the ratio of the quantities on the two occasions. Thus it is an unequivocal quantity index for the complex of commodities, unchanged as to price and differing only as to quantity.
>
> "Let it be noted that the mere algebraic form of these expressions shows at once the logic of the problem of finding these two indices is identical" (Sir George H. Knibbs, 1924, pp. 43–44).

[26]Irving Fisher (1922, p. 53) used the terminology "weighted by a hybrid value," while Walsh (1932, p. 657) used the term "hybrid weights."

[27]Note that the ith share defined by equation (15.16) in this case is the hybrid share $s_i = p_i^0 q_i^1 \Big/ \sum_{i=1}^{n} p_i^0 q_i^1$, which uses the prices of period 0 and the quantities of period 1.

[28]Note that we have chosen the mean function $m(q_i^0,q_i^1)$ to be the same for each item i. We assume that $m(a,b)$ has the following two properties: $m(a,b)$ is a positive and continuous function, defined for all positive numbers a and b, and $m(a,a) = a$ for all $a > 0$.

tion, the Lowe price index in equation (15.15) becomes

$$(15.17)\quad P_{Lo}(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 m(q_i^0,q_i^1)}{\sum_{j=1}^{n} p_j^0 m(q_j^0,q_j^1)}.$$

15.28 To determine the functional form for the mean function m, it is necessary to impose some *tests* or *axioms* on the pure price index defined by equation (15.17). As in Section C.1, we ask that P_{Lo} satisfy the *time reversal test*, equation (15.13) above. Under this hypothesis, it is immediately obvious that the mean function m must be a *symmetric mean*;[29] that is, m must satisfy the following property: $m(a,b) = m(b,a)$ for all $a > 0$ and $b > 0$. This assumption still does not pin down the functional form for the pure price index defined by equation (15.17) above. For example, the function $m(a,b)$ could be the *arithmetic mean*, $(1/2)a + (1/2)b$, in which case equation (15.17) reduces to the *Marshall* (1887) *Edgeworth* (1925) *price index* P_{ME}, which was the pure price index preferred by Knibbs (1924, p. 56):

$$(15.18)\quad P_{ME}(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 \left\{\left(q_i^0+q_i^1\right)/2\right\}}{\sum_{j=1}^{n} p_j^0 \left\{\left(q_j^0+q_j^1\right)/2\right\}}.$$

15.29 On the other hand, the function $m(a,b)$ could be the *geometric mean*, $(ab)^{1/2}$, in which case equation (15.17) reduces to the *Walsh* (1901, p. 398; 1921a, p. 97) *price index*, P_W:[30]

$$(15.19)\quad P_W(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 \sqrt{q_i^0 q_i^1}}{\sum_{j=1}^{n} p_j^0 \sqrt{q_j^0 q_j^1}}.$$

15.30 There are many other possibilities for the mean function m, including the mean of order r, $[(1/2)a^r + (1/2)b^r\,]^{1/r}$ for $r \neq 0$. To completely determine the functional form for the pure price index P_{Lo}, it is necessary to impose at least one additional test or axiom on $P_{Lo}(p^0,p^1,q^0,q^1)$.

15.31 There is a potential problem with the use of the Marshall-Edgeworth price index, equation (15.18), that has been noticed in the context of using the formula to make international comparisons of prices. If the price levels of a very large country are compared with the price levels of a small country using equation (15.18), then the quantity vector of the large country may totally overwhelm the influence of the quantity vector corresponding to the small country.[31] In technical terms, the Marshall-Edgeworth formula is not homogeneous of degree 0 in the components of both q^0 and q^1. To prevent this problem from occurring in the use of the pure price index $P_K(p^0,p^1,q^0,q^1)$ defined by equation (15.17), it is asked that P_{Lo} satisfy the following *invariance to proportional changes in current quantities test:*[32]

$$(15.20)\quad P_{Lo}(p^0,p^1,q^0,\lambda q^1) = P_{Lo}(p^0,p^1,q^0,q^1)$$
for all p^0, p^1, q^0, q^1 and all $\lambda > 0$.

The two tests, the time reversal test in equation (15.13) and the invariance test in equation (15.20), enable one to determine the precise functional form for the pure price index P_{Lo} defined by equation (15.17) above: the pure price index P_K must be the Walsh index P_W defined by equation (15.19).[33]

15.32 To be of practical use by statistical agencies, an index number formula must be able to be expressed as a function of the base-period revenue shares, s_i^0; the current-period revenue shares, s_i^1;

[29]For more on symmetric means, see Diewert (1993c, p. 361).

[30]Walsh endorsed P_W as being the best index number formula: “We have seen reason to believe formula 6 better than formula 7. Perhaps formula 9 is the best of the rest, but between it and Nos. 6 and 8 it would be difficult to decide with assurance” (C.M. Walsh, 1921a, p. 103). His formula 6 is P_W defined by equation (15.19), and his 9 is the Fisher ideal defined by equation (15.12) above. The *Walsh quantity index*, $Q_W(p^0,p^1,q^0,q^1)$, is defined as $P_W(q^0,q^1,p^0,p^1)$; that is, prices and quantities in equation (15.19) are interchanged. If the Walsh quantity index is used to deflate the value ratio, an implicit price index is obtained, which is Walsh’s formula 8.

[31]This is not likely to be a severe problem in the time-series context where the change in quantity vectors going from one period to the next is small.

[32]This is the terminology used by Diewert (1992a, p. 216). Vogt (1980) was the first to propose this test.

[33]See Section 7 in Diewert (2001).

and the n price ratios, p_i^1/p_i^0. The Walsh price index defined by equation (15.19) above can be rewritten in this format:

$$(15.21)\quad P_W(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1\sqrt{q_i^0 q_i^1}}{\sum_{j=1}^{n} p_j^0\sqrt{q_j^0 q_j^1}}$$

$$= \frac{\sum_{i=1}^{n} (p_i^1/\sqrt{p_i^0 p_i^1})\sqrt{s_i^0 s_i^1}}{\sum_{j=1}^{n} (p_j^0/\sqrt{p_j^0 p_j^1})\sqrt{s_j^0 s_j^1}}$$

$$= \frac{\sum_{i=1}^{n} \sqrt{s_i^0 s_i^1}\sqrt{p_i^1/p_i^0}}{\sum_{j=1}^{n} \sqrt{s_j^0 s_j^1}\sqrt{p_j^0/p_j^1}}.$$

C.3 Conclusions

15.33 The approach taken to index number theory in this section was to consider averages of various fixed-basket price indices. The first approach was to take an evenhanded average of the two primary fixed-basket indices: the Laspeyres and Paasche price indices. These two primary indices are based on pricing out the baskets that pertain to the two periods (or locations) under consideration. Taking an average of them led to the Fisher ideal price index P_F defined by equation (15.12) above. The second approach was to average the basket quantity weights and then price out this average basket at the prices pertaining to the two situations under consideration. This approach led to the Walsh price index P_W defined by equation (15.19) above. Both these indices can be written as a function of the base-period revenue shares, s_i^0; the current-period revenue shares, s_i^1; and the n price ratios, p_i^1/p_i^0. Assuming that the statistical agency has information on these three sets of variables, which index should be used? Experience with normal time-series data has shown that these two indices will not differ substantially, and thus it is a matter of choice which of these indices is used in practice.[34] Both these indices are examples of *superlative indices*, which will be defined in Chapter 17. However, note that both these indices treat the data pertaining to the two situations in a *symmetric* manner. Hill commented on superlative price indices and the importance of a symmetric treatment of the data as follows:

> Thus economic theory suggests that, in general, a symmetric index that assigns equal weight to the two situations being compared is to be preferred to either the Laspeyres or Paasche indices on their own. The precise choice of superlative index—whether Fisher, Törnqvist or other superlative index—may be of only secondary importance as all the symmetric indices are likely to approximate each other, and the underlying theoretic index fairly closely, at least when the index number spread between the Laspeyres and Paasche is not very great. (Peter Hill, 1993, p. 384)[35]

D. Annual Weights and Monthly Price Indices

D.1 Lowe index with monthly prices and annual base-year quantities

15.34 It is now necessary to discuss a major practical problem with the theory of basket-type indices. Up to now, it has been assumed that the quantity vector $q \equiv (q_1,q_2,\ldots,q_n)$ that appeared in the definition of the Lowe index, $P_{Lo}(p^0,p^1,q)$ defined by equation (15.15), is either the base-period quantity vector q^0 or the current-period quantity vector q^1 or an average of the two. In fact, in terms of actual statistical agency practice, the quantity vector q is usually taken to be an annual quantity vector that refers to a *base year b*, say, that is before the base period for the prices, period 0. Typically, a statistical agency will produce a PPI at a monthly or quarterly frequency, but, for the sake of definiteness, a monthly frequency will be assumed in what follows. Thus, a typical price index will have the form $P_{Lo}(p^0,p^t,q^b)$, where p^0 is the price vector pertaining to the base-period month for prices, month 0; p^t is the price vector pertaining to the current-period month for prices, month t, say;

[34]Diewert (1978, pp. 887–89) showed that these two indices will approximate each other to the second order around an equal price and quantity point. Thus, for normal time-series data where prices and quantities do not change much (continued) going from the base period to the current period, the indices will approximate each other quite closely.

[35]See also Peter Hill (1988).

and q^b is a reference basket quantity vector that refers to the base year b, which is equal to or before month 0.[36] Note that this Lowe index $P_{Lo}(p^0,p^t,q^b)$ is *not* a true Laspeyres index (because the annual quantity vector q^b is not equal to the monthly quantity vector q^0 in general).[37]

15.35 The question is this: why do statistical agencies *not* pick the reference quantity vector q in the Lowe formula to be the monthly quantity vector q^0 that pertains to transactions in month 0 (so that the index would reduce to an ordinary Laspeyres price index)? There are two main reasons:

- Most economies are subject to seasonal fluctuations, and so picking the quantity vector of month 0 as the reference quantity vector for all months of the year would not be representative of transactions made throughout the year.
- Monthly household quantity or revenue weights are usually collected by the statistical agency using an establishment survey with a relatively small sample. Hence, the resulting weights are usually subject to very large sampling errors, and so standard practice is to average these monthly revenue or quantity weights over an entire year (or in some cases, over several years), in an attempt to reduce these sampling errors. In other instances, where an establishment census is used, the reported revenue weights are for an annual period.

The index number problems that are caused by seasonal monthly weights will be studied in more detail in Chapter 22. For now, it can be argued that the use of annual weights in a monthly index number formula is simply a method for dealing with the seasonality problem.[38]

15.36 One problem with using annual weights corresponding to a perhaps distant year in the context of a monthly PPI must be noted at this point. If there are systematic (but divergent) trends in product prices, and consumers or businesses increase their purchases of products that decline (relatively) in price and decrease their purchases of products that increase (relatively) in price, then the use of distant quantity weights will tend to lead to an upward bias in this Lowe index compared with one that used more current weights, as will be shown below. This observation suggests that statistical agencies should get up-to-date weights on an ongoing basis.

15.37 It is useful to explain how the annual quantity vector q^b could be obtained from monthly revenues on each product during the chosen base year b. Let the month m revenue of the reference population in the base year b for product i be $v_i^{b,m}$, and let the corresponding price and quantity be $p_i^{b,m}$ and $q_i^{b,m}$, respectively. Value, price, and quantity for each product are related by the following equations:

$$(15.22)\quad v_i^{b,m} = p_i^{b,m} q_i^{b,m};\ i = 1,...,n;\ m = 1,...,12.$$

For each product i, the annual total q_i^b can be obtained by price-deflating monthly values and summing over months in the base year b as follows:

$$(15.23)\quad q_i^b = \sum_{m=1}^{12} \frac{v_i^{b,m}}{p_i^{b,m}} = \sum_{m=1}^{12} q_i^{b,m};\ i = 1,...,n,$$

where equation (15.22) was used to derive equation (15.23). In practice, the above equations will be evaluated using aggregate revenues over closely related products, and the price $p_i^{b,m}$ will be the month m price index for this elementary product group i in year b relative to the first month of year b.

15.38 For some purposes, it is also useful to have annual prices by product to match the annual quantities defined by equation (15.23). Following national income accounting conventions, a reason-

[36]Month 0 is called the price reference period, and year b is called the weight reference period.

[37]Triplett (1981, p. 12) defined the Lowe index, calling it a Laspeyres index, and calling the index that has the weight reference period equal to the price reference period a pure Laspeyres index. Triplett also noted the hybrid share representation for the Lowe index defined by equation (15.15) and equation (15.16). Triplett noted that the ratio of two Lowe indices using the same quantity weights was also a Lowe index.

[38]In fact, using the Lowe index $P_{Lo}(p^0,p^t,q^b)$ in the context of seasonal products corresponds to Bean and Stine's (1924, p. 31) Type A index number formula. Bean and (continued)

Stine made three additional suggestions for price indices in the context of seasonal products. Their contributions will be evaluated in Chapter 22.

able[39] price p_i^b to match the annual quantity q_i^b is the value of total revenue for product i in year b divided by q_i^b. Thus, we have

$$(15.24)\ p_i^b \equiv \sum_{m=1}^{12} v_i^{b,m} \Big/ q_i^b\ ;\ i = 1,\ldots,n$$

$$= \frac{\sum_{m=1}^{12} v_i^{b,m}}{\sum_{m=1}^{12} v_i^{b,m} \Big/ p_i^{b,m}}\quad ;\text{ using equation (15.23)}$$

$$= \left[\sum_{m=1}^{12} s_i^{b,m} (p_i^{b,m})^{-1}\right]^{-1},$$

where the share of annual revenue on product i in month m of the base year is

$$(15.25)\ \ s_i^{b,m} \equiv \frac{v_i^{b,m}}{\sum_{k=1}^{12} v_i^{b,k}};\ i = 1,\ldots,n.$$

Thus, the annual base-year price for product i, p_i^b, turns out to be a monthly revenue-weighted *harmonic mean* of the monthly prices for product i in the base year, $p_i^{b,1}, p_i^{b,2},\ldots, p_i^{b,12}$.

15.39 Using the annual product prices for the base year defined by equation (15.24), a vector of these prices can be defined as $p^b \equiv [p_1^b,\ldots,p_n^b]$. Using this definition, the Lowe index can be expressed as a ratio of two Laspeyres indices where the price vector p^b plays the role of base-period prices in each of the two Laspeyres indices:

$$(15.26)\ \ P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \frac{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b} = \frac{\sum_{i=1}^{n} s_i^b (p_i^t / p_i^b)}{\sum_{i=1}^{n} s_i^b (p_i^0 / p_i^b)}$$

$$= P_L(p^b, p^t, q^b) / P_L(p^b, p^0, q^b),$$

where the Laspeyres formula P_L was defined by equation (15.5) above. Thus, the above equation shows that the Lowe monthly price index comparing the prices of month 0 with those of month t using the quantities of base year b as weights, $P_{Lo}(p^0,p^t,q^b)$, is equal to the Laspeyres index that compares the prices of month t with those of year b, $P_L(p^b,p^t,q^b)$, divided by the Laspeyres index that compares the prices of month 0 with those of year b, $P_L(p^b,p^0,q^b)$. Note that the Laspeyres index in the numerator can be calculated if the base-year product revenue shares, s_i^b, are known along with the price ratios that compare the prices of product i in month t, p_i^t, with the corresponding annual average prices in the base year b, p_i^b. The Laspeyres index in the denominator can be calculated if the base-year product revenue shares, s_i^b, are known along with the price ratios that compare the prices of product i in month 0, p_i^0, with the corresponding annual average prices in the base year b, p_i^b.

15.40 Another convenient formula for evaluating the Lowe index, $P_{Lo}(p^0,p^t,q^b)$, uses the hybrid weights formula, equation (15.15). In the present context, the formula becomes

$$(15.27)\ \ P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \frac{\sum_{i=1}^{n} (p_i^t / p_i^0) p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^{0b},$$

where the hybrid weights s_i^{0b} using the prices of month 0 and the quantities of year b are defined by

$$(15.28)\ \ s_i^{0b} \equiv \frac{p_i^0 q_i^b}{\sum_{j=1}^{n} p_j^0 q_j^b};\ i = 1,\ldots,n$$

[39] Hence, these annual product prices are essentially unit-value prices. Under conditions of high inflation, the annual prices defined by equation (15.24) may no longer be reasonable or representative of prices during the entire base year because the revenues in the final months of the high-inflation year will be somewhat artificially blown up by general inflation. Under these conditions, the annual prices and annual product revenue shares should be interpreted with caution. For more on dealing with situations where there is high inflation within a year, see Peter Hill (1996).

$$= \frac{p_i^b q_i^b (p_i^0 / p_i^b)}{\sum_{j=1}^{n} \left[p_j^b q_j^b (p_j^0 / p_j^b) \right]}.$$

Equation (15.28) shows how the base-year revenues, $p_i^b q_i^b$, can be multiplied by the product price indices, p_i^0/p_i^b, to calculate the hybrid shares.

15.41 One additional formula for the Lowe index, $P_{Lo}(p^0,p^t,q^b)$, will be exhibited. Note that the Laspeyres decomposition of the Lowe index defined by the third line in equation (15.26) involves the very long-term price relatives, p_i^t/p_i^b, that compare the prices in month t, p_i^t, with the possibly distant base-year prices, p_i^b. Further, the hybrid share decomposition of the Lowe index defined by the third line in equation (15.27) involves the long-term monthly price relatives, p_i^t/p_i^0, which compare the prices in month t, p_i^t, with the base month prices, p_i^0. Both these formulas are not satisfactory in practice because of the problem of sample attrition: each month, a substantial fraction of products disappears from the marketplace, and thus it is useful to have a formula for updating the previous month's price index using just month-over-month price relatives. In other words, long-term price relatives disappear at a rate that is too large in practice to base an index number formula on their use. The Lowe index for month $t + 1$, $P_{Lo}(p^0,p^{t+1},q^b)$, can be written in terms of the Lowe index for month t, $P_{Lo}(p^0,p^t,q^b)$, and an updating factor as follows:

$$(15.29)\ P_{Lo}(p^0, p^{t+1}, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \right] \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right]$$

$$= P_{Lo}(p^0, p^t, q^b) \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right]$$

$$= P_{Lo}(p^0, p^t, q^b) \left[\frac{\sum_{i=1}^{n} \left(\frac{p_i^{t+1}}{p_i^t} \right) p_i^t q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right]$$

$$= P_{Lo}(p^0, p^t, q^b) \left[\sum_{i=1}^{n} \left(\frac{p_i^{t+1}}{p_i^t} \right) s_i^{tb} \right],$$

where the hybrid weights s_i^{tb} are defined by

$$(15.30)\ s_i^{tb} \equiv \frac{p_i^t q_i^b}{\sum_{j=1}^{n} p_j^t q_j^b};\ i = 1,...,n.$$

Thus, the required updating factor, going from month t to month $t + 1$, is the chain-linked index $\sum_{i=1}^{n} s_i^{tb} \left(p_i^{t+1} / p_i^t \right)$, which uses the hybrid share weights s_i^{tb} corresponding to month t and base year b.

15.42 The Lowe index $P_{Lo}(p^0,p^t,q^b)$ can be regarded as an approximation to the ordinary Laspeyres index, $P_L(p^0,p^t,q^0)$, that compares the prices of the base month 0, p^0, with those of month t, p^t, using the quantity vector of month 0, q^0, as weights. There is a relatively simple formula that relates these two indices. To explain this formula, it is first necessary to make a few definitions. Define the ith price relative between month 0 and month t as

$$(15.31)\ r_i \equiv p_i^t / p_i^0;\ i = 1,...,n.$$

The ordinary Laspeyres price index, going from month 0 to t, can be defined in terms of these price relatives as follows:

$$(15.32)\ P_L(p^0, p^t, q^0) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \frac{\sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right) s_i^0$$

$$= \sum_{i=1}^{n} s_i^0 r_i \equiv r^*,$$

where the month 0 revenue shares s_i^0 are defined as follows:

$$(15.33)\quad s_i^0 \equiv \frac{p_i^0 q_i^0}{\sum_{j=1}^{n} p_j^0 q_j^0};\ i=1,...,n.$$

15.43 Define the *i*th quantity relative t_i as the ratio of the quantity of product i used in the base year b, q_i^b, to the quantity used in month 0, q_i^0, as follows:

$$(15.34)\quad t_i \equiv q_i^b / q_i^0;\ i=1,...,n.$$

The Laspeyres quantity index, $Q_L(q^0,q^b,p^0)$, that compares quantities in year b, q^b, with the corresponding quantities in month 0, q^0, using the prices of month 0, p^0, as weights can be defined as a weighted average of the quantity ratios t_i as follows:

$$(15.35)\quad Q_L(q^0,q^b,p^0) \equiv \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \frac{\sum_{i=1}^{n}\left(\frac{q_i^b}{q_i^0}\right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \sum_{i=1}^{n}\left(\frac{q_i^b}{q_i^0}\right) s_i^0 \text{ ; using equation (15.34)}$$

$$= \sum\nolimits_{i=1}^{n} s_i^0 t_i \equiv t^*$$

15.44 Using equation (A15.2.4) in Appendix 15.2, the relationship between the Lowe index $P_{Lo}(p^0,p^t,q^b)$ that uses the quantities of year b as weights to compare the prices of month t with month 0 and the corresponding ordinary Laspeyres index $P_L(p^0,p^t,q^0)$ that uses the quantities of month 0 as weights is defined as

$$(15.36)\quad P_{Lo}(p^0,p^t,q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= P_L(p^0,p^t,q^0) + \frac{\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0}{Q_L(q^0,q^b,p^0)}.$$

Thus, the Lowe price index using the quantities of year b as weights, $P_{Lo}(p^0,p^t,q^b)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0,p^t,q^0)$, plus a covariance term $\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0$ between the price relatives $r_i \equiv p_i^{\,t}/\,p_i^0$ and the quantity relatives $t_i \equiv q_i^b/q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0,q^b,p^0)$ between month 0 and base year b.

15.45 Equation (15.36) shows that the Lowe price index will coincide with the Laspeyres price index if the covariance or correlation between the month 0 to t price relatives $r_i \equiv p_i^t/p_i^0$ and the month 0 to year b quantity relatives $t_i \equiv q_i^b/q_i^0$ is zero. Note that this covariance will be zero under three different sets of conditions:

- If the month t prices are proportional to the month 0 prices so that all $r_i = r^*$,
- If the base year b quantities are proportional to the month 0 quantities so that all $t_i = t^*$, and
- If the distribution of the relative prices r_i is independent of the distribution of the relative quantities t_i.

The first two conditions are unlikely to hold empirically, but the third is possible, at least approximately, if purchasers do not systematically change their purchasing habits in response to changes in relative prices.

15.46 If this covariance in equation (15.36) is negative, then the Lowe index will be less than the Laspeyres, and, finally, if the covariance is positive, then the Lowe index will be greater than the Laspeyres index. Although the sign and magnitude of the covariance term is ultimately an empirical matter, it is possible to make some reasonable conjectures about its likely sign. If the base year b precedes the price reference month 0 *and there are long-term trends in prices*, then it is likely that this covariance is *positive,* and hence that the Lowe index will exceed the corresponding Laspeyres price index;[40] that is,

(15.37) $P_{Lo}(p^0, p^t, q^b) > P_L(p^0, p^t, q^0)$.

To see why this covariance is likely to be positive, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t / p_i^0) - r^*$ is positive. With normal substitution responses,[41] q_i^t / q_i^0 less an average quantity change of this type (t^*) is likely to be negative, or, upon taking reciprocals, q_i^0 / q_i^t less an average quantity change of this (reciprocal) type is likely to be positive. But if the long-term upward trend in prices has persisted back to the base year b, then $t_i - t^* \equiv (q_i^b / q_i^0) - t^*$ is also likely to be positive. Hence, the covariance will be positive under these circumstances. Moreover, the more distant is the weight reference year b from the price reference month 0, the bigger the residuals $t_i - t^*$ will likely be and the bigger will be the positive covariance. Similarly, the more distant is the current-period month t from the base-period month 0, the bigger the residuals $r_i - r^*$ will likely be and the bigger will be the positive covariance. *Thus, under the assumptions that there are long-term trends in prices and normal substitution responses, the Lowe index will normally be greater than the corresponding Laspeyres index*.

15.47 Define the Paasche index between months 0 and t as follows:

$$(15.38)\quad P_P(p^0, p^t, q^t) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t}.$$

As was discussed in Section C.1, a reasonable target index to measure the price change going from month 0 to t is some sort of symmetric average of the Paasche index $P_P(p^0,p^t,q^t)$ defined by equation (15.38) and the corresponding Laspeyres index $P_L(p^0,p^t,q^0)$ defined by equation (15.32). Adapting equation (A15.1.5) in Appendix 15.1, the relationship between the Paasche and Laspeyres indices can be written as follows:

$$(15.39)\quad P_P(p^0, p^t, q^t) = P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*) s_i^0}{Q_L(q^0, q^t, p^0)},$$

where the price relatives $r_i \equiv p_i^t / p_i^0$ are defined by equation (15.31) and their share-weighted average r^* by equation (15.32), and the u_i, u^* and Q_L are defined as follows:

$$(15.40)\quad u_i \equiv q_i^t / q_i^0;\ i = 1,...,n,$$

$$(15.41)\quad u^* \equiv \sum_{i=1}^{n} s_i^0 u_i = Q_L(q^0, q^t, p^0),$$

and the month 0 revenue shares s_i^0 are defined by equation (15.33). Thus, u^* is equal to the Laspeyres quantity index between months 0 and t. This means that the Paasche price index that uses the quantities of month t as weights, $P_P(p^0,p^t,q^t)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0,p^t,q^0)$, plus a covariance term $\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*) s_i^0$ between the price relatives $r_i \equiv p_i^t / p_i^0$ and the quantity relatives $u_i \equiv q_i^t / q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0,q^t,p^0)$ between month 0 and month t.

15.48 Although the sign and magnitude of the covariance term is again an empirical matter, it is possible to make a reasonable conjecture about its likely sign. If *there are long-term trends in prices, and purchasers respond normally to price changes in their purchases*, then it is likely that this covari-

[40]It is also necessary to assume that purchasers have normal substitution effects in response to these long-term trends in prices; that is, if a product increases (relatively) in price, its quantity purchased will decline (relatively), and if a product decreases relatively in price, its quantity purchased will increase relatively. This reflects the normal "market equilibrium" response to changes in supply.

[41]Walsh (1901, pp. 281–82) was well aware of substitution effects, as can be seen in the following comment that noted the basic problem with a fixed-basket index that uses the quantity weights of a single period: "The argument made by the arithmetic averagist supposes that we buy the same quantities of every class at both periods in spite of the variation in their prices, which we rarely, if ever, do. As a rough proposition, we—a community—generally spend more on articles that have risen in price and get less of them, and spend less on articles that have fallen in price and get more of them."

ance is *negative,* and hence the Paasche index will be less than the corresponding Laspeyres price index; that is,

(15.42) $P_P(p^0, p^t, q^t) < P_L(p^0, p^t, q^0)$.

To see why this covariance is likely to be negative, suppose that there is a long-term upward trend in the price of product i [42] so that $r_i - r^* \equiv (p_i^t / p_i^0) - r^*$ is positive. With normal substitution responses, q_i^t / q_i^0 less an average quantity change of this type (u^*) is likely to be negative. Hence, $u_i - u^* \equiv (q_i^t / q_i^0) - u^*$ is likely to be negative. Thus, the covariance will be negative under these circumstances. Moreover, the more distant is the base month 0 from the current-month t, the bigger in magnitude the residuals $u_i - u^*$ will likely be and the bigger in magnitude will be the negative covariance.[43] Similarly, the more distant is the current-period month t from the base-period month 0, the bigger the residuals $r_i - r^*$ will likely be and the bigger in magnitude will be the covariance. *Thus, under the assumptions that there are long-term trends in prices and normal substitution responses, the Laspeyres index will be greater than the corresponding Paasche index*, with the divergence likely growing as month t becomes more distant from month 0.

15.49 Putting the arguments in the three previous paragraphs together, it can be seen that under the assumptions that there are long-term trends in prices and normal substitution responses, the Lowe price index between months 0 and t will exceed the corresponding Laspeyres price index, which in turn will exceed the corresponding Paasche price index; that is, under these hypotheses,

(15.43)
$P_{Lo}(p^0, p^t, q^b) > P_L(p^0, p^t, q^0) > P_P(p^0, p^t, q^t)$.

Thus, if the long-run target price index is an average of the Laspeyres and Paasche indices, it can be seen that the Laspeyres index will have an *upward bias* relative to this target index, and the Paasche index will have a *downward bias*. In addition, *if the base year* b *is prior to the price reference month, month 0, then the Lowe index will also have an upward bias relative to the Laspeyres index and hence also to the target index*.

D.2 Lowe index and midyear indices

15.50 The discussion in the previous paragraph assumed that the base year b for quantities preceded the base month for prices, month 0. However, if the current-period month t is quite distant from the base month 0, then it is possible to think of the base year b as referring to a year *that lies between months 0 and* t. If the year b does fall between months 0 and t, then the Lowe index becomes a *midyear index*.[44] The Lowe midyear index no longer has the upward biases indicated by the inequalities in equation (15.43) under the assumption of long-term trends in prices and normal substitution responses by quantities.

15.51 It is now assumed that the base-year quantity vector q^b corresponds to a year that lies between months 0 and t. Under the assumption of long-term trends in prices and normal substitution effects so that there are also long-term trends in quantities (in the opposite direction to the trends in prices so that if the ith product price is trending up, then the corresponding ith quantity is trending down), it is likely that the intermediate-year quan-

[42] The reader can carry through the argument if there is a long-term relative decline in the price of the ith product. The argument required to obtain a negative covariance requires that there be some differences in the long-term trends in prices; that is, if all prices grow (or fall) at the same rate, we have price proportionality, and the covariance will be zero.

[43] However, $Q_L = u^*$ may also be growing in magnitude, so the net effect on the divergence between P_L and P_P is ambiguous.

[44] This concept can be traced to Peter Hill (1998, p. 46): "When inflation has to be measured over a specified sequence of years, such as a decade, a pragmatic solution to the problems raised above would be to take the middle year as the base year. This can be justified on the grounds that the basket of goods and services purchased in the middle year is likely to be much more representative of the pattern of consumption over the decade as a whole than baskets purchased in either the first or the last years. Moreover, choosing a more representative basket will also tend to reduce, or even eliminate, any bias in the rate of inflation over the decade as a whole as compared with the increase in the CoL index." Thus, in addition to introducing the concept of a midyear index, Hill also introduced the idea of *representativity bias*. For additional material on midyear indices, see Schultz (1999) and Okamoto (2001). Note that the midyear index concept could be viewed as a close competitor to Walsh's (1901, p. 431) multiyear fixed-basket index, where the quantity vector was chosen to be an arithmetic or geometric average of the quantity vectors in the period.

tity vector will lie between the monthly quantity vectors q^0 and q^t. The midyear Lowe index, $P_{Lo}(p^0,p^t,q^b)$, and the Laspeyres index going from month 0 to t, $P_L(p^0,p^t,q^0)$, will still satisfy the exact relationship given by equation (15.36). Thus, $P_{Lo}(p^0,p^t,q^b)$ will equal $P_L(p^0,p^t,q^0)$ plus the covariance term $\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0 \Big/ Q_L(q^0,q^b,p^0)$, where $Q_L(q^0,q^b,p^0)$ is the Laspeyres quantity index going from month 0 to t. This covariance term is likely to be negative, so that

$$(15.44)\quad P_L(p^0,p^t,q^0) > P_{Lo}(p^0,p^t,q^b).$$

To see why this covariance is likely to be negative, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t / p_i^0) - r^*$ is positive. With normal substitution responses, q_i will tend to decrease relatively over time, and since q_i^b is assumed to be between q_i^0 and q_i^t, q_i^b/q_i^0 less an average quantity change of this type, r^* is likely to be negative. Hence $u_i - u^* \equiv (q_i^b / q_i^0) - t^*$ is likely to be negative. Thus, the covariance is likely to be negative under these circumstances. *Under the assumptions that the quantity base year falls between months 0 and* t *and that there are long-term trends in prices and normal substitution responses, the Laspeyres index will normally be larger than the corresponding Lowe midyear index*, with the divergence likely growing as month t becomes more distant from month 0.

15.52 It can also be seen that under the above assumptions, the midyear Lowe index is likely to be greater than the Paasche index between months 0 and t; that is,

$$(15.45)\quad P_{Lo}(p^0,p^t,q^b) > P_P(p^0,p^t,q^t).$$

To see why the above inequality is likely to hold, think of q^b starting at the month 0 quantity vector q^0 and then trending smoothly to the month t quantity vector q^t. When $q^b = q^0$, the Lowe index becomes the Laspeyres index $P_L(p^0,p^t,q^0)$. When $q^b = q^t$, the Lowe index becomes the Paasche index $P_P(p^0,p^t,q^t)$. Under the assumption of trending prices and normal substitution responses to these trending prices, it was shown earlier that the Paasche index will be less than the corresponding Laspeyres price index; that is, that $P_P(p^0,p^t,q^t)$ was less than $P_L(p^0,p^t,q^0)$; recall equation (15.42). Thus, under the assumption of smoothly trending prices and quantities between months 0 and t, and *assuming that* q^b *is between* q^0 *and* q^t, we will have

$$(15.46)\quad P_P(p^0,p^t,q^t) < P_{Lo}(p^0,p^t,q^b) < P_L(p^0,p^t,q^0).$$

Thus, if the base year for the Lowe index is chosen to be between the base month for the prices, month 0, and the current month for prices, month t, and there are trends in prices with corresponding trends in quantities that correspond to normal substitution effects, then the resulting Lowe index is likely to lie between the Paasche and Laspeyres indices going from months 0 to t. If the trends in prices and quantities are smooth, then choosing the base year halfway between periods 0 and t should give a Lowe index that is approximately halfway between the Paasche and Laspeyres indices and hence will be very close to an ideal target index between months 0 and t. This basic idea has been implemented by Okamoto (2001) using Japanese consumer data, and he found that the resulting midyear indices approximated the corresponding Fisher ideal indices very closely.

15.53 It should be noted that these midyear indices can be computed only on a retrospective basis; that is, they cannot be calculated in a timely fashion as can Lowe indices that use a base year before month 0. Thus, midyear indices cannot be used to replace the more timely Lowe indices. However, these timely Lowe indices are likely to have an upward bias even bigger than the usual Laspeyres upward bias compared with an ideal target index, which was taken to be an average of the Paasche and Laspeyres indices.

15.54 All of the inequalities derived in this section rest on the assumption of long-term trends in prices (and corresponding economic responses in quantities). If there are no systematic long-run trends in prices and only random fluctuations around a common trend in all prices, then the above inequalities are not valid, and the Lowe index using a prior base year will probably provide a perfectly adequate approximation to both the Paasche and Laspeyres indices. However, there are some reasons for believing that some long-run trends in prices exist:

(i) The computer chip revolution of the past 40 years has led to strong downward trends in the prices of products that use these chips intensively. As new uses for chips are developed, the share of products that are chip-intensive has grown, which implies that what used to be a relatively minor problem has become a major problem.

(ii) Other major scientific advances have had similar effects. For example, the invention of fiber-optic cable (and lasers) has led to a downward trend in telecommunications prices as obsolete technologies based on copper wire are gradually replaced.

(iii) Since the end of World War II, a series of international trade agreements have dramatically reduced tariffs around the world. These reductions, combined with improvements in transportation technologies, have led to a rapid growth of international trade and remarkable improvements in international specialization. Manufacturing activities in the more developed economies have gradually been outsourced to lower-wage countries, leading to deflation in goods prices in most countries. However, many services cannot be readily outsourced,[45] and so on average the price of services trends upward while the price of goods trends downward.

(iv) At the microeconomic level, there are tremendous differences in growth rates of firms. Successful firms expand their scale, lower their costs, and cause less successful competitors to wither away with their higher prices and lower volumes. This leads to a systematic negative correlation between changes in item prices and the corresponding changes in item volumes that can be very large.

Thus, there is some a priori basis for assuming long-run divergent trends in prices and hence some basis for concern that a Lowe index that uses a base year for quantity weights that is prior to the base month for prices may be upward biased, compared with a more ideal target index.

[45]However some services can be internationally outsourced; for example, call centers, computer programming, and airline maintenance.

D.3 Young index

15.55 Recall the definitions for the base-year quantities, q_i^b, and the base-year prices, p_i^b, given by equation (15.23) and equation (15.24). The base-year revenue shares can be defined in the usual way as follows:

$$(15.47)\quad s_i^b \equiv \frac{p_i^b q_i^b}{\sum_{k=1}^{n} p_k^b q_k^b}; \ i = 1,...,n.$$

Define the vector of base-year revenue shares in the usual way as $s^b \equiv [s_1^b,\ldots,s_n^b]$. These base-year revenue shares were used to provide an alternative formula for the base year b Lowe price index going from month 0 to t defined in equation (15.26) as $P_{Lo}(p^0,p^t,q^b) = \sum_{i=1}^{n} s_i^b (p_i^t / p_i^b) \Big/ \sum_{i=1}^{n} s_i^b (p_i^0 / p_i^b)$. Rather than using this index as their short-run target index, many statistical agencies use the following closely related index:

$$(15.48)\quad P_Y(p^0, p^t, s^b) \equiv \sum_{i=1}^{n} s_i^b \left(p_i^t / p_i^0 \right).$$

This type of index was first defined by the English economist Arthur Young (1812).[46] Note that there is a change in focus when the Young index is used compared with the indices proposed earlier in this chapter. Up to this point, the indices proposed have been of the fixed-basket type (or averages of such indices), where a *product basket* that is somehow representative for the two periods being compared is chosen and then "purchased" at the prices of the two periods, and the index is taken to be the ratio of these two costs. On the other hand, for the Young index, one instead chooses *representative revenue shares* that pertain to the two periods under consideration and then uses these shares to calculate the overall index as a share-weighted average of the individual price ratios, p_i^t / p_i^0. Note that this share-weighted average of price ratios view of index number theory is a bit different from the view taken at the beginning of this chapter, which viewed the index number problem as the problem of decomposing a value ratio into the product of two terms, one of which expresses the amount of

[46]Walsh (1901, p. 536; 1932, p. 657) attributes this formula to Young.

price change between the two periods and the other that expresses the amount of quantity change.[47]

15.56 Statistical agencies sometimes regard the Young index defined above as an approximation to the Laspeyres price index $P_L(p^0,p^t,q^0)$. Hence, it is of interest to see how the two indices compare. Defining the long-term monthly price relatives going from month 0 to t as $r_i \equiv p_i^t/p_i^0$ and using equations (15.32) and (15.48),

$$(15.49)\quad P_Y(p^0,p^t,s^b) - P_L(p^0,p^t,q^0)$$
$$\equiv \sum_{i=1}^{n} s_i^b\left(\frac{p_i^t}{p_i^0}\right) - \sum_{i=1}^{n} s_i^0\left(\frac{p_i^t}{p_i^0}\right)$$
$$= \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]r_i$$
$$= \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left[r_i - r^*\right] + r^*\sum_{i=1}^{n}\left[s_i^b - s_i^0\right]$$
$$= \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left[r_i - r^*\right],$$

since $\sum_{i=1}^{n} s_i^b = \sum_{i=1}^{n} s_i^0 = 1$ and defining

$$r^* \equiv \sum_{i=1}^{n} s_i^0 r_i \; = P_L\left(p^0,p^t,q^0\right).$$

Thus, the Young index $P_Y(p^0,p^t,s^b)$ is equal to the Laspeyres index $P_L(p^0,p^t,q^0)$ plus the *covariance* between the difference in the annual shares pertaining to year b and the month 0 shares, $s_i^b - s_i^0$, and the deviations of the relative prices from their mean, $r_i - r^*$.

15.57 It is no longer possible to guess the likely sign of the covariance term. The question is no longer whether the *quantity* demanded goes down as the price of product i goes up (the answer to this question is usually yes) but does the *share* of revenue go down as the price of product i goes up? The answer depends on the elasticity of demand for the product. However, let us provisionally assume that there are long-run trends in product prices, and if the trend in prices for product i is above the mean, then the revenue share for the product trends *down* (and vice versa). Thus, we are assuming high elasticities or very strong substitution effects. Assuming also that the base year b is before month 0, then under these conditions, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t / p_i^0) - r^*$ is positive. With the assumed very elastic purchaser substitution responses, s_i will tend to decrease relatively over time. Since s_i^b is assumed to be before s_i^0, s_i^0 is expected to be less than s_i^b , or $s_i^b - s_i^0$ will likely be positive. Thus, the covariance is likely to be *positive* under these circumstances. *Hence with long-run trends in prices and very elastic responses of purchasers to price changes, the Young index is likely to be greater than the corresponding Laspeyres index.*

15.58 Assume that there are long-run trends in product prices. If the trend in prices for product i is above the mean, then suppose that the revenue share for the product trends *up* (and vice versa). Thus, we are assuming low elasticities or very weak substitution effects. Assume also that the base year b is before month 0, and suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t / p_i^0) - r^*$ is positive. With the assumed very inelastic substitution responses, s_i will tend to increase relatively over time, and, since s_i^b is assumed to be before s_i^0, we will have s_i^0 greater than s_i^b , or $s_i^b - s_i^0$ is negative. Thus, the covariance is likely to be *negative* under

[47]Irving Fisher's 1922 book is famous for developing the value ratio decomposition approach to index number theory, but his introductory chapters took the share-weighted average point of view: "An index number of prices, then, shows the *average percentage change* of prices from one point of time to another" (1922, p. 3). Fisher went on to note the importance of economic weighting: "The preceding calculation treats all the commodities as equally important; consequently, the average was called 'simple'. If one commodity is more important than another, we may treat the more important as though it were two or three commodities, thus giving it two or three times as much 'weight' as the other commodity" (1922, p. 6). Walsh (1901, pp. 430–31) considered both approaches: "We can either (1) draw some average of the total money values of the classes during an epoch of years, and with weighting so determined employ the geometric average of the price variations [ratios]; or (2) draw some average of the mass quantities of the classes during the epoch, and apply to them Scrope's method." Scrope's method is the same as using the Lowe index. Walsh (1901, pp. 88–90) consistently stressed the importance of weighting price ratios by their economic importance (rather than using equally weighted averages of price relatives). Both the value ratio decomposition approach and the share-weighted average approach to index number theory will be studied from the axiomatic perspective in the following chapter; see also Sections C and E in Chapter 16.

these circumstances. *Hence with long-run trends in prices and very inelastic responses of purchasers to price changes, the Young index is likely to be less than the corresponding Laspeyres index*.

15.59 The previous two paragraphs indicate that, a priori, it is not known what the likely difference between the Young index and the corresponding Laspeyres index will be. If elasticities of substitution are close to 1, then the two sets of revenue shares, s_i^b and s_i^0, will be close to each other and the difference between the two indices will be close to zero. However, if monthly revenue shares have strong seasonal components, then the annual shares s_i^b could differ substantially from the monthly shares s_i^0.

15.60 It is useful to have a formula for updating the previous month's Young price index using only month-over-month price relatives. The Young index for month $t + 1$, $P_Y(p^0,p^{t+1},s^b)$, can be presented in terms of the Lowe index for month t, $P_Y(p^0,p^t,s^b)$, and an updating factor as follows:

$$(15.50)\quad P_Y(p^0,p^{t+1},s^b) \equiv \sum_{i=1}^{n} s_i^b \left(\frac{p_i^{t+1}}{p_i^0}\right)$$

$$= P_Y(p^0,p^t,s^b)\frac{\sum_{i=1}^{n} s_i^b (p_i^{t+1}/p_i^0)}{\sum_{i=1}^{n} s_i^b (p_i^t/p_i^0)}$$

$$= P_Y(p^0,p^t,s^b)\frac{\sum_{i=1}^{n} p_i^b q_i^b (p_i^{t+1}/p_i^0)}{\sum_{i=1}^{n} p_i^b q_i^b (p_i^t/p_i^0)};$$

using equation (15.47)

$$= P_Y(p^0,p^t,s^b)\frac{\sum_{i=1}^{n} p_i^b q_i^b \left(\frac{p_i^t}{p_i^0}\right)\left(\frac{p_i^{t+1}}{p_i^t}\right)}{\sum_{i=1}^{n} p_i^b q_i^b (p_i^t/p_i^0)}$$

$$= P_Y(p^0,p^t,s^b)\left[\sum_{i=1}^{n} s_i^{b0t} (p_i^{t+1}/p_i^t)\right],$$

where the hybrid weights s_i^{b0t} are defined by

$$(15.51)\quad s_i^{b0t} \equiv \frac{p_i^b q_i^b (p_i^t/p_i^0)}{\sum_{k=1}^{n} p_k^b q_k^b (p_k^t/p_k^0)}$$

$$= \frac{s_i^b (p_i^t/p_i^0)}{\sum_{k=1}^{n} s_k^b (p_k^t/p_k^0)};\ i = 1,\ldots,n.$$

Thus, the hybrid weights s_i^{b0t} can be obtained from the base-year weights s_i^b by updating them; that is, by multiplying them by the price relatives (or indices at higher levels of aggregation), p_i^t / p_i^0. Thus, the required updating factor, going from month t to month $t + 1$, is the chain-linked index, $\sum_{i=1}^{n} s_i^{b0t} (p_i^{t+1}/p_i^t)$, which uses the hybrid revenue-share weights s_i^{b0t} defined by equation (15.51).

15.61 Even if the Young index provides a close approximation to the corresponding Laspeyres index, it is difficult to recommend the use of the Young index as a final estimate of the change in prices going from period 0 to t, just as it was difficult to recommend the use of the Laspeyres index as the *final* estimate of inflation going from period 0 to t. Recall that the problem with the Laspeyres index was its lack of symmetry in the treatment of the two periods under consideration. That is, using the justification for the Laspeyres index as a good fixed-basket index, there was an identical justification for the use of the Paasche index as an equally good fixed-basket index to compare periods 0 and t. The Young index suffers from a similar lack of symmetry with respect to the treatment of the base period. The problem can be explained as follows. The Young index, $P_Y(p^0,p^t,s^b)$, defined by equation (15.48), calculates the price change between months 0 and t, treating month 0 as the base. But there is no particular reason to treat month 0 as the base month other than convention. Hence, if we treat month t as the base and use the same formula to measure the price change from month t back to month 0, the index $P_Y(p^0,p^t,s^b) = \sum_{i=1}^{n} s_i^b (p_i^0/p_i^t)$ would be appropriate. This estimate of price change can then be made comparable to the original Young index by taking its reciprocal, leading to the following *rebased Young index,*[48] $P_Y^*(p^0,p^t,s^b)$, defined as

[48]Using Irving Fisher's (1922, p. 118) terminology, $P_Y^*(p^0,p^t,s^b) \equiv 1/[P_Y(p^t,p^0,s^b)]$ is the *time antithesis* of the original Young index, $P_Y(p^0,p^t,s^b)$.

$$(15.52)\ P_Y^*(p^0, p^t, s^b) \equiv 1 \Big/ \sum_{i=1}^{n} s_i^b (p_i^0 / p_i^t) = \left[\sum_{i=1}^{n} s_i^b (p_i^t / p_i^0)^{-1} \right]^{-1}.$$

Thus, the rebased Young index, $P_Y{}^*(p^0,p^t,s^b)$, that uses the current month as the initial base period is a *share-weighted harmonic mean* of the price relatives going from month 0 to month t, whereas the original Young index, $P_Y(p^0,p^t,s^b)$, is a *share-weighted arithmetic mean* of the same price relatives.

15.62 Fisher argued that an index number formula should give the same answer no matter which period was chosen as the base:

> Either one of the two times may be taken as the "base". Will it make a difference which is chosen? Certainly, it *ought* not and our Test 1 demands that it shall not. More fully expressed, the test is that the formula for calculating an index number should be such that it will give the same ratio between one point of comparison and the other point, *no matter which of the two is taken as the base*. (Irving Fisher, 1922, p. 64)

15.63 The problem with the Young index is that not only does it not coincide with its rebased counterpart, but there is a definite inequality between the two indices, namely

$$(15.53)\ P_Y^*(p^0, p^t, s^b) \le P_Y(p^0, p^t, s^b),$$

with a strict inequality provided that the period t price vector p^t is not proportional to the period 0 price vector p^0.[49] Thus, a statistical agency that uses the direct Young index $P_Y(p^0,p^t,s^b)$ will generally show a higher inflation rate than a statistical agency that uses the same raw data but uses the rebased Young index, $P_Y{}^*(p^0,p^t,s^b)$.

15.64 The inequality in equation (15.53) does not tell us by how much the Young index will exceed its rebased time antithesis. However, in Appendix 15.3, it is shown that to the accuracy of a certain second-order Taylor series approximation, the following relationship holds between the direct Young index and its time antithesis:

$$(15.54)\ P_Y(p^0, p^t, s^b) \approx P_Y^*(p^0, p^t, s^b) + P_Y(p^0, p^t, s^b)\ \mathrm{Var}\ e,$$

where Var e is defined as

$$(15.55)\ \mathrm{Var}\ e \equiv \sum_{i=1}^{n} s_i^b \left[e_i - e^* \right]^2.$$

The deviations e_i are defined by $1 + e_i = r_i / r^*$ for $i = 1,\ldots,n$ where the r_i and their weighted mean r^* are defined by

$$(15.56)\ r_i \equiv p_i^t / p_i^0;\ i = 1,\ldots,n\quad ,$$

$$(15.57)\ r^* \equiv \sum_{i=1}^{n} s_i^b r_i\ ,$$

which turns out to equal the direct Young index, $P_Y(p^0,p^t,s^b)$. The weighted mean of the e_i is defined as

$$(15.58)\ e^* \equiv \sum_{i=1}^{n} s_i^b e_i\ ,$$

which turns out to equal 0. Hence, the more dispersion there is in the price relatives p_i^t / p_i^0, to the accuracy of a second-order approximation, the more the direct Young index will exceed its counterpart that uses month t as the initial base period rather than month 0.

[49]These inequalities follow from the fact that a harmonic mean of M positive numbers is always equal to or less than the corresponding arithmetic mean; see Walsh (1901, p. 517) or Irving Fisher (1922, pp. 383–84). This inequality is a special case of Schlömilch's (1858) Inequality; see Hardy, Littlewood and Polyá (1934, p. 26). Walsh (1901, pp. 330–32) explicitly noted the inequality in equation (15.53) and also noted that the corresponding geometric average would fall between the harmonic and arithmetic averages. Walsh (1901, p. 432) computed some numerical examples of the Young index and found big differences between it and his best indices, even using weights that were representative for the periods being compared. Recall that the Lowe index becomes the Walsh index when geometric (continued) mean quantity weights are chosen, and so the Lowe index can perform well when representative weights are used. This is not necessarily the case for the Young index, even using representative weights. Walsh (1901, p. 433) summed up his numerical experiments with the Young index as follows: "In fact, Young's method, in every form, has been found to be bad."

15.65 Given two a priori equally plausible index number formulas that give different answers, such as the Young index and its time antithesis, Irving Fisher (1922, p. 136) generally suggested taking the geometric average of the two indices.[50] A benefit of this averaging is that the resulting formula will satisfy the time reversal test. Thus, rather than using *either* the base period 0 Young index, $P_Y(p^0,p^t,s^b)$, *or* the current period t Young index, $P_Y{}^*(p^0,p^t,s^b)$, which is always below the base period 0 Young index if there is any dispersion in relative prices, it seems preferable to use the following index, which is the *geometric average* of the two alternatively based Young indices:[51]

$$(15.59)\; P_Y^{**}(p^0,p^t,s^b) \equiv \left[P_Y(p^0,p^t,s^b)P_Y^*(p^0,p^t,s^b)\right]^{1/2}.$$

If the base-year shares s_i^b happen to coincide with both the month 0 and month t shares, s_i^0 and s_i^t, respectively, the time-rectified Young index $P_Y{}^{**}(p^0,p^t,s^b)$ defined by equation (15.59) will coincide with the Fisher ideal price index between months 0 and t, $P_F(p^0,p^t,q^0,q^t)$ (which will also equal the Laspeyres and Paasche indices under these conditions). Note also that the index $P_Y{}^{**}$ defined by equation (15.59) can be produced on a timely basis by a statistical agency.

[50]"We now come to a third use of these tests, namely, to 'rectify' formulae, i.e., to derive from any given formula which does not satisfy a test another formula which does satisfy it; …. This is easily done by 'crossing', that is, by averaging antitheses. If a given formula fails to satisfy Test 1 [the time reversal test], its time antithesis will also fail to satisfy it; but the two will fail, as it were, in opposite ways, so that a cross between them (obtained by *geometrical* averaging) will give the golden mean which does satisfy" (Irving Fisher, 1922, p. 136). Actually, the basic idea behind Fisher's rectification procedure was suggested by Walsh, who was a discussant for Fisher (1921), where Fisher gave a preview of his 1922 book: "We merely have to take any index number, find its antithesis in the way prescribed by Professor Fisher, and then draw the geometric mean between the two" (Correa Moylan Walsh, 1921b, p. 542).

[51]This index is a base-year weighted counterpart to an equally weighted index proposed by Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992a, p. 140) in the context of elementary index number formulas. See Chapter 20 for further discussion of this unweighted index.

E. Divisia Index and Discrete Approximations

E.1 Divisia price and quantity indices

15.66 The second broad approach to index number theory relies on the assumption that price and quantity data change in a more or less continuous way.

15.67 Suppose that the price and quantity data on the n products in the chosen domain of definition can be regarded as continuous functions of (continuous) time, say, $p_i(t)$ and $q_i(t)$ for $i = 1,\ldots,n$. The value of producer revenue at time t is $V(t)$ defined in the obvious way as

$$(15.60)\quad V(t) \equiv \sum_{i=1}^{n} p_i(t)q_i(t).$$

15.68 Now suppose that the functions $p_i(t)$ and $q_i(t)$ are differentiable. Then both sides of equation (15.60) can be differentiated with respect to time to obtain

$$(15.61)\quad V'(t) = \sum_{i=1}^{n} p_i'(t)q_i(t) + \sum_{i=1}^{n} p_i(t)q_i'(t).$$

Divide both sides of equation (15.61) through by $V(t)$ and, using equation (15.60), the following equation is obtained:

$$(15.62)\; V'(t)/V(t) = \frac{\sum_{i=1}^{n} p_i'(t)q_i(t) + \sum_{i=1}^{n} p_i(t)q_i'(t)}{\sum_{j=1}^{n} p_j(t)q_j(t)} = \sum_{i=1}^{n}\frac{p_i'(t)}{p_i(t)}s_i(t) + \sum_{i=1}^{n}\frac{q_i'(t)}{q_i(t)}s_i(t),$$

where the time t revenue share on product i, $s_i(t)$, is defined as

$$(15.63)\quad s_i(t) \equiv \frac{p_i(t)q_i(t)}{\sum_{m=1}^{n} p_m(t)q_m(t)} \quad \text{for } i = 1,\ldots,n.$$

15.69 François Divisia (1926, p. 39) argued as follows: *suppose* the aggregate value at time t,

$V(t)$, can be written as the product of a time t price-level function, $P(t)$, say, multiplied by a time t quantity-level function, $Q(t)$, say; that is, we have

$$(15.64)\quad V(t) = P(t)Q(t).$$

Suppose, further, that the functions $P(t)$ and $Q(t)$ are differentiable. Then, differentiating equation (15.64) yields

$$(15.65)\quad V'(t) = P'(t)Q(t) + P(t)Q'(t).$$

Dividing both sides of equation (15.65) by $V(t)$ and using equation (15.64) leads to the following equation:

$$(15.66)\quad \frac{V'(t)}{V(t)} = \frac{P'(t)}{P(t)} + \frac{Q'(t)}{Q(t)}.$$

15.70 Divisia compared the two expressions for the logarithmic value derivative, $V'(t)/V(t)$, given by equation (15.62) and equation (15.66). He simply *defined* the logarithmic rate of change of the *aggregate price level*, $P'(t)/P(t)$, as the first set of terms on the right-hand side of equation (15.62), and he simply *defined* the logarithmic rate of change of the *aggregate quantity level*, $Q'(t)/Q(t)$, as the second set of terms on the right-hand side of equation (15.62). That is, he made the following definitions:

$$(15.67)\quad \frac{P'(t)}{P(t)} \equiv \sum_{i=1}^{n} s_i(t)\frac{p_i'(t)}{p_i(t)};$$

$$(15.68)\quad \frac{Q'(t)}{Q(t)} \equiv \sum_{i=1}^{n} s_i(t)\frac{q_i'(t)}{q_i(t)}.$$

15.71 Equations (15.67) and (15.68) are reasonable definitions for the proportional changes in the aggregate price and quantity (or quantity) levels, $P(t)$ and $Q(t)$.[52] The problem with these definitions is that economic data are not collected in *continuous* time; they are collected in *discrete* time. In other words, even though transactions can be thought of as occurring in continuous time, no producer records his or her purchases as they occur in continuous time; rather, purchases over a finite time period are cumulated and then recorded. A similar situation occurs for producers or sellers of products; firms cumulate their sales over discrete periods of time for accounting or analytical purposes. If it is attempted to approximate continuous time by shorter and shorter discrete time intervals, empirical price and quantity data can be expected to become increasingly erratic, since consumers make purchases only at discrete points of time (and producers or sellers of products make sales only at discrete points of time). However, it is still of some interest to approximate the continuous time price and quantity levels, $P(t)$ and $Q(t)$ defined implicitly by equations (15.67) and (15.68), by discrete time approximations. This can be done in two ways. Either methods of numerical approximation can be used or assumptions about the path taken by the functions $p_i(t)$ and $q_i(t)$ ($i = 1,\ldots,n$) through time can be made. The first strategy is used in the following section. For discussions of the second strategy, see Vogt (1977; 1978), Van Ijzeren (1987, pp. 8–12), Vogt and Barta (1997), and Balk (2000).

15.72 There is a connection between the Divisia price and quantity levels, $P(t)$ and $Q(t)$, and the economic approach to index number theory. However, this connection is best made after one has studied the economic approach to index number theory in Chapter 17. Since this material is rather technical, it appears in Appendix 17.1.

E.2 Discrete approximations to continuous-time Divisia index

15.73 To make operational the continuous time Divisia price and quantity levels, $P(t)$ and $Q(t)$ defined by the differential equations (15.67) and (15.68), it is necessary to convert to discrete time. Divisia (1926, p. 40) suggested a straightforward method for doing this conversion, which we now outline.

15.74 Define the following price and quantity (forward) differences

$$(15.69)\quad \Delta P \equiv P(1) - P(0);$$

$$(15.70)\quad \Delta p_i \equiv p_i(1) - p_i(0);\ i = 1,\ldots,n.$$

Using the above definitions

[52]If these definitions are applied (approximately) to the Young index studied in the previous section, then it can be seen that for the Young price index to be consistent with the Divisia price index, the base-year shares should be chosen to be average shares that apply to the entire time period between months 0 and t.

(15.71) $$\frac{P(1)}{P(0)} = \frac{P(0)+\Delta P}{P(0)}$$

$$= 1 + \frac{\Delta P}{P(0)} \approx 1 + \frac{\sum_{i=1}^{n} \Delta p_i q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)},$$

using equation (15.67) when $t = 0$ and approximating $p_i(0)$ by the difference Δp_i

$$= \frac{\sum_{i=1}^{n} \{p_i(0) + \Delta p_i\} q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)}$$

$$= \frac{\sum_{i=1}^{n} p_i(1) q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)}$$

$$= P_L(p^0, p^1, q^0, q^1),$$

where $p^t \equiv [p_1(t),\ldots,p_n(t)]$ and $q^t \equiv [q_1(t),\ldots,q_n(t)]$ for $t = 0,1$. Thus, it can be seen that Divisia's discrete approximation to his continuous-time price index is just the Laspeyres price index, P_L, defined by equation (15.5).

15.75 But now a problem noted by Frisch (1936, p. 8) occurs: instead of approximating the derivatives by the discrete (forward) differences defined by equations (15.69) and (15.70), other approximations could be used and a wide variety of discrete time approximations can be obtained. For example, instead of using forward differences and evaluating the index at time $t = 0$, one could use backward differences and evaluate the index at time $t = 1$. These backward differences are defined as

(15.72) $$\Delta_b p_i \equiv p_i(0) - p_i(1);\ i = 1,\ldots,n.$$

This use of backward differences leads to the following approximation for $P(0) / P(1)$:

(15.73) $$\frac{P(0)}{P(1)} = \frac{P(1)+\Delta_b P}{P(1)}$$

$$= 1 + \frac{\Delta_b P}{P(1)} \approx 1 + \frac{\sum_{i=1}^{n} \Delta_b p_i q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}$$

using equation (15.67) when $t = 1$ and approximating $p_i(1)$ by the difference $\Delta_b p_i$:

$$= \frac{\sum_{i=1}^{n} \{p_i(1) + \Delta_b p_i\} q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}$$

$$= \frac{\sum_{i=1}^{n} p_i(0) q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}$$

$$= \frac{1}{P_P(p^0, p^1, q^0, q^1)},$$

where P_P is the Paasche index defined by equation (15.6). Taking reciprocals of both sides of equation (15.73) leads to the following discrete approximation to $P(1) / P(0)$:

(15.74) $$\frac{P(1)}{P(0)} \approx P_P.$$

15.76 Thus, as Frisch[53] noted, both the Paasche and Laspeyres indices can be regarded as (equally valid) approximations to the continuous-time Divisia price index.[54] Since the Paasche and Laspeyres indices can differ considerably in some empirical applications, it can be seen that Divisia's idea is not all that helpful in determining a *unique* discrete time index number formula.[55] What is useful about the Divisia indices is the idea that as the discrete unit of time gets smaller, discrete ap-

[53]"As the elementary formula of the chaining, we may get Laspeyres' or Paasche's or Edgeworth's or nearly any other formula, according as we choose the approximation principle for the steps of the numerical integration (Ragnar Frisch, 1936, p. 8).

[54]Diewert (1980, p. 444) also obtained the Paasche and Laspeyres approximations to the Divisia index using a somewhat different approximation argument. He also showed how several other popular discrete time index number formulas could be regarded as approximations to the continuous-time Divisia index.

[55]Trivedi (1981) systematically examined the problems involved in finding a best discrete time approximation to the Divisia indices using the techniques of numerical analysis. However, these numerical analysis techniques depend on the assumption that the true continuous-time micro price functions, $p_i(t)$, can be adequately represented by a polynomial approximation. Thus, we are led to the conclusion that the best discrete time approximation to the Divisia index depends on assumptions that are difficult to verify.

proximations to the Divisia indices can approach meaningful economic indices under certain conditions. Moreover, if the Divisia concept is accepted as the correct one for index number theory, then the corresponding correct discrete time counterpart might be taken as a weighted average of the chain price relatives pertaining to the adjacent periods under consideration, where the weights are somehow representative for the two periods under consideration.

F. Fixed-Base versus Chain Indices

15.77 This section[56] discusses the merits of using the chain system for constructing price indices in the time series context versus using the fixed-base system.[57]

15.78 The chain system[58] measures the change in prices going from one period to another using a bilateral index number formula involving the prices and quantities pertaining to the two adjacent periods. These one-period rates of change (the links in the chain) are then cumulated to yield the relative levels of prices over the entire period under consideration. Thus, if the bilateral price index is P, the chain system generates the following pattern of price levels for the first three periods:

$$(15.75)\quad 1,\ P(p^0,p^1,q^0,q^1),\\ P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2).$$

15.79 On the other hand, the fixed-base system of price levels using the same bilateral index number formula P simply computes the level of prices in period t relative to the base period 0 as $P(p^0,p^t,q^0,q^t)$. Thus, the fixed-base pattern of price levels for periods 0, 1, and 2 is

$$(15.76)\quad 1,\ P(p^0,p^1,q^0,q^1),\ P(p^0,p^2,q^0,q^2).$$

15.80 Note that in both the chain system and the fixed-base system of price levels defined by equations (15.75) and (15.76), the base-period price level is equal to 1. The usual practice in statistical agencies is to set the base-period price level equal to 100. If this is done, then it is necessary to multiply each of the numbers in equations (15.75) and (15.76) by 100.

15.81 Because of the difficulties involved in obtaining current-period information on quantities (or, equivalently, on revenues), many statistical agencies loosely base their PPI on the use of the Laspeyres formula in equation (15.5) and the fixed-base system. Therefore, it is of some interest to look at the possible problems associated with the use of fixed-base Laspeyres indices.

15.82 The main problem with the use of fixed-base Laspeyres indices is that the period 0 fixed basket of products that is being priced out in period t often can be quite different from the period t basket. Thus, if there are systematic *trends* in at least some of the prices and quantities[59] in the index basket, the fixed-base Laspeyres price index, $P_L(p^0,p^t,q^0,q^t)$, can be quite different from the corresponding fixed-base Paasche price index, $P_P(p^0,p^t,q^0,q^t)$.[60] This means that both indices are likely to be an inadequate representation of the movement in average prices over the time period under consideration.

15.83 The fixed-base Laspeyres quantity index cannot be used forever; eventually, the base-period quantities q^0 are so far removed from the current-period quantities q^t that the base must be changed.

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[56]This section is based largely on the work of Peter Hill (1988; 1993, pp. 385–90).

[57]The results in Appendix 17.1 provide some theoretical support for the use of chain indices in that it is shown that under certain conditions, the Divisia index will equal an economic index. Hence, any discrete approximation to the Divisia index will approach the economic index as the time period gets shorter. Thus, under certain conditions, chain indices will approach an underlying economic index.

[58]The chain principle was introduced independently into the economics literature by Lehr (1885, pp. 45–6) and Marshall (1887, p. 373). Both authors observed that the chain system would mitigate the difficulties because of the introduction of new products into the economy, a point also mentioned by Peter Hill (1993, p. 388). Irving Fisher (1911, p. 203) introduced the term "chain system."

[59]Examples of rapidly downward trending prices and upward trending quantities are computers, electronic equipment of all types, Internet access, and telecommunication charges.

[60] Note that $P_L(p^0,p^t,q^0,q^t)$ will equal $P_P(p^0,p^t,q^0,q^t)$ if *either* the two quantity vectors q^0 and q^t are proportional *or* the two price vectors p^0 and p^t are proportional. Thus, to obtain a difference between the Paasche and Laspeyres indices, nonproportionality in *both* prices and quantities is required.

Chaining is merely the limiting case where the base is changed each period.[61]

15.84 The main advantage of the chain system is that under normal conditions, chaining will reduce the spread between the Paasche and Laspeyres indices.[62] These indices provide an asymmetric perspective on the amount of price change that has occurred between the two periods under consideration, and it could be expected that a single point estimate of the aggregate price change should lie between these two estimates. Thus, the use of either a chained Paasche or Laspeyres index will usually lead to a smaller difference between the two and hence to estimates that are closer to the "truth."[63]

15.85 Peter Hill (1993, p. 388), drawing on his earlier research (1988, pp. 136–37) and that of Szulc (1983), noted that it is not appropriate to use the chain system when prices oscillate, or "bounce," to use Szulc's (1983, p. 548) term. This phenomenon can occur in the context of regular seasonal fluctuations or in the context of price wars. However, in the context of roughly monotonically changing prices and quantities, Peter Hill (1993, p. 389) recommended the use of chained symmetrically weighted indices (see Section C). The Fisher and Walsh indices are examples of symmetrically weighted indices.

15.86 It is possible to be a bit more precise regarding under which conditions one should or should not chain. Basically, one should chain if the prices and quantities pertaining to adjacent periods are *more similar* than the prices and quantities of more distant periods, since this strategy will lead to a narrowing of the spread between the Paasche and Laspeyres indices at each link.[64] One needs a measure of how similar are the prices and quantities pertaining to two periods. The similarity measures could be *relative* or *absolute*. In the case of absolute comparisons, two vectors of the same dimension are similar if they are identical and dissimilar otherwise. In the case of relative comparisons, two vectors are similar if they are proportional and dissimilar if they are nonproportional.[65] Once a similarity measure has been defined, the prices and quantities of each period can be compared using this measure, and a "tree" or path that links all of the observations can be constructed where the most similar observations are compared using a bilateral index number formula.[66] R. J. Hill (1995) defined the price structures between the two countries to be more dissimilar the bigger is

[61]Regular seasonal fluctuations can cause monthly or quarterly data to "bounce," using Szulc's (1983) term, and chaining bouncing data can lead to a considerable amount of index drift. That is, if after 12 months, prices and quantities return to their levels of a year earlier, then a chained monthly index will usually not return to unity. Hence, the use of chained indices for "noisy" monthly or quarterly data is not recommended without careful consideration.

[62]See Diewert (1978, p. 895) and Peter Hill (1988; 1993, pp. 387–88).

[63]This observation will be illustrated with an artificial data set in Chapter 19.

[64]Walsh, in discussing whether fixed-base or chained index numbers should be constructed, took for granted that the precision of all reasonable bilateral index number for-
(continued)

mulas would improve, provided that the two periods or situations being compared were more similar and, for this reason, favored the use of chained indices: "The question is really, in which of the two courses [fixed-base or chained index numbers] are we likely to gain greater exactness in the comparisons actually made? Here the probability seems to incline in favor of the second course; for the conditions are likely to be less diverse between two contiguous periods than between two periods say, fifty years apart" (Correa Moylan Walsh, 1901, p. 206). Walsh (1921a, pp. 84–85) later reiterated his preference for chained index numbers. Fisher also made use of the idea that the chain system would usually make bilateral comparisons between price and quantity data that were more similar and hence the resulting comparisons would be more accurate: "The index numbers for 1909 and 1910 (each calculated in terms of 1867–1877) are compared with each other. But direct comparison between 1909 and 1910 would give a different and more valuable result. To use a common base is like comparing the relative heights of two men by measuring the height of each above the floor, instead of putting them back to back and directly measuring the difference of level between the tops of their heads" (Irving Fisher, 1911, p. 204). "It seems, therefore, advisable to compare each year with the next, or, in other words, to make each year the base year for the next. Such a procedure has been recommended by Marshall, Edgeworth and Flux. It largely meets the difficulty of non-uniform changes in the Q's, for any inequalities for successive years are relatively small" (Irving Fisher, 1911, pp. 423–24).

[65]Diewert (2002b) takes an axiomatic approach to defining various indices of absolute and relative dissimilarity.

[66]Irving Fisher (1922, pp. 271–76) hinted at the possibility of using spatial linking; that is, of linking countries similar in structure. However, the modern literature has grown due to the pioneering efforts of R.J. Hill (1995; 1999a; 1999b; 2001). R.J. Hill (1995) used the spread between the Paasche and Laspeyres price indices as an indicator of similarity and showed that this criterion gives the same results as a criterion that looks at the spread between the Paasche and Laspeyres quantity indices.

the spread between P_L and P_P; that is, the bigger is max $\{P_L/P_P, P_P/P_L\}$. The problem with this measure of dissimilarity in the price structures of the two countries is that it could be the case that $P_L = P_P$ (so that the R. J. Hill measure would register a maximal degree of similarity), but p^0 could be very different from p^t. Thus, there is a need for a more systematic study of similarity (or dissimilarity) measures to pick the best one that could be used as an input into R. J. Hill's (1999a; 1999b; 2001) spanning tree algorithm for linking observations.

15.87 The method of linking observations explained in the previous paragraph based on the similarity of the price and quantity structures of any two observations may not be practical in a statistical agency context, since the addition of a new period may lead to a reordering of the previous links. However, the above scientific method for linking observations may be useful in deciding whether chaining is preferable or whether fixed-base indices should be used while making month-to-month comparisons within a year.

15.88 Some index number theorists have objected to the chain principle on the grounds that it has no counterpart in the spatial context:

> They [chain indices] only apply to intertemporal comparisons, and in contrast to direct indices they are not applicable to cases in which no natural order or sequence exists. Thus, the idea of a chain index, for example, has no counterpart in interregional or international price comparisons, because countries cannot be sequenced in a "logical" or "natural" way (there is no $k + 1$ nor $k - 1$ country to be compared with country k). (Peter von der Lippe, 2001, p. 12)[67]

This is correct, but R.J. Hill's approach does lead to a natural set of spatial links. Applying the same approach to the time-series context will lead to a set of links between periods that may not be month to month, but it will in many cases justify year-over-year linking of the data pertaining to the same month. This problem will be reconsidered in Chapter 22.

15.89 It is of some interest to determine if there are index number formulas that give the same answer when either the fixed-base or chain system is used. Comparing the sequence of chain indices defined by equation (15.75) above to the corresponding fixed-base indices, it can be seen that we will obtain the same answer in all three periods if the index number formula P satisfies the following functional equation for all price and quantity vectors:

$$(15.77)\quad P(p^0, p^2, q^0, q^2) = P(p^0, p^1, q^0, q^1) \times P(p^1, p^2, q^1, q^2).$$

If an index number formula P satisfies equation (15.77), then P satisfies the *circularity test*.[68]

15.90 If it is assumed that the index number formula P satisfies certain properties or tests in addition to the circularity test above,[69] then Funke, Hacker, and Voeller (1979) showed that P must have the following functional form credited originally to Konüs and Byushgens[70] (1926, pp. 163–66):[71]

[67]It should be noted that von der Lippe (2001, pp. 56–8) is a vigorous critic of all index number tests based on symmetry in the time series context, although he is willing to accept symmetry in the context of making international comparisons. "But there are good reasons *not* to insist on such criteria in the *intertemporal* case. When no symmetry exists between 0 and t, there is no point in interchanging 0 and t" (Peter von der Lippe, 2001, p. 58).

[68]The test name is credited to Irving Fisher (1922, p. 413), and the concept was originally credited to Westergaard (1890, pp. 218–19).

[69]The additional tests are (i) positivity and continuity of $P(p^0,p^1,q^0,q^1)$ for all strictly positive price and quantity vectors p^0,p^1,q^0,q^1; (ii) the identity test; (iii) the commensurability test; (iv) $P(p^0,p^1,q^0,q^1)$ is positively homogeneous of degree 1 in the components of p^1 ; and (v) $P(p^0,p^1,q^0,q^1)$ is positively homogeneous of degree zero in the components of q^1.

[70]Konüs and Byushgens show that the index defined by equation (15.78) is exact for Cobb-Douglas (1928) preferences; see also Pollak (1983a, pp. 119–20). The concept of an exact index number formula will be explained in Chapter 17.

[71]This result can be derived using results in Eichhorn (1978, pp. 167–68) and Vogt and Barta (1997, p. 47). A simple proof can be found in Balk (1995). This result vindicates Irving Fisher's (1922, p. 274) intuition. He asserted that "the only formulae which conform perfectly to the circular test are index numbers which have *constant weights*…" Irving Fisher (1922, p. 275) went on to assert, "But, clearly, constant weighting is not theoretically correct. If we compare 1913 with 1914, we need one set of weights; if we compare 1913 with 1915, we need, theoretically at least, another set of weights. … Similarly, turning
(continued)

(15.78) $$P_{KB}(p^0, p^1, q^0, q^1) \equiv \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0} \right)^{\alpha_i},$$

where the n constants α_i satisfy the following restrictions:

(15.79) $$\sum_{i=1}^{n} \alpha_i = 1 \quad \text{and} \quad \alpha_i > 0 \ \text{ for } i = 1,...,n \ .$$

Thus, under very weak regularity conditions, the only price index satisfying the circularity test is a weighted geometric average of all the individual price ratios, the weights being constant through time.

15.91 An interesting special case of the family of indices defined by equation (15.78) occurs when the weights α_i are all equal. In this case, P_{KB} reduces to the Jevons (1865) index:

(15.80) $$P_J(p^0, p^1, q^0, q^1) \equiv \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}}.$$

15.92 The problem with the indices defined by Konüs and Byushgens and Jevons is that the individual price ratios, p_i^1 / p_i^0, have weights (either α_i or $1/n$) that are *independent* of the economic importance of product i in the two periods under consideration. Put another way, these price weights are independent of the quantities of product i consumed or the revenues on product i during the two periods. Hence, these indices are not really suitable for use by statistical agencies at higher levels of aggregation when revenue share information is available.

15.93 The above results indicate that it is not useful to ask that the price index P satisfy the circularity test *exactly*. However, it is of some interest to find index number formulas that satisfy the circularity test to some degree of *approximation*, since the use of such an index number formula will lead to measures of aggregate price change that are more or less the same whether we use the chain or fixed-base systems. Irving Fisher (1922, p. 284) found that deviations from circularity using his data set and the Fisher ideal price index P_F defined by equation (15.12) above were quite small. This relatively high degree of correspondence between fixed-base and chain indices has been found to hold for other symmetrically weighted formulas like the Walsh index P_W defined by equation (15.19).[72] Thus, in most time-series applications of index number theory where the base year in fixed-base indices is changed every five years or so, it will not matter very much whether the statistical agency uses a fixed-base price index or a chain index, provided that a symmetrically weighted formula is used.[73] This, of course, depends on the length of the time series considered and the degree of variation in the prices and quantities as we go from period to period. The more prices and quantities are subject to large fluctuations (rather than smooth trends), the less the correspondence.[74]

15.94 It is possible to give a theoretical explanation for the approximate satisfaction of the circularity test for symmetrically weighted index number formulas. Another symmetrically weighted formula is the Törnqvist index P_T.[75] The natural logarithm of this index is defined as follows:

(15.81) $$\ln P_T(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{p_i^1}{p_i^0}\right),$$

where the period t revenue shares s_i^t are defined by equation (15.7) above. Alterman, Diewert, and Feenstra (1999, p. 61) show that if the logarithmic

from time to space, an index number for comparing the United States and England requires one set of weights, and an index number for comparing the United States and France requires, theoretically at least, another."

[72] See, for example, Diewert (1978, p. 894). Walsh (1901, pp. 424 and 429) found that his three preferred formulas all approximated each other very well, as did the Fisher ideal for his artificial data set.

[73] More specifically, most superlative indices (which are symmetrically weighted) will satisfy the circularity test to a high degree of approximation in the time series context. See Chapter 17 for the definition of a superlative index. It is worth stressing that fixed-base Paasche and Laspeyres indices are very likely to diverge considerably over a five-year period if computers (or any other product that has price and quantity trends different from the trends in the other products) are included in the value aggregate under consideration. See Chapter 19 for some empirical evidence on this topic.

[74] Again, see Szulc (1983) and Peter Hill (1988).

[75] This formula was implicitly introduced in Törnqvist (1936) and explicitly defined in Törnqvist and Törnqvist (1937).

price ratios ln (p_i^t / p_i^{t-1}) trend linearly with time t, and the revenue shares s_i^t also trend linearly with time, then the Törnqvist index P_T will satisfy the circularity test exactly.[76] Since many economic time series on prices and quantities satisfy these assumptions approximately, then the Törnqvist index will satisfy the circularity test approximately. As will be seen in Chapter 19, generally the Törnqvist index closely approximates the symmetrically weighted Fisher and Walsh indices, so that for many economic time series (with smooth trends), all three of these symmetrically weighted indices will satisfy the circularity test to a high enough degree of approximation so that it will not matter whether we use the fixed-base or chain principle.

15.95 Walsh (1901, p. 401; 1921a, p. 98; 1921b, p. 540) introduced the following useful variant of the circularity test:

$$(15.82)\quad 1 = P(p^0, p^1, q^0, q^1)P(p^1, p^2, q^1, q^2) \ldots P(p^T, p^0, q^T, q^0).$$

The motivation for this test is the following. Use the bilateral index formula $P(p^0,p^1,q^0,q^1)$ to calculate the change in prices going from period 0 to 1, use the same formula evaluated at the data corresponding to periods 1 and 2, $P(p^1,p^2,q^1,q^2)$, to calculate the change in prices going from period 1 to 2, … . Use $P(p^{T-1},p^T,q^{T-1},q^T)$ to calculate the change in prices going from period $T - 1$ to T. Introduce an artificial period $T + 1$ that has exactly the price and quantity of the initial period 0 and use $P(p^T,p^0,q^T,q^0)$ to calculate the change in prices going from period T to 0. Finally, multiply all these indices, and, since we end up where we started, then the product of all of these indices should ideally be 1. Diewert (1993a, p. 40) called this test a *multiperiod identity test*.[77] Note that if $T = 2$ (so that the number of periods is 3 in total), then Walsh's test reduces to Fisher's (1921, p. 534; 1922, p. 64) *time reversal test*.[78]

15.96 Walsh (1901, pp. 423–33) showed how his circularity test could be used to evaluate the worth of a bilateral index number formula. He invented artificial price and quantity data for five periods and added a sixth period that had the data of the first period. He then evaluated the right-hand side of equation (15.82) for various formulas, $P(p^0,p^1,q^0,q^1)$, and determined how far from unity the results were. His best formulas had products that were close to 1.[79]

15.97 This same framework is often used to evaluate the efficacy of chained indices versus their direct counterparts. Thus, if the right-hand side of equation (15.82) turns out to be different from unity, the chained indices are said to suffer from "chain drift." If a formula does suffer from chain drift, it is sometimes recommended that fixed-base indices be used in place of chained ones. However, this advice, if accepted, would *always* lead to the adoption of fixed-base indices, provided that the bilateral index formula satisfies the identity test, $P(p^0,p^0,q^0,q^0) = 1$. Thus, it is not recommended that Walsh's circularity test be used to decide whether fixed-base or chained indices should be calculated. However, it is fair to use Walsh's circularity test as he originally used it; that is, as an approximate method for deciding the force of a particular index number formula. To decide whether to chain or use fixed-base indices, one should decide on the basis of how similar are the observations being compared and choose the method that will best link the most similar observations.

15.98 Various properties, axioms, or tests that an index number formula could satisfy have already been introduced in this chapter. In the following chapter, the test approach to index number theory will be studied in a more systematic manner.

[76] This exactness result can be extended to cover the case when there are monthly proportional variations in prices and the revenue shares have constant seasonal effects in addition to linear trends; see Alterman, Diewert, and Feenstra (1999, p. 65).

[77] Walsh (1921a, p. 98) called his test the *circular test,* but since Irving Fisher also used this term to describe his transitivity test defined earlier by equation (15.77), it seems best to stick to Irving Fisher's terminology since it is well established in the literature.

[78] Walsh (1921b, pp. 540–41) noted that the time reversal test was a special case of his circularity test.

[79] This is essentially a variant of the methodology that Irving Fisher (1922, p. 284) used to check how well various formulas corresponded to his version of the circularity test.

Appendix 15.1: Relationship Between Paasche and Laspeyres Indices

15.99 Recall the notation used in Section B.2. Define the ith relative price or price relative r_i and the ith quantity relative t_i as follows:

$$\text{(A15.1.1)} \quad r_i \equiv \frac{p_i^1}{p_i^0}; \quad t_i \equiv \frac{q_i^1}{q_i^0}; \quad i = 1,...,n.$$

Using equation (15.8) above for the Laspeyres price index P_L and equation (A15.1.1), we have

$$\text{(A15.1.2)} \quad P_L = \sum_{i=1}^{n} r_i s_i^0 \equiv r^*;$$

that is, we define the "average" price relative r^* as the base-period revenue share-weighted average of the individual price relatives, r_i .

15.100 Using equation (15.6) for the Paasche price index P_P, we have

$$\text{(A15.1.3)} \quad P_P \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{m=1}^{n} p_m^0 q_m^1}$$

$$= \frac{\sum_{i=1}^{n} r_i t_i p_i^0 q_i^0}{\sum_{m=1}^{n} t_m p_m^0 q_m^0} \quad \text{using equation (A15.1.1)}$$

$$= \frac{\sum_{i=1}^{n} r_i t_i s_i^0}{\sum_{m=1}^{n} t_m s_m^0}$$

$$= \left\{ \frac{1}{\sum_{m=1}^{n} t_m s_m^0} \sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0 \right\} + r^*,$$

using equation (A15.1.2) and $\sum_{i=1}^{n} s_i^0 = 1$ and where the average quantity relative t^* is defined as

$$\text{(A15.1.4)} \quad t^* \equiv \sum_{i=1}^{n} t_i s_i^0 = Q_L \ ,$$

where the last equality follows using equation (15.11), the definition of the Laspeyres quantity index Q_L.

15.101 Taking the difference between P_P and P_L and using equation (A15.1.2) – equation (A15.1.4) yields

$$\text{(A15.1.5)} \quad P_P - P_L = \frac{1}{Q_L} \sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0.$$

Now let r and t be discrete random variables that take on the n values r_i and t_i, respectively. Let s_i^0 be the joint probability that $r = r_i$ and $t = t_i$ for $i = 1,\ldots,n,$ and let the joint probability be 0 if $r = r_i$ and $t = t_j$ where $i \neq j$. It can be verified that the summation $\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0$ on the right-hand side of equation (A15.1.5) is the covariance between the price relatives r_i and the corresponding quantity relatives t_i. This covariance can be converted into a correlation coefficient.[80] If this covariance is negative, which is the usual case in the consumer context, then P_P will be less than P_L. If it is positive, which will occur in the situations where supply conditions are fixed (as in the fixed-input *output* price index), but demand is changing, then P_P will be greater than P_L.

Appendix 15.2: Relationship Between Lowe and Laspeyres Indices

15.102 Recall the notation used in Section D.1. Define the ith relative price relating the price of product i of month t to month 0, r_i, and the ith quantity relative, t_i, relating quantity of product i in base year b to month 0, t_i, as follows:

$$\text{(A15.2.1)} \quad r_i \equiv \frac{p_i^t}{p_i^0} \quad t_i \equiv \frac{q_i^b}{q_i^0}; \quad i = 1,\ldots,n.$$

[80]See Bortkiewicz (1923, pp. 374–75) for the first application of this correlation coefficient decomposition technique.

As in Appendix 15.1, the Laspeyres price index $P_L(p^0,p^t,q^0)$ can be defined as r^*, the month 0 revenue share-weighted average of the individual price relatives r_i defined in equation (A15.2.1), except that the month t price, p_i^t, now replaces period 1 price, p_i^1, in the definition of the ith price relative r_i :

$$\text{(A15.2.2)}\quad r^* \equiv \sum_{i=1}^{n} r_i s_i^0 = P_L .$$

15.103 The average quantity relative t^* relating the quantities of base year b to those of month 0 is defined as the month 0 revenue share-weighted average of the individual quantity relatives t_i defined in equation (A15.2.1):

$$\text{(A15.2.3)}\quad t^* \equiv \sum_{i=1}^{n} t_i s_i^0 = Q_L\ ,$$

where $Q_L = Q_L(q^0,q^b,p^0)$ is the Laspeyres quantity index relating the quantities of month 0, q^0, to those of the year b, q^b, using the prices of month 0, p^0, as weights.

15.104 Using equation (15.26), the Lowe index comparing the prices in month t with those of month 0, using the quantity weights of the base year b, is equal to

$$\text{(A15.2.4)}\quad P_{Lo}(p^0,p^t,q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \frac{\sum_{i=1}^{n} p_i^t t_i q_i^0}{\sum_{i=1}^{n} p_i^0 t_i q_i^0} \quad \text{using equation (A15.2.1)}$$

$$= \left\{\frac{\sum_{i=1}^{n} p_i^t t_i q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}\left\{\frac{\sum_{i=1}^{n} p_i^0 t_i q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}^{-1}$$

$$= \left\{\frac{\sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) t_i p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\} \Big/ t^*$$

using equation (A15.2.3)

$$= \left\{\frac{\sum_{i=1}^{n} r_i t_i p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\} \Big/ t^* \quad \text{using equation (A15.2.1)}$$

$$= \frac{\sum_{i=1}^{n} r_i t_i s_i^0}{t^*} = \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{\sum_{i=1}^{n} r^* t_i s_i^0}{t^*}$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{r^*\left[\sum_{i=1}^{n} t_i s_i^0\right]}{t^*}$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{r^*\left[t^*\right]}{t^*}$$

using equation (A15.2.3)

$$= \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{t^*} + \frac{\sum_{i=1}^{n} (r_i - r^*) t^* s_i^0}{t^*} + r^*$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{t^*} + \frac{t^*\left[\sum_{i=1}^{n} r_i s_i^0 - r^*\right]}{t^*} + r^*$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{t^*} + r^* \quad \text{since} \sum_{i=1}^{n} r_i s_i^0 = r^*$$

$$= P_L(p^0,p^t,q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{Q_L(q^0,q^b,p^0)}\ ,$$

since using equation (A15.2.2), r^* equals the Laspeyres price index, $P_L(p^0,p^t,q^0)$, and using equation (A15.2.3), t^* equals the Laspeyres quantity index, $Q_L(q^0,q^b,p^0)$. Thus, equation (A15.2.4) tells us that the Lowe price index using the quantities of year b as weights, $P_{Lo}(p^0,p^t,q^b)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0,p^t,q^0)$, plus a covariance term $\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0$ between the price relatives $r_i \equiv p_i^t / p_i^0$ and the quantity relatives $t_i \equiv q_i^b / q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0,q^b,p^0)$ between month 0 and base year b.

Appendix 15.3: Relationship Between Young Index and Its Time Antithesis

15.105 Recall that the direct Young index, $P_Y(p^0,p^t,s^b)$, was defined by equation (15.48) and its time antithesis, $P_Y*(p^0,p^t,s^b)$, was defined by equation (15.52). Define the *i*th relative price between months 0 and *t* as

(A15.3.1) $r_i \equiv p_i^t / p_i^0$; $i = 1,...,n$,

and define the weighted average (using the base-year weights s_i^b) of the r_i as

(A15.3.2) $r^* \equiv \sum_{i=1}^{n} s_i^b r_i$,

which turns out to equal the direct Young index, $P_Y(p^0,p^t,s^b)$. Define the deviation e_i of r_i from their weighted average r^* using the following equation:

(A15.3.3) $r_i = r^*(1+e_i)$; $i = 1,...,n.$

If equation (A15.3.3) is substituted into equation (A15.3.2), the following equations are obtained:

(A15.3.4) $r^* \equiv \sum_{i=1}^{n} s_i^b r^*(1+e_i)$

$$= r^* + r^* \sum_{i=1}^{n} s_i^b e_i \text{ , since } \sum_{i=1}^{n} s_i^b = 1 .$$

(A15.3.5) $e^* \equiv \sum_{i=1}^{n} s_i^b e_i = 0.$

Thus, the weighted mean e^* of the deviations e_i equals 0.

15.106 The direct Young index, $P_Y(p^0,p^t,s^b)$, and its time antithesis, $P_Y*(p^0,p^t,s^b)$, can be written as functions of r^*, the weights s_i^b and the deviations of the price relatives e_i as follows:

(A15.3.6) $P_Y(p^0,p^t,s^b) = r^*$;

(A15.3.7) $P_Y^*(p^0,p^t,s^b) = \left[\sum_{i=1}^{n} s_i^b \left\{r^*(1+e_i)\right\}^{-1}\right]^{-1}$

$$= r^* \left[\sum_{i=1}^{n} s_i^b (1+e_i)^{-1}\right]^{-1} .$$

15.107 Now, regard $P_Y*(p^0,p^t,s^b)$ as a function of the vector of deviations, $e \equiv [e_1,\ldots,e_n]$, say, $P_Y*(e)$. The second-order Taylor series approximation to $P_Y*(e)$ around the point $e = 0_n$ is given by the following expression:[81]

(A15.3.8) $P_Y^*(e)$

$$\approx r^* + r^* \sum_{i=1}^{n} s_i^b e_i + r^* \sum_{i=1}^{n}\sum_{j=1}^{n} s_i^b s_j^b e_i e_j - r^* \sum_{i=1}^{n} s_i^b [e_i]^2$$

$$= r^* + r^* 0 + r^* \sum_{i=1}^{n} s_i^b \left[\sum_{j=1}^{n} s_j^b e_j\right] e_i - r^* \sum_{i=1}^{n} s_i^b \left[e_i - e^*\right]^2$$

using equation (A15.3.5)

$$= r^* + r^* \sum_{i=1}^{n} s_i^b [0] e_i - r^* \sum_{i=1}^{n} s_i^b \left[e_i - e^*\right]^2$$

using equation (A15.3.5)

$$= P_Y(p^0,p^t,s^b) - P_Y(p^0,p^t,s^b) \sum_{i=1}^{n} s_i^b \left[e_i - e^*\right]^2$$

using equation (A15.3.6)

$$= P_Y(p^0,p^t,s^b) - P_Y(p^0,p^t,s^b) \text{ Var } e ,$$

where the weighted sample variance of the vector e of price deviations is defined as

(A15.3.9) $\text{Var } e \equiv \sum_{i=1}^{n} s_i^b \left[e_i - e^*\right]^2$.

15.108 Rearranging equation (A15.3.8) gives the following approximate relationship between the direct Young index $P_Y(p^0,p^t,s^b)$ and its time antithesis $P_Y*(p^0,p^t,s^b)$, to the accuracy of a second-order Taylor series approximation about a price point where the month *t* price vector is proportional to the month 0 price vector:

(A15.3.10) $P_Y(p^0,p^t,s^b)$

$$\approx P_Y^*(p^0,p^t,s^b) + P_Y(p^0,p^t,s^b) \text{ Var } e.$$

Thus, to the accuracy of a second-order approximation, the direct Young index will exceed its time

[81]This type of second-order approximation is credited to Dalén (1992a, p. 143) for the case $r^* = 1$ and to Diewert (1995a, p. 29) for the case of a general r^*.

antithesis by a term equal to the direct Young index times the weighted variance of the deviations of the price relatives from their weighted mean. Thus, the bigger the dispersion in relative prices, the more the direct Young index will exceed its time antithesis.

16. Axiomatic and Stochastic Approaches to Index Number Theory

A. Introduction

16.1 As Chapter 15 demonstrated, it is useful to be able to evaluate various index number formulas that have been proposed in terms of their properties. If a formula turns out to have rather undesirable properties, then doubt is cast on its suitability as a target index that could be used by a statistical agency. Looking at the mathematical properties of index number formulas leads to the *test* or *axiomatic approach to index number theory*. In this approach, desirable properties for an index number formula are proposed; then it is determined whether any formula is consistent with these properties or tests. An ideal outcome is that the proposed tests are desirable and completely determine the functional form for the formula.

16.2 The axiomatic approach to index number theory is not completely straightforward, since choices have to be made in two dimensions:

- The index number framework must be determined; and
- Once the framework has been decided upon, the tests or properties that should be imposed on the index number need to be determined.

The second point is straightforward: different price statisticians may have different ideas about what tests are important, and alternative sets of axioms can lead to alternative best index number functional forms. This point must be kept in mind while reading this chapter, since there is no universal agreement on what is the best set of reasonable axioms. Hence, the axiomatic approach can lead to more than one best index number formula.

16.3 The first point about choices listed above requires further discussion. In the previous chapter, for the most part, the focus was on *bilateral index number theory*; that is, it was assumed that prices and quantities for the same n commodities were given for two periods, and the object of the index number formula was to compare the overall level of prices in one period with that of the other period. In this framework, both sets of price and quantity vectors were regarded as variables that could be *independently varied*, so that, for example, variations in the prices of one period did not affect the prices of the other period or the quantities in either period. The emphasis was on comparing the overall cost of a fixed basket of quantities in the two periods or taking averages of such fixed-basket indices. This is an example of an index number framework.

16.4 But other index number frameworks are possible. For example, instead of decomposing a value ratio into a term that represents price change between the two periods times another term that represents quantity change, one could attempt to decompose a value aggregate for one period into a single number that represents the price level in the period times another number that represents the quantity level in the period. In the first variant of this approach, the price index number is supposed to be a function of the n product prices pertaining to that aggregate in the period under consideration, and the quantity index number is supposed to be a function of the n product quantities pertaining to the aggregate in the period. The resulting price index function was called an *absolute index number* by Frisch (1930, p. 397), a *price level* by Eichhorn (1978, p. 141), and a *unilateral price index* by Anderson, Jones, and Nesmith (1997, p. 75). In a second variant of this approach, the price and quantity functions are allowed to depend on *both* the price *and* quantity vectors pertaining to the period under consideration.[1] These two variants of unilateral index number theory will be considered in Section B.[2]

[1]Eichhorn (1978 p. 144) and Diewert (1993d, p. 9) considered this approach.

[2]In these unilateral index number approaches, the price and quantity vectors are allowed to vary independently. In (continued)

16.5 The remaining approaches in this chapter are largely bilateral approaches; that is, the prices and quantities in an aggregate are compared for two periods. In Sections C and E, the value ratio decomposition approach is taken.[3] In Section C, the bilateral price and quantity indices, $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$, are regarded as functions of the price vectors pertaining to the two periods, p^0 and p^1, and the two quantity vectors, q^0 and q^1. Not only do the axioms or tests that are placed on the price index $P(p^0,p^1,q^0,q^1)$ reflect reasonable price index properties, some of them have their origin as reasonable tests on the quantity index $Q(p^0,p^1,q^0,q^1)$. The approach in Section C simultaneously determines the best price and quantity indices.

16.6 In Section D, attention is shifted to the *price ratios* for the *n* commodities between periods 0 and 1, $r_i \equiv p_i^1/p_i^0$ for $i = 1,\ldots,n$. In the *unweighted stochastic approach to index number theory*, the price index is regarded as an evenly weighted average of the *n* price relatives or ratios, r_i. Carli (1804; originally published in 1764) and Jevons (1863, 1865) were the early pioneers in this approach to index number theory, with Carli using the arithmetic average of the price relatives and Jevons endorsing the geometric average (but also considering the harmonic average). This approach to index number theory will be covered in Section D.1. This approach is consistent with a statistical approach that regards each price ratio r_i as a random variable with mean equal to the underlying price index.

16.7 A major problem with the unweighted average of price relatives approach to index number theory is that it does not take into account the economic importance of the individual commodities in the aggregate. Arthur Young (1812) did advocate some form of rough weighting of the price relatives according to their relative value over the period being considered, but the precise form of the required value weighting was not indicated.[4] However, it was Walsh (1901, pp. 83–121; 1921a, pp. 81–90) who stressed the importance of weighting the individual price ratios, where the weights are functions of the associated values for the commodities in each period, and each period is to be treated symmetrically in the resulting formula:

> What we are seeking is to average the variations in the exchange value of one given total sum of money in relation to the several classes of goods, to which several variations [price ratios] must be assigned weights proportional to the relative sizes of the classes. Hence the relative sizes of the classes at both the periods must be considered. (Correa Moylan Walsh, 1901, p. 104)

> Commodities are to be weighted according to their importance, or their full values. But the problem of axiometry always involves at least two periods. There is a first period and there is a second period which is compared with it. Price variations[5] have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period or those of the second? Or should there be a combination of the two sets? There is no reason for preferring either the first or the second. Then the combination of both would seem to be the proper answer. And this combination itself involves an averaging of the weights of the two periods. (Correa Moylan Walsh, 1921a, p. 90)

16.8 Thus, Walsh was the first to examine in some detail the rather intricate problems[6] in decid-

yet another index number framework, prices are allowed to vary freely, but quantities are regarded as functions of the prices. This leads to the *economic approach to index number theory*, which will be considered in more depth in Chapters 17 and 18.

[3]Recall Section B in Chapter 15 for an explanation of this approach.

[4]Walsh (1901, p. 84) refers to Young's contributions as follows: "Still, although few of the practical investigators have actually employed anything but even weighting, they have almost always recognized the theoretical need of allowing for the relative importance of the different classes ever since this need was first pointed out, near the commencement of the century just ended, by Arthur Young. … Arthur Young advised simply that the classes should be weighted according to their importance."

[5]A price variation is a price ratio or price relative in Walsh's terminology.

[6]Walsh (1901, pp. 104–105) realized that it would not do to simply take the arithmetic average of the values in the two periods, $[v_i^0 + v_i^1]/2$, as the correct weight for the *i*th price relative r_i since, in a period of rapid inflation, this would give too much importance to the period that had the highest prices, and he wanted to treat each period symmetrically: "But such an operation is manifestly wrong. In the first place, the sizes of the classes at each period are reckoned in the money of the period, and if it happens that the (continued)

ing how to weight the price relatives pertaining to an aggregate, taking into account the economic importance of the commodities in the two periods being considered. Note that the type of index number formulas that he was considering was of the form $P(r,v^0,v^1)$, where r is the vector of price relatives that has ith component $r_i = p_i^1/p_i^0$ and v^t is the period t value vector that has ith component $v_i^t = p_i^t q_i^t$ for $t = 0,1$. His suggested solution to this weighting problem was not completely satisfactory, but he did at least suggest a useful framework for a price index as a value-weighted average of the n price relatives. The first satisfactory solution to the weighting problem was obtained by Theil (1967, pp. 136–37), and his solution will be explained in Section D.2.

16.9 It can be seen that one of Walsh's approaches to index number theory[7] was an attempt to determine the best weighted average of the price relatives, r_i. This is equivalent to using an axiomatic approach to try to determine the best index of the form $P(r,v^0,v^1)$. This approach will be considered in Section E below.[8]

16.10 Recall that in Chapter 15, the Young and Lowe indices were introduced. These indices do not fit precisely into the bilateral framework because the value or quantity weights used in these indices do not necessarily correspond to the values or quantities that pertain to either of the periods that correspond to the price vectors p^0 and p^1. In Section F, the axiomatic properties of these two indices with respect to their price variables will be studied.

B. The Levels Approach to Index Number Theory

B.1 Axiomatic approach to unilateral price indices

16.11 Denote the price and quantity of product n in period t by p_i^t and q_i^t, respectively, for $i = 1,2,\ldots,n$ and $t = 0,1,\ldots,T$. The variable q_i^t is interpreted as the total amount of product i transacted within period t. In order to conserve the value of transactions, it is necessary that p_i^t be defined as a unit value; that is, p_i^t must be equal to the value of transactions in product i for period t divided by the total quantity transacted, q_i^t. In principle, the period of time should be chosen so that variations in product prices within a period are quite small compared with their variations between periods.[9]

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exchange value of money has fallen, or prices in general have risen, greater influence upon the result would be given to the weighting of the second period; or if prices in general have fallen, greater influence would be given to the weighting of the second period. Or in a comparison between two countries greater influence would be given to the weighting of the country with the higher level of prices. But it is plain that *the one period, or the one country, is as important, in our comparison between them, as the other, and the weighting in the averaging of their weights should really be even*." However, Walsh was unable to come up with Theil's (1967) solution to the weighting problem, which was to use the average revenue share $[s_i^0 + s_i^1]/2$, as the correct weight for the ith price relative in the context of using a weighted geometric mean of the price relatives.

[7] Walsh also considered basket-type approaches to index number theory, as was seen in Chapter 15.

[8] In Section E, rather than starting with indices of the form $P(r,v^0,v^1)$, indices of the form $P(p^0,p^1,v^0,v^1)$ are considered. However, if the invariance to changes in the units of measurement test is imposed on this index, it is equivalent to studying indices of the form $P(r,v^0,v^1)$. Vartia (1976a) also used a variation of this approach to index number theory.

[9] This treatment of prices as unit values over time follows Walsh (1901, p. 96; 1921a, p. 88) and Fisher (1922, p. 318). Fisher and Hicks both had the idea that the length of the period should be short enough so that variations in price within the period could be ignored as the following quotations indicate: "Throughout this book 'the price' of any commodity or 'the quantity' of it for any one year was assumed given. But what is such a price or quantity? Sometimes it is a single quotation for January 1 or July 1, but usually it is an average of several quotations scattered throughout the year. The question arises: On what principle should this average be constructed? The *practical* answer is *any* kind of average since, ordinarily, the variation during a year, so far, at least, as prices are concerned, are too little to make any perceptible difference in the result, whatever kind of average is used. Otherwise, there would be ground for subdividing the year into quarters or months until we reach a small enough period to be considered practically a point. The quantities sold will, of course, vary widely. What is needed is their sum for the year (which, of course, is the same thing as the simple arithmetic average of the per annum rates for the separate months or other subdivisions). In short, the simple arithmetic average, both of prices and of quantities, may be used. Or, if it is worth while to put any finer point on it, we may take the weighted arithmetic average for the prices, the weights being the quantities sold" (Irving Fisher, 1922, p. 318). "I shall define a week as that period of time during which variations in prices can be neglected. For theoretical purposes this means that prices will be supposed to change, not continuously, but at short intervals. The calendar length of the week is of course quite arbitrary; by taking it to be very short, our theoretical (continued)

For $t = 0,1,\ldots,T$, and $i = 1,\ldots,n$, define the value of transactions in product i as $v_i^t \equiv p_i^t q_i^t$ and define the *total value of transactions in period t* as

$$(16.1)\; V^t \equiv \sum_{i=1}^{n} v_i^t = \sum_{i=1}^{n} p_i^t q_i^t \;,\;\; t = 0,1,...,T.$$

16.12 Using the notation above, the following *levels version of the index number problem* is defined as follows: for $t = 0,1,\ldots,T$, find scalar numbers P^t and Q^t such that

$$(16.2)\; V^t = P^t Q^t \;,\;\; t = 0,1,...,T.$$

16.13 The number P^t is interpreted as an aggregate period t price level, while the number Q^t is interpreted as an aggregate period t quantity level. The aggregate price level P^t is allowed to be a function of the period t price vector, p^t, while the aggregate period t quantity level Q^t is allowed to be a function of the period t quantity vector, q^t. As a result we have the following:

$$(16.3)\; P^t = c(p^t) \;\text{ and }\; Q^t = f(q^t)\;,\;\; t = 0,1,...,T.$$

16.14 The functions c and f are to be determined somehow. Note that equation (16.3) requires that the functional forms for the price aggregation function c and for the quantity aggregation function f be independent of time. This is a reasonable requirement, since there is no reason to change the method of aggregation as time changes.

16.15 Substituting equations (16.3) and (16.2) into equation (16.1) and dropping the superscript t means that c and f must satisfy the following functional equation for all strictly positive price and quantity vectors:

$$(16.4)\;\; c(p)f(q) = \sum_{i=1}^{n} p_i q_i \;\;,$$

for all $p_i > 0$ and for all $q_i > 0$.

16.16 It is natural to assume that the functions $c(p)$ and $f(q)$ are positive if all prices and quantities are positive:

$$(16.5)\;\; c(p_1,...,p_n) > 0 \;;\; f(q_1,...,q_n) > 0$$

if all $p_i > 0$ and for all $q_i > 0$.

16.17 Let 1_n denote an n dimensional vector of ones. Then equation (16.5) implies that when $p = 1_n$, $c(1_n)$ is a positive number, a for example, and when $q = 1_n$, then $f(1_n)$ is also a positive number, b for example; that is, equation (16.5) implies that c and f satisfy

$$(16.6)\;\; c(1_n) = a > 0 \;;\; f(1_n) = b > 0.$$

16.18 Let $p = 1_n$ and substitute the first expression in equation (16.6) into (16.4) in order to obtain the following equation:

$$(16.7)\;\; f(q) = \sum_{i=1}^{n} \frac{q_i}{a} \;\; \text{for all } q_i > 0.$$

16.19 Now let $q = 1_n$ and substitute the second part of equation (16.6) into (16.4) in order to obtain the following equation:

$$c(p) = \sum_{i=1}^{n} \frac{p_i}{b} \;\text{ for all } p_i > 0.$$

16.20 Finally substitute equations (16.7) and (16.8) into the left-hand side of equation (16.4) and the following equation is obtained:

$$(16.9)\;\; \left(\sum_{i=1}^{n} \frac{p_i}{b}\right) \left(\sum_{i=1}^{n} \frac{q_i}{a}\right) = \sum_{i=1}^{n} p_i q_i \;,$$

for all $p_i > 0$ and for all $q_i > 0$. If n is greater than 1, it is obvious that equation (16.9) cannot be satisfied for all strictly positive p and q vectors. Thus, if the number of commodities n exceeds 1, then there are no functions c and f that satisfy equations (16.4) and (16.5).[10]

16.21 Thus, this levels test approach to index number theory comes to an abrupt halt; it is fruitless to look for price- and quantity-level functions, $P^t = c(p^t)$ and $Q^t = f(q^t)$, that satisfy equations (16.2) or (16.4) and also satisfy the very reasonable positivity requirements in equation (16.5).

scheme can be fitted as closely as we like to that ceaseless oscillation which is a characteristic of prices in certain markets" (John Hicks, 1946, p. 122).

[10]Eichhorn (1978, p. 144) established this result.

16.22 Note that the levels price index function, $c(p^t)$, did not depend on the corresponding quantity vector q^t, and the levels quantity index function, $f(q^t)$, did not depend on the price vector p^t. Perhaps this is the reason for the rather negative result obtained above. As a result, in the next section, the price and quantity functions are allowed to be functions of both p^t and q^t.

B.2 A second axiomatic approach to unilateral price indices

16.23 In this section, the goal is to find functions of $2n$ variables, $c(p,q)$ and $f(p,q)$ such that the following counterpart to equation (16.4) holds:

$$(16.10)\quad c(p,q)f(p,q)=\sum_{i=1}^{n}p_iq_i\ ,$$

for all $p_i > 0$ and for all $q_i > 0$.

16.24 Again, it is natural to assume that the functions $c(p,q)$ and $f(p,q)$ are positive if all prices and quantities are positive:

$$(16.11)\quad c(p_1,...,p_n;q_1,...,q_n)>0\ ;$$
$$f(p_1,...,p_n;q_1,...,q_n)>0\ ,$$

if all $p_i > 0$ and for all $q_i > 0$.

16.25 The present framework does not distinguish between the functions c and f, so it is necessary to require that these functions satisfy some reasonable properties. The first property imposed on c is that this function be homogeneous of degree 1 in its price components:

$$(16.12)\quad c(\lambda p,q)=\lambda\, c(p,q)\ \text{ for all } \lambda > 0.$$

Thus, if all prices are multiplied by the positive number λ, then the resulting price index is λ times the initial price index. A similar linear homogeneity property is imposed on the quantity index f; that is, f is to be homogeneous of degree 1 in its quantity components:

$$(16.13)\quad f(p,\lambda q)=\lambda\, f(p,q)\ \text{ for all } \lambda > 0.$$

16.26 Note that the properties in equations (16.10), (16.11), and (16.13) imply that the price index $c(p,q)$ has the following homogeneity property with respect to the components of q:

$$(16.14)\quad c(p,\lambda q)=\sum_{i=1}^{n}\frac{p_i\lambda q_i}{f(p,\lambda q)}\ \text{ where } \lambda > 0.$$
$$=\sum_{i=1}^{n}\frac{p_i\lambda q_i}{\lambda f(p,q)}\ \text{ using equation (16.3)}$$
$$=\sum_{i=1}^{n}\frac{p_iq_i}{f(p,q)}$$
$$=c(p,q)\ \text{ using equations (16.10) and (16.11)}$$

Thus $c(p,q)$ is homogeneous of degree 0 in its q components.

16.27 A final property that is imposed on the levels price index $c(p,q)$ is the following: Let the positive numbers d_i be given. Then it is asked that the price index be invariant to changes in the units of measurement for the n commodities, so that the function $c(p,q)$ has the following property:

$$(16.15)\quad c(d_1p_1,...,d_np_n;q_1/d_1,...,q_n/d_n)$$
$$=c(p_1,...,p_n;q_1,...,q_n).$$

16.28 It is now possible to show that the properties in equations (16.10), (16.11), (16.12), (16.14), and (16.15) on the price-levels function $c(p,q)$ are inconsistent; that is, there is no function of $2n$ variables $c(p,q)$ that satisfies these quite reasonable properties.[11]

16.29 To see why this is so, apply equation (16.15), setting $d_i = q_i$ for each i, to obtain the following equation:

$$(16.16)\quad c(p_1,...,p_n;q_1,...,q_n)$$
$$=c(p_1q_1,...,p_nq_n;1,...,1).$$

If $c(p,q)$ satisfies the linear homogeneity property in equation (16.12) so that $c(\lambda p,q) = \lambda c(p,q)$, then equation (16.16) implies that $c(p,q)$ is also linearly homogeneous in q, so that $c(p,\lambda q) = \lambda c(p,q)$. But this last equation contradicts equation (16.14), which establishes the impossibility result.

16.30 The rather negative results obtained in Section B.1 and this section indicate that it is fruitless to pursue the axiomatic approach to the deter-

[11]This proposition is due to Diewert (1993d, p. 9), but his proof is an adaptation of a closely related result due to Eichhorn (1978, pp. 144–45).

mination of price and quantity levels, where both the price and quantity vector are regarded as independent variables.[12] Therefore, in the following sections of this chapter, the axiomatic approach to the determination of a *bilateral price index* of the form $P(p^0,p^1,q^0,q^1)$ will be pursued.

C. First Axiomatic Approach to Bilateral Price Indices

C.1 Bilateral indices and some early tests

16.31 In this section, the strategy will be to assume that the bilateral price index formula, $P(p^0,p^1,q^0,q^1)$, satisfies a sufficient number of reasonable tests or properties so that the functional form for p is determined.[13] The word bilateral[14] refers to the assumption that the function p depends only on the data pertaining to the two situations or periods being compared; that is, p is regarded as a function of the two sets of price and quantity vectors, (p^0,p^1,q^0,q^1), that are to be aggregated into a single number that summarizes the overall change in the n price ratios, $p_1^1/p_1^0,\ldots, p_n^1/p_n^0$.

16.32 The value ratio decomposition approach to index number theory will be taken; that is, along with the price index $P(p^0,p^1,q^0,q^1)$, there is a companion quantity index $Q(p^0,p^1,q^0,q^1)$ such that the product of these two indices equals the value ratio between the two pe- riods.[15] Thus, throughout this section, it is assumed that p and q satisfy the following *product test*:

$$(16.17)\quad V^1/V^0 = P(p^0,p^1,q^0,q^1) \times Q(p^0,p^1,q^0,q^1)\,.$$

The period t values, V^t, for $t = 0,1$ are defined by equation (16.1). Equation (16.17) means that as soon as the functional form for the price index p is determined, then equation (16.17) can be used to determine the functional form for the quantity index Q. However, a further advantage of assuming that the product test holds is that if a reasonable test is imposed on the quantity index Q, then equation (16.17) can be used to translate this test on the quantity index into a corresponding test on the price index P.[16]

16.33 If $n = 1$, so that there is only one price and quantity to be aggregated, then a natural candidate for p is p_1^1/p_1^0 , the single-price ratio, and a natural candidate for q is q_1^1/q_1^0 , the single-quantity ratio. When the number of products or items to be aggregated is greater than 1, index number theorists have proposed over the years properties or tests that the price index p should satisfy. These properties are generally multidimensional analogues to the one good price index formula, p_1^1/p_1^0. In sections C.2 through C.6, 20 tests are listed that turn out to characterize the Fisher ideal price index.

16.34 It will be assumed that every component of each price and quantity vector is positive; that is, $p^t >> 0_n$ and $q^t >> 0_n$ [17] for $t = 0,1$. If it is desired to set $q^0 = q^1$, the common quantity vector is denoted by q; if it is desired to set $p^0 = p^1$, the common price vector is denoted by p.

16.35 The first two tests are not very controversial, so they will not be discussed in detail.

T1—*Positivity*:[18] $P(p^0,p^1,q^0,q^1) > 0$.

T2—*Continuity*:[19] $P(p^0,p^1,q^0,q^1)$ is a continuous function of its arguments.

16.36 The next two tests are somewhat more controversial.

T3—*Identity or Constant Prices Test*:[20]
$P(p,p,q^0,q^1) = 1$.

[12]Recall that in the economic approach, the price vector p is allowed to vary independently, but the corresponding quantity vector q is regarded as being determined by p.

[13]Much of the material in this section is drawn from Sections 2 and 3 of Diewert (1992a). For more recent surveys of the axiomatic approach, see Balk (1995) and Auer (2001).

[14]Multilateral index number theory refers to the case where there are more than two situations whose prices and quantities need to be aggregated.

[15]See Section B of Chapter 15 for more on this approach, which was initially due to I. Fisher (1911, p. 403; 1922).

[16]This observation was first made by Fisher (1911, pp. 400–406). See alsoVogt (1980) and Diewert (1992a).

[17]Notation: $q >> 0_n$ means that each component of the vector q is positive; $q \geq 0_n$ means each component of q is nonnegative; and $q > 0_n$ means $q \geq 0_n$ and $q \neq 0_n$.

[18]Eichhorn and Voeller (1976, p. 23) suggested this test.

[19]Fisher (1922, pp. 207–15) informally suggested this.

[20]Laspeyres (1871, p. 308), Walsh (1901, p. 308), and Eichhorn and Voeller (1976, p. 24) have all suggested this test. Laspeyres came up with this test or property to dis-
(continued)

16.37 That is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the quantity vectors are. The controversial part of this test is that the two quantity vectors are allowed to be different.[21]

T4—*Fixed-Basket or Constant Quantities Test*:[22]

$$P(p^0, p^1, q, q) = \frac{\sum_{i=1}^{n} p_i^1 q_i}{\sum_{i=1}^{n} p_i^0 q_i}.$$

That is, if quantities are constant during the two periods so that $q^0 = q^1 \equiv q$, then the price index should equal the revenue in the constant basket in period 1, $\sum_{i=1}^{n} p_i^1 q_i$, divided by the revenue in the basket in period 0, $\sum_{i=1}^{n} p_i^0 q_i$.

16.38 If the price index p satisfies test T4 and p and q jointly satisfy the product test, equation (16.17), then it is easy to show[23] that q must satisfy the identity test $Q(p^0,p^1,q,q) = 1$ for all strictly positive vectors p^0,p^1,q. This *constant quantities test* for q is also somewhat controversial, since p^0 and p^1 are allowed to be different.

credit the ratio of unit-values index of Drobisch (1871a), which does not satisfy this test. This test is also a special case of Fisher's (1911, pp. 409–10) price proportionality test.

[21]Usually, economists assume that given a price vector p, the corresponding quantity vector q is uniquely determined. Here, the same price vector is used, but the corresponding quantity vectors are allowed to be different.

[22]The origins of this test go back at least 200 years to the Massachusetts legislature, which used a constant basket of goods to index the pay of Massachusetts soldiers fighting in the American Revolution; see Willard Fisher (1913). Other researchers who have suggested the test over the years include Lowe (1823, Appendix, p. 95), Scrope (1833, p. 406), Jevons (1865), Sidgwick (1883, pp. 67–68), Edgeworth (1925, p. 215; originally published in 1887), Marshall (1887, p. 363), Pierson (1895, p. 332), Walsh (1901, p. 540; 1921b, pp. 543–44), and Bowley (1901, p. 227). Vogt and Barta (1997, p. 49) correctly observe that this test is a special case of Fisher's (1911, p. 411) proportionality test for quantity indices, which Fisher (1911, p. 405) translated into a test for the price index using the product test in equation (15.3).

[23]See Vogt (1980, p. 70).

C.2 Homogeneity tests

16.39 The following four tests restrict the behavior of the price index p as the scale of any one of the four vectors p^0,p^1,q^0,q^1 changes.

T5—*Proportionality in Current Prices*:[24]
$P(p^0,\lambda p^1,q^0,q^1) = \lambda P(p^0,p^1,q^0,q^1)$ for $\lambda > 0$.

16.40 That is, if all period 1 prices are multiplied by the positive number λ, then the new price index is λ times the old price index. Put another way, the price index function $P(p^0,p^1,q^0,q^1)$ is (positively) homogeneous of degree 1 in the components of the period 1 price vector p^1. Most index number theorists regard this property as a fundamental one that the index number formula should satisfy.

16.41 Walsh (1901) and Fisher (1911, p. 418; 1922, p. 420) proposed the related proportionality test $P(p,\lambda p,q^0,q^1) = \lambda$. This last test is a combination of T3 and T5; in fact, Walsh (1901, p. 385) noted that this last test implies the identity test T3.

16.42 In the next test, instead of multiplying all period 1 prices by the same number, all period 0 prices are multiplied by the number λ.

T6—*Inverse Proportionality in Base-Period Prices*:[25]
$P(\lambda p^0,p^1,q^0,q^1) = \lambda^{-1}P(p^0,p^1,q^0,q^1)$ for $\lambda > 0$.

That is, if all period 0 prices are multiplied by the positive number λ, then the new price index is $1/\lambda$ times the old price index. Put another way, the price index function $P(p^0,p^1,q^0,q^1)$ is (positively) homogeneous of degree minus 1 in the components of the period 0 price vector p^0.

16.43 The following two homogeneity tests can also be regarded as invariance tests.

T7—*Invariance to Proportional Changes in Current Quantities*:
$P(p^0,p^1,q^0,\lambda q^1) = P(p^0,p^1,q^0,q^1)$ for all $\lambda > 0$.

That is, if current-period quantities are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index

[24]This test was proposed by Walsh (1901, p. 385), Eichhorn and Voeller (1976, p. 24), and Vogt (1980, p. 68).

[25]Eichhorn and Voeller (1976, p. 28) suggested this test.

function $P(p^0,p^1,q^0,q^1)$ is (positively) homogeneous of degree zero in the components of the period 1 quantity vector q^1. Vogt (1980, p. 70) was the first to propose this test,[26] and his derivation of the test is of some interest. Suppose the quantity index q satisfies the quantity analogue to the price test T5; that is, suppose q satisfies $Q(p^0,p^1,q^0,\lambda q^1) = \lambda Q(p^0,p^1,q^0,q^1)$ for $\lambda > 0$. Then using the product test in equation (16.17), it can be seen that p must satisfy T7.

T8—*Invariance to Proportional Changes in Base Quantities*:[27]
$P(p^0,p^1,\lambda q^0,q^1) = P(p^0,p^1,q^0,q^1)$ for all $\lambda > 0$.

That is, if base-period quantities are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0,p^1,q^0,q^1)$ is (positively) homogeneous of degree 0 in the components of the period 0 quantity vector q^0. If the quantity index q satisfies the following counterpart to T8: $Q(p^0,p^1,\lambda q^0,q^1) = \lambda^{-1}Q(p^0,p^1,q^0,q^1)$ for all $\lambda > 0$, then using equation (16.17), the corresponding price index p must satisfy T8. This argument provides some additional justification for assuming the validity of T8 for the price index function P.

16.44 T7 and T8 together impose the property that the price index p does not depend on the *absolute* magnitudes of the quantity vectors q^0 and q^1.

C.3 Invariance and symmetry tests

16.45 The next five tests are invariance or symmetry tests. Fisher (1922, pp. 62–63, 458–60) and Walsh (1901, p. 105; 1921b, p. 542) seem to have been the first researchers to appreciate the significance of these kinds of tests. Fisher (1922, pp. 62–63) spoke of fairness, but it is clear that he had symmetry properties in mind. It is perhaps unfortunate that he did not realize that there were more symmetry and invariance properties than the ones he proposed; if he had realized this, it is likely that he would have been able to provide an axiomatic characterization for his ideal price index, as will be done in Section C.6. The first invariance test is that the price index should remain unchanged if the *ordering* of the commodities is changed:

T9—*Commodity Reversal Test* (or invariance to changes in the ordering of commodities):
$P(p^{0*},p^{1*},q^{0*},q^{1*}) = P(p^0,p^1,q^0,q^1)$,

where p^{t*} denotes a permutation of the components of the vector p^t, and q^{t*} denotes the same permutation of the components of q^t for $t = 0,1$. This test is due to Irving Fisher (1922, p. 63);[28] it is one of his three famous reversal tests. The other two are the time reversal test and the factor reversal test, which will be considered below.

16.46 The next test asks that the index be invariant to changes in the units of measurement.

T10—*Invariance to Changes in the Units of Measurement* (commensurability test):

$$P(\alpha_1 p_1^0,\ldots,\alpha_n p_n^0;\ \alpha_1 p_1^1,\ldots,\alpha_n p_n^1;\ \alpha_1^{-1} q_1^0,\ldots,\alpha_n^{-1} q_n^0;\ \alpha_1^{-1} q_1^1,\ldots,\alpha_n^{-1} q_n^1)$$
$$= P(p_1^0,\ldots,p_n^0;\ p_1^1,\ldots,p_n^1;\ q_1^0,\ldots,q_n^0;\ q_1^1,\ldots,q_n^1)$$

for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

That is, the price index does not change if the units of measurement for each product are changed. The concept of this test comes from Jevons (1863, p. 23) and the Dutch economist Pierson (1896, p. 131), who criticized several index number formulas for not satisfying this fundamental test. Fisher (1911, p. 411) first called this test *the change of units test*, and later (Fisher, 1922, p. 420) he called it the *commensurability test*.

16.47 The next test asks that the formula be invariant to the period chosen as the base period.

T11—*Time Reversal Test*:
$P(p^0,p^1,q^0,q^1) = 1/P(p^1,p^0,q^1,q^0)$.

That is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index. In the one good case when the price index is simply

[26]Fisher (1911, p. 405) proposed the related test $P(p^0,p^1,q^0,\lambda q^0) = P(p^0,p^1,q^0,q^0) = \sum_{i=1}^{n} p_i^1 q_i^0 \Big/ \sum_{i=1}^{n} p_i^0 q_i^0$.

[27]This test was proposed by Diewert (1992a, p. 216).

[28]"This [test] is so simple as never to have been formulated. It is merely taken for granted and observed instinctively. Any rule for averaging the commodities must be so general as to apply interchangeably to all of the terms averaged" (Irving Fisher, 1922, p. 63).

the single-price ratio, this test will be satisfied (as are all of the other tests listed in this section). When the number of goods is greater than one, many commonly used price indices fail this test; for example, the Laspeyres (1871) price index, P_L, defined by equation (15.5) in Chapter 15, and the Paasche (1874) price index, P_P, defined by equation (15.6) in Chapter 15, both fail this fundamental test. The concept of the test comes from Pierson (1896, p. 128), who was so upset with the fact that many of the commonly used index number formulas did not satisfy this test that he proposed that the entire concept of an index number should be abandoned. More formal statements of the test were made by Walsh (1901, p. 368; 1921b, p. 541) and Fisher (1911, p. 534; 1922, p. 64).

16.48 The next two tests are more controversial, since they are not necessarily consistent with the economic approach to index number theory. However, these tests are quite consistent with the weighted stochastic approach to index number theory to be discussed later in this chapter.

T12—*Quantity Reversal Test* (quantity weights symmetry test): $P(p^0,p^1,q^0,q^1) = P(p^0,p^1,q^1,q^0)$.

That is, if the quantity vectors for the two periods are interchanged, then the price index remains invariant. This property means that if quantities are used to weight the prices in the index number formula, then the period 0 quantities q^0 and the period 1 quantities q^1 must enter the formula in a symmetric or evenhanded manner. Funke and Voeller (1978, p. 3) introduced this test; they called it the *weight property*.

16.49 The next test is the analogue to T12 applied to quantity indices:

T13—*Price Reversal Test* (price weights symmetry test):[29]

$$(16.18)\quad \left(\frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}\right) \Big/ P(p^0,p^1,q^0,q^1)$$

$$= \left(\frac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^0}\right) \Big/ P(p^1,p^0,q^0,q^1)\,.$$

Thus, if we use equation (16.17) to define the quantity index q in terms of the price index P, then it can be seen that T13 is equivalent to the following property for the associated quantity index Q:

$$(16.19)\quad Q(p^0,p^1,q^0,q^1) = Q(p^1,p^0,q^0,q^1).$$

That is, if the price vectors for the two periods are interchanged, then the quantity index remains invariant. Thus, if prices for the same good in the two periods are used to weight quantities in the construction of the quantity index, then property T13 implies that these prices enter the quantity index in a symmetric manner.

C.4 Mean value tests

16.50 The next three tests are mean value tests.

T14—*Mean Value Test for Prices*:[30]

$$(16.20)\quad \min_i(p_i^1/p_i^0 : i = 1,...,n) \le P(p^0,p^1,q^0,q^1) \le \max_i(p_i^1/p_i^0 : i = 1,...,n)\,.$$

That is, the price index lies between the minimum price ratio and the maximum price ratio. Since the price index is supposed to be interpreted as a kind of average of the n price ratios, p_i^1/p_i^0, it seems essential that the price index p satisfy this test.

16.51 The next test is the analogue to T14 applied to quantity indices:

T15—*Mean Value Test for Quantities*:[31]

$$(16.21)\quad \min_i(q_i^1/q_i^0 : i = 1,...,n) \le \frac{(V^1/V^0)}{P(p^0,p^1,q^0,q^1)} \le \max_i(q_i^1/q_i^0 : i = 1,...,n)\,,$$

[29]This test was proposed by Diewert (1992a, p. 218).

[30]This test seems to have been first proposed by Eichhorn and Voeller (1976, p. 10).

[31]This test was proposed by Diewert (1992a, p. 219).

where V^t is the period t value for the aggregate defined by equation (16.1) above. Using the product test in equation (16.17) to define the quantity index q in terms of the price index P, it can be seen that T15 is equivalent to the following property for the associated quantity index Q:

$$(16.22)\quad \min_i (q_i^1/q_i^0 \;:\; i=1,...,n) \le Q(p^0,p^1,q^0,q^1) \le \max_i (q_i^1/q_i^0 \;:\; i=1,...,n)\,.$$

That is, the implicit quantity index q defined by p lies between the minimum and maximum rates of growth q_i^1/q_i^0 of the individual quantities.

16.52 In Section C of Chapter 15, it was argued that it was reasonable to take an average of the Laspeyres and Paasche price indices as a single best measure of overall price change. This point of view can be turned into a test:

T16—*Paasche and Laspeyres Bounding Test*:[32] The price index p lies between the Laspeyres and Paasche indices, P_L and P_P, defined by equations (15.5) and (15.6) in Chapter 15.

A test could be proposed where the implicit quantity index q that corresponds to p via equation (16.17) is to lie between the Laspeyres and Paasche quantity indices, Q_P and Q_L, defined by equations (15.10) and (15.11) in Chapter 15. However, the resulting test turns out to be equivalent to test T16.

C.5 Monotonicity tests

16.53 The final four tests are monotonicity tests; that is, how should the price index $P(p^0,p^1,q^0,q^1)$ change as any component of the two price vectors p^0 and p^1 increases or as any component of the two quantity vectors q^0 and q^1 increases?

T17—*Monotonicity in Current Prices*: $P(p^0,p^1,q^0,q^1) < P(p^0,p^2,q^0,q^1)$ if $p^1 < p^2$.

That is, if some period 1 price increases, then the price index must increase, so that $P(p^0,p^1,q^0,q^1)$ is increasing in the components of p^1. This property was proposed by Eichhorn and Voeller (1976, p. 23), and it is a reasonable property for a price index to satisfy.

T18—*Monotonicity in Base Prices*: $P(p^0,p^1,q^0,q^1) > P(p^2,p^1,q^0,q^1)$ if $p^0 < p^2$.

That is, if any period 0 price increases, then the price index must decrease, so that $P(p^0,p^1,q^0,q^1)$ is decreasing in the components of p^0 . This quite reasonable property was also proposed by Eichhorn and Voeller (1976, p. 23).

T19—*Monotonicity in Current Quantities*:
If $q^1 < q^2$, then

$$(16.23)\quad \left(\frac{\sum_{i=1}^n p_i^1 q_i^1}{\sum_{i=1}^n p_i^0 q_i^0}\right)\Bigg/ P(p^0,p^1,q^0,q^1) < \left(\frac{\sum_{i=1}^n p_i^1 q_i^2}{\sum_{i=1}^n p_i^0 q_i^0}\right)\Bigg/ P(p^0,p^1,q^0,q^2)\,.$$

T20—*Monotonicity in Base Quantities*: If $q^0 < q^2$, then

$$(16.24)\quad \left(\frac{\sum_{i=1}^n p_i^1 q_i^1}{\sum_{i=1}^n p_i^0 q_i^0}\right)\Bigg/ P(p^0,p^1,q^0,q^1) > \left(\frac{\sum_{i=1}^n p_i^1 q_i^1}{\sum_{i=1}^n p_i^0 q_i^2}\right)\Bigg/ P(p^0,p^1,q^2,q^1)\,.$$

16.54 Let q be the implicit quantity index that corresponds to p using equation (16.17). Then it is found that T19 translates into the following inequality involving Q:

$$(16.25)\quad Q(p^0,p^1,q^0,q^1) < Q(p^0,p^1,q^0,q^2) \quad \text{if } q^1 < q^2.$$

That is, if any period 1 quantity increases, then the implicit quantity index q that corresponds to the price index p must increase. Similarly, we find that T20 translates into:

[32]Bowley (1901, p. 227) and Fisher (1922, p. 403) both endorsed this property for a price index.

$$(16.26)\quad Q(p^0,p^1,q^0,q^1) > Q(p^0,p^1,q^2,q^1) \text{ if } q^0 < q^2.$$

That is, if any period 0 quantity increases, then the implicit quantity index q must decrease. Tests T19 and T20 are due to Vogt (1980, p. 70).

16.55 This concludes the listing of tests. In the next section, it is asked whether any index number formula $P(p^0,p^1,q^0,q^1)$ exists that can satisfy all 20 tests.

C.6 Fisher ideal index and test approach

16.56 It can be shown that the only index number formula $P(p^0,p^1,q^0,q^1)$ that satisfies tests T1–T20 is the Fisher ideal price index P_F, defined as the geometric mean of the Laspeyres and Paasche indices:[33]

$$(16.27)\quad P_F(p^0,p^1,q^0,q^1) \equiv \left[P_L(p^0,p^1,q^0,q^1)\right]^{1/2} \times \left[P_P(p^0,p^1,q^0,q^1)\right]^{1/2}.$$

To prove this assertion, it is relatively straightforward to show that the Fisher index satisfies all 20 tests.

16.57 The more difficult part of the proof is showing that it is the *only* index number formula that satisfies these tests. This part of the proof follows from the fact that if p satisfies the positivity test T1 and the three reversal tests, T11–T13, then p must equal P_F. To see this, rearrange the terms in the statement of test T13 into the following equation:

$$(16.28)\quad \frac{\sum_{i=1}^{n} p_i^1 q_i^1 / \sum_{i=1}^{n} p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^1 / \sum_{i=1}^{n} p_i^1 q_i^0} = \frac{P(p^0,p^1,q^0,q^1)}{P(p^1,p^0,q^0,q^1)}$$

$$= \frac{P(p^0,p^1,q^0,q^1)}{P(p^1,p^0,q^1,q^0)},$$

using T12, the quantity reversal test

$$= P(p^0,p^1,q^0,q^1)P(p^0,p^1,q^0,q^1),$$

using T11, the time reversal test.

Now take positive square roots on both sides of equation (16.28), and it can be seen that the left-hand side of the equation is the Fisher index $P_F(p^0,p^1,q^0,q^1)$ defined by equation (16.27) and the right-hand side is $P(p^0,p^1,q^0,q^1)$. Thus, if p satisfies T1, T11, T12, and T13, it must equal the Fisher ideal index P_F.

16.58 The quantity index that corresponds to the Fisher price index using the product test in equation (16.17) is Q_F , the Fisher quantity index, defined by equation (15.14) in Chapter 15.

16.59 It turns out that P_F satisfies yet another test, T21, which was Irving Fisher's (1921, p. 534; 1922. pp. 72–81) third reversal test (the other two being T9 and T11):

T21—*Factor Reversal Test* (functional form symmetry test):

$$(16.29)\quad P(p^0,p^1,q^0,q^1)P(q^0,q^1,p^0,p^1) = \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}.$$

A justification for this test is the following: assume $P(p^0,p^1,q^0,q^1)$ is a good functional form for the price index; then if the roles of prices and quantities are reversed, $P(q^0,q^1,p^0,p^1)$ ought to be a good functional form for a quantity index (which seems to be a correct argument). The product, therefore, of the price index $P(q^0,q^1,p^0,p^1)$ and the quantity index $Q(q^0,q^1,p^0,p^1) = P(q^0,q^1,p^0,p^1)$ ought to equal the value ratio, V^1/V^0 . The second part of this argument does not seem to be valid; consequently, many researchers over the years have objected to the factor reversal test. However, if one is willing to embrace T21 as a basic test, Funke and Voeller (1978, p. 180) showed that the only index number function $P(q^0,q^1,p^0,p^1)$ that satisfies T1 (positivity),

[33] See Diewert (1992a, p. 221).

T11 (time reversal test), T12 (quantity reversal test), and T21 (factor reversal test) is the Fisher ideal index P_F defined by equation (16.27). Thus, the price reversal test T13 can be replaced by the factor reversal test in order to obtain a minimal set of four tests that lead to the Fisher price index.[34]

C.7 Test performance of other indices

16.60 The Fisher price index P_F satisfies all 20 of the tests listed in Sections C.1–C.5. Which tests do other commonly used price indices satisfy? Recall the Laspeyres index P_L, equation (15.5); the Paasche index P_P, equation (15.6); the Walsh index P_W, equation (15.19); and the Törnqvist index P_T, equation (15.81) in Chapter 15.

16.61 Straightforward computations show that the Paasche and Laspeyres price indices, P_L and P_P, fail only the three reversal tests, T11, T12, and T13. Since the quantity and price reversal tests, T12 and T13, are somewhat controversial and can be discounted, the test performance of P_L and P_P seems at first glance to be quite good. However, the failure of the time reversal test, T11, is a severe limitation associated with the use of these indices.

16.62 The Walsh price index, P_W, fails four tests: T13, the price reversal test; T16, the Paasche and Laspeyres bounding test; T19, the monotonicity in current quantities test; and T20, the monotonicity in base quantities test.

16.63 Finally, the Törnqvist price index P_T fails nine tests: T4, the fixed-basket test; T12 and T13, the quantity and price reversal tests; T15, the mean value test for quantities; T16, the Paasche and Laspeyres bounding test; and T17–T20, the four monotonicity tests. Thus, the Törnqvist index is subject to a rather high failure rate from the viewpoint of this axiomatic approach to index number theory.[35]

[34]Other characterizations of the Fisher price index can be found in Funke and Voeller (1978) and Balk (1985, 1995).

[35]However, it will be shown later in Chapter 19 that the Törnqvist index approximates the Fisher index quite closely using normal time-series data that are subject to relatively smooth trends. Under these circumstances, the Törnqvist index can be regarded as passing the 20 tests to a reasonably high degree of approximation.

16.64 The tentative conclusion that can be drawn from these results is that from the viewpoint of this particular bilateral test approach to index numbers, the Fisher ideal price index P_F appears to be best because it satisfies all 20 tests.[36] The Paasche and Laspeyres indices are next best if we treat each test as being equally important. However, both of these indices fail the very important time reversal test. The remaining two indices, the Walsh and Törnqvist price indices, both satisfy the time reversal test, but the Walsh index emerges as the better one because it passes 16 of the 20 tests, whereas the Törnqvist satisfies only 11 tests.

C.8 Additivity test

16.65 There is an additional test that many national income accountants regard as very important: the *additivity test*. This is a test or property that is placed on the implicit quantity index $Q(q^0,q^1,p^0,p^1)$ that corresponds to the price index $P(q^0,q^1,p^0,p^1)$ using the product test in equation (16.17). This test states that the implicit quantity index has the following form:

$$(16.30)\quad Q(p^0,p^1,q^0,q^1)=\frac{\sum_{i=1}^{n}p_i^*q_i^1}{\sum_{m=1}^{n}p_m^*q_m^0},$$

where the common across-periods *price* for product *i*, p_i^* for $i = 1,\ldots,n$, can be a function of all $4n$ prices and quantities pertaining to the two periods or situations under consideration, p^0,p^1,q^0,q^1. In the literature on making multilateral comparisons (that is, comparisons among more than two situations), it is quite common to assume that the quantity comparison between any two regions can be made using the two regional quantity vectors, q^0 and q^1, and a common reference price vector, $p^* \equiv (p_1^*,\ldots,p_n^*)$.[37]

[36]This assertion needs to be qualified: there are many other tests that we have not discussed, and price statisticians could differ on the importance of satisfying various sets of tests. Some references that discuss other tests are Auer (2001; 2002), Eichhorn and Voeller (1976), Balk (1995), and Vogt and Barta (1997). In Section E, it is shown that the Törnqvist index is ideal for a different set of axioms.

[37]Hill (1993, pp. 395–97) termed such multilateral methods *the block approach*, while Diewert (1996a, pp. 250–51) used the term *average price approaches*. Diewert (1999b, (continued)

16.66 Different versions of the additivity test can be obtained if further restrictions are placed on precisely which variables each reference price p_i* depends. The simplest such restriction is to assume that each p_i* depends only on the product i prices pertaining to each of the two situations under consideration, p_i^0 and p_i^1. If it is further assumed that the functional form for the weighting function is the same for each product, so that $p_i* = m(p_i^0, p_i^1)$ for $i = 1,…,n$, then we are led to the *unequivocal quantity index* postulated by Knibbs (1924, p. 44).

16.67 The theory of the *unequivocal quantity index* (or the *pure quantity index*)[38] parallels the theory of the pure price index outlined in Section C.2 of Chapter 15. An outline of this theory is now given. Let the pure quantity index Q_K have the following functional form:

$$(16.31) \quad Q_K(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} q_i^1 m(p_i^0, p_i^1)}{\sum_{k=1}^{n} q_k^0 m(p_k^0, p_k^1)}.$$

It is assumed that the price vectors p^0 and p^1 are strictly positive, and the quantity vectors q^0 and q^1 are nonnegative but have at least one positive component.[39] The problem is to determine the functional form for the averaging function m if possible. To do this, it is necessary to impose some tests or properties on the pure quantity index Q_K. As was the case with the pure price index, it is reasonable to ask that the quantity index satisfy the *time reversal test*:

$$(16.32) \quad Q_K(p^1, p^0, q^1, q^0) = \frac{1}{Q_K(p^0, p^1, q^0, q^1)}.$$

16.68 As was the case with the theory of the unequivocal price index, it can be seen that if the unequivocal quantity index Q_K is to satisfy the time reversal test of equation (16.32), the mean function in equation (16.31) must be *symmetric*. It is also asked that Q_K satisfy the following *invariance to proportional changes in current prices test*.

$$(16.33) \quad Q_K(p^0, \lambda p^1, q^0, q^1) = Q_K(p^0, p^1, q^0, q^1) \text{ for all } p^0, p^1, q^0, q^1 \text{ and all } \lambda > 0.$$

16.69 The idea behind this invariance test is this: the quantity index $Q_K(p^0,p^1,q^0,q^1)$ should depend only on the *relative* prices in each period. It should not depend on the amount of inflation between the two periods. Another way to interpret equation (16.33) is to look at what the test implies for the corresponding implicit price index, P_{IK}, defined using the product test of equation (16.17). It can be shown that if Q_K satisfies equation (16.33), then the corresponding implicit price index P_{IK} will satisfy test T5, the *proportionality in current prices test*. The two tests in equations (16.32) and (16.33) determine the precise functional form for the pure quantity index Q_K defined by equation (16.31): the *pure quantity index* or Knibbs' *unequivocal quantity index* Q_K must be the Walsh quantity index Q_W[40] defined by

$$(16.34) \quad Q_W(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} q_i^1 \sqrt{p_i^0 p_i^1}}{\sum_{k=1}^{n} q_k^0 \sqrt{p_k^0 p_k^1}}.$$

16.70 Thus, with the addition of two tests, the pure price index P_K must be the Walsh price index P_W defined by equation (15.19) in Chapter 15. With the addition of the same two tests (but applied to quantity indices instead of price indices), the pure quantity index Q_K must be the Walsh quantity index Q_W defined by equation (16.34). However, note that the product of the Walsh price and quantity indices is *not* equal to the revenue ratio, V^1/V^0. Thus, believers in the pure or unequivocal price and quantity index concepts *have to choose one of these two concepts*; they cannot apply both simultaneously.[41]

16.71 If the quantity index $Q(q^0,q^1,p^0,p^1)$ satisfies the additivity test in equation (16.30) for some

p. 19) used the term *additive multilateral system*. For axiomatic approaches to multilateral index number theory, see Balk (1996a, 2001) and Diewert (1999b).

[38] Diewert (2001) used this term.

[39] It is assumed that $m(a,b)$ has the following two properties: $m(a,b)$ is a positive and continuous function, defined for all positive numbers a and b; and $m(a,a) = a$ for all $a > 0$.

[40] This is the quantity index that corresponds to the price index 8 defined by Walsh (1921a, p. 101).

[41] Knibbs (1924) did not notice this point!

price weights p_i^*, then the percentage change in the quantity aggregate, $Q(q^0,q^1,p^0,p^1) - 1$, can be rewritten as follows:

$$(16.35)\quad Q(p^0,p^1,q^0,q^1)-1=\frac{\sum_{i=1}^{n}p_i^*q_i^1}{\sum_{m=1}^{n}p_m^*q_m^0}-1 =\frac{\sum_{i=1}^{n}p_i^*q_i^1-\sum_{m=1}^{n}p_m^*q_m^0}{\sum_{m=1}^{n}p_m^*q_m^0} =\sum_{i=1}^{n}w_i(q_i^1-q_i^0),$$

where the *weight* for product *i,* w_i, is defined as

$$(16.36)\quad w_i \equiv \frac{p_i^*}{\sum_{m=1}^{n}p_m^*q_m^0};\quad i=1,...,n.$$

Note that the change in product *i* going from situation 0 to situation 1 is $q_i^1 - q_i^0$. Thus, the *i*th term on the right-hand side of equation (16.35) is the *contribution of the change in product* i *to the overall percentage change in the aggregate going from period 0 to 1*. Business analysts often want statistical agencies to provide decompositions like equation (16.35) so they can decompose the overall change in an aggregate into sector-specific components of change.[42] Thus, there is a demand on the part of users for additive quantity indices.

16.72 For the Walsh quantity index defined by equation (16.34), the *i*th weight is

$$(16.37)\quad w_{W_i} \equiv \frac{\sqrt{p_i^0p_i^1}}{\sum_{m=1}^{n}q_m^0\sqrt{p_m^0p_m^1}};\quad i=1,...,n.$$

Thus, the Walsh quantity index Q_W has a percentage decomposition into component changes of the form in equation (16.35), where the weights are defined by equation (16.37).

16.73 It turns out that the Fisher quantity index Q_F defined by equation (15.14) in Chapter 15 also has an additive percentage change decomposition of the form given by equation (16.35).[43] The *i*th weight w_{Fi} for this Fisher decomposition is rather complicated and depends on the Fisher quantity index $Q_F(p^0,p^1,q^0,q^1)$ as follows:[44]

$$(16.38)\quad w_{F_i} \equiv \frac{w_i^0+(Q_F)^2w_i^1}{1+Q_F};\quad i=1,...,n,$$

where Q_F is the value of the Fisher quantity index, $Q_F(p^0,p^1,q^0,q^1)$, and the period *t* normalized price for product *i,* w_i^t, is defined as the period *i* price p_i^t divided by the period *t* revenue on the aggregate:

$$(16.39)\quad w_i^t \equiv \frac{p_i^t}{\sum_{m=1}^{n}p_m^tq_m^t};\quad t = 0,1;\quad i = 1,\ldots,n.$$

16.74 Using the weights w_{Fi} defined by equations (16.38) and (16.39), the following exact decomposition is obtained for the Fisher ideal quantity index:[45]

$$(16.40)\quad Q_F(p^0,p^1,q^0,q^1)-1=\sum_{i=1}^{n}w_{F_i}(q_i^1-q_i^0).$$

Thus, the Fisher quantity index has an additive percentage change decomposition.

16.75 Due to the symmetric nature of the Fisher price and quantity indices, it can be seen that the Fisher price index P_F defined by equation (16.27)

[42]Business and government analysts also often demand an analogous decomposition of the change in price aggregate into sector-specific components that add up.

[43]The Fisher quantity index also has an additive decomposition of the type defined by equation (16.30) due to Van Ijzeren (1987, p. 6). The *i*th reference price p_i^* is defined as $p_i^* \equiv (1/2)p_i^0 + (1/2)p_i^1/P_F(p^0,p^1,q^0,q^1)$ for $i = 1,\ldots,n$ and where P_F is the Fisher price index. This decomposition was also independently derived by Dikhanov (1997). The Van Ijzeren decomposition for the Fisher quantity index is currently being used by the Bureau of Economic Analysis; see Moulton and Seskin (1999, p. 16) and Ehemann, Katz, and Moulton (2002).

[44]This decomposition was obtained by Diewert (2002a) and Reinsdorf, Diewert, and Ehemann (2002). For an economic interpretation of this decomposition, see Diewert (2002a).

[45]To verify the exactness of the decomposition, substitute equation (16.38) into equation (16.40) and solve the resulting equation for Q_F. It is found that the solution is equal to Q_F defined by equation (15.14) in Chapter 15.

also has the following additive percentage change decomposition:

$$(16.41) \quad P_F(p^0,p^1,q^0,q^1) - 1 = \sum_{i=1}^{n} v_{F_i}(p_i^1 - p_i^0),$$

where the product i weight v_{Fi} is defined as

$$(16.42) \quad v_{F_i} \equiv \frac{v_i^0 + (P_F)^2 v_i^1}{1+P_F}; \quad i = 1,...,n,$$

where P_F is the value of the Fisher price index, $P_F(p^0,p^1,q^0,q^1)$, and the period t normalized quantity for product i, v_i^t, is defined as the period i quantity q_i^t divided by the period t revenue on the aggregate:

$$(16.43) \quad v_i^t \equiv \frac{q_i^t}{\sum_{m=1}^{n} p_m^t q_m^t}; \quad t = 0,1; \quad i = 1,\ldots,n.$$

16.76 The above results show that the Fisher price and quantity indices have exact additive decompositions into components that give the contribution to the overall change in the price (or quantity) index of the change in each price (or quantity).

D. Stochastic Approach to Price Indices

D.1 Early unweighted stochastic approach

16.77 The stochastic approach to the determination of the price index can be traced back to the work of Jevons (1863, 1865) and Edgeworth (1888) over a hundred years ago.[46] The basic idea behind the (unweighted) stochastic approach is that each price relative, p_i^1/p_i^0 for $i = 1,2,\ldots,n$ can be regarded as an estimate of a common inflation rate α between periods 0 and 1;[47] that is, it is assumed that

$$(16.44) \quad \frac{p_i^1}{p_i^0} = \alpha + \varepsilon_i; \quad i = 1,2,...,n,$$

where α is the common inflation rate and the ε_i are random variables with mean 0 and variance σ^2. The least squares or maximum likelihood estimator for α is the *Carli* (1804) *price index* P_C defined as

$$(16.45) \quad P_C(p^0,p^1) \equiv \sum_{i=1}^{n} \frac{1}{n} \frac{p_i^1}{p_i^0}.$$

A drawback of the Carli price index is that it does not satisfy the time reversal test, that is, $P_C(p^1,p^0) \neq 1/P_C(p^0,p^1)$.[48]

16.78 Now change the stochastic specification and assume that the logarithm of each price relative, $\ln(p_i^1/p_i^0)$, is an unbiased estimate of the logarithm of the inflation rate between periods 0 and 1, β, say. The counterpart to equation (16.44) is:

$$(16.46) \quad \ln\left(\frac{p_i^1}{p_i^0}\right) = \beta + \varepsilon_i; \quad i = 1,2,...,n,$$

where $\beta \equiv \ln \alpha$ and the ε_i are independently distributed random variables with mean 0 and variance σ^2. The least-squares or maximum-likelihood estimator for β is the logarithm of the geometric mean of the price relatives. Hence, the corresponding estimate for the common inflation rate α[49]

[46]For references to the literature, see Diewert (1993a, pp. 37–38; 1995a; 1995b).

[47]"In drawing our averages the independent fluctuations will more or less destroy each other; the one required variation of gold will remain undiminished" (W. Stanley Jevons, 1863, p. 26).

[48]In fact, Fisher (1922, p. 66) noted that $P_C(p^0,p^1)P_C(p^1,p^0) \geq 1$ unless the period 1 price vector p^1 is proportional to the period 0 price vector p^0; that is, Fisher showed that the Carli index has a definite upward bias. He urged statistical agencies not to use this formula. Walsh (1901, pp. 331 and 530) also discovered this result for the case $n = 2$.

[49]Greenlees (1999) pointed out that although $\frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{p_i^1}{p_i^0}\right)$ is an unbiased estimator for β, the corresponding exponential of this estimator, P_J defined by equation (16.47), will generally *not* be an unbiased estimator for α under our stochastic assumptions. To see this, let $x_i = \ln(p_i^1/p_i^0)$. Taking expectations, we have: $Ex_i = \beta = \ln \alpha$. Define the positive, convex function f of one variable x by $f(x) \equiv e^x$. By Jensen's (1906) inequality, $Ef(x) \geq f(Ex)$. Letting x equal the random variable x_i, this inequality becomes $E(p_i^1/p_i^0) = Ef(x_i) \geq f(Ex_i) = f(\beta) = e^{\beta} = e^{\ln \alpha} = \alpha$. Thus, for each n, $E(p_i^1/p_i^0) \geq \alpha$, and it can be seen that the Jevons (continued)

is the *Jevons* (1865) *price index* P_J defined as follows:

$$(16.47)\ P_J(p^0, p^1) \equiv \prod_{i=1}^{n} \sqrt[n]{\frac{p_i^1}{p_i^0}}.$$

16.79 The Jevons price index P_J does satisfy the time reversal test and thus is much more satisfactory than the Carli index P_C. However, both the Jevons and Carli price indices suffer from a fatal flaw: each price relative p_i^1/p_i^0 is regarded as *being equally important* and is given an equal weight in the index number equations (16.45) and (16.47). Keynes was particularly critical of this *unweighted stochastic approach* to index number theory.[50] He directed the following criticism toward this approach, which was vigorously advocated by Edgeworth (1923):

> Nevertheless I venture to maintain that such ideas, which I have endeavoured to expound above as fairly and as plausibly as I can, are root-and-branch erroneous. The "errors of observation", the "faulty shots aimed at a single bull's eye" conception of the index number of prices, Edgeworth's "objective mean variation of general prices", is the result of confusion of thought. There is no bull's eye. There is no moving but unique centre, to be called the general price level or the objective mean variation of general prices, round which are scattered the moving price levels of individual things. There are all the various, quite definite, conceptions of price levels of composite commodities appropriate for various purposes and inquiries which have been scheduled above, and many others too. There is nothing else. Jevons was pursuing a mirage.
>
> What is the flaw in the argument? In the first place it assumed that the fluctuations of individual prices round the "mean" are "random" in the sense required by the theory of the combination of independent observations. In this theory the divergence of one "observation" from the true position is assumed to have no influence on the divergences of other "observations". But in the case of prices, a movement in the price of one product necessarily influences the movement in the prices of other commodities, whilst the magnitudes of these compensatory movements depend on the magnitude of the change in revenue on the first product as compared with the importance of the revenue on the commodities secondarily affected. Thus, instead of "independence", there is between the "errors" in the successive "observations" what some writers on probability have called "connexity", or, as Lexis expressed it, there is "sub-normal dispersion".
>
> We cannot, therefore, proceed further until we have enunciated the appropriate law of connexity. But the law of connexity cannot be enunciated without reference to the relative importance of the commodities affected—which brings us back to the problem that we have been trying to avoid, of weighting the items of a composite commodity. (John Maynard Keynes, 1930, pp. 76–77)

The main point Keynes seemed to be making in the quotation above is that prices in the economy are not independently distributed from each other and from quantities. In current macroeconomic terminology, Keynes can be interpreted as saying that a macroeconomic shock will be distributed across all prices and quantities in the economy through the normal interaction between supply and demand; that is, through the workings of the general equilibrium system. Thus, Keynes seemed to be leaning toward the economic approach to index number theory (even before it was developed to any great extent), where quantity movements are functionally related to price movements. A second point that Keynes made in the above quotation is that there is no such thing as *the* inflation rate; there are only price changes that pertain to well-specified sets of commodities or transactions; that is, the domain of definition of the price index must be carefully specified.[51] A final point that Keynes made is that price movements must be weighted by their economic importance; that is, by quantities or revenues.

price index will generally have an upward bias under the usual stochastic assumptions.

[50]Walsh (1901, p. 83) also stressed the importance of proper weighting according to the economic importance of the commodities in the periods being compared: "But to assign uneven weighting with approximation to the relative sizes, either over a long series of years or for every period separately, would not require much additional trouble; and even a rough procedure of this sort would yield results far superior to those yielded by even weighting. It is especially absurd to refrain from using roughly reckoned uneven weighting on the ground that it is not accurate, and instead to use even weighting, which is much more inaccurate."

[51]See Section B in Chapter 15 for additional discussion on this point.

16.80 In addition to the above theoretical criticisms, Keynes also made the following strong empirical attack on Edgeworth's unweighted stochastic approach:

> The Jevons-Edgeworth "objective mean variation of general prices," or "indefinite" standard, has generally been identified, by those who were not as alive as Edgeworth himself was to the subtleties of the case, with the purchasing power of money—if only for the excellent reason that it was difficult to visualise it as anything else. And since any respectable index number, however weighted, which covered a fairly large number of commodities could, in accordance with the argument, be regarded as a fair approximation to the indefinite standard, it seemed natural to regard any such index as a fair approximation to the purchasing power of money also.
>
> Finally, the conclusion that all the standards "come to much the same thing in the end" has been reinforced "inductively" by the fact that rival index numbers (all of them, however, of the wholesale type) have shown a considerable measure of agreement with one another in spite of their different compositions. … On the contrary, the tables given above (pp. 53, 55) supply strong presumptive evidence that over long period as well as over short periods the movements of the wholesale and of the consumption standards respectively are capable of being widely divergent. (John Maynard Keynes, 1930, pp. 80–81)

In the quotation above, Keynes noted that the proponents of the unweighted stochastic approach to price change measurement were comforted by the fact that all of the then existing (unweighted) indices of wholesale prices showed broadly similar movements. However, Keynes showed empirically that his wholesale price indices moved quite differently than his consumer price indices.

16.81 In order to overcome these criticisms of the unweighted stochastic approach to index numbers, it is necessary to

- Have a definite domain of definition for the index number; and
- Weight the price relatives by their economic importance.[52]

16.82 In the following section, alternative methods of weighting will be discussed.

D.2 Weighted stochastic approach

16.83 Walsh (1901, pp. 88–89) seems to have been the first index number theorist to point out that a sensible stochastic approach to measuring price change means that individual price relatives should be weighted according to their economic importance or *their transactions' value* in the two periods under consideration:

> It might seem at first sight as if simply every price quotation were a single item, and since every commodity (any kind of commodity) has one price-quotation attached to it, it would seem as if price-variations of every kind of commodity were the single item in question. This is the way the question struck the first inquirers into price-variations, wherefore they used simple averaging with even weighting. But a price-quotation is the quotation of the price of a generic name for many articles; and one such generic name covers a few articles, and another covers many. … A single price-quotation, therefore, may be the quotation of the price of a hundred, a thousand, or a million dollars' worth, of the articles that make up the commodity named. Its weight in the averaging, therefore, ought to be according to these money-unit's worth. (Correa Moylan Walsh, 1921a, pp. 82–83)

However, Walsh did not give a convincing argument on exactly how these economic weights should be determined.

16.84 Theil (1967, pp. 136–37) proposed a solution to the lack of weighting in the Jevons index, P_J, defined by equation (16.47). He argued as follows. Suppose we draw price relatives at random in such a way that each dollar of revenue in the base period has an equal chance of being selected. Then the probability that we will draw the ith price relative is equal to $s_i^0 \equiv p_i^0 q_i^0 \Big/ \sum_{k=1}^{n} p_k^0 q_k^0$, the period

[52]Walsh (1901, pp. 82–90; 1921a, pp. 82–83) also objected to the lack of weighting in the unweighted stochastic approach to index number theory.

0 revenue share for product *i*. Then the overall mean (period 0 weighted) logarithmic price change is $\sum_{i=1}^{n} s_i^0 \ln\left(p_i^1/p_i^0\right)$.[53] Now repeat the above mental experiment and draw price relatives at random in such a way that each dollar of revenue in period 1 has an equal probability of being selected. This leads to the overall mean (period 1 weighted) logarithmic price change of $\sum_{i=1}^{n} s_i^1 \ln\left(p_i^1/p_i^0\right)$.[54] Each of these measures of overall logarithmic price change seems equally valid, so we could argue for taking a symmetric average of the two measures in order to obtain a final single measure of overall logarithmic price change. Theil[55] argued that a nice, symmetric index number formula can be obtained if the probability of selection for the *n*th price relative is made equal to the arithmetic average of the period 0 and 1 revenue shares for product *n*. Using these probabilities of selection, Theil's final measure of overall logarithmic price change was

$$(16.48)\quad \ln P_T(p^0,p^1,q^0,q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1)\ln(\frac{p_i^1}{p_i^0}).$$

Note that the index P_T defined by equation (16.48) is equal to the Törnqvist index defined by equation (15.81) in Chapter 15.

16.85 A statistical interpretation of the right-hand side of equation (16.48) can be given. Define the *i*th logarithmic price ratio r_i by:

$$(16.49)\quad r_i \equiv \ln(\frac{p_i^1}{p_i^0}) \quad \text{for } i = 1,\ldots,n.$$

Now define the discrete random variable—we will call it *R*—as the random variable that can take on the values r_i with probabilities $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1,\ldots,n$. Note that since each set of revenue shares, s_i^0 and s_i^1, sums to 1 over *i*, the probabilities ρ_i will also sum to 1. It can be seen that the expected value of the discrete random variable *R* is

$$(16.50)\quad \mathrm{E}[R] \equiv \sum_{i=1}^{n} \rho_i r_i = \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1)\ln(\frac{p_i^1}{p_i^0}) = \ln P_T(p^0,p^1,q^0,q^1).$$

Thus, the logarithm of the index P_T can be interpreted as *the expected value of the distribution of the logarithmic price ratios* in the domain of definition under consideration, where the *n* discrete price ratios in this domain of definition are weighted according to Theil's probability weights, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1,\ldots,n$.

16.86 Taking antilogs of both sides of equation (16.48), the Törnqvist- (1936, 1937) Theil price index, P_T, is obtained.[56] This index number formula has a number of good properties. In particular, P_T satisfies the proportionality in current prices test (T5) and the time reversal test (T11) discussed in Section C. These two tests can be used to justify Theil's (arithmetic) method of forming an average of the two sets of revenue shares in order to obtain his probability weights, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1,\ldots,n$. Consider the following *symmetric mean class of logarithmic index number formulas*:

$$(16.51)\quad \ln P_S(p^0,p^1,q^0,q^1) \equiv \sum_{i=1}^{n} m(s_i^0,s_i^1)\ln(\frac{p_i^1}{p_i^0}),$$

where $m(s_i^0,s_i^1)$ is a positive function of the period 0 and 1 revenue shares on product *i*, s_i^0 and s_i^1, respectively. In order for P_S to satisfy the time reversal test, it is necessary for the function *m* to be

[53]In Chapter 19, this index will be called the *geometric Laspeyres index*, P_{GL}. Vartia (1978, p. 272) referred to this index as the *logarithmic Laspeyres index*. Yet another name for the index is the *base-weighted geometric index*.

[54]In Chapter 19, this index will be called the *geometric Paasche index*, P_{GP}. Vartia (1978, p. 272) referred to this index as the *logarithmic Paasche index*. Yet another name for the index is the *current-period weighted geometric index*.

[55]"The price index number defined in (1.8) and (1.9) uses the *n* individual logarithmic price differences as the basic ingredients. They are combined linearly by means of a two stage random selection procedure: First, we give each region the same chance (½) of being selected, and second, we give each dollar spent in the selected region the same chance ($1/m_a$ or $1/m_b$) of being drawn" (Henri Theil, 1967, p. 138).

[56]The sampling bias problem studied by Greenlees (1999) does not occur in the present context because there is no sampling involved in equation (16.50): the sum of the $p_i^t q_i^t$ over *i* for each period *t* is assumed to equal the value aggregate V^t for period *t*.

symmetric. Then it can be shown[57] that for P_S to satisfy test T5, m must be the arithmetic mean. This provides a reasonably strong justification for Theil's choice of the mean function.

16.87 The stochastic approach of Theil has another advantageous symmetry property. Instead of considering the distribution of the price ratios $r_i = \ln(p_i^1/p_i^0)$, we could also consider the distribution of the *reciprocals* of these price ratios, say,

$$(16.52) \quad t_i \equiv \ln\frac{p_i^0}{p_i^1} = \ln\left(\frac{p_i^1}{p_i^0}\right)^{-1} = -\ln\frac{p_i^1}{p_i^0} = -r_i \quad \text{for } i = 1,\ldots,n.$$

The symmetric probability, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$, can still be associated with the ith reciprocal logarithmic price ratio t_i for $i = 1,\ldots,n$. Now define the discrete random variable, t, say, as the random variable that can take on the values t_i with probabilities $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1,\ldots,n$. It can be seen that the expected value of the discrete random variable t is

$$(16.53) \quad \mathrm{E}[T] \equiv \sum_{i=1}^{n}\rho_i t_i$$
$$= -\sum_{i=1}^{n} r_i t_i \quad \text{using equation (16.52)}$$
$$= -\mathrm{E}[R] \quad \text{using equation (16.50)}$$
$$= -\ln P_T(p^0, p^1, q^0, q^1).$$

Thus, it can be seen that the distribution of the random variable t is equal to minus the distribution of the random variable R. Hence, it does not matter whether the distribution of the original logarithmic price ratios, $r_i \equiv \ln(p_i^1/p_i^0)$, is considered or the distribution of their reciprocals, $t_i \equiv \ln(p_i^1/p_i^0)$, is considered: essentially the same stochastic theory is obtained.

16.88 It is possible to consider weighted stochastic approaches to index number theory where the distribution of the price ratios, p_i^1/p_i^0, is considered rather than the distribution of the logarithmic price ratios, $\ln(p_i^1/p_i^0)$. Thus, again following in the footsteps of Theil, suppose that price relatives are drawn at random in such a way that each dollar of revenue in the *base period* has an equal chance of being selected. Then the probability that the ith price relative will be drawn is equal to s_i^0, the period 0 revenue share for product i. Thus, the overall mean (period 0 weighted) price change is

$$(16.54) \quad P_L(p^0, p^1, q^0, q^1) = \sum_{i=1}^{n} s_i^0 \frac{p_i^1}{p_i^0},$$

which turns out to be the Laspeyres price index, P_L. This stochastic approach is the natural one for studying *sampling problems* associated with implementing a Laspeyres price index.

16.89 Take the same hypothetical situation and draw price relatives at random in such a way that each dollar of revenue in period 1 has an equal probability of being selected. This leads to the overall mean (period 1 weighted) price change equal to

$$(16.55) \quad P_{Pal}(p^0, p^1, q^0, q^1) = \sum_{i=1}^{n} s_i^1 \frac{p_i^1}{p_i^0}.$$

This is known as the Palgrave (1886) index number formula.[58]

16.90 It can be verified that neither the Laspeyres nor the Palgrave price indices satisfy the time reversal test, T11. Thus, again following in the footsteps of Theil, it might be attempted to obtain a formula that satisfied the time reversal test by taking a symmetric average of the two sets of shares. Thus, consider the following class of *symmetric mean index number formulas*:

$$(16.56) \quad P_m(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} m(s_i^0, s_i^1)\frac{p_i^1}{p_i^0},$$

where $m(s_i^0, s_i^1)$ is a symmetric function of the period 0 and 1 revenue shares for product i, s_i^0 and s_i^1, respectively. In order to interpret the right-hand side of equation (16.56) as an expected value of the price ratios p_i^1/p_i^0, it is necessary that

[57] See Diewert (2000) and Balk and Diewert (2001).

[58] It is formula number 9 in Fisher's (1922, p. 466) listing of index number formulas.

(16.57) $$\sum_{i=1}^{n} m(s_i^0, s_i^1) = 1.$$

However, in order to satisfy equation (16.57), m must be the arithmetic mean.[59] With this choice of m, equation (16.56) becomes the following (unnamed) index number formula, P_u:

(16.58) $$P_u(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1)\frac{p_i^1}{p_i^0}.$$

Unfortunately, the unnamed index P_u does not satisfy the time reversal test either.[60]

16.91 Instead of considering the distribution of the price ratios, p_i^1/p_i^0, the distribution of the *reciprocals* of these price ratios could be considered. The counterparts to the asymmetric indices defined earlier by equations (16.54) and (16.55) are now $\sum_{i=1}^{n} s_i^0\left(p_i^0/p_i^1\right)$ and $\sum_{i=1}^{n} s_i^1\left(p_i^0/p_i^1\right)$, respectively. These are (stochastic) price indices going *backward* from period 1 to 0. In order to make these indices comparable with other previous forward-looking indices, take the reciprocals of these indices (which lead to harmonic averages) and the following two indices are obtained:

(16.59) $$P_{HL}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} s_i^0 \frac{p_i^0}{p_i^1}},$$

(16.60) $$P_{HP}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} s_i^1 \frac{p_i^0}{p_i^1}}$$

$$= \frac{1}{\sum_{i=1}^{n} s_i^1 \left(\frac{p_i^1}{p_i^0}\right)^{-1}} = P_P(p^0, p^1, q^0, q^1),$$

using equation (15.9) in Chapter 15. Thus, the reciprocal stochastic price index defined by equation (16.60) turns out to equal the fixed-basket Paasche price index, P_P. This stochastic approach is the natural one for studying *sampling problems* associated with implementing a Paasche price index. The other asymmetrically weighted reciprocal stochastic price index defined by equation (16.59) has no author's name associated with it, but it was noted by Irving Fisher (1922, p. 467) as his index number formula 13. Vartia (1978, p. 272) called this index the *harmonic Laspeyres index*, and his terminology will be used.

16.92 Now consider the class of symmetrically weighted reciprocal price indices defined as

(16.61) $$P_{mr}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} m(s_i^0, s_i^1)\left(\frac{p_i^1}{p_i^0}\right)^{-1}},$$

where, as usual, $m(s_i^0, s_i^1)$ is a homogeneous symmetric mean of the period 0 and 1 revenue shares on product i. However, none of the indices defined by equations (16.59)–(16.61) satisfy the time reversal test.

16.93 The fact that Theil's index number formula P_T satisfies the time reversal test leads to a preference for Theil's index as the best weighted stochastic approach.

16.94 The main features of the weighted stochastic approach to index number theory can be summarized as follows. It is first necessary to pick two periods and a transaction's domain of definition. As usual, each value transaction for each of the n commodities in the domain of definition is split up into price and quantity components. Then, assuming there are no new commodities or no disappearing commodities, there are n price relatives p_i^1/p_i^0 pertaining to the two situations under consideration along with the corresponding $2n$ revenue shares. The weighted stochastic approach just assumes that these n relative prices, or some transformation of these price relatives, $f(p_i^1/p_i^0)$, have a discrete statistical distribution, where the ith probability, $\rho_i = m(s_i^0, s_i^1)$, is a function of the revenue shares pertaining to product i in the two situations under consideration, s_i^0 and s_i^1. Different price indices result, depending on how one chooses the

[59]For a proof of this assertion, see Balk and Diewert (2001).

[60]In fact, this index suffers from the same upward bias as the Carli index in that $P_u(p^0,p^1,q^0,q^1)P_u(p^1,p^0,q^1,q^0) \geq 1$. To prove this, note that the previous inequality is equivalent to $[P_u(p^1,p^0,q^1,q^0)]^{-1} \leq P_u(p^0,p^1,q^0,q^1)$, and this inequality follows from the fact that a weighted harmonic mean of n positive numbers is equal to or less than the corresponding weighted arithmetic mean; see Hardy, Littlewood, and Pólya (1934, p. 26).

functions f and m. In Theil's approach, the transformation function f was the natural logarithm, and the mean function m was the simple unweighted arithmetic mean.

16.95 There is a third aspect to the weighted stochastic approach to index number theory: one must decided what *single number* best summarizes the distribution of the n (possibly transformed) price relatives. In the analysis above, the *mean* of the discrete distribution was chosen as the best summary measure for the distribution of the (possibly transformed) price relatives, but other measures are possible. In particular, the *weighted median* or various *trimmed means* are often suggested as the best measure of central tendency because these measures minimize the influence of outliers. However, a detailed discussion of these alternative measures of central tendency is beyond the scope of this chapter. Additional material on stochastic approaches to index number theory and references to the literature can be found in Clements and Izan (1981, 1987), Selvanathan and Rao (1994), Diewert (1995b), Cecchetti (1997), and Wynne (1997, 1999).

16.96 Instead of taking the above stochastic approach to index number theory, it is possible to take the same raw data that are used in this approach but use them with an axiomatic approach. Thus, in the following section, the price index is regarded as a value-weighted function of the n price relatives, and the test approach to index number theory is used in order to determine the functional form for the price index. Put another way, the axiomatic approach in the next section looks at the *properties* of alternative descriptive statistics that aggregate the individual price relatives (weighted by their economic importance) into summary measures of price change in an attempt to find the best summary measure of price change. Thus, the axiomatic approach pursued in Section E can be viewed as a branch of the theory of descriptive statistics.

E. Second Axiomatic Approach to Bilateral Price Indices

E.1 Basic framework and some preliminary tests

16.97 As was mentioned in Section A, one of Walsh's approaches to index number theory was an attempt to determine the best weighted average of the price relatives, r_i.[61] This is equivalent to using an axiomatic approach to try to determine the best index of the form $P(r,v^0,v^1)$, where v^0 and v^1 are the vectors of revenues on the n commodities during periods 0 and 1.[62] However, rather than starting off with indices of the form $P(r,v^0,v^1)$, indices of the form $P(p^0,p^1,v^0,v^1)$ will be considered, since this framework will be more comparable to the first bilateral axiomatic framework taken in Section C. If the invariance to changes in the units of measurement test is imposed on an index of the form $P(p^0,p^1,v^0,v^1)$, then $P(p^0,p^1,v^0,v^1)$ can be written in the form $P(r,v^0,v^1)$.

16.98 Recall that the product test, equation (16.17), was used in order to define the quantity index, $Q(p^0,p^1,q^0,q^1) \equiv V^1/[V^0P(p^0,p^1,q^0,q^1)]$, that corresponded to the bilateral price index $P(p^0,p^1,q^0,q^1)$. A similar product test holds in the present framework; that is, given that the functional form for the price index $P(p^0,p^1,v^0,v^1)$ has been determined, then the corresponding *implicit quantity index* can be defined in terms of p as follows:

[61]Fisher also took this point of view when describing his approach to index number theory: "An index number of the prices of a number of commodities is an average of their price relatives. This definition has, for concreteness, been expressed in terms of prices. But in like manner, an index number can be calculated for wages, for quantities of goods imported or exported, and, in fact, for any subject matter involving divergent changes of a group of magnitudes. Again, this definition has been expressed in terms of time. But an index number can be applied with equal propriety to comparisons between two places or, in fact, to comparisons between the magnitudes of a group of elements under any one set of circumstances and their magnitudes under another set of circumstances" (Irving Fisher, 1922, p. 3). However, in setting up his axiomatic approach, Fisher imposed axioms on the price and quantity indices written as functions of the two price vectors, p^0 and p^1, and the two quantity vectors, q^0 and q^1; that is, he did not write his price index in the form $P(r,v^0,v^1)$ and impose axioms on indices of this type. Of course, in the end, his ideal price index turned out to be the geometric mean of the Laspeyres and Paasche price indices, and, as was seen in Chapter 15, each of these indices can be written as revenue share-weighted averages of the n price relatives, $r_i \equiv p_i^1/p_i^0$.

[62]Chapter 3 in Vartia (1976a) considered a variant of this axiomatic approach.

$$(16.62)\ Q(p^0,p^1,v^0,v^1) \equiv \frac{\sum_{i=1}^{n} v_i^1}{\left(\sum_{i=1}^{n} v_i^0\right) P(p^0,p^1,v^0,v^1)}.$$

16.99 In Section C, the price and quantity indices $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$ were determined *jointly*; that is, not only were axioms imposed on $P(p^0,p^1,q^0,q^1)$, but they were also imposed on $Q(p^0,p^1,q^0,q^1)$, and the product test in equation (16.17) was used to translate these tests on q into tests on P. In Section E, this approach will not be followed: only tests on $P(p^0,p^1,v^0,v^1)$ will be used in order to determine the best price index of this form. Thus, there is a parallel theory for quantity indices of the form $Q(q^0,q^1,v^0,v^1)$ where it is attempted to find the best value-weighted average of the quantity relatives, q_i^1/q_i^0.[63]

16.100 For the most part, the tests that will be imposed on the price index $P(p^0,p^1,v^0,v^1)$ in this section are counterparts to the tests that were imposed on the price index $P(p^0,p^1,v^0,v^1)$ in Section C. It will be assumed that every component of each price and value vector is positive; that is, $p^t >> 0_n$ and $v^t >> 0_n$ for $t = 0,1$. If it is desired to set $v^0 = v^1$, the common revenue vector is denoted by v; if it is desired to set $p^0 = p^1$, the common price vector is denoted by p.

16.101 The first two tests are straightforward counterparts to the corresponding tests in Section C.

T1—*Positivity*: $P(p^0,p^1,v^0,v^1) > 0$.

T2—*Continuity*: $P(p^0,p^1,v^0,v^1)$ is a continuous function of its arguments.

T3—*Identity or Constant Prices Test*:
$$P(p,p,v^0,v^1) = 1.$$

That is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the value vectors are. Note that the two value vectors are allowed to be different in the above test.

E.2 Homogeneity tests

16.102 The following four tests restrict the behavior of the price index p as the scale of any one of the four vectors p^0,p^1,v^0,v^1 changes.

T4—*Proportionality in Current Prices*:
$P(p^0,\lambda p^1,v^0,v^1) = \lambda P(p^0,p^1,v^0,v^1)$ for $\lambda > 0$.

That is, if all period 1 prices are multiplied by the positive number λ, then the new price index is λ times the old price index. Put another way, the price index function $P(p^0,p^1,v^0,v^1)$ is (positively) homogeneous of degree 1 in the components of the period 1 price vector p^1. This test is the counterpart to test T5 in Section C.

16.103 In the next test, instead of multiplying all period 1 prices by the same number, all period 0 prices are multiplied by the number λ.

T5—*Inverse Proportionality in Base-Period Prices*:
$P(\lambda p^0,p^1,v^0,v^1) = \lambda^{-1} P(p^0,p^1,v^0,v^1)$ for $\lambda > 0$.

That is, if all period 0 prices are multiplied by the positive number λ, then the new price index is $1/\lambda$ times the old price index. Put another way, the price index function $P(p^0,p^1,v^0,v^1)$ is (positively) homogeneous of degree minus 1 in the components of the period 0 price vector p^0. This test is the counterpart to test T6 in Section C.

16.104 The following two homogeneity tests can also be regarded as invariance tests.

T6—*Invariance to Proportional Changes in Current-Period Values*:
$P(p^0,p^1,v^0,\lambda v^1) = P(p^0,p^1,v^0,v^1)$ for all $\lambda > 0$.

That is, if current-period values are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0,p^1,v^0,v^1)$ is (positively) homogeneous of degree 0 in the components of the period 1 value vector v^1.

[63]It turns out that the price index that corresponds to this best quantity index, defined as $P^*(p^0,p^1,v^0,v^1) \equiv \sum_{i=1}^{n} \ln v_i^1 \Big/ \left[\sum_{i=1}^{n} \ln v_i^0 Q\left(q^0,q^1,v^0,v^i\right)\right]$, will not equal the best price index, $P(p^0,p^1,v^0,v^1)$. Thus, the axiomatic approach in Section E generates separate best price and quantity indices whose product does not equal the value ratio in general. This is a disadvantage of the second axiomatic approach to bilateral indices compared with the first approach studied in Section C.

T7—*Invariance to Proportional Changes in Base-Period Values*:
$P(p^0,p^1,\lambda v^0,v^1) = P(p^0,p^1,v^0,v^1)$ for all $\lambda > 0$.

That is, if base-period values are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0,p^1,v^0,v^1)$ is (positively) homogeneous of degree 0 in the components of the period 0 value vector v^0.

16.105 T6 and T7 together impose the property that the price index p does not depend on the *absolute* magnitudes of the value vectors v^0 and v^1. Using test T6 with $\lambda = 1\Big/\sum_{i=1}^{n} v_i^1$, and using test T7 with $\lambda = 1\Big/\sum_{i=1}^{n} v_i^0$, it can be seen that p has the following property:

$$(16.63)\quad P(p^0,p^1,v^0,v^1) = P(p^0,p^1,s^0,s^1),$$

where s^0 and s^1 are the vectors of revenue shares for periods 0 and 1; that is, the ith component of s^t is $s_i^t \equiv v_i^t\Big/\sum_{k=1}^{n} v_k^t$ for $t = 0,1$. Thus, the tests T6 and T7 imply that the price index function p is a function of the two price vectors p^0 and p^1 and the two vectors of revenue shares, s^0 and s^1.

16.106 Walsh suggested the spirit of tests T6 and T7 as the following quotation indicates:

> What we are seeking is to average the variations in the exchange value of one given total sum of money in relation to the several classes of goods, to which several variations [i.e., the price relatives] must be assigned weights proportional to the relative sizes of the classes. Hence the relative sizes of the classes at both the periods must be considered. (Correa Moylan Walsh, 1901, p. 104)

16.107 Walsh also realized that weighting the ith price relative r_i by the arithmetic mean of the value weights in the two periods under consideration, $(1/2)[v_i^0 + v_i^1]$, would give too much weight to the revenues of the period that had the highest level of prices:

> At first sight it might be thought sufficient to add up the weights of every class at the two periods and to divide by two. This would give the (arithmetic) mean size of every class over the two periods together. But such an operation is manifestly wrong. In the first place, the sizes of the classes at each period are reckoned in the money of the period, and if it happens that the exchange value of money has fallen, or prices in general have risen, greater influence upon the result would be given to the weighting of the second period; or if prices in general have fallen, greater influence would be given to the weighting of the first period. Or in a comparison between two countries, greater influence would be given to the weighting of the country with the higher level of prices. But it is plain that *the one period, or the one country, is as important, in our comparison between them, as the other, and the weighting in the averaging of their weights should really be even*. (Correa Moylan Walsh, 1901, pp. 104–05)

16.108 As a solution to the above weighting problem, Walsh (1901, p. 202; 1921a, p. 97) proposed the following *geometric price index*:

$$(16.64)\quad P_{GW}(p^0,p^1,v^0,v^1) \equiv \prod_{i=1}^{n}\left(\frac{p_i^1}{p_i^0}\right)^{w(i)},$$

where the ith weight in the above formula was defined as

$$(16.65)\quad w(i) \equiv \frac{(v_i^0 v_i^1)^{1/2}}{\sum_{k=1}^{n}(v_k^0 v_k^1)^{1/2}} = \frac{(s_i^0 s_i^1)^{1/2}}{\sum_{k=1}^{n}(s_k^0 s_k^1)^{1/2}},\quad i = 1,\ldots,n.$$

The second part of equation (16.65) shows that Walsh's geometric price index $P_{GW}(p^0,p^1,v^0,v^1)$ can also be written as a function of the revenue share vectors, s^0 and s^1; that is, $P_{GW}(p^0,p^1,v^0,v^1)$ is homogeneous of degree 0 in the components of the value vectors v^0 and v^1, and so $P_{GW}(p^0,p^1,v^0,v^1) = P_{GW}(p^0,p^1,s^0,s^1)$. Thus, Walsh came very close to deriving the Törnqvist-Theil index defined earlier by equation (16.48).[64]

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[64]One could derive Walsh's index using the same arguments as Theil except that the geometric average of the revenue shares $(s_i^0 s_i^1)^{1/2}$ could be taken as a preliminary (continued)

E.3 Invariance and symmetry tests

16.109 The next five tests are *invariance* or *symmetry tests*, and four of them are direct counterparts to similar tests in Section C. The first invariance test is that the price index should remain unchanged if the *ordering* of the commodities is changed.

T8—*Commodity Reversal Test* (or invariance to changes in the ordering of commodities):
$P(p^0*,p^1*,v^0*,v^1*) = P(p^0,p^1,v^0,v^1)$,

where p^t* denotes a permutation of the components of the vector p^t and v^t* denotes the same permutation of the components of v^t for $t = 0,1$.

16.110 The next test asks that the index be invariant to changes in the units of measurement.

T9—*Invariance to Changes in the Units of Measurement* (commensurability test):
$P(\alpha_1 p_1^0,...,\alpha_n p_n^0; \alpha_1 p_1^1,...,\alpha_n p_n^1; v_1^0,...,v_n^0; v_1^1,...,v_n^1)$
$= P(p_1^0,...,p_n^0; p_1^1,...,p_n^1; v_1^0,...,v_n^0; v_1^1,...,v_n^1)$
for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

That is, the price index does not change if the units of measurement for each product are changed. Note that the revenue on product i during period t, v_i^t, does not change if the unit by which product i is measured changes.

16.111 Test T9 has a very important implication. Let $\alpha_1 = 1/p_1^0, \ldots, \alpha_n = 1/p_n^0$ and substitute these values for the α_i into the definition of the test. The following equation is obtained:

$$(16.66)\quad P(p^0,p^1,v^0,v^1) = P(1_n,r,v^0,v^1) \equiv P^*(r,v^0,v^1),$$

where 1_n is a vector of ones of dimension n, and r is a vector of the price relatives; that is, the ith component of r is $r_i \equiv p_i^1/p_i^0$. Thus, if the commensurability test T9 is satisfied, then the price index $P(p^0,p^1,v^0,v^1)$, which is a function of $4n$ variables, can be written as a function of $3n$ variables, $P^*(r, v^0,v^1)$, where r is the vector of price relatives and $P^*(r, v^0,v^1)$ is defined as $P(1_n,r,v^0,v^1)$.

16.112 The next test asks that the formula be invariant to the period chosen as the base period.

T10—*Time Reversal Test*: $P(p^0,p^1,v^0,v^1) = 1/P(p^1,p^0,v^1,v^0)$.

That is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index. Obviously, in the one good case when the price index is simply the single-price ratio, this test will be satisfied (as are all of the other tests listed in this section).

16.113 The next test is a variant of the circularity test that was introduced in Section F of Chapter 15.[65]

T11—*Transitivity in Prices for Fixed-Value Weights*:
$P(p^0,p^1,v^r,v^s)P(p^1,p^2,v^r,v^s) = P(p^0,p^2,v^r,v^s)$.

In this test, the revenue-weighting vectors, v^r and v^s, are held constant while making all price comparisons. However, given that these weights are held constant, then the test asks that the product of the index going from period 0 to 1, $P(p^0,p^1,v^r,v^s)$, times the index going from period 1 to 2, $P(p^1,p^2,v^r,v^s)$, should equal the direct index that compares the prices of period 2 with those of period 0, $P(p^0,p^2,v^r,v^s)$. Clearly, this test is a many-product counterpart to a property that holds for a single price relative.

16.114 The next test in this section captures the idea that the value weights should enter the index number formula in a symmetric manner.

T12—*Quantity Weights Symmetry Test*:
$P(p^0,p^1,v^0,v^1) = P(p^0,p^1,v^1,v^0)$.

probability weight for the ith logarithmic price relative, $\ln r_i$. These preliminary weights are then normalized to add up to unity by dividing by their sum. It is evident that Walsh's geometric price index will closely approximate Theil's index using normal time-series data. More formally, regarding both indices as functions of p^0,p^1,v^0,v^1, it can be shown that $P_W(p^0,p^1,v^0,v^1)$ approximates $P_T(p^0,p^1,v^0,v^1)$ to the second order around an equal price (that is, $p^0 = p^1$) and quantity (that is, $q^0 = q^1$) point.

[65]See equation (15.77) in Chapter 15.

That is, if the revenue vectors for the two periods are interchanged, then the price index remains invariant. This property means that if values are used to weight the prices in the index number formula, then the period 0 values v^0 and the period 1 values v^1 must enter the formula in a symmetric or even-handed manner.

E.4 Mean value test

16.115 The next test is a *mean value test*.

T13—*Mean Value Test for Prices*:

$$(16.67)\ \min_i (p_i^1/p_i^0 : i = 1,...,n) \le P(p^0,p^1,v^0,v^1) \le \max_i (p_i^1/p_i^0 : i = 1,...,n).$$

That is, the price index lies between the minimum price ratio and the maximum price ratio. Since the price index is to be interpreted as an average of the n price ratios, p_i^1/p_i^0, it seems essential that the price index p satisfy this test.

E.5 Monotonicity tests

16.116 The next two tests in this section are *monotonicity tests*; that is, how should the price index $P(p^0,p^1,v^0,v^1)$ change as any component of the two price vectors p^0 and p^1 increases?

T14—*Monotonicity in Current Prices*:

$$P(p^0,p^1,v^0,v^1) < P(p^0,p^2,v^0,v^1) \text{ if } p^1 < p^2.$$

That is, if some period 1 price increases, then the price index must increase (holding the value vectors fixed), so that $P(p^0,p^1,v^0,v^1)$ is increasing in the components of p^1 for fixed p^0, v^0, and v^1.

T15—*Monotonicity in Base Prices*:

$$P(p^0,p^1,v^0,v^1) > P(p^2,p^1,v^0,v^1) \text{ if } p^0 < p^2.$$

That is, if any period 0 price increases, then the price index must decrease, so that $P(p^0,p^1,v^0,v^1)$ is decreasing in the components of p^0 for fixed p^1, v^0 and v^1.

E.6 Weighting tests

16.117 The preceding tests are not sufficient to determine the functional form of the price index; for example, it can be shown that both Walsh's geometric price index $P_{GW}(p^0,p^1,v^0,v^1)$ defined by equation (16.65) and the Törnqvist-Theil index $P_T(p^0,p^1,v^0,v^1)$ defined by equation (16.48) satisfy all of the above axioms. At least one more test, therefore, will be required in order to determine the functional form for the price index $P(p^0,p^1,v^0,v^1)$.

16.118 The tests proposed thus far do not specify exactly how the revenue share vectors s^0 and s^1 are to be used in order to weight, for example, the first price relative, p_1^1/p_1^0. The next test says that only the revenue shares s_1^0 and s_1^1 pertaining to the first product are to be used in order to weight the prices that correspond to product 1, p_1^1 and p_1^0.

T16—*Own-Share Price Weighting*:

$$(16.68)\ P\left(p_1^0,1,...,1\,;\,p_1^1,1,...,1\,;\,v^0,v^1\right) = f\left(p_1^0,p_1^1,\left[v_1^0\Big/\sum_{k=1}^n v_k^0\right],\left[v_1^1\Big/\sum_{k=1}^n v_k^1\right]\right).$$

Note that $v_1^t\Big/\sum_{k=1}^n v_k^t$ equals s_1^t, the revenue share for product 1 in period t. This test says that if all of the prices are set equal to 1 except the prices for product 1 in the two periods, but the revenues in the two periods are arbitrarily given, then the index depends only on the two prices for product 1 and the two revenue shares for product 1. The axiom says that a function of 2 + 2n variables is actually only a function of four variables.[66]

16.119 If test T16 is combined with test T8, the commodity reversal test, then it can be seen that p has the following property:

$$(16.69)\ P(1,...,1,p_i^0,1,...,1\,;\,1,...,1,p_i^1,1,...,1\,;\,v^0\,;\,v^1) = f\left(p_1^0,p_1^1,\left[v_1^0\Big/\sum_{k=1}^n v_k^0\right],\left[v_1^1\Big/\sum_{k=1}^n v_k^1\right]\right), i = 1,...,n.$$

Equation (16.69) says that if all of the prices are set equal to 1 except the prices for product i in the two periods, but the revenues in the two periods are arbitrarily given, then the index depends only on the two prices for product i and the two revenue shares for product i.

[66]In the economics literature, axioms of this type are known as separability axioms.

16.120 The final test that also involves the weighting of prices is the following:

T17—*Irrelevance of Price Change with Tiny Value Weights*:

$$(16.70)\quad P(p_1^0,1,...,1\,;\,p_1^1,1,...,1\,;\,0,v_2^0,...,v_n^0\,;\,0,v_2^1,...,v_n^1)=1.$$

The test T17 says that if all of the prices are set equal to 1 except the prices for product 1 in the two periods, and the revenues on product 1 are 0 in the two periods but the revenues on the other commodities are arbitrarily given, then the index is equal to 1.[67] Roughly speaking, if the value weights for product 1 are tiny, then it does not matter what the price of product 1 is during the two periods.

16.121 If test T17 is combined with test T8, the product reversal test, then it can be seen that p has the following property: for $i = 1,\ldots,n$:

$$(16.71)\quad P(1,...,1,p_i^0,1,...,1\,;\,1,...,1,p_i^1,1,...,1\,;\,v_1^0,...,0,...,v_n^0\,;\,v_1^1,...,0,...,v_n^1)=1\,.$$

Equation (16.71) says that if all of the prices are set equal to 1 except the prices for product i in the two periods, and the revenues on product i are 0 during the two periods but the other revenues in the two periods are arbitrarily given, then the index is equal to 1.

16.122 This completes the listing of tests for the weighted average of price relatives approach to bilateral index number theory. It turns out that these tests are sufficient to imply a specific functional form for the price index as will be seen in the next section.

[67]Strictly speaking, since all prices and values are required to be positive, the left-hand side of equation (16.70) should be replaced by the limit as the product 1 values, v_1^0 and v_1^1, approach 0.

E.7 Törnqvist-Theil price index and second test approach to bilateral indices

16.123 In Appendix 16.1, it is shown that if the number of commodities n exceeds two and the bilateral price index function $P(p^0,p^1,v^0,v^1)$ satisfies the 17 axioms listed above, then p must be the Törnqvist-Theil price index $P_T(p^0,p^1,v^0,v^1)$ defined by equation (16.48).[68] Thus, the 17 properties or tests listed in Section E provide an axiomatic characterization of the Törnqvist-Theil price index, just as the 20 tests listed in Section C provided an axiomatic characterization of the Fisher ideal price index.

16.124 There is a parallel axiomatic theory for quantity indices of the form $Q(p^0,p^1,v^0,v^1)$ that depend on the two quantity vectors for periods 0 and 1, q^0 and q^1, as well as on the corresponding two revenue vectors, v^0 and v^1. Thus, if $Q(p^0,p^1,v^0,v^1)$ satisfies the quantity counterparts to tests T1–T17, then q must be equal to the Törnqvist-Theil quantity index $Q_T(q^0,q^1,v^0,v^1)$, defined as follows:

$$(16.72)\quad \ln Q_T(q^0,q^1,v^0,v^1)\equiv\sum_{i=1}^{n}\frac{1}{2}(s_i^0+s_i^1)\ln\left(\frac{q_i^1}{q_i^0}\right),$$

where, as usual, the period t revenue share on product i, s_i^t, is defined as $v_1^t \Big/ \sum_{k=1}^{n} v_k^t$ for $i = 1,\ldots,n$ and $t = 0,1$.

16.125 Unfortunately, the implicit Törnqvist-Theil price index $P_{IT}(q^0,q^1,v^0,v^1)$, which corresponds to the Törnqvist-Theil quantity index Q_T defined by equation (16.72) using the product test, is *not* equal to the direct Törnqvist-Theil price index $P_T(p^0,p^1,v^0,v^1)$ defined by equation (16.48). The product test equation that defines P_{IT} in the present context is given by the following equation:

[68]The Törnqvist-Theil price index satisfies all 17 tests, but the proof in Appendix 16.1 did not use all of these tests to establish the result in the opposite direction: tests T5, T13, T15, and either T10 or T12 were not required in order to show that an index satisfying the remaining tests must be the Törnqvist-Theil price index. For alternative characterizations of the Törnqvist-Theil price index, see Balk and Diewert (2001) and Hillinger (2002).

(16.73) $P_{IT}(q^0,q^1,v^0,v^1)$

$$\equiv \frac{\sum_{i=1}^{n} v_i^1}{\left(\sum_{i=1}^{n} v_i^0\right) Q_T(q^0,q^1,v^0,v^1)}.$$

The fact that the direct Törnqvist-Theil price index P_T is not in general equal to the implicit Törnqvist-Theil price index P_{IT} defined by equation (16.73) is a bit of a disadvantage compared with the axiomatic approach outlined in Section C, which led to the Fisher ideal price and quantity indices as being best. Using the Fisher approach meant that it was not necessary to decide whether one wanted a best price index or a best quantity index: the theory outlined in Section C determined both indices simultaneously. However, in the Törnqvist-Theil approach outlined in this section, it is necessary to *choose* whether one wants a best price index or a best quantity index.[69]

16.126 Other tests are, of course, possible. A counterpart to test T16 in Section C, the Paasche and Laspeyres bounding test, is the following *geometric Paasche and Laspeyres bounding test*:

(16.74) $P_{GL}(p^0,p^1,v^0,v^1)$
$\le P(p^0,p^1,v^0,v^1) \le P_{GP}(p^0,p^1,v^0,v^1)$ or
$P_{GP}(p^0,p^1,v^0,v^1)$
$\le P(p^0,p^1,v^0,v^1) \le P_{GL}(p^0,p^1,v^0,v^1)$,

where the logarithms of the geometric Laspeyres and geometric Paasche price indices, P_{GL} and P_{GP}, are defined as follows:

(16.75) $\ln P_{GL}(p^0,p^1,v^0,v^1) \equiv \sum_{i=1}^{n} s_i^0 \ln\left(\frac{p_i^1}{p_i^0}\right)$,

(16.76) $\ln P_{GP}(p^0,p^1,v^0,v^1) \equiv \sum_{i=1}^{n} s_i^1 \ln\left(\frac{p_i^1}{p_i^0}\right)$.

As usual, the period t revenue share on product *i*, s_i^t, is defined as $v_1^t \Big/ \sum_{k=1}^{n} v_k^t$ for $i = 1,\ldots,n$ and $t = 0,1$. It can be shown that the Törnqvist-Theil price index $P_T(p^0,p^1,v^0,v^1)$ defined by equation (16.48) satisfies this test, but the geometric Walsh price index $P_{GW}(p^0,p^1,v^0,v^1)$ defined by equation (16.65) does not satisfy it. The geometric Paasche and Laspeyres bounding test was not included as a primary test in Section E because, a priori, it was not known what form of averaging of the price relatives (for example, geometric, arithmetic, or harmonic) would turn out to be appropriate in this test framework. The test equation (16.74) is an appropriate one if it has been decided that geometric averaging of the price relatives is the appropriate framework. The geometric Paasche and Laspeyres indices correspond to extreme forms of value weighting in the context of geometric averaging, and it is natural to require that the best price index lie between these extreme indices.

16.127 Walsh (1901, p. 408) pointed out a problem with his geometric price index P_{GW} defined by equation (16.65), which also applies to the Törnqvist-Theil price index $P_T(p^0,p^1,v^0,v^1)$ defined by equation (16.48): these geometric-type indices do not give the right answer when the quantity vectors are constant (or proportional) over the two periods. In this case, Walsh thought that the right answer must be the Lowe index, which is the ratio of the costs of purchasing the constant basket during the two periods. Put another way, the geometric indices P_{GW} and P_T do not satisfy T4, the fixed-basket test, in Section C above. What, then, was the argument that led Walsh to define his geometric average type index P_{GW}? It turns out that he was led to this type of index by considering another test, which will now be explained.

16.128 Walsh (1901, pp. 228–31) derived his test by considering the following simple framework. Let there be only two commodities in the index, and suppose that the revenue share on each product is equal in each of the two periods under consideration. The price index under these conditions is equal to $P(p_1^0,p_2^0;p_1^1,p_2^1;v_1^0,v_2^0;v_1^1,v_2^1) = P^*(r_1,r_2;1/2,1/2;1/2,1/2) \equiv m(r_1,r_2)$, where $m(r_1,r_2)$ is a symmetric mean of the two price relatives,

[69]Hillinger (2002) suggested taking the geometric mean of the direct and implicit Törnqvist-Theil price indices in order to resolve this conflict. Unfortunately, the resulting index is not best for either set of axioms that were suggested in this section.

$r_1 \equiv p_1^1/p_1^0$ and $r_2 \equiv p_2^1/p_2^0$.[70] In this framework, Walsh then proposed the following *price-relative reciprocal test*:

$$(16.77)\quad m(r_1, r_1^{-1}) = 1.$$

Thus, if the value weighting for the two commodities is equal over the two periods, and the second price relative is the reciprocal of the first price relative I_1, then Walsh (1901, p. 230) argued that the overall price index under these circumstances ought to equal 1, since the relative fall in one price is exactly counterbalanced by a rise in the other, and both commodities have the same revenues in each period. He found that the geometric mean satisfied this test perfectly, but the arithmetic mean led to index values greater than 1 (provided that r_1 was not equal to 1), and the harmonic mean led to index values that were less than 1, a situation that was not at all satisfactory.[71] Thus, he was led to some form of geometric averaging of the price relatives in one of his approaches to index number theory.

16.129 A generalization of Walsh's result is easy to obtain. Suppose that the mean function, $m(r_1,r_2)$, satisfies Walsh's reciprocal test, equation (16.77), and, in addition, m is a homogeneous mean, so that it satisfies the following property for all $r_1 > 0$, $r_2 > 0$, and $\lambda > 0$:

$$(16.78)\quad m(\lambda r_1, \lambda r_2) = \lambda m(r_1, r_2).$$

Let $r_1 > 0$, $r_2 > 0$. Then

$$(16.79)\quad m(r_1, r_2) = \left(\frac{r_1}{r_1}\right) m(r_1, r_2)$$

$$= r_1 m\left(\frac{r_1}{r_1}, \frac{r_2}{r_1}\right), \text{ using equation (16.78) with } \lambda = \frac{1}{r_1}$$

$$= r_1 m\left(1, \frac{r_2}{r_1}\right) = r_1 f\left(\frac{r_2}{r_1}\right),$$

where the function of one (positive) variable $f(z)$ is defined as

$$(16.80)\quad f(z) \equiv m(1, z).$$

Using equation (16.77):

$$(16.81)\quad 1 = m(r_1, r_1^{-1}) = \left(\frac{r_1}{r_1}\right) m(r_1, r_1^{-1}) = r_1 m(1, r_1^{-2}),$$

using equation (16.78) with $\lambda = \dfrac{1}{r_1}$.

Using equation (16.80), equation (16.81) can be rearranged in the following form:

$$(16.82)\quad f(r_1^{-2}) = r_1^{-1}.$$

Letting $z \equiv r_1^{-2}$ so that $z^{1/2} = r_1^{-1}$, equation (16.82) becomes

$$(16.83)\quad f(z) = z^{1/2}.$$

Now substitute equation (16.83) into equation (16.79) and the functional form for the mean function $m(r_1,r_2)$ is determined:

$$(16.84)\quad m(r_1, r_2) = r_1 f\left(\frac{r_2}{r_1}\right) = r_1 \left(\frac{r_2}{r_1}\right)^{1/2} = r_1^{1/2} r_2^{1/2}.$$

Thus, the geometric mean of the two price relatives is the only homogeneous mean that will satisfy Walsh's price-relative reciprocal test.

16.130 There is one additional test that should be mentioned. Fisher (1911, p. 401) introduced this test in his first book that dealt with the test approach to index number theory. He called it the *test of determinateness as to prices* and described it as follows:

> A price index should not be rendered zero, infinity, or indeterminate by an individual price becoming zero. Thus, if any product should in 1910 be a glut on the market, becoming a "free good," that fact ought not to render the index number for 1910 zero. (Irving Fisher, 1911, p. 401)

[70] Walsh considered only the cases where m was the arithmetic, geometric, and harmonic means of r_1 and r_2.

[71] "This tendency of the arithmetic and harmonic solutions to run into the ground or to fly into the air by their excessive demands is clear indication of their falsity" (Correa Moylan Walsh, 1901, p. 231).

In the present context, this test could be interpreted to mean the following: if any single price p_i^0 or p_i^1 tends to zero, then the price index $P(p^0,p,v^0,v^1)$ should not tend to zero or plus infinity. However, with this interpretation of the test, which regards the values v_i^t as remaining constant as the p_i^0 or p_i^1 tends to zero, none of the commonly used index number formulas would satisfy this test. As a result, this test should be interpreted as a test that applies to price indices $P(p^0,p^1,q^0,q^1)$ of the type that were studied in Section C, which is how Fisher intended the test to apply. Thus, Fisher's price determinateness test should be interpreted as follows: if any single price p_i^0 or p_i^1 tends to zero, then the price index $P(p^0,p,q^0,q^1)$ should not tend to zero or plus infinity. With this interpretation of the test, it can be verified that Laspeyres, Paasche, and Fisher indices satisfy this test, but the Törnqvist-Theil price index will not satisfy this test. Thus, when using the Törnqvist-Theil price index, *care must be taken to bound the prices away from zero in order to avoid a meaningless index number value*.

16.131 Walsh was aware that geometric average type indices like the Törnqvist-Theil price index P_T or Walsh's geometric price index P_{GW} defined by equation (16.64) become somewhat unstable[72] as individual price relatives become very large or small:

> Hence in practice the geometric average is not likely to depart much from the truth. Still, we have seen that when the classes [that is, revenues] are very unequal and the price variations are very great, this average may deflect considerably. (Correa Moylan Walsh, 1901, p. 373)

> In the cases of moderate inequality in the sizes of the classes and of excessive variation in one of the prices, there seems to be a tendency on the part of the geometric method to deviate by itself, becoming untrustworthy, while the other two methods keep fairly close together. (Correa Moylan Walsh, 1901, p. 404)

16.132 Weighing all of the arguments and tests presented in Sections C and E of this chapter, it seems that there may be a slight preference for the use of the Fisher ideal price index as a suitable target index for a statistical agency, but opinions can differ on which set of axioms is the most appropriate to use in practice.

[72]That is, the index may approach zero or plus infinity.

F. Test Properties of Lowe and Young Indices

16.133 In Chapter 15, the Young and Lowe indices were defined. In the present section, the axiomatic properties of these indices with respect to their price arguments will be developed.[73]

16.134 Let $q^b \equiv [q_1^b,...,q_n^b]$ and $p^b \equiv [p_1^b,...,p_n^b]$ denote the quantity and price vectors pertaining to some base year. The corresponding *base-year revenue shares* can be defined in the usual way as

$$(16.85)\quad s_i^b \equiv \frac{p_i^b q_i^b}{\sum_{k=1}^{n} p_k^b q_k^b}\ ,\ i=1,...,n.$$

Let $s^b \equiv [s_1^b,...,s_n^b]$ denote the vector of base-year revenue shares. The Young (1812) price index between periods 0 and t is defined as follows:

$$(16.86)\quad P_Y(p^0,p^t,s^b) \equiv \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0}\right).$$

The Lowe (1823, p. 316) price index[74] between periods 0 and t is defined as follows:

[73]Baldwin (1990, p. 255) worked out a few of the axiomatic properties of the Lowe index.

[74]This index number formula is also precisely Bean and Stine's (1924, p. 31) Type A index number formula. Walsh (1901, p. 539) initially mistakenly attributed Lowe's formula to G. Poulett Scrope (1833), who wrote *Principles of Political Economy* in 1833 and suggested Lowe's formula without acknowledging Lowe's priority. But in his discussion of Fisher's (1921) paper, Walsh (1921b, pp. 543–44) corrects his mistake on assigning Lowe's formula: "What index number should you then use? It should be this: $\sum q\, p_1 / \sum q\, p_0$. This is the method used by Lowe within a year or two of one hundred years ago. In my [1901] book, I called it Scope's index number; but it should be called Lowe's. Note that in it are used quantities neither of a base year nor of a subsequent year. The quantities used should be rough estimates of what the quantities were throughout the period or epoch."

$$(16.87)\ P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{k=1}^{n} p_k^0 q_k^b} = \frac{\sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^b}\right)}{\sum_{k=1}^{n} s_k^b \left(\frac{p_k^0}{p_k^b}\right)}.$$

16.135 Drawing on those that have been listed in Sections C and E, we highlight 12 desirable axioms for price indices of the form $P(p^0,p^1)$. The period 0 and t price vectors, p^0 and p^t, are presumed to have strictly positive components.

T1—*Positivity Test*: $P(p^0,p^t) > 0$ if all prices are positive.

T2—*Continuity Test*: $P(p^0,p^t)$ is a continuous function of prices.

T3—*Identity Test*: $P(p^0,p^0) = 1$.

T4—*Homogeneity Test for Period t Prices*: $P(p^0,\lambda p^t) = \lambda P(p^0,p^t)$ for all $\lambda > 0$.

T5—*Homogeneity Test for Period 0 Prices*: $P(\lambda p^0,p^t) = \lambda^{-1} P(p^0,p^t)$ for all $\lambda > 0$.

T6—*Commodity Reversal Test*: $P(p^t,p^0) = P(p^{0*},p^{t*})$, where p^{0*} and p^{t*} denote the same permutation of the components of the price vectors p^0 and p^t.[75]

T7—*Invariance to Changes in the Units of Measurement or the Commensurability Test*: $P(\alpha_1 p_1^0,...,\alpha_n p_n^0;\ \alpha_1 p_1^t,...,\alpha_n p_n^t) = P(p_1^0,...,p_n^0;\ p_1^t,...,p_n^t)$ for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

T8—*Time Reversal Test*: $P(p^t,p^0) = 1/P(p^0,p^t)$.

T9—*Circularity or Transitivity Test*: $P(p^0,p^2) = P(p^0,p^1)P(p^1,p^2)$.

T10—*Mean Value Test*: $\min\{p_i^t/p_i^0 : i = 1,\ldots,n\} \le P(p^t,p^0) \le \max\{p_i^t/p_i^0 : i = 1,\ldots,n\}$.

T11—*Monotonicity Test with Respect to Period t Prices*: $P(p^0,p^t) < P(p^0,p^{t*})$ if $p^t < p^{t*}$.

[75]In applying this test to the Lowe and Young indices, it is assumed that the base-year quantity vector q^b and the base-year share vector s^b are subject to the same permutation.

T12—*Monotonicity Test with Respect to Period 0 Prices*: $P(p^0,p^t) > P(p^{0*},p^t)$ if $p^0 < p^{0*}$.

16.136 It is straightforward to show that the Lowe index defined by equation (16.87) satisfies all 12 of the axioms or tests listed above. Hence, the Lowe index has very good axiomatic properties with respect to its price variables.[76]

16.137 It is straightforward to show that the Young index defined by equation (16.86) satisfies 10 of the 12 axioms, failing T8, the time reversal test, and T9, the circularity test. Thus, the axiomatic properties of the Young index are definitely inferior to those of the Lowe index.

Appendix 16.1: Proof of Optimality of Törnqvist-Theil Price Index in Second Bilateral Test Approach

16.138 Define $r_i \equiv p_i^1/p_i^0$ for $i = 1,\ldots,n$. Using T1, T9, and equation (16.66), $P(p^0,p^1,v^0,v^1) = P^*(r, v^0,v^1)$. Using T6, T7, and equation (16.63):

$$(A16.1)\ P(p^0, p^1, v^0, v^1) = P^*(r, s^0, s^1),$$

where s^t is the period t revenue share vector for $t = 0,1$.

16.139 Let $x \equiv (x_1,\ldots,x_n)$ and $y \equiv (y_1,\ldots,y_n)$ be strictly positive vectors. The transitivity test T11 and equation (A16.1) imply that the function P^* has the following property:

$$(A16.2)\ P^*(x; s^0, s^1) P^*(y; s^0, s^1) = P^*(x_1 y_1, ..., x_n y_n; s^0, s^1).$$

16.140 Using T1, $P^*(r,s^0,s^1) > 0$ and using T14, $P^*(r, s^0,s^1)$ is strictly increasing in the components of r. The identity test T3 implies that

$$(A16.3)\ P^*(1_n, s^0, s^1) = 1,$$

[76]From the discussion in Chapter 15, it will be recalled that the main problem with the Lowe index occurs if the quantity weight vector q^b is not representative of the quantities that were purchased during the time interval between periods 0 and 1.

where 1_n is a vector of ones of dimension n. Using a result due to Eichhorn (1978, p. 66), it can be seen that these properties of P^* are sufficient to imply that there exist positive functions $\alpha_i(s^0,s^1)$ for $i = 1,\ldots,n$ such that P^* has the following representation:

$$\text{(A16.4)}\quad \ln P^*(r,s^0,s^1) = \sum_{i=1}^{n} \alpha_i(s^0,s^1)\ln r_i .$$

16.141 The continuity test T2 implies that the positive functions $\alpha_i(s^0,s^1)$ are continuous. For $\lambda > 0$, the linear homogeneity test T4 implies that

$$\text{(A16.5)}\quad \ln P^*(\lambda r,s^0,s^1) = \ln\lambda + \ln P^*(r,s^0,s^1)$$
$$= \sum_{i=1}^{n} \alpha_i(s^0,s^1)\ln\lambda r_i,\ \text{ using equation (A16.4)}$$
$$= \sum_{i=1}^{n} \alpha_i(s^0,s^1)\ln\lambda + \sum_{i=1}^{n} \alpha_i(s^0,s^1)\ln r_i$$
$$= \sum_{i=1}^{n} \alpha_i(s^0,s^1)\ln\lambda + \ln P^*(r,s^0,s^1),$$
using equation (A16.4).

Equating the right-hand sides of the first and last lines in (A16.5) shows that the functions $\alpha_i(s^0,s^1)$ must satisfy the following restriction:

$$\text{(A16.6)}\quad \sum_{i=1}^{n} \alpha_i(s^0,s^1) = 1,$$

for all strictly positive vectors s^0 and s^1.

16.142 Using the weighting test T16 and the commodity reversal test T8, equation (16.69) holds. Equation (16.69) combined with the commensurability test T9 implies that P^* satisfies the following equation:

$$\text{(A16.7)}\quad P^*(1,\ldots,1,r_i,1,\ldots,1\,;s^0,s^1)$$
$$= f(1,r_i,s^0,s^1)\ ;\ i = 1,\ldots,n ,$$

for all $r_i > 0$, where f is the function defined in test T16.

16.143 Substitute equation (A16.7) into equation (A16.4) in order to obtain the following system of equations:

$$\text{(A16.8)}\quad P^*(1,\ldots,1,r_i,1,\ldots,1\,;s^0,s^1)$$
$$= f(1,r_i,s^0,s^1)$$
$$= \alpha_i(s^0,s^1)\ln r_i\ ;\ i = 1,\ldots,n .$$

But the first part of equation (A16.8) implies that the positive continuous function of $2n$ variables $\alpha_i(s^0,s^1)$ is constant with respect to all of its arguments except s_i^0 and s_i^1, and this property holds for each i. Thus, each $\alpha_i(s^0,s^1)$ can be replaced by the positive continuous function of two variables $\beta_i(s_i^0,s_i^1)$ for $i = 1,\ldots,n$.[77] Now replace the $\alpha_i(s^0,s^1)$ in equation (A16.4) with the $\beta_i(s_i^0,s_i^1)$ for $i = 1,\ldots,n$ and the following representation for P^* is obtained:

$$\text{(A16.9)}\quad \ln P^*(r,s^0,s^1) = \sum_{i=1}^{n} \beta_i(s_i^0,s_i^1)\ln r_i .$$

16.144 Equation (A16.6) implies that the functions $\beta_i(s_i^0,s_i^1)$ also satisfy the following restrictions:

$$\text{(A16.10)}\quad \sum_{i=1}^{n} s_i^0 = 1\ ;\ \sum_{i=1}^{n} s_i^1 = 1$$
$$\text{implies } \sum_{i=1}^{n} \beta_i(s_i^0,s_i^1) = 1 .$$

16.145 Assume that the weighting test T17 holds, and substitute equation (16.71) into (A16.9) in order to obtain the following equation:

$$\text{(A16.11)}\quad \beta_i(0,0)\ln\left(\frac{p_i^1}{p_i^0}\right) = 0\ ;\ i = 1,\ldots,n .$$

Since the p_i^1 and p_i^0 can be arbitrary positive numbers, it can be seen that equation (A16.11) implies

$$\text{(A16.12)}\quad \beta_i(0,0) = 0\ ;\ i = 1,\ldots,n.$$

16.146 Assume that the number of commodities n is equal to or greater than 3. Using equations (A16.10) and (A16.12), Theorem 2 in Aczél (1987, p. 8) can be applied and the following functional form for each of the $\beta_i(s_i^0,s_i^1)$ is obtained:

[77]More explicitly, $\beta_1(s_1^0,s_1^1) \equiv \alpha_1(s_1^0,1,\ldots,1;s_1^1,1,\ldots,1)$ and so on. That is, in defining $\beta_1(s_1^0,s_1^1)$, the function $\alpha_1(s_1^0,1,\ldots,1;s_1^1,1,\ldots,1)$ is used where all components of the vectors s^0 and s^1 except the first are set equal to an arbitrary positive number like 1.

$$\text{(A16.13)}\quad \beta_i(s_i^0, s_i^1) = \gamma\, s_i^0 + (1-\gamma)s_i^1;\quad i = 1,...,n,$$

where γ is a positive number satisfying $0 < \gamma < 1$.

16.147 Finally, the time reversal test T10 *or* the quantity weights symmetry test T12 can be used to show that γ must equal ½. Substituting this value or γ back into equation (A16.13) and then substituting those equations back into equation (A16.9), the functional form for P^*, and hence p, is determined as

$$\text{(A16.14)}\quad \ln P(p^0, p^1, v^0, v^1) = \ln P^*(r, s^0, s^1) = \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1)\ln\left(\frac{p_i^1}{p_i^0}\right).$$

17. Economic Approach

A. Introduction

A.1 Setting the Stage

17.1 The *family of PPIs* provides price indices to deflate parts of the system of national accounts. As is well known,[1] there are three distinct approaches to measuring GDP:

- The production approach,
- The expenditure or final demand approach, and
- The income approach.

The production approach[2] to calculating nominal GDP involves calculating the value of outputs produced by an industry and subtracting the value of intermediate inputs (or intermediate consumption, to use the national accounting term) used in the industry. This difference in value is called the industry's *value added*. Summing these industry estimates of value added leads to an estimate of national GDP. PPIs are used to separately deflate both industry outputs and industry intermediate inputs.[3] A PPI also is used to deflate an industry's nominal value added into value added at constant prices.

17.2 The economic approach to the PPI begins not at the industry level, but at the *establishment* level. An establishment is the PPI counterpart to a *household* in the theory of the CPI. An establishment is an economic entity that undertakes *production* or *productive activity* at a specific geographic location in the country and is capable of providing basic accounting information on the prices and quantities of the outputs it produces and on the inputs it uses during an accounting period. This chapter focuses on establishments that undertake production under a *for-profit* motivation. In Chapter 14, it was shown that the *1993 SNA* output in the production account is broken down into market output (P.11), output for own final use (P.12), and other nonmarket output (P.13). The latter includes output of government and nonprofit institutions serving households distributed free or sold at prices not economically significant. The PPI covers all types of domestically produced or processed goods and services that are valued at market prices and thus excludes P.13.

17.3 *Production* is an activity that transforms or combines material inputs into other material outputs (for example, agricultural, mining, manufacturing, or construction activities) or transports materials from one location to another. Production also includes storage activities, which in effect transport materials in the same location from one time period to another. Finally, production also includes the use and creation of services of all types.[4]

17.4 There are two major problems with the above definition of an establishment. The first is that many production units at specific geographic locations do not have the capability of providing basic accounting information on their inputs used and outputs produced. These production units may simply be a division or single plant of a large firm, and detailed accounting information on prices may be available only at the head office (or not at all). If this is the case, the definition of an establishment is modified to include production units at a number of specific geographic locations in the country instead of just one location. The important aspect of the definition of an establishment is that it be able to provide accounting information on prices and quantities.[5] A second problem is that

[1]See Eurostat and others (1993) or Bloem, Dippelsman, and Maehle (2001, p. 17).

[2]Early contributors to this approach include Bowley (1922, p. 2), Rowe (1927, p. 173), Burns (1930, pp. 247–50), and Copeland (1932, pp. 3–5).

[3]Additional material relating national accounting aggregates to PPIs may be found in Chapter 14.

[4]See Peter Hill (1999) for a taxonomy for services.

[5]In this modified definition of an establishment, it is generally a smaller collection of production units than a *firm*, since a firm may be multinational. Thus, another way of de-
(continued)

while the establishment may be able to report accurate quantity information, its price information may be based on *transfer prices* set by a head office. These transfer prices are *imputed prices* and may not be very closely related to market prices.[6]

17.5 Thus the problems involved in obtaining the correct commodity prices for establishments are generally more difficult than the corresponding problems associated with obtaining market prices for households. However, in this chapter, these problems will be ignored, and it will be assumed that representative market prices are available for each output produced by an establishment and for each intermediate input used by the same establishment for at least two accounting periods.[7]

17.6 The economic approach to PPIs requires that establishment output prices *exclude* any indirect taxes that various layers of government might levy on outputs produced by the establishment. These indirect taxes are excluded because firms do not get to keep these tax revenues, even though they may collect them for governments. Thus, these taxes are not part of establishment revenue streams. On the other hand, the economic approach to PPIs requires that establishment intermediate input prices *include* any indirect taxes that governments might levy on these inputs used by the establishment. The reason for including these taxes is that they are actual costs paid by the establishment. These conventions on the treatment of indirect taxes on production are consistent with those specified in Section B.1 of Chapter 2.

17.7 For the first sections of this chapter, an *output price index*, an *intermediate input price index*, and a *value-added deflator*[8] will be defined for a *single establishment* from the economic perspective. In subsequent sections, aggregation will take place over establishments to define national counterparts to these establishment price indices.

17.8 Some notation is required. Consider the case of an establishment that produces N commodities during two periods, periods 0 and 1. Denote the period t *output price vector* by $p_y^t \equiv [p_{y1}^t,...,p_{yN}^t]$ and the corresponding period t *output quantity vector* by $y^t \equiv [y_1^t,...,y_N^t]$, for $t = 0,1$. Assume that the establishment uses M commodities as intermediate inputs during periods 0 and 1. An *intermediate input* is an input produced by another establishment in the country or an imported (nondurable) commodity.[9] The period t *intermediate*

fining an establishment for our purposes is as follows: an establishment is the smallest aggregate of national production units able to provide accounting information on its inputs and outputs for the time period under consideration.

[6]For many highly specialized intermediate inputs in a multistage production process using proprietary technologies, market prices may simply not exist. Furthermore, several alternative concepts could be used to define transfer prices; see Diewert (1985) and Eden (1998). The *1993 SNA* (paragraph 6.82) notes that for deliveries between establishments belonging to the same enterprise

> Goods and services that one establishment provides to a different establishment belonging to the same enterprise are counted as part of the output of the producing establishment. Such goods and services may be used for intermediate consumption by the receiving establishment, but they also could be used for gross fixed capital formation. The goods and services should be valued by the producing establishment at current basic prices; the receiving establishment should value them at the same prices plus any additional transportation costs paid to third parties. The use of artificial transfer prices employed for internal accounting purposes within the enterprise should be avoided, if possible.

The difficulties in ascertaining such prices are recognized however:

> From an accounting point of view it can be difficult to partition a vertically integrated enterprise into establishments because values have to be imputed for the outputs from the earlier stages of production which are not actually sold on the market and which become intermediate inputs into later stages. Some of these enterprises may record the intra-enterprise deliveries at prices that reflect market values, but others may not. Even if adequate data are available on the costs incurred at each stage of production, it may be difficult to decide what is the appropriate way in which to allocate the operating surplus of the enterprise among the various stages. One possibility is that a uniform rate of profit could be applied to the costs incurred at each stage (*1993 SNA*, paragraph 5.33).

[7]These pricing problems are pursued in Chapter 6, where the concept of a market price for each product produced by an establishment during the accounting period under consideration is the value of production for that product divided by the quantity produced during that period; that is, the price is the average price for that product.

[8]While the value-added price index is just like any other price index in its definition, it is commonly referred to as the "value-added deflator," and the *Manual* will observe this common terminology.

[9]However, capital inputs or durable inputs are excluded from the list of intermediate inputs. A durable input is an input whose contribution to production lasts more than one accounting period. This makes the definition of a durable

(continued)

input price vector is denoted by $p_x^t \equiv [p_{x1}^t,...,p_{xM}^t]$ and the corresponding period *t intermediate input quantity vector* by $x^t \equiv [x_1^t,...,x_M^t]$ for $t = 0,1$. Finally, it is assumed that that the establishment uses the services of *K primary inputs* during periods 0 and 1. The period *t primary input vector* used by the establishment is denoted by $z^t \equiv [z_1^t,...,z_K^t]$ for $t = 0,1$.

17.9 Note it is assumed that the list of commodities produced by the establishment and the list of inputs used by the establishment *remains the same* over the two periods for which a price comparison is wanted. In real life, the list of commodities used and produced by an establishment does not remain constant over time. New commodities appear and old commodities disappear. The reasons for this churning of commodities include the following:

(i) Producers substitute new processes for older ones in response to changes in relative prices, and some of these new processes use new inputs.
(ii) Technical progress creates new processes or products, and the new processes use inputs not used in previous periods.
(iii) Seasonal fluctuations in the demand (or supply) of commodities cause some commodities to be unavailable in certain periods of the year.

The introduction of new commodities is dealt with in Chapter 21 and the problems associated with seasonal commodities in Chapter 22. In the present chapter, these complications are ignored, and it is assumed that the list of commodities remains the *same* over the two periods under consideration. It also will be assumed that all establishments are present in both periods under consideration; that is, there are no new or disappearing establishments.[10]

17.10 When convenient, the above notation will be simplified to match the notation used in Chapters 15 and 16. Thus, when studying the output price index, $p_y^t \equiv [p_{y1}^t,...,p_{yN}^t]$ and $y^t \equiv [y_1^t,...,y_N^t]$ will be replaced by $p^t \equiv [p_1^t,...,p_N^t]$ and $q^t \equiv [q_1^t,...,q_N^t]$; when studying the input price index, $p_x^t \equiv [p_{x1}^t,...,p_{xM}^t]$ and $x^t \equiv [x_1^t,...,x_M^t]$ will be replaced by $p^t \equiv [p_1^t,...,p_M^t]$ and $q^t \equiv [q_1^t,...,q_M^t]$; and when studying the value-added deflator, the composite vector of output and input prices $[p_y^t,p_x^t]$, will be replaced by $p^t \equiv [p_1^t,...,p_N^t]$; and the vector of net outputs $[y^t,-x^t]$, by $q^t \equiv [q_1^t,...,q_N^t]$ for $t = 0,1$ in each case. Thus, the appropriate definition for p^t and q^t depends on the context.

17.11 To most practitioners in the field, our basic framework, which assumes that detailed price and quantity data are available for each of the possibly millions of establishments in the economy, will seem to be utterly unrealistic. However, two answers can be directed at this very valid criticism:

- The spread of the computer and the ease of storing transaction data have made the assumption that the statistical agency has access to detailed price and quantity data less unrealistic. With the cooperation of businesses, it is now possible to calculate price and quantity indices of the type studied in Chapters 15 and

input dependent on the length of the accounting period. However, by convention, an input is classified as being durable if it lasts longer than two or three years. Thus, an intermediate input is a nondurable input that is also not a primary input. Durable inputs are classified as primary inputs even if they are produced by other establishments. Other primary inputs include labor, land, and natural resource inputs.

[10]Rowe (1927, pp. 174–75) was one of the first economists to appreciate the difficulties statisticians faced when attempting to construct price or quantity indices of production: "In the construction of an index of production there are three inherent difficulties which, inasmuch as they are almost insurmountable, impose on the accuracy of the index, limitations, which under certain circumstances may be somewhat serious. The first is that many of the products of industry are not capable of quantitative measurement. This difficulty appears in its most serious form in the case of the engineering industry. ... The second inherent difficulty is that the output of an industry, even when quantitatively measurable, may over a series of years change qualitatively as well as quantitatively. Thus during the last twenty years there has almost certainly been a tendency towards an improvement in the average quality of the yarn and cloth produced by the cotton industry .... The third inherent difficulty lies in the inclusion of new industries which develop importance as the years go on." These three difficulties still exist today: think of the difficulties involved in measuring the outputs of the insurance and gambling industries; an increasing number of industries produce outputs that are one of a kind, and, hence, price and quantity comparisons are necessarily difficult if not impossible; and, finally, the huge increases in research and development expenditures by firms and governments have led to ever increasing numbers of new products and industries. Chapter 8 considers the issues for index compilation arising from new and disappearing goods and services, as well as establishments.

16 using very detailed data on prices and quantities.[11]

- Even if it is not realistic to expect to obtain detailed price and quantity data for every transaction made by every establishment in the economy on a monthly or quarterly basis, it is still necessary to accurately specify the *universe* of transactions in the economy. Once the target universe is known, sampling techniques can be applied in order to reduce data requirements.

A.2 An overview of the chapter

17.12 In this subsection, a brief overview of the contents of this chapter will be given. In Section B, the economic theory of the *output price index* for an establishment is outlined. This theory is credited primarily to Fisher and Shell (1972) and Archibald (1977). Various bounds to the output price index are developed, along with some useful approximations to the theoretical output price index. Diewert's (1976) theory of *superlative indices* is outlined. A superlative index can be evaluated using observable price and quantity data, but under certain conditions it can give exactly the same answer as the theoretical output price index.

17.13 In the previous two chapters, the Fisher (1922) ideal price index and the Törnqvist (1936) price index emerged as being supported by the test and stochastic approaches to index number theory, respectively. These two indices also will emerge as very good choices from the economic perspective. However, a practical drawback to their use is that current-period information on quantities is required, information that the statistical agency will usually not have on a current-period basis. Hence, in Section E, recent suggestions for *approximating* these indices are looked at using only current information on prices; that is, it is assumed that current-period information on quantities is not available.

17.14 Finally, in Appendix 17.1, the relationship between the Divisia price index introduced in Chapter 15 and an economic output price index is considered.

B. Fisher-Shell Output Price Index: The Case of One Establishment

B.1 Fisher-Shell output price index and observable bounds

17.15 This subsection includes an outline of the theory of the output price index for a single establishment developed by Fisher and Shell (1972) and Archibald (1977). This theory is the producer theory counterpart to the theory of the cost-of-living index for a single consumer (or household) that was first developed by the Russian economist, Konüs (1924). These economic approaches to price indices rely on the assumption of (competitive) *optimizing behavior* on the part of economic agents (consumers or producers). Thus, in the case of the output price index, given a vector of output prices p^t that the agent faces in a given time period t, it is assumed that the corresponding hypothetical quantity vector q^t is the solution to a revenue maximization problem that involves the producer's production function f or production possibilities set. (Hereafter the terms *value of output* and *revenue* are used interchangeably, inventory changes being ignored.)

17.16 In contrast to the axiomatic approach to index number theory, the economic approach does *not* assume that the two quantity vectors $q^0 \equiv [q_1^0,...,q_N^0]$ and $q^1 \equiv [q_1^1,...,q_N^1]$ are independent of the two price vectors $p^0 \equiv [p_1^0,...,p_N^0]$ and $p^1 \equiv [p_1^1,...,p_N^1]$. In the economic approach, the period 0 quantity vector q^0 is determined by the producer's period 0 production function and the period 0 vector of prices p^0 that the producer faces, and the period 1 quantity vector q^1 is determined by the producer's period 1 production function f and the period 1 vector of prices p^1.

17.17 Before the output price index is defined for an establishment, it is necessary to describe the establishment's technology in period t. In the economics literature, it is traditional to describe the technology of a firm or industry in terms of a production function, which reveals the maximum amount of output that can be produced using a given vector of inputs. However, since most estab-

[11] An early study that computed Fisher ideal indices for a distribution firm in western Canada for seven quarters aggregating over 76,000 inventory items is found in Diewert and Smith (1994).

lishments produce more than one output, it is more convenient to describe the establishment's technology in period t by means of a *production possibilities set* S^t. The set S^t describes what output vectors q can be produced in period t if the establishment has at its disposal the vector of inputs $v \equiv [x,z]$, where x is a vector of intermediate inputs and z is a vector of primary inputs. Thus, if $[q,v]$ belongs to S^t, then the nonnegative output vector q can be produced by the establishment in period t if it can use the nonnegative vector v of inputs.

17.18 Let $p \equiv (p_1,\ldots p_N)$ denote a vector of positive output prices that the establishment might face in period t, and let $v \equiv [x,z]$ be a nonnegative vector of inputs that the establishment might have available for use during period t. Then the establishment's *revenue function* using period t technology is defined as the solution to the following revenue maximization problem:

$$(17.1)\ R^t(p,v) \equiv \max_q \{\sum_{n=1}^{N} p_n q_n : q \text{ belongs to } S^t(v)\}.$$

Thus, $R^t(p,v)$ is the maximum value of output, $\sum_{n=1}^{N} p_n q_n$, that the establishment can produce, given that it faces the vector of output prices p and the vector of inputs v is available for use, using the period t technology.[12]

17.19 The period t revenue function R^t can be used to define the establishment's *period* t *technology output price index* P^t between any two periods, say, period 0 and period 1, as follows:

$$(17.2)\ P^t(p^0,p^1,v) = R^t(p^1,v) / R^t(p^0,v),$$

where p^0 and p^1 are the vectors of output prices that the establishment faces in periods 0 and 1, respectively, and v is a reference vector of intermediate and primary inputs.[13] If $N = 1$ so that the establishment produces only one output, then it can be shown that the output price index collapses to the single-output price relative between periods 0 and 1, p_1^1/ p_1^0. In the general case, note that the output price index defined by equation (17.2) is a ratio of hypothetical revenues that the establishment could realize, given that it has the period t technology and the vector of inputs v to work with. The numerator in equation (17.2) is the maximum revenue that the establishment could attain if it faced the output prices of period 1, p^1, while the denominator in equation (17.2) is the maximum revenue that the establishment could attain if it faced the output prices of period 0, p^0. Note that all of the variables in the numerator and denominator functions are exactly the same, except that the output price vectors differ. This is a defining characteristic of an economic price index: all environmental variables are held constant with the exception of the prices in the domain of definition of the price index.

17.20 Note that there are a wide variety of price indices of the form equation (17.2), depending on which reference technology t and reference input vector v is chosen. Thus, there is not a single economic price index of the type defined by equation (17.2): there is an entire *family* of indices.

17.21 Usually, interest lies in two special cases of the general definition of the output price index in equation (17.2): (i) $P^0(p^0,p^1,v^0)$, which uses the period 0 technology set and the input vector v^0 that was actually used in period 0, and (ii) $P^1(p^0,p^1,v^1)$, which uses the period 1 technology set and the input vector v^1 that was actually used in period 1. Let q^0 and q^1 be the observed output vectors for the establishment in periods 0 and 1, respectively. If

[12]The function R^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli, 1978 and 1991; or Woodland, 1982). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include (i) the *gross profit function*, see Gorman (1968); (ii) the *restricted profit function*, see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*, see Diewert (1973 and 1974a). The mathematical properties of the revenue function are laid out in these references.

[13]This concept of the output price index (or a closely related variant) was defined by Fisher and Shell (1972, pp. 56–58), Samuelson and Swamy (1974, pp. 588–92), Archibald (1977, pp. 60–61), Diewert (1980, pp. 460–61; 1983a, p. 1055), and Balk (1998a, pp. 83–89). Readers who are familiar with the theory of the true cost-of-living index will note that the output price index defined by equation (17.2) is analogous to the *true cost-of-living index,* which is a ratio of cost functions, say, $C(u,p^1)/C(u,p^0)$, where u is a reference utility level: r replaces C, and the reference utility level u is replaced by the vector of reference variables (t,v). For references to the theory of the true cost-of-living index, see Konüs (1924), Pollak (1983a), or the CPI counterpart to this *Manual*, ILO and others (2004).

there is revenue-maximizing behavior on the part of the establishment in periods 0 and 1, then observed revenue in periods 0 and 1 should be equal to $R^0(p^0,v^0)$ and $R^1(p^1,v^1)$, respectively; that is, the following equalities should hold:

$$(17.3)\ R^0(p^0,v^0) = \sum_{n=1}^{N} p_n^0 q_n^0 \text{ and } R^1(p^1,v^1) = \sum_{n=1}^{N} p_n^1 q_n^1 .$$

17.22 Under these revenue-maximizing assumptions, Fisher and Shell (1972, pp. 57–58) and Archibald (1977, p. 66) have shown that the two theoretical indices, $P^0(p^0,p^1,v^0)$ and $P^1(p^0,p^1,v^1)$ described in (i) and (ii) above, satisfy equations (17.4) and (17.5):

$$(17.4)\ P^0(p^0,p^1,v^0) \equiv R^0(p^1,v^0) / R^0(p^0,v^0),$$

using equation (17.2)

$$= R^0(p^1,v^0) / \sum_{n=1}^{N} p_n^0 q_n^0 ,$$

using equation (17.3)

$$\geq \sum_{n=1}^{N} p_n^1 q_n^0 \Big/ \sum_{n=1}^{N} p_n^0 q_n^0 ,$$

since q^0 is feasible for the maximization problem, which defines $R^0(p^1,v^0)$, and so

$$R^0(p^1,v^0) \geq \sum_{n=1}^{N} p_n^1 q_n^0$$
$$\equiv P_L(p^0,p^1,q^0,q^1),$$

where P_L is the Laspeyres (1871) price index. Similarly,

$$(17.5)\ P^1(p^0,p^1,v^1) \equiv R^1(p^1,v^1) / R^1(p^0,v^1),$$

using equation (17.2)

$$= \sum_{n=1}^{N} p_n^1 q_n^1 \Big/ R^1(p^0,v^1),$$

using equation (17.3)

$$\leq \sum_{n=1}^{N} p_n^1 q_n^1 \Big/ \sum_{n=1}^{N} p_n^0 q_n^1 ,$$

since q^1 is feasible for the maximization problem, which defines $R^1(p^0,v^1)$, and so

$$R^1(p^0,v^1) \geq \sum_{n=1}^{N} p_n^0 q_n^1$$
$$\equiv P_P(p^0,p^1,q^0,q^1),$$

where P_P is the Paasche (1874) price index. Thus, the inequality in equation (17.4) says that the observable Laspeyres index of output prices P_L is a *lower bound* to the theoretical output price index $P^0(p^0,p^1,v^0)$, and the inequality (17.5) says that the observable Paasche index of output prices P_P is an *upper bound* to the theoretical output price index $P^1(p^0,p^1,v^1)$. Note that these inequalities are in the *opposite direction* compared with their counterparts in the theory of the true cost-of-living index.[14]

17.23 It is possible to illustrate the two inequalities in equations (17.4) and (17.5) if there are only two commodities; see Figure 17.1, which is based on diagrams credited to Hicks (1940, p. 120) and Fisher and Shell (1972, p. 57).

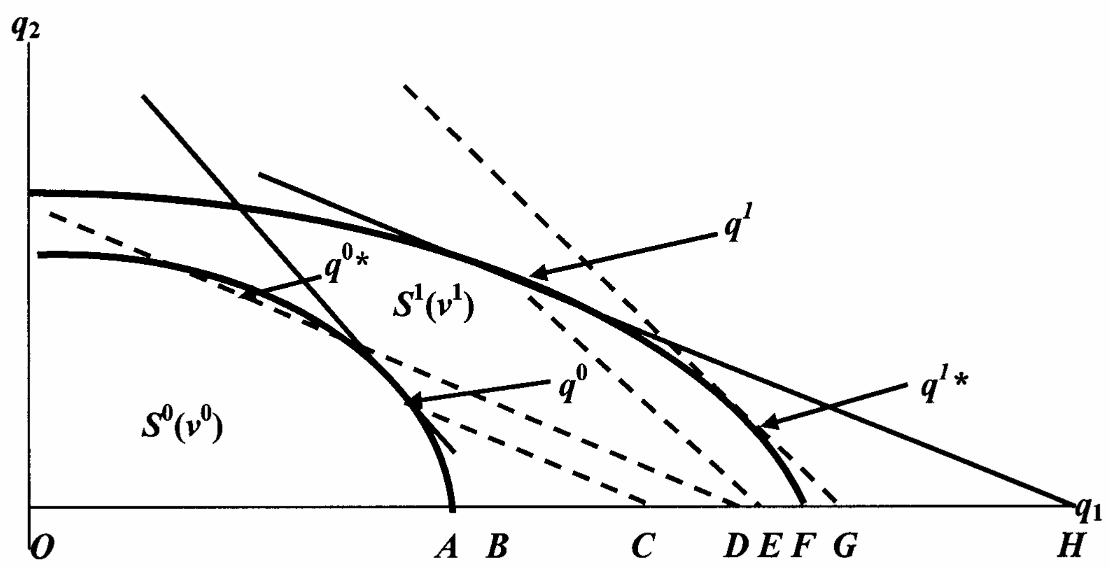
17.24 First the inequality in equation (17.4) is illustrated for the case of two outputs both produced in both periods. The solution to the period 0 revenue maximization problem is the vector q^0, and the straight line through B represents the revenue line that is just tangent to the period 0 output production possibilities set, $S^0(v^0) \equiv \{(q_1,q_2,v^0)$ belongs to $S^0\}$. The curved line through q^0 and A is the frontier to the producer's period 0 output production possibilities set $S^0(v^0)$. The solution to the period 1 revenue maximization problem is the vector q^1, and the straight line through H represents the revenue line that is just tangent to the period 1 output production possibilities set, $S^1(v^1) \equiv \{(q_1,q_2,v^1)$ belongs to $S^1\}$. The curved line through q^1 and F is the frontier to the producer's period 1 output production possibilities set $S^1(v^1)$. The point q^{0*} solves the hypothetical maximization problem of maximizing revenue when facing the period 1 price vector $p^1 = (p_1^1,p_2^1)$ but using the period 0 technology and input vector. This is given by $R^0(p^1,v^0) = p_1^1 q_1^{0*} + p_2^1 q_2^{0*}$, and the dashed line through D is the corresponding isorevenue line $p_1^1 q_1 + p_2^1 q_2 = R^0(p^1,v^0)$. Note that the hypothetical revenue line through D is parallel to the actual period 1 revenue line through H. From equation (17.4), the hypothetical Fisher-Shell output price index, $P^0(p^0,p^1,v^0)$, is $R^0(p^1,v^0)$ / $[p_1^0 q_1^0 + p_2^0 q_2^0]$, while the ordinary Laspeyres output price index is $[p_1^1 q_1^0$

[14]This is because the optimization problem in the cost-of-living theory is a cost *minimization* problem as opposed to our present revenue *maximization* problem. The method of proof used to derive equations (17.4) and (17.5) dates back to Konüs (1924), Hicks (1940), and Samuelson (1950).

$+ p_2^1 q_2^0] / [p_1^0 q_1^0 + p_2^0 q_2^0]$. Since the denominators for these two indices are the same, the difference between the indices is due to the differences in their numerators. In Figure 17.1, this difference in the numerators is expressed by the fact that the revenue line through *C* lies *below* the parallel revenue line through *D*. Now, if the producer's period 0 output production possibilities set were block-shaped with a vertex at q^0, then the producer would not change production patterns in response to a change in the relative prices of the two commodities while using the period 0 technology and inputs. In this case, the hypothetical vector q^{0*} would coincide with q^0, the dashed line through *D* would coincide with the dashed line through *C*, and the true output price index, $P^0(p^0,p^1,v^0)$, would coincide with the ordinary Laspeyres price index. However, block-shaped production possibilities sets are generally not consistent with producer behavior; that is, when the price of a commodity increases, producers generally supply more of it. Thus, in the general case, there will be a gap between the points *C* and *D*. The magnitude of this gap represents the amount of *substitution bias* between the true index and the corresponding Laspeyres index; that is, the Laspeyres index generally will be *less* than the corresponding true output price index, $P^0(p^0,p^1,v^0)$.

17.25 Figure 17.1 also can be used to illustrate the inequality (17.5) for the two-output case. Note that technical progress or increases in input availability have caused the period 1 output production possibilities set $S^1(v^1) \equiv \{(q_1,q_2) : [q_1,q_2,v^1]$ belongs to $S^1\}$ to be much bigger than the corresponding period 0 output production possibilities set $S^0(v^0) \equiv$

Figure 17.1. Laspeyres and Paasche Bounds to the Output Price Index



$\{(q_1,q_2) : [q_1,q_2,v^0]$ belongs to $S^0\}$.[15] Note also that the dashed lines through E and G are parallel to the period 0 isorevenue line through B. The point q^{1*} solves the hypothetical problem of maximizing revenue using the period 1 technology and inputs when facing the period 0 price vector $p^0 = (p_1^0,p_2^0)$. This is given by $R^1(p^0,v^1) = p_1^0q_1^{1*} + p_2^0q_2^{1*}$, and the dashed line through G is the corresponding isorevenue line $p_1^1q_1 + p_2^1q_2 = R^1(p^0,v^1)$. From equation (17.5), the theoretical output price index using the period 1 technology and inputs is $[p_1^1q_1^1 + p_2^1q_2^1] / R^1(p^0,v^1)$, while the ordinary Paasche price index is $[p_1^1q_1^1 + p_2^1q_2^1] / [p_1^0q_1^1 + p_2^0q_2^1]$. Since the numerators for these two indices are the same, the difference between the indices is due to the differences in their denominators. In Figure 17.1, this difference in the denominators is expressed by the fact that the revenue line through E lies *below* the parallel cost line through G. The magnitude of this difference represents the amount of *substitution bias* between the true index and the corresponding Paasche index; that is, the Paasche index generally will be *greater* than the corresponding true output price index using current-period technology and inputs, $P^1(p^0,p^1,v^1)$. Note that this inequality goes in the opposite direction to the previous inequality (17.4). The reason for this change in direction is that one set of differences between the two indices takes place in the numerators of the two indices (the Laspeyres inequalities), while the other set takes place in the denominators of the two indices (the Paasche inequalities).

17.26 Equations (17.4) and (17.5) have two problems:

- *Two* equally valid economic price indices, $P^0(p^0,p^1,v^0)$ and $P^1(p^0, p^1 ,v^1)$, can be used to describe the amount of price change that took place between periods 0 and 1, whereas the public will demand that the statistical agency produce a *single* estimate of price change between the two periods.
- Only *one-sided* observable bounds to these two theoretical price indices[16] result from this analysis, and for most practical purposes, *two-sided* bounds are required.

The following subsection shows a possible solution to these two problems.

B.2 Fisher ideal index as an average of observable bounds

17.27 It is possible to define a theoretical output price index that falls *between* the observable Paasche and Laspeyres price indices. To do this, first define a hypothetical revenue function, $\pi(p,\alpha)$, that corresponds to the use of an α weighted average of the technology sets $S^0(v^0)$ and $S^1(v^1)$ for periods 0 and 1 as the reference technology:

$$(17.6)\ R(p,\alpha) \equiv \max_q \{\sum_{n=1}^{N} p_nq_n : q \text{ belongs to } (1-\alpha)S^0(v^0) + \alpha S^1(v^1)\}.$$

Thus, the revenue maximization problem in equation (17.6) corresponds to the use of a weighted average of the period 0 and period 1 technology sets, where the period 0 vector gets the weight $1-\alpha$ and the period 1 vector gets the weight α, where α is a number between 0 and 1.[17] The meaning of the weighted average technology set in equation (17.6) can be explained in terms of Figure 17.1 as follows. As α changes continuously from 0 to 1, the output production possibilities set changes in a continuous manner from the set $S^0(v^0)$ (whose frontier is the curve that ends in the point A) to the set $S^1(v^1)$ (whose frontier is the curve that ends in the point F). Thus, for any α between 0 and 1, a hypothetical establishment output production possibilities set is obtained that lies between the base-period set $S^0(v^0)$ and the current-period set $S^1(v^1)$. For each α, this hypothetical output production

[15]However, the validity of equation (17.5) does not depend on the relative position of the two output production possibilities sets. To obtain the strict inequality version of equation (17.5), two things are needed: (i) the frontier of the period 1 output production possibilities set to be "curved" and (ii) relative output prices to change going from period 0 to 1, so that the two price lines through G and H in Figure 17.1 are tangent to *different* points on the frontier of the period 1 output production possibilities set.

[16]The Laspeyres output price index is a lower bound to the theoretical index $P^0(p^0,p^1,v^0)$, while the Paasche output price index is an upper bound to the theoretical index $P^1(p^0,p^1,v^1)$.

[17]When $\alpha = 0$, $R(p,0) = R^0(p,v^0)$, and when $\alpha = 1$, $R(p,1) = R^1(p,v^1)$.

possibilities set can be used as the constraint set for a theoretical output price index.

17.28 The new revenue function in definition (17.6) is now used in order to define the following family (indexed by α) of theoretical net output price indices:

(17.7) $P(p^0,p^1,\alpha) \equiv R(p^1,\alpha) / R(p^0,\alpha)$.

The important advantage that theoretical output price indices of the form in equations (17.2) or (17.7) have over the traditional Laspeyres and Paasche output price indices P_L and P_P is that these theoretical indices deal adequately with *substitution effects*; that is, when an output price increases, the producer supply should increase, holding inputs and the technology constant.[18]

17.29 Diewert (1983a, pp. 1060–61) showed that, under certain conditions,[19] there exists an α between 0 and 1 such that the theoretical output price index defined by equation (17.7) lies between the observable (in principle) Paasche and Laspeyres output indices, P_P and P_L ; that is, there exists an α such that

(17.8) $P_L \le P(p^0,p^1,\alpha) \le P_P$ or
$P_P \le P(p^0,p^1,\alpha) \le P_L$.

17.30 The fact that the Paasche and Laspeyres output price indices provide upper and lower bounds to a "true" output price $P(p^0,p^1,\alpha)$ in equation (17.8) is a more useful and important result than the one-sided bounds on the "true" indices that were derived in equations (17.4) and (17.5). If the observable (in principle) Paasche and Laspeyres indices are not too far apart, then taking a symmetric average of these indices should provide a good approximation to an economic output price index, where the reference technology is somewhere between the base- and current-period technologies. The precise symmetric average of the Paasche and Laspeyres indices was determined in Section C.1 of Chapter 15 on axiomatic grounds and led to the geometric mean, the Fisher price index, P_F:

(17.9) $P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0,q^1) \times P_P(p^0,p^1,q^0,q^1)]^{1/2}$.

Thus, the Fisher ideal price index receives a fairly strong justification as a good approximation to an unobservable theoretical output price index.[20]

17.31 The bounds given by equations (17.4), (17.5), and (17.8) are the best that can be obtained on economic output price indices without making further assumptions. In the next subsection, further assumptions are made on the two technology sets S^0 and S^1 or, equivalently, on the two revenue functions $R^0(p,v)$ and $R^1(p,v)$. With these extra assumptions, it is possible to determine the geometric average of the two theoretical output price indices that are of primary interest, $P^0(p^0,p^1,v^0)$ and $P^1(p^0,p^1,v^1)$.

B.3 Törnqvist index as an approximation to an economic output price index

17.32 An alternative to the Laspeyres and Paasche indices defined in equations (17.4) and

[18]This is a normal output substitution effect. However, empirically, it will often happen that observed period-to-period decreases in price are not accompanied by corresponding decreases in supply. However, these abnormal "substitution" effects can be rationalized as the effects of technological progress. For example, suppose the price of computer chips decreases substantially going from period 0 to 1. If the technology were constant over these two periods, one would expect domestic producers to decrease their supply of chips going from period 0 to 1. In actual fact, the opposite happens, because technological progress has led to a sharp reduction in the cost of producing chips, which is passed on to demanders of chips. Thus the effects of technological progress cannot be ignored in the theory of the output price index. The counterpart to technological change in the theory of the cost-of-living index is taste change, which is often ignored.

[19]Diewert adapted a method of proof credited originally to Konüs (1924) in the consumer context. Sufficient conditions on the period 0 and 1 technology sets for the result to hold are given in Diewert (1983a, p. 1105). The exposition of the material in Sections B.2, B.3, and C.1 also draws on Chapter 2 in Alterman, Diewert, and Feenstra (1999).

[20]Note that Irving Fisher (1922) constructed Laspeyres, Paasche, and Fisher output price indices for his U.S. data set. Fisher also adopted the view that the product of the price and quantity index should equal the value ratio between the two periods under consideration, an idea that he had already formulated (1911, p. 403). He did not consider explicitly the problem of deflating value added, but by 1930, his ideas on deflation and measuring quantity growth being essentially the same problem had spread to the problem of deflating nominal value added; see Burns (1930).

(17.5) or the Fisher index defined by equation (17.9) is to use the Törnqvist (1936) Theil (1967) price index P_T, whose natural logarithm is defined as follows:

$$(17.10)\ \ln P_T(p^0,p^1,q^0,q^1) = \sum_{n=1}^{N} (1/2)\left(s_n^0 + s_n^1\right)\ln\left(p_n^1/p_n^0\right),$$

where $s_n^t = p_n^t q_n^t \Big/ \sum_{n=1}^{N} p_n^t q_n^t$ is the revenue share of commodity n in the total value of sales in period t.

17.33 Recall the definition of the period t revenue function, $R^t(p,v)$, defined earlier by equation (17.1). Now assume that the period t revenue function has the following *translog functional form*[21] for $t = 0,1$:

$$(17.11)\ \ln R^t(p,v) = \alpha_0^t + \sum_{n=1}^{N} \alpha_n^t \ln p_n + \sum_{m=1}^{M+K} \beta_m^t \ln v_m + \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{N} \alpha_{nj}^t \ln p_n \ln p_j + \sum_{n=1}^{N}\sum_{m=1}^{M+K} \beta_{nm}^t \ln p_n \ln v_m + \frac{1}{2}\sum_{m=1}^{M+K}\sum_{k=1}^{M+K} \gamma_{mk}^t \ln v_m \ln v_k,$$

where the α_n^t coefficients satisfy the restrictions:

$$(17.12)\ \sum_{n=1}^{N} \alpha_n^t = 1 \text{ for } t = 0,1,$$

and the α_{nj}^t coefficients satisfy the following restrictions:[22]

$$(17.13)\ \sum_{n=1}^{N} \alpha_{nj}^t = 0, \text{ for } t = 0,1 \text{ and } n = 1,2,\ldots,N.$$

The equations (17.12) and (17.13) are necessary to ensure that $R^t(p,v)$ is linearly homogeneous in the components of the output price vector p (which is a property that a revenue function must satisfy.)[23] Note that at this stage of our argument, the coefficients that characterize the technology in each period (the αs, βs and γs) are allowed to be completely different in each period. It also should be noted that the translog functional form is an example of a *flexible* functional form;[24] that is, it can approximate an arbitrary technology to the second order.

17.34 A result in Caves, Christensen, and Diewert (1982, p. 1410) now can be adapted to the present context: if the quadratic price coefficients in equation (17.11) are equal across the two periods of the index number comparison (that is, $\alpha_{nj}^0 = \alpha_{nj}^1$ for all n,j), then the geometric mean of the economic output price index that uses period 0 technology and the period 0 input vector v^0, $P^0(p^0,p^1,v^0)$, and the economic output price index that uses period 1 technology and the period 1 input vector v^1, $P^1(p^0,p^1,v^1)$, is *exactly equal* to the Törnqvist output price index P_T defined by equation (17.10) above; that is,

$$(17.14)\ P_T(p^0,p^1,q^0,q^1) = [P^0(p^0,p^1,v^0) \times P^1(p^0,p^1,v^1)]^{1/2}.$$

The assumptions required for this result seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period, and our assumptions are consistent with technological progress occurring between the two periods being compared. Because the index number formula P_T is *exactly* equal to the geometric mean of two theoretical economic output price indices, and it corresponds to a flexible functional form, the Törnqvist output price index number formula is said to be *superlative,* following the terminology used by Diewert (1976).

17.35 In the following section, additional superlative output price formulas are derived. However, this section concludes with a few words of caution on the applicability of the economic approach to PPIs.

[21]This functional form was introduced and named by Christensen, Jorgenson, and Lau (1971). It was adapted to the revenue function or profit function context by Diewert (1974a).

[22]It also is assumed that the symmetry conditions $\alpha_{nj}^t = \alpha_{jn}^t$ for all n,j and for $t = 0,1$, and $\gamma_{mk}^t = \gamma_{km}^t$ for all m,k, and for $t = 0,1$ are satisfied.

[23]See Diewert (1973 and 1974a) for the regularity conditions that a revenue or profit function must satisfy.

[24]The concept of flexible functional form was introduced by Diewert (1974a, p. 113).

17.36 The above economic approaches to the theory of output price indices have been based on the assumption that producers take the prices of their outputs as given fixed parameters that they cannot affect by their actions. However, a *monopolistic supplier* of a commodity will be well aware that the average price that can be obtained in the market for the commodity will depend on the number of units supplied during the period. Thus, under noncompetitive conditions when outputs are monopolistically supplied (or when intermediate inputs are monopsonistically demanded), the economic approach to PPIs breaks down. The problem of modeling noncompetitive behavior does not arise in the economic approach to CPIs because a single household usually does not have much control over the prices it faces in the marketplace.

17.37 The economic approach to producer output price indices can be modified to deal with certain monopolistic situations. The basic idea is credited to Frisch (1936, 14–15), and it involves linearizing the demand functions a producer faces in each period around the observed equilibrium points in each period and then calculating shadow prices that replace market prices. Alternatively, one can assume that the producer is a markup monopolist and simply adds a markup or premium to the relevant marginal cost of production.[25] However, to implement these techniques, econometric methods usually will have to be employed, and, hence, these methods are not really suitable for use by statistical agencies, except in very special circumstances when the problem of noncompetitive behavior is thought to be very significant and the agency has access to econometric resources.

B.4 Fisher ideal index revisited

17.38 In Section B.2, a justification was provided for the Fisher ideal index. It was argued, from the economic approach, that an appropriate index defined from economic theory should fall between Laspeyres and Paasche indices. On axiomatic grounds, the Fisher ideal index was then proposed as the best average of these two formulas. The justification for the Törnqvist index in Section B.3 was quite different. The theory of exact and superlative index numbers was used to justify its use. In the previous section, equation (17.14) showed that if the revenue function took a translog functional form, equation (17.11), then a theoretical price index based on this form would correspond exactly with the Törnqvist output price index, which is a price index number formula based on observable price and quantity data. Moreover, since the translog function is one form of a class of flexible functional forms, the Törnqvist output price index number formula was said to be *superlative,* following the terminology used by Diewert (1976). Flexible functional forms can approximate an arbitrary twice continuously differentiable linearly homogeneous functional form to the second order, which is an attractive property of an index number. Bear in mind that Laspeyres and Paasche correspond to revenue functions that have restrictive Leontief forms, which allow no substitution, and the geometric Laspeyres and Paasche indices correspond to Cobb-Douglas forms, which restrict the elasticity of substitution to unity. The translog production technology is a form that allows for wider substitution possibilities and that can, to the second order, approximate a range of functional forms. The economic theory of index numbers provided a direct link between formulas used in practice and the implicit underlying economic behavior they represent. Diewert (1973) showed that if the functional form assumed is not flexible, it implicitly imposes restrictions on the elasticity of substitution. Index numbers that do not correspond to flexible functional forms, that is, are not superlative, are restrictive in this sense. In this section the findings for the Fisher ideal index are outlined. That is, the Fisher index, although justified on a mix of economic and axiomatic principles in Section B.2, is revisited here using the exact and superlative approach to economic index numbers. It will be shown that its derivation, while analogous to that of the Törnqvist index, requires more restrictive assumptions. In Section B.5 the findings on superlative indices are generalized.

17.39 The approach of the previous section is followed for the Fisher ideal index. However, first it is assumed that a linear homogeneous aggregator function exists for outputs. An additional (and considerably more restrictive) assumption is being invoked here than that required for the Törnqvist index: that outputs are said to be homogeneously weakly separable from the other commodities in the production function. The intuitive meaning of

[25] See Diewert (1993b, pp. 584–90) for a more detailed description of these techniques for modeling monopolistic behavior and for additional references to the literature.

the separability assumption defined by equation (17.15) is that an *output aggregate* $q \equiv f(q_1,...,q_N)$ *exists*; that is, a measure of the aggregate contribution to production of the amounts q_1 of the first output, q_2 of the second output, ..., and q_N of the Nth output is the number $q = f(q_1,q_2,...,q_N)$. Note that it is assumed that the linearly homogeneous output aggregator function f does not depend on t. These assumptions are quite restrictive from the viewpoint of empirical economics,[26] but strong assumptions are required to obtain the existence of output aggregates.

17.40 A *unit revenue function,*[27] r can be defined as follows:

$$(17.15)\ r(p) \equiv \max_q \{ \sum_{n=1}^{N} p_n q_n : f(q) = 1\},$$

where $p \equiv [p_1,...,p_N]$ and $q \equiv [q_1,...,q_N]$. Thus $r(p)$ is the maximum revenue the establishment can make, given that it faces the vector of output prices p and is asked to produce a combination of outputs $[q_1,...,q_N] = q$ that will produce a unit level of aggregate output. Under the separability assumptions, the theoretical price index $r(p^1) / r(p^0)$ is a ratio of unit revenue functions.

17.41 Instead of starting with the a translog function for the revenue function of the Törnqvist index, the assumption of the Fisher ideal index is that the *unit* revenue function takes a homogeneous quadratic form given by

$$(17.16)\ r(p_1,\ldots,p_N) \equiv \left[\sum_{i=1}^{N}\sum_{k=1}^{N} b_{ik} p_i p_k \right]^{1/2},$$

where the parameters b_{ik} satisfy the following symmetry conditions:

$$(17.17)\ b_{ik} = b_{ki} \text{ for all } i \text{ and } k.$$

Differentiating $r(p)$ defined by equation (17.16) with respect to p_i yields the following equations:

$$(17.18)\ r_i(p) = \left(\frac{1}{2}\right)\left[\sum_{i=1}^{N}\sum_{k=1}^{N} b_{ik} p_i p_k \right]^{-1/2} 2\sum_{k=1}^{N} b_{ik} p_k\ ;$$

$i = 1,\ldots,N$ and using equation (17.16),

$$= \sum_{k=1}^{N} b_{ik} p_k \Big/ r\ (p),$$

where $r_i(p) \equiv \partial r(p^t) / \partial p_i$. To obtain the first equation in equation (17.18), it is necessary to use the symmetry conditions, equation (17.17). The second equation in equation (17.18) now is evaluated at the observed period t price vector $p^t \equiv (p_1^t,\ldots,p_N^t)$, and dividing both sides of the resulting equation by $r(p^t)$ yields

$$(17.19)\ \frac{r_i(p^t)}{r(p^t)} = \frac{\sum_{k=1}^{N} b_{ik} p_k^t}{\left[r(p^t)\right]^2},\ t = 0,1\ ;\ i = 1,\ldots,N.$$

The above equation defines a theoretical price index. It now is required to relate this theoretical price index, which comes from a particular functional form for the unit revenue function, that is, a homogeneous quadratic form, to an index number formula that can be used in practice. To do this, it is necessary to assume the establishment is maximizing revenue during the two periods, subject to the constraints of technology, and that the unit revenue function is differentiable, and to apply Hotelling's lemma: that the partial derivative of a unit revenue function with respect to an output price is proportional to the equilibrium output quantity.

$$(17.20)\ \frac{q_n^t}{\sum_{k=1}^{N} p_k^t q_k^t} = \frac{\left[\partial r(p^t) / \partial(p_n)\right]}{r\ (p^t)};\ n = 1,\ldots,N\ ;$$

$$t = 0,1.$$

[26] Suppose that in period 0, the vector of inputs v^0 produces the vector of outputs q^0. Our separability assumptions imply that the same vector of inputs v^0 could produce *any* vector of outputs q such that $f(q) = f(q^0)$. In real life, as q varied, one would expect that the corresponding input requirements also would vary instead of remaining fixed.

[27] An alternative approach, which reaches the same conclusions, is to start with assuming the producer's aggregator function takes this quadratic form and, assuming outputs are homogeneously weakly separable from the other commodities in the production function, applies Wold's identity. It then can be shown that the Fisher ideal quantity index corresponds exactly to a homogeneous quadratic aggregator. Using the product rule, the unit revenue function can be derived to yield analogous results for the Fisher ideal price index.

In words, equation (17.20) says that the vector of period t establishment outputs q^t , divided by period t establishment revenues $\sum_{k=1}^{N} p_k^t q_k^t$, is equal to the vector of first-order partial derivatives of the establishment unit revenue function $\nabla r(p^t) \equiv [\partial r(p^t)/\partial p_1,...,\partial r(p^t)/\partial p_N]$, divided by the period t unit revenue function $r(p^t)$.

Now recall the definition of the Fisher ideal price index P_F defined by equations (15.12) or (17.9):

$$(17.21)\ P_F(p^0,p^1,q^0,q^1) = \left[\sum_{i=1}^{N} p_i^1 q_i^0 \Big/ \sum_{k=1}^{N} p_k^0 q_k^0\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^0 q_k^1\right]^{1/2}$$

substituting for $q_{nt} \Big/ \sum_{k=1}^{N} p_{kt} q_{kt}$ from equation (17.20) for $t = 0$

$$= \left[\sum_{i=1}^{N} p_i^1 r_i\left(p^0\right) \Big/ r\left(p^0\right)\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^0 q_k^1\right]^{1/2}$$

$$= \left[\sum_{i=1}^{N} p_i^1 r_i\left(p^0\right) \Big/ r\left(p^0\right)\right]^{1/2} \Bigg/ \left[\sum_{i=1}^{N} p_i^0 q_i^1 \Big/ \sum_{k=1}^{N} p_k^1 q_k^1\right]^{1/2}$$

and for $q_{nt} \Big/ \sum_{k=1}^{N} p_{kt} q_{kt}$ from equation (17.20) for $t = 1$

$$= \frac{\left[\sum_{i=1}^{N} p_i^1 r_i\left(p^0\right) \Big/ r\left(p^0\right)\right]^{1/2}}{\left[\sum_{i=1}^{N} p_i^0 r_i\left(p^1\right) \Big/ r\left(p^1\right)\right]^{1/2}}$$

and using equation (17.19)

$$= \frac{\left[\sum_{i=1}^{N}\sum_{k=1}^{N} b_{ik} p_k^0 p_i^1 \Big/ \left[r\left(p^0\right)\right]^2\right]^{1/2}}{\left[\sum_{i=1}^{N}\sum_{k=1}^{N} b_{ik} p_k^1 p_i^0 \Big/ \left[r\left(p^1\right)\right]^2\right]^{1/2}}$$

using equation (17.17) and canceling terms

$$= \left[1 \Big/ \left[r\left(p^0\right)\right]^2\right]^{1/2} \Bigg/ \left[1 \Big/ \left[r\left(p^1\right)\right]^2\right]^{1/2}$$

$$= r\left(p^1\right) \Big/ r\left(p^0\right).$$

Thus, under the assumption that the producer engages in revenue-maximizing behavior during periods 0 and 1 and has technologies that satisfy the separability assumption, and that the unit revenue function is homogeneous quadratic, then the Fisher ideal price index P_F is *exactly* equal to the true price index, $r(p^1) / r(p^0)$.[28]

17.42 Since the homogeneous quadratic unit revenue function *r(p)* defined by equation (17.16) is also a flexible functional form, the fact that the Fisher ideal price index P_F exactly equals the true price index $r(p^1) / r(p^0)$ means that P_F is a *superlative index number formula*.[29]

B.5 Superlative output price indices

B.5.1 A general class of superlative output price indices

17.43 There are many other superlative index number formulas; that is, there exist many quantity indices $Q(p^0,p^1,q^0,q^1)$ that are exactly equal to $f(q^1) / f(q^0)$ and many price indices $P(p^0,p^1,q^0,q^1)$ that are exactly equal to $r(p^1) / r(p^0)$, where the aggregator function f or the unit revenue function r is a flexible functional form. Two families of superlative indices are defined below—quantity indices and price indices.

17.44 Suppose that the producer's output aggregator function[30] is the *following quadratic mean of order r aggregator function*:

$$(17.22)\ \ f^r\left(q_1,...,q_N\right) \equiv \left[\sum_{i=1}^{N}\sum_{k=1}^{N} a_{ik} q_i^{r/2} q_k^{r/2}\right]^{1/r},$$

where the parameters a_{ik} satisfy the symmetry conditions $a_{ik} = a_{ki}$ for all i and k, and the parameter r satisfies the restriction $r \neq 0$. Diewert (1976,

[28]This result was obtained by Diewert (1976, pp. 133–34) in the consumer context.

[29]Note that the Fisher index P_F is exact for the unit revenue function defined by equation (17.16). These two output aggregator functions do not coincide in general. However, if the N by N symmetric matrix **A** of the a_{ik} has an inverse, then it readily can be shown that the N by N matrix **B** of the b_{ik} will equal $\mathbf{A}^{-1}$.

[30]This terminology is credited to Diewert (1976, p. 129). This functional form was first defined by Denny (1974) as a unit cost function.

p. 130) showed that the aggregator function f^r defined by equation (17.22) is a flexible functional form; that is, it can approximate an arbitrary twice continuously differentiable linearly homogeneous functional form to the second order. Note that when $r = 2$, r^r equals the homogeneous quadratic function defined by equation (17.16) above.

17.45 Define the *quadratic mean of order* r *quantity index* Q^r by

$$(17.23)\ Q^r\left(p^0,p^1,q^0,q^1\right) \equiv \left[\sum_{i=1}^{n} s_i^0\left(\frac{q_i^1}{q_i^0}\right)^{r/2}\right]^{1/r}\left[\sum_{i=1}^{n} s_i^1\left(\frac{q_i^1}{q_i^0}\right)^{-r/2}\right]^{1/-r},$$

where $s_i^t = p_i^t q_i^t \Big/ \sum_{i=1}^{N} p_i^t q_i^t$ is the period t revenue share for output i as usual. It can be verified that when $r = 2$, Q^r simplifies into Q_F, the Fisher ideal quantity index.

17.46 Using exactly the same techniques as were used in Section B.3, it can be shown that Q^r is exact for the aggregator function f^r defined by equation (17.22); that is,

$$(17.24)\ Q^r(p^0,p^1,q^0,q^1) = f^r\left(q^1\right)\Big/f^r\left(q^0\right).$$

Thus, under the assumption that the producer engages in revenue-maximizing behavior during periods 0 and 1 and has technologies that satisfy a linearly homogeneous aggregator function for outputs[31] where the output aggregator function $f(q)$ is defined by equation (17.22), then the quadratic mean of order r quantity index Q_F is *exactly* equal to the true quantity index, $f^r(q^1) / f^r(q^0)$.[32] Since Q^r is exact for f^r, and f^r is a flexible functional form, the quadratic mean of order r quantity index Q^r is a *superlative index* for each $r \neq 0$. Thus, there are an infinite number of superlative quantity indices.

17.47 For each quantity index Q^r, the product test in equation (15.3) can be used to define the corresponding *implicit quadratic mean of order* r *price index* P^r*:

$$(17.25)\ P^{r*}(p^0,p^1,q^0,q^1) \equiv \sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \left[p_i^0 q_i^0\, Q^r\left(p^0,p^1,q^0,q^1\right)\right] = r^{r*}\left(p^1\right)\Big/r^{r*}\left(p^0\right),$$

where r^r* is the unit revenue function that corresponds to the aggregator function f^r defined by equation (17.22). For each $r \neq 0$, the implicit quadratic mean of order r price index P^r* is also a superlative index.

17.48 When $r = 2$, Q^r defined by equation (17.23) simplifies to Q_F, the Fisher ideal quantity index, and P^r* defined by equation (17.25) simplifies to P_F, the Fisher ideal price index. When $r = 1$, Q^r defined by equation (17.23) simplifies to

$$(17.26)\ Q^1(p^0,p^1,q^0,q^1) \equiv \left[\sum_{i=1}^{n} s_i^0\left(\frac{q_i^1}{q_i^0}\right)^{1/2}\right]^1 \Big/ \left[\sum_{i=1}^{n} s_i^1\left(\frac{q_i^1}{q_i^0}\right)^{-1/2}\right]^{-1}$$
$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0}\left[\sum_{i=1}^{N} p_i^0 q_i^0\left(\frac{q_i^1}{q_i^0}\right)^{1/2}\right]^1 \Big/ \left[\sum_{i=1}^{N} p_i^1 q_i^1\left(\frac{q_i^1}{q_i^0}\right)^{-1/2}\right]^{-1}$$
$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0}\left[\sum_{i=1}^{N} p_i^0\left(q_i^0 q_i^1\right)^{1/2}\right]^1 \Big/ \left[\sum_{i=1}^{N} p_i^1\left(q_i^0 q_i^1\right)^{-1/2}\right]^{-1}$$
$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0}\left[\sum_{i=1}^{N} p_i^1\left(q_i^0 q_i^1\right)^{1/2}\right] \Big/ \left[\sum_{i=1}^{N} p_i^0\left(q_i^0 q_i^1\right)^{1/2}\right]$$
$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0} \Big/ \left[P_W\left(p^0,p^1,q^0,q^1\right)\right],$$

where P_W is the *Walsh price index* defined previously by equation (15.19) in Chapter 15. Thus P^1* is equal to P_W, the *Walsh price index*, and hence it is also a superlative price index.

[31]This method for justifying aggregation over commodities is due to Shephard (1953, pp. 61–71). It is assumed that $f(q)$ is an increasing, positive, and convex function of q for positive q. Samuelson and Swamy (1974) and Diewert (1980, pp. 438–42) also developed this approach to index number theory.

[32]See Diewert (1976, p. 130).

17.49 Suppose the producer's unit revenue function[33] is the following quadratic mean of order r unit revenue function:

$$(17.27)\ r^r(p_1,\ldots,p_n) \equiv \left[\sum_{i=1}^{N}\sum_{k=1}^{N} b_{ik}\, p_i^{r/2} p_k^{r/2}\right]^{1/r},$$

where the parameters b_{ik} satisfy the symmetry conditions $b_{ik} = b_{ki}$ for all i and k and the parameter r satisfies the restriction $r \neq 0$. Diewert (1976, p. 130) showed that the unit revenue function r^r defined by equation (17.27) is a flexible functional form; i.e., it can approximate an arbitrary twice continuously differentiable linearly homogeneous functional form to the second order. Note again that when $r = 2$, r^r equals the homogeneous quadratic function defined by equation (17.16) above.

17.50 Define the quadratic mean of order r price index P^r by:

$$(17.28)\ P^r(p^0,p^1,q^0,q^1) \equiv \left[\sum_{i=1}^{n} s_i^0 \left(\frac{p_i^1}{p_i^0}\right)^{r/2}\right]^{1/r} \left[\sum_{i=1}^{n} s_i^1 \left(\frac{p_i^1}{p_i^0}\right)^{-r/2}\right]^{-1/r}$$

where $s_i^t = p_i^t q_i^t \Big/ \sum_{i=1}^{N} p_i^t q_i^t$ is the period t revenue share for output i as usual. It can be verified that when $r = 2$, P^r simplifies into P_F, the Fisher ideal price index.

17.51 Using exactly the same techniques as were used in Section B.3, it can be shown that P^r is exact for the unit revenue function r^r defined by (17.27); that is,

$$(17.29)\ P^r(p^0,p^1,q^0,q^1) = r^r\left(p^t\right) \Big/ r^r\left(p^0\right).$$

Thus, under the assumption that the producer engages in revenue-maximizing behavior during periods 0 and 1 and has technologies that are homogeneously weakly separable where the output aggregator function $f(q)$ corresponds to the unit revenue function $r^r(p)$ defined by (17.27), then the quadratic mean of order r price index P^r is *exactly* equal to the true output price index, $r^r(p^1)/r^r(p^0)$.[34] Since P^r is exact for r^r and r^r is a flexible functional form, that the quadratic mean of order r price index P^r is a *superlative index* for each $r \neq 0$. Thus there are an infinite number of superlative price indices.

17.52 For each price index P^r, the product test (15.3) can be used in order to define the corresponding *implicit quadratic mean of order r quantity index* $Q^{r}*$:

$$(17.30)\ Q^r{*}(p^0,p^1,q^0,q^1) \equiv \sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \left\{p_i^0 q_i^0 P^r\left(p^0,p^1,q^0,q^1\right)\right\} = f^{r*}\left(p^1\right) \Big/ f^{r*}\left(p^0\right)$$

where $f^{r}*$ is the aggregator function that corresponds to the unit cost function r^r defined by (17.27) above.[35] For each $r \neq 0$, the implicit quadratic mean of order r quantity index Q^r* is also a superlative index.

17.53 When $r = 2$, P^r defined by (17.28) simplifies to P_F, the Fisher ideal price index and Q^r* defined by (17.30) simplifies to Q_F, the Fisher ideal quantity index. When $r = 1$, P^r defined by (17.28) simplifies to:

$$(17.31)\ P^1(p^0,p^1,q^0,q^1)$$

$$\equiv \left[\sum_{i=1}^{n} s_i^0 \left(\frac{p_i^1}{p_i^0}\right)^{1/2}\right]^1 \Bigg/ \left[\sum_{i=1}^{n} s_i^1 \left(\frac{p_i^1}{p_i^0}\right)^{-1/2}\right]^{-1}$$

$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0} \left[\sum_{i=1}^{N} p_i^0 q_i^0 \left(\frac{p_i^1}{p_i^0}\right)^{1/2}\right]^1 \Bigg/ \left[\sum_{i=1}^{N} p_i^1 q_i^1 \left(\frac{p_i^1}{p_i^0}\right)^{-1/2}\right]^{-1}$$

[33]Again, the approach here is by way of a unit revenue function. An alternative formulation is via a quadratic mean of order r superlative quantity index. Using the product rule, the quantity index defines an implicit quadratic mean of order r price index that also is a superlative index.

[34]See Diewert (1976, pp. 133–34).

[35]The function f^r* can be defined by using r^r as follows: $f^r{*}(q) \equiv \max_p \left\{\sum_{i=1}^{n} p_i q_i : r^r\left(p\right) = 1\right\}$.

$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0} \left[\sum_{i=1}^{N} q_i^0 \left(p_i^0 p_i^1 \right)^{1/2} \right]^1 \Bigg/ \left[\sum_{i=1}^{N} q_i^1 \left(p_i^0 p_i^1 \right)^{-1/2} \right]^{-1}$$

$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0} \left[\sum_{i=1}^{N} q_i^1 \left(p_i^0 p_i^1 \right)^{1/2} \right] \Bigg/ \left[\sum_{i=1}^{N} q_i^0 \left(p_i^0 p_i^1 \right)^{1/2} \right]$$

$$= \frac{\sum_{i=1}^{N} p_i^1 q_i^1}{\sum_{i=1}^{N} p_i^0 q_i^0} \Bigg/ \left[Q_W \left(p^0, p^1, q^0, q^1 \right) \right],$$

where Q_W is the *Walsh quantity index* defined previously by equation (16.34) in Chapter 16. Thus $Q^{1}*$ is equal to Q_W, the Walsh (1901; 1921) quantity index, and hence it is also a superlative quantity index.

17.54 Essentially, the economic approach to index number theory provides reasonably strong justifications for the use of the Fisher price index P_F defined by equation (15.12) or equation (17.9), the Törnqvist-Theil price index P_T defined by equation (16.22) or equation (17.10), and the quadratic mean of order r price indices P^r defined by equation (17.28) (when $r = 1$, this index is the Walsh price index defined by equation [15.19] in Chapter 15). It is now necessary to ask if it matters which one of these formulas is chosen as best.

B.5.2 Approximation properties of superlative indices

17.55 The analysis in this chapter has led to three superlative index number formulas, the Fisher price index, the Törnqvist-Theil price index, and the Walsh price index, all of which appear to have good properties from the viewpoint of the economic approach to index number theory.

17.56 Two questions arise as a consequence of these results:

- Does it matter which formula is chosen?
- If it does matter, which formula should be chosen?

With respect to the first question, the justifications for the Törnqvist index presented in Section B.3 are stronger than the justifications for the other superlative indices presented in Section B.2, because the economic derivation did not rely on restrictive separability assumptions. The justification for the Fisher index, however, took a different form. Economic theory established that Laspeyres and Paasche bounded a true index, and axiomatic grounds were found for the Fisher being the best average of the two. However, Diewert (1978, p. 888) showed that the three superlative index number formulas listed approximate each other to the second order around any point where the two price vectors, p^0 and p^1, are equal and where the two quantity vectors, q^0 and q^1, are equal. He concluded that "all superlative indices closely approximate each other" (Diewert, 1978, p. 884).

17.57 However, the above conclusion requires a caveat. The problem is that the quadratic mean of order r price indices P^r is a (continuous) function of the parameter r. Hence, as r becomes very large in magnitude, the index P^r can differ substantially from, say, $P^2 = P_F$, the Fisher ideal index. In fact, using equation (17.28) and the limiting properties of means of order r,[36] R.J. Hill (2000, p. 7) showed that P^r has the following limit as r approaches plus or minus infinity:

$$(17.32)\ \lim_{r\to+\infty} P^r(p^0,p^1,q^0,q^1) = \lim_{r\to-\infty} P^r(p^0,p^1,q^0,q^1) = [\min_i\{p_i^1/p_i^0\}\max_i\{p_i^1/p_i^0\}]^{1/2}.$$

Thus for r large in magnitude, P^r can differ substantially from the Törnqvist-Theil price index, the Walsh price index, and the Fisher ideal index.[37]

17.58 Although R.J. Hill's theoretical and empirical results demonstrate conclusively that all superlative indices will not necessarily closely approximate each other, there is still the question of how well the more commonly used superlative indices will approximate each other. All of the commonly used superlative indices, P^r and P^r*, fall into the interval $0 \le r \le 2$. Diewert (1980,

[36]See Hardy, Littlewood, and Polyá (1934). Actually, Allen and Diewert (1981, p. 434) obtained the result (17.32) but they did not appreciate its significance.

[37]R.J. Hill (2000) documents this for two data sets. His time-series data consists of annual expenditure and quantity data for 64 components of U.S. GDP from 1977 to 1994. For this data set, Hill (2000, p. 16) found that "superlative indices can differ by more than a factor of two (i.e., by more than 100 percent), even though Fisher and Törnqvist never differ by more than 0.6 percent."

p. 451) showed that the Törnqvist index P_T is a limiting case of P^r as r tends to 0. R.J. Hill (2000, p. 16) summarized how far apart the Törnqvist and Fisher indices were making all possible bilateral comparisons between any two data points for his time-series data set as follows:

> The superlative spread $S(0,2)$ is also of interest since, in practice, Törnqvist ($r = 0$) and Fisher ($r = 2$) are by far the two most widely used superlative indices. In all 153 bilateral comparisons, $S(0,2)$ is less than the Paasche-Laspeyres spread and on average, the superlative spread is only 0.1 percent. It is because attention, until now, has focused almost exclusively on superlative indices in the range $0 \leq r \leq 2$ that a general misperception has persisted in the index number literature that all superlative indices approximate each other closely.

17.59 Thus for R.J. Hill's time-series data set covering 64 components of U.S. GDP from 1977 to 1994 and making all possible bilateral comparisons between any two years, the Fisher and Törnqvist price indices differed by only 0.1 percent on average. This close correspondence is consistent with the results of other empirical studies using annual time-series data.[38] Additional evidence on this topic may be found in Chapter 19.

17.60 A reasonably strong justification has been provided by the economic approach for a small group of index numbers: the *Fisher ideal index* $P_F = P^2 = P^2*$ defined by equation (15.12) or equation (17.9), the *Törnqvist-Theil index* P_T defined by equations (17.10) or (15.81), and the *Walsh index* P_W defined by equation (15.19) (which is equal to the implicit quadratic mean of order r price indices P^r* defined by equation (17.25) when $r = 1$). They share the property of being *superlative* and approximate each other to the second order around any point. This indicates that for normal time-series data, these three indices will give virtually the same answer. The economic approach gave particular support to the Fisher and Törnqvist-Theil indices, albeit on different grounds. The Fisher index was advocated as the only symmetrically weighted average of Laspeyres and Paasche bounds that satisfied the time reversal test. Economic theory argued for the existence of Laspeyres and Paasche bounds on a suitable true theoretical index. The support for the Törnqvist index arose from its requiring less restrictive assumptions to show it was superlative than the Fisher and Walsh indices. The Törnqvist-Theil index seemed to be best from the stochastic viewpoint, and the Fisher ideal index was supported from the axiomatic viewpoint in that it best satisfied the quite reasonable tests presented. The Walsh index seemed to be best from the viewpoint of the pure price index. To determine precisely which one of these three alternative indices to use as a theoretical target or actual index, the statistical agency will have to decide which approach to bilateral index number theory is most consistent with its goals. It is reassuring that, as illustrated in Chapter 19, for normal time series data, these three indices give virtually the same answer.

C. Economic Approach to an Intermediate Input Price Index for an Establishment

17.61 Attention now is turned to the economic theory of the intermediate input price index for an establishment. This theory is analogous to the economic theory of the output price index explained in Section B but now uses the *joint cost function* or the *conditional cost function* C in place of the revenue function r that was used in Section B. Section E will continue the analysis in a similar vein for the value-added deflator. The approach in this section for the intermediate input price index is analogous to the Konüs (1924) theory for the true cost-of-living index in consumer theory.

17.62 Recall that the set $S^t(v^t)$ describes what output vectors y can be produced in period t if the establishment has at its disposal the vector of inputs $v \equiv [x,z]$, where x is a vector of intermediate inputs and z is a vector of primary inputs. Thus if $[y,x,z]$ belongs to S^t, then the nonnegative output vector y can be produced by the establishment in period t, if it can use the nonnegative vector x of intermediate inputs and the nonnegative vector z of primary inputs.

17.63 Let $p_x \equiv (p_{x1},\ldots p_{xM})$ denote a positive vector of intermediate input prices that the establishment might face in period t, let y be a nonnegative vector of output targets, and let z be a nonnegative vector of primary inputs that the establishment might have available for use during period t. Then the establishment's *conditional cost function* using

[38]See, for example, Diewert (1978, p. 894) or Fisher (1922), which is reproduced in Diewert (1976, p. 135).

period t technology is defined as the solution to the following intermediate input cost minimization problem:

(17.33) $C^t(p_x,y,z) \equiv \min_x \{\sum_{m=1}^{M} p_{xm}x_m : [y,x,z]$ belongs to $S^t\}$.

Thus $C^t(p_x,y,z)$ is the minimum intermediate input cost, $\sum_{m=1}^{M} p_{xm}x_m$, that the establishment must pay to produce the vector of outputs y, given that it faces the vector of intermediate input prices p_x and the vector of primary inputs z is available for use, using the period t technology.[39]

17.64 To make the notation for the intermediate input price index comparable to the notation used in Chapters 15 and 16 for price and quantity indices, in the remainder of this subsection the intermediate input price vector p_x is replaced by the vector p, and the vector of intermediate quantities x is replaced by the vector q. Thus $C^t(p_x,y,z)$ is rewritten as $C^t(p,y,z)$.

17.65 The period t conditional cost function C^t can be used to define the economy's *period* t *technology intermediate input price index* P^t between any two periods, say, period 0 and period 1, as follows:

(17.34) $P^t(p^0,p^1,y,z) = C^t(p^1,y,z) / C^t(p^0,y,z)$,

where p^0 and p^1 are the vectors of intermediate input prices that the establishment faces in periods 0 and 1, respectively; y is a reference vector of outputs that the establishment must produce, and z is a reference vector of primary inputs.[40] If $M = 1$, so that there is only one intermediate input that the establishment uses, then it can be shown that the intermediate input price index collapses to the single intermediate input price relative between periods 0 and 1, p_1^1/p_1^0. In the general case, note that the intermediate input price index defined by equation (17.34) is a ratio of hypothetical intermediate input costs that the establishment must pay to produce the vector of outputs y, given that it has the period t technology and the vector of primary inputs v to work with. The numerator in equation (17.34) is the minimum intermediate input cost that the establishment could attain if it faced the intermediate input prices of period 1, p^1, while the denominator in equation (17.34) is the minimum intermediate input cost that the establishment could attain if it faced the output prices of period 0, p^0. Note that all variables in the numerator and denominator of equation (17.34) are held constant except the vectors of intermediate input prices.

17.66 As was the case with the theory of the output price index, there are a wide variety of price indices in equation (17.34) depending on which reference vector (t,y,z) is chosen (the reference technology is indexed by t, the reference output vector is indexed by y, and the reference primary input vector is indexed by z). As in the theory of the output price index, two special cases of the general definition of the intermediate input price index, equation (17.34), are of interest: (i) $P^0(p^0,p^1,y^0,z^0)$, which uses the period 0 technology set, the output vector y^0 produced in period 0, and the primary input vector z^0 used in period 0; and (ii) $P^1(p^0,p^1,y^1,z^1)$, which uses the period 1, technology set, the output vector y^1 produced in period 1, and the primary input vector z^1 used in period 1. Let I^0 and q^1 be the observed intermediate input vectors for the establishment in periods 0 and 1, respectively. If there is cost-minimizing behavior on the part of the producer in periods 0 and 1, then the observed intermediate input cost in periods 0 and 1 should equal $C^0(p^0,y^0,z^0)$ and $C^1(p^1,y^1,z^1)$, respectively; that is, the following equalities should hold:

(17.35) $C^0(p^0,y^0,z^0) = \sum_{m=1}^{M} p_m^0 q_m^0$ and

$$C^1(p^1,y^1,z^1) = \sum_{m=1}^{M} p_m^1 q_m^1 .$$

17.67 Under these cost-minimizing assumptions, adapt the arguments of Fisher and Shell (1972, pp. 57–58) and Archibald (1977, p. 66) to show that the two theoretical indices, $P^0(p^0,p^1,y^0,z^0)$ and

[39]See McFadden (1978) for the mathematical properties of a conditional cost function. Alternatively, note that $-C^t(p_x,y,z)$ has the same mathematical properties as the revenue function R^t defined earlier in this chapter.

[40]This concept of the intermediate input price index is analogous to the import price index defined in Alterman, Diewert, and Feenstra (1999). If the vector of primary inputs is omitted from equation (17.34), then the resulting intermediate input price index reduces to the physical production cost index defined by Court and Lewis (1942–43, p. 30).

$P^1(p^0,p^1,y^1,z^1)$ described in (i) and (ii) above, satisfy the inequalities of equations (17.36) and (17.37):

(17.36) $P^0(p^0,p^1,y^0,z^0) \equiv C^0(p^1,y^0,z^0) / C^0(p^0,y^0,z^0)$

using equation (17.34)

$$= C^0(p^1,y^0,z^0) \Big/ \sum_{m=1}^{M} p_m^0 q_m^0$$

using equation (17.35)

$$\leq \sum_{m=1}^{M} p_m^1 q_m^0 \Big/ \sum_{m=1}^{M} p_m^0 q_m^0 ,$$

since q^0 is feasible for the minimization problem that defines $C^0(p^1,y^0,z^0)$, and so

$$C^0(p^1,y^0,z^0) \leq \sum_{m=1}^{M} p_m^1 q_m^0 \equiv P_L(p^0,p^1,q^0,q^1),$$

where P_L is the Laspeyres intermediate input price index. Similarly,

(17.37) $P^1(p^0,p^1,y^1,z^1) \equiv C^1(p^1,y^1,z^1) / C^1(p^0,y^1,z^1)$

using equation (17.34)

$$= \sum_{m=1}^{M} p_m^1 q_m^1 \Big/ C^1\left(p^0, y^1, z^1\right)$$

using equation (17.35)

$$\geq \sum_{m=1}^{M} p_m^1 q_m^1 \Big/ \sum_{m=1}^{M} p_m^0 q_m^1 ,$$

since q^1 is feasible for the minimization problem that defines $C^1(p^0,y^1,z^1)$, and so

$$C^1(p^0,y^1,z^1) \leq \sum_{m=1}^{M} p_m^0 q_m^1 \equiv P_P(p^0,p^1,q^0,q^1),$$

where P_P is the Paasche price index. Thus, equation (17.36) says that the observable Laspeyres index of intermediate input prices, P_L , is an *upper bound* to the theoretical intermediate input price index, $P^0(p^0,p^1,y^0,z^0)$, and the equation (17.37) says that the observable Paasche index of intermediate input prices, P_P, is a *lower bound* to the theoretical intermediate input price index, $P^1(p^0,p^1,y^1,z^1)$. Note that these inequalities are the reverse of earlier equations (17.4) and (17.5) found for the output price index, but the new inequalities are analogous to their counterparts in the theory of the true cost-of-living index.

17.68 As was the case in Section B.2, it is possible to define a theoretical intermediate input price index that falls *between* the observable Paasche and Laspeyres intermediate input price indices. To do this, first define a *hypothetical intermediate input cost function*, $C(p,\alpha)$, that corresponds to the use of an α weighted average of the technology sets $S^0(y^0,z^0)$ and $S^1(y^1,z^1)$ for periods 0 and 1 as the reference technology and that uses an α-weighted average of the period 0 and period 1 output vectors y^0 and y^1 and primary input vectors z^0 and z^1 as the reference output and primary input vectors:

(17.38) $C(p,\alpha)$

$$\equiv \min q \; \{\sum_{m=1}^{M} p_m q_m : q \text{ belongs to } (1-\alpha)\, S^0(y^0,z^0) + \alpha\, S^1(y^1,z^1)\}.$$

Thus, the intermediate input cost minimization problem in equation (17.38) corresponds to the intermediate output target $(1-\alpha)y^0 + \alpha y^1$ and the use of an average of the period 0 and 1 primary input vectors z^0 and z^1 , where the period 0 vector gets the weight $1-\alpha$ and the period 1 vector gets the weight α. An average is used of the period 0 and period 1 technology sets, where the period 0 set gets the weight $1-\alpha$ and the period 1 set gets the weight α, where α is a number between 0 and 1. The new intermediate input cost function defined by equation (17.38) now can be used to define the following *family of theoretical intermediate input price indices*:

(17.39) $P(p^0,p^1,\alpha) \equiv C(p^1,\alpha) / C(p^0,\alpha)$.

17.69 Adapting the proof of Diewert (1983a, pp. 1060–61) shows that there exists an α between 0 and 1 such that the theoretical intermediate input price index defined by equation (17.39) lies between the observable (in principle) Paasche and Laspeyres intermediate input price indices, P_P and P_L ; that is, there exists an α such that

(17.40) $P_L \leq P(p^0,p^1,\alpha) \leq P_P$
or $P_P \leq P(p^0,p^1,\alpha) \leq P_L$.

17.70 If the Paasche and Laspeyres indices are numerically close to each other, then equation (17.40) tells us that a true economic intermediate input price index is fairly well determined, and a reasonably close approximation to the true index can be found by taking a symmetric average of

P_L and P_P such as the geometric average, which again leads to Irving Fisher's (1922) ideal price index, P_F, defined earlier by equation (17.40).

17.71 It is worth noting that the above theory of the economic intermediate input price indices was very general; in particular, no restrictive functional form or separability assumptions were made on the technology.

17.72 The translog technology assumptions used in Section B.3 to justify the use of the Törnqvist-Theil output price index as an approximation to a theoretical output price index can be adapted to yield a justification for the use of the Törnqvist-Theil intermediate input price index as an approximation to a theoretical intermediate input price index. Recall the definition of the period t conditional intermediate input cost function, $C^t(p_x,y,z)$, defined by equation (17.33). Replace the vector of intermediate input prices p_x by the vector p, and define the $N + K$ vector u as $u \equiv [y,z]$. Now assume that the period t conditional cost function has the following *translog functional form* : for t = 0,1 :

(17.41) $\ln C^t(p,u)$

$$= \alpha_0^t + \sum_{m=1}^{M} \alpha_m^t \ln p_m + \sum_{j=1}^{N+K} \beta_j^t \ln u_j$$
$$+ \frac{1}{2}\sum_{m=1}^{M}\sum_{j=1}^{M} \alpha_{mj}^t \ln p_m \ln p_j$$
$$+ \sum_{m=1}^{M}\sum_{n=1}^{N+K} \beta_{mn}^t \ln p_m \ln u_n$$
$$+ \frac{1}{2}\sum_{n=1}^{N+K}\sum_{k=1}^{N+K} \gamma_{nk}^t \ln u_n \ln u_k \ ,$$

where the $\alpha_n{}^t$ and the $\gamma_n{}^t$ coefficients satisfy the following restrictions:

(17.42) $\alpha_{mj}{}^t = \alpha_{jm}{}^t$ for all m,j and for $t = 0,1$;

(17.43) $\gamma_{nk}{}^t = \gamma_{kn}{}^t$ for all k,n and for $t = 0,1$;

(17.44) $\sum_{m=1}^{M} \alpha_m^t = 1$ for $t = 0,1$; and

(17.45) $\sum_{m=1}^{M} \alpha_m^t = 0$ for $t = 0,1$ and $m = 1,2,\ldots,M$.

The restrictions in equations (17.44) and (17.45) are necessary to ensure that $C^t(p,u)$ is linearly homogeneous in the components of the intermediate input price vector p (which is a property that a conditional cost function must satisfy). Note that at this stage of our argument the coefficients that characterize the technology in each period (the αs, βs, and γs) are allowed to be completely different in each period.

17.73 Adapting the result in Caves, Christensen, and Diewert (1982b, p. 1410) to the present context;[41] if the quadratic price coefficients in equation (17.41) are equal across the two periods where an index number comparison (that is, $\alpha_{mj}{}^0 = \alpha_{mj}{}^1$ for all m,j) is being made, then the geometric mean of the economic intermediate input price index that uses period 0 technology, the period 0 output vector y^0, and the period 0 vector of primary inputs z^0, $P^0(p^0,p^1,y^0,z^0)$, and the economic intermediate input price index that uses period 1 technology, the period 1 output vector y^1, and the period 1 primary input vector z^1, $P^1(p^0,p^1,y^1,z^1)$, is *exactly* equal to the Törnqvist intermediate input price index P_T defined by equation (17.10);[42] that is,

(17.46) $P_T(p^0,p^1,q^0,q^1)$
$$= [P^0(p^0,p^1,y^0,z^0)\, P^1(p^0,p^1,y^1,z^1)]^{1/2} .$$

17.74 As was the case with our previous result in equation (17.40), the assumptions required for the result (17.46) seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period, and our assumptions are consistent with comparing technological progress occurring between the two periods. Because the index number formula P_T is *exactly* equal to the geometric mean of two theoretical economic intermediate input price index, and this corresponds to a flexible functional form, the Törnqvist intermediate input index number formula is said to be *superlative*.

[41]The Caves, Christensen, and Diewert translog exactness result is slightly more general than a similar translog exactness result obtained earlier by Diewert and Morrison (1986, p. 668); Diewert and Morrison assumed that all of the quadratic terms in equation (17.41) were equal during the two periods under consideration, whereas Caves, Christensen, and Diewert assumed only that $\alpha_{mj}{}^0 = \alpha_{mj}{}^1$ for all m,j.

[42]In the present context, output prices are replaced by intermediate input prices, and the number of terms in the summation of terms defined by equation (17.10) is changed from N to M.

17.75 It is possible to adapt the analysis of the output price index that was developed in Sections C.3 and C.4 to the intermediate input price index, and show that the two families of superlative output price indices, P^{r*} defined by equation (17.25) and P^{r} defined by equation (17.23), also are superlative intermediate input price indices. However, the details are omitted here since to derive these results, rather restrictive separability restrictions are required on the technology of the establishment.[43]

17.76 In the following section, the analysis presented in this section is modified to provide an economic approach to the value-added deflator.

D. Economic Approach to the Value-Added Deflator for an Establishment

17.77 Attention now is turned to the economic theory of the value-added deflator for an establishment. This theory is analogous to the economic theory of the output price index explained in Section B, but now the *profit function* π is used in place of the revenue function r used in Section B.

17.78 Recall that the set S^t describes which output vectors y can be produced in period t if the establishment has at its disposal the vector of inputs $[x,z]$, where x is a vector of intermediate inputs and z is a vector of primary inputs. Thus, if $[y,x,z]$ belongs to S^t, then the nonnegative output vector y can be produced by the establishment in period t, if it can use the nonnegative vector x of intermediate inputs and the nonnegative vector z of primary inputs.

17.79 Let $p_y \equiv (p_{y1},\ldots p_{yN})$ and $p_x \equiv (p_{x1},\ldots p_{xM})$ denote positive vectors of output and intermediate input prices that the establishment might face in period t, and let z be a nonnegative vector of primary inputs that the establishment might have available for use during period t. Then the establishment's *(gross) profit function* or *net revenue function* using period t technology is defined as the solution to the following net revenue maximization problem:

$$(17.47)\ \pi^t(p_y,p_x,z) \equiv \max_{y,x} \left\{ \sum_{n=1}^{N} p_{yn} y_n - \sum_{m=1}^{M} p_{xm} x_m \quad : (y,x) \text{ belongs to } S^t(z) \right\},$$

where, as usual, $y \equiv [y_1,\ldots,y_N]$ is an output vector and $x \equiv [x_1,\ldots,x_M]$ is an intermediate input vector. Thus, $\pi^t(p_y,p_x,z)$ is the maximum output revenue, $\sum_{n=1}^{N} p_{yn} y_n$, less intermediate input cost, $\sum_{m=1}^{M} p_{xm} x_m$, that the establishment could generate, given that it faces the vector of output prices p_y and the vector of intermediate input prices p_x, and given that the vector of primary inputs z is available for use, using the period t technology.[44]

17.80 To make the notation for the value-added deflator comparable to the notation used in Chapters 15 and 16 for price and quantity indices, in the remainder of this subsection, the *net output price vector* p is defined as $p \equiv [p_y,p_x]$, and the *net output quantity vector* q is defined as $q \equiv [y,-x]$. Thus, all output and intermediate input prices are positive, output quantities are positive, but intermediate inputs are indexed with a minus sign. With these definitions, $\pi^t(p_y,p_x,z)$ can be rewritten as $\pi^t(p,z)$.

17.81 The period t profit function π^t can be used to define the economy's *period* t *technology value added deflator* P^t between any two periods, say, period 0 and period 1, as follows:[45]

$$(17.48)\ P^t(p^0,p^1,z) = \pi^t(p^1,z) / \pi^t(p^0,z),$$

where p^0 and p^1 are the $N + M$ dimensional vectors of net output prices that the establishment faces in periods 0 and 1, and z is a reference vector of primary inputs. Note that all variables in the numerator and denominator of equation (17.48) are held

[43] The counterpart to our earlier separability assumption in equation (17.15) is now $z_1 = F^t(y,x,z_2,\ldots,z_K) = G^t(y,f(x),z_2,\ldots,z_K)$ for $t = 0,1$, where the intermediate input aggregator function f is linearly homogeneous and independent of t.

[44] The profit function π^t has the same mathematical properties as the revenue function R^t.

[45] If there are no intermediate inputs, this concept reduces to Archibald's (1977) fixed-input quantity output price index. In the case where there is no technical progress between the two periods, this concept reduces to Diewert's (1980, pp. 455–61) (net) output price deflator. Diewert (1983a) considered the general concept, which allows for technical progress between periods.

constant, except the vectors of net output (output and intermediate input) prices.

17.82 As was the case with the theory of output price index, there are various price indices of the form of equation (17.48), depending on which reference vector (t,z) is chosen. The analysis follows that of the output price index in Section B. As in the theory of the output price index, interest lies in two special cases of the general definition of the intermediate input price index of the form of equation (17.48): a theoretical index that uses the period 0 technology set and the primary input vector z^0 used in period 0, and one that uses the period 1 technology set and the primary input vector z^1 used in period 1. The observable Laspeyres index of output and intermediate input prices P_L is shown to be a *lower bound* to the former theoretical value-added deflator, and the observable Paasche index of output and intermediate input prices P_P is an *upper bound* to the latter theoretical value-added deflator.[46] These inequalities go in the same direction as the earlier inequalities of equations (17.4) and (17.5) obtained for the output price index.

17.83 As was the case in Section B.2, it is possible to define a value-added deflator that falls *between* the observable Paasche and Laspeyres value-added deflators. To do this, a *hypothetical net revenue function*, $\pi(p,\alpha)$, is defined to correspond to an α-weighted average of the period 0 and 1 technology sets, and an α-weighted average of the primary input vectors z^0 and z^1 is used as the reference primary input vector.

17.84 Following the arguments made for the output price index if the Paasche and Laspeyres indices are numerically close, then a true economic value-added deflator is fairly well determined. A reasonably close approximation to the true index is a symmetric average of P_L and P_P , such as the geometric average, which again leads to Irving Fisher's ideal price index.[47]

17.85 The translog technology assumptions used in Section B.3 to justify the use of the Törnqvist-Theil output price index as an approximation to a theoretical output price index can be adapted to yield a justification for the use of the Törnqvist-Theil value-added price index as an approximation to a theoretical value-added deflator. Recall the definition of the period t net revenue function, $\pi^t(p_y,p_x,z)$, defined by equation (17.47). Replace the vectors of output prices p_y and the vector of intermediate input prices p_x by the vector $p \equiv [p_y,p_x]$, and assume that the period t net revenue function has the *translog functional form.* Following the argument for the output price index, if the quadratic price coefficients are equal across the two periods, Törnqvist value-added deflator is exactly equal to this form of the theoretical index. Because the index number formula is *exactly* equal to an underlying *flexible* functional form, the Törnqvist value-added deflator formula is *superlative*. As was the case with the output price index, the assumptions required for this finding seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period, and the assumptions are consistent with technological progress occurring between the two periods being compared.

17.86 It is possible to adapt the analysis of the output price index developed in Sections B.4 and B.5 to the value-added deflator and show that the family of superlative output price indices, P^r defined by equation (17.28), also are superlative

[46]To derive this inequality, the hypothetical value added $\sum_{n=1}^{N+M} p_n^0 q_n^1 \equiv \sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1$ must be positive to establish the inequality in (17.4). If the periods 0 and 1 are quite distant in time, or if there are dramatic changes in output or intermediate input prices between the two periods, this hypothetical value added can be negative. In this case, one can try to use the chain principle to break up the large price and quantity changes that occurred between periods 0 and 1 into a series of smaller changes. With smaller changes, there is a better chance that the hypothetical value-added series will remain positive. This seems consistent with the advice of Burns (1930, p. 256) on this topic. Under certain circumstances, Bowley (1922, p. 256) raised the possibility of a negative nominal value added. Burns (1930, p. 257) noted that this anomaly will generally disappear with higher aggregations across establishments or industries.

[47]Burns (1930, pp. 244–47) noted that the Laspeyres, Paasche, and Fisher value-added deflators could be used to deflate nominal net output or value added into real measures. Burns (1930, p. 247) also noted that that a Fisher ideal production aggregate built up as the product of the Laspeyres and Paasche quantity indices (the "index" method) would give the same answer as deflating the nominal value-added ratio by the Fisher price index (the "deflating" method).

value-added deflators.[48] However, the details are omitted here because to derive these results, rather restrictive separability restrictions are required on the technology of the establishment.[49]

17.87 Attention now is turned to the problems involved in aggregating over establishments to form national output, intermediate input, and value-added deflators.

E. Approximations to Superlative Indices: Midyear Indices

17.88 A practical problem with superlative indices is that they always require current-period information on *quantities* as well as prices to be implemented. In the following section, a recent suggestion is looked at for approximating superlative indices when information on current-period quantities is not available.

17.89 Recall equations (15.18) and (15.19) in Section C.2 of Chapter 15, which defined the Walsh (1901, p. 398; 1921a, p. 97) and Marshall (1887) Edgeworth (1925) price index between periods 0 and 1, $P_W(p^0,p^1,q^0,q^1)$ and $P_{ME}(p^0,p^1,q^0,q^1)$, respectively. In Section C.4 it was indicated that the Walsh price index is a superlative index. On the other hand, although the Marshall-Edgeworth price index is not superlative, Diewert (1978, p. 897) showed that it will approximate any superlative index to the second order around a point where the base- and current-period price and quantity vectors are equal,[50] so that P_{ME} usually will approximate a superlative index fairly closely. In this section, some recent results credited to Schultz (1999) and Okamoto (2001) will be drawn on to show how various *midyear price indices* can approximate Walsh or Marshall-Edgeworth indices fairly closely under certain conditions. As shall be seen, midyear indices do not rely on quantity weights for the current and base periods; rather, they use quantity weights from years that lie between the base period and current period, and, hence, they can be produced on a timely basis. It is noted that the account is given in terms of using midperiod *quantity* weights, although equivalent indices could also be defined using midperiod *revenue shares* using appropriate definitions of indices in the terms given, for example, for Laspeyres and Paasche in equations (15.8) and (15.9), respectively.

17.90 Let t be an even positive integer. Then Schultz (1999) defined a *midyear price index*, which compares the price vector in period t, p^t, with the corresponding price vector in period 0, p^0, as follows:

$$(17.49)\ P_S(p^0,p^t,q^{t/2}) \equiv \sum_{n=1}^{N} p_n^t q_n^{t/2} \Big/ \sum_{n=1}^{N} p_n^0 q_n^{t/2} ,$$

where $q^{t/2}$ is the quantity vector that pertains to the intermediate period, $t/2$. The definition for a midyear price index when t is odd (and greater than 2) is a bit trickier. Okamoto (2001) defined *arithmetic-type* and *geometric-type midyear price indices* comparing prices in period 0 with period t, where

[48]The value-added aggregator function that corresponds to equation (17.55) is now $f^r(y,x)$. For this functional form, all quantities must be positive, and hence the prices of the outputs must be taken to be positive and the prices of intermediate inputs must be negative for the exactness result of equation (17.56) to hold. For the unit net revenue function that now corresponds to equation (17.27), all prices must be positive, output quantities positive, and intermediate input quantities negative for the exactness result (17.29) to hold.

[49]The counterpart to the earlier separability assumption in equation (17.15) is now $z_1 = F^t(y,x,z_2,...,z_K) = G^t(f(y,x),z_2,...,z_K)$ for $t = 0,1$, where the output and intermediate input aggregator function f is linearly homogeneous and independent of t. This type of separability assumption was first made by Sims (1969). Under this separability assumption, the family of value-added deflators defined by equation (17.48) simplify to $r(p^1) / r(p^0)$, where the *unit net revenue function* is defined by $r(p) \equiv \max_q \left\{ \sum_{n=1}^{N+M} p_n q_n : f(q_1,...,q_{N+M}) = 1 \right\}$. Note that these deflators are independent of quantities. Under this separability assumption, the quantity index that corresponds to this real value-added price index is $f(y^1,x^1)/f(y^0,x^0)$, and thus this index depends *only* on quantities. Sims (1977, p. 129) emphasizes that if measures of real net output are to depend only on the quantity vectors of outputs produced and intermediate inputs used, then it will be necessary to make a separability assumption. Since these separability assumptions are very restrictive from an empirical point of view, the economic approaches to the PPI have been developed so they do not rely on separability assumptions.

[50]As usual, this result can be generalized to points of approximation where $p^1 = \alpha p^0$ and $q^1 = \beta q^0$; that is, points where the period 1 price vector is proportional to the period 0 price vector and where the period 1 quantity vector is proportional to the period 0 quantity vector.

t is odd by equations (17.50) and (17.51), respectively:

$$(17.50)\ P_{OA}(p^0,p^t,q^{(t-1)/2},q^{(t+1)/2}) \equiv \frac{\sum_{n=1}^{N} p_n^t (½)\left(q_n^{(t-1)/2} + q_n^{(t+1)/2}\right)}{\sum_{n=1}^{N} p_n^0 (½)\left(q_n^{(t-1)/2} + q_n^{(t+1)/2}\right)}$$

$$(17.51)\ P_{OG}(p^0,p^t,q^{(t-1)/2},q^{(t+1)/2}) \equiv \frac{\sum_{n=1}^{N} p_n^t \left(q_n^{(t-1)/2} + q_n^{(t+1)/2}\right)^{1/2}}{\sum_{n=1}^{N} p_n^0 \left(q_n^{(t-1)/2} + q_n^{(t+1)/2}\right)^{1/2}}.$$

Each of the price indices defined by equation (17.50) and equation (17.51) is of the fixed-basket type. In the arithmetic-type index defined by (17.50), the fixed-basket quantity vector is the simple arithmetic average of the two quantity vectors that pertain to the intermediate periods, $(t-1)/2$ and $(t+1)/2$, whereas in the geometric-type index defined by equation (17.51), the reference quantity vector is the geometric average of these two intermediate period quantity vectors.

17.91 Okamoto (2001) used the above definitions to define the following sequence of *fixed-base (arithmetic-type) midyear price indices*:

$$(17.52)\ 1, P_{ME}(p^0,p^1,q^0,q^1), P_S(p^0,p^2,q^1), P_{OA}(p^0,p^3,q^1,q^2), P_S(p^0,p^4,q^2), P_{OA}(p^0,p^5,q^2,q^3), \ldots .$$

Thus, in period 0, the index is set equal to 1. In period 1, the index is set equal to the Marshall-Edgeworth price index between periods 0 and 1, $P_{ME}(p^0,p^1,q^0,q^1)$ (which is the only index number in the above sequence that requires information on current-period quantities). In period 2, the index is set equal to the Schultz midyear index, $P_S(p^0,p^2,q^1)$, defined by equation (17.49), which uses the quantity weights of the prior period 1, q^1. In period 3, the index is set equal to the arithmetic Okamoto midyear index, $P_{OA}(p^0,p^3,q^1,q^2)$, defined by equation (17.50), which uses the quantity weights of the two prior periods, q^1 and q^2, and so on.

17.92 Okamoto (2001) also used the above definitions to define the following sequence of *fixed-base (geometric-type) midyear price indices*:

$$(17.53)\ 1, P_W(p^0,p^1,q^0,q^1), P_S(p^0,p^2,q^1), P_{OG}(p^0,p^3,q^1,q^2), P_S(p^0,p^4,q^2), P_{OG}(p^0,p^5,q^2,q^3), \ldots .$$

Thus, in period 0, the index is set equal to 1. In period 1, the index is set equal to the Walsh price index between periods 0 and 1, $P_W(p^0,p^1,q^0,q^1)$ (which is the only index number in the sequence that requires information on current period quantities). In period 2, the index is set equal to the Schultz midyear index, $P_S(p^0,p^2,q^1)$. In period 3, the index is set equal to the (geometric-type) Okamoto midyear index, $P_{OG}(p^0,p^3,q^1,q^2)$, defined by equation (17.51), which uses the quantity weights of the two prior periods, q^1 and q^2 , and so on.

17.93 It is also possible to define *chained* sequences[51] of midyear indices that are counterparts to the fixed-base sequences defined by equations (17.52) and (17.53). Thus a chained counterpart to equation (17.52) can be defined as follows:

$$(17.54)\ 1, P_{ME}(p^0,p^1,q^0,q^1), P_S(p^0,p^2,q^1), P_{ME}(p^0,p^1,q^0,q^1)P_S(p^1,p^3,q^2), P_S(p^0,p^2,q^1)P_S(p^2,p^4,q^3), P_{ME}(p^0,p^1,q^0,q^1)P_S(p^1,p^3,q^2)P_S(p^3,p^5,q^4), P_S(p^0,p^2,q^1)P_S(p^2,p^4,q^3)P_S(p^4,p^6,q^5), \ldots .$$

A chained counterpart to equation (17.53) can be defined as follows:

$$(17.55)\ 1, P_W(p^0,p^1,q^0,q^1), P_S(p^0,p^2,q^1), P_W(p^0,p^1,q^0,q^1)P_S(p^1,p^3,q^2), P_S(p^0,p^2,q^1)P_S(p^2,p^4,q^3), P_W(p^0,p^1,q^0,q^1)P_S(p^1,p^3,q^2)P_S(p^3,p^5,q^4), P_S(p^0,p^2,q^1)P_S(p^2,p^4,q^3)P_S(p^4,p^6,q^5), \ldots .$$

Note that equations (17.54) and (17.55) differ only in the use of the Marshall-Edgeworth index, $P_{ME}(p^0,p^1,q^0,q^1)$, to compare prices in period 1 with period 0, versus the Walsh index, $P_W(p^0,p^1,q^0,q^1)$, which is also used to compare prices for the same two periods. Otherwise, only the basic Schultz midyear formula, $P_S(p^t,p^{t+2},q^{t+1})$, is used in both equations (17.54) and (17.55).

[51]See Section F in Chapter 15 for a review of chained indices.

17.94 Using Canadian and Japanese data, Schultz (1999) and Okamoto (2001) showed that midyear index number sequences like those defined by equations (17.54) and (17.55) are reasonably close to their superlative Fisher ideal counterparts.

17.95 In addition to the above empirical results, some theoretical results can be generated that support the use of midyear indices as approximations to superlative indices.[52] The theoretical results presented rely on specific assumptions about how the quantity vectors q^t change over time. Two such specific assumptions will be made.

17.96 It now is assumed that there are *linear trends in quantities* over the sample period; that is, it is assumed that

$$(17.56)\ q^t = q^0 + t\alpha\ ;\ t = 1,...,T,$$

where $\alpha \equiv [\alpha_1,..., \alpha_N]$ is a vector of constants. Hence, for t even, using equation (17.56), it follows that

$$(17.57)\ (½)q^0 + (½)q^t = (½)q^0 + (½)\left[q^0 + t\alpha\right] = q^0 + \left(t/2\right)\alpha = q^{t/2}.$$

Similarly, for t odd (and greater than 2), it follows that

$$(17.58)\ (½)q^0 + (½)q^t = (½)q^0 + (½)\left[q^0 + t\alpha\right] = (½)q^0 + (½)\left[q^0 + \left\{(½)(t-1) + (½)(t+1)\right\}\alpha\right] = (½)\left[q^0 + ½(t-1)\alpha\right] + (½)\left[q^0 + ½(t+1)\alpha\right] = (½)q^{(t-1)/2} + (½)q^{(t+1)/2}.$$

Thus, under the linear time trends in quantities equation (17.56), it can be shown, using equations (17.57) and (17.58), that the Schultz midyear and the Okamoto arithmetic-type midyear indices all equal their Marshall-Edgeworth counterparts; that is,

$$(17.59)\ P_S(p^0,p^t,q^{t/2}) = P_{ME}(p^0,p^t,q^0,q^t) \text{ for } t \text{ even;}$$

$$(17.60)\ P_{OA}(p^0,p^t,q^{(q-1)/2},q^{(q+1)/2}) = P_{ME}(p^0,p^t,q^0,q^t) \text{ for } t \text{ odd.}$$

Thus, under the linear trends equation (17.56), the fixed-base and chained arithmetic-type sequences of midyear indices, equations (17.52) and (17.54), respectively, become the following sequences of Marshall-Edgeworth indices:[53]

$$(17.60)\ 1, P_{ME}(p^0,p^1,q^0,q^1), P_{ME}(p^0,p^2,q^0,q^2), P_{ME}(p^0,p^3,q^0,q^3), P_{ME}(p^0,p^4,q^0,q^4), ...;$$

$$(17.61)\ 1, P_{ME}(p^0,p^1,q^0,q^1), P_{ME}(p^0,p^2,q^0,q^2), P_{ME}(p^0,p^1,q^0,q^1)P_{ME}(p^1,p^3,q^1,q^3), P_{ME}(p^0,p^2,q^0,q^2)P_{ME}(p^2,p^4,q^2,q^4), P_{ME}(p^0,p^1,q^0,q^1)P_{ME}(p^1,p^3,q^1,q^3)P_{ME}(p^3,p^5,q^3,q^5), ...$$

17.97 The second specific assumption about the behavior of quantities over time is that quantities change at *geometric rates* over the sample period; that is, it is assumed that

$$(17.62)\ q_n^t = (1+g_n)^t q_n^0\ n = 1,...,N\ ;\ t = 1,...,T,$$

where g_n is the geometric growth rate for quantity n. Hence, for t even, using equation (17.62),

$$(17.63)\left[q_n^0 q_n^t\right]^{1/2} = (1+g_n)^{t/2} q_n^0 = q_n^{t/2}.$$

For t odd (and greater than 2), again using (17.62),

$$(17.64)\ \left[q_n^0 q_n^t\right]^{1/2} = (1+g_n)^{t/2} q_n^0 = (1+g_n)^{(1/4)\left[(t-1)(t+1)\right]} q_n^0 = \left[q_n^{(t-1)/2} q_n^{(t+1)/2}\right]^{1/2}.$$

Using equations (17.63) and (17.64), it can be shown that, if quantities grow geometrically, then the Schultz midyear and the Okamoto geometric-type midyear indices all equal their Walsh counterparts; that is,

[52]Okamoto (2001) also makes some theoretical arguments relying on the theory of Divisia indices to show why midyear indices might approximate superlative indices.

[53]Recall that Marshall-Edgeworth indices are not actually superlative, but they will usually approximate their superlative Fisher counterparts fairly closely using "normal" time-series data.

(17.65) $P_S(p^0,p^t,q^{t/2}) = P_W(p^0,p^t,q^0,q^t)$ for t even;

(17.66) $P_{OG}(p^0,p^t,q^{(q-1)/2}, q^{(q+1)/2}) = P_W(p^0,p^t,q^0,q^t)$ for t odd.

Thus, under the geometric growth rates equation (17.62), the fixed-base and chained geometric-type sequences of midyear indices, equations (17.53) and (17.55), respectively, become the following sequences of Walsh price indices:

(17.67) $1, P_W(p^0,p^1,q^0,q^1), P_W(p^0,p^2,q^0,q^2), P_W(p^0,p^3,q^0,q^3), P_W(p^0,p^4,q^0,q^4), \ldots;$

(17.68) $1, P_W(p^0,p^1,q^0,q^1), P_W(p^0,p^2,q^0,q^2), P_W(p^0,p^1,q^0,q^1)P_W(p^1,p^3,q^1,q^3), P_W(p^0,p^2,q^0,q^2)P_W(p^2,p^4,q^2,q^4), P_W(p^0,p^1,q^0,q^1)P_W(p^1,p^3,q^1,q^3)P_W(p^3,p^5,q^3,q^5), \ldots .$

17.98 Since the Walsh price indices are superlative, the results in this section show that if quantities are trending in a very smooth manner, then it is likely that superlative indices can be approximated fairly closely without having a knowledge of current-period quantities (but provided that lagged quantity vectors can be estimated on a continuous basis).

17.99 It seems very likely that the midyear indices will approximate superlative indices to a much higher degree of approximation than chained or fixed-base Laspeyres indices.[54] However, the real choice may not be between computing Laspeyres indices versus midyear indices but in producing midyear indices, in a timely manner versus waiting a year or two to produce actual superlative indices. However, there is always the danger that when price or quantity trends suddenly change, the midyear indices considered could give rather misleading advanced estimates of a superlative index. However, if this limitation of midyear indices is kept in mind, it seems that it would generally be useful for statistical agencies to compute midyear indices on an experimental basis.[55]

Appendix 17.1: Relationship Between Divisia and Economic Approaches

17.100 Divisia's approach to index number theory relied on the theory of differentiation. Thus, it does not appear to have any connection with economic theory. However, starting with Ville (1946), a number of economists[56] have established that the Divisia price and quantity indices *do* have a connection with the economic approach to index number theory. This connection is outlined in the context of output price indices.

17.101 The approach taken to the output price index is similar to that taken in Section C.1. Thus, it is assumed that there is a linearly homogeneous *output aggregator function*, $f(q) = f(q_1,\ldots,q_N)$, that aggregates the N individual outputs that the establishment produces into an aggregate output, $q = f(q)$.[57] It is assumed further that in period t, the producer maximizes the revenue that it can achieve, given that it faces the period t aggregator constraint, $f(q) = f(q^t)$, where q^t is the observed period t output vector produced by the establishment. Thus, the observed period t production vector q^t is assumed to solve the following period t revenue maximization problem:

(A17.1) $R(Q^t,p^t)$

$$\equiv \max_q \left\{ \sum_{i=1}^{N} p_i^t q_i \ : f(q_1,\ldots,q_N) = Q^t \right\}$$

$$= \sum_{i=1}^{N} p_i^t q_i^t \ ; \ t = 0,1,\ldots,T,$$

where the period t output aggregate Q^t is defined as $Q^t \equiv f(q^t)$, and $q^t \equiv [q_1^t,\ldots,q_N^t]$ is the establishment's period t observed output vector. The period

[54]It is clear that the midyear index methodologies could be regarded as very simple forecasting schemes to *estimate* the current period quantity vector based on past time series of quantity vectors. Viewed in this way, these midyear methods could be greatly generalized using time-series forecasting methods.

[55]Okamoto (2001) notes that in the 2000 Japanese CPI revision, midyear indices and chained Laspeyres indices will be added as a set of supplementary indices to the usual fixed-base Laspeyres price index.

[56]See, for example, Malmquist (1953, p. 227), Wold (1953, pp. 134–47), Solow (1957), Jorgenson and Griliches (1967), and Hulten (1973). See Balk (2000) for a comprehensive survey of work on Divisia price and quantity indices.

[57]Recall the separability assumptions (17.15).

t price vector for the N outputs that the establishment produces is $p^t \equiv [p_1^t,...,p_N^t]$. Note that the solution to the period t revenue maximization problem defines the *producer's revenue function*, $R(Q^t,p^t)$.

17.102 As in Section B.4, it is assumed that f is (positively) linearly homogeneous for strictly positive quantity vectors. Under this assumption, the producer's revenue function, $R(Q,p)$, decomposes into $Qr(p)$, where $r(p)$ is the *producer's unit revenue function*; see equation (17.16) in Section B.4. Using this assumption, it is found that the observed period t revenue, $\sum_{i=1}^{N} p_i^t q_i^t$, has the following decomposition:

(A17.2) $\sum_{i=1}^{N} p_i^t q_i^t = r\left(p^t\right) f\left(q^t\right)$ for $t = 0,1,\dots,T$.

Thus, the period t total revenue for the N commodities in the aggregate, $\sum_{i=1}^{N} p_i^t q_i^t$, decomposes into the product of two terms, $r(p^t)f(q^t)$. The period t unit revenue, $r(p^t)$, can be identified as *the period t price level* P^t, and the period t output aggregate, $f(q^t)$, as *the period* t *quantity level* Q^t.

17.103 The economic price level for period t, $P^t \equiv c(p^t)$, defined in the previous paragraph now is related to the Divisia price level for time t, $P(t)$, that was defined in Chapter 15 by the differential equation (15.29). As in Section D.1 of Chapter 15, now think of the prices as being continuous, differentiable functions of time, $p_i(t)$ say, for $i = 1,\dots,N$. Thus, the unit revenue function can be regarded as a function of time t as well; that is, define the unit revenue function as a function of t as

(A17.3) $r^*(t) \equiv r[p_1(t),p_2(t),\dots,p_N(t)]$.

Assuming that the first-order partial derivatives of the unit revenue function r exist, the logarithmic derivative of $r^*(t)$ can be calculated as follows:

(A17.4) $d \ln r^*(t)/dt \equiv \left[1/r^*(t)\right] dr^*(t) dt$

$$= \left[1/r^*(t)\right] \sum_{n=1}^{N} r_i\left[p_1(t), p_2(t),..., p_N(t)\right]$$

using equation (A17.3),

where

$$r_i[p_1(t),p_2(t),\dots,p_N(t)] \equiv \partial r[p_1(t),p_2(t),\dots,p_N(t)]/\partial p_i$$

is the partial derivative of the unit revenue function with respect to the ith price, p_i, and $p_i'(t) \equiv dp_i(t)/dt$ is the time derivative of the ith price function, $p_i(t)$. Using Hotelling's (1932, p. 594) lemma, the producer's revenue-maximizing supply for commodity i at time t is

(A17.5) $q_i(t) = Q(t)\, r_i[p_1(t),p_2(t),\dots,p_N(t)]$
for $i = 1,\dots,N$,

where the aggregate output level at time t is $Q(t) = f[q_1(t),q_2(t),\dots,q_N(t)]$. The continuous-time counterpart to equation (A17.2) is that total revenue at time t is equal to the output aggregate, $Q(t)$, times the period t unit revenue, $r^*(t)$; that is,

(A17.6) $\sum_{i=1}^{N} p_i(t) q_i(t) = Q(t) r^*(t)$

$$= Q(t) r\left[p_1(t), p_2(t),..., p_N(t)\right].$$

Now the logarithmic derivative of the Divisia price level $P(t)$ can be written as (recall equation 15.29 in Chapter 15)

(A17.7) $P'(t) / P(t) = \dfrac{\sum_{i=1}^{N} p_i'(t) q_i(t)}{\sum_{i=1}^{N} p_i(t) q_i(t)} \sum_{i=1}^{N} p$,

$$= \frac{\sum_{i=1}^{N} p_i'(t) q_i(t)}{Q(t) r^*(t)}, \text{ using (A17.6)}$$

$$= \frac{\sum_{i=1}^{N} p_i(t)\left\{Q(t) r_i\left[p_1(t), p_2(t),..., p_N(t)\right]\right\}}{Q(t) r^*(t)},$$

using equation (A17.5)

$$= \sum_{i=1}^{N} r_i\left[p_1(t), p_2(t),..., p_N(t)\right] p_i' \Big/ r^*(t)$$

$$= \left[1/r^*(t)\right] dr^*(t)/dt,$$

using equation (A17.4)

$$\equiv r^{*\prime}(t)/r^*(t).$$

Thus, under the above continuous-time revenue-maximizing assumptions, the Divisia price level, $P(t)$, is essentially equal to the unit revenue function evaluated at the time t prices, that is,

$r^*(t) \equiv r[p_1(t),p_2(t),\ldots,p_N(t)]$.

17.104 If the Divisia price level $P(t)$ is set equal to the unit revenue function $r^*(t) \equiv r[p_1(t),p_2(t),\ldots,p_N(t)]$, then from equation (A17.2) it follows that the Divisia quantity level $Q(t)$ defined in Chapter 15 by equation (15.30) will equal the producer's output aggregator function regarded as a function of time, $f^*(t) \equiv f[q_1(t),\ldots,q_N(t)]$. Thus, under the assumption that the producer is continuously maximizing the revenue that can be achieved given an aggregate output target where the output aggregator function is linearly homogeneous, it has been shown that the Divisia price and quantity levels $P(t)$ and $Q(t)$, defined implicitly by the differential equations (15.29) and (15.30) in Chapter 15, are essentially equal to the producer's unit revenue function $r^*(t)$ and output aggregator function $f^*(t)$, respectively.[58] These are rather remarkable equalities since, in principle, given the functions of time, $p_i(t)$ and $q_i(t)$, the differential equations can be solved numerically,[59] and hence $P(t)$ and $Q(t)$ are in principle observable (up to some normalizing constants).

17.105 For more on the Divisia approach to index number theory, see Vogt (1977; 1978) and Balk (2000).

[58]The scale of the output aggregator and unit revenue functions are not uniquely determined by the differential equations (15.29) and (15.30); that is, given $f(q)$ and $r(p)$, one can replace these functions by $\alpha f(q)$ and $(1/\alpha)r(p)$, respectively, and still satisfy equations (15.29) and (15.30) in Chapter 15.

[59]See Vartia (1983).

18. Aggregation Issues

A. Introduction

18.1 In Chapter 15 a basic *index number problem* was identified: how exactly should the microeconomic information involving possibly millions of prices and quantities be aggregated into a smaller number of price and quantity variables? The primary concern of Chapters 15 to 17 was in deciding on an appropriate index number formula so that a *value ratio* pertaining to two periods of time could be decomposed into a component that measures the overall *change in prices* between the two periods (this is the *price index*) times a term that measures the overall *change in quantities* between the two periods (this is the *quantity index*). The summation for these indices was over items. For the fixed-basket and Divisia approach of Chapter 15 and the axiomatic and stochastic approach of Chapter 16, no distinction was drawn between aggregation over items produced by a single establishment, industry, or the economy as a whole. Microeconomic theory regarding the behavior of the establishment in a market was introduced in Chapter 17, and index number formulas were derived that corresponded to specific theoretical assumptions. There was nothing explicit in the analysis to suggest that the same findings would not hold when aggregation took place for outputs, inputs, or the value added of *all* establishments in the economy. Section B of this chapter examines the extent to which the various conclusions reached in Chapter 17 remain valid at an aggregate, economy level. The aggregation of price indices for establishments into national price indices is considered in turn for the output price index, input price index, and value-added deflator.[1] The details of the analysis for the output price index are given in Section B.1, but since a similar methodology is used for the input price index and value-added deflator, only the conclusions are given in Sections B.2 and B.3, respectively.

18.2 Section C notes that, in practice, PPIs tend to be calculated in two stages: first, commodities within establishments are calculated, and second, the commodity and establishment results are used as inputs for aggregation across commodities and establishments to provide industry, product group, and overall PPI results. Section C addresses whether indices calculated this way are consistent in aggregation; that is, if they have the same values whether calculated in a single operation or in two stages.

18.3 Section D considers the relationship between the three PPIs and, in particular, that separate deflation of inputs by the input price index and outputs by the output price index provide the components for the double-deflated value-added index. Section D also outlines a number of equivalent methods that may be used to derive estimates of double-deflated value added for a particular production unit. These are based on the separate deflation by price indices of input and output values, the separate escalation of input and output reference period values by quantity indices, and the use of value-added price and quantity indices. In Section E, the use of value-added price and quantity indices is reconsidered for two-stage aggregation over *industries* (rather than over commodities in a single industry as in Section D) to see if it is consistent with aggregation in a single stage. Finally, Section F considers under what conditions *national value-added price and quantity indices* will be identical to the corresponding *final-demand price and quantity indices*. Note that the final-demand indices are calculated using just the components of final demand, whereas the national value-added indices are constructed by aggregating outputs and intermediate inputs over all industries.

[1]While the value-added price index is just like any other price index in its definition, it is commonly referred to as the "value-added deflator," and the *Manual* will observe this common terminology.

B. Aggregation over Establishments

B.1 National output price index

18.4 An analysis undertaken for aggregation over products at the establishment level for the output price index in Chapter 17, Section B, will now be extended to aggregation over establishments. Assume now that there are E establishments in the economy (or industry, if the goal is to obtain an industry aggregate). The goal in this section is to obtain a national output price index that compares output prices in period 1 to those in period 0 and aggregates over these establishments.

18.5 For $e = 1,2,...,E$, let $p^e \equiv (p_1^e,\ldots p_N^e)$ denote a positive vector of output prices that establishment e might face in period t, and let $v^e \equiv [x^e,z^e]$ be a nonnegative vector of inputs that establishment e might have available for use during period t. Denote the period t technology set for establishment e by S^{et}. As in Chapter 17, Section B.1, *the revenue function for establishment* e can be defined using the period t technology as follows:

$$(18.1)\ R^{et}(p^e,v^e) \equiv \max_q \left\{ \sum_{n=1}^{N} p_n^e q_n : (q) \text{ belongs to } S^{et}(v^e) \right\},$$

where $e = 1,...,E$ and $t = 0,1$.

Now define the *national revenue function*, $R^t(p^1,...,p^E,v^1,...,v^E)$, using period t technologies as the sum of the period t establishment revenue functions R^{et} defined by equation (18.2):

$$(18.2)\ R^t(p^1,...,p^E,v^1,...,v^E) \equiv \sum_{e=1}^{E} R^{et}\left(p^e, v^e\right).$$

Simplify the notation by defining the national price vector p as $p \equiv [p^1,...,p^E]$ and the national input vector v as $v \equiv [v^1,...,v^E]$. With this new notation, $R^t(p^1,...,p^E,v^1,...,v^E)$ can be written as $R^t(p,v)$. Thus, $R^t(p,v)$ is the maximum value of output, $\sum_{e=1}^{E}\sum_{n=1}^{N} p_n^e q_n^e$, that all establishments in the economy can produce, given that establishment e faces the vector of output prices p^e and given that the vector of inputs v^e is available for use by establishment e, using the period t technologies.

18.6 The period t national revenue function R^t can be used to define the *national output price index* using the period t technologies P^t between any two periods, say, period 0 and period 1, as follows:

$$(18.3)\ P^t(p^0,p^1,v) = R^t\left(p^1, v\right) \big/ R^t\left(p^0, v\right),$$

where $p^0 \equiv [p^{10},p^{20},...,p^{E0}]$ and $p^1 \equiv [p^{11},p^{21},...,p^{E1}]$ are the national vectors of output prices that the various establishments face in periods 0 and 1, respectively, and $v \equiv [v^1,v^2,...,v^E]$ is a reference vector of intermediate and primary inputs for each establishment in the economy.[2] The numerator in equation (18.3) is the maximum revenue that the economy could attain (using inputs v) if establishments faced the output prices of period 1, p^1, while the denominator in equation (18.3) is the maximum revenue that establishments could attain (using inputs v) if they faced the output prices of period 0, p^0. Note that all the variables in the numerator and denominator functions are exactly the same, except that the output price vectors differ.

18.7 As was the case of a single establishment studied in Chapter 17, Section B.1, there are a wide variety of price indices of the form in equation (18.3), depending on which reference technology t and reference input vector v are chosen. Thus, there is not a single economic price index of the type defined by equation (18.3)—there is an entire *family* of indices.

18.8 As usual, interest lies in two special cases of the general definition of the output price index in equation (18.3): (i) $P^0(p^0,p^1,v^0)$, which uses the period 0 establishment technology sets and the input vector v^0 that was actually used in period 0, and (ii) $P^1(p^0,p^1,v^1)$, which uses the period 1 establishment technology sets and the input vector v^1 that was actually used in period 1. Let q^{e0} and q^{e1} be the observed output vectors for the establishments in periods 0 and 1, respectively, for $e = 1,...,E$. If there is revenue-maximizing behavior on the part of each establishment in periods 0 and 1, then the sum of observed establishment revenues in periods 0 and 1 should be equal to $R^0(p^0,v^0)$ and $R^1(p^1,v^1)$, respectively; that is, the following equalities should hold:

[2] This concept for an economywide producer output price index may be found in Diewert (2001).

(18.4) $R^0(p^0,v^0) = \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e0}$

and $R^1(p^1,v^1) = \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e1}$.

Under these revenue-maximizing assumptions, adapting the arguments of F.M. Fisher and Shell (1972, pp. 57–58) and Archibald (1977, p. 66), Diewert (2001) showed that the two theoretical indices, $P^0(p^0,p^1,v^0)$ and $P^1(p^0,p^1,v^1)$, described in (i) and (ii) above, satisfy the following inequalities of equations (18.5) and (18.6):

(18.5) $P^0(p^0,p^1,v^0) \equiv R^0\left(p^1,v^0\right) \big/ R^0\left(p^0,v^0\right)$

using equation (18.3)

$$= R^0\left(p^1,v^0\right) \Big/ \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e0}$$

using equation (18.4)

$$\geq \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e0} \Big/ \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e0} ,$$

since q^{e0} is feasible for the maximization problem that defines $R^{e0}(p^{e1},v^{e0})$, and so

$$R^{e0}\left(p^{e1},v^{e0}\right) \geq \sum_{n=1}^{N} p_n^{e1} q_n^{e0} \quad \text{for } e = 1,\ldots,E$$

$$\equiv P_L(p^0,p^1,q^0,q^1),$$

where P_L is the Laspeyres output price index, which treats each commodity produced by each establishment as a separate commodity. Similarly,

(18.6) $P^1(p^0,p^1,v^1) \equiv R^1\left(p^1,v^1\right) \big/ R^1\left(p^0,v^1\right)$

using equation (18.3)

$$= \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e1} \Big/ R^1\left(p^0,v^1\right)$$

using equation (18.4)

$$\leq \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e1} \Big/ \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e1} ,$$

since q^{e1} is feasible for the maximization problem that defines $R^{e1}(p^{e0},v^{e1})$ and so

$$R^{e1}\left(p^{e0},v^{e1}\right) \geq \sum_{n=1}^{N} p_n^{e0} q_n^{e1} \quad \text{for } e = 1,\ldots,E$$

$$\equiv P_P(p^0,p^1,q^0,q^1),$$

where P_P is the Paasche output price index, which treats each commodity produced by each establishment as a separate commodity. Thus, equation (18.5) says that the observable Laspeyres index of output prices P_L is a *lower bound* to the theoretical national output price index $P^0(p^0,p^1,v^0)$, and equation (18.6) says that the observable Paasche index of output prices P_P is an *upper bound* to the theoretical national output price index $P^1(p^0,p^1,v^1)$.

18.9 It is possible to relate the Laspeyres-type *national* output price index $P^0(p^0,p^1,v^0)$ to the *individual establishment* Laspeyres-type output price indices $P^{e0}(p^{e0},p^{e1},v^{e0})$, defined as follows:

(18.7) $P^{e0}(p^{e0},p^{e1},v^{e0})$

$$\equiv R^{e0}\left(p^{e1},v^{e0}\right) \big/ R^{e0}\left(p^{e0},v^{e0}\right)$$

$$= R^{e0}\left(p^{e1},v^{e0}\right) \Big/ \sum_{n=1}^{N} p_n^{e0} q_n^{e0} \geq$$

for $e = 1,\ldots,E$,

where the establishment period 0 technology revenue functions R^{e0} were defined above by equation (18.1), and assumptions in equation (18.4) were used to establish the second set of equalities; that is, the assumption that each establishment's observed period 0 revenues, $\sum_{n=1}^{N} p_n^{e0} q_n^{e0}$, are equal to the optimal revenues, $R^{e0}(p^{e0},v^{e0})$. Now define *the revenue share of establishment* e *in national revenue for period 0* as

(18.8) $S_e{}^0 \equiv \sum_{n=1}^{N} p_n^{e0} q_n^{e0} \Big/ \sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e0}$; $e = 1,\ldots,E.$

Using the definition of the Laspeyres-type national output price index $P^0(p^0,p^1,v^0)$, equation (18.3), for $(t,v) = (0,v^0)$, and using also equation (18.2),

(18.9) $P^0(p^0,p^1,v^0)$

$$\equiv \sum_{e=1}^{E} R^{e0}\left(p^{e1},v^{e0}\right) \Big/ \sum_{e=1}^{E} R^{e0}\left(p^{e0},v^{e0}\right)$$

$$= \frac{\sum_{e=1}^{E} R^{e0}\left(p^{e0},v^{e0}\right) \left(\dfrac{R^{e0}\left(p^{e1},v^{e0}\right)}{R^{e0}\left(p^{e0},v^{e0}\right)}\right)}{\sum_{e=1}^{E} R^{e0}\left(p^{e0},v^{e0}\right)}$$

$$= \sum_{e=1}^{E} S_e^0 \left[\frac{R^{e0}\left(p^{e1}, v^{e0}\right)}{R^{e0}\left(p^{e0}, v^{e0}\right)} \right]$$

using equation (18.8)

$$= \sum_{i=1}^{E} S_e^0 P^{e0}\left(p^{e0}, p^{e1}, v^{e0}\right)$$

using equation (18.7).

Thus, the Laspeyres-type national output price index $P^0(p^0,p^1,v^0)$ is equal to a base-period establishment revenue *share-weighted average* of the individual establishment Laspeyres-type output price indices $P^{e0}(p^{e0},p^{e1},v^{e0})$.

18.10 It is also possible to relate the Paasche-type *national* output price index $P^1(p^0,p^1,v^1)$ to the *individual establishment* Paasche-type output price indices $P^{e1}(p^{e0},p^{e1},v^{e1})$, defined as follows:

$$(18.10)\ P^{e1}(p^{e0},p^{e1},v^{e1}) \equiv \frac{R^{e1}\left(p^{e1}, v^{e1}\right)}{R^{e1}\left(p^{e0}, v^{e1}\right)}$$

$$= \sum_{n=1}^{N} p_n^{e1} q_n^{e1} \Big/ R^{e1}\left(p^{e0}, v^{e1}\right);$$

$e = 1,\ldots,E,$

where the establishment period 1 technology revenue functions R^{e1} were defined above by equation (18.1), and assumptions in equation (18.4) are used to establish the second set of equalities; that is, the assumption that each establishment's observed period 1 revenues, $\sum_{n=1}^{N} p_n^{e1} q_n^{e1}$, are equal to the optimal revenues, $R^{e1}(p^{e1},v^{e1})$. Now, define *the revenue share of establishment* e *in national revenue for period 1* as

$$(18.11)\ S_e^1 \equiv \sum_{n=1}^{N} p_n^{e1} q_n^{e1} \Big/ \sum_{i=1}^{E} \sum_{n=1}^{N} p_n^{i1} q_n^{i1}\ ;\ e = 1,\ldots,E.$$

Using the definition of the Paasche-type national output price index $P^1(p^0,p^1,v^1)$, equation (18.3), for $(t,v) = (1,v^1)$, and using also equation (18.2),

$$(18.12)\ P^1(p^0,p^1,v^1)$$

$$\equiv \sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right) \Big/ \sum_{e=1}^{E} R^{e1}\left(p^{e0}, v^{e1}\right)$$

$$= \left\{ \left[\sum_{e=1}^{E} R^{e1}\left(p^{e0}, v^{e1}\right) \Big/ \sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right) \right] \right\}^{-1}$$

$$= \left\{ \sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right) \frac{\sum_{e=1}^{E} R^{e1}\left(p^{e0}, v^{e1}\right) \Big/ \sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right)}{\sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right)} \right\}^{-1}$$

$$= \left\{ \sum_{e=1}^{E} S_e^1 \left[\frac{\sum_{e=1}^{E} R^{e1}\left(p^{e1}, v^{e1}\right)}{\sum_{e=1}^{E} R^{e1}\left(p^{e0}, v^{e1}\right)} \right]^{-1} \right\}^{-1}$$

$$= \left\{ \sum_{i=1}^{E} S_e^1 \left[P^{e1}\left(p^{e0}, p^{e1}, v^{e1}\right) \right]^{-1} \right\}^{-1}$$

using equation (18.10).

Thus, the Paasche-type national output price index $P^1(p^0,p^1,v^1)$ is equal to *a period 1 establishment revenue share-weighted harmonic average* of the individual establishment Paasche-type output price indices $P^{e1}(p^{e0},p^{e1},v^{e1})$.

18.11 As was the case in Chapter 17, Section B.2, it is possible to define a national output price index that falls *between* the observable Paasche and Laspeyres national output price indices. To do this, first a *hypothetical revenue function*, $R^e(p^e,\alpha)$, is defined for each establishment that corresponds to the use of an α-weighted average of the technology sets $S^{e0}(v^0)$ and $S^{e1}(v^1)$ (with their associated input vectors) for periods 0 and 1 as the reference technologies and input vectors:

$$(18.13)\ R^e(p^e,\alpha)$$

$$\equiv \max_q \{ \sum_{n=1}^{N} p_n^e q_n : q \text{ belongs to}$$

$$(1-\alpha)S^{e0}(v^0) + \alpha S^{e1}(v^1)\};\ e = 1,\ldots,E.$$

Once the establishment hypothetical revenue functions have been defined by equation (18.13), *the intermediate technology national revenue function* $R^t(p^1,\ldots,p^E,v^1,\ldots,v^E)$ can be defined as the sum of the period t intermediate technology establishment revenue functions R^e defined by equation (18.13):

(18.14) $R(p^1,...,p^E,\alpha) \equiv \sum_{e=1}^{E} R^e\left(p^e,\alpha\right)$.

Again, simplify the notation by defining the national price vector p as $p \equiv [p^1,...,p^E]$. With this new notation, $R(p^1,...,p^E,\alpha)$ can be written as $R(p,\alpha)$. Now, use the national revenue function defined by equation (18.14) in order to define the following family of *theoretical national output price indices*:

(18.15) $P(p^0,p^1,\alpha) \equiv R(p^1,\alpha) / R(p^0,\alpha)$.

18.12 As usual, the proof of Diewert (1983a, pp. 1060–61) can be adapted to show that there exists an α between 0 and 1 such that a theoretical national output price index defined by equation (18.15) lies between the observable (in principle) Paasche and Laspeyres national output price indices defined in equations (18.5) and (18.6), P_P and P_L ; that is, there exists an α such that

(18.16) $P_L \le P(p^0,p^1,\alpha) \le P_P$ or
$P_P \le P(p^0,p^1,\alpha) \le P_L$.

If the Paasche and Laspeyres indices are numerically close to each other, then equation (18.16) tells us that a true national output price index is fairly well determined and that a reasonably close approximation can be found to the true index by taking a symmetric average of P_L and P_P, such as the geometric average, which again leads to Irving Fisher's (1922) ideal price index, P_F, defined earlier by equation (17.9).

18.13 The above theory for the national output price indices is very general; in particular, no restrictive functional form or separability assumptions were made on the establishment technologies.

18.14 The translog technology assumptions used in Chapter 17, Section B.3 to justify the use of the Törnqvist-Theil output price index for a single establishment as an approximation to a theoretical output price index for a single establishment can be adapted to yield a justification for the use of a national Törnqvist-Theil output price index as an approximation to a theoretical national output price index.

18.15 Recall the definition of the national period t national revenue function, $R^t(p,v) \equiv R^t(p^1,...,p^E,v^1,...,v^E)$, defined earlier by equation (18.2) above. Assume that the period t national revenue function has the following *translog functional form* for $t = 0,1$:

(18.17) $\ln R^t(p,v)$

$$= \alpha_0^t + \sum_{n=1}^{NE}\alpha_n^t \ln p_n + \sum_{m=1}^{(N+K)E}\beta_m^t \ln v_m + \frac{1}{2}\sum_{n=1}^{NE}\sum_{j=1}^{NE}\alpha_{nj}^t \ln p_n \ln p_j + \sum_{n=1}^{NE}\sum_{m=1}^{(N+K)E}\beta_{nm}^t \ln p_n \ln v_m + \frac{1}{2}\sum_{m=1}^{(N+K)E}\sum_{k=1}^{(N+K)E}\gamma_{mk}^t \ln v_m \ln v_k ,$$

where the α_n^t coefficients satisfy the restrictions

(18.18) $\sum_{n=1}^{NE}\alpha_n^t = 1$ for $t = 0,1$,

and the α_{nj}^t coefficients satisfy the following restrictions:[3]

(18.19) $\sum_{n=1}^{NE}\alpha_{nj}^t = 0$ for $t = 0,1$ and n = 1,2,…,NE.

Note that the national output price vector p in equation (18.17) has dimension equal to NE, the number of outputs times the number of establishment—that is, $p \equiv [p_1,...,p_N; \; p_{N+1},...,p_{2N}; \; ...; \; p_{(E-1)N+1},...,p_{NE}] = [p_1^1,...,p_N^1; \; p_1^2,...,p_N^2; \; ...; \; p_1^E,...,p_N^E]$. Similarly, the national input vector v in equation (18.17) has dimension equal to $(M + K)E$, the number of intermediate and primary inputs in the economy times the number of establishments.[4] The restrictions in equations (18.18) and (18.19)

[3]It is also assumed that the symmetry conditions $\alpha_{nj}^t = \alpha_{jn}^t$ for all n,j and for $t = 0,1$ and $\gamma_{mk}^t = \gamma_{km}^t$ for all m,k and for $t = 0,1$ are satisfied.

[4]It has also been implicitly assumed that each establishment can produce each of the N outputs in the economy and that each establishment uses all $M + K$ inputs in the economy. These restrictive assumptions can readily be relaxed, but only at the cost of notational complexity. All that is required is that each establishment produce the same set of outputs in each period.

are necessary to ensure that $R^t(p,v)$ is linearly homogeneous in the components of the output price vector p (which is a property that a revenue function must satisfy). Note that at this stage of the argument, the coefficients that characterize the technology in each period (the αs, βs, and γs) are allowed to be completely different in each period. Also note that the translog functional form is an example of a *flexible* functional form;[5] that is, it can approximate an arbitrary technology to the second order.

18.16 Define the national revenue share for establishment e and output n for period t as follows:

$$(18.20)\ s_n^{et} \equiv \sum_{n=1}^{N} p_n^{et} q_n^{et} \Big/ \sum_{i=1}^{E}\sum_{j=1}^{N} p_j^{it} q_j^{it}\ ;\ n = 1,\ldots,N\ ;$$
$$e = 1,\ldots,E\ ;\ t = 0,1.$$

Using the above establishment revenue shares and the establishment output price relatives, $p_n^{e1}/\,p_n^{e0}$, define the logarithm of the *national Törnqvist-Theil output price index* P_T (Törnqvist, 1936; Törnqvist and Törnqvist, 1937; and Theil, 1967) as follows:

$$(18.21)\ \ln P_T(p^0,p^1,q^0,q^1) \equiv \sum_{e=1}^{E}\sum_{n=1}^{N} (½)\left(s_n^{e0} + s_n^{e1}\right)\ln\left(p_n^{e1}\big/p_n^{e0}\right).$$

18.17 Recall Theil's (1967) weighted stochastic approach to index number theory explained in Section D.2 of Chapter 16. In the present context, the discrete random variable R takes on the NE values for the logarithms of the establishment output price ratios between periods 0 and 1, $\ln\left(p_n^{e1}\big/p_n^{e0}\right)$, with probabilities $\left(\frac{1}{2}\right)\left(s_n^{e0} + s_n^{e1}\right)$. Thus, the right-hand side of equation (18.21) can also be interpreted as the *mean* of this distribution of economywide logarithmic output price relatives.

18.18 A result in Caves, Christensen, and Diewert (1982b, p. 1410) can be adapted to the present context: if the quadratic price coefficients in equation (18.17) are equal across the two periods where an index number comparison (that is, $\alpha_{ij}^{\ 0} = \alpha_{ij}^{\ 1}$ for all i,j) is made, then the geometric mean of the national output price index that uses period 0 technology and the period 0 input vector v^0, $P^0(p^0,p^1,v^0)$, and the national output price index that uses period 1 technology and the period 1 input vector v^1, $P^1(p^0,p^1,v^1)$, is *exactly equal* to the Törnqvist output price index P_T defined by equation (18.21) above; that is,

$$(18.22)\ P_T(p^0,p^1,q^0,q^1) = \left[P^0\left(p^0,p^1,v^0\right)P^1\left(p^0,p^1,v^1\right)\right]^{1/2}.$$

As usual, the assumptions required for this result seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period, and our assumptions are consistent with technological progress occurring between the two periods being compared. Because the index number formula P_T is *exactly* equal to the geometric mean of two theoretical economic output price indices and this corresponds to a flexible functional form, the Törnqvist national output price index number formula is *superlative* following the terminology used by Diewert (1976).

18.19 There are *four important results* in this section, which can be summarized as follows. Define the national Laspeyres output price index as follows:

$$(18.23)\ P_L(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e0}}{\sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e0}}.$$

Then, this national Laspeyres output price index is a *lower bound* to the economic output price index $P^0(p^0,p^1,v^0) \equiv R^0(p^1,v^0)\ /\ R^0(p^0,v^0)$, where the national revenue function $R^0(p,v^0)$ using the period 0 technology and input vector v^0 is defined by equations (18.1) and (18.2).

18.20 Define the national Paasche output price index as follows:

$$(18.24)\ P_P(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e1} q_n^{e1}}{\sum_{e=1}^{E}\sum_{n=1}^{N} p_n^{e0} q_n^{e1}}.$$

[5]In fact, the assumption that the period t national revenue function $R^t(p,v)$ has the translog functional form defined by equation (18.17) may be regarded as an approximation to the true technology, since equation (18.17) has not imposed any restrictions on the national technology, implied by the fact that the national revenue function is equal to the sum of the establishment revenue functions.

Then, this national Paasche output price index is an *upper bound* to the economic output price index $P^1(p^0,p^1,v^1) \equiv R^1(p^1,v^1) / R^1(p^0,v^1)$, where the national revenue function $R^1(p,v^1)$ using the period 1 technology and input vector v^1 is defined by equations (18.1) and (18.2).

18.21 Define the *national Fisher output price index* P_F as the square root of the product of the national Laspeyres and Paasche indices defined above:

$$(18.25)\ P_F(p^0,p^1,q^0,q^1) = \left[P_L\left(p^0,p^1,q^0,q^1\right)P_P\left(p^0,p^1,q^0,q^1\right)\right]^{1/2}.$$

Then, usually, the national Fisher output price index will be a good approximation to an economic output price index based on a revenue function that uses a technology set and an input vector that is intermediate to the period 0 and 1 technology sets and input vectors.

18.22 Under the assumption that the period 0 and 1 national revenue functions have translog functional forms, then the geometric mean of the national output price index that uses period 0 technology and the period 0 input vector v^0, $P^0(p^0,p^1,v^0)$, and the national output price index that uses period 1 technology and the period 1 input vector v^1, $P^1(p^0,p^1,v^1)$, is *exactly equal* to the Törnqvist output price index P_T defined by equation (18.21) above; that is, equation (18.22) holds true.

18.23 This section concludes with an observation. Economic justifications have been presented for the use of the national Fisher output price index, $P_F(p^0,p^1,q^0,q^1)$, defined by equation (18.25), and for the use of the national Törnqvist output price index, $P_T(p^0,p^1,q^0,q^1)$, defined by equation (18.21). The results in Chapter 17, Section B.5, indicate that for normal time-series data, these two indices will give virtually the same answer.

B.2 National intermediate input price index

18.24 The theory of the intermediate input price index for a single establishment that was developed in Chapter 17, Section C, can be extended to the case where there are E establishments in the economy. The techniques used for this extension are very similar to the techniques used in Section B.1 above, so it is not necessary to replicate this work here.

18.25 The observable national Laspeyres index of intermediate input prices P_L is found to be an *upper bound* to the theoretical national intermediate input price index using period 0 technology and inputs, and the observable national Paasche index of intermediate input prices P_P is a *lower bound* to the theoretical national intermediate input price index using period 1 technology and inputs.

18.26 As was the case in Section B.1, it is possible to define a theoretical national intermediate input price index that falls *between* the observable Paasche and Laspeyres national intermediate input price indices. The details are omitted, although they follow along the lines used in Section B.1. Usually, the *national Fisher intermediate input price index* P_F, defined as the square root of the product of the national Laspeyres and Paasche indices, will be a good approximation to this economic intermediate input price index. Such an index is based on a national cost function that uses establishment technology sets, target establishment output vectors, and establishment primary input vectors intermediate to the period 0 and 1 technology sets, observed output vectors, and observed primary input vectors.

18.27 The translog technology assumptions used in Section B.1 to justify the use of the Törnqvist-Theil intermediate input price index for a single establishment as an approximation to a theoretical intermediate input price index for a single establishment can be adapted to yield a justification for the use of a national Törnqvist-Theil intermediate input price index as an approximation to a theoretical national intermediate input price index.

B.3 National value-added deflator

18.28 In this section, it is the theory of the *value-added deflator for a single establishment* developed in Chapter 17, Section D, that is drawn on and extended to the case where there are E establishments in the economy. The techniques used for this extension are, again, very similar to the techniques used in Section B.1, except that an establishment net revenue functions π^{et} is used in place of establishment revenue functions R^{et}.

18.29 The observable Laspeyres index of net output prices is shown to be a *lower bound* to the theoretical national value-added deflator based on period 0 technology and inputs, and the observable Paasche index of net output prices is an *upper bound* to the theoretical national value-added deflator based on period 1 technology and inputs.

18.30 Constructing industry indices, such as Laspeyres and Paasche, from individual establishment indices, and national indices from individual industry indices requires weights. It should be noted that *establishment shares of national value added* are used for national value-added deflators, whereas *establishment shares of the national value of (gross) outputs produced* were used in Section B.1 for national output price indices. Results supporting the use of Fisher's ideal index and the Törnqvist index arise from arguments similar to those presented for the national output price index.

18.31 Recall Theil's (1967) weighted stochastic approach to index number theory that was explained in Section D.2 of Chapter 16. If his approach is adapted to the present context, then the discrete random variable R would take on the $(N + M)E$ values for the logarithms of the establishment net output price relatives between periods 0 and 1, $\ln\left(p_n^{e1}/p_n^{e0}\right)$, with probabilities $\left(\frac{1}{2}\right)\left(s_n^{e0}+s_n^{e1}\right)$. Thus, under this interpretation of the stochastic approach, it would appear that the right-hand side of the Törnqvist-Theil index could be interpreted as the *mean* of this distribution of economywide logarithmic output and intermediate input price relatives. However, in the present context, this stochastic interpretation for the Törnqvist-Theil net output price formula breaks down because the shares $\left(\frac{1}{2}\right)\left(s_n^{e0}+s_n^{e1}\right)$ are negative when n corresponds to an intermediate input.

C. Laspeyres, Paasche, Superlative Indices and Two-Stage Aggregation

18.32 The above analysis has been conducted as if the aggregation had been undertaken in a single stage. Most statistical agencies use the Laspeyres formula to aggregate prices in two stages. At the first stage of aggregation, the Laspeyres formula is used to aggregate components of the overall index (for example, agricultural output prices, other primary industry output prices, manufacturing prices, service output prices). Then, at the second stage of aggregation, these component subindices are further combined into the overall index. The following question then naturally arises: does the index computed in two stages coincide with the index computed in a single stage? This question is initially addressed in the context of the Laspeyres formula.[6]

18.33 Now, suppose that the price and quantity data for period t, p^t and q^t, can be written in terms of j subvectors as follows:

$$(18.26)\ p^t = (p^{t1}, p^{t2}, \ldots, p^{tJ})\ ; \quad q^t = (q^{t1}, q^{t2}, \ldots, q^{tJ})\ ;\ t = 0,1,$$

where the dimensionality of the subvectors p^{tj} and q^{tj} is N_j for $j = 1,2,\ldots,J$ with the sum of the dimensions N_j equal to N. These subvectors correspond to the price and quantity data for subcomponents of the producer output price index for period t. The analysis is undertaken for output price indices here, but similar conclusions hold for input price indices. Construct subindices for each of these components going from period 0 to 1. For the base period, the price for each of these subcomponents, say, P_j^0 for $j = 1,2,\ldots J$, is set equal to 1, and the corresponding base-period subcomponent quantities, say, Q_j^0 for $j = 1,2,\ldots,J$, are set equal to the base-period value of production for that subcomponent. For $j = 1,2,\ldots,J$; that is,

$$(18.27)\ P_j^0 \equiv 1\ ;\ Q_j^0 \equiv \sum_{i=1}^{Nj} p_i^{1j} q_i^{0j} \quad \text{for } j = 1,2,\ldots,J.$$

Now use the Laspeyres formula to construct a period 1 price for each subcomponent, say, P_j^1 for $j = 1,2,\ldots,J$, of the producer price index. Since the dimensionality of the subcomponent vectors, p^{tj} and q^{tj}, differ from the dimensionality of the complete period t vectors of prices and quantities, p^t and q^t, different symbols for these subcomponent Laspeyres indices will be used, say, P_L^j for $j = 1,2,\ldots J$. Thus, the period 1 subcomponent prices are defined as follows:

[6]Much of the initial material in this section is adapted from Diewert (1978) and Alterman, Diewert, and Feenstra (1999). See also Vartia (1976a; 1976b) and Balk (1996b) for a discussion of alternative definitions for the two-stage aggregation concept and references to the literature on this topic.

$$(18.28)\ P_j^1 \equiv P_L^j(p^{0j},p^{1j},q^{0j},q^{1j}) \equiv \frac{\sum_{i=1}^{Nj} p_i^{1j} q_i^{0j}}{\sum_{i=1}^{Nj} p_i^{0j} q_i^{0j}}$$

for $j = 1,2,\ldots,J$.

Once the period 1 prices for the j subindices have been defined by equation (18.28), then corresponding subcomponent period 1 quantities Q_j^1 for $j = 1,2,\ldots J$ can be defined by deflating the period 1 subcomponent values $\sum_{i=1}^{Nj} p_i^{1j} q_i^{1j}$ by the prices P_j^1 defined by equation (18.28); that is,

$$(18.29)\ Q_j^1 \equiv \sum_{i=1}^{Nj} p_i^{1j} q_i^{1j} \Big/ P_j^1 \text{ for } j = 1,2,\ldots,J.$$

Subcomponent price and quantity vectors for each period $t = 0,1$ can now be defined using equations (18.27) to (18.29). Thus, define the period 0 and 1 subcomponent price vectors P^0 and P^1 as follows:

$$(18.30)\ P^0 = (P_1^0, P_2^0,\ldots,P_J^0) \equiv 1_J\ ;\ P^1 = (P_1^1, P_2^1,\ldots,P_J^1),$$

where 1_J denotes a vector of ones of dimension j, and the components of P^1 are defined by equation (18.28). The period 0 and 1 subcomponent quantity vectors Q^0 and Q^1 are defined as follows:

$$(18.31)\ Q^0 = (Q_1^0, Q_2^0,\ldots,Q_J^0)\ ;\quad Q^1 = (Q_1^1, Q_2^1,\ldots,Q_J^1),$$

where the components of Q^0 are defined in equation (18.27) and the components of Q^1 are defined by equation (18.29). The price and quantity vectors in equations (18.30) and (18.31) represent the results of the first-stage aggregation. These vectors can now be used as inputs into the second-stage aggregation problem; that is, the Laspeyres price index formula can be applied using the information in equations (18.30) and (18.31) as inputs into the index number formula. Since the price and quantity vectors that are inputs into this second-stage aggregation problem have dimension j instead of the single-stage formula that used vectors of dimension N_j, a different symbol is needed for our new Laspeyres index, which is chosen to be P_L^*. Thus, the Laspeyres price index computed in two stages can be denoted as $P_L^*(P^0,P^1,Q^0,Q^1)$. It is now appropriate to ask whether this two-stage Laspeyres index equals the corresponding single-stage index P_L studied in the previous sections of this chapter; that is, whether

$$(18.32)\ P_L^*(P^0,P^1,Q^0,Q^1) = P_L(p^0,p^1,q^0,q^1).$$

If the Laspeyres formula is used at each stage of each aggregation, the answer to the above question is yes: straightforward calculations show that the Laspeyres index calculated in two stages equals the Laspeyres index calculated in one stage. The answer is also yes if the Paasche formula is used at each stage of aggregation; that is, the Paasche formula is consistent in aggregation just like the Laspeyres formula.

18.34 Now suppose the Fisher or Törnqvist formula is used at each stage of the aggregation; that is, in equation (18.28), suppose the Laspeyres formula $P_L^j(p^{0j},p^{1j},q^{0j},q^{1j})$ is replaced by the Fisher formula $P_F^j(p^{0j},p^{1j},q^{0j},q^{1j})$ or by the Törnqvist formula $P_T^j(p^{0j},p^{1j},q^{0j},q^{1j})$, and in equation (18.32), $P_L^*(P^0,P^1,Q^0,Q^1)$ is replaced by P_F^* (or by P_T^*) and $P_L(p^0,p^1,q^0,q^1)$ replaced by P_F (or by P_T). Then, do counterparts to the two-stage aggregation result for the Laspeyres formula, equation (18.32)? The answer is no; it can be shown that, in general,

$$(18.33)\ P_F^*(P^0,P^1,Q^0,Q^1) \neq P_F(p^0,p^1,q^0,q^1) \text{ and } P_T^*(P^0,P^1,Q^0,Q^1) \neq P_T(p^0,p^1,q^0,q^1).$$

Similarly, it can be shown that the quadratic mean of order r index number formula P^r defined by equation (17.28) and the implicit quadratic mean of order r index number formula P^{r*} defined by equation (17.25) are also not consistent in aggregation.

18.35 However, even though the Fisher and Törnqvist formulas are not *exactly* consistent in aggregation, it can be shown that these formulas are *approximately* consistent in aggregation. More specifically, it can be shown that the two-stage Fisher formula P_F^* and the single-stage Fisher formula P_F in equation (18.33), both regarded as functions of the $4N$ variables in the vectors p^0,p^1,q^0,q^1, approximate each other to the second order around a point where the two price vectors are equal (so that $p^0 = p^1$) and where the two quantity vectors are equal (so that $q^0 = q^1$). A similar result holds for the two-stage and single-stage Törnqvist indices in equation (18.33).[7] As it was shown in the previous section, the single-stage Fisher and Törnqvist indices have a similar approximation property, and so all four indices in equation (18.33) approximate each other to the second order around an equal (or proportional) price and quantity point. Thus, for normal time-series data, single-stage and two-stage Fisher and Törnqvist indices usually will be numerically very close.[8] This result for an artificial data set is illustrated in Chapter 19.

18.36 Similar approximate consistency in aggregation results (to the results for the Fisher and Törnqvist formulas explained in the previous paragraph) can be derived for the *quadratic mean of order* r *indices*, P^r, and for the implicit quadratic mean of order r indices, P^{r*}; see Diewert (1978, p. 889). However, the results of R.J. Hill (2000) again imply that *the second-order approximation property of the single-stage quadratic mean of order* r *index* P^r *to its two-stage counterpart will break down as* r *approaches either plus or minus infinity*. To see this, consider a simple example where there are only four commodities in total. Let the first price relative p_1^1 / p_1^0 be equal to the positive number a, let the second two price relatives p_i^1 / p_i^0 equal b, and let the last price relative p_4^1 / p_4^0 equal c where it is assumed that $a < c$ and $a \le b \le c$. Using R.J. Hill's result in equation (17.32), the limiting value of the single-stage index is

$$(18.34)\ \lim_{r\to+\infty} P^r(p^0,p^1,q^0,q^1) = \lim_{r\to-\infty} P^r(p^0,p^1,q^0,q^1) = [\min_i\{p_i^1 / p_i^0\}\max_i\{p_i^1 / p_i^0\}]^{1/2} = [ac]^{1/2}.$$

If commodities 1 and 2 are aggregated into a subaggregate and commodities 3 and 4 into another subaggregate, using R.J. Hill's result in equation (17.32) again, it is found that the limiting price index for the first subaggregate is $[ab]^{1/2}$ and the limiting price index for the second subaggregate is $[bc]^{1/2}$. Now, apply the second stage of aggregation and use R.J. Hill's result once again to conclude that the limiting value of the two-stage aggregation using P^r as our index number formula is $[ab^2c]^{1/4}$. Thus, the limiting value as r tends to plus or minus infinity of the single-stage aggregate over the two-stage aggregate is $[ac]^{1/2} / [ab^2c]^{1/4} = [ac/b^2]^{1/4}$. Now b can take on any value between a and c, and the ratio of the single-stage limiting P^r to its two-stage counterpart can take on any value between $[c/a]^{1/4}$ and $[a/c]^{1/4}$. Since c/a is less than 1 and a/c is greater than 1, it can be seen that the ratio of the single-stage to the two-stage index can be arbitrarily far from 1 as r becomes large in magnitude with an appropriate choice of the numbers a, b, and c.

18.37 The results in the previous paragraph show that caution is required in assuming that *all* superlative indices will be approximately consistent in aggregation. However, for the three most commonly used superlative indices (the Fisher ideal P_F, the Törnqvist-Theil P_T, and the Walsh P_W), the available empirical evidence indicates that these indices satisfy the consistency-in-aggregation property to a sufficiently high enough degree of approximation that users will not be unduly troubled by any inconsistencies.[9]

18.38 A similar analysis could be undertaken for *input price indices*, and similar conclusions would hold. The value-added deflator is considered in the next subsection.

D. Value-Added Deflators—Relationships Between Producer Price Indices

D.1 Output price, intermediate input price, and deflation of value added

18.39 Let the vectors of output price, output quantity, intermediate input price, and intermediate input price vectors for an establishment[10] in period t be denoted by p_y^t, y^t, p_x^t, and x^t, respectively, for $t = 0,1$. Suppose a bilateral index number formula P is used to construct an establishment output price

[7]See Diewert (1978, p. 889), who used some results credited to Vartia (1976a; 1976b).

[8]For an empirical comparison of the four indices, see Diewert (1978, pp. 894–95). For the Canadian consumer data considered there, the chained two-stage Fisher in 1971 was 2.3228 and the corresponding chained two-stage Törnqvist was 2.3230, the same values as for the corresponding single-stage indices.

[9]See Chapter 19 for additional evidence on this topic.

[10]Instead of "establishment," one could substitute the words "industry" or "national economy."

index, $P(p_y^0,p_y^1,y^0,y^1)$, an establishment intermediate input price index, $P(p_x^0,p_x^1,x^0,x^1)$, and an establishment *value-added deflator*, $P(p^0,p^1,q^0,q^1)$ where, as usual, $p^t \equiv [p_y^t,p_x^t]$ and $q^t \equiv [y^t,-x^t]$ for $t = 0,1$. Two related questions arise:

- How is the *value-added deflator* related to the output price index and the intermediate input price index?
- How can the output price index and the intermediate input price index be combined to obtain a *value-added deflator*?

Answers to the above questions can be obtained using the two-stage aggregation procedure explained in Section C.

18.40 In the present application of the two-stage aggregation procedure explained in Section C, let $j = 2$. The price and quantity vectors p^{tj} and q^{tj} that appeared in equation (18.26) are now defined as follows:

(18.35) $p^{t1} \equiv p_y^t$; $p^{t2} \equiv p_x^t$; $q^{t1} \equiv y^t$; $q^{t2} \equiv -x^t$; $t = 0,1$.

Thus, the first group of commodities aggregated in the first stage of aggregation are the outputs y^t of the establishment, and the second group of commodities aggregated in the first stage of aggregation are minus the intermediate inputs $-x^t$ of the establishment.

18.41 The base-period first-stage aggregate prices and quantities, P_j^0 and Q_j^0, that appeared in equation (18.27) are now defined as follows:

(18.36) $$P_1^0 = P_2^0 \equiv 1 ;\quad Q_1^0 \equiv \sum_{n=1}^{N} p_{yn}^0 y_n^0 ;\quad Q_2^0 \equiv -\sum_{m=1}^{M} p_{xm}^0 x_m^0 .$$

Note that Q_1^0 is the base-period value of outputs produced by the establishment, and Q_2^0 is minus the value of intermediate inputs used by the establishment in period 0.

18.42 Now, use a chosen index number formula to construct an output price index, $P(p_y^0,p_y^1,y^0,y^1)$, and an intermediate input price index, $P(p_x^0,p_x^1,x^0,x^1)$. These two numbers are set equal to the aggregate price of establishment output P_1^1 and the aggregate price of intermediate input P_2^1 in period 1; that is, the bilateral index number formula P is used to form the following counterparts to equation (18.28) in Section C:

(18.37) $P_1^1 \equiv P(p_y^0,p_y^1,y^0,y^1)$; $P_2^1 \equiv P(p_x^0,p_x^1,x^0,x^1)$.

18.43 Finally, the following counterparts to equation (18.29) generate the period 1 output quantity aggregate Q_1^1 and minus the period 1 input aggregate Q_2^1:

(18.38) $$Q_1^1 \equiv \sum_{n=1}^{N} p_{yn}^1 y_n^1 \Big/ P_1^1 = \sum_{n=1}^{N} p_{yn}^1 y_n^1 \Big/ P\left(p_y^0, p_y^1, y^0, y^1\right);$$
$$Q_2^1 \equiv -\sum_{m=1}^{M} p_{xm}^1 x_m^1 \Big/ P_2^1 = -\sum_{m=1}^{M} p_{xm}^1 x_m^1 \Big/ P\left(p_x^0, p_x^1, x^0, x^1\right).$$

Thus, the period 1 output aggregate, Q_1^1, is equal to the value of period 1 production, $\sum_{n=1}^{N} p_{yn}^1 y_n^1$, divided by the output price index, $P(p_y^0,p_y^1,y^0,y^1)$. The period 1 intermediate input aggregate, Q_2^1, is equal to minus the period 1 cost of intermediate inputs, $\sum_{m=1}^{M} p_{xm}^1 x_m^1$, divided by the intermediate input price index, $P(p_x^0,p_x^1,x^0,x^1)$. Thus, the period 1 output and intermediate input quantity aggregates are constructed by deflating period 1 value aggregates by an appropriate price index, which may be considered to be a type of double-deflation procedure.

18.44 Following equation (18.30), the period 0 and 1 subcomponent price vectors P^0 and P^1 and the period 0 and 1 subcomponent quantity vectors Q^0 and Q^1 are defined as follows:

(18.39) $$P^0 \equiv [P_1^0,P_2^0] ;\ P^1 \equiv [P_1^1,P_2^1] ;\quad Q^0 \equiv [Q_1^0,Q_2^0] ;\ Q^1 \equiv [Q_1^1,Q_2^1].$$

Finally, given the aggregate prices and quantity vectors defined in equation (18.39), again make use of the chosen bilateral index number formula P, and calculate the *two-stage value-added deflator* for the establishment, $P(P^0,P^1,Q^0,Q^1)$. The construction of this two-stage value-added deflator provides an answer to the second question asked above; that is, how can the output price index and the intermediate input price index be combined to obtain a value-added deflator?

18.45 It is now necessary to ask whether the two-stage value-added deflator that was just constructed, $P(P^0,P^1,Q^0,Q^1)$, using the bilateral index number formula P in both stages of aggregation, is equal to the value-added deflator that was constructed in a single-stage aggregation, $P(p^0,p^1,q^0,q^1)$, using the same index number formula P. That is, ask whether

$$(18.40)\ P(P^0,P^1,Q^0,Q^1) = P(p^0,p^1,q^0,q^1).$$

The answer to this question is yes, if the Laspeyres or Paasche price index is used at each stage of aggregation; that is, if $P = P_L$ or if $P = P_P$. The answer is no if a superlative price index is used at each stage of aggregation; that is, if $P = P_F$ or if $P = P_T$. However, using the results explained in Section C, the difference between the right-hand and left-hand sides of equation (18.40) will be very small if the Fisher or Törnqvist-Theil formulas, P_F or P_T, are used consistently at each stage of aggregation. Thus, using a superlative index number formula to construct output price, intermediate input price, and value-added deflators comes at the cost of small inconsistencies as prices are aggregated up in two or more stages of aggregation, whereas the Laspeyres and Paasche formulas are exactly consistent in aggregation. However, the use of the Laspeyres or Paasche formulas also comes at a cost: these indices will have an indeterminate amount of substitution bias compared with their theoretical counterparts,[11] whereas superlative indices will be largely free of substitution bias.

D.2 Laspeyres and Paasche value-added deflators

18.46 Given the importance of Paasche and Laspeyres price indices in statistical agency practice, it is worth writing out explicitly the value-added deflator using the two-stage aggregation procedure explained above when these two indices are used as the basic index number formula. If the Laspeyres formula is used, the two sides of equation (18.40) become

$$(18.41)\ P_L(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^0 - \sum_{m=1}^{M} p_{xm}^1 x_m^0}{\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0}$$
$$= s_y^0 \left(\frac{\sum_{n=1}^{N} p_{yn}^1 y_n^0}{\sum_{n=1}^{N} p_{yn}^0 y_n^0} \right) + s_x^0 \left(\frac{\sum_{m=1}^{M} p_{xm}^1 x_m^0}{\sum_{m=1}^{M} p_{xm}^0 x_m^0} \right)$$
$$= s_y^0\, P_L(p_y^0,p_y^1,y^0,y^1) + s_x^0\, P_L(p_x^0,p_x^1,x^0,x^1),$$

where the period 0 output share s_y^0 and the period 0 intermediate input share s_x^0 are defined as follows:

$$(18.42)\ s_y^0 = \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^0}{\sum_{n=1}^{N} p_{yn}^1 y_n^0 - \sum_{m=1}^{M} p_{xm}^1 x_m^0} = \frac{P_1^0 Q_1^0}{\left(P_1^0 Q_1^0 + P_2^0 Q_2^0\right)};$$

$$s_x^0 \equiv \frac{-\sum_{m=1}^{M} p_{xm}^0 x_m^0}{\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0} = \frac{P_2^0 Q_2^0}{\left(P_1^0 Q_1^0 + P_2^0 Q_2^0\right)}.$$

Note that s_y^0 will be greater than 1, and s_x^0 will be negative. Thus, equation (18.41) says that the Laspeyres value-added deflator can be written as a weighted average of the Laspeyres output price index, $P_L(p_y^0,p_y^1,y^0,y^1)$, and the Laspeyres intermediate input price index, $P_L(p_x^0,p_x^1,x^0,x^1)$. Although the weights sum to 1, s_x^0 is negative and s_y^0 is greater than 1, so these weights are rather unusual.

18.47 There is an analogous two-stage decomposition for the Paasche value-added deflator:

$$(18.43)\ P_P(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1}{\sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1}$$

[11]Recall Figure 17.1, which illustrated substitution biases for the Laspeyres and Paasche output price indices.

$$= 1 \Bigg/ \left(\frac{\sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1} \right)$$

$$= 1 \Bigg/ \left[s_y^1 \left(\frac{\sum_{n=1}^{N} p_{yn}^0 y_n^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^1} \right) + s_x^1 \left(\frac{\sum_{m=1}^{M} p_{xm}^0 x_m^1}{\sum_{m=1}^{M} p_{xm}^1 x_m^1} \right) \right]$$

$$= \{s_y^1[P_P(p_y^0,p_y^1,y^0,y^1)]^{-1} + s_x^1[P_P(p_x^0,p_x^1,x^0,x^1)]^{-1}\}^{-1}$$

where the period 1 output share s_y^1 and the period 1 intermediate input share s_x^1 are defined as follows:

$$(18.44)\ s_y^1 \equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1} = \frac{P_1^1 Q_1^1}{\left(P_1^1 Q_1^1 + P_2^1 Q_2^1\right)};$$

$$s_x^1 \equiv \frac{-\sum_{m=1}^{M} p_{xm}^1 x_m^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1} = \frac{P_2^1 Q_2^1}{\left(P_1^1 Q_1^1 + P_2^1 Q_2^1\right)}.$$

Note that s_y^1 will be greater than 1, and s_x^1 will be negative. Thus, equation (18.43) says that the Paasche value-added deflator can be written as a weighted *harmonic* average of the Paasche output price index, $P_P(p_y^0,p_y^1,y^0,y^1)$, and the Paasche intermediate input price index, $P_P(p_x^0,p_x^1,x^0,x^1)$.

18.48 The analysis presented in this section on the relationships between the output price, the intermediate input price, and the value-added deflator for an establishment can be extended to the industry or national levels.

D.3 Value-added deflators and double-deflation method for constructing real value added

18.49 In the previous section, it was shown how the Paasche and Laspeyres value-added deflators for an establishment were related to the Paasche and Laspeyres output and intermediate input price indices for an establishment. In this section, this analysis will be extended to look at the problems involved in using these indices to deflate nominal values into real values. Having defined a value-added deflator, $P(p^0,p^1,q^0,q^1)$, using some index number formula, equation (15.4) in Chapter 15 can be used to define a corresponding quantity index, $Q(p^0,p^1,q^0,q^1)$, which can be interpreted as the growth rate for real value added from period 0 to 1; that is, given P, Q can be defined as follows:

$$(18.45)\ Q(p^0,p^1,q^0,q^1) \equiv \left[\frac{V^1}{V^0}\right] P\left(p^0, p^1, q^0, q^1\right),$$

where V^t is the nominal establishment value added for period $t = 0,1$.

18.50 When the Laspeyres value-added deflator, $P_L(p^0,p^1,q^0,q^1)$, is used as the price index in equation (18.45), the resulting quantity index Q is the Paasche value-added quantity index Q_P defined as follows:

$$(18.46)\ Q_P(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^0 - \sum_{m=1}^{M} p_{xm}^1 x_m^0}.$$

When the Paasche value-added deflator, $P_P(p^0,p^1,q^0,q^1)$, is used as the price index in equation (18.45), the resulting quantity index Q is the Laspeyres value-added quantity index Q_L defined as follows:

$$(18.47)\ Q_L(p^0,p^1,q^0,q^1) \equiv \frac{\sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1}{\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0}.$$

18.51 Given a generic value-added quantity index, $Q(p^0,p^1,q^0,q^1)$, real value added in period 1 at the prices of period 0, rva^1, can be defined as the period 0 nominal value added of the establishment escalated by the value-added quantity index Q; that is,

$$(18.48)\ rva^1 \equiv V^0\ Q(p^0,p^1,q^0,q^1)$$

$$= \sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \quad Q\left(p^0, p^1, q^0, q^1\right).$$

18.52 If the Laspeyres value-added quantity index $Q_L(p^0,p^1,q^0,q^1)$ defined by equation (18.47) is used as the escalator of nominal value added in equation (18.48), the following rather interesting decomposition for the resulting period 1 real value added at period 0 prices is obtained:

(18.49) rva^1

$$\equiv \sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \quad Q_L\left(p^0, p^1, q^0, q^1\right)$$

$$= \sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1$$

using equation (18.47)

$$= \sum_{n=1}^{N} p_{yn}^0 y_n^0 \left(\frac{\sum_{n=1}^{N} p_{yn}^0 y_n^1}{\sum_{n=1}^{N} p_{yn}^0 y_n^0} \right) - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \left(\frac{\sum_{m=1}^{M} p_{xm}^0 x_m^1}{\sum_{m=1}^{M} p_{xm}^0 x_m^0} \right)$$

$$\equiv \sum_{n=1}^{N} p_{yn}^0 y_n^0 \, Q_L\left(p_y^0, p_y^1, q_y^0, q_y^1\right) - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \, Q_L\left(p_x^0, p_x^1, q_x^0, q_x^1\right).$$

Thus, period 1 real value added at period 0 prices, rva^1, is defined as period 0 nominal value added, $\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0$, escalated by the Laspeyres value-added quantity index, $Q_L(p^0,p^1,q^0,q^1)$, defined by equation (18.47). But the last line of equation (18.49) shows that rva^1 is also equal to the period 0 value of production, $\sum_{n=1}^{N} p_{yn}^0 y_n^0$, escalated by the Laspeyres output quantity index,[12] $Q_L\left(p_y^0, p_y^1, q_y^0, q_y^1\right)$, minus the period 0 intermediate input cost, $\sum_{m=1}^{M} p_{xm}^0 x_m^0$, escalated by the Laspeyres intermediate input quantity index, $Q_L\left(p_x^0, p_x^1, q_x^0, q_x^1\right)$.

[12]The use of the Laspeyres output quantity index can be traced back to Bowley (1921, p. 203).

18.53 Using equation (18.45) yields the following formula for the Laspeyres value-added quantity index, Q_L, in terms of the Paasche value-added deflator, P_P:

$$(18.50)\quad Q_L(p^0,p^1,q^0,q^1) = \left[\frac{V^1}{V^0}\right] \Big/ P_P\left(p^0, p^1, q^0, q^1\right).$$

Now, substitute equation (18.50) into the first line of equation (18.49) to obtain the following alternative decomposition for the period 1 real value added at period 0 prices, rva^1:

$$(18.51)\quad rva^1 \equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1}{P_P\left(p^0, p^1, q^0, q^1\right)}$$

$$= \sum_{n=1}^{N} p_{yn}^0 y_n^1 - \sum_{m=1}^{M} p_{xm}^0 x_m^1$$

using equation (18.43)

$$= \sum_{n=1}^{N} p_{yn}^1 y_n^1 \Bigg/ \left(\frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1}{\sum_{n=1}^{N} p_{yn}^0 y_n^1} \right) - \sum_{m=1}^{M} p_{xm}^1 x_m^1 \Bigg/ \left(\frac{\sum_{m=1}^{M} p_{xm}^1 x_m^1}{\sum_{m=1}^{M} p_{xm}^0 x_m^1} \right)$$

$$\equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1}{P_P\left(p_y^0, p_y^1, q_y^0, q_y^1\right)} - \frac{\sum_{m=1}^{M} p_{xm}^1 x_m^1}{P_P\left(p_x^0, p_x^1, q_x^0, q_x^1\right)}.$$

Thus, period 1 real value added at period 0 prices, rva^1, is equal to period 1 nominal value added, $\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1$, deflated by the Paasche *value-added deflator*, $P_P\left(p^0, p^1, q^0, q^1\right)$, defined by equation (18.43). But the last line of equation (18.51) shows that rva^1 is also equal to the period 1 value of production, $\sum_{n=1}^{N} p_{yn}^1 y_n^1$, deflated by the Paasche output price index, $P_P\left(p_y^0, p_y^1, q_y^0, q_y^1\right)$, minus the period 1 intermediate input cost, $\sum_{m=1}^{M} p_{xm}^1 x_m^1$, deflated by the Paasche intermediate input price index, $P_P\left(p_x^0, p_x^1, q_x^0, q_x^1\right)$. Thus, the use of the Paasche value-added deflator leads to a measure of period 1 real value added at period 0 prices, rva^1, that is equal to period 1 deflated output minus period 1 deflated intermediate input. Hence, this method for constructing a real value-added measure is called the *double-deflation method*.[13] The method of double deflation has been subject to some criticism. Peter Hill (1996) has shown that errors in measurement in the individual components, reflected in a higher variance of price changes, can lead to even larger errors in the double-deflated value added, since the subtraction of the two variances compounds the overall error.

18.54 There is a less well-known method of double deflation that reverses the above roles of the Paasche and Laspeyres indices. Instead of expressing real value added in period 1 at the prices of period 0, it is also possible to define real value added in period 0 at the prices of period 1, rva^0. Using this methodology, given a generic value-added quantity index, the counterpart to equation (18.48) is

$$(18.52)\quad rva^0 \equiv V^1 \big/ Q\left(p^0, p^1, q^0, q^1\right) = \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1}{Q\left(p^0, p^1, q^0, q^1\right)}.$$

Thus, to obtain period 0 real value added at the prices of period 1, rva^0, take the nominal period 1 value added, V^1, and deflate it by the value-added quantity index, $Q(p^0,p^1,q^0,q^1)$.

18.55 If the Paasche value-added quantity index, $Q_P(p^0,p^1,q^0,q^1)$, defined by equation (18.46) above is used as the deflator of nominal value added in (18.52), the following interesting decomposition for the resulting period 0 real value added at period 1 prices is obtained:

$$(18.53)\quad rva^0 \equiv \left[\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1\right] \Big/ Q_P\left(p^0, p^1, q^0, q^1\right)$$

$$= \left[\sum_{n=1}^{N} p_{yn}^1 y_n^0 - \sum_{m=1}^{M} p_{xm}^1 x_m^0\right]$$

using equation (18.46)

$$= \left[\sum_{n=1}^{N} p_{yn}^1 y_n^1\right] \Bigg/ \left(\frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1}{\sum_{n=1}^{N} p_{yn}^1 y_n^0}\right) - \sum_{m=1}^{M} p_{xm}^1 x_m^1 \Bigg/ \left(\frac{\sum_{m=1}^{M} p_{xm}^1 x_m^1}{\sum_{m=1}^{M} p_{xm}^1 x_m^0}\right)$$

$$\equiv \frac{\sum_{n=1}^{N} p_{yn}^1 y_n^1}{Q_P\left(p_y^0, p_y^1, y^0, y^1\right)} - \frac{\sum_{m=1}^{M} p_{xm}^1 x_m^1}{Q_P\left(p_x^0, p_x^1, x^0, x^1\right)}.$$

Thus, period 0 real value added at period 1 prices, rva^0, is defined as period 1 nominal value added, $\sum_{n=1}^{N} p_{yn}^1 y_n^1 - \sum_{m=1}^{M} p_{xm}^1 x_m^1$, deflated by the Paasche value-added quantity index, $Q_P(p^0,p^1,q^0,q^1)$, defined by equation (18.46). But the last line of equation (18.53) shows that rva^0 is also equal to the period 1 value of production, $\sum_{n=1}^{N} p_{yn}^1 y_n^1$, deflated by the Paasche output quantity index, $Q_P(p_y^0,p_y^1,y^0,y^1)$, minus the period 1 intermediate input cost, $\sum_{m=1}^{M} p_{xm}^1 x_m^1$, deflated by the Paasche intermediate input quantity index, $Q_P(p_x^0,p_x^1,x^0,x^1)$.

18.56 Using equation (18.45) yields the following formula for the Paasche value-added quantity index, Q_P, in terms of the Laspeyres value-added deflator, P_L:

$$(18.54)\quad Q_P(p^0,p^1,q^0,q^1) = \left[\frac{V^1}{V^0}\right] \Big/ P_L\left(p^0, p^1, q^0, q^1\right).$$

Now, substitute equation (18.54) into the first line of equation (18.53) to obtain the following alternative decomposition for the period 0 real value added at period 1 prices, rva^0:

[13] See Schreyer (2001, p. 32). A great deal of useful material in this book will be of interest to price statisticians.

(18.55) rva^0

$$\equiv \left[\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0\right] P_L\left(p^0, p^1, q^0, q^1\right)$$

$$= \left[\sum_{n=1}^{N} p_{yn}^1 y_n^0 - \sum_{m=1}^{M} p_{xm}^1 x_m^0\right]$$

using equation (18.41)

$$= \left[\sum_{n=1}^{N} p_{yn}^0 y_n^0\right]\left(\frac{\sum_{n=1}^{N} p_{yn}^1 y_n^0}{\sum_{n=1}^{N} p_{yn}^0 y_n^0}\right) - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \left(\frac{\sum_{m=1}^{M} p_{xm}^1 x_m^0}{\sum_{m=1}^{M} p_{xm}^0 x_m^0}\right)$$

$$= \sum_{n=1}^{N} p_{yn}^0 y_n^0 \; P_L\left(p_y^0, p_y^1, y^0, y^1\right) - \sum_{m=1}^{M} p_{xm}^0 x_m^0 \; P_L\left(p_x^0, p_x^1, x^0, x^1\right).$$

Thus, period 0 real value added at period 1 prices, rva^0, is equal to period 0 nominal value added, $\sum_{n=1}^{N} p_{yn}^0 y_n^0 - \sum_{m=1}^{M} p_{xm}^0 x_m^0$, escalated by the Laspeyres *value-added deflator*, $P_L(p^0,p^1,q^0,q^1)$, defined by equation (18.41). But the last line of equation (18.55) shows that rva^0 is also equal to the period 0 value of production, $\sum_{n=1}^{N} p_{yn}^0 y_n^0$, escalated by the Laspeyres output price index, $P_L(p_y^0,p_y^1,y^0,y^1)$, minus the period 0 intermediate input cost, $\sum_{m=1}^{M} p_{xm}^0 x_m^0$, escalated by the Laspeyres intermediate input price index, $P_L(p_x^0,p_x^1,x^0, x^1)$.[14]

E. Aggregation of Establishment Deflators into a National Value-Added Deflator

18.57 Once establishment value-added deflators have been constructed for each establishment, there remains the problem of aggregating up these deflators into an industry or regional or national value-added deflator. Only the national aggregation problem is considered in this section, but the same logic will apply to the regional and industry aggregation problems.[15]

18.58 Let the vectors of output price, output quantity, intermediate input price, and intermediate input price vectors for an establishment e in period t be denoted by p_y^{et}, y^{et}, p_x^{et}, and x^{et}, respectively, for $t = 0,1$ and $e = 1,...,E$. As usual, the net price and net quantity vectors for establishment e in period t are defined as $p^{et} \equiv [p_y^{et},p_x^{et}]$ and $q^{et} \equiv [y^{et},-x^{et}]$ for $t = 0,1$ and $e = 1,...,E$. Suppose that a bilateral index number formula P is used to construct a value-added deflator, $P(p^{e0},p^{e1},q^{e0},q^{e1})$, for establishment e where $e = 1,...,E$. Our problem is somehow to aggregate up these establishment indices into a national value-added deflator.

18.59 The two-stage aggregation procedure explained in Section C above is used to do this aggregation. The first stage of the aggregation of price and quantity vectors is for the establishment net output price vectors, p^{et}, and the establishment net output quantity vectors, q^{et}. These establishment price and quantity vectors are combined into national price and quantity vectors, p^t and q^t, as follows:[16]

(18.56) $p^t = (p^{1t}, p^{2t}, \ldots ,p^{Et})$; $q^t = (q^{1t}, q^{1t}, \ldots ,q^{Et})$; $t = 0,1$.

For each establishment e, its aggregate price of value added P_e^0 in the base period is set equal to 1, and the corresponding establishment e base-period quantity of value added Q_e^0 is defined as the establishment's period 0 value added; that is,

(18.57) $P_e^0 \equiv 1$; $Q_e^0 \equiv \sum_{i=1}^{N+M} p_i^{e0} q_i^{e0}$ for $e = 1,2,\ldots,E$.

Now, the chosen price index formula P is used to construct a period 1 price for the price of value added for each establishment e, say, P_e^1 for $e = 1,2,\ldots,E$:

[14]This method for constructing real value-added measures was used by Phillips (1961, p. 320).

[15]The algebra developed in Section E can also be applied to the problem of aggregating establishment or industry output or intermediate input price indices into national output or intermediate input price indices.

[16]Equation (18.56) is the counterpart to equation (18.26) in Section C. Equations (18.57)–(18.61) are counterparts to equations (18.27)–(18.38) in Sections C and D.

(18.58) $P_e^1 \equiv P(p^{e0},p^{e1},q^{e0},q^{e1})$ for $e = 1,2,\ldots E$.

Once the period 1 prices for the E establishments have been defined by equation (18.58), then corresponding establishment e period 1 quantities Q_e^1 can be defined by deflating the period 1 establishment values $\sum_{i=1}^{N+M} p_i^{e1} q_i^{e1}$ by the prices P_e^1 defined by equation (18.58); that is,

(18.59) $Q_e^1 \equiv \sum_{i=1}^{N+M} p_i^{e1} q_i^{e1} \Big/ P_e^1$ for $e = 1,2,\ldots,E$.

The aggregate establishment price and quantity vectors for each period $t = 0,1$ can be defined using equations (18.57) to (18.59). Thus, the period 0 and 1 establishment value-added price vectors P^0 and P^1 are defined as follows:

(18.60) $P^0 = (P_1^0, P_2^0,\ldots,P_E^0) \equiv 1_E$;
$P^1 = (P_1^1, P_2^1,\ldots,P_E^1)$,

where 1_E denotes a vector of ones of dimension E, and the components of P^1 are defined by equation (18.58). The period 0 and 1 establishment value-added quantity vectors Q^0 and Q^1 are defined as

(18.61) $Q^0 = (Q_1^0, Q_2^0,\ldots,Q_E^0)$;
$Q^1 = (Q_1^1, Q_2^1,\ldots,Q_E^1)$,

where the components of Q^0 are defined in equation (18.57) and the components of Q^1 are defined in equation (18.59). The price and quantity vectors in equations (18.60) and (18.61) represent the results of the first-stage aggregation (over commodities within an establishment). These vectors can now be inputs into the second-stage aggregation problem (which aggregates over establishments); that is, our chosen price index formula can be applied using the information in equations (18.60) and (18.61) as inputs into the index number formula. The resulting two-stage aggregation national value-added deflator is $P(P^0,P^1,Q^0,Q^1)$. It should be asked whether this two-stage index equals the corresponding single-stage index $P(p^0,p^1,q^0,q^1)$ that treats each output or intermediate input produced or used by each establishment as a separate commodity, using the same index number formula P. That is, it is asked whether

(18.62) $P(P^0,P^1,Q^0,Q^1) = P(p^0,p^1,q^0,q^1)$.

18.60 If the Laspeyres or Paasche formula is used at each stage of each aggregation, the answer to the above question is yes. Thus, in particular, the national Laspeyres value-added deflator that is constructed in a single stage of aggregation, $P_L(p^0,p^1,q^0,q^1)$, is equal to the two-stage Laspeyres value-added deflator, $P_L(P^0,P^1,Q^0,Q^1)$, where the Laspeyres formula is used in equation (18.58) to construct establishment value-added deflators in the first stage of aggregation. If a superlative formula is used at each stage of aggregation, the answer to the above consistency-in-aggregation question is no: equation (18.62) using a superlative P will hold only approximately. However, if the Fisher, Walsh, or Törnqvist price index formulas are used at each stage of aggregation, the differences between the right- and left-hand sides of equation (18.62) will be very small using normal time series data.

F. National Value-Added Deflator versus Final-Demand Deflator

18.61 In this section, we ask whether there are any relationships between the *national value-added deflator* defined in the preceding sections of this chapter and the *national deflator for final-demand expenditures*. In particular, we look for conditions that will imply that the two deflators are exactly equal.

18.62 Assume that the commodity classification for intermediate inputs is exactly the same as the commodity classification for outputs, so that, in particular, N, the number of outputs, is equal to M, the number of intermediate inputs. This assumption is not restrictive, since if N is chosen to be large enough, all produced intermediate inputs can be accommodated in the expanded output classification.[17] With this change in assumptions, the

[17]It is not necessary to assume that each establishment or sector of the economy produces all outputs and uses all intermediate inputs in each of the two periods being compared. All that is required is that if an output is not produced in one period by establishment e, then that output is also not produced in the other period. Similarly, it is required that if an establishment does not use a particular intermediate input in one period, then it also does not use it in the other period.

same notation can be used as was used in the previous section. Thus, let the vectors of output price, output quantity, intermediate input price, and intermediate input price vectors for an establishment e in period t be denoted by p_y^{et}, y^{et}, p_x^{et}, and x^{et}, respectively, for $t = 0,1$ and $e = 1,...,E$. As usual, the net price and net quantity vectors for establishment e in period t are defined as $p^{et} \equiv [p_y^{et},p_x^{et}]$ and $q^{et} \equiv [y^{et}, -x^{et}]$ for $t = 0,1$ and $e = 1,...,E$. Again, define the national price and quantity vectors, p^t and q^t, as

$$p^t \equiv (p^{1t}; p^{2t}; \dots ;p^{Et}) = (p_y^{1t},p_x^{1t};p_y^{2t},p_x^{2t};\dots;p_y^{Et},p_x^{Et}) \text{ and}$$
$$q^t \equiv (q^{1t}, q^{1t},\dots,q^{Et}) = (y^{1t},-x^{1t},y^{2t},-x^{2t};\dots;y^{Et},-x^{Et}) \text{ for } t = 0,1.$$

As in the previous section, an index number formula P is chosen and the national value-added deflator denoted as $P(p^0,p^1,q^0,q^1)$.

18.63 Using the above notation, the period tN by E *make matrix* for the economy, Y^t, and the period tN by E *use matrix*, X^t, are defined as follows:

$$(18.63)\quad Y^t \equiv [y^{1t},y^{2t},...,y^{Et}] \; ; X^t \equiv [x^{1t},x^{2t},...,x^{Et}] \; ; \; t = 0,1.$$

The *period* t *final-demand vector* for the economy, f^t, can be defined by summing up all the establishment output vectors y^{et} in the period t make matrix and subtracting all the establishment intermediate input-demand vectors x^{et} in the period t use matrix; that is, define f^t by[18]

$$(18.64)\; f^t \equiv \sum_{e=1}^{E} y^{et} - \sum_{e=1}^{E} x^{et} \; ; \; t = 0,1.$$

18.64 Final-demand prices are required to match up with the components of the period t final-demand quantity vector $f^t = [f_1^t,...,f_N^t]$. The net value of production for commodity n in period t divided by the net deliveries of this commodity to final demand f_n^t is *the period* t *final-demand unit value for commodity* n, p_{fn}^t:

$$(18.65)\quad p_{fn}^t \equiv \frac{\sum_{e=1}^{E} p_{yn}^{et} y_n^{et} - \sum_{e=1}^{E} p_{xn}^{et} x_n^{et}}{f_n^t} \; ; \; n = 1,...,N \; ; \; t = 0,1.$$

If equation (18.64) is to hold so that production minus intermediate input use equals deliveries to final demand for each commodity in period t, and if the value of production minus the value of intermediate demands is to equal the value of final demand for each commodity in period t, then the value-added prices defined by equation (18.65) must be used as final-demand prices.

18.65 Define the vector of period t final-demand prices as $p_f^t \equiv [p_{f1}^t,p_{f2}^t,...,p_{fN}^t]$ for $t = 0,1$, where the components p_{fn}^t are defined by equation (18.65). The corresponding final-demand quantity vector f^t has already been defined by equation (18.64). Hence, a generic price index number formula P can be taken to form the *final-demand deflator*, $P(p_f^0,p_f^1,f^0,f^1)$. It is now asked whether this final-demand deflator is equal to the national value-added deflator $P(p^0,p^1,q^0,q^1)$ defined in Section B.3; that is, whether

$$(18.66)\; P(p_f^0,p_f^1,f^0,f^1) = P(p^0,p^1,q^0,q^1).$$

Note that the dimensionality of each price and quantity vector that occurs in the left-hand side of equation (18.66) is N (the number of commodities in our output classification), while the dimensionality of each price and quantity vector that occurs in the right-hand side of equation (18.66) is $2NE$, where E is the number of establishments (or industries or sectors that have separate price and quantity vectors for both outputs and intermediate inputs) that are aggregating over.

18.66 The answer to the question asked in the previous paragraph is no; in general, it will *not* be the case that the final-demand deflator is equal to the national value-added deflator.

18.67 However, under certain conditions, equation (18.66) will hold as an equality. A set of conditions is now developed. The first assumption is that all establishments face the same vector of prices p^t in period t for both the outputs that they

[18] Components of f^t can be negative if the corresponding commodity is being imported into the economy during period t, or if the component corresponds to the change in an inventory item.

produce and for the intermediate inputs that they use. That is, it is assumed[19]

(18.67) $p_y^{et} = p_x^{et} = p^t$; $e = 1,...,E$; $t = 0,1$.

If assumptions in equation (18.67) hold, then it is easy to verify that the vector of period t final-demand prices p_f^t defined above by equation (18.65) is also equal to the vector of period t basic prices p^t.

18.68 If assumptions in equation (18.67) hold and the price index formula used in both sides of equation (18.66) is the *Laspeyres formula*, then it can be verified that equation (18.66) will hold as an equality; that is, the Laspeyres final-demand deflator will be equal to the national Laspeyres value-added deflator. To see why this is so, use the Laspeyres formula in equation (18.66) and, for the left-hand side of the index, collect all the quantity terms both in the numerator and denominator of the index that correspond to the common establishment price for the nth commodity, $p_n^t = p_{yn}^{et} = p_{xn}^{et}$, for $e = 1,...,E$. Using equation (18.64) for $t = 0$, the resulting sum of collected quantity terms will sum to f_n^0. Since this is true for $n = 1,...,N$, it can be seen that the left-hand side of the Laspeyres index is equal to the right-hand side of the Laspeyres index.

18.69 If assumptions in equation (18.67) hold and the price index formula used in both sides of equation (18.66) is the *Paasche formula*, then it can be verified that equation (18.66) will also hold as an equality; that is, the Paasche final-demand deflator will equal the national Paasche value-added deflator. To see why this is so, use the Paasche formula in equation (18.66) and, for the left-hand side of the index, collect all the quantity terms both in the numerator and denominator of the index that correspond to the common establishment price for the nth commodity, $p_n^t = p_{yn}^{et} = p_{xn}^{et}$, for $e = 1,...,E$. Using equation (18.64) for t =1, the resulting sum of collected quantity terms will sum to f_n^1. Since this is true for $n = 1,...,N$, it can be seen that the left-hand side of the Paasche index is equal to the right-hand side of the Paasche index.

18.70 The results in the previous two paragraphs imply that the national value-added deflator will equal the final-demand deflator provided that Paasche or Laspeyres indices are used and provided that assumptions in equation (18.67) hold. But these two results immediately imply that if equation (18.67) holds and Fisher ideal price indices are used, then an important equality is obtained—that the Fisher national value-added deflator is equal to the Fisher final-demand deflator.

18.71 Recall equation (18.21) of the national Törnqvist-Theil output price index P_T in Section B.1 above. The corresponding national Törnqvist-Theil value-added deflator P_T was defined in Section B.3. Make assumptions in equation (18.67), start with the national Törnqvist-Theil value-added deflator, and collect all the exponents that correspond to the common price relative for commodity n, p_n^1 / p_n^0. Using equation (18.65), the sum of these exponents will equal the exponent for the nth price term, $p_{fn}^1 / p_{fn}^0 = p_n^1 / p_n^0$, in the Törnqvist-Theil final-demand deflator. Since this equality holds for all $n = 1,...,N$, the equality of the national value-added deflator to the final-demand deflator is also obtained if the Törnqvist formula P_T is used on both sides of equation (18.66).

18.72 Summarizing the above results, it has been shown that the national value-added deflator is equal to the final-demand deflator, provided that all establishments face the same vector of prices in each period for both the outputs that they produce and for the intermediate inputs that they use, and provided that either the Laspeyres, Paasche, Fisher, or Törnqvist price index formula is used for both deflators.[20] However, these results were established ignoring the existence of indirect taxes and subsidies that may be applied to the outputs and intermediate inputs of each establishment. It is necessary to extend the initial results to deal with situations where there are indirect taxes on deliveries to final demand and indirect taxes on the use of intermediate inputs.

18.73 Again, it is assumed that all establishments face the same prices for their inputs and outputs, but it is now assumed that their deliveries

[19]Under these hypotheses, the vector of producer prices p^t can be interpreted as the vector of *basic producer prices* that appears in the *1993 SNA*.

[20]This result does not carry over if we use the Walsh price index formula.

to the final-demand sector are *taxed*.[21] Let τ_n^t be the period t ad valorem commodity tax rate on deliveries to final demand of commodity n for $t = 0,1$ and $n = 1,...,N$.[22] Thus, the period t final-demand price for commodity n is now

$$(18.68)\quad p_{fn}^t = p_n^t(1 + \tau_n^t)\ ;\ n = 1,...,N\ ;\ t = 0,1.$$

These tax-adjusted final-demand prices defined by equation (18.68) can be used to form new vectors of final-demand price vectors, $p_f^t \equiv [p_{f1}^t,...,p_{fN}^t]$ for $t = 0,1$. The corresponding final-demand quantity vectors, f^0 and f^1, are still defined by the commodity balance equation (18.64). Now, pick an index number formula P and form the *final-demand deflator* $P(p_f^0,p_f^1,f^0,f^1)$ using the new tax-adjusted prices, p_f^0,p_f^1. If the commodity tax rates τ_n^t are substantial, the new final-demand deflator $P(p_f^0,p_f^1,f^0,f^1)$ can be substantially different from the national value-added deflator $P(p^0,p^1,q^0,q^1)$ defined earlier in this section (because all the commodity tax terms are missing from the national value-added deflator).

[21]Hicks (1940, p. 106) appears to have been the first to note that the treatment of indirect taxes in national income accounting depends on the purpose for which the calculation is to be used. Thus, for measuring productivity, Hicks (1940, p. 124) advocated using prices that best represented marginal costs and benefits from the perspective of producers—that is, basic prices should be used. On the other hand, if the measurement of economic welfare is required, Hicks (1940, pp. 123–24) advocated the use of prices that best represent marginal utilities of consumers—that is, final-demand prices should be used. Bowley (1922, p. 8) advocated the use of final-demand prices, but he implicitly took a welfare point of view: "To the purchaser of whisky, tobacco and entertainment tickets, the goods bought are worth what he pays; it is indifferent to him whether the State or the producer gets the money."

[22]If commodity n is subsidized during period t, then τ_n^t can be set equal to minus the subsidy rate. In most countries, the commodity tax regime is much more complex than we have modeled it above, in that some sectors of final demand are taxed differently than other sectors; for example, exported commodities are generally not taxed or are taxed more lightly than other final-demand sectors. To deal with these complications, it would be necessary to decompose the single final-demand sector into a number of sectors (for example, the familiar $C + I + G + X - M$ decomposition) where the tax treatment in each sector is uniform. In this disaggregated framework, tariffs on imported goods and services can readily be accommodated. There are additional complications owing to the existence of commodity taxes that fall on intermediate inputs. To deal adequately with all these complications would require a rather extended discussion. The purpose here is to indicate to the reader that the national value-added deflator is closely connected to the final-demand deflator.

18.74 However, it is possible to adjust our national value-added deflator in an attempt to make it more comparable to the final-demand deflator. Recall that the price and quantity vectors, p^t and q^t, that appear in the national value-added deflator are defined as follows:[23]

$$(18.69)\quad p^t \equiv [p_y^{1t},p_x^{1t};p_y^{2t},p_x^{2t};...;p_y^{Et},p_x^{Et}]\ ;\ t = 0,1;$$
$$q^t \equiv [y^{1t},-x^{1t};y^{2t},-x^{2t};...;y^{Et},-x^{Et}]\ ;\ t = 0,1;$$

where p_y^{et} is the vector of output prices that establishment e faces in period t, p_x^{et} is the vector of input prices that establishment e faces in period t, y^{et} is the production vector for establishment e in period t, and x^{et} is the vector of intermediate inputs used by establishment e during period t. The adjustment made to the national value-added deflator is that an additional N artificial commodities are added to the list of outputs and inputs that the national value-added deflator aggregates over. Define the price and quantity of the nth extra *artificial commodity* as follows:

$$(18.70)\quad p_n^{At} \equiv p_n^t\tau_n^t\ ;\ q_n^{At} \equiv f_n^t\ ;\ n = 1,...,N\ ;\ t = 0,1.$$

Thus, the period t price of the nth artificial commodity is just the product of the nth basic price, p_n^t, times the nth commodity tax rate in period t, τ_n^t. The period t quantity for the nth artificial commodity is simply equal to period t final demand for commodity n, f_n^t. Note that the period t value of all N artificial commodities is just equal to period t commodity tax revenue. Define the period t price and quantity vectors for the artificial commodities in the usual way; that is, $p^{At} \equiv [p_1^{At},...,p_N^{At}]$ and $q^{At} \equiv [q_1^{At},...,q_N^{At}] = f^t$, $t = 0,1$. Now, add the extra price vector p^{At} to the initial period t price vector p^t that was used in the national value-added deflator, and add the extra quantity vector q^{At} to the initial period t quantity vector q^t that was used in the national value-added deflator. That is, define the *augmented national price and quantity vectors*, p^{t*} and q^{t*} as follows:

$$(18.71)\quad p^{t*} \equiv [p^t,p^{At}]\ ;\ q^{t*} \equiv [q^t,q^{At}]\ ;\ t = 0,1.$$

[23]Under assumptions in equation (18.67), the definition of p^t simplifies dramatically.

Using the augmented price and quantity vectors defined above, calculate a *new tax-adjusted national value-added deflator* using the chosen index number formula, $P(p^0*,p^1*,q^0*,q^1*)$, and ask whether it will equal the final-demand deflator, $P(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$ using the new tax-adjusted prices, $p_f^{\,0},p_f^{\,1}$, defined by equation (18.68). That is, ask whether the following equality holds:

(18.72) $P(p^0*,p^1*,q^0*,q^1*) = P(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$.

18.75 Choose P to be P_L, the Laspeyres formula, and evaluate the left-hand side of equation (18.72). Using assumptions in equation (18.67), collect all terms in the numerator of the Laspeyres national value-added deflator, $P_L(p^0*,p^1*,q^0*,q^1*)$, that correspond to the nth commodity price $p_n^{\,1}$. Using equation (18.64) for $t = 0$, it is found that the sum of these terms involving $p_n^{\,1}$ is $p_n^{\,1}(1 + \tau_n^{\,1})f_n^{\,0}$, which is equal to the nth term in the numerator of the final-demand deflator, $P_L(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$. In a similar fashion, collect all terms in the denominator of the Laspeyres national value-added deflator, $P_L(p^0*,p^1*,q^0*,q^1*)$, that correspond to the nth commodity price $p_n^{\,0}$. Using equation (18.64) for $t = 0$, it is found that the sum of these terms involving $p_n^{\,0}$ is $p_n^{\,0}(1 + \tau_n^{\,1})f_n^{\,0}$, which is equal to the nth term in the denominator of the final-demand deflator, $P_L(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$. Thus, equation (18.72) does hold as an exact equality under the above assumptions if the Laspeyres price index is used for each of the deflators.

18.76 Now choose P to be P_P, the Paasche formula, and evaluate the left-hand side of equation (18.72). Using assumptions in equation (18.67), collect all terms in the numerator of the Paasche national value-added deflator, $P_P(p^0*,p^1*,q^0*,q^1*)$, that correspond to the nth commodity price $p_n^{\,1}$. Using equation (18.64) for $t = 1$, it is found that the sum of these terms involving $p_n^{\,1}$ is $p_n^{\,1}(1 + \tau_n^{\,1})f_n^{\,1}$, which is equal to the nth term in the numerator of the final-demand deflator, $P_P(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$. In a similar fashion, collect all terms in the denominator of the Paasche national value-added deflator, $P_P(p^0*,p^1*,q^0*,q^1*)$, that correspond to the nth commodity price $p_n^{\,0}$. Using equation (18.64) for $t = 1$, it is found that the sum of these terms involving $p_n^{\,0}$ is $p_n^{\,0}(1 + \tau_n^{\,1})f_n^{\,1}$, which is equal to the nth term in the denominator of the final-demand deflator, $P_P(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$. Thus, equation (18.72) does hold as an exact equality under the above assumptions if the Paasche price index is used for each of the deflators. Putting this result together with the result in the previous paragraph, we see that under the above assumptions, equation (18.72) also holds as an exact equality if the Fisher index is used for both the final-demand deflator and tax-adjusted national value-added deflator, which is built up using industry information.

18.77 Finally, choose P to be P_T, the Törnqvist-Theil formula for a price index, and evaluate both sides of equation (18.79). In general, this time an exact equality is *not* obtained between the national Törnqvist-Theil tax-adjusted value-added deflator $P_T(p^0*,p^1*,q^0*,q^1*)$ and the Törnqvist-Theil final-demand deflator $P_T(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$.

18.78 However, if the extra assumption—in addition to equation (18.67), the assumption of equal basic prices across industries—is made that the commodity tax rates are equal in periods 0 and 1 so that

(18.73) $\tau_n^{\,0} = \tau_n^{\,1}$ for $n = 1,...,N$,

then it can be shown that the national Törnqvist-Theil tax-adjusted value-added deflator $P_T(p^0*,p^1*,q^0*,q^1*)$ and the Törnqvist-Theil final-demand deflator $P_T(p_f^{\,0},p_f^{\,1},f^{\,0},f^{\,1})$ are *exactly equal*.

The last few results can be modified to work in reverse: that is, start with the final-demand deflator and make some adjustments to it using artificial commodities. Then the resulting tax-adjusted final-demand deflator can equal the original unadjusted national value-added deflator. To implement this reverse procedure, it is necessary to add an additional N artificial commodities to the list of outputs and inputs that the final-demand deflator aggregates over. Define the price and quantity of the nth extra *artificial commodity* as follows:

(18.74) $p_n^{\,At} \equiv p_n^{\,t}\tau_n^{\,t}$; $q_n^{\,At} \equiv -f_n^{\,t}$; $n = 1,...,N$; $t = 0,1$.

Thus, the period t price of the nth artificial commodity is just the product of the nth basic price, $p_n^{\,t}$, times the nth commodity tax rate in period t, $\tau_n^{\,t}$. The period t quantity for the nth artificial commodity is simply equal to minus period t final demand for commodity n, $-f_n^{\,t}$. Note that the period t value of all N artificial commodities is just equal to *minus* period t commodity tax revenue. Define the period t price and quantity vectors for the artificial commodities in the usual way; that is, $p^{At} \equiv [p_1^{At},...,p_N^{At}]$ and $q^{At} \equiv [q_1^{At},...,q_N^{At}] = f^{\,t}$, $t = 0,1$. The extra price vector p^{At} is now added to the old period t price vector p_f^t that was used in the final-demand deflator, and the extra quantity vector q^{At} is added to the initial period t quantity vector $f^{\,t}$ that was used in the final-demand deflator. That is, define the *augmented final-demand price and quantity vectors*, p^t* and $f^{\,t}*$, as follows:

(18.75) $p_f^{\,t}* \equiv [p_f^{\,t},p^{At}]$; $f^{\,t}* \equiv [f^{\,t},q^{At}]$; $t = 0,1.$

Using the augmented price and quantity vectors defined above, a *new tax-adjusted final-demand deflator* is calculated using the chosen index number formula, $P(p_f^{\,0}*,p_f^{\,1}*,f^{\,0}*,f^{\,1}*)$, and the question asked is whether it will equal our initial *national value-added deflator* (that did not make any tax adjustments for commodity taxes on final demands), $P(p^0,p^1,q^0,q^1)$; that is, ask whether the following equality holds:

(18.76) $P(p_f^{\,0}*,p_f^{\,1}*,f^{\,0}*,f^{\,1}*) = P(p^0,p^1,q^0,q^1).$

18.79 Under the assumption that all establishments face the same prices, it can be shown *that the tax-adjusted final-demand deflator will exactly equal the national value-added deflator*, provided that the index number formula in equation (18.76) is chosen to be the Laspeyres, Paasche, or Fisher formulas, P_L, P_P, or P_F. In general, equation (18.76) will not hold as an exact equality if the Törnqvist-Theil formula, P_T, is used. However, if the commodity tax rates are equal in periods 0 and 1, so that assumptions in equation (18.73) hold in addition to assumptions in equation (18.67), then it can be shown that equation (18.76) will hold as an exact equality when P is set equal to P_T, the Törnqvist-Theil formula. These results are of some practical importance for the following reason. Most countries do not have adequate surveys that will support a complete system of value-added price indices for each sector of the economy.[24] Adequate information is generally available that will enable the statistical agency to calculate the final-demand deflator. However, for measuring the productivity of the economy using the economic approach to index number theory, the national value-added deflator is the preferred deflator.[25] The results that have just been stated show how the final-demand deflator can be modified to give a close approximation to the national value-added deflator under certain conditions.

18.80 It has always been a bit of a mystery how tax payments should be decomposed into price and quantity components in national accounting theory. The results presented in this section may be helpful in suggesting reasonable decompositions under certain conditions.

[24]In particular, information on the prices and quantities of intermediate inputs used by sector are generally lacking. These data deficiencies were noted by Fabricant (1938, pp. 566–70) many years ago, and he indicated some useful methods that are still used today in attempts to overcome these data deficiencies.

[25]See Schreyer (2001) for more explanation.

19. Price Indices Using an Artificial Data Set

A. Introduction

19.1 In order to give the reader some idea of how much the various index numbers might differ using a real data set, all of the major indices defined in the previous chapters are computed using an artificial data set consisting of prices and quantities for eight commodities over five periods (see Section B).[1] The period can be thought of as between one and five years. The trends in the data are generally more pronounced than one would see in the course of a year. The eight commodities can be thought of as the net deliveries to the final demand sector of all industries in the economy. The first six commodities are outputs and correspond to the usual private consumption plus government consumption plus investment plus export deliveries to final demand, whereas the last two commodities are imports (and hence are indexed with a negative sign).

19.2 In Section C, the same final-demand data set is used in order to compute the midyear indices that were described in Chapter 17. Recall that these indices have an important practical advantage over superlative indices because they can be computed using current data on prices and lagged data on quantities (or equivalently, using lagged data on expenditures).

19.3 In Section D, the additive percentage change decompositions for the Fisher ideal price index that were discussed in Section C.8 of Chapter 16 are illustrated using the final-demand data set on eight commodities.

19.4 In Section E.1, price and quantity data for three industrial sectors of the economy are presented. This industrial data set is consistent with the final-demand data set listed in Section B.1 below. Sections E.2 through E.4 construct value-added deflators for these three industries. Only the Laspeyres, Paasche, Fisher, and Törnqvist formulas are considered in Section E and subsequent sections since these are the formulas that are likely to be used in practice.

19.5 In Section F, the industry data are used in order to construct national output price indices, national intermediate input price deflators, and national value-added deflators. The construction of a national value-added deflator by aggregating the national output and intermediate input price indices is undertaken in Section F.4. This two-stage national value-added deflator is then compared with its single-stage counterpart and also with the final-demand deflator constructed in Section B.

B. Price Indices for Final-Demand Components

B.1 Final-demand data set

19.6 The price and quantity data for net deliveries to final demand are listed in Tables 19.1 and 19.2 below. For convenience, the period t nominal expenditures, $p^t \cdot q^t \equiv \sum_{i=1}^{8} p_i^t q_i^t$, have been listed along with the corresponding period t expenditure shares, $s_i^t \equiv p_i^t q_i^t / p^t \cdot q^t$, in Table 19.3. Typically, the statistical agency will not have quantity data available; only price and expenditure data will be collected. However, given the information in Table 19.3, the period t net expenditure shares s_n^t may be multiplied by period t total net expenditures $p^t \cdot q^t$ in order to obtain final-demand expenditures by commodity. Then these commodity expenditures may be divided by the corresponding prices in Ta-

[1] Lowe and Young indices are not calculated for this data set; however, they are available in Chapter 19 of the *Consumer Price Index Manual* (International Labour Organization and others, 2004) to allow comparisons with the other major indices.

ble 19.1 in order to obtain the implicit quantities listed in Table 19.2.[2]

19.7 The trends that are built into the tables can be explained as follows. Think of the first four commodities as the final-demand consumption of various classes of *goods* in some economy, while the next two commodities are the consumption of two classes of *services*. Think of the first good as *agricultural consumption and exports*. The final-demand quantity for this good mildly fluctuates around 30 units of output, while its price fluctuates more violently around 1. However, as the rest of the economy grows, the share of agricultural output declines to about one-half of its initial share. The second good is *energy consumption* in final demand. The quantity of this good trends up gently during the five periods with some fluctuations. However, note that the price of energy fluctuates wildly from period to period.[3] The third good is *traditional manufactures*. There are rather high inflation rates for this commodity for periods 2 and 3, which diminish to a very low inflation rate by the end of our sample period.[4] The final-demand consumption of traditional manufactured goods is more or less static in our data set. The fourth commodity is *high-technology manufactured goods*; for example, computers, video cameras, compact discs, etc. The demand for these high-tech commodities grows tenfold over our sample period, while the final period price is only one-fifth of the first-period price. The fifth commodity is *traditional services*. The price trends for this commodity are similar to traditional manufactures, except that the period-to-period inflation rates are a bit higher. However, the demand for traditional services is growing much more strongly than for traditional manufactures. Our sixth commodity is *high-technology services*; for example, telecommunications, wireless phones, Internet services, stock market trading, etc. For this final commodity, the price is trending downward very strongly to end up at 20 percent of the starting level, while demand increases fivefold. The final two commodities are *energy imports* and *imports of high-technology manufactured goods*. Since imports are intermediate inputs to the economy as a whole, the quantities for these last two commodities are indexed with minus signs. The prices and quantities for the two imported commodities are more or less proportional to the corresponding final consumption demand prices and quantities. The movements of prices and quantities in this artificial data set are more pronounced than the year-to-year movements that would be encountered in a typical country. However, they do illustrate the problem that is facing compilers of the producer price index: namely, *year-to-year price and quantity movements are far from being proportional across commodities, so the choice of index number formula will matter*.

19.8 Every price statistician is familiar with the *Laspeyres index*, P_L, defined by equation (15.5) in the main text of Chapter 15, and the *Paasche index*, P_P, defined by equation (15.6). These indices are listed in Table 19.4 along with the two unweighted indices that were considered in Chapters 15 and 16: the *Carli index* defined by equation (16.45) and the *Jevons index* defined by equation (16.47). The indices in Table 19.4 compare the prices in period t with the prices in period 1; that is, they are *fixed-base indices*. Thus, the period t entry for the Carli index, P_C, is simply the arithmetic mean of the eight price relatives, $\sum_{i=1}^{8}(1/8)\left(p_i^t/p_i^1\right)$, while the period t entry for the Jevons index, P_J, is the geometric mean of the eight price relatives, $\prod_{i=1}^{8}\left(p_i^t/p_i^1\right)^{1/8}$.

19.9 Note that by period 5, the spread between the fixed-base Laspeyres and Paasche price indices is fairly large: P_L is equal to 1.6343 while P_P is 1.2865, *a spread of about 27 percent*. Since these indices have exactly the same *theoretical* justification, it can be seen that the choice of index number formula matters a great deal. There is also a substantial spread between the two unweighted indices by period 5: the fixed-base Carli index is equal to 0.9125, while the fixed-base Jevons index is 0.6373, *a spread of about 43 percent*. However,

[2]Typically, the prices will be price relatives or averages of price relatives, but if the base period is equal to period 1, then these relative prices will all be unity in period 1.

[3]This is an example of the price-bouncing phenomenon noted by Szulc (1983). Note that the fluctuations in the price of energy that have been built into our data set are not that unrealistic: in the recent past, the price of a barrel of crude oil has fluctuated from US$10 to US$37. Note that agricultural prices also bounce but not as violently.

[4]This corresponds roughly to the experience of most industrialized countries over a period starting in 1973 and ending in the mid 1990s. Thus, roughly five years of price movement are compressed into one of the periods.

Table 19.1. Prices for Eight Commodities

	Final Demand of Goods				Services		Imports	
Period t	Agriculture exports p_1^t	Energy p_2^t	Traditional manufacturing p_3^t	High-tech manufacturing p_4^t	Traditional services p_5^t	High-tech services p_6^t	Energy imports p_7^t	High-tech imports p_8^t
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.3	2.0	1.3	0.7	1.4	0.8	2.1	0.7
3	1.0	1.0	1.5	0.5	1.7	0.6	1.0	0.5
4	0.7	0.5	1.6	0.3	1.9	0.4	0.6	0.3
5	1.0	1.0	1.7	0.2	2.0	0.2	1.0	0.2

Table 19.2. Quantities for Eight Commodities

	Final Demand of Goods				Services		Imports	
Period t	Agriculture exports q_1^t	Energy q_2^t	Traditional manufacturing q_3^t	High-tech manufacturing q_4^t	Traditional services q_5^t	High-tech services q_6^t	Energy imports q_7^t	High-tech imports q_8^t
1	30	10	40	10	45	5	–28	–7
2	28	8	39	13	47	6	–20	–9
3	30	11	38	30	50	8	–29	–21
4	32	14	39	60	56	13	–35	–42
5	29	12	40	100	65	25	–30	–70

Table 19.3. Net Expenditures and Net Expenditure Shares for Eight Commodities

		Final Demand of Goods				Services		Imports	
Period t	$p^t \cdot q^t$	Agriculture exports s_1^t	Energy s_2^t	Traditional manufacturing s_3^t	High-tech manufacturing s_4^t	Traditional services s_5^t	High-tech services s_6^t	Energy imports s_7^t	High-tech imports s_8^t
1	105.0	0.2857	0.0952	0.3810	0.0952	0.4286	0.0476	–0.2667	–0.0667
2	134.5	0.2706	0.1190	0.3770	0.0677	0.4892	0.0357	–0.3123	–0.0468
3	163.3	0.1837	0.0674	0.3491	0.0919	0.5205	0.0294	–0.1776	–0.0643
4	187.8	0.1193	0.0373	0.3323	0.0958	0.5666	0.0277	–0.1118	–0.0671
5	220.0	0.1318	0.0545	0.3091	0.0909	0.5909	0.0227	–0.1364	–0.0636

Table 19.4. Fixed-Base Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.2875	1.1853
3	1.4571	1.3957	0.9750	0.8868
4	1.5390	1.3708	0.7875	0.6240
5	1.6343	1.2865	0.9125	0.6373

Table 19.5. Chained Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.2875	1.1853
3	1.3743	1.4834	1.0126	0.8868
4	1.4374	1.5349	0.7406	0.6240
5	1.4963	1.5720	0.8372	0.6373

more troublesome than this spread is the fact that *the unweighted indices are far below both the Paasche and Laspeyres indices* by period 5.[5] Thus, when there are divergent trends in both prices and quantities, it will usually be the case that unweighted price indices will give very different answers than their weighted counterparts. Since none of the index number theories considered in previous chapters supported the use of unweighted indices, the use of unweighted formulas is not recommended for aggregation at the higher level, that is, when data on weights are available. However, in Chapter 20, aggregation at the lower level is considered for weights that are unavailable, and the use of unweighted index number formulas will be revisited. Finally, note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[6]

19.10 It is of interest to recalculate the four indices listed in Table 19.4 using the *chain principle* rather than the *fixed-base principle*. Our expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chained indices are listed in Table 19.5.

19.11 It can be seen comparing Tables 19.4 and 19.5 that chaining eliminated about three-fourths of the spread between the fixed-base Paasche and Laspeyres indices for period 5. However, even the chained Paasche and Laspeyres indices differ by about 8 percent in period 3, so the choice of index number formula still matters. In Table 19.4, the fixed-base Laspeyres exceeds the fixed-base

[5]The reason for this is that when using weighted indices, the imports of high-technology goods are offset by the final-demand expenditures on high-technology goods to a large extent; that is, commodities 6 and 8 have the same dramatic downward price trends, but their quantity trends are opposite in sign and cancel each other out to a large extent. However, when calculating the unweighted indices, this cancellation does not occur, and the downward trends in the prices of commodities 6 and 8 get a much higher implicit weight in the unweighted indices.

[6]This is the Theorem of the Arithmetic and Geometric Mean; see Hardy, Littlewood, and Polyá (1934) and Chapter 20.

Table 19.6. Asymmetrically Weighted Fixed-Base Indices

Period *t*	P_{PAL}	P_{GP}	P_L	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1520	1.1852	1.1552	1.1811	1.2009	1.1906
3	1.5133	1.4676	1.4571	1.4018	1.3957	1.3212
4	1.6628	1.5661	1.5390	1.4111	1.3708	1.2017
5	1.7673	1.6374	1.6343	1.4573	1.2865	1.0711

Paasche, while in Table 19.5, the positions are reversed for the respective chained indices. Such differences for fixed-base Laspeyres and Paasche were shown in Appendix 15.1 of Chapter 15 to depend on the sign of the correlation between relative price changes and average quantity changes.[7] Note that chaining did not affect the Jevons index. This is an advantage of the index, but the lack of weighting is a fatal flaw. The truth would be expected to lie between the Paasche and Laspeyres indices and in Table 19.5. However, the unweighted Jevons index is far below this acceptable range. Note that chaining did not affect the Carli index in a systematic way for our particular data set: in period 3, the chained Carli index is above the corresponding fixed-base Carli, but in periods 4 and 5, the chained Carli index is below the fixed-base Carli.

19.12 A systematic comparison of all of the *asymmetrically weighted price indices* is now undertaken. The *fixed-base indices* are listed in Table 19.6. The fixed-base *Laspeyres* and *Paasche indices*, P_L and P_P, are the same as those indices listed in Table 19.4. The *Palgrave index*, P_{PAL}, is defined by equation (16.55). The indices denoted by P_{GL} and P_{GP} are the *geometric Laspeyres* and *geometric Paasche indices*,[8] which are special cases of the fixed-weight geometric indices defined by Konüs and Byushgens (1926); see equations (16.75) and (16.76). For the *geometric Laspeyres index*, P_{GL}, let the weights α_i be the *base-period expenditure shares*, s_i^1. This index should be considered an alternative to the fixed-base Laspeyres index, since each of these indices makes use of the same information set. For the *geometric Paasche index*, P_{GP}, let the weights α_i be the *current-period expenditure shares*, s_i^t. Finally, the index P_{HL} is the *harmonic Laspeyres index* that was defined by equation (16.59).

19.13 By looking at the period 5 entries in Table 19.6, it can be seen that the spread between all of these fixed-base asymmetrically weighted indices has grown to be even larger than our earlier spread of 27 percent between the fixed-base Paasche and Laspeyres indices. In Table 19.6, the period 5 Palgrave index is about 1.65 times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, *this illustrates the point that due to the nonproportional growth of prices and quantities in most economies today, the choice of index number formula is very important*.

19.14 If there were no negative quantities in the final-demand vectors, then it is possible to explain why certain elements of the indices in Table 19.6

[7]Forsyth and Fowler (1981, p. 234) show how the relative positions of fixed and chained Laspeyres depend on the sign of their respective correlation coefficients. With the former, it is the correlation between price changes and quantities for periods 0 and *t*; with the latter, it is that between periods $t-1$ and *t*. The latter are more likely to take account of substitution effects leading to differences between the two.

[8]Vartia (1978, p. 272) used the terms *logarithmic Laspeyres* and *logarithmic Paasche*, respectively.

Table 19.7. Asymmetrically Weighted Indices Using the Chain Principle

Period t	P_{PAL}	P_{GP}	P_L	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1520	1.1852	1.1552	1.1811	1.2009	1.1906
3	1.3444	1.4050	1.3743	1.4569	1.4834	1.6083
4	1.4229	1.4730	1.4374	1.5057	1.5349	1.6342
5	1.4942	1.5292	1.4963	1.5510	1.5720	1.6599

are bigger than others. If all weights are positive, it can be shown that a *weighted arithmetic mean* of N numbers is equal to or greater than the corresponding *weighted geometric mean* of the same N numbers, which in turn is equal to or greater than the corresponding *weighted harmonic mean* of the same N numbers.[9] It can be seen that the three indices P_{PAL}, P_{GP}, and P_P all use the current-period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1), but P_{PAL} is a weighted *arithmetic* mean of these price relatives, P_{GP} is a weighted *geometric* mean of these price relatives, and P_P is a weighted *harmonic* mean of these price relatives. Thus, if there are no negative components in final demand, we have the following, according to Schlömilch's inequality:[10]

(19.1) $P_{PAL} \geq P_{GP} \geq P_P$.

However, due to the existence of imports in each period (which leads to negative quantities for these components of the final-demand vector), the inequalities in equation (19.1) are not necessarily true. Viewing Table 19.6, it can be seen that the inequalities in equation (19.1) hold for periods 3, 4, and 5 but not for period 2. It can also be verified that the three indices P_L, P_{GL}, and P_{HL} all use the base-period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1), but P_L is a weighted *arithmetic* mean of these price relatives, P_{GL} is a weighted *geometric* mean of these price relatives, and P_{HL} is a weighted *harmonic* mean of these price relatives. If all of these shares were nonnegative, then we have the following, according to Schlömilch's inequality:[11]

(19.2) $P_L \geq P_{GL} \geq P_{HL}$.

However, due to the existence of imports in each period, the inequalities in equation (19.2) are not necessarily true. Viewing Table 19.6, it can be seen that the inequalities in equation (19.2) hold for periods 3, 4, and 5 but not for period 2.

19.15 Now continue with the systematic comparison of all of the *asymmetrically weighted price indices*. These indices that use the *chain principle* are listed in Table 19.7. Viewing Table 19.7, it can be seen that the use of the chain principle dramatically reduced the spread between all of the asymmetrically weighted indices compared with their fixed-base counterparts in Table 19.6. For period 5, the spread between the smallest and largest asymmetrically weighted fixed-base index was 65 percent, but for the period 5 chained indices, this spread was reduced to 11 percent.

19.16 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory; that is, as was seen in Chapter 17, a superlative index is exact for a representation of the producer's production function or the corresponding unit revenue function that can provide a second-order approximation to arbitrary technologies that satisfy certain regularity conditions. In Chapters

[9]This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood, and Polyá (1934, chapter 11).

[10]These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[11]These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

Table 19.8. Symmetrically Weighted Fixed-Base Indices

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1831	1.1827	1.1814	1.1778	1.1781	1.1788
3	1.4343	1.4339	1.4327	1.4261	1.4264	1.4248
4	1.4866	1.4840	1.4820	1.4525	1.4549	1.4438
5	1.5447	1.5320	1.5193	1.4500	1.4604	1.4188

Table 19.9. Symmetrically Weighted Indices Using the Chain Principle

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1831	1.1827	1.1814	1.1778	1.1781	1.1788
3	1.4307	1.4257	1.4298	1.4278	1.4288	1.4290
4	1.4893	1.4844	1.4889	1.4853	1.4861	1.4862
5	1.5400	1.5344	1.5387	1.5337	1.5342	1.5338

Table 19.10. Fixed-Base Superlative Single-Stage and Two-Stage Indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1778	1.1830	1.1831	1.1837	1.1814	1.1835	1.1827	1.1829
3	1.4261	1.4259	1.4343	1.4351	1.4327	1.4341	1.4339	1.4325
4	1.4525	1.4713	1.4866	1.4974	1.4820	1.4990	1.4840	1.4798
5	1.4500	1.4366	1.5447	1.5440	1.5193	1.5208	1.5320	1.5191

15–17, four primary superlative indices were considered:

- The *Fisher ideal price index,* P_F, defined by equation (15.12);
- The *Walsh price index,* P_W, defined by equation (15.19) (this price index also corresponds to the quantity index Q^1 defined by equation [17.26]);[12]
- The *Törnqvist-Theil price index,* P_T, defined by equation (15.81); and
- The *implicit Walsh price index,* P_{IW}, that corresponds to the Walsh quantity index Q_W defined by equation (16.34).

These four symmetrically weighted superlative price indices are listed in Table 19.8 using the fixed-base principle. Also listed in this table are

[12]Since square roots of negative quantities are not feasible, the sign conventions are changed when calculating this index: change the negative quantities into positive quantities and change the corresponding positive prices into negative prices.

two symmetrically weighted (but not superlative) price indices:[13]

- The Marshall-Edgeworth price index, P_{ME}, defined by equation (15.18) and
- The Drobisch price index, P_{DR}, defined in Paragraph 15.19.

19.17 Note that the Drobisch index P_{DR} is always equal to or greater than the corresponding Fisher index P_F. This follows from the facts that the Fisher index is the geometric mean of the Paasche and Laspeyres indices while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices; an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed-base asymmetrically weighted indices, Table 19.6, with the symmetrically weighted indices, Table 19.8, *it can be seen that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices*. The spread was 1.7673/1.0711 = 1.65 for the asymmetrically weighted indices but only 1.5447/1.4188 = 1.09 for the symmetrically weighted indices. If the analysis is restricted to the superlative indices listed for period 5 in Table 19.8, then this spread is further reduced to 1.5447/1.4500 = 1.065; that is, the spread between the fixed-base superlative indices is only 6.5 percent compared with the fixed-base spread between the Paasche and Laspeyres indices of 27 percent (1.6343/1.2865 = 1.27). The spread between the superlative indices can be expected to be further reduced by using the chain principle.

19.18 The symmetrically weighted indices are recomputed using the chain principle. The results may be found in Table 19.9. A quick glance at Table 19.9 shows that *the combined effect of using both the chain principle as well as symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles*. The spread between all of the symmetrically weighted indices in period 5 is only 1.5400/1.5337 = 1.004 or 0.4 percent, which is the same as the spread between the four superlative indices in period 5.[14]

19.19 The results listed in Table 19.9 reinforce the numerical results tabled in R.J. Hill (2000) and Diewert (1978, p. 894): *the most commonly used chained superlative indices will generally give approximately the same numerical results*.[15] This is in spite of the erratic nature of the fluctuations in the data in Tables 19.1 to 19.3. In particular, the chained Fisher, Törnqvist, and Walsh indices will generally approximate each other very closely.

19.20 Attention is now turned to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see Section C of Chapter 17 for a discussion of the issues and a listing of the formulas used. In our artificial data set, the first four commodities are aggregated into a *goods aggregate*, the next two commodities into a *services aggregate,* and the last two commodities into an imports aggregate. In the second stage of aggregation, these three price and quantity components will be aggregated into a net final-demand price index.

19.21 The results are reported in Table 19.10 for our two-stage aggregation procedure using period 1 as the *fixed base* for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indexes, P_W and P_{IW}. Viewing Table 19.10, it can be seen that the fixed-base single-stage superlative indices generally approximate their fixed-base two-stage counterparts fairly closely. The divergence between the single-stage Fisher index P_F and its two-stage counterpart P_{F2S} in period 5 is 1.4500/1.4366 = 1.008 or 0.8 percent. The other divergences are even less.

[13]Diewert (1978, p. 897) showed that the Drobisch-Sidgwick-Bowley price index approximates any superlative index to the second order around an equal price and quantity point; that is, P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall-Edgeworth index P_{ME} is also pseudo-superlative.

[14]On average over the last four periods, the chain Fisher and the chain Törnqvist indices differed by 0.0046 percentage points.

[15]More precisely, the superlative quadratic mean of order *r* price indices P^r defined by equation (17.28) and the implicit quadratic mean of order *r* price indices $P^{r}*$ defined by equation (17.25) will generally closely approximate each other provided that *r* is in the interval $0 \leq r \leq 2$. Note that when one or more of the quantities being aggregated is negative (as in the present situation), the sign conventions are changed when calculating Q^r or $P^{r}*$: change the negative sign on import quantities to positive and make the import prices negative.

Table 19.11. Chained Superlative Single-Stage and Two-Stage Indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1778	1.1830	1.1831	1.1837	1.1814	1.1835	1.1827	1.1829
3	1.4278	1.4448	1.4307	1.4309	1.4298	1.4378	1.4257	1.4282
4	1.4853	1.5059	1.4893	1.4907	1.4889	1.4991	1.4844	1.4871
5	1.5337	1.5556	1.5400	1.5419	1.5387	1.5499	1.5344	1.5372

19.22 Using *chained indices*, the results are reported in Table 19.11 for our two-stage aggregation procedure. Again, the single-stage approach and its two-stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indexes, P_W and P_{IW}. Viewing Table 19.11, it can be seen that the chained single-stage superlative indices generally approximate their fixed-base two-stage counterparts quite closely. The divergence between the chained single-stage Fisher index P_F and its two-stage counterpart P_{F2S} in period 5 is 1.5556/1.5337 = 1.014 or 1.4 percent. The other divergences are all less than this. Given the large dispersion in period-to-period price movements, these two-stage aggregation errors are not large. However, the important point that emerges from Table 19.11 is that *the use of the chain principle has reduced the spread across all eight single-stage and two-stage superlative indices* compared with their fixed-base counterparts in Table 19.10. The maximum spread for the period 5 chained index values is 1.4 percent, while the maximum spread for the period 5 fixed-base index values is 7.5 percent.

C. Midyear Indices

19.23 The next formulas to illustrate using our artificial data set are the arithmetic- and geometric-type midyear indices defined in Section E of Chapter 17. Recall that these indices are due to Schultz (1998) and Okamoto (2001). Basically, midyear indices are fixed-basket indices, where the basket of quantities being priced is midway between the base period and the current period. If the current period t less the base period 1 is an even integer, then the quantity vector $q^{(t-1)/2}$ is used as the midyear basket. If the current period t less the base period 1 is an odd integer, then the midyear basket is an average of the two midyear quantity vectors, $q^{t/2}$ and $q^{(t/2)+1}$. If the arithmetic average of these two midyear baskets is taken, the sequence of *fixed-base arithmetic-type midyear indices*, $P_{OSA}{}^t$, is obtained, defined by equation (17.50) in Chapter 17. If the geometric average of these two midyear baskets is taken, the sequence of *fixed-base geometric-type midyear indices* is obtained, $P_{OSG}{}^t$, defined by equation (17.51) in Chapter 17.[16] Recall also that going from period 1 to period 2, the period 2 *midyear arithmetic-type index number* $P_{OSA}{}^2$ is equal to $P_{ME}(p^1,p^2,q^1,q^2)$, the Marshall- (1887) Edgeworth (1925) price index for period 2. In addition, the period 2 *midyear geometric-type index number* $P_{OSG}{}^2$ is equal to $P_W(p^1,p^2,q^1,q^2)$, the Walsh (1901) price index for period 2.[17]

19.24 The two sequences of *fixed-base midyear price indices*, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, along with the corresponding *fixed-base Fisher, Törnqvist, and Walsh price indices*, $P_F{}^t$, $P_T{}^t$, and $P_W{}^t$, respectively, are listed in Table 19.12. Note that for odd t, the arithmetic- and geometric-type midyear indices, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, coincide. This is as it should be because when t is odd, both indices are set equal to

[16]Since the quantity vectors have two negative components (and thus, one cannot take square roots of these negative components), the sign conventions need to be changed when evaluating these geometric-type midyear indices; make all quantities positive but change the prices of the import components from positive to negative. Thus, when calculating a geometric-type midyear index where it is necessary to take the geometric average of two midyear quantity vectors, the same sign conventions are used as when calculating Walsh price indices where the same problem occurred.

[17]As usual, when calculating this Walsh price index, switch the signs of the negative import quantities to positive signs and make the corresponding import prices negative.

Table 19.12. Fixed-Base Arithmetic- and Geometric-Type Midyear Indices

Period t	P_{OSA}	P_{OSG}	P_F	P_T	P_W
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1788	1.1814	1.1778	1.1831	1.1814
3	1.4286	1.4286	1.4261	1.4343	1.4327
4	1.4747	1.4783	1.4525	1.4866	1.4820
5	1.5385	1.5385	1.4500	1.5447	1.5193

Table 19.13. Chained Arithmetic- and Geometric-Type Midyear Indices

Period t	P_{OSA}	P_{OSG}	P_F	P_T	P_W
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1788	1.1814	1.1778	1.1831	1.1814
3	1.4286	1.4286	1.4278	1.4307	1.4298
4	1.5230	1.5263	1.4853	1.4893	1.4889
5	1.5388	1.5388	1.5337	1.5400	1.5387

the Schultz midyear index, since there is a single unique midyear basket in this case. The two sequences of midyear indices differ only for even t, since in the even case, there are two midyear baskets and a decision must be made on arithmetic or geometric averaging of these baskets. Note also that the Walsh index for period 2 is equal to the corresponding geometric-type midyear index, since this is true by construction. Finally, note that with the exception of the Fisher fixed-base index, P_F, the fixed-base indices listed in Table 19.12 approximate each other surprisingly well, given the tremendous variability that was built into the underlying data set. The relatively low results for the fixed-base Fisher index may arise from the relatively low results for the fixed-base Paasche index in Table 19.4 and its high spread. When chained-base Laspeyres and Paasche indices were calculated in Table 19.5, the spread was much less, with the Paasche index being pulled up *above* the Laspeyres index to a figure quite close to the Törnqvist and Walsh indices. This seems to suggest that the relatively low Paasche fixed-base index result in Table 19.4, and thus, the fixed-base Fisher index in Table 19.12, was biased downward.

19.25 The chained counterparts to the indices listed in Table 19.12 are now considered. Recall that the chained sequence of arithmetic- and geometric-type midyear indices was defined by equations (17.54) and (17.55), respectively, in Chapter 17. The two sequences of *chained midyear price indices*, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, along with the corresponding *chained Fisher, Törnqvist, and Walsh price indices*, $P_F{}^t$, $P_T{}^t$, and $P_W{}^t$, respectively, are listed in Table 19.13. Note that for odd t, the chained arithmetic- and geometric-type midyear indices, $P_{OSA}{}^t$ and $P_{OSG}{}^t$, coincide. This is as it should be because when t is odd, both indices are set equal to chained Schultz midyear indices. What is striking in looking at Table 19.13 is how close the chained midyear indices are to their chained superlative counterparts for odd periods. For year 5, the maximum spread among the five indices is the spread between the chained Fisher and Törnqvist indices, which was only 1.5400/1.5337 = 1.004 or 0.4 percent. The explanation for this rather remarkable result is that for odd periods, the underlying price and quantity data have fairly smooth trends; and, under these circumstances, the midyear indices would be expected to approximate the superlative Walsh index rather closely, as was

Table 19.14. An Additive Percentage Change Decomposition of the Fisher Index

Period t	P_F–1	$v_{F1}\Delta p_1$	$v_{F2}\Delta p_2$	$v_{F3}\Delta p_3$	$v_{F4}\Delta p_4$	$v_{F5}\Delta p_5$	$v_{F6}\Delta p_6$	$v_{F7}\Delta p_7$	$v_{F8}\Delta p_8$
2	0.1778	0.0791	0.0816	0.1079	–0.0316	0.1678	–0.0101	–0.2389	0.0220
3	0.2122	–0.0648	–0.0716	0.0571	–0.0331	0.1084	–0.0105	0.2037	0.0231
4	0.0403	–0.0541	–0.0363	0.0224	–0.0519	0.0616	–0.0121	0.0744	0.0363
5	0.0326	0.0459	0.0326	0.0198	–0.0396	0.0302	–0.0187	–0.0653	0.0277

indicated in Chapter 17. However, for periods 2 and 4, the underlying data bounce considerably, so the trends in the data switch abruptly. Therefore, under these conditions, it is expected that the midyear indices could deviate from their superlative counterparts. This expectation is borne out by looking at the entries for period 4 in Table 19.12, where the two midyear indices are about 2 to 3 percent higher than their chained superlative counterparts.

19.26 The conclusion that emerges from Tables 19.12 and 19.13 is that midyear indices approximate their superlative counterparts surprisingly well but not perfectly. Given the large amount of variability in the underlying price and quantity data, it appears that the *midyear indices could be used to give very good advanced estimates of superlative indices*, which cannot necessarily be evaluated on a timely basis.

D. Additive Percentage Change Decompositions for the Fisher Index

19.27 The final formulas that are illustrated using the artificial final expenditures data set are the *additive percentage change decompositions* for the Fisher ideal index that were discussed in Section C.8 of Chapter 16. The *chain links* for the Fisher price index will first be decomposed using the Diewert (2002a) decomposition formulas shown in equations (16.41) through (16.43). The results of the decomposition are listed in Table 19.14. Thus, $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t - 1$ to t, and *the decomposition factor* $v_{Fi}\Delta p_i = v_{Fi}\,(p_i^t - p_i^{t-1})$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for i = 1,2,…,8. Viewing Table 19.14, it can be seen that the price index going from period 1 to 2 grew 17.78 percent, and the major contributors to this change were the increases in the price of commodity 1, finally demanded agricultural products (7.91 percentage points); commodity 2, finally demanded energy (8.16 percentage points); commodity 3, finally demanded traditional manufactures (10.79 percentage points); commodity 5, traditional services (16.78 percentage points); and commodity 7, energy imports (–23.89 percentage points). The sum of the last eight entries for period 2 in Table 19.14 is equal to 0.1778, the percentage increase in the Fisher price index going from period 1 to 2. Note that although the price of energy imports *increased* dramatically in period 2, the contribution to the overall price change is *negative* due to the fact that the quantity of energy imports is indexed with a negative sign. Similarly, although the price of high-technology imports *decreased* dramatically in period 2, the contribution to the overall price change is *positive* due to the fact that the quantity of high-technology imports is indexed with a negative sign.[18] Care must be taken, therefore, in interpreting the last two columns of Table 19.14, because there are negative quantities for some components of the aggregate.[19] It can be seen that a big price change in a particular component i combined with a big expenditure share in the

[18]Since the expenditure share of high-technology imports is small, the large decrease in price does not translate into a large change in the overall Fisher price index for final-demand expenditures.

[19]The counterintuitive numbers in the last two columns of Table 19.14 help to explain why the deflator for final-demand expenditures (or the GDP deflator as it is commonly known) is not a satisfactory indicator of inflationary pressures in the economy because a large *increase* in the relative price of imported goods leads to a *decrease* in the index.

Table 19.15. Van Ijzeren's Decomposition of the Fisher Price Index

Period t	P_F-1	$v_{F1}{*}\Delta p_1$	$v_{F2}{*}\Delta p_2$	$v_{F3}{*}\Delta p_3$	$v_{F4}{*}\Delta p_4$	$v_{F5}{*}\Delta p_5$	$v_{F6}{*}\Delta p_6$	$v_{F7}{*}\Delta p_7$	$v_{F8}{*}\Delta p_8$
2	0.1778	0.0804	0.0834	0.1094	–0.0317	0.1697	–0.0101	–0.2454	0.0220
3	0.2122	–0.0652	–0.0712	0.0577	–0.0322	0.1091	–0.0105	0.2021	0.0225
4	0.0403	–0.0540	–0.0361	0.0224	–0.0515	0.0615	–0.0121	0.0741	0.0360
5	0.0326	0.0458	0.0326	0.0197	–0.0393	0.0300	–0.0186	–0.0652	0.0275

two periods under consideration will lead to a big decomposition factor, v_{Fi}.

19.28 Our final set of computations illustrates the *additive percentage change decomposition* for the Fisher ideal index that is due to Van Ijzeren (1987, p. 6), mentioned in Section C.8 of Chapter 16.[20] The *price* counterpart to the *additive decomposition* for a *quantity* index, shown in equation (16.35), is:

$$(19.3)\ P_F(p^0,p^1,q^0,q^1) = \frac{\sum_{i=1}^{8} q_{Fi}^{*} p_i^t}{\sum_{i=1}^{8} q_{Fi}^{*} p_i^0},$$

where the reference quantities need to be defined somehow. Van Ijzeren (1987, p. 6) showed that the following reference weights provided an *exact additive representation for the Fisher ideal price index*:

$$(19.4)\quad q_{Fi}^{*} \equiv \left(\frac{1}{2}\right) q_i^0 + \left[\left(\frac{1}{2}\right) q_i^t \Big/ Q_F\left(p^0, p^1, q^0, q^1\right)\right];$$

$i = 1,2,\ldots,8$,

where Q_F is the overall Fisher quantity index. Thus, using the Van Ijzeren quantity weights $q_{Fi}*$, the *following Van Ijzeren additive percentage change decomposition for the Fisher price index* is obtained:

$$(19.5)\ P_F(p^0,p^1,q^0,q^1) - 1 = \frac{\sum_{i=1}^{8} q_{Fi}^{*} p_i^1}{\sum_{i=1}^{8} q_{Fi}^{*} p_i^0} - 1$$

$$= \sum_{i=1}^{8} v_{Fi}^{*}\left(p_i^1 - p_i^0\right),$$

where the *Van Ijzeren weight* for commodity i, $v_{Fi}*$, is defined as

$$(19.6)\ v_{Fi}{*} \equiv \frac{q_{Fi}^{*}}{\sum_{i=1}^{8} q_{Fi}^{*} p_i^0};\ i = 1,\ldots,8.$$

The *chain links* for the Fisher price index will again be decomposed using equations (19.2) to (19.4) listed above. The results of the decomposition are listed in Table 19.15. Thus, $P_F - 1$ is the *percentage change in the Fisher ideal chain link* going from period $t-1$ to t and the *Van Ijzeren decomposition factor* $v_{Fi}{*}\Delta p_i$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1,2,\ldots,8$.

19.29 Comparing the entries in Tables 19.14 and 19.15, it can be seen that the differences between the Diewert and Van Ijzeren decompositions of the Fisher price index are *very small*. This is somewhat surprising given the very different nature of the two decompositions.[21] As was mentioned in Section C.8 of Chapter 16, the Van Ijzeren decomposition of the chain Fisher *quantity* index is used

[20] It was also independently derived by Dikhanov (1997) and used by Ehemann, Katz, and Moulton (2002).

[21] The terms in Diewert's decomposition can be given economic interpretations, whereas the terms in the other decomposition are more difficult to interpret from an economic perspective. However, Reinsdorf, Diewert, and Ehemann (2002) show that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and the two quantity vectors are equal.

by the Bureau of Economic Analysis in the United States.[22]

E. Industry Price Indices

E.1 Industry data set

19.30 A highly simplified economy consisting of three industrial sectors is considered. The three sectors are the *agricultural sector* (or primary sector), the *manufacturing sector* (or secondary sector), and the *services sector* (or tertiary sector). It is assumed that all transactions go through the services sector. This might appear to be a bit unusual initially. However, recall that transportation services reside in the services sector. Hence, imported goods are delivered as intermediate inputs to the agricultural and manufacturing sectors using service transportation inputs, or they are delivered directly to the final-demand sector—again using service sector transportation, storage, retailing, and wholesaling services. Similarly, the agricultural sector produces unprocessed food that is delivered by the services sector to the manufacturing sector for further processing and packaging. That manufactured food output is then again delivered by the services sector to the final-demand sector.[23]

19.31 Three outputs and intermediate inputs are distinguished for the agricultural sector. The first commodity is agricultural output delivered to the services sector. This is the only output of this sector. There are two intermediate inputs used in the agricultural sector: commodity 2 is deliveries of nonenergy materials (fertilizer, etc.) to agriculture from the services sector, and commodity 3 is deliveries of energy from the services sector to agriculture. These prices and quantities are denoted by p_n^{At} and q_n^{At} for $n = 1, 2, 3$ and $t = 1,...,5$. Note that q_1^{At} is positive (because commodity 1 is an output) and q_2^{At} and q_3^{At} are negative (since commodities 2 and 3 in the agriculture sector are intermediate inputs). The data for the agriculture sector for five periods are listed in Table 19.16 (on the next page).

[22]See Ehemann, Katz, and Moulton (2002).

[23]Our treatment of industrial transactions is an extension of Kohli's (1978) approach to modeling the treatment of imports as flowing first through the production sector of the economy rather than being directly delivered to final demand or other industrial sectors.

19.32 Two outputs and three intermediate inputs are distinguished for the manufacturing sector, five commodities in all.

- Commodity 1 is processed agricultural output delivered to the services sector;
- Commodity 2 is traditional manufactures delivered to the services sector;
- Commodity 3 is deliveries of transported agricultural intermediate inputs delivered from the services sector;
- Commodity 4 is deliveries of energy from services to manufacturing; and
- Commodity 5 is inputs of business services.

These prices and quantities are denoted by p_n^{Mt} and q_n^{Mt} for $n = 1,...,5$ and $t = 1,...,5$. Note that q_1^{Mt} and q_2^{Mt} are positive (because these commodities are outputs) and q_3^{Mt}, q_4^{Mt}, and q_5^{Mt} are negative (since commodities 3, 4, and 5 in the manufacturing sector are intermediate inputs). The data for the manufacturing sector for five periods are listed in Table 19.17 (on the next page).

19.33 Eleven service sector outputs and five service sector intermediate inputs, or 16 commodities in all, are distinguished. The 11 *outputs* are listed as follows:

- Commodity 1 is food deliveries to final demand;
- Commodity 2 is energy deliveries to final demand;
- Commodity 3 is traditional manufacturing deliveries to final demand;
- Commodity 4 is deliveries of high-technology manufactured goods to final demand;
- Commodity 5 is delivery of personal services to final demand;
- Commodity 6 is deliveries of high-technology services to final demand;
- Commodity 7 is deliveries of materials to agriculture;
- Commodity 8 is deliveries of energy to agriculture;
- Commodity 9 is delivery of materials to manufacturing;
- Commodity 10 is deliveries of energy to manufacturing; and
- Commodity 11 is deliveries of business services to manufacturing.

Table 19.16. Price and Quantity Data for the Agriculture Sector

Period	p_1^A	p_2^A	p_3^A	q_1^A	q_2^A	q_3^A
1	1.0	1.0	1.0	20.0	–3.0	–6.0
2	1.5	1.4	2.2	16.0	–2.0	–4.0
3	1.1	1.6	1.1	20.0	–3.0	–5.0
4	0.6	1.4	0.7	23.0	–3.0	–6.0
5	1.0	1.7	1.1	19.0	–3.0	–5.0

Table 19.17. Price and Quantity Data for the Manufacturing Sector

Period	p_1^M	p_2^M	p_3^M	p_4^M	p_5^M	q_1^M	q_2^M	q_3^M	q_4^M	q_5^M
1	1.0	1.0	1.0	1.0	1.0	26.0	36.0	–22.0	–6.0	–8.0
2	1.3	1.2	1.4	2.0	1.2	23.0	35.0	–19.0	–5.0	–9.0
3	1.1	1.4	1.1	1.1	1.6	26.0	34.0	–22.0	–5.0	–10.0
4	0.8	1.5	0.7	0.8	1.8	27.0	35.0	–23.0	–5.0	–11.0
5	1.0	1.6	1.0	1.1	1.9	25.0	36.0	–21.0	–5.0	–11.0

Table 19.18. Price Data for the Services Sector

t	p_1^S	p_2^S	p_3^S	p_4^S	p_5^S	p_6^S	p_7^S	p_8^S	p_9^S	p_{10}^S	p_{11}^S	p_{12}^S	p_{13}^S	p_{14}^S	p_{15}^S	p_{16}^S
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.3	2.0	1.3	0.7	1.4	0.8	1.4	2.2	1.4	2.0	1.2	2.1	0.7	1.5	1.3	1.2
3	1.0	1.0	1.5	0.5	1.7	0.6	1.6	1.1	1.1	1.1	1.6	1.0	0.5	1.1	1.1	1.4
4	0.7	0.5	1.6	0.3	1.9	0.4	1.4	0.7	0.7	0.8	1.8	0.6	0.3	0.6	0.8	1.5
5	1.0	1.0	1.7	0.2	2.0	0.2	1.7	1.1	1.0	1.1	1.9	1.0	0.2	1.0	1.0	1.6

The five *intermediate inputs* into the services sector are listed as follows:

- Commodity 12 is imports of energy into the economy;
- Commodity 13 is imports of high-technology manufactures into the economy;
- Commodity 14 is deliveries of agricultural output to services;
- Commodity 15 is deliveries of processed food from manufacturing to services; and
- Commodity 16 is deliveries of traditional manufacturing to services.

These prices and quantities are denoted by p_n^{St} and q_n^{St} for $n = 1,...,16$ and $t = 1,...,5$. Note that q_1^{St} to q_{11}^{St} are positive (because these commodities are outputs) and q_{12}^{St} to q_{16}^{St} are negative (since these commodities in the services sector are intermediate inputs). The service sector price and quantity data for the 16 commodities are listed in Tables 19.18 and 19.19, respectively.

Table 19.19. Quantity Data for the Services Sector

t	q_1^S	q_2^S	q_3^S	q_4^S	q_5^S	q_6^S	q_7^S	q_8^S	q_9^S	q_{10}^S	q_{11}^S	q_{12}^S	q_{13}^S	q_{14}^S	q_{15}^S	q_{16}^S
1	30	10	40	10	45	5	3	6	22	6	8	–28	–7	–20	–26	–36
2	28	8	39	13	47	6	2	4	19	5	9	–20	–9	–16	–23	–35
3	30	11	38	30	50	8	3	5	22	5	10	–29	–21	–20	–26	–34
4	32	14	39	60	56	13	3	6	23	5	11	–35	–42	–23	–27	–35
5	29	12	40	100	65	25	3	5	21	5	11	–30	–70	–19	–25	–36

Table 19.20. Agriculture Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1455	1.2400	1.1918	1.2000
3	0.9636	0.9750	0.9693	0.9679
4	0.3273	0.3857	0.3553	0.3472
5	0.7545	0.7636	0.7591	0.7478

Table 19.21. Agriculture Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Price Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1455	1.2400	1.1918	1.2000
3	0.9238	0.9803	0.9516	0.9579
4	0.3395	0.3808	0.3596	0.3584
5	0.7104	0.8646	0.7837	0.7758

19.34 The sectoral data above satisfy the conventions of national income accounting in that every value transaction (which is of the form $p_n^{et}q_n^{et}$, where e denotes a sector and n denotes a commodity) in each sector has a *matching transaction* in another sector for each period and each sector. It should be noted that no attempt has been made to balance the supply and demand for each commodity across sectors; put another way, no attempt has been made to produce *balanced input output tables in real terms,* commodity by commodity across sectors. In order to produce such constant dollar input output tables, it is necessary to make assumptions about margins in each sector; a primary commodity is, for example, transformed as it progresses from the agriculture sector to the various downstream sectors. However, these margins are not constant from period to period, which makes it difficult to interpret constant dollar input output tables. Moreover, as goods are transformed through the manufacturing process, they often lose their initial identities, which again makes it difficult to interpret a constant dollar input output table. The approach used in this chapter avoids all of these problems by focusing on transactions between each pair of sectors in the industrial classification. For each pair of sectors, these intersector transactions can be further classified using a commodity classification, which is what has been done in the data set above, but there is no attempt to have a uniform commodity classification across all sectors.

19.35 In the next three subsections, value-added deflators for each of the three industrial sectors are calculated. Only fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist indices will be computed, since these are the ones most likely to be used in practice.

E.2 Value-added deflators for the agriculture sector

19.36 The data listed in Table 19.16 for the agriculture sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.20 (on preceding page).

19.37 From Table 19.20 it can be seen that all four value-added deflators are close to each other for the odd periods; but for the even periods (when agricultural and energy prices *bounce* or are quite different from their longer term normal values), the Paasche and Laspeyres indices differ considerably. However, for all periods, the two superlative indices are quite close to each other.

19.38 The data listed in Table 19.16 for the agriculture sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist price value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.21 (on preceding page).

19.39 It can be seen, comparing Tables 19.20 and 19.21, that the chained indices show considerably *more* variation than their fixed-base counterparts. Here is an example of a sector where chaining does not reduce the spread between the Paasche and Laspeyres value-added deflators. The reason why chaining does not reduce the spread is that agriculture is an example of a sector where price bouncing is much more important than divergent trends in relative prices. The commodities that have divergent prices are high-technology goods and services, and the agriculture sector does not use or produce these commodities. Even though chaining did not reduce the spread between the Paasche and Laspeyres indices for the agriculture sector, it can be seen that the chained Fisher and Törnqvist price indices are still close to each other, although they are somewhat higher than their fixed-base counterparts for the later periods.

E.3 Value-added deflators for the manufacturing sector

19.40 The data listed in Table 19.17 for the manufacturing sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.22 .

19.41 From Table 19.22, it can be seen that the divergence between the fixed-base Laspeyres and Paasche value-added deflators for the manufacturing sector grows steadily from period 3 when it is 3.6 percent to period 5 when it is 4.4 percent. However, the divergence between the two superlative value-added deflators is quite small for all periods.

19.42 The data listed in Table 19.17 for the manufacturing sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.23.

19.43 Comparing Tables 19.22 and 19.23, it can be seen that chaining did *not* reduce the spread between the Paasche and Laspeyres value-added deflators for the manufacturing sector; the spread between these two chained indices in period 5 is 7.0 percent, whereas it was only 4.4 percent for the corresponding fixed-base indices. The explanation for this result is the same as it was for agriculture: (traditional) manufacturing is an example of a sector where the bouncing behavior of energy prices is much more important than divergent trends in relative prices. The commodities that have divergent prices are high-technology goods and services, and the traditional manufacturing sector does not use or produce these commodities. Comparing Tables 19.22 and 19.23, it can also be seen that chaining did *not* reduce the spread between the Fisher and Törnqvist value-added deflators for the manufacturing sector. Again, bouncing energy prices explain this result. However, the chained Fisher and Törnqvist price indices are still quite close to each other.

E.4 Value-added deflators for the services sector

19.44 The data listed in Tables 19.18 and 19.19 for the services sector are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods *t* equal to 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.24.

Table 19.22. Manufacturing Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period *t*	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	0.9462	0.9800	0.9629	0.9599
3	1.3615	1.3261	1.3437	1.3425
4	1.5462	1.4870	1.5163	1.5265
5	1.5308	1.4667	1.4984	1.4951

Table 19.23. Manufacturing Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period *t*	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	0.9462	0.9800	0.9629	0.9599
3	1.2937	1.3711	1.3318	1.3430
4	1.4591	1.5476	1.5027	1.5217
5	1.4335	1.5345	1.4832	1.5013

Table 19.24. Services Sector Fixed-Base Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period *t*	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2368	1.2675	1.2521	1.2561
3	1.5735	1.4768	1.5244	1.5344
4	1.7324	1.4820	1.6023	1.6555
5	1.8162	1.2971	1.5348	1.6547

Table 19.25. Services Sector Chained Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2368	1.2675	1.2521	1.2561
3	1.4763	1.6056	1.5396	1.5324
4	1.6104	1.7331	1.6706	1.6662
5	1.6364	1.7410	1.6879	1.6870

19.45 From Table 19.24, it can be seen that the divergence between the fixed-base Laspeyres and Paasche value-added deflators for the services sector grows steadily from period 2 when it is 2.5 percent to period 5 when it is 40.0 percent. However, the divergence between the two superlative value-added deflators is much smaller but does grow over time to reach 7.8 percent in period 5.

19.46 The data listed in Tables 19.18 and 19.19 for the services sector are used to calculate chained Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators for periods t equal 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.25.

19.47 Comparing Tables 19.24 and 19.25, it can be seen that chaining has substantially reduced the spread between the Paasche and Laspeyres value-added deflators for the services sector. In period 5, the divergence between the chained Paasche and Laspeyres is only 6.4 percent, compared with the 40 percent divergence between their fixed-base counterparts. Similarly, chaining has reduced the spread between the two superlative indices; in period 5, the chained Fisher and Törnqvist value-added deflators differ only by 0.05 percent, compared with the 7.8 percent divergence between their fixed-base counterparts. Chaining reduces divergences between the four indices for the services sector because several outputs and intermediate inputs for this sector have strongly divergent trends in their prices. This divergent prices effect overwhelms the effects of bouncing agricultural and energy prices.

F. National Producer Price Indices

F.1 The national output price index

19.48 In order to construct a national output price index, all that is required is to collect the outputs from each of the three industrial sectors and apply normal index number theory to these value flows. There is 1 output in the agriculture sector, 2 outputs in the manufacturing sector, and 11 outputs in the services sector, or 14 outputs in all. The price and quantity data pertaining to these 14 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist output price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.26.

19.49 Since there are divergent trends in the relative prices of outputs in the economy, it should come as no surprise that the Paasche and Laspeyres output price indices grow farther apart over time, reaching a difference of 25.7 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 7.2 percent in period 5. The expectation is that chaining will reduce these divergences.

19.50 The price and quantity data pertaining to the 14 sectoral outputs in the economy are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist output price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.27.

19.51 Comparing Tables 19.26 and 19.27, it can be seen that chaining has indeed reduced the differences between the various national output price

Table 19.26. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Output Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3551	1.3295	1.3422	1.3424
3	1.2753	1.2226	1.2487	1.2575
4	1.1622	1.0305	1.0944	1.1203
5	1.3487	1.0697	1.2011	1.2880

Table 19.27. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Output Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3551	1.3295	1.3422	1.3424
3	1.3033	1.2477	1.2752	1.2751
4	1.1806	1.1119	1.1457	1.1456
5	1.3404	1.2221	1.2799	1.2813

indices. The period 5 difference between the Paasche and Laspeyres price indices is only 9.7 percent, compared with a difference of 25.7 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist price indices is only 0.1 percent, compared with a difference of 7.2 percent for their fixed-base counterparts.

F.2 The national intermediate input price index

19.52 In order to construct a national intermediate input price index, it is necessary only to collect the intermediate inputs from each of the three industrial sectors and apply normal index number theory to these value flows.[24] There are 2 intermediate inputs in the agriculture sector, 3 intermediate inputs in the manufacturing sector, and 5 intermediate inputs in the services sector, or 10 intermediate inputs in all. The price and quantity data pertaining to these 10 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist intermediate input price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.28.

19.53 Since there are divergent trends in the relative prices of intermediate inputs in the economy, it should come as no surprise that the Paasche and Laspeyres intermediate input price indices grow farther apart over time, reaching a difference of 28.6 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 6.7 percent in period 5. The expectation is that chaining will reduce these divergences.

19.54 The price and quantity data pertaining to the 10 sectoral intermediate inputs in the economy

[24]In this section, the negative quantities are changed into positive quantities.

Table 19.28. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Intermediate Input Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4846	1.4310	1.4575	1.4582
3	1.1574	1.1069	1.1319	1.1397
4	0.9179	0.8086	0.8615	0.8817
5	1.1636	0.9049	1.0261	1.0997

Table 19.29. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Intermediate Input Producer Price Indices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4846	1.4310	1.4575	1.4582
3	1.2040	1.1168	1.1596	1.1597
4	0.9485	0.8627	0.9046	0.9052
5	1.1759	1.0296	1.1003	1.1030

are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist intermediate input price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.29.

19.55 Comparing Tables 19.28 and 19.29, it can be seen that chaining has reduced the differences between the Paasche and Laspeyres intermediate input price indices. The period 5 difference between the chained Paasche and Laspeyres price indices is 12.4 percent, compared to a difference of 28.6 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist price indices is only 0.2 percent, compared to a difference of 6.7 percent for their fixed-base counterparts.

F.3 The national value-added deflator

19.56 In order to construct a national value-added deflator, all that is needed is to collect all of the outputs and intermediate inputs from each of the three industrial sectors and apply normal index number theory to these value flows. There are 2 intermediate inputs and 1 output in the agriculture sector, 2 outputs and 3 intermediate inputs in the manufacturing sector, and 11 outputs and 5 intermediate inputs in the services sector, or 24 commodities in all. The price and quantity data pertaining to these 24 commodities are used to calculate *fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.30.

19.57 Since there are divergent trends in the relative prices of outputs and intermediate inputs in the economy, it should come as no surprise that the fixed-base Paasche and Laspeyres value-added deflators grow farther apart over time, reaching a difference of 27.0 percent in period 5. The two superlative indices show a similar diverging trend, reaching a difference of 6.5 percent in period 5. As usual, our expectation is that chaining will reduce these divergences.

Table 19.30. Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1778	1.1831
3	1.4571	1.3957	1.4261	1.4343
4	1.5390	1.3708	1.4525	1.4866
5	1.6343	1.2865	1.4500	1.5447

Table 19.31. Chained National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1778	1.1831
3	1.3743	1.4834	1.4278	1.4307
4	1.4374	1.5349	1.4853	1.4893
5	1.4963	1.5720	1.5337	1.5400

19.58 The price and quantity data pertaining to the 24 sectoral outputs and intermediate inputs in the economy are used again to calculate *chained* Laspeyres, Paasche, Fisher, and Törnqvist national value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. The results are listed in Table 19.31.

19.59 Comparing Tables 19.30 and 19.31, it can be seen that chaining has reduced the differences between the Paasche and Laspeyres deflators. The period 5 difference between the chained Paasche and Laspeyres deflators is 5.1 percent, compared to a difference of 27.0 percent for their fixed-base counterparts. Similarly, the period 5 difference between the chained Fisher and Törnqvist deflators is only 0.4 percent, compared to a difference of 6.5 percent for their fixed-base counterparts.

19.60 At the beginning of this chapter, the Laspeyres, Paasche, Fisher, and Törnqvist *final-demand deflators* were calculated using a fixed base in Tables 19.4 and 19.8 and using the chain principle in Tables 19.5 and 19.9. If these final-demand deflators are compared with their *national value-added deflator* counterparts listed in Tables 19.30 and 19.31, the reader will find that *these two types of deflator give exactly the same answer*. It was assumed that *all transactions are classified on a bilateral sectoral basis*; that is, all transactions between each pair of sectors in the economy are tracked. Under these conditions, if any of the commonly used index number formulas are used, then it can be shown that the final-demand deflator will be *exactly equal* to the national value-added deflator.[25]

[25]The index number formula used must be consistent with either Hicks' (1946, pp. 312–13) or Leontief's (1936) aggregation theorems. That is, if all prices vary in strict proportion across the two periods under consideration, then the price index is equal to this common factor of proportionality (Hicks); or if all quantities vary in strict proportion across the two periods under consideration, then the quantity index that corresponds to the price index is equal to this common factor of proportionality (Leontief). See Allen and Diewert (1981, p. 433) for additional material on these aggregation theorems.

Table 19.32. Two-Stage Fixed-Base National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1815	1.1830
3	1.4571	1.3957	1.4259	1.4379
4	1.5390	1.3708	1.4510	1.5018
5	1.6343	1.2865	1.4485	1.5653

Table 19.33. Two-Stage Chained National Laspeyres, Paasche, Fisher, and Törnqvist Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1552	1.2009	1.1815	1.1830
3	1.3743	1.4834	1.4281	1.4277
4	1.4374	1.5349	1.4853	1.4861
5	1.4963	1.5720	1.5342	1.5368

F.4 National two-stage aggregation

19.61 The national output price index and the national intermediate input price index have been constructed. It is natural to use the two-stage aggregation explained in Section D of Chapter 17 to aggregate these two indices into a national value-added deflator. This result can then be compared with the national value-added deflator that was obtained in the previous section (which was a single-stage aggregation procedure). This comparison is undertaken in this section.

19.62 Using the computations made in the previous section and the theory outlined in Section D of Chapter 17, *two-stage fixed-base* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively, were constructed. The resulting two-stage national value-added deflators are listed in Table 19.32.

19.63 Comparing the two-stage value-added deflators listed in Table 19.32 with the corresponding single-stage deflators listed in Table 19.30, it can be seen that the Paasche and Laspeyres estimates *are exactly the same*, but there are some small differences between the single-stage and two-stage Fisher and Törnqvist value-added deflators. For period 5, the difference in the two fixed-base Fisher deflators is only 0.1 percent, and the difference in the two fixed-base Törnqvist deflators is 1.3 percent.

19.64 Using the computations made in the previous section and the theory outlined in Section D of Chapter 17, *two-stage chained* Laspeyres, Paasche, Fisher, and Törnqvist value-added deflators, P_L^t, P_P^t, P_F^t, and P_T^t, respectively, were constructed. The resulting two-stage national value-added deflators are listed in Table 19.33.

19.65 Comparing the two-stage chained value-added deflators listed in Table 19.33 with the corresponding chained single-stage deflators listed in Table 19.31, it can be seen that the Paasche and Laspeyres estimates *are exactly the same*, but there are some small differences between the single-stage and two-stage Fisher and Törnqvist value-added deflators. For period 5, the difference in the chained Fisher deflators is only 0.03 percent, and the difference in the two chained Törnqvist deflators is 0.2 percent. Thus, chaining has led to a closer correspondence between the single-stage and two-stage national value-added deflators.

20. Elementary Indices

A. Introduction

20.1 In all countries, the calculation of an output PPI proceeds in two (or more) stages. In the first stage of calculation, *elementary price indices* are estimated for the *elementary aggregates* of a PPI. In the second and higher stages of aggregation, these elementary price indices are combined to obtain higher-level indices using information on the net output on each elementary aggregate as weights. An elementary aggregate consists of the revenue from a small and relatively homogeneous set of commodities defined within the industrial classification used in the PPI. Samples of prices are collected within each elementary aggregate, so that elementary aggregates serve as strata for sampling purposes.

20.2 Data on the revenues, or quantities, of different goods and services are typically not available within an elementary aggregate. Since there are no quantity or revenue weights, most of the index number theory outlined from Chapter 15 to 19 is not directly applicable. As was noted in Chapter 1, an elementary price index is a more primitive concept that often relies on price data only.

20.3 The question of which is the most appropriate formula to use to estimate an elementary price index is considered in this chapter. The quality of a PPI depends heavily on the quality of the elementary indices, which are the basic building blocks from which PPIs are constructed.

20.4 As was explained in Chapter 6, compilers have to select *representative commodities* within an elementary aggregate and then collect a sample of prices for each of the representative commodities, usually from a sample of different establishments. The individual commodities whose prices actually are collected are described as the *sampled commodities*. Their prices are collected over a succession of time periods. An elementary price index is therefore typically calculated from two sets of matched price observations. It is assumed in this chapter that there are no missing observations and no changes in the quality of the commodities sampled, so that the two sets of prices are perfectly matched. The treatment of new and disappearing commodities, and of quality change, is a separate and complex issue that is discussed in detail in Chapters 7, 8, and 21 of the *Manual*.

20.5 Even though quantity or revenue weights are usually not available to weight the individual elementary price quotes, it is useful to consider an *ideal framework* where such information is available. This is done in Section B. The problems involved in aggregating narrowly defined price quotes over *time* also are discussed in this section. Thus, the discussion in Section B provides a theoretical target for practical elementary price indices constructed using only information on prices.

20.6 Section C introduces the main elementary index formulas used in practice, and Section D develops some numerical relationships between the various indices. Chapters 15 to 17 developed the various approaches to index number theory when information on both prices and quantities was available. It also is possible to develop axiomatic, economic, or sampling approaches to elementary indices, and these three approaches are discussed below in Sections E, F, and G. Section H develops a simple statistical approach to elementary indices that resembles a highly simplified hedonic regression model. Section I concludes with an overview of the various results.[1]

B. Ideal Elementary Indices

20.7 The aggregates covered by a CPI or a PPI usually are arranged in the form of a tree-like hierarchy, such as COICOP or NACE. Any *aggregate* is a set of economic transactions pertaining to a set of commodities over a specified time period. Every economic transaction relates to the change of own-

[1]This chapter draws heavily on the recent contributions of Dalén (1992a), Balk (1994, 1998b, 2002) and Diewert (1995a, 2002a, 2002b).

ership of a specific, well-defined product (good or service) at a particular place and date, and comes with a quantity and a price. The price index for an aggregate is calculated as a weighted average of the price indices for the subaggregates, the (net output) weights and type of average being determined by the index formula. One can descend in such a hierarchy as far as available information allows the weights to be decomposed. The lowest-level aggregates are called *elementary* aggregates. They are basically of two types:

(i) Those for which all detailed price and quantity information is available, and
(ii) Those for which the statistician, considering the operational cost and the response burden of getting detailed price and quantity information about all the transactions, decides to make use of a representative sample of commodities or respondents.

The practical relevance of studying this topic is large. Since the elementary aggregates form the building blocks of a CPI or a PPI, the choice of an inappropriate formula at this level can have a tremendous impact on the overall index.

20.8 In this section, it will be assumed that detailed price and quantity information is available for all transactions pertaining to the elementary aggregate for the two time periods under consideration. This assumption allows us to define an *ideal elementary aggregate*. Subsequent sections will relax this strong assumption about the availability of detailed price and quantity data on transactions, but it is necessary to have a theoretically ideal target for the practical elementary index.

20.9 The detailed price and quantity data, although perhaps not available to the statistician, are, in principle, available in the outside world. It is frequently the case that at the respondent level (that is, at the firm level), some aggregation of the individual transactions information has been executed, usually in a form that suits the respondent's financial or management information system. This respondent determined level of information could be called the *basic information level*. This is, however, not necessarily the finest level of information that could be made available to the price statistician. One could always ask the respondent to provide more disaggregated information. For instance, instead of monthly data, one could ask for weekly data; or, whenever appropriate, one could ask for regional instead of global data; or, one could ask for data according to a finer product classification. The only natural barrier to further disaggregation is the individual transaction level.[2]

20.10 It is now necessary to discuss a problem that arises when detailed data on *individual transactions* are available. This may occur at the individual establishment level, or even for individual production runs. Recall that in Chapter 15, the price and quantity indexes, $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$, were introduced. These (bilateral) price and quantity indices decomposed the value ratio V^1/V^0 into a price change part $P(p^0,p^1,q^0,q^1)$ and a quantity change part $Q(p^0,p^1,q^0,q^1)$. In this framework, it was taken for granted that the period t price and quantity for product i, p_i^t and q_i^t, were well defined. However, these definitions are not straightforward, since individual purchasers may buy the *same* product during period *t at different prices*. Similarly, consider the sales of a particular establishment, *when the same product may sell at very different prices during the course of the period*. Hence before a traditional bilateral price index of the form $P(p^0,p^1,q^0,q^1)$ considered in previous chapters of this *Manual* can be applied, there is a nontrivial *time aggregation problem* to obtain the basic prices p_i^t and q_i^t that are the components of the price vectors p^0 and p^1 and the quantity vectors q^0 and q^1. Walsh[3] (1901, 1921a) and Davies (1924, 1932) suggested a solution in a CPI context to this time aggregation problem: the appropriate quantity at this very first stage of aggregation is the *total quantity purchased* of the narrowly defined product, and the corresponding price is the value of

[2]See Balk (1994) for a similar approach.

[3]Walsh explained his reasoning as follows: "Of all the prices reported of the same kind of article, the average to be drawn is the arithmetic; and the prices should be weighted according to the relative mass quantities that were sold at them (1901, p. 96). "Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principal market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities" (1921a, p. 88).

purchases of this product divided by the total amount purchased, which is a *narrowly defined unit value*. The appropriate unit value for a PPI context is the value of revenue divided by the total amount sold. In more recent times, other researchers have adopted the Walsh and Davies solution to the time aggregation problem.[4] Note that this solution to the time aggregation problem has the following advantages:

(i) The quantity aggregate is intuitively plausible, being the total quantity of the narrowly defined products sold by establishments during the time period under consideration, and
(ii) The product of the price times quantity equals the total revenue or value sold by the establishment during the time period under consideration.

This solution will be adopted to the time aggregation problem as a valid concept for the price and quantity at this first stage of aggregation.

20.11 Having decided on an appropriate theoretical definition of price and quantity for a product at the very lowest level of aggregation (that is, a narrowly defined unit value and the total quantity sold of that product by the individual establishment), it is now necessary to consider how to aggregate these narrowly defined elementary prices and quantities into an overall elementary aggregate. Suppose that there are M lowest-level items, or specific products, in this chosen elementary category. Denote the period t quantity of product m by $q_m{}^t$ and the corresponding time aggregated unit value by $p_m{}^t$ for $t = 0,1$ and for products $m = 1,2,...,M$. Define the period t quantity and price vectors as $q^t \equiv [q_1{}^t,q_2{}^t,...,q_M{}^t]$ and $p^t \equiv [p_1{}^t,p_2{}^t,...,p_M{}^t]$ for $t = 0,1$. It is now necessary to choose a theoretically ideal index number formula $P(p^0,p^1,q^0,q^1)$ that will aggregate the individual product prices into an overall aggregate price relative for the M products in the chosen elementary aggregate. However, this problem of choosing a functional form for $P(p^0,p^1,q^0,q^1)$ is identical to the overall index number problem that was addressed in Chapters 15 to 17. In these chapters, four different approaches to index number theory were studied that led to specific index number formulas as being best from each perspective. From the viewpoint of *fixed-basket approaches*, it was found that the Fisher (1922) and Walsh (1901) price indexes, P_F and P_W, appeared to be best. From the viewpoint of the *test approach*, the Fisher index appeared to be best. From the viewpoint of the *stochastic approach* to index number theory, the Törnqvist-Theil (Theil, 1967) index number formula P_T emerged as being best. Finally, from the viewpoint of the *economic approach* to index number theory, the Walsh price index P_W, the Fisher ideal index P_F, and the Törnqvist-Theil index number formula P_T were all regarded as being equally desirable. It also was shown that the same three index number formulas numerically approximate each other very closely, so it will not matter very much which of these alternative indexes is chosen.[5] Hence, the *theoretically ideal elementary index number formula* is taken to be one of the three formulas $P_F(p^0,p^1,q^0,q^1)$, $P_W(p^0,p^1,q^0,q^1)$, or $P_T(p^0,p^1,q^0,q^1)$, where the period t quantity of product m, $q_m{}^t$, is the total quantity of that narrowly defined product produced by the establishment during period t, and the corresponding price for product m is $p_m{}^t$, the time aggregated unit value for $t = 0,1$ and for products $m = 1,....,M$.

20.12 In the following section, various practical elementary price indices will be defined. These indices do not have quantity weights and thus are functions only of the price vectors p^0 and p^1, which contain time aggregated unit values for the M products in the elementary aggregate for periods 0 and 1. Thus, when a practical elementary index number formula, say, $P_E(p^0,p^1)$, is compared with an ideal elementary price index, say, the Fisher price index $P_F(p^0,p^1,q^0,q^1)$, then obviously P_E will differ from P_F because the prices are not weighted according to their economic importance in the practical elementary formula. Call this difference between the two index number formulas *formula approximation error*.

20.13 Practical elementary indices are subject to two other types of error:

[4] See, for example, Szulc (1987, p. 13), Dalén (1992a, p. 135), Reinsdorf (1994b), Diewert (1995a, pp. 20–21), Reinsdorf and Moulton (1997), and Balk (2002).

[5] Theorem 5 in Diewert (1978, p. 888) showed that P_F, P_T, and P_W will approximate each other to the second order around an equal price and quantity point; see Diewert (1978, p. 894), R.J. Hill (2000), and Chapter 19, Section B, for some empirical results.

- The statistical agency may not be able to collect information on all M prices in the elementary aggregate; that is, only a *sample* of the M prices may be collected. Call the resulting divergence between the incomplete elementary aggregate and the theoretically ideal elementary index the *sampling error*.
- Even if a price for a narrowly defined product is collected by the statistical agency, it may not be equal to the theoretically appropriate time aggregated unit value price. This use of an inappropriate price at the very lowest level of aggregation gives rise to *time aggregation error*.

20.14 In Section G, a sampling framework for the collection of prices that can reduce the above three types of error will be discussed. In Section C, the five main elementary index number formulas are defined, and in Section D, various numerical relationships between these five indices are developed. Sections E and F develop the axiomatic and economic approaches to elementary indices, and the five main elementary formulas used in practice will be evaluated in light of these approaches.

C. Elementary Indices Used in Practice

20.15 Suppose that there are M lowest-level products or specific products in a chosen elementary category. Denote the period t price of product m by p_m^t for $t = 0,1$ and for products $m = 1,2,...,M$. Define the period t price vector as $p^t \equiv [p_1^t,p_2^t,...,p_M^t]$ for $t = 0,1$.

20.16 The first widely used elementary index number formula is from the French economist Dutot (1738):

$$(20.1)\ P_D(p^0,p^1) \equiv \left[\sum_{m=1}^{M}\frac{1}{M}\left(p_m^1\right)\right] \Big/ \left[\sum_{m=1}^{M}\frac{1}{M}\left(p_m^0\right)\right] = \left[\sum_{i=1}^{M}\left(p_m^1\right)\right] \Big/ \left[\sum_{i=1}^{M}\left(p_m^0\right)\right].$$

Thus the Dutot elementary price index is equal to the arithmetic average of the M period 1 prices divided by the arithmetic average of the M period 0 prices.

20.17 The second widely used elementary index number formula is from the Italian economist Carli (1804):

$$(20.2)\ P_C(p^0,p^1) \equiv \sum_{m=1}^{M}\frac{1}{M}\left(p_m^1 \big/ p_m^0\right).$$

Thus the Carli elementary price index is equal to the *arithmetic* average of the M product price ratios or price relatives, p_m^1 / p_m^0 .

20.18 The third widely used elementary index number formula is from the English economist Jevons (1863):

$$(20.3)\ P_J(p^0,p^1) \equiv \prod_{m=1}^{M}\left(p_m^1 \big/ p_m^0\right)^{1/M} .$$

Thus the Jevons elementary price index is equal to the *geometric* average of the M product price ratios or price relatives, p_m^1 / p_m^0 .

20.19 The fourth elementary index number formula P_H is the *harmonic* average of the M product price relatives, and it was first suggested in passing as an index number formula by Jevons (1865, p. 121) and Coggeshall (1887):

$$(20.4)\ P_H(p^0,p^1) \equiv \left[\sum_{m=1}^{M}\frac{1}{M}\left(p_m^1 \big/ p_m^0\right)^{-1}\right]^{-1} .$$

20.20 Finally, the fifth elementary index number formula is the geometric average of the Carli and harmonic formulas; that is, it is the *geometric mean of the arithmetic and harmonic means of the* M *price relatives*:

$$(20.5)\ P_{CSWD}(p^0,p^1) \equiv \sqrt{P_C(p^0,p^1)\,P_H(p^0,p^1)} .$$

This index number formula was first suggested by Fisher (1922, p. 472) as his formula 101. Fisher also observed that, empirically for his data set, P_{CSWD} was very close to the Jevons index P_J, and these two indices were his best unweighted index number formulas. In more recent times, Carruthers, Sellwood, and Ward (1980, p. 25) and

Dalén (1992a, p. 140) also proposed P_{CSWD} as an elementary index number formula.

20.21 Having defined the most commonly used elementary formulas, the question now arises: which formula is best? Obviously, this question cannot be answered until desirable properties for elementary indices are developed. This will be done in a systematic manner in Section E, but in the present section, one desirable property for an elementary index will be noted: the *time reversal test*, noted in Chapter 15. In the present context, this test for the elementary index $P(p^0,p^1)$ becomes

(20.6) $P(p^0,p^1)\ P(p^1,p^0) = 1$.

20.22 This test says that if the prices in period 2 revert to the initial prices of period 0, then the product of the price change going from period 0 to 1, $P(p^0,p^1)$, times the price change going from period 1 to 2, $P(p^1,p^0)$, should equal unity; that is, under the stated conditions, the index should end up where it started. It can be verified that the Dutot; Jevons; and Carruthers, Sellwood, and Ward indices, P_D, P_J , and P_{CSWD}, all satisfy the time reversal test, but the Carli and harmonic indices, P_C and P_H, fail this test. In fact, these last two indices fail the test in the following *biased* manner:

(20.7) $P_C(p^0,p^1)\ P_C(p^1,p^0) \geq 1$,

(20.8) $P_H(p^0,p^1)\ P_H(p^1,p^0) \leq 1$,

with strict inequalities holding in formulas (20.7) and (20.8), provided that the period 1 price vector p^1 is not proportional to the period 0 price vector p^0.[6] Thus the Carli index will generally have an upward bias while the harmonic index will generally have a downward bias. Fisher (1922, pp. 66 and 383) seems to have been the first to establish the upward bias of the Carli index,[7] and he made the following observations on its use by statistical agencies:

> In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic-type of index number, it will have served a useful purpose (Irving Fisher, 1922, pp. 29–30).

20.23 In the following section, some numerical relationships between the five elementary indices defined in this section will be established. Then, in the subsequent section, a more comprehensive list of desirable properties for elementary indices will be developed, and the five elementary formulas will be evaluated in light of these properties or tests.

D. Numerical Relationships Between the Frequently Used Elementary Indices

20.24 It can be shown[8] that the Carli, Jevons, and harmonic elementary price indices satisfy the following inequalities:

(20.9) $P_H(p^0,p^1) \leq P_J(p^0,p^1) \leq P_C(p^0,p^1)$;

that is, the harmonic index is always equal to or less than the Jevons index, which in turn is always equal to or less than the Carli index. In fact, the strict inequalities in formula (20.9) will hold, provided that the period 0 vector of prices, p^0, is not proportional to the period 1 vector of prices, p^1.

20.25 The inequalities in formula (20.9) do not tell us by how much the Carli index will exceed the Jevons index and by how much the Jevons index will exceed the harmonic index. Hence, in the remainder of this section, some approximate relationships among the five indices defined in the previous section will be developed, which will provide some practical guidance on the relative magnitudes of each of the indices.

20.26 The first approximate relationship derived is between the Carli index P_C and the Dutot index

[6] These inequalities follow from the fact that a harmonic mean of M positive numbers is always equal to or less than the corresponding arithmetic mean; see Walsh (1901, p. 517) or Fisher (1922, pp. 383–84). This inequality is a special case of Schlömilch's Inequality; see Hardy, Littlewood, and Polyá (1934, p. 26).

[7] See also Pigou (1924, pp. 59 and 70), Szulc (1987, p. 12), and Dalén (1992a, p. 139). Dalén (1994, pp. 150–51) provides some nice intuitive explanations for the upward bias of the Carli index.

[8] Each of the three indices P_H, P_J, and P_C is a mean of order r where r equals −1, 0, and 1, respectively, and so the inequalities follow from Schlömilch's inequality; see Hardy, Littlewood, and Polyá (1934, p. 26).

P_D. For each period t, define the *arithmetic mean of the M prices* pertaining to that period as follows:

$$(20.10)\ p^{t*} \equiv \sum_{m=1}^{M} \frac{1}{M}\left(p_m^t\right);\ t = 0,1.$$

Now define the *multiplicative deviation of the* m*th price in period* t *relative to the mean price in that period*, e_m^t, as follows:

$$(20.11)\ p_m^{\ t} = p^{t*}(1+e_m^t);\ \ m = 1,...,M\ ;\ t = 0,1.$$

Note that formula (20.10) and formula (20.11) imply that the deviations $e_m^{\ t}$ sum to zero in each period; that is,

$$(20.12)\ \sum_{m=1}^{M} \frac{1}{M}\left(e_m^t\right) = 0\ ;\ t = 0,1.$$

Note that the Dutot index can be written as the ratio of the mean prices, p^{1*}/p^{0*}; that is,

$$(20.13)\ P_D(p^0,p^1) = p^{1*}/p^{0*}.$$

Now substitute formula (20.11) into the definition of the Jevons index, formula (20.3):

$$(20.14)\ P_J(p^0,p^1) = \prod_{m=1}^{M}\left[p^{1*}\left(1+e_m^1\right)\Big/p^{0*}\left(1+e_m^0\right)\right]^{1/M}$$

$$= \left(p^{1*}/p^{0*}\right)\prod_{m=1}^{M}\left[\left(1+e_m^1\right)\Big/\left(1+e_m^0\right)\right]^{1/M}$$

$$= P_D(p^0,p^1)\,f(e^0,e^1),\ \text{using formula (20.13)},$$

where $e^t \equiv [e_1^{\ t},..., e_m^t]$ for $t = 0$ and 1, and the function f is defined as follows:

$$(20.15)\ f(e^0,e^1) \equiv \prod_{m=1}^{M}\left[\left(1+e_m^1\right)\Big/\left(1+e_m^0\right)\right]^{1/M}.$$

Expand $f(e^0,e^1)$ by a second-order Taylor series approximation around $e^0 = 0_M$ and $e^1 = 0_M$. Using formula (20.12), it can be verified[9] that the following second-order approximate relationship between P_J and P_D results:

$$(20.16)\ P_J(p^0,p^1)$$

$$\approx P_D(p^0,p^1)\left[1 + (1/2M)e^0e^0 - (1/2M)e^1e^1\right]$$

$$= P_D(p^0,p^1)\left[1 + (1/2)\,\text{var}\left(e^0\right) - (1/2)\,\text{var}\left(e^1\right)\right],$$

where var(e^t) is the variance of the period t multiplicative deviations; that is, for $t = 0,1$:

$$(20.17)\ \text{var}(e^t) \equiv \left(1/M\right)\sum_{m=1}^{M}\left(e_m^t - e^{t*}\right)^2$$

$$= \left(1/M\right)\sum_{m=1}^{M}\left(e_m^t\right)^2,$$

since $e^{t*} = 0$ using equation (20.12)

$$= \left(1/M\right)\ e^te^t.$$

20.27 Under normal conditions,[10] the variance of the deviations of the prices from their means in each period is likely to be approximately constant, and so under these conditions, the Jevons price index will approximate the Dutot price index to the second order. With the exception of the Dutot formula, the remaining four elementary indices defined in Section C are functions of the relative prices of the M products being aggregated. This fact is used to derive some approximate relationships between these four elementary indices. Thus define the m*th price relative* as

$$(20.18)\ r_m \equiv p_m^1/p_m^0\ \ ;\ m = 1,...,M.$$

20.28 Define the arithmetic mean of the m price relatives as

$$(20.19)\ r^* \equiv \left(1/M\right)\sum_{m=1}^{M}\left(r_m\right) = P_C(p^0,p^1),$$

[9]This approximate relationship was first obtained by Carruthers, Sellwood, and Ward (1980, p. 25).

[10]If there are significant changes in the overall inflation rate, some studies indicate that the variance of deviations of prices from their means also can change. Also, if M is small, there will be sampling fluctuations in the variances of the prices from period to period.

where the last equality follows from the definition of formula (20.2) of the Carli index. Finally, define the *deviation* e_m *of the* m*th price relative* r_m *from the arithmetic average of the* M *price relatives* r^* as follows:

(20.20) $r_m = r^*(1 + e_m)$; $m = 1,...,M$.

20.29 Note that formula (20.19) and formula (20.20) imply that the deviations e_m sum to zero; that is,

(20.21) $\sum_{m=1}^{M}\left(e_m\right) = 0.$

Now substitute formula (20.20) into the definitions of P_C, P_J, P_H, and P_{CSWD}, formulas (20.2) to (20.5), to obtain the following representations for these indices in terms of the vector of deviations, $e \equiv [e_1,...,e_M]$:

(20.22) $P_C(p^0,p^1) = \sum_{m=1}^{M}\left(\frac{1}{M}(r_m)\right) = r^*\ 1 \equiv r^* f_C(e)$;

(20.23) $P_J(p^0,p^1) = \prod_{m=1}^{M}(r_m)^{1/M} = r^* \prod_{m=1}^{M}(1+e_m)^{1/M} \equiv r^* f_J(e)$;

(20.24) $P_H(p^0,p^1) = \left[\sum_{m=1}^{M}\left(\frac{1}{M}(r_m)\right)^{-1}\right]^{-1} = r^* \left[\sum_{m=1}^{M}\left(\frac{1}{M}(1+e_m)\right)^{-1}\right]^{-1} \equiv r^* f_H(e)$;

(20.25) $P_{CSWD}(p^0,p^1) = \sqrt{P_C(p^0,p^1)P_H(p^0,p^1)} = r^* \sqrt{f_C(e) f_H(e)} \equiv r^* f_{CSWD}(e)$,

where the last equation in formulas (20.22) to (20.25) serves to define the deviation functions, $f_C(e)$, $f_J(e)$, $f_H(e)$, and $f_{CSW}(e)$. The second-order Taylor series approximations to each of these functions around the point $e = 0_M$ are

(20.26) $f_C(e) \approx 1$;

(20.27) $f_J(e) \approx 1 - (\frac{1}{2} M) e \cdot e = 1 - (\frac{1}{2})\mathrm{var}(e)$;

(20.28) $f_H(e) \approx 1 - (\frac{1}{M}) e \cdot e = 1 - \mathrm{var}(e)$;

(20.29) $f_{CSWD}(e) \approx 1 - (\frac{1}{2} M) e \cdot e = 1 - (\frac{1}{2})\mathrm{var}(e)$;

where repeated use is made of formula (20.21) in deriving the above approximations.[11] Thus to the second order, the Carli index P_C will *exceed* the Jevons and Carruthers, Sellwood, and Ward indices, P_J and P_{CSWD}, by $(\frac{1}{2})r^*\mathrm{var}(e)$, which is one-half of the variance of the M price relatives p_m^1/p_m^0. Much like the second order, the harmonic index P_H will *lie below* the Jevons and Carruthers, Sellwood, and Ward indices, P_J and P_{CSWD}, by one-half of the variance of the M price relatives p_m^1/p_m^0.

20.30 Thus, empirically, it is expected that the Jevons and Carruthers, Sellwood, and Ward indices will be very close to each other. Using the previous approximation result formula (20.16), it is expected that the Dutot index P_D also will be fairly close to P_J and P_{CSWD}, with some fluctuations over time because of changing variances of the period 0 and 1 deviation vectors e^0 and e^1. Thus, it is expected that these three elementary indices will give similar numerical answers in empirical applications. On the other hand, the Carli index can be expected to be substantially *above* these three indices, with the degree of divergence growing as the variance of the M price relatives grows. Similarly, the harmonic index can be expected to be substantially *below* the three middle indices, with the degree of divergence growing as the variance of the M price relatives grows.

E. The Axiomatic Approach to Elementary Indices

20.31 Recall that in Chapter 16, the axiomatic approach to bilateral price indices, $P(p^0,p^1,q^0,q^1)$, was developed. In the present chapter, the elementary price index $P(p^0,p^1)$ depends only on the period 0 and 1 price vectors, p^0 and p^1, not on the period 0 and 1 quantity vectors, q^0 and q^1. One approach to obtaining new tests (T) or axioms for an elementary index is to look at the 20 or so axioms listed in Chapter 16 for bilateral price indices $P(p^0,p^1,q^0,q^1)$, and adapt those axioms to the present context; that is, use the old bilateral tests for $P(p^0,p^1,q^0,q^1)$ that do not depend on the quantity

[11]These second-order approximations are from Dalén (1992a, p. 143) for the case $r^* = 1$ and Diewert (1995a, p. 29) for the case of a general r^*.

vectors q^0 and q^1 as tests for an elementary index $P(p^0,p^1)$.[12]

20.32 The first eight tests or axioms are reasonably straightforward and uncontroversial:

T1: *Continuity*: $P(p^0,p^1)$ is a continuous function of the M positive period 0 prices $p^0 \equiv [p_1^0,...,p_M^0]$ and the M positive period 1 prices $p^1 \equiv [p_1^1,...,p_M^1]$.

T2: *Identity*: $P(p,p) = 1$; that is, if the period 0 price vector equals the period 1 price vector, then the index is equal to unity.

T3: *Monotonicity in Current-Period Prices*: $P(p^0,p^1) < P(p^0,p)$ if $p^1 < p$; that is, if any period 1 price increases, then the price index increases.

T4: *Monotonicity in Base-Period Prices*: $P(p^0,p^1) > P(p,p^1)$ if $p^0 < p$; that is, if any period 0 price increases, then the price index decreases.

T5: *Proportionality in Current-Period Prices*: $P(p^0,\lambda p^1) = \lambda P(p^0,p^1)$ if $\lambda > 0$; that is, if all period 1 prices are multiplied by the positive number λ, then the initial price index is also multiplied by λ.

T6: *Inverse Proportionality in Base-Period Prices*: $P(\lambda p^0,p^1) = \lambda^{-1} P(p^0,p^1)$ if $\lambda > 0$; that is, if all period 0 prices are multiplied by the positive number λ, then the initial price index is multiplied by $1/\lambda$.

T7: *Mean Value Test*: $\min_m \{ p_m^1 / p_m^0 : m = 1,...,M\} \le P(p^0,p^1) \le \max_m \{ p_m^1 / p_m^0 : m = 1,...,M\}$; that is, the price index lies between the smallest and largest price relatives.

T8: *Symmetric Treatment of Establishments/ Products*: $P(p^0,p^1) = P(p^0*,p^1*)$, where p^0* and p^1* denote the *same* permutation of the components of p^0 and p^1; that is, if there is a change in ordering of the establishments from which the price quotations (or products within establishments) are obtained for the two periods, then the elementary index remains unchanged.

20.33 Eichhorn (1978, p. 155) showed that tests T1, T2, T3, and T5 imply T7, so that not all of the above tests are logically independent. The following tests are more controversial and are not necessarily accepted by all price statisticians.

T9: *The Price-Bouncing Test*: $P(p^0,p^1) = P(p^0*,p^1**)$, where p^0* and p^1** denote possibly *different* permutations of the components of p^0 and p^1; that is, if the ordering of the price quotes for both periods is changed in possibly different ways, then the elementary index remains unchanged.

20.34 Obviously, test T8 is a special case of test T9, where in test T8 the two permutations of the initial ordering of the prices are restricted to be the same. Thus test T9 implies test T8. Test T9 is from Dalén (1992a, p. 138), who justified this test by suggesting that the price index should remain unchanged if outlet (for CPIs) prices "bounce" in such a manner that the outlets are just exchanging prices with each other over the two periods. While this test has some intuitive appeal, it is not consistent with the idea that outlet prices should be matched to each other in a one-to-one manner across the two periods. If elementary aggregates contain thousands of individual products that differ not only by outlet, there still is less reason to maintain this test.

20.35 The following test was also proposed by Dalén (1992a) in the elementary index context:

T10: *Time Reversal*: $P(p^1,p^0) = 1/P(p^0,p^1)$; that is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index.

20.36 Since many price statisticians approve of the Laspeyres price index in the bilateral index context, and this index does not satisfy the time reversal test, it is obvious that not all price statisticians would regard the time reversal test in the elementary index context as being a fundamental test that must be satisfied. Nevertheless, many other price statisticians do regard this test as fundamental, since it is difficult to accept an index that gives a different answer if the ordering of time is reversed.

T11: *Circularity*: $P(p^0,p^1)P(p^1,p^2) = P(p^0,p^2)$; that is, the price index going from period 0 to 1, times

[12]This was the approach used by Diewert (1995a, pp. 5–17), who drew on the earlier work of Eichhorn (1978, pp. 152–60) and Dalén (1992a).

the price index going from period 1 to 2, equals the price index going from period 0 to 2 directly.

20.37 The circularity and identity tests imply the time reversal test (just set $p^2 = p^0$). Thus, the circularity test is essentially a strengthening of the time reversal test, so price statisticians who did not accept the time reversal test are unlikely to accept the circularity test. However, if there are no obvious drawbacks to accepting the circularity test, it would seem to be a very desirable property: it is a generalization of a property that holds for a single price relative.

T12: *Commensurability*:
$P(\lambda_1 p_1^0,...,\lambda_M p_M^0;\lambda_1 p_1^1,...,\lambda_M p_M^1)$
$= P(p_1^0,...,p_M^0; p_1^1,...,p_M^1)$
$= P(p^0,p^1)$ for all $\lambda_1 > 0,..., \lambda_M > 0$;

that is, if the units of measurement for each product in each establishment are changed, then the elementary index remains unchanged.

20.38 In the bilateral index context, virtually every price statistician accepts the validity of this test. However, in the elementary context, this test is more controversial. If the M products in the elementary aggregate are homogeneous, then it makes sense to measure all of the products in the same units. The very essence of homogeneity is that quantities can be added up in an economically meaningful way. Hence, if the unit of measurement is changed, then test T12 should restrict all of the λ_m to be the same number (say, λ) and the test T12 becomes

(20.30) $P(\lambda p^0,\lambda p^1) = P(p^0,p^1)$; $\lambda > 0$.

This modified test T12 will be satisfied if tests T5 and T6 are satisfied. Thus, if the products in the elementary aggregate are very homogeneous, then there is no need for test T12.

20.39 However, in actual practice, there usually will be thousands of individual products in each elementary aggregate, and the hypothesis of product homogeneity is not warranted. Under these circumstances, it is important that the elementary index satisfy the commensurability test, since the units of measurement of the heterogeneous products in the elementary aggregate are arbitrary and hence *the price statistician can change the index simply by changing the units of measurement for some of the products*.

20.40 This completes the listing of the tests for an elementary index. There remains the task of evaluating how many tests each of the five elementary indices defined in Section C passed.

20.41 The Jevons elementary index, P_J , satisfies *all* of the tests, and hence emerges as being best from the viewpoint of the axiomatic approach to elementary indices.

20.42 The Dutot index, P_D , satisfies all of the tests with the important exception of the commensurability test T12, which it fails. Heterogeneous products in the elementary aggregate constitute a rather serious failure, and price statisticians should be careful in using this index under these conditions.

20.43 The geometric mean of the Carli and harmonic elementary indices, P_{CSWD}, fails only the price-bouncing test T9 and the circularity test T11. The failure of these two tests is probably not a fatal failure, so this index could be used by price statisticians if, for some reason, they decided not to use the Jevons formula. It particularly would be suited to those who favor the test approach for guidance in choosing an index formula. As observed in Section D, numerically, P_{CSWD} will be very close to P_J.

20.44 The Carli and harmonic elementary indices, P_C and P_H, fail the price-bouncing test T9, the time reversal test T10, and the circularity test T11, and pass the other tests. The failure of tests T9 and T11 is not a fatal failure, but the failure of the time reversal test T10 is rather serious, so price statisticians should be cautious in using these indices.

F. The Economic Approach to Elementary Indices

20.45 Recall the notation and discussion in Section B. First, it is necessary to recall some of the basics of the economic approach from Chapter 17. This allowed the aggregator functions representing the producing technology and the behavioral assumptions of the economic agents implicit in different formulas to be identified. The more realistic these were, the more support was given to the corresponding index number formula. The economic approach helps identify what the target index should be.

20.46 Suppose that each establishment producing products in the elementary aggregate has a set of inputs, and the linearly homogeneous aggregator function $f(q)$ describes what output vector $q \equiv [q_1,...,q_M]$ can be produced from the inputs. Further assume that each establishment engages in revenue-maximizing behavior in each period. Then, as was seen in Chapter 17, it can be shown that that certain specific functional forms for the aggregator $f(q)$ or its dual unit revenue function $R(p)$[13] lead to specific functional forms for the price index, $P(p^0,p^1,q^0,q^1)$, with

(20.31) $$P(p^0,p^1,q^0,q^1) = R(p^1)/R(p^0).$$

20.47 Suppose that the establishments have aggregator functions f defined as follows:[14]

(20.32) $$f(q_1,...,q_M) \equiv \max_m \{q_m/\alpha_m : m = 1,...,M\},$$

where the α_m are positive constants. Then under these assumptions, it can be shown that equation (20.31) becomes[15]

(20.33) $$R(p^1)/R(p^0) = p^1q^0/p^0q^0 = p^1q^1/p^0q^1,$$

and the quantity vector of products produced during the two periods must be proportional; that is,

(20.34) $$q^1 = \lambda q^0 \text{ for some } \lambda > 0.$$

20.48 From the first equation in formula (20.33), it can be seen that the true output price index, $R(p^1)/R(p^0)$, under assumptions of formula (20.32) about the aggregator function f, is equal to the Laspeyres price index, $P_L(p^0,p^1,q^0,q^1) \equiv p^1{\cdot}q^0 / p^0{\cdot}q^0$. The Paasche formula $P_P(p^0,p^1,q^0,q^1) \equiv p^1q^1/p^0q^1$ is equally justified under formula (20.34).

20.49 Formula (20.32) on f thus justifies the Laspeyres and Paasche indices as being the "true" elementary aggregate from the economic approach to elementary indices. Yet this is a restrictive assumption, at least from an economic viewpoint, that relative quantities produced do not vary with relative prices. Other less restrictive assumptions on technology can be made. For example, as shown in Section B.3, Chapter 17, certain assumptions on technology justify the Törnqvist price index, P_T, whose logarithm is defined as

(20.35) $$\ln P_T(p^0,p^1,q^0,q^1) \equiv \sum_{i=1}^{M} \frac{\left(s_i^0 + s_i^1\right)}{2} \ln\left(\frac{p_i^1}{p_i^0}\right).$$

20.50 Suppose now that product revenues are proportional for each product over the two periods so that

(20.36) $p_m^{\,1}q_m^{\,1} = \lambda\, p_m^{\,0}q_m^{\,0}$ for $m = 1,...,M$ and for some $\lambda > 0$.

Under these conditions, the base-period revenue shares $s_m^{\,0}$ will equal the corresponding period 1 revenue shares $s_m^{\,1}$, as well as the corresponding $\beta(m)$; that is, formula (20.36) implies

(20.37) $$s_m^{\,0} = s_m^{\,1} \equiv \beta(m)\ ;\ m = 1,...,M.$$

Under these conditions, the Törnqvist index reduces to the following weighted Jevons index:

(20.38) $$P_J(p^0,p^1,\beta(1),...,\beta(M)) = \prod_{m=1}^{M}\left(p_m^1/p_m^0\right)^{\beta(m)}.$$

20.51 Thus, if the relative prices of products in a Jevons index are weighted using weights proportional to base-period (which equals current-period) revenue shares in the product class, then the Jevons index defined by equation (20.38) is equal to the following approximation to the Törnqvist index:

(20.39) $$P_J(p^0,p^1,s^0) \equiv \prod_{m=1}^{M}\left(p_m^1/p_m^0\right)^{s_m^0}.$$

20.52 In Section G, the sampling approach shows how, under various sample designs, elementary index number formulas have implicit weighting systems. Of particular interest are sample designs where products are sampled with probabilities proportionate to quantity or revenue shares in

[13]The unit revenue function is defined as $R(p) \equiv \max_q \{p{\cdot}q : f(q) = 1\}$.

[14]The preferences that correspond to this f are known as *Leontief* (1936) or *no substitution* preferences.

[15]See Pollak (1983a).

either period. Under such circumstances, quantity weights are implicitly introduced, so that the sample elementary index is an estimate of a population-weighted index. The economic approach then provides a basis for deciding whether the economic assumptions underlying the resulting population estimates are reasonable. For example, the above results show that the sample Jevons elementary index can be justified as an approximation to an underlying Törnqvist price index for a homogeneous elementary aggregate *under a price sampling scheme with probabilities of selection proportionate to base-period revenue shares*.

20.53 Two assumptions have been outlined here: the assumption that the quantity vectors pertaining to the two periods under consideration are proportional, formula (20.34), and the assumption that revenues are proportional over the two periods, formula (20.36).

20.54 The choice between formulas depends not only on the sample design used, but also on the relative merits of the proportional quantities versus proportional revenues assumption. Similar considerations apply to the economic theory of the CPI (or an intermediate input PPI), except that the aggregator function describes the preferences of a cost-minimizing purchaser. In this context, index number theorists have debated the relative merits of the proportional quantities versus proportional expenditures assumption for a long time. Authors who thought that the proportional expenditures assumption was more likely empirical include Jevons (1865, p. 295) and Ferger (1931, p. 39; 1936, p. 271). These early authors did not have the economic approach to index number theory at their disposal, but they intuitively understood, along with Pierson (1895, p. 332), that substitution effects occurred and, hence, the proportional expenditures assumption was more plausible than the proportional quantities assumption. This is because *cost-minimizing consumers* will purchase fewer sampled products with above-average price increases; the quantities can be expected to fall rather than remain constant. Such a decrease in quantities combined with the increase in price makes the assumption of constant expenditures more tenable. However, this is for the economic theory of CPIs. In Chapter 17, the economic theory of PPIs argued that *revenue-maximizing establishments* will produce *more* sampled products with above-average price increases, making assumptions of constant revenues less tenable. However, the theory presented in Chapter 17 also indicated that technical progress was a complicating factor largely absent in the consumer context.

20.55 If quantities supplied move proportionally over time, then this is consistent with a Leontief technology, and the use of a Laspeyres index is perfectly consistent with the economic approach to the output price index. On the other hand, if the probabilities used for sampling of prices for the Jevons index are taken to be the arithmetic average of the period 0 and 1 product revenue shares, and narrowly defined unit values are used as the price concept, then the weighted Jevons index becomes an ideal type of elementary index discussed in Section B. In general, the biases introduced by the use of an unweighted formula cannot be assessed accurately unless information on weights for the two periods is somehow obtained.

G. The Sampling Approach to Elementary Indices

20.56 It can now be shown how various elementary formulas can estimate this Laspeyres formula under alternative assumptions about the sampling of prices.

20.57 To justify the use of the Dutot elementary formula, consider the expected value of the Dutot index when sampling with *base-period product inclusion probabilities* equal to the sales quantities of product m in the base period relative to total sales quantities of all products in the product class in the base period. Assume *that these definitions require that all products in the product class have the same units*.[16]

20.58 The expected value of the sample Dutot index is[17]

$$(20.40)\quad \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^0}{\sum_{m=1}^{M} q_m^0}\right) \Bigg/ \left(\frac{\sum_{m=1}^{M} p_m^0 q_m^0}{\sum_{m=1}^{M} q_m^0}\right),$$

[16]The inclusion probabilities are meaningless unless the products are homogeneous.

[17]There is a technical bias since $E(x/y)$ is approximated by $E(x)/E(y)$, but this will approach zero as m gets larger.

which is the familiar Laspeyres index,

$$(20.41)\quad \sum_{m=1}^{M} p_m^1 q_m^0 \Big/ \sum_{m=1}^{M} p_m^0 q_m^0 \equiv P_L(p^0,p^1,q^0,q^1).$$

20.59 Now it is easy to see how this sample design could be turned into a rigorous sampling framework for sampling prices in the particular product class under consideration. If product prices in the product class were sampled proportionally to their base-period probabilities, then the Laspeyres index formula (20.41) could be estimated by a probability-weighted Dutot index, where the probabilities are defined by their base-period quantity shares. In general, with an appropriate sampling scheme, the use of the Dutot formula at the elementary level of aggregation *for homogeneous products* can be perfectly consistent with a Laspeyres index concept. Put otherwise, under this sampling design, the expectation of the sample Dutot is equal to the population Laspeyres.

20.60 The Dutot formula also can be consistent with a Paasche index concept at the elementary level of aggregation. If sampling is with *period 1 item inclusion probabilities,* the expectation of the sample Dutot is equal to

$$(20.42)\quad \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1}\right) \Big/ \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1}\right),$$

which is the familiar Paasche formula,

$$(20.43)\quad \sum_{m=1}^{M} p_m^1 q_m^1 \Big/ \sum_{m=1}^{M} p_m^0 q_m^1 \equiv P_P(p^0,p^1,q^0,q^1).$$

20.61 Put otherwise, under this sampling design, the expectation of the sample Dutot is equal to the population Paasche index. Again, it is easy to see how this sample design could be turned into a rigorous sampling framework for sampling prices in the particular product class under consideration. If product prices in the product class were sampled proportionally to their period 1 probabilities, then the Paasche index formula (20.43) could be estimated by the probability-weighted Dutot index. In general, with an appropriate sampling scheme, the use of the Dutot formula at the elementary level of aggregation (*for a homogeneous elementary aggregate*) can be perfectly consistent with a Paasche index concept.[18]

20.62 Rather than use the fixed-basket representations for the Laspeyres and Paasche indexes, the revenue-share representations for the Laspeyres and Paasche indexes could be used along with the revenue shares $s_m{}^0$ or $s_m{}^1$ as probability weights for price relatives. Under sampling proportional to base-period revenue shares, the expectation of the Carli index is

$$(20.44)\quad P_C(p^0,p^1,s^0) \equiv \sum_{m=1}^{M} s_m^0 \ln\left(p_m^1 \big/ p_m^0\right),$$

which is the population Laspeyres index. Of course, formula (20.44) does not require the assumption of homogeneous products as did formula (20.40) and formula (20.42). On the other hand, one can show analogously that under sampling proportional to period 1 revenue shares, the expectation of the reciprocal of the sample harmonic index is equal to the reciprocal of the population Paasche index, and thus that the expectation of the sample harmonic index,

$$(20.45)\quad P_H(p^0,p^1,s^1) \equiv \left[\sum_{m=1}^{M} s_m^1 \left(p_m^1 \big/ p_m^0\right)^{-1}\right]^{-1},$$

will be equal to the Paasche index.

20.63 The above results show that the sample Dutot elementary index can be justified as an approximation to an underlying population Laspeyres or Paasche price index for a homogeneous elementary aggregate *under appropriate price sampling schemes*. The above results also show that the sample Carli and harmonic elementary indexes can be justified as approximations to an underlying population Laspeyres or Paasche price index for a heterogeneous elementary aggregate *under appropriate price sampling schemes*.

[18]Of course, the Dutot index as an estimate of a population Paasche index will differ from the Dutot index as an estimate of a population Laspeyres index because of representativity or substitution bias.

20.64 Thus, if the relative prices of products in the product class under consideration are sampled using weights that are proportional to the arithmetic average of the base- and current-period revenue shares in the product class, then the expectation of this sample Jevons index is equal to the population Törnqvist index formula (20.35).

20.65 S*ample elementary indices* sampled under appropriate probability designs were capable of approximating various population economic elementary indices, with the approximation becoming exact as the sampling approached complete coverage. Conversely, it can be seen that, in general, it will be impossible for a sample *elementary price index*, of the type defined in Section C, to provide an unbiased estimate of the theoretical population ideal elementary price index defined in Section B, even if all product prices in the elementary aggregate were sampled. Hence, rather than just sampling prices, it will be necessary for the price statistician to collect information on the *transaction values* (or quantities) associated with the sampled prices to form sample elementary aggregates that will approach the target ideal elementary aggregate as the sample size becomes large. Thus instead of just collecting a sample of prices, it will be necessary to collect corresponding sample quantities (or values) so that a sample Fisher, Törnqvist, or Walsh price index can be constructed. This sample-based superlative elementary price index will approach the population ideal elementary index as the sample size becomes large. This approach to the construction of elementary indices in a sampling context was recommended by Pigou (1924, pp. 66–7), Fisher (1922, p. 380), Diewert (1995a, p. 25), and Balk (2002).[19] In particular, Pigou (1924, p. 67) suggested that the sample-based Fisher ideal price index be used to deflate the value ratio for the aggregate under consideration to obtain an estimate of the quantity ratio for the aggregate under consideration.

20.66 Until fairly recently, it was not possible to determine how close an unweighted elementary index, defined in Section C, was to an ideal elementary aggregate. However, with the availability of *scanner data* (that is, of detailed data on the prices and quantities of individual products that are sold in retail outlets), it has been possible to compute ideal elementary aggregates for some product strata and compare the results with statistical agency estimates of price change for the same class of products. Of course, the statistical agency estimates of price change usually are based on the use of the Dutot, Jevons, or Carli formulas. These studies relate to CPIs, the data collected from the bar-code readers of retail outlets. But the concern here is with the discrepancy between unweighted and weighted indices used at this elementary aggregate level, and the discrepancies are sufficiently large to merit highlighting in this PPI context. The following quotations summarize many of these scanner data studies:

> A second major recent development is the willingness of statistical agencies to experiment with scanner data, which are the electronic data generated at the point of sale by the retail outlet and generally include transactions prices, quantities, location, date and time of purchase and the product described by brand, make or model. Such detailed data may prove especially useful for constructing better indexes at the elementary level. Recent studies that use scanner data in this way include Silver (1995), Reinsdorf (1996), Bradley, Cook, Leaver and Moulton (1997), Dalén (1997), de Haan and Opperdoes (1997) and Hawkes (1997). Some estimates of elementary index bias (on an annual basis) that emerged from these studies were: 1.1 percentage points for television sets in the United Kingdom; 4.5 percentage points for coffee in the United States; 1.5 percentage points for ketchup, toilet tissue, milk and tuna in the United States; 1 percentage point for fats, detergents, breakfast cereals and frozen fish in Sweden; 1 percentage point for coffee in the Netherlands and 3 percentage points for coffee in the United States respectively. These bias estimates incorporate both elementary and outlet substitution biases and are significantly higher than our earlier ballpark estimates of .255 and .41 percentage points. On the other hand, it is unclear to what extent these large bias estimates can be generalized to other commodities (Diewert, 1998a, pp. 54–55).
>
> Before considering the results it is worth commenting on some general findings from scanner data. It is stressed that the results here are for an experiment in which the same data were used to compare different methods. The results for the U.K. Retail Prices Index can not be fairly compared since they are based on quite different practices and data, their data being collected by

[19]Balk (2002) provides the details for this sampling framework.

price collectors and having strengths as well as weaknesses (Fenwick, Ball, Silver and Morgan (2002)). Yet it is worth following up on Diewert's (2002c) comment on the U.K. Retail Prices Index electrical appliances section, which includes a wide variety of appliances, such as irons, toasters, refrigerators, etc. which went from 98.6 to 98.0, a drop of 0.6 percentage points from January 1998 to December 1998. He compares these results with those for washing machines and notes that "..it may be that the non washing machine components of the electrical appliances index increased in price enough over this period to cancel out the large apparent drop in the price of washing machines but I think that this is somewhat unlikely." A number of studies on similar such products have been conducted using scanner data for this period. Chained Fishers indices have been calculated from the scanner data, (the RPI (within year) indices are fixed-base Laspeyres ones), and have been found to fall by about 12% for televisions (Silver and Heravi, 2001a), 10% for washing machines (Table 7 below), 7.5% for dishwashers, 15% for cameras and 5% for vacuum cleaners (Silver and Heravi, 2001b). These results are quite different from those for the RPI section and suggest that the washing machine disparity, as Diewert notes, may not be an anomaly. Traditional methods and data sources seem to be giving much higher rates for the CPI than those from scanner data, though the reasons for these discrepancies were not the subject of this study (Silver and Heravi, 2002, p. 25).

20.67 These quotations summarize the results of many elementary aggregate index number studies based on the use of scanner data. These studies indicate that when detailed price and quantity data are used to compute superlative indexes or hedonic indexes for an expenditure category, the resulting measures of price change are often below the corresponding official statistical agency estimates of price change for that category. Sometimes the measures of price change based on the use of scanner data are *considerably below* the corresponding official measures.[20] These results indicate that there may be large gains in the precision of elementary indices if a *weighted* sampling framework is adopted.

20.68 Is there a simple intuitive explanation for the above empirical results? The empirical work is on CPIs, and the behavioral assumptions relate to such indices, though they equally apply to input PPIs. Furthermore, the analysis can be undertaken readily based on the behavioral assumptions underlying output PPIs, its principles being more important. A partial explanation may be possible by looking at the dynamics of product demand. In any market economy, firms and outlets sell products that are either declining or increasing in price. Usually, the products that decline in price experience an increase in sales. Thus, the expenditure shares associated with products declining in price usually increase, and the reverse is true for products increasing in price. Unfortunately, elementary indices cannot pick up the effects of this negative correlation between price changes and the induced changes in expenditure shares, because elementary indices depend only on prices and not on expenditure shares.

20.69 An example can illustrate this point. Suppose that there are only three products in the elementary aggregate, and that in period 0, the price of each product is $p_m^0 = 1$, and the expenditure share for each product is equal, so that $s_m^0 = 1/3$ for $m = 1,2,3$. Suppose that in period 1, the price of product 1 increases to $p_1^1 = 1 + i$, the price of product 2 remains constant at $p_2^1 = 1$, and the price of product 3 decreases to $p_3^1 = (1 + i)^{-1}$, where the product 1 rate of increase in price is $i > 0$. Suppose further that the expenditure share of product 1 decreases to $s_1^1 = (⅓) - \sigma$, where σ is a small number between 0 and ⅓, and the expenditure share of product 3 increases to $s_3^1 = (⅓) + \sigma$. The expenditure share of product 2 remains constant at $s_2^1 = ⅓$. The five elementary indices, defined in Section C, all can be written as functions of the product 1 inflation rate i (which is also the product 3 deflation rate) as follows:

$$(20.46)\ P_J(p^0,p^1) = \left[(1+i)(1+i)^{-1}\right]^{1/3} = 1 \equiv f_J(i)\ ;$$

[20]However, scanner data studies do not always show large potential biases in official CPIs. Masato Okamoto of the National Statistics Center in Japan informed us in a personal communication that a large-scale internal study was undertaken. Using scanner data for about 250 categories of processed food and daily necessities collected over the period 1997 to 2000, it was found that the indices based on (continued) scanner data averaged only about 0.2 percentage points below the corresponding official indices per year. Japan uses the Dutot formula at the elementary level in its official CPI.

(20.47) $P_C(p^0,p^1) = \frac{1}{3}(1+i) + \frac{1}{3} + \frac{1}{3}(1+i)^{-1}$
$\equiv f_C(i)$;

(20.48) $P_H(p^0,p^1) = \frac{1}{3}(1+i)^{-1} + \frac{1}{3} + \frac{1}{3}(1+i)^{-1}$
$\equiv f_H(i)$;

(20.49) $P_{CSWD}(p^0,p^1) = \sqrt{P_C(p^0,p^1)\,P_H(p^0,p^1)}$
$\equiv f_{CSW}(i)$;

(20.50) $P_D(p^0,p^1) = \frac{1}{3}(1+i) + \frac{1}{3} + \frac{1}{3}(1+i)^{-1}$
$\equiv f_D(i)$.

20.70 Note that in this particular example, the Dutot index $f_D(i)$ turns out to equal the Carli index $f_C(i)$. The second-order Taylor series approximations to the five elementary indices formulas (20.46) to (20.50) are given by formulas (20.51) to (20.55) below:

(20.51) $f_J(i) = 1$;
(20.52) $f_C(i) \approx 1 + \frac{1}{3}i^2$;
(20.53) $f_H(i) \approx 1 - \frac{1}{3}i^2$;
(20.54) $f_{CSW}(i) \approx 1$;
(20.55) $f_D(i) \approx 1 + \frac{1}{3}i^2$.

Thus for small i, the Carli and Dutot indices will be slightly greater than 1,[21] the Jevons and Carruthers, Sellwood, and Ward indices will be approximately equal to 1, and the harmonic index will be slightly less than 1. Note that the first-order Taylor series approximation to all five indices is 1; that is, to the accuracy of a first-order approximation, all five indices equal unity.

20.71 Now calculate the Laspeyres, Paasche, and Fisher indices for the elementary aggregate:

(20.56) $P_L = \frac{1}{3}(1+i) + \frac{1}{3} + \frac{1}{3}(1+i)^{-1} \equiv f_L(i)$;

(20.57) P_P
$= \left[(\frac{1}{3} - \sigma)(1+i) + \frac{1}{3} + (\frac{1}{3} + \sigma)(1+i)^{-1}\right]^{-1}$
$\equiv f_P(i)$;

(20.58) $P_F = \sqrt{P_L \bullet P_P} \equiv f_F(i)$.

First-order Taylor series approximations to the above indices formulas (20.56) to (20.58) around $i = 0$ are given by formulas (20.59)–(20.61):

(20.59) $f_L(i) \approx 1$;

(20.60) $f_P(i) \approx 1 - 2\sigma\, i$;

(20.61) $f_F(i) \approx 1 - \sigma\, i$.

An ideal elementary index for the three products is the Fisher ideal index $f_F(i)$. The approximations in formulas (20.51) to (20.55) and formula (20.61) show that the Fisher index will lie below all five elementary indices by the amount $\sigma\, i$ using first-order approximations to all six indices. *Thus all five elementary indices will have an approximate upward bias equal to* σ i *compared with an ideal elementary aggregate*.

20.72 Suppose that the annual product inflation rate for the product rising in price is equal to 10 percent, so that $i = .10$ (and, hence, the rate of price decrease for the product decreasing in price is approximately 10 percent as well). If the expenditure share of the increasing price product declines by 5 percentage points, then $\sigma = .05$, and the annual approximate upward bias in all five elementary indices is $\sigma\, i = .05 \times .10 = .005$ or one-half of a percentage point. If i increases to 20 percent and σ increases to 10 percent, then the approximate bias increases to $\sigma\, i = .10 \times .20 = .02$, or 2 percent.

20.73 The above example is highly simplified, but more sophisticated versions of it are capable of explaining at least some of the discrepancy between official elementary indices and superlative indices calculated by using scanner data for an expenditure class. Basically, elementary indices defined without using associated quantity or value weights are incapable of picking up shifts in expenditure shares induced by fluctuations in product prices.[22] To eliminate this problem, it will be necessary to sample values along with prices in both the base and comparison periods.

[21]Recall the approximate relationship in formula (20.16) in Section C between the Dutot and Jevons indices. In the example, $\mathrm{var}(e^0) = 0$, whereas $\mathrm{var}(I^1) > 0$. This explains why the Dutot index is not approximately equal to the Jevons index in the example.

[22]Put another way, elementary indices are subject to substitution or representativity bias.

20.74 In the following section, a simple regression-based approach to the construction of elementary indices is outlined, and, again, the importance of weighting the price quotes will emerge from the analysis.

H. A Simple Stochastic Approach to Elementary Indices

20.75 Recall the notation used in Section B. Suppose the prices of the M products for period 0 and 1 are equal to the right-hand sides of formulas (20.62) and (20.63) below:

(20.62) $p_m{}^0 = \beta_m$; $m = 1,\ldots,M$;

(20.63) $p_m{}^1 = \alpha\beta_m$; $m = 1,\ldots,M$,

where α and the β_m are positive parameters. Note that there are two M prices on the left-hand sides of equations (20.62) and (20.63) but only $M + 1$ parameters on the right-hand sides of these equations. The basic hypothesis in equations (20.62) and (20.63) is that the two price vectors p^0 and p^1 are proportional (with $p^1 = \alpha p^0$, so that α is the factor of proportionality) except for random multiplicative errors, and, hence, α represents the underlying elementary price aggregate. If logarithms are taken of both sides of equations (20.62) and (20.63) and some random errors $e_m{}^0$ and $e_m{}^1$ added to the right-hand sides of the resulting equations, the following *linear regression model* results:

(20.64) $\ln p_m{}^0 = \delta_m + e_m{}^0$; $m = 1,\ldots,M$;

(20.65) $\ln p_m{}^1 = \gamma + \delta_m + e_m{}^1$; $m = 1,\ldots,M$,

where

(20.66) $\gamma \equiv \ln \alpha$ and $\delta_m \equiv \ln \beta_m$; $m = 1,\ldots,M$.

20.76 Note that equations (20.64) and (20.65) can be interpreted as a highly simplified *hedonic regression model*.[23] The only characteristic of each product is the product itself. This model is also a special case of the *country product dummy method* for making international comparisons among the prices of different countries.[24] A major advantage of this regression method for constructing an elementary price index is that *standard errors* for the index number α can be obtained. This advantage of the stochastic approach to index number theory was stressed by Selvanathan and Rao (1994).

20.77 It can be verified that the least-squares estimator for γ is

(20.67) $$\gamma^* \equiv \sum_{m=1}^{M} 1/M \ln\left(p^1/p^0\right).$$

If γ^* is exponentiated, then the following estimator for the elementary aggregate α is obtained:

(20.68) $$\alpha^* \equiv \prod_{m=1}^{M}\left(p_m^2/p_m^1\right)^{1/M} \equiv P_J(p^1,p^2),$$

where $P_J(p^0,p^1)$ is the *Jevons elementary price index* defined in Section C above. Thus, there is a regression model-based justification for the use of the Jevons elementary index.

20.78 Consider the following unweighted *least-squares model*:

(20.69) $$\min_{\gamma,\ \delta\text{'s}} \sum_{m=1}^{M}\left(\ln p_m^1 - \delta_m\right)^2 + \sum_{m=1}^{M}\left(\ln p_m^0 - \gamma - \delta_m\right)^2 .$$

It can be verified that the γ solution to the unconstrained minimization problem (20.69) is the γ^* defined by (20.67).

20.79 There is a problem with the unweighted least-squares model defined by formula (20.69): the logarithm of each price quote is given exactly the *same weight* in the model, no matter what the revenue on that product was in each period. This is obviously unsatisfactory, since a price that has very little economic importance (that is, a low revenue share in each period) is given the same weight in the regression model compared with a very important product. Thus, it is useful to consider the following *weighted least-squares model:*

[23]See Chapters 7, 8, and 21 for material on hedonic regression models.

[24]See Summers (1973). In our special case, there are only two "countries," which are the two observations on the prices of the elementary aggregate for two periods.

(20.70) $\min_{\gamma, \delta\text{'s}}$
$$\sum_{m=1}^{M} s_m^0 \left(\ln p_m^0 - \delta_m\right)^2 + \sum_{m=1}^{M} s_m^1 \left(\ln p_m^1 - \gamma - \delta_m\right)^2,$$

where the period t revenue share on product m is defined in the usual manner as

(20.71) $$s_m^t \equiv \frac{p_m^t q_m^t}{\sum_{m=1}^{M} p_m^t q_m^t}; \; t = 0,1 \; ; \; m = 1,\ldots,M.$$

Thus, in the model (20.70), the logarithm of each product price quotation in each period is weighted by its revenue share in that period.

20.80 The γ solution to (20.70) is

(20.72) $$\gamma^{**} = \sum_{m=1}^{M} h\left(s_m^0, s_m^1\right) \ln\left(p_m^1 / p_m^0\right),$$

where

(20.73) $$h(a,b) \equiv \left[\frac{1}{2}a^{-1} + \frac{1}{2}b^{-1}\right]^{-1} = 2ab / \left[a+b\right],$$

and $h(a,b)$ is the *harmonic mean* of the numbers a and b. Thus γ^{**} is a share-weighted average of the logarithms of the price ratios p_m^1/p_m^0. If γ^{**} is exponentiated, then an estimator α^{**} for the elementary aggregate α is obtained.

20.81 How does α^{**} compare with the three ideal elementary price indices defined in Section B? It can be shown[25] that α^{**} approximates those three indices to the second order around an equal price and quantity point; that is, for most data sets, α^{**} will be very close to the Fisher, Törnqvist, and Walsh elementary indices.

20.82 The results in this section provide some weak support for the use of the Jevons elementary index, but they provide much stronger support for the use of weighted elementary indices of the type defined in Section B above. The results in this section also provide support for the use of value or quantity weights in hedonic regressions.

I. Conclusions

20.83 The main results in this chapter can be summarized as follows:

(i) To define a "best" elementary index number formula, it is necessary to have a target index number concept. In Section B, it is suggested that normal bilateral index number theory applies at the elementary level as well as at higher levels, and hence the target concept should be one of the Fisher, Törnqvist, or Walsh formulas.

(ii) When aggregating the prices of the same narrowly defined product within a period, the narrowly defined unit value is a reasonable target price concept.

(iii) The axiomatic approach to traditional elementary indices (that is, no quantity or value weights are available) supports the use of the Jevons formula under all circumstances. If the products in the elementary aggregate are very homogeneous (that is, they have the same unit of measurement), then the Dutot formula can be used. In the case of a heterogeneous elementary aggregate (the usual case), the Carruthers, Sellwood, and Ward formula can be used as an alternative to the Jevons formula, but both will give much the same numerical answers.

(iv) The Carli index has an upward bias and the harmonic index has a downward bias.

(v) All five unweighted elementary indices are not really satisfactory. A much more satisfactory approach would be to collect quantity or value information along with price information and form sample superlative indices as the preferred elementary indices.

(vi) A simple hedonic regression approach to elementary indices supports the use of the Jevons formula. However, a more satisfactory approach is to use a weighted hedonic regression approach. The resulting index will closely approximate the ideal indices defined in Section B.

[25] Use the techniques discussed in Diewert (1978).

21. Quality Change and Hedonics

21.1 Chapters 15 to 20 cover theoretical issues relating to the choice of index number formulas and are based on a simplifying assumption: that the aggregation was over the same matched $i = 1....n$ items in the two periods being compared. This meets the needs of the discussion of alternative index number formulas, since a measure of price change between two periods requires the quality of each item to remain the same. The practical compilation of PPIs involves defining the *price basis* (quality specification and terms of sale) of a sample of items in an initial period and monitoring the prices of this matched sample over time, so that only "pure" price changes are measured, not price changes tainted by changes in quality. In practice, this matching becomes imperfect. The quality of what is produced *does* change, and, furthermore, new goods (and services) appear on the market that the matched sampling ignores. The relative price changes of these new goods may differ from those of the existing ones, leading to bias in the index if they are excluded. In this chapter, a theoretical framework is outlined that extends the definition of items to include their quality characteristics. The focus of the chapter is on the *economic* theory of the market for quality characteristics and its practical manifestation in hedonic regression outlined in Chapter 7, Section E.4. This provides a *background* for the more practical issues relating to quality adjustments in Chapter 7 and item substitution in Chapter 8.

A. New and Disappearing Items and Quality Change

21.2 The assumption in the previous chapters was that the same set of items was being compared in each period.[1] Such a set can be considered as a sample from all the matched items available in periods 0 and t—the *intersection universe*, which includes only matched items. Yet, for many commodity markets, old items disappear and new items appear. Constraining the sample to be drawn from this intersection universe is unrealistic. Establishments may produce an item in period 0, but it may not be sold in subsequent periods t.[2] New items may be introduced after period 0 that cannot be compared with a corresponding item in period 0. These items may be variants of the old existing ones or provide totally new services that cannot be directly compared with anything that previously existed. This universe of all items in periods 0 and t is the dynamic *double universe.*

21.3 There is a third universe from which prices might be sampled: a *replacement universe*. The prices reported by establishments are those for an agreed *price basis*—a detailed description of the item being sold and the terms of the transaction. The price basis for items in period 0 are first determined, and then their prices are monitored in subsequent periods. If the item is discontinued and there are no longer prices to record for a particular price basis, prices of a comparable replacement item may be used to continue the series of prices. This universe is a *replacement universe* that starts with the base-period universe, but it also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

21.4 When a comparable replacement is unavailable, a noncomparable one may be selected. In this case, an explicit adjustment has to be made to the price of either the old or the replacement item for the quality difference. Since the replacement is of a different quality than the old item, it is likely to have a different price basis. Alternatively, assumptions may be made so that the price change of the old item (had it continued to exist) follows those of other items, keeping to the matched universe. In this second case, an implicit adjustment is being made for quality changes, so that the differ-

[1] The terminology is credited to Dalén (1998); see also Appendix 8.1.

[2] Its absence may be temporary, being a seasonal item, and specific issues and methods for such temporarily unavailable items are considered in Chapter 9. The concern here is with items that disappear permanently.

ence in price changes for the group and the old item (had it continued to exist) is equivalent to their quality differences.[3] What is stressed here is that the problem of missing items is the problem of adjusting prices for quality differences.

21.5 Three practical problems emerge. First is the problem of explicit quality adjustment between a replacement and old item. The item is no longer produced, a replacement is found that is not strictly comparable in quality, the differences in quality are identified, and a price has to be put on these differences if the series of prices for the new replacement item are to be used to continue those of the old series.

21.6 Second, in markets where the turnover of items is high, the sample space selected from the matched universe is going to become increasingly unrepresentative of the dynamic universe, as argued in detail in Chapter 8. Even the replacement universe may be inappropriate, as it will be made of series carrying with them quality adjustments in each period whose overall accuracy, given the rapidly changing technology, may be tenuous. In such cases, it may be that prices are no longer collected from a matched sample but from a sample of the main items available in each period, even though they are of a different quality. A comparison between the average prices of such items would be biased if, say, the quality of the items was improving. The need for, and details of, mechanisms to remove the effects of such changes from the average price comparisons were discussed in some detail in Chapter 7, Section G.

21.7 Finally, there is the problem of new and disappearing goods and services—when the new item is not a variant of the old but provides a completely new service. It is not possible to use it as a replacement for an old item by adjusting a price for the quality differential because what it provides is, by definition, something new.

21.8 There are a number of approaches to quality adjustment, and these are considered in Chapter 7. One of the approaches is to make explicit adjustments to prices for the quality difference between the old and replacement item using the coefficients from hedonic regression equations. *Hedonic regressions* are regressions of the prices of individual models of a product on their characteristics—for example, the prices of television sets on screen size, stereo sound, and text retrieval. The coefficients on such variables provide estimates of the monetary values of different quantifiable characteristics of the product. They can be used to adjust the price of a noncomparable replacement item for quality differences compared with the old item—for example, the replacement television set may have text-retrieval facilities that the previous version did not. Yet, it is important that a clear understanding exists of the meaning of such estimated coefficients if they are to be used for quality adjustment, especially given that their use is being promoted.[4] To understand what these estimated parameters mean, it is first necessary to conceive of products as aggregates of their characteristics because, unlike items, characteristics have no separate prices attached to them. The price of the item is the price of a "tied" bundle of characteristics. One must also consider what determines the prices of these characteristics. Economic theory points toward examining demand and supply factors (Sections B.2 and B.3) and the interaction of the two to determine an equilibrium price (Section B.4). Having developed the analytical framework for such prices, it is then necessary to see what interpretation the economic theoretic framework allows us to put on these calculated coefficients (Section B.5). It will be seen that unless there is uniformity of buyers' tastes or suppliers' technologies, an identification problem prevents an unambiguous supply or demand interpretation. Borrowing a framework by Diewert (2002d), a demand-side interpretation that assumes firms are competitive price takers is provided, which, under this user-value approach, shows the assumptions required to generate such meaningful coefficients (Section B.6). Yet, all the aforementioned analysis assumes competitive behavior, an assumption relaxed in Section B.7.

21.9 Chapter 7, Section G, recommends two main approaches for handling industries with rapid turnover of items. If the sample in period 0 is soon outdated, the matched universe and even replacement is increasingly unrepresentative of the double universe, and repeated sampling from the double universe is required. In this case, either chained in-

[3]Such methods and their assumptions are outlined in detail in Chapter 15.

[4]Boskin and others (1996; 1998) and Shultze and Mackie (2002).

dices are advised in Chapter 7, Section G.3, or one of a number of *hedonic indices*, described in Chapter 7, Section G.2. Such indices differ from the use of hedonic regression for adjusting prices for quality differences for a missing item. These indices use hedonic regressions, say, by including a dummy variable for time on the right-hand side of the equation to estimate the quality-adjusted price change, as outlined below in Section C and in Chapter 7, Section G.2. They build on the theory outlined in Chapter 17 and Chapter 8, Section B. The economic theory of output price indices outlined in Chapter 21 is developed to include those tied bundles of a good that can be defined in terms of their characteristics as an item in the revenue function. *Theoretical output price indices* are defined that include changes in the prices of characteristics. Yet, as with the output price indices for goods considered in Chapter 17, there are many formulations that hedonic indices can take, and analogous issues and formulas arise here when discussing alternative approaches in Sections C.3–C.6.

21.10 The estimation of hedonic regressions and the testing of their statistical properties are facilitated by the availability of user-friendly, yet powerful, statistical and econometric software. There are many standard issues in the estimation of regression equations, which can be examined by the diagnostics tests available in such software, as discussed in Kennedy (2003) and Maddala (1988). However, there are issues on functional form, the use of weighted least-squares estimators, and specifications that are quite specific to the estimation of hedonic equations. While many of these are taken up in Chapter 7, where an illustration is provided, Appendix 21.1 considers some of the theoretical issues. See also Gordon (1990), Griliches (1990), and Triplett (1990).

21.11 Finally, in Section D, economic theory will be used to advise on the problem of new and disappearing goods and services. This problem arises where differences between existing goods and services and the new goods and services are substantive and cannot be meaningfully compared with an old item, even with a quality adjustment. The economic theory of reservation prices will be considered and some issues about its practical implementation expressed.

B. Hedonic Prices and Implicit Markets

B.1 Items as tied bundles of characteristics

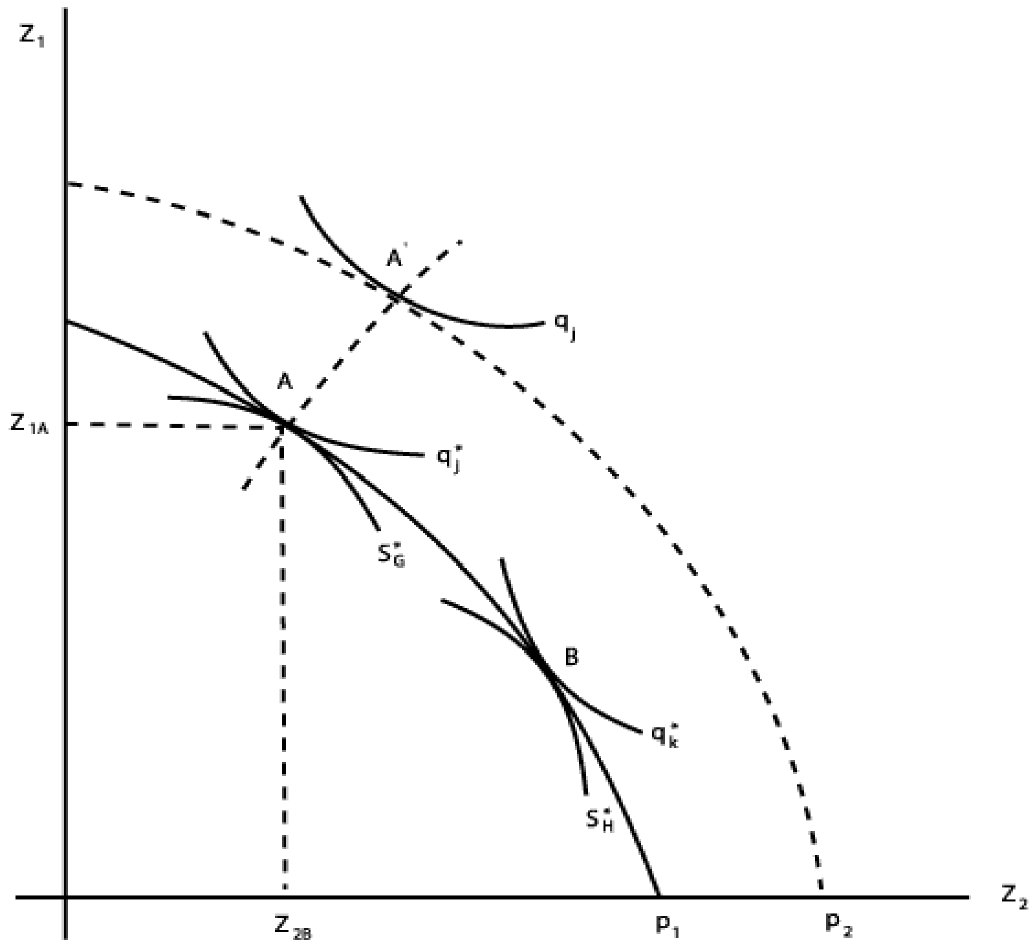
21.12 A *hedonic regression* is a regression equation that relates the prices of items, p, to the quantities of characteristics, given by the vector $z = (z_1, z_2, \ldots, z_n)$, that is,

$$(21.1)\ p(z) = p(z_1, z_2, \ldots, z_n),$$

where the items are defined in terms of varying amounts of their characteristics. In practice, what will be observed for each item or variant of the commodity is its price, a set of its characteristics, and possibly the quantity and, thus, value sold. Empirical work in this area has been concerned with two issues: estimating how the price of an item changes as a result of unit changes in each characteristic—that is, the estimated coefficients of equation (21.1)—and estimating the demand and supply functions for each characteristic. The depiction of an item as a basket of characteristics, each characteristic having its own implicit (shadow) price, requires in turn the specification of a market for such characteristics, since prices result from the workings of markets. Houthakker (1952), Becker (1965), Lancaster (1966), and Muth (1966) have identified the demand for items in terms of their characteristics. The sale of an item is the sale of a tied bundle of characteristics to consumers, whose economic behavior in choosing between items is depicted as one of choosing between bundles of characteristics.[5] However, Rosen (1974) further developed the analysis by providing a structural market framework in terms of both producers and consumers. There are two sides: demand and supply. How much of each characteristic is supplied and consumed is determined by the interaction of the demand for characteristics by consumers and the supply of characteristics by producers. These are considered in turn.

[5]The range of items is assumed to be continuous in terms of the combinations of characteristics that define it. A noncontinuous case can be depicted where the price functions are piecewise linear, and an optimal set of characteristics is obtained by combining the purchases of different items (Lancaster, 1971; Gorman, 1980).

Figure 21.1. Consumption and Production Decisions for Combinations of Characteristics



B.2 Consumer or demand side

21.13 Figure 21.1, adapted from Triplett (1987, p. 634), presents a simplified version of the characteristic space between two characteristics. The hedonic surfaces p_1 and p_2 in that figure trace out all the combinations of the two characteristics z_1 and z_2 that can be purchased at prices p_1 and p_2. An indifference curve q_j* maps the combinations of z_1 and z_2 that the consumer is indifferent against purchasing; that is, the consumer will derive the same utility from any point on the curve. The tangency of q_j* with p_1 at A is the solution to the utility maximization problem for a given budget (price p_1) and tastes (reflected in q_j*).

21.14 The slope of the hedonic surface is the marginal cost of acquiring the combination of characteristics, and the slope of the utility function is the marginal utility gained from their purchase. The tangency at A is the utility-maximizing combination of characteristics to be purchased at that price. If consumers purchased any other combination of characteristics in the space of Figure 21.1, it would either cost them more to do so or lead to a lower level of utility. Position A', for example, has more of both z_1 and z_2, and the consumer receives a higher level of utility being on q_j, but the consumer also has to have a higher budget and pays p_2 for being there. Note that the hedonic surface depicted here is nonlinear, so that relative characteristic prices are not fixed. The consumer with tastes q_k* chooses characteristic set B at p_1. Thus, the data observed in the market depend on the set of tastes. Triplett (2002) has argued that if tastes were all the same, then only one model of a personal computer would be purchased. But in the real world more than one model does exist, reflecting heterogeneous tastes and income levels. Rosen (1974) shows that of all the characteristic combinations and prices at which they may be offered, the hedonic surface traces out an envelope[6] of tangencies including q_j* and q_k* on p_1 in Figure 21.1. This envelope is simply a description of the locus of the points chosen. Since rational consumers who optimize are assumed, these are the points that will be observed in the market and are thus used to estimate the hedonic regression. Note further that points A and B alone will not allow the regression to determine the price of z_1 relative to z_2, since the observed data will be two combinations of outputs at the same price. However, the locus of points on an expansion path $A\,A'$ would allow this to be determined. There may be expansion paths for consumers with different tastes, such as B, and this may give rise to conflicting valuations, so that the overall parameter estimates determined by the regression from transactions observed in the market are an amalgam of such data. And this would just be a reflection of the reality of economic life. What arises from this exposition is the fact that the form of the hedonic function is determined in part by the distribution of buyers and their tastes in the market.

21.15 The exposition is now formalized to include parameters for tastes and a numeraire commodity[7] against which combinations of other aggregates are selected following Rosen (1974). The hedonic function $p(z)$ describes variation in the market price of the items in terms of their characteristics. The consumer purchase decision is assumed to be based on utility maximization behavior, the utility function being given by $U(z, x;\alpha)$, where x is a numeraire commodity, the maximization of utility being subject to a budget constraint given by income y measured as $y = x + p(z)$ (the amount spent on the numeraire commodity and the hedonic commodities), and α is a vector of the features of the individual consumer that describe their tastes. Consumers maximize their utility by selecting a combination of quantities of x and characteristics z subject to a budget constraint. The market is assumed to be competitive, and consumers are described as price takers; they purchase only the one item, so their purchase decision does not influence the market price. The price they pay for a combination of characteristics, vector z, is given by $p(z)$. Since they are optimizing consumers, the combination chosen is such that

(21.2) $[\partial U(z, y - p(z);\alpha) / \partial z_i] / [\partial U(z, y - p(z);\alpha) / \partial x] = \partial p(z) / \partial z_i \equiv p_i(z)$,

where $\partial p(z) / \partial z_i$ is the first derivative of the hedonic function in equation (21.1) with respect to each z characteristic. The coefficients of the hedonic function are equal to their shadow price p_i, which measures the utility derived from that characteristic relative to the numeraire good for given budgets and tastes.

21.16 A *value function* θ can be defined as the value of expenditure a consumer with tastes α is willing to pay for alternative values of z at a given utility u and income y, represented by $\theta(z;u,y,\alpha)$. It defines a family of indifference curves relating the z_i to foregone x, money. For individual characteristics z_i, θ is the marginal rate of substitution between z_i and money, or the implicit marginal valuation the consumer with tastes α puts on z_i at a given utility level and income. It is an indication of the reservation demand price[8] for additional units

[6]An envelope is more formally defined by letting $f(x,y,k) = 0$ be an implicit function of x and y. The form of the function is assumed to depend on k, the tastes in this case. A different curve corresponds to each value of k in the xy plane. The envelope of this family of curves is itself a curve with the property that it is tangent to each member of the family. The equation of the envelope is obtained by taking the partial derivative of $f(x,y,k)$ with respect to k and eliminating k from the two equations $f(x,y,k) = 0$ and $fk(x,y,k) = 0$. (See Osgood, 1925.)

[7]The numeraire commodity represents all other goods and services consumed—it represents the "normal" nonhedonic commodities. The price of x is set equal to unity; $p(z)$ and income are measured in these units.

[8]This is the hypothetical price that makes the demand for the characteristic equal to zero; that is, it is the price that, (continued)

of z_i.[9] The price in the market is $p(z)$, and utility is maximized when $\theta(z;u,y,\alpha) = p(z)$; that is, the purchase takes place where the surface of the indifference curve θ is tangent to the hedonic price surface. If different buyers have different value functions (tastes), some will buy more of a characteristic than others for a given price function, as illustrated in Figure 21.1.

21.17 The joint distribution function of tastes and income sets out a family of value functions, each of which, when tangential to the price function, depicts a purchase and simultaneously defines the price function whose envelope is the market hedonic price function. The points of purchase traced out by the hedonic function thus depend on the budget of the individual and the tastes of the individual consumer purchasing an individual set of characteristics. If demand functions are to be traced out, the joint probability distribution of consumers with particular budgets and tastes occurring in the market needs to be specified, that is, $F(y, \alpha)$. This function, along with equation (21.1), allows the demand equations to be represented for each characteristic.

B.3 Producer or supply side

21.18 Referring again to Figure 21.1, it also shows the production side. In Chapter 17, Section B.1, a revenue-maximizing producer was considered whose revenue maximization problem was given by equation (17.1);[10]

(21.3) $R(p,v) \equiv \max_q \left[\sum_{n=1}^{N} p_n q_n : q \text{ belongs to } S(v) \right]$,

where $R(p,v)$ is the maximum value of output, $\sum_{n=1}^{N} p_n q_n$, that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v is available for use, using the period t technology. Figure 17.1 illustrated in goods-space how the producer would choose between different combinations of outputs, q_1 and q_2. In Figure 21.1, the characteristics-space problem is analogous to the goods-space one with producers choosing here between combinations of z_1 and z_2 to produce for a particular level of technology and inputs $S(v)$. For a particular producer with level of inputs and technology S^*_G facing a price surface p_1, the optimal production combination is at A. However, a different producer with technology and inputs S^*_H facing a price surface p_1 would produce at B. At these points, the marginal cost of z_1 with respect to z_2 is equal to its marginal price from the hedonic surface as depicted by the tangency of the point. Production under these circumstances at any other combination would not be optimal. The envelope of tangencies such as S^*_G and S^*_H trace out the production decisions that would be observed in the market from optimizing, price-taking producers and are used as data for estimating the hedonic regressions. The hedonic function can be seen to be determined, in part, by the distribution of technologies of producers, including their output scale.

21.19 Rosen (1974) formalizes the producer side, whereby price-taking producers are assumed to have cost functions described by $C(M, z; \tau)$,[11] where $Q =,Q(z)$ is the output scale-number of units produced by an establishment offering specifications of an item with characteristics z. They have to decide which items to produce, that is, which package of z. To do this, a cost minimization problem is solved that requires τ, equivalent to $S(v)$ above, a vector of the technology of each producer that describes the output combinations each producer can produce with given input costs using its factors of production and the factor prices. It is the variation in τ across producers that distinguishes producer A's decision about which combination of z to produce from that of producer B in Figure 21.1. Producers are optimizers who seek to maximize profits given by

(21.4) $Q\, p(z) - C(Q,z;\tau)$

by selecting Q and z optimally. The supplying market is assumed to be competitive, and producers are price takers, so the producers cannot influence price by their production decision. Their deci-

when inserted into the demand function, sets demand to zero.

[9]The utility function is assumed strictly concave so that θ is concave in z, and the value function is increasing in z_i at a decreasing rate.

[10]The time superscripts are not relevant in this context.

[11]The cost function is assumed to be convex with no indivisibilities. The marginal cost of producing one more item of a given combination of characteristics is assumed to be positive and increasing, and, similarly, the marginal cost of increasing production of each component characteristic is positive and nondecreasing.

sion about how much to produce of each z is determined by the price of z, assuming that the producer can vary Q and z in the short run.[12] Dividing equation (21.4) by Q and setting it equal to zero, the first-order profit-maximizing conditions are given by

$$(21.5)\quad \frac{\partial p}{\partial z_i} = p_i = \frac{C_{zi}(Q,z;\tau)}{Q},$$

where $p = p(z_1, z_2, \ldots, z_n)$ from equation (21.1).

21.20 The *marginal unit revenue* from producing characteristic z_i is given by its shadow price in the price function and its marginal cost of production. In the producer case, the probability distribution of the technologies of firms, $G(\tau)$, is necessary if the overall quantity supplied of items with given characteristic sets are to be revealed. Since it is a profit maximization problem to select the optimal combination of characteristics to produce, marginal revenue from the additional attributes must equal their marginal cost of production per unit sold. Quantities are produced up to the point where unit revenues $p(z)$ equal marginal production costs, evaluated at the optimum bundle of characteristics supplied.

21.21 While for consumers a *value function* was considered, producers require an *offer function* $\phi(z;\pi,\tau)$. The offer price is the price the seller is willing to accept for various designs at constant profit level π, when quantities produced are optimally chosen, while $p(z)$ is the maximum price obtainable from those models in the market. Producer equilibrium is characterized by a tangency between a profit characteristics indifference surface and the market characteristics price surface, where $p_i(z_i) = \phi_{zi}(z;\pi,\tau)$ and $p\ (z) = \phi_z(z;\pi,\tau)$. Since there is a distribution of technologies $G(\tau)$, the producer equilibrium is characterized by a family of offer functions that envelop the market hedonic price function. The varying τ will depend on different factor prices for items produced in different countries, multiproduct firms with economies of scale, and differences in the technology, whether the quality of capital, labor, or intermediate inputs and their organization. Different values of τ will define a family of production surfaces.

B.4 Equilibrium

21.22 The theoretical framework first defined each item as a point on a plane of several dimensions made up by the $z_1, z_2, \ldots, z_n$ quality characteristics; each item was a combination of values $z_1, z_2, \ldots, z_n$. If only two characteristics defined the item, then each point in the positive space of Figure 21.1 would define an item. The characteristics were not bought individually but as bundles of characteristics tied together to make up an item. It was assumed that the markets were differentiated so that there was a wide range of choices to be made.[13] The market was also assumed to be perfectly competitive with consumers and producers as price takers undertaking optimizing behavior to decide which items (tied sets of characteristics) to buy and sell. Competitive markets in characteristics and optimizing behavior are assumed so that the quantity demanded of characteristics z must equal the quantity supplied. It has been shown that consumers' and producers' choices or "locations" on the plane will be dictated by consumer tastes and producer technology. Tauchen and Witte (2001, p. 4) show that the hedonic price function will differ across markets in accordance with the means and variances (and in some cases also higher moments) of the distributions of household and firm characteristics.

21.23 Rosen (1974, p. 44) notes that a buyer and seller are perfectly matched when their respective value and offer functions are tangential. The common gradient at that point is given by the gradient of the market-clearing implicit price function $p(z)$. The consumption and production decisions were seen in the value and offer functions to be jointly determined, for given $p(z)$, by $F(y, \alpha)$ and $G(\tau)$. In competitive markets there is a simultaneity in the determination of the hedonic equation, since the distribution of $F(y, \alpha)$ and $G(\tau)$ help determined the quantities demanded and supplied and

[12]Rosen (1974) considered two other supply characterizations: the short run in which only Q is variable, and a long run in which plants can be added and retired. The determination of equilibrium supply and demand is not straightforward. A function $p(z)$ is required such that market demand for all z will equate to market supply and clear the market. But demand and supply depend on the whole $p(z)$, since any adjustment to prices to equate demand and supply for one combination of items will induce substitutions and changes for others. Rosen (1974, pp. 44–48) discusses this in some detail.

[13]So that choices among combinations of z are continuous, assume further that z possesses continuous second-order derivatives.

also the slope of the function. Although the decisions made by consumers and producers are as price takers, the prices taken are those from the hedonic function. There is a sense in which the hedonic function and its shadow prices emerge from the operations of the market. The product markets implicitly reveal the hedonic function. Since consumers and producers are optimizers in competitive markets, the hedonic function, in principle, gives the minimum price of any bundle of characteristics. Given all of this, Rosen (1974, p. 44) asked: what do hedonic prices mean?

B.5 What do hedonic prices mean?

21.24 It would be convenient if, for PPI construction, the estimated coefficients from hedonic regressions were estimates of the marginal production cost or producer value of a characteristic or, for CPI construction, they were estimates of the marginal utility from a characteristic or user value. But theory tells us that this is not the case and that the interpretation is not clear.

21.25 There was an erroneous perception in the 1960s that the coefficients from hedonic methods represented user values as opposed to resource costs. Rosen (1974), as has been shown, found that hedonic coefficients generally reflect both user values and resource costs; both supply and demand situations. The ratios of these coefficients may reflect consumers' marginal rates of substitution or producers' marginal rates of substitution (transformation) for characteristics. There is what is referred to in econometrics as an "identification"' problem in which the observed prices and quantities are jointly determined by supply and demand considerations, and their underlying effects cannot be separated. The data collected on prices jointly arise from variations in demand by different consumers with different tastes and preferences, and from variations in supply by producers with different technologies.

21.26 First, it is necessary to come to terms with this simultaneity problem. Hedonic regressions are an increasingly important analytical tool, one implicitly promoted by the attention given to it in this *Manual* but also promoted in separate manuals by organizations such as the OECD (see Triplett, 2002) and Eurostat (2001), and widely used by the U.S. Bureau of Labor Statistics (Kokoski, Waehrer, and Rozaklis, 2001, and Moulton, 2001b). So how do economists writing on the subject shrug their intellectual shoulders in light of these findings?

21.27 Rosen (1974, p. 43) refers to the hedonic function as "...a joint envelope of a family of value functions and another family of offer functions. An envelope function by itself reveals nothing about the underlying members that generate it; and they in turn constitute the generating structure of the observations."

21.28 Griliches (1988, p. 120) notes the following:

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions *per se*, though it can also be useful for these purposes….what is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

21.29 Triplett (1987) states, "It is well-established—but still not widely understood—that the form of $h(\cdot)$ [the hedonic function] cannot be derived from the form of $Q(\cdot)$ and $t(\cdot)$ [utility and production functions], nor does $h(\cdot)$ represent a "reduced form" of supply and demand functions derived from $Q(\cdot)$ and $t(\cdot)$.

21.30 Diewert (2003, p. 320), with his focus on the consumer side, says,

> Thus, I am following Muellbauer's (1974, p. 977) example where he says that his "approach is unashamedly one-sided; only the demand side is treated…Its subject matter is therefore rather different from that of the recent paper by Sherwin Rosen. The supply side and simultaneity problems which may arise are ignored."

Diewert (2003) has also considered the theoretical PPI indices with a focus on the producer side. He bases the optimizing problem the establishments face when deciding on which combinations of characteristics to produce, however, on the consumer's valuations, giving them precedence. There are many industries in which firms are effective price takers, and the prices taken are dictated by the consumer side rather than by cost and technological considerations. In Section B.6 this framework is outlined, which allows a more straightforward development of the theory of hedonic index numbers for PPIs.

21.31 Second, the theoretical framework allows the conditions to be considered under which the hedonic coefficients are determined by only demand side or supply side factors—the circumstances under which clear explanations would be valid. The problem is that because the coefficients of a hedonic function are the outcome of the interaction of consumer- and producer-optimizing conditions, it is not possible to interpret the function only in terms of, say, producer marginal costs or consumer marginal values. However, suppose the *production technology* τ was the same for each producing establishment. Buyers differ but sellers are identical. Then, instead of a confusing family of offer functions, there is a unique offer function with the hedonic function describing the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. The offer function becomes $p(z)$, since there is no distribution of τ to confuse it. There are different tastes on the consumer side, and so what appears in the market is the result of firms trying to satisfy consumer preferences all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. In Figure 21.1 only the expansion path traced out by, say, S_H* akin to $A A'$, would be revealed. Now, suppose sellers differ, but *buyers' tastes* α are identical. Here the family of *value functions* collapses to be revealed as the hedonic function $p(z)$, which identifies the structure of demand, such as $A A'$ in Figure 21.1.[14] Section B.6 uses Diewert's (2003) approach in following a representative consumer, rather than consumers with different tastes, so that the demand side alone can be identified. Triplett (1987, p. 632) notes that of these possibilities, uniformity of technologies is the most likely, especially when access to technology is unrestricted in the long run, while uniformity of tastes is unlikely. There may be, of course, segmented markets where tastes are more uniform to which specific sets of items are tailored and for which hedonic equations can be estimated for individual segments.[15] In some industries there may be a prior expectation of uniformity of tastes against uniformity of technologies and interpretation of coefficients will accordingly follow. In many cases, however, the interpretation may be more problematic.

21.32 Third, issues relating to the estimation of the underlying supply and demand functions for characteristics have implications for the estimation of hedonic functions. In Appendix 21.1, identification and estimation issues will be considered in this light. Finally, the subsequent concern with new products in Section D of this chapter refers to demand functions. However, attention is now turned to hedonic *indices*. In the next section, these are noted to have a quite different application than that for the quality adjustment of noncomparable replacement items.

B.6 An alternative hedonic theoretical formulation

21.33 This section is based on a formulation by Diewert (2002d). It assumes competitive price-taking behavior on the part of firms. In this approach, the user's valuations of the various models that could be produced flow to producers via the hedonic function in the same way that output prices are taken, as given in the usual theory of the output price index. It is necessary to set up the establishment's revenue maximization problem assuming that it produces a single output, but in each period, the establishment has a choice of which type of model it could produce. Let the model be identified by a K dimensional vector of character-

[14]Correspondingly, if the supply curves were perfectly inelastic, so that a change in price would not affect the supply of any of the differentiated products, then the variation in prices underlying the data and feeding the hedonic estimates would be determined by demand factors. The coefficients would provide estimates of user values. Similarly, if the supplying market were perfectly competitive, the estimates would be of resource costs. None of the price differences between differentiated items would be due to, say, novel configurations of characteristics, and no temporary monopoly profit would be achieved as a reward for this, or as a results of the exercise of market power. See Berndt (1983).

[15]Berry, Levinsohn, and Pakes (1995) provide a detailed and interesting example for automobiles in which makes are used as market segments, while Tauchen and Witte (2001) provide a systematic theoretical study of estimation issues for supply, demand, and hedonic functions where consumers and producers and their transactions are indexed across communities.

istics, $z \equiv [z_1,...,z_K]$. Before tackling the establishment's revenue maximization problem, it is necessary to characterize the set of output prices that the establishment faces in period t as a function of the characteristics of the model that the establishment might produce. It is assumed that in period t, the demanders of the output of the establishment have a cardinal utility function, $f^t(z)$, that enables each demander to determine that the value of a model with the vector of characteristics $z^1 \equiv [z_1^1,...,z_K^1]$ compared with a model with characteristics vector $z^2 \equiv [z_1^2,...,z_K^2]$ is $f^t(z^1) / f^t(z^2)$. Thus, in period t, demanders are *willing to pay* the amount of money $P^t(z)$ for a model with the vector of characteristics z, where

$$(21.6)\ \Pi^t(z) \equiv p^t \cdot f^t(z),\ \ t = 0,1.$$

The scalar ρ^t is inserted into the willingness-to-pay function because, under certain restrictions, ρ^t can be interpreted as a period t price for the entire family of hedonic models that might be produced in period t. These restrictions are

$$(21.7)\ f^0 = f^1,$$

that is, the *model relative utility functions* f^t are identical for the two periods under consideration. We will make use of the specific assumption in equation (21.7) later.

21.34 In what follows, it is assumed that econometric estimates for the period 0 and 1 *hedonic model price functions*, Π^0 and Π^1, are available, although we will also consider the case where only an estimate for Π^0 is available.[16] Now, consider an establishment that produces a single model in each period in the marketplace that is characterized by the hedonic model price functions, $\Pi^t(z)$, for periods $t = 0,1$. Suppose that in period t, the establishment has the *production function* F^t, where

$$(21.8)\ q = F^t(z,v)$$

is the number of models, each with vector of characteristics z, that can be produced if the vector of inputs v is available for use by the establishment in period t. As is usual in the economic approach to index numbers, we assume a competitive model, where each establishment takes output prices as fixed parameters beyond its control. In this case, there is an entire schedule of model prices that the establishment takes as given instead of just a single price in each period. Thus, it is assumed that if the establishment decides to produce a model with the vector of characteristics z, then it can sell any number of units of this model in period t at the price $\Pi^t(z) \equiv \rho \cdot f^t(z)$. Note that the establishment is allowed to choose which model type to produce in each period.

21.35 Now, define the establishment's *revenue function*, R, assuming the establishment is facing the period s hedonic price function $\Pi^s = \rho^s f^s$ and is using the vector of inputs v and has access to the period t production function F^t:

$$\begin{aligned}(21.9)\ & R(\rho^s f^s, F^t, Z^t, v) \\ & \equiv \max_{q,z} \{\rho^s f^s(z)q : q = F^t(z,v)\ ;\ z \text{ belongs to } Z^t\} \\ & = \max_{z} \{\rho^s f^s(z)F^t(z,v) : z \text{ belongs to } Z^t\},\end{aligned}$$

where Z^t is a *technologically feasible set of model characteristics* that can be produced in period t. The second line follows from the line above by substituting the production-function constraint into the objective function.

21.36 The actual period t revenue maximization problem that the establishment faces is defined by the revenue function in equation (21.9), except that we replace the period s hedonic price function $\rho^s f^s$ by the period t hedonic price function $\rho^t f^t$, and the generic input quantity vector v is replaced by the observed period t input quantity vector used by the establishment, v^t. Further assume that the establishment produces q^t units of a single model with characteristics vector z^t and that $[q^t,z^t]$ solves the period t revenue maximization problem—that is, $[q^t,z^t]$ is a solution to[17]

[16]We will need some identifying restrictions to identify the parameters of f^0 and f^1 along with ρ^0 and ρ^1. One common model sets $\rho^0 = 1$ and $f^0 = f^1$. A more general model sets $\rho^0 = 1$ and $f^0(z^*) = f^1(z^*)$ for a reference characteristics vector, $z^* \equiv [z_1^*,...,z_K^*]$.

[17]If the establishment is competitively optimizing with respect to its choice of inputs as well, then the period t input vector v^t, along with q^t and z^t, is a solution to the following period t profit maximization problem for the estab- (continued)

(21.10) $R(\rho^t f^t, F^t, Z^t, v^t)$
$$\equiv \max_{q,z} \{\rho^t f^t(z)q : q = F^t(z,v^t) ; z \text{ belongs to } Z^t\}; \ t = 0,1$$
$$= \rho^t f^t(z^t)q^t,$$

where the period t establishment output q^t is equal to

(21.11) $q^t = F^t(z^t,v^t) ; \ t = 0,1.$

Now, a family of *Konüs-type hedonic output price indices P* between periods 0 and 1 can be defined as follows:

(21.12) $P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v)$
$$\equiv R(\rho^1 f^1, F^t, Z^t, v) / R(\rho^0 f^0, F^t, Z^t, v).$$

21.37 Thus, a particular member of the above family of indices is equal to the establishment's revenue ratio, where the revenue in the numerator of equation (21.12) uses the hedonic model price function for period 1, and the revenue in the denominator of equation (21.12) uses the hedonic model price function for period 0. For both revenues, however, the technology of period t is used (that is, F^t and Z^t are used in both revenue maximization problems), and the same input quantity vector v is used. This is the usual definition for an economic output price index, except that instead of a single price facing the producer in each period, we have a whole family of model prices facing the establishment in each period. Note that the only variables that are different in the numerator and denominator of equation (21.12) are the two hedonic model price functions facing the establishment in periods 0 and 1.

21.38 The right-hand side of equation (21.12) looks a bit complex. However, if the assumption in equation (21.7) holds (that is, the period 0 and 1 hedonic model price functions are identical except for the multiplicative scalars ρ^0 and ρ^1), then equation (21.12) reduces to the very simple ratio, ρ^1 / ρ^0. To see this, use equations (21.12) and (21.10) as follows:

lishment: $\max_{q,z,v} \{\rho^t f^t(z)q - w^t \bullet v : q = F^t(z,v) ; z \text{ belongs to } Z^t\}$, where w^t is a vector of input prices that the establishment faces in period t and $w^t \bullet v$ denotes the inner product of the vectors w^t and v. It is possible to rework our analysis presented below, conditioning on an input price vector rather than on an input quantity vector.

(21.13) $P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v)$
$$\equiv R(\rho^1 f^1, F^t, Z^t, v) / R(\rho^0 f^0, F^t, Z^t, v)$$

$$= \max_z \{\rho^1 f^1(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\} / \max_z \{\rho^0 f^0(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\}$$

$$= \max_z \{\rho^1 f^0(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\} / \max_z \{\rho^0 f^0(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\}$$

using equation (21.7)

$$= [\rho^1 / \rho^0]\max_z \{\rho^0 f^0(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\} / \max_z \{\rho^0 f^0(z)F^t(z,v^t) ; z \text{ belongs to } Z^t\}$$

assuming ρ^0 and ρ^1 are positive and canceling terms

$$= \rho^1 / \rho^0.$$

This is a very useful result, since many hedonic regression models have been successfully estimated using equation (21.7). Under this assumption, *all* the theoretical hedonic establishment output price indices reduce to the observable ratio, ρ^1 / ρ^0.

21.39 We return to the general case where the assumption in equation (21.7) is not made. As usual, it is always of interest to specialize equation (21.12) to the special cases where the conditioning variables that are held constant in the numerator and denominator of equation (21.12), F^t, Z^t, and v, are equal to the period 0 and 1 values for these variables, namely, F^0, Z^0, and v^0, and F^1, Z^1, and v^1. Thus, define the *Laspeyres-type hedonic output price index* between periods 0 and 1 for our establishment as follows:

(21.14) $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$
$$\equiv R(\rho^1 f^1, F^0, Z^0, v^0) / R(\rho^0 f^0, F^0, Z^0, v^0)$$
$$= R(\rho^1 f^1, F^0, Z^0, v^0) / \rho^0 f^0(z^0)q^0,$$

using equation (21.10) for $t = 0$

$$= \max_z \{\rho^1 f^1(z)F^0(z,v^0) ; z \text{ belongs to } Z^0\} / \rho^0 f^0(z^0)q^0$$

using equation (21.9)

$$\geq \rho^1 f^1(z^0)F^0(z^0,v^0) / \rho^0 f^0(z^0)q^0$$

since z^0 is feasible for the maximization problem

$= \rho^1 f^1(z^0)q^0 / \rho^0 f^0(z^0)q^0$

using equation (21.11) for $t = 0$

$= \rho^1 f^1(z^0)/\rho^0 f^0(z^0)$
$\equiv P_{HL}$.

where the *observable hedonic Laspeyres output price index* P_{HL} is defined as

(21.15) $P_{HL} \equiv \rho^1 f^1(z^0) / \rho^0 f^0(z^0)$.

Thus, the inequality in equation (21.14) says that the unobservable theoretical Laspeyres-type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ is bounded from below by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0, and f^1) hedonic Laspeyres output price index P_{HL}. The inequality in equation (21.14) is the hedonic counterpart to a standard Laspeyres-type inequality for a theoretical output price index.

21.40 It is of modest interest to rewrite P_{HL} in terms of the observable model prices for the establishment in periods 0 and 1. Denote these prices by P^0 and P^1, respectively. Using equation (21.6),

(21.16) $P^0 = \rho^0 f^0(z^0)$ and $P^1 = \rho^1 f^1(z^1)$.

Now, rewriting equation (21.15) as follows,

(21.17) $P_{HL} \equiv \rho^1 f^1(z^0) / \rho^0 f^0(z^0)$
$= \rho^1 f^1(z^1)[f^1(z^0) / f^1(z^1)] / \rho^0 f^0(z^0)$
$= P^1[f^1(z^0)/f^1(z^1)] / P^0$

using equation (21.16)

$= [P^1/f^1(z^1)] / [P^0/f^1(z^0)]$.

The prices $P^1 / f^1(z^1)$ and $P^0 / f^1(z^0)$ can be interpreted as *quality-adjusted model prices* for the establishment in periods 1 and 0, respectively, using the hedonic regression pertaining to period 1 to do the quality adjustment.

21.41 In the theoretical hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ defined by equation (21.14) above, we conditioned on F^0 (the base-period production function), Z^0 (the base-period set of models that were technologically feasible in period 0), and v^0 (the establishment's base-period input vector). We now define a companion period 1 theoretical hedonic output price that conditions on the period 1 variables, F^1, Z^1, v^1. Thus, define the *Paasche-type hedonic output price index* between periods 0 and 1 for an establishment as follows:[18]

(21.18) $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$
$\equiv R(\rho^1 f^1, F^1, Z^1, v^1) / R(\rho^0 f^0, F^1, Z^1, v^1)$
$= \rho^1 f^1(z^1)q^1 / R(\rho^0 f^0, F^1, Z^1, v^1)$

using equation (21.10) for $t = 1$

$= \rho^1 f^1(z^1)q^1$
$/ \max_z \{\rho^0 f^0(z)F^1(z,v^1);\ z \text{ belongs to } Z^1\}$

using equation (21.9)
$\leq \rho^1 f^1(z^1)q^1 / \rho^0 f^0(z^1)F^1(z^1,v^1)$

since z^1 is feasible for the maximization problem

$= \rho^1 f^1(z^1)q^1 / \rho^0 f^0(z^1)q^1$

using equation (21.11) for $t = 1$

$= \rho^1 f^1(z^1) / \rho^0 f^0(z^1)$
$\equiv P_{HP}$,

where the *observable hedonic Paasche output price index* P_{HP} is defined as

(21.19) $P_{HP} \equiv \rho^1 f^1(z^1) / \rho^0 f^0(z^1)$.

Thus, the inequality in equation (21.18) says that the unobservable theoretical Paasche-type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$ is bounded from above by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0, and f^1) hedonic Paasche output price index P_{HP}. The inequality in equation (21.18) is the hedonic counterpart to a standard Paasche-type inequality for a theoretical output price index.

21.42 Again, it is of interest to rewrite P_{HP} in terms of the observable model prices for the establishment in periods 0 and 1. Rewrite equation (21.19) as follows:

(21.20) $P_{HP} \equiv \rho^1 f^1(z^1) / \rho^0 f^0(z^1)$
$= \rho^1 f^1(z^1) / \{\rho^0 f^0(z^0)[f^0(z^1) / f^0(z^0)]\}$

[18] Assume that all ρ^t, $f^t(z)$, and $F^t(z,v^t)$ are positive for $t = 0,1$.

$$= P^1/\{P^0[f^0(z^1)/f^0(z^0)]\}$$

using equation (21.16)

$$= [P^1/f^0(z^1)]/[P^0/f^0(z^0)].$$

The prices $P^1 / f^0(z^1)$ and $P^0 / f^0(z^0)$ can be interpreted as *quality-adjusted model prices* for the establishment in periods 1 and 0, respectively, using the hedonic regression pertaining to period 0 to do the quality adjustment.

21.43 It is possible to adapt a technique originally credited to Konüs (1924) and obtain a theoretical hedonic output price index that lies between the observable Laspeyres and Paasche bounding indices, P_{HL} and P_{HP}, defined above. Recall the definition of the revenue function, $R(\rho^s f^s, F^t, Z^t, v)$, defined by equation (21.9) above. Instead of using either F^0, Z^0, v^0 or F^1, Z^1, v^1 as reference production functions, feasible characteristics sets, and input vectors for the establishment in equation (21.12), use a *convex combination* or *weighted average* of these variables in our definition of a theoretical hedonic output price index. Thus, for each scalar λ between 0 and 1, define the theoretical hedonic output price index between periods 0 and 1, $P(\lambda)$, as follows:

$$(21.21)\ P(\lambda) \equiv R(\rho^1 f^1, (1-\lambda)F^0 + \lambda F^1, (1-\lambda)Z^0 + \lambda Z^1, (1-\lambda)v^0 + \lambda v^1) / R(\rho^0 f^0, (1-\lambda)F^0 + \lambda F^1, (1-\lambda)Z^0 + \lambda Z^1, (1-\lambda)v^0 + \lambda v^1)$$

$$= \max_z\{\rho^1 f^1(z)[(1-\lambda)F^0(z,(1-\lambda)v^0 + \lambda v^1) + \lambda F^1(z,(1-\lambda)v^0 + \lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0 + \lambda Z^1\} / \max_z\{\rho^0 f^0(z)[(1-\lambda)F^0(z,(1-\lambda)v^0 + \lambda v^1) + \lambda F^1(z,(1-\lambda)v^0 + \lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0 + \lambda Z^1\}.$$

When $\lambda = 0$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$, the Laspeyres-type hedonic output price index defined by equation (21.14) above. Thus, using the inequality in equation (21.14), we have

$$(21.22)\ P(0) \geq P_{HL},$$

where P_{HL} is equal to $\rho^1 f^1(z^0)/\rho^0 f^0(z^0)$, the observable Laspeyres hedonic output price index defined by equation (21.15) above. When $\lambda = 1$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$, the Paasche-type hedonic output price index defined by equation (21.18) above. Thus, using the inequality in equation (21.18), we have

$$(21.23)\ P(1) \leq P_{HP} P_{HL},$$

where P_{HP} is equal to $\rho^1 f^1(z^1) / \rho^0 f^0(z^1)$, the observable Paasche hedonic output price index defined by equation (21.20) above.

21.44 If $P(\lambda)$ is a continuous function of λ between 0 and 1, then we can adapt the proof of Diewert (1983a, pp. 1060–61), which in turn is based on a technique of proof by Konüs (1924), and show that there exists a λ^* such that $0 \leq \lambda^* \leq 1$, and either

$$(21.24)\ P_{HL} \leq P(\lambda^*) \leq P_{HP} \text{ or } P_{HP} \leq P(\lambda^*) \leq P_{HL},$$

that is, there exists a theoretical hedonic output price index between periods 0 and 1 using a technology that is intermediate to the technology of the establishment between periods 0 and 1, $P(\lambda^*)$ that lies *between* the observable[19] Laspeyres and Paasche hedonic output price indices, P_{HL} and P_{HP}. However, to obtain this result, we need conditions on the hedonic model price functions, $\rho^0 f^0(z)$ and $\rho^1 f^1(z)$, on the production functions, $F^0(z,v)$ and $F^1(z,v)$, and on the feasible characteristics sets, Z^0 and Z^1, that will ensure that the maximum functions in the numerator and denominator in the last equality of equation (21.21) are continuous in λ. Sufficient conditions to guarantee continuity are[20]

- The production functions $F^0(z,v)$ and $F^1(z,v)$ are positive and jointly continuous in z,v,
- The hedonic model price functions $f^0(z)$ and $f^1(z)$ are positive and continuous in z,
- ρ^0 and ρ^1 are positive, and
- The sets of feasible characteristics Z^0 and Z^1 are convex, closed, and bounded.

21.45 A theoretical output price index has been defined that is bounded by two observable indices. It is natural to take a symmetric mean of the

[19]We need estimates of the hedonic model price functions for both periods to implement these "observable" indices.

[20]The result follows using Debreu's (1952, pp. 889–90; 1959, p. 19) Maximum Theorem.

bounds to obtain a best single number that will approximate the theoretical index. Thus, let $m(a,b)$ be a symmetric homogeneous mean of the two positive numbers a and b. We want to find a best $m(P_{HL},P_{HP})$. If we want the resulting index, $m(P_{HL},P_{HP})$, to satisfy the time reversal test, then we can adapt the argument of Diewert (1997, p. 138) and show that the resulting $m(a,b)$ must be the geometric mean, $a^{1/2}b^{1/2}$. Thus, a good candidate to best approximate a theoretical hedonic output price index is the following observable *Fisher hedonic output price index*:

$$(21.25)\ P_{HF} \equiv [P_{HL}P_{HP}]^{1/2} = [\rho^1 f^1(z^0) / \rho^0 f^0(z^0)]^{1/2}[\rho^1 f^1(z^1) / \rho^0 f^0(z^1)]^{1/2}$$

using equations (21.15) and (21.21)

$$= [\rho^1 / \rho^0][f^1(z^0) / f^0(z^0)]^{1/2}[f^1(z^1)/f^0(z^1)]^{1/2} .$$

Note that P_{HF} reduces to ρ^1 / ρ^0 if $f^0 = f^1$; that is, if the hedonic model price functions are identical for each of the two periods under consideration, except for the proportional factors, $\rho^1$and ρ^0.

21.46 Instead of using equations (21.15) and (21.17) in the first line of equation (21.7), equations (21.17) and (21.20) can be used. The resulting formula for the Fisher hedonic output price index is

$$(21.26)\ P_{HF} \equiv [P_{HL}P_{HP}]^{1/2} = \{[P^1/f^1(z^1)] / [P^0/f^1(z^0)]\}^{1/2} \times \{[P^1/f^0(z^1)] / [P^0/f^0(z^0)]\}^{1/2}.$$

Equation (21.26) is preferred. It is the geometric mean of two sets of quality-adjusted model price ratios, using the hedonic regression in each of the two periods to do one of the quality adjustments.

21.47 The above theory, for the quality adjustment of establishment output prices, is not perfect. It has two weak parts:

- Using a convex combination of the two reference period technologies may not appeal to everyone, and
- Our technique for converting the bounds to a single number is only one method out of many.

21.48 The initial Laspeyres-type bounds and Paasche-type bounds formalizes the bounds outlined in Section C.5 below and referred to in Section C.2. The quality adjustments in equations (21.13) and (21.14) will be seen from this approach to be made using the user's model valuation functions, $f^0(z)$ and $f^1(z)$. Producers' costs or production functions enter into the quality adjustment only to determine z^0 and z^1; that is, only to determine which models the establishment will produce. Hence, establishments that have different technologies, primary inputs, or face different input prices will in general choose to produce different models in the same period. The choice problem has been modeled here only facing a single establishment, although the generalization should be straightforward.

B.7 Markups and imperfect competition

21.49 In Section B.5 it was shown there was some ambiguity in the interpretation of hedonic coefficients. A user-value or resource-cost interpretation was possible if there was uniformity in buyers' tastes or suppliers' technologies, respectively. In Section B.6 an assumption of price-taking behavior on the part of firms was introduced and a formal setting given to a user-value interpretation, albeit involving some restrictive assumptions. Yet the approaches in Sections B.5 and B.6 both assume perfectly competitive behavior, and the discussion extends now to the effects of markups in imperfect competition. Feenstra (1995) notes that in imperfect competition, when pricing is above marginal cost, the hedonic function should include a term for the price-cost markup.

21.50 Pakes (2001) has developed the argument focusing on the study of new products as the result of prior investments in product development and marketing. A competitive marginal cost-pricing assumption would require that either (i) products with identical characteristics are developed from such investments, so that the law of one price for these identical products will eliminate any margin, or (ii) all products lose their investment (markup) in the new products. Neither of these is reasonable. Indeed, varying markups are a feature of differentiated products (see Feenstra and Levinsohn, 1995, for example). Pakes (2001) argued that markups should change over time. When new products are introduced, the improvements, and associated markups, are directed to characteristics where markups have previously been high. The markups on existing products with these characteristics will fall, and hedonic coefficients will thus change over time. Pakes (2001) also argued that there may be an ambiguity as to the signs of the coefficients—that there is no economic reason to expect a positive relationship between price and a desirable characteristic. Such a conclusion would be at odds with a resource-cost or user-value approach. If the characteristics being compared are *vertical*—that is, they are characteristics of which everyone would like more—then we can expect the sign to be positive. However, Pakes (2001) has argued that the sign on *horizontal* characteristics—that is, for which the ordering of the desirable amounts of characteristics is not the same for all consumers—can be negative. The entry of new products aimed at some segments of the market may drive down the markup on products with more desirable attributes. For example, some consumers may have a preference for television sets with smaller screen sizes and be willing to pay a premium price. Indeed, the required technology for the production of these sets may have required increased investment and, thus, increased expected markups. It may be that the quality of the picture on these sets is such that it drives down the price of large-sized sets, resulting in an inverse relationship between price and screen size, where the latter is taken as one variable over the full range of screen sizes. Prior (to the modeling) information on the two markets would allow the regression equation to be appropriately specified, with dummy slope and intercepts for the ranges of screen sizes with new and old technologies.

21.51 Pakes (2001) takes the view that no meaning can be attributed to estimated coefficients and predicted values should be used for price comparisons of models of different quality attributes, rather than the individual coefficients. There are many good reasons for this, as discussed in Chapter 7, Section E.4.3 and Section G.2.2, and Appendix 21.1. Yet, it must be stressed that for vertical characteristics the coefficients may be quite meaningful, and even for horizontal characteristics or new characteristics, embodied with the latest research and development, some sense can be made by recourse to the above considerations. But again, theory does not support any easy answer to the interpretation of the coefficients from hedonic regressions. Their grace is that they emanate from market data, from the often complex interaction of demand and supply and strategic pricing decisions. That theory warns us not to give simplistic interpretations to such coefficients, and allows an understanding of the factors underlying them, is a strength of theory. Yet hedonic regression coefficients remain and are generally regarded (Shultze and Mackie, 2002) as the most promising objective basis for estimating the marginal value of quality dimensions of products, even though a purist interpretation is beyond their capability.[21]

C. Hedonic Indices

C.1 The need for such indices

21.52 In Section A it was noted that hedonic functions are required for two purposes with regard to a quality adjustment. The first is when an item is no longer produced and the replacement item, whose price is used to continue the series, is of a different quality from the original price basis. The differences in quality can be established in terms of different values of a subset of the z price-determining variables. The coefficients from the hedonic regressions, as estimates of the monetary value of additional units of each quality component z, can then be used to adjust the price of the old item so that it is comparable with the price of the new[22]—so that, again, like is compared with like. This process could be described as "patching," in that an adjustment is needed to the price of the old (or new replacement) series for the quality differences, to enable the new series to be patched onto the old. A second use of hedonic functions referred to in Section A is for estimating *hedonic indices.* These are suitable when the pace and scale of replacements of items is substantial and an extensive use of patching might (i) lead to extensive errors if there were some error or bias in the quality adjustment process and (ii) lead to sampling from a biased replacement universe as outlined in Section A. Hedonic indices use data in each period

[21]Diewert (2002f) goes further in suggesting positive sign restrictions are imposed on the coefficients in the econometric estimation.

[22]Mechanisms for such adjustments are varied, as outlined in Chapter 7, Section E.4.3, and Triplett (2002). They include using the coefficients from the salient set of characteristics or using the predicted values from the regression as a whole and, in either case, making the adjustment to the old for comparison with the new, or to the new for comparison with the old, or some effective average of the two.

from a sample of items that should include those with a substantial share of sales revenue—sampling in each period from the double universe. There is no need to establish a price basis and for respondents to keep quoting prices from that basis. What is required are samples of items to be re-drawn in each month along with information on their prices, characteristics z_i, and, possibly, quantities or values. The identification of multiple characteristics in the hedonic regressions controls for quality differences, as opposed to the matching of price quotes on the same price basis by the respondents. A number of procedures for estimating hedonic indices are briefly considered below.

C.2 Theoretical characteristics of price indices

21.53 In Chapter 17 theoretical output price indices were defined and practical index number formulas considered as estimates of these indices. Theoretical output index numbers are defined here not just on the goods produced, but also on their characteristics. $R(p,S(v))$ was defined in Chapter 17 as the maximum value of output that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v (using technology S) is available for use. The establishment's *output price index P* between any two periods, say, period 0 and period 1, was defined as

$$(21.27)\ P(p^0,p^1,v) = R(p^1, S(v)) / R(p^0, S(v))\ ,$$

where p^0 and p^1 are the vectors of output prices that the establishment faces in periods 0 and 1, respectively, and $S(v)$ is a reference vector of technology using v intermediate and primary inputs.[23] For theoretical indices in characteristic space, the revenue functions are *also* defined over goods made up of bundles of characteristics represented by the hedonic function[24]

$$(21.28)\ P(p^0,p^1,v, z^0, z^1) = \frac{R(p_1,\ p(z_1),\ S(v))}{R(p_0,\ p(z_0),\ S(v))}\ .$$

21.54 The output price index defined by equation (21.28) is a ratio of hypothetical revenues that the establishment could realize, with a given technology and vector of inputs v to work with. Equation (21.28) incorporates substitution effects: if the prices of some characteristics increase more than others, then the revenue-maximizing establishment can switch its output mix of characteristics in favor of such characteristics. The numerator in equation (21.28) is the maximum revenue that the establishment could attain if it faced the output prices and implicit hedonic shadow prices of period 1, p^1 and $p(z^1)$, while the denominator in equation (21.28) is the maximum revenue that the establishment could attain if it faced the output and characteristic's prices of period 0, p^0 and $p(z^0)$. Note that all the variables in the numerator and denominator functions are exactly the same, except that the output price and characteristics price vectors differ. This is a defining characteristic of an output price index: the technology and inputs are held constant. As with the economic indices in Chapter 15, there is an entire *family* of indices depending on which reference technology and reference input vector v that is chosen. In Section C.5 some explicit formulations will be considered, including a base-period 0 reference technology and inputs and a current-period 1 reference technology and inputs analogous to the derivation of Laspeyres and Paasche in Chapter 17, Section B.1. Before considering such hedonic indices in Section C.5, two simpler formulations are first considered in Sections C.3 and C.4: hedonic regressions using dummy variables on time and period-on-period hedonic indices. They are simpler and widely used because they require no information on quantities or weights. Yet, their interpretation from economic

[23]This concept of the output price index (or a closely related variant) was defined by F.M. Fisher and Shell (1972, pp. 56–58), Samuelson and Swamy (1974, pp. 588–92), Archibald (1977, pp. 60–61), Diewert (1980, pp. 460–61; 1983a, p. 1055), and Balk (1998b, pp. 83–89). Readers who are familiar with the theory of the true cost-of-living index will note that the output price index defined by equation (17.2) is analogous to the true cost-of-living index, which is a ratio of cost functions, say, $C(u,p1)$ / $C(u,p0)$, where u is a reference utility level: R replaces C, and the reference utility level u is replaced by the vector of reference variables $S(v)$. For references to the theory of the true cost-of-living index, see Konüs (1924), Pollak (1983a), or ILO and others (2004), which is the CPI counterpart to this *Manual*.

[24]Triplett (1987) and Diewert (2002d), following Pollak (1975), consider a two-stage budgeting process whereby that portion of utility concerned with items defined as characteristics has its theoretical index defined in terms of a cost-minimizing selection of characteristics, conditioned on an optimum output level for composite and hedonic commodities. These quantities are then fed back into the second-stage overall revenue maximization.

theory is therefore more limited. However, as will be shown, weighted formulations are possible using a WLS estimator, although they are first considered in their unweighted form.

C.3 Hedonic regressions and dummy variables on time

21.55 Let there be K characteristics of a product, and let model or item i of the product in period t have the vector of characteristics $z_i^t \equiv [z_{i1}^t,...,z_{iK}^t]$ for i = 1,....,K and t = 1,...,T. Denote the price of model i in period t by p_i^t. A hedonic regression of the price of model i in period t on its characteristics set z_i^t is given by

$$(21.29)\quad \ln p_i^t = \gamma_0 + \sum_{t=2}^{T} \gamma_t D_t + \sum_{k=1}^{K} \beta_k z_{ik}^t + \varepsilon_i^t ,$$

where D_t are dummy variables for the time periods, D_2 being 1 in period t = 2, zero otherwise; D_3 being 1 in period t = 3, zero otherwise, and so on. The coefficients γ_t are estimates of quality-adjusted price changes, having controlled for the effects of variation in quality (via $\sum_{k=1}^{K} \gamma_k z_{tki}$)—although see Goldberger (1968) and Teekens and Koerts (1972) for the adjustment for estimation bias.

21.56 The above approach uses the dummy variables on time to compare prices in period 1 with prices in each subsequent period. In doing so, the γ parameters are constrained to be constant over the period t = 1,…,T. Such an approach is fine retrospectively, but in real time the index may be estimated as a fixed-base or chained-base formulation. The *fixed-base* formulation would estimate the index for period 1 and 2, $I_{1,2}$, using equation (21.29) for t = 1, 2; the index for period 3, $I_{1,3}$, would use equation (21.29) for t = 1, 3; for period 4, $I_{1,4}$, using equation (21.29) for t = 1, 4; and so forth. In each case the index constrains the parameters to be the same over the current and base period. A fixed-base, bilateral comparison using equation (21.29) makes use of the constrained parameter estimates over the two periods of the price comparison. A *chained* formulation would estimate $I_{1,4}$, for example, as the product of a series of links: $I_{1,4} = I_{1,2} \times I_{2,3} \times I_{3,4}$.[25] Each successive binary comparison or link is combined by successive multiplication. The index for each link is estimated using equation (21.24). Because the periods of time being compared are close, it is generally more likely that the constraining of parameters required by chained-time dummy hedonic indices is considered to be less severe than that required of their fixed-base counterparts.

21.57 There is no explicit weighting in these formulations, and this is a serious disadvantage. In practice, cutoff sampling might be employed to include only the most important items. If sales data are available, a WLS (weighted by sales quantities—see Appendix 21.1) estimator instead of an OLS estimator should be used.[26]

C.4 Period-on-period hedonic indices

21.58 An alternative approach to comparing period 1 and t is to estimate a hedonic regression for period t and insert the values of the characteristics of each model existing in period 1 into the period t regression to predict, for each item, its price, $\hat{p}_i^t(z_i^1)$. This would generate predictions of the price of items existing in period 1*,* at period t shadow prices, $\hat{p}_i^t\left(z_i^1\right)$, i = 1,...,N . These prices (or an average) can be compared with (the average of) the actual prices of models i = 1,…..N models in period 1. The averages may be arithmetic, as in a Dutot index, or geometric, as in a Jevons index. The arithmetic formulation is defined as follows:

$$(21.30a)\quad \frac{\sum_{i=1}^{N} (1/N)\hat{p}_i^t(z_i^1)}{\sum_{i=1}^{N} (1/N)p_i^1(z_i^1)} .$$

21.59 Alternatively, the characteristics of models existing in period t can be inserted into a regression for period 1. Predicted prices of period t items generated at period 1 shadow prices (or an

[25] Chapter 15, Section F contains a detailed account of chained indices.

[26] Ioannidis and Silver (1999) and Bode and van Dalen (2001) compared the results from these different estimators, finding notable differences, but not in all cases (see also Silver and Heravi, 2002).

average) can be compared with (the average of) the actual prices in period t:

$$(21.30b)\quad \frac{\sum_{i=1}^{N}(1/N)p_i^t(z_i^{\,t})}{\sum_{i=1}^{N}(1/N)\hat{p}_i^{\,1}(z_i^{\,t})}.$$

21.60 For a fixed-base bilateral comparison using either equation (21.30a) or (21.30b), the hedonic equation need be estimated only for one period. The denominator in equation (21.30a) is the average observed price in period 1, which should be equal to the average price a hedonic regression based on period 1 data will predict using period 1 characteristics. The numerator, however, requires an estimated hedonic regression to predict period 1 characteristics at period t hedonic prices. Similarly, in equation (21.30b), a hedonic regression is required only for the denominator. For reasons analogous to those explained in Chapters 15, 16, and 17, a symmetric average of these indices should have some theoretical support.

21.61 Note that all the indices described in Sections C.1 and C.2 use all the data available in each period. If there is a new item, for example, in period 4, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old items drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure, unlike using matched data and hedonic adjustments on noncomparable replacements when items are no longer produced.

21.62 As with the dummy variable approach, there is no need for matched data. Yet there is also no explicit weighting in these formulations and this is a serious disadvantage. Were data on quantities or values available, it is immediately apparent that such weights could be attached to the individual $i = 1,\ldots N$ prices or their estimates. This is considered in the next section.

C.5 Superlative and exact hedonic indices

21.63 In Chapter 17 Laspeyres and Paasche bounds were defined on a theoretical basis, as were superlative indices, which treat both periods symmetrically. These superlative formulas, in particular the Fisher index, were also seen in Chapter 16 to have desirable axiomatic properties. Furthermore, the Fisher index was supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds and was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index seemed to be best from the stochastic viewpoint and also did not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices were found to correspond to (be *exact* for) underlying (Leontief) aggregator functions with no substitution possibilities, while superlative indices were exact for flexible functional forms including the quadratic and translog forms for the Fisher and Törnqvist indices, respectively. If data on prices, characteristics, *and quantities* are available, analogous approaches and findings arise for hedonic indices (see Fixler and Zieschang, 1992a, and Feenstra, 1995). Exact bounds on such an index were defined by Feenstra (1995). Consider the theoretical index in equation (21.28), but now defined only over items in terms of their characteristics. The prices are still of items, but they are wholly defined through $p(z)$. An arithmetic aggregation for a linear hedonic equation finds a Laspeyres lower bound (as quantities supplied are *increased* with increasing relative prices) is given by

$$(21.31a)\quad \frac{R(p(z)_t,\, S(v)_{t-1})}{R(p(z)_{t-1},\, S(v)_{t-1})} \geq \frac{\sum_{i=1}^{N} x_{it-1}\hat{p}_{it}}{\sum_{i=1}^{N} x_{it-1}p_{it-1}} = \sum_{i=1}^{N} s_{i,t-1}\left(\frac{\hat{p}_{it}}{p_{it-1}}\right),$$

where $R(.)$ denotes the revenue at a set of output prices, input quantities, v, and technology, S, following the fixed-input output price index model. The price comparison is evaluated at a fixed level of period $t-1$ technology and inputs. s_{it-1} are the shares in total value of output of product i in period $t-1$, where $s_{it\text{-}1} = x_{it-1}p_{it-1}\Big/\sum_{i=1}^{N} x_{it-1}p_{it-1}$, and

$$(21.31b)\quad \hat{p}_{it} \equiv p_{it} - \sum_{i=1}^{N}\beta_{kt}(z_{ikt} - z_{ikt-1})$$

are prices in periods t adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. As noted in Appendix 21.1, β_{kt} may be estimated using a WLS estimator where the weights are the sales quantities. The summation is over the same *i* in both periods, since replacements are included when items are missing and equation (21.31b) adjusts their prices for quality differences.

21.64 A Paasche upper bound is estimated as

(21.32a)

$$\frac{R(p_t,\, S(v)_t)}{R(p_{t-1},\, S(v)_t)} \le \frac{\sum_{i=1}^{N} x_{it}\hat{p}_{it}}{\sum_{i=1}^{N} x_{it}p_{it-1}} = \left[\sum_{i=1}^{N} s_{it}'\left(\frac{\hat{p}_{it}}{p_{it-1}}\right)\right]^{-1}$$

where $s_{it}' = x_{it}\hat{p}_{it} \Big/ \sum_{i=1}^{N} x_{it}\hat{p}_{it}$, and

(21.32b) $\hat{p}_{it-1} \equiv p_{it-1} + \sum_{i=1}^{N} \beta_{kt-1}(z_{ikt} - z_{ikt-1})$,

which are prices in periods $t - 1$ adjusted for the sum of the changes in each quality characteristic weighted by its respective coefficients derived from a linear hedonic regression.

21.65 Following from the inequalities in Chapter 17 where the Laspeyres P_L and Paasche P_P form bounds on their true, economic theoretic indexes,

(21.33) $P_L \le P(p^0,p^1,\alpha) \le P_P$
or $P_L \le P(p^0,p^1,\alpha) \ge P_P$.

21.66 The SEHI approach thus first applies the coefficients from hedonic regressions to changes in the characteristics to adjust observed prices for quality changes (equations 21.31b and 21.32b). Second, it incorporates a weighting system using data on the value of output of each model and its characteristics, rather than treating each model as equally important (equations 21.31a and 21.32a). Finally, it has a direct correspondence to formulation defined from economic theory.

21.67 Semilogarithmic hedonic regressions would supply a set of β coefficients suitable for use with these base-period and current-period geometric bounds:

(21.34a) $$\prod_{i=1}^{N}\left(\frac{p_{it}}{\hat{p}_{it-1}}\right)^{s_{it}} \ge \frac{R(p(z)_t,\, q,\, T)}{R(p(z)_{t-1},\, q,\, T)} \ge \prod_{i=1}^{N}\left(\frac{\hat{p}_{it}}{p_{it-1}}\right)^{s_{it-1}}$$

(21.34b) $$\hat{p}_{it-1} \equiv p_{it-1}\exp[\sum_{i=1}^{N}\beta_{k\,t-1}(z_{ikt} - z_{ikt-1})]$$

$$\hat{p}_{it} \equiv p_{it}\exp[-\sum_{i=1}^{N}\beta_{k\,t}(z_{ikt} - z_{ikt-1})].$$

21.68 In equation (21.34a) the two bounds on their respective theoretical indices have been shown to be brought together. The calculation of such indices is no small task. For examples of its application, see Silver and Heravi (2002; 2003) and Chapter 7, Section G.2, for comparisons over time, and Kokoski, Moulton, and Zieschang (1999) for price comparisons across areas of a country.

21.69 The above has illustrated how weighted index number formulas might be constructed using data on prices, quantities, and characteristics for an item when the data are not matched. But what of unweighted indices, which was the concern of the initial section of this chapter? What correspondence do the unweighted hedonic indices outlined in Sections C.3 and C.4 above have to the unweighted index number formulas outlined at the start of this chapter?

C.6 Unweighted hedonic indices and unweighted index number formulas

21.70 Triplett (2002) argues and Diewert (2003) shows formally that an unweighted geometric mean Jevons index for matched data gives the same result as a logarithmic hedonic index run on the same data. There is simply no point in estimating hedonic indices using *matched* data. Those involved in the matching have worked to ensure that no quality adjustment is necessary. An index from a dummy variable hedonic regression such as equation (21.29), but in log-log form, for matched models can be shown (Aizcorbe, Corrado, and Doms, 2001) to equal

$$(21.35)\ \ln p_t / p_{t-1} = \sum_{m \in M_t} (\ln p_{mt} - Z_m)/M_t - \sum_{m \in M_{t-1}} (\ln p_{mt-1} - Z_m)/M_{t-1},$$

where m is the matched sample and Z_t and Z_{t-1} are in principle the quality adjustments to the dummy variables for time in equation (21.29), that is, $\sum_{k=1}^{K} \gamma_k z_{tk}$. Equation (21.35) is simply the difference between two geometric means of quality-adjusted prices. The sample space $m = M_t = M_{t-1}$ is the same model in each period. Consider the introduction of a new model n introduced in period t with no counterpart in $t-1$ and the demise of an old model o so it has no counterpart in t. So in period t, M_t is composed of the period t matched items m and the new items n, and in period $t-1$, M_{t-1} is composed of the period $t-1$ matched items m and the old items. Silver and Heravi (2002) have shown the dummy variable hedonic comparison to now be

$$
\begin{aligned}
(21.36)\ \ln p_t/p_{t-1} &= [m/(m+n) \sum_m (\ln p_{mt} - Z_m)/m \\
&\quad + n/(m+n) \sum_n (\ln p_{nt} - Z_n)/n] \\
&\quad - [m/(m+o) \sum_m (\ln p_{mt-1} - Z_m)/m \\
&\quad + o/(m+o) \sum_o (\ln p_{ot-1} - Z_o)/o] \\
&= [m/(m+n) \sum_m (\ln p_{mt} - Z_m)/m \\
&\quad - m/(m+o) \sum_m (\ln p_{mt-1} - Z_m)/m] \\
&\quad + [n/(m+n) \sum_n (\ln p_{nt} - Z_n)/n \\
&\quad - o/(m+o) \sum_o (\ln p_{ot-1} - Z_o)/o].
\end{aligned}
$$

21.71 Consider the second expression in equation (21.36). First there is the change for m matched observations. This is the change in mean prices of matched models m in period t and $t-1$ adjusted for quality. Note that the weight in period t for this matched component is the proportion of matched observations to all observations in period t. And, similarly, for period $t-1$, the matched weight depends on how many unmatched old observations are in the sample. In the last line of equation (21.36), the change is between the unmatched new and the unmatched old mean (quality-adjusted) prices in periods t and $t-1$. Thus, matched methods can be seen to ignore the last line in equation (21.36) and will thus differ from the hedonic dummy variable approach. The hedonic dummy variable approach in its inclusion of unmatched old and new observations can be seen from equation (21.36) possibly to differ from a geometric mean of matched prices changes. The extent of any difference depends, in this unweighted formulation, on the proportions of old and new items leaving and entering the sample and on the price changes of old and new items relative to those of matched items. If the market for commodities is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes (see Silver and Heravi, 2002, and Berndt, Ling, and Kyle, 2003, for examples). Different market behavior will lead to different forms of bias. There is a second way in which the results will differ. Index number formulas provide weights for the price changes. The Carli index, for example, weights each observation equally, while the Dutot index weights each observation according to its relative price in the base period. The Jevons index, with no assumptions as to economic behavior, weights each observation equally. Silver (2002) has argued, however, that the weight given to each observation in an ordinary least-squares regression also depends on the characteristics of the observations, some observations with unusual characteristics having more leverage. In this way, the results from the two approaches may differ even more.

D. New Goods and Services

21.72 This section briefly highlights issues relating to the incorporation of new goods into the index. Practical issues were outlined in Chapter 8, Section D.3. The term *new goods* will be used here to refer to those that provide a substantial and substantive change in what is provided, as opposed to more of a currently available set of service flows, such as a new model of an automobile that has a bigger engine. In this latter instance, there is a continuation of a service and production flow, and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of new goods as against quality changes is that the former cannot be easily linked to existing items as a continuation of an existing resource base and service flow, because of the very nature of their "newness." There are alternative definitions; Oi (1997) directs the problem of defining new goods to that of defining a monopoly. If there is no close substitute, the good is new. A monopoly supplier may be able to supply an item with new combinations of the hedonic *z* characteristics because of a new technology and have a monopoly power in doing so, but in practice the new good can be linked via the hedonic characteristics set to the existing ones. In this practical sense, such goods are not considered new for the purposes of the *Manual*.

21.73 Merkel (2000, p. 6) takes a similar practical line in devising a classification scheme that will meet the practical needs of PPI compilation. He considers *evolutionary* and *revolutionary* goods. The former are defined as

> …extensions of existing goods. From a production inputs standpoint, evolutionary goods are similar to pre-existing goods. They are typically produced on the same production line and/or use largely the same production inputs and processes as pre-existing goods. Consequently, in theory at least, it should be possible to quality adjust for any differences between a pre-existing good and an evolutionary good.

21.74 In contrast, revolutionary goods are goods that are substantially different from pre-existing goods. They are generally produced on entirely new production lines or with substantially new production inputs and processes than those used to produce preexisting goods. These differences make it virtually impossible, both from a theoretical and practical standpoint, to quality adjust between a revolutionary good and any preexisting good.

21.75 The main concern regarding the incorporation of new goods into the PPI is the decision on the need and timing for their inclusion. Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new products may lead to errors in the measurement of price changes if the unusual price movements at critical stages in the product life cycles are ignored. There are practical approaches to the early adoption of both evolutionary and revolutionary goods. These are outlined in Chapter 8, Section D.3. For evolutionary goods, such strategies include the rebasing of the index, resampling of items, and introduction of new goods as directed *sample substitutions* (Merkel, 2000). Also of use are hedonic quality adjustments and indices outlined in Chapter 7, Section E.4, and Section C above that facilitate the incorporation of such evolutionary goods, since they possess a similar characteristics set to existing ones but deliver different quantities of these characteristics. The modified short-run or chained framework outlined in Chapter 7, Sections G–H may also be more appropriate for product areas with high turnover of items. These approaches can incorporate the price change of new goods into the index as soon as prices are available for two successive periods, although issues relating to the proper weighting of such changes may remain.

21.76 However, for revolutionary goods, substitution may not be appropriate. First, they may not be able to be defined within the existing classification systems. Second, they may be primarily produced by a new establishment, which will require extending the sample to such establishments. Third, there will be no previous items to match against and make a quality adjustment to prices, since by definition, they are substantially different from preexisting goods. And, finally, there is no weight to attach to the new establishment or item(s). *Sample augmentation* is appropriate for revolutionary goods, as opposed to sample substitution for evolutionary goods. It is necessary to bring the new revolutionary goods into the sample in addition to what exists. This may involve extending the classification, the sample of establishments, and item list within new or existing establishments (Merkel, 2000).

Appendix 21.1: Some Econometric Issues

21.77 Hedonic regression estimates were seen in Chapter 7 to have potential use for the quality adjustment of prices. There are a number of issues that arise from the specification and estimation of hedonic regressions, the use of diagnostic statistics, and courses of action when the standard OLS assumptions are seen to break down. Many of these issues are standard econometric ones and not the subject of this *Manual*. This is not to say, they are unimportant. The use of hedonic regressions will require some econometric or statistical expertise, but suitable texts are generally available. See Berndt (1991)—particularly the chapter on hedonic regressions—and Maddala (1988) and Kennedy (2003), among many others. Modern statistical and econometric software have adequate diagnostic tests for testing when OLS assumptions break down. There remain, however, some specific issues that merit attention, although it must be stressed that these points are over and above, and should not be taken to diminish, the important standard econometric issues found in econometric texts.

Identification and appropriate estimators

21.78 Wooldridge (1996, pp. 400–01) has shown on standard econometric grounds that the estimation of supply and demand functions by OLS is biased *and this bias carries over to the estimation of the hedonic function.* It is first useful to consider estimation issues in the supply and demand functions. These functions are rarely estimated in practice. The more common approach is to estimate offer functions, with the marginal price offered by the firm dependent on chosen attributes (product characteristics) and firm characteristics, and to estimate *bid* or value functions, with the marginal prices paid by a consumer dependent on chosen attributes and consumer characteristics.[27] As noted earlier, the observed prices and quantities are the result of the interaction of structural demand and supply equations and the distributions of producer technologies and consumer tastes, and they cannot reveal the parameters of these offer and value functions. Rosen (1974, pp. 50–51) suggested a procedure for determining these parameters. Since these estimates are conditioned on tastes (α) and technologies (τ), the estimation procedure needs to include empirical measures or "proxy variables" of α and τ. For the tastes α of consumers, the empirical counterparts may be sociodemographic and economic variables, which may include age, income, education, and geographical region. For technologies τ, variables may include technologies and factor prices. First, the hedonic equation is estimated without these variables in the normal manner using the best-fitting functional form. This is to represent the price function consumers and producers face when making their decisions. Then, an implicit marginal price function is computed for each characteristic as $\partial p(z)/\partial z_i = \hat{p}_i(z)$, where $\hat{p}(z)$ is the estimated hedonic equation. Bear in mind that in normal demand and supply studies for *products*, the prices are observed in the market. For *characteristics* they are unobserved, and this first stage must be to estimate the parameters from the hedonic regression. The actual values of each z_i bought and sold is then inserted into each implicit marginal price function to yield a numerical value for each characteristic. These marginal values are used in the second stage[28] of estimation as endogenous variables for the estimation of the demand side:

(A21.1) $\hat{p}_i(z) = F(z1,\ldots,zK, \alpha^*)$,

where α^* are the proxy variables for tastes, and the supply side:

(A21.2) $\hat{p}_i(z) = F(z1,\ldots,zK, \tau^*)$,

where τ^* are the proxy variables for technologies. The variables τ^* drop out when there is no variation in technologies and $\hat{p}_i(z)$ is an estimate of the offer function. Similarly the variables α^* drop out when sellers differ and buyers are identical and cross-section estimates trace out compensated demand functions.

21.79 Epple (1987) has argued that Rosen's modeling strategy is likely to give rise to inappropriate estimation procedures of the demand and supply parameters. The hedonic approach to estimating the demand for characteristics has a difficulty arising from the fact that marginal prices are likely to be endogenous—they depend on the amount of each characteristic consumed and must be estimated from the hedonic function rather than observed directly. There are two resulting problems. First, there is an identification problem (see Epple, 1987) because both the marginal price of a characteristic and the inverse bid depend on the levels of characteristics consumed. Second, if important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients on measured characteristics will be biased.

[27]These are equivalent to inverse demand (supply) functions, with the prices dependent on the quantities demanded (supplied) and the individual consumer (producer) characteristics.

[28]This two-stage approach is common in the literature, though Wooldridge (1996) discusses the joint estimation of the hedonic and demand and supply side functions as a system.

This applies to all econometric models, but it is particularly relevant to hedonic models; on this point, see Wooldridge (1996, pp. 400–01) in particular. The equilibrium conditions for characteristic prices imply functional relationships among the characteristics of demanders, suppliers, and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with the included variables of the model (see also Bartik, 1988, on this point). The bias arises because buyers are differentiated by characteristics (y, α) and sellers by technologies τ. The type of item buyers will purchase is related to (y, α) and the type sellers provide to τ. On the plane of combinations of z transacted, the equilibrium ones chosen may be systematically related; the characteristics of buyers are related to those of sellers. Epple (1987) uses the example of stereo equipment: the higher income of some buyers leads to purchases of high-quality equipment, and the technical competence of sellers leads them to provide it. The consumer and producer characteristics may be correlated.

21.80 Wooldridge (1996, pp. 400–01) suggests that individual consumer and firm characteristics such as income, education, and input prices should be used as instruments in estimating hedonic functions. In addition, variables other than a good's characteristics should be included as instruments if they are price determining, such as geographical location—say, proximity to ports, good road systems, climate, and so on. Communities of economic agents are assumed, within which consumers consume and producers produce for each other at prices that vary across communities for identical goods. Variables on the characteristics of the communities will not in themselves enter the demand and supply equation but are price determining for observed prices recorded across communities. Tauchen and Witte (2001) provide a systematic investigation of the conditions under which consumer and producer and community characteristics will affect the hedonic parameter estimates for a single-regression equation estimated across all communities. A key concern is whether the hedonic price function error term represents factors that are unobserved by both the economic agents and the researcher, or by the researcher only. In the latter case, the error term may be correlated with the product attributes, and instrumental variable estimation is required. If the error term is *not* correlated with the product characteristics—preferences are quasi-linear—then a properly specified hedonic regression, including community-specific characteristics or appropriate slope dummies, can be estimated using OLS. In other cases, depending on the correlation between consumer and producer characteristics, assumptions about the error term and the method of incorporating community characteristics into the regression, instrumental variables, including consumer or producer or community dummy or characteristics, may need to be used.

Functional form

21.81 Triplett (1987; 2002) argues that neither classical utility theory nor production theory can specify the functional form of the hedonic function.[29] This point dates back to Rosen (1974, p. 54) who describes the observations as being "...a joint-envelope function and cannot by themselves identify the structure of consumer preferences and producer technologies that generate them." A priori judgments about what the form should look like may be based on ideas about how consumers and production technologies respond to price changes. These judgments are difficult to make when the observations are jointly determined by demand and supply factors but not impossible in rare instances. However, it is complicated when pricing is with a markup, the extent of which may vary over the life cycle of a product. Some tied combinations of characteristics will have higher markups than others. New-item introductions are likely to be attracted to these areas of characteristic space, and this will have the effect of increasing supply and thus lowering the markup and price (Cockburn and Anis, 1998; Feenstra, 1995, p. 647; and Triplett, 1987, p. 38). This again must be taken into account in any a priori reasoning—not an easy or straightforward matter.

21.82 It may be that in some cases the hedonic function's functional form will be very straightforward. For example, prices on the websites for options for products are often additive. The underlying cost and utility structure are unlikely to jointly generate such linear functions, but the pro-

[29] Arguea, Hsiao, and Taylor (1994) propose a linear form on the basis of arbitrage for characteristics, held to be likely in competitive markets, although Triplett (2002) argues that this is unlikely to be a realistic scenario in most commodity markets.

ducer or consumer is also paying for the convenience of selling in this way and are willing to bear losses or make gains if the cost or utility at higher values of z are priced lower or are worth more than the price set. But, in general, the data should convey what the functional form should look like, and imposing artificial structures simply leads to specification bias. For examples of econometric testing of hedonic functional form, see Cassel and Mendelsohn (1985); Cropper, Deck, and McConnell (1988); Rasmussen and Zuehlke (1990); Bode and van Dalen (2001); and Curry, Morgan, and Silver (2001).

21.83 The three forms prevalent in the literature are linear, semilogarithmic, and double-logarithmic (log-log). A number of studies have used econometric tests, in the absence of a clear theoretical statement, to choose among them. There have been a large number of hedonic studies, and, as illustrated in Curry, Morgan, and Silver (2001), in many of these the quite simple forms do well, at least in terms of the $\bar{R}^2$ presented, and the parameters accord with a priori reasoning, usually on the consumer side. Of the three popular forms, some are favored in testing. For example, Murray and Sarantis (1999) favored the semilogarithmic form, while in others—for example Hoffmann (1998)—the three functional forms were found to scarcely differ in terms of their explanatory power. That the parameters from these simple forms accord with a priori reasoning, usually from the consumer side, is promising, but researchers should be aware that such matters are not assured. Of the three forms, the semilogarithmic form has much to commend it. The interpretation of its coefficients is quite straightforward—the coefficients represent proportionate changes in prices arising from a unit change in the value of the characteristic.[30] This is a useful formulation, since quality adjustments are usually undertaken by making multiplicative instead of additive adjustments (see Chapter 7, Section C.3). The semilogarithmic form, unlike the log-log model, can also incorporate dummy variables for characteristics that are either present, $z_i = 1$, or not, $z_i = 0$.[31]

21.84 More complicated forms are possible. Simple forms have the virtue of parsimony and allow more efficient estimates to be made for a given sample. However, parsimony is not something to be achieved at the cost of misspecification bias. First, if the hedonic function is estimated across multiple independent markets, then interaction terms are required (see Mendelsohn, 1984, for fishing sites). Excluding them is tantamount to omitting variables and inappropriately constraining the estimated coefficients of the regression. Tauchen and Witte (2001) have outlined the particular biases that can arise from such omitted variables in hedonic studies. Second, it may be argued that the functional form should correspond to the aggregator for the index—linear for a Laspeyres index, logarithmic for a geometric Laspeyres index, translog for a Törnqvist index, and quadratic for a Fisher index (Chapter 17). However, as Triplett (2002) notes, the purpose of estimating hedonic regressions is to adjust prices for quality differences, and imposing a functional form on the data that is inconsistent with the data might create an error in the quality adjustment procedure. Yet, as Diewert (2002f) notes, flexible functional forms encompass these simple forms. The log-log form is a special case of the translog form as in equation (17.11), and the semi-log form is a special case of the semi-log quadratic form as in equation (17.16). If there are a priori reasons to expect interaction terms for specific characteristics, as illustrated in the example in Chapter 7, Section E.4, then these more general forms allow this, and the theory of hedonic functions neither dictates the form of the hedonic form nor restricts it.

[30]It is noted that the anti-log of the OLS-estimated coefficients are not unbiased—the estimation of semilogarithmic functions as transformed linear regressions requires an adjustment to provide minimum-variance unbiased estimates of parameters of the conditional mean. A standard adjustment is to add one-half of the coefficient's squared standard error to the estimated coefficient (Goldberger, 1968, and Teekens and Koerts, 1972).

[31]Diewert (2002f) argues against the linear form on the grounds that, while the hedonic model is linear, the estimation required is of a nonlinear *regression* model, and the semi-log and log-log models are linear *regression* models. He also notes that semi-log form has the disadvantage against the log-log of not being able to impose constraints of constant returns to scale. Diewert (2002d) also argues for the use of nonparametric functional forms and the estimation of linear generalized dummy variable hedonic regression models. This has been taken up in Curry, Morgan, and Silver (2001), who use neural networks that are shown to work well, although the variable set required for their estimation has to be relatively small.

Changing tastes and technologies

21.85 The estimates of the coefficients may change over time. Some of this will be attributed to sampling error, especially if multicollinearity is present, as discussed below. But, in other cases, it may be a genuine reflection of changes in tastes and technologies. If a subset of the estimated coefficients from a hedonic regression is to be used to quality-adjust a noncomparable replacement price, then the use of estimated out-of-date coefficients from some previous period to adjust the prices of the new replacement model would be inappropriate. There would be a need to update the indices as regularly as the changes demanded.[32] For estimating hedonic indices, the matter is more complicated. The coefficients in a simple dummy time-period model as in Section C.3 now have different estimates of the parameters in each period. Silver (1999), using a simple example, shows how the estimate of quality-adjusted price change from such a dummy variable model requires a reference basket of characteristics. This is apparent for the hedonic imputation indices where separate indices using base- and current-period characteristics are estimated. A symmetric average of such indices is considered appropriate. A hedonic index based on a time dummy variable implicitly constrains the estimated coefficients from the base and current periods to be the same. Diewert (2003) formalizes the problem of choosing the reference characteristics when comparing prices over time when the parameters of the hedonic function may themselves be changing over time. He finds the results of hedonic indices to *not* be invariant to the choice of reference-period characteristic vector set z. The use of a sales- (quantity-) weighted average vector of characteristics proposed by Silver (1999) is considered, but Diewert notes that over long time periods this may become unrepresentative.[33] Of course, if the dummy variable approach is used in a chained formulation as outlined in Section C.3, the weighted averages of characteristics remain reasonably up to date, though chaining has its own pros and cons (see Chapter 15). A fixed-base alternative noted by Diewert (2003) is to use a Laspeyres-type comparison with the base-period parameter set, and a Paasche-type current-period index with the current-period parameter set, and take the geometric mean of the two indices for reasons similar to those given in Chapter 17, Section B.3. The resulting Fisher-type index is similar to that given in equation (21.32) proposed by Feenstra (1995).[34] A feature of the time dummy approach is that it implicitly takes a symmetric average of the coefficients by constraining them to be the same. But what if, as is more likely the case, only base-period hedonic regression coefficients are available? Since hedonic indices based on a symmetric average of the coefficients are desirable, the spread or difference between estimates based on either a current- or a reference-period characteristics set is an indication of potential bias, and estimates of such spread may be undertaken retrospectively. If the spread is large, estimates based on the use of a single period's characteristics set, say, the current period, should be treated with caution. More regular updating of the hedonic regressions is likely to reduce spread because the periods being compared will be closer and the characteristics of the items in the periods compared more similar.

Weighting

21.86 OLS estimators implicitly treat each item as being of equal importance, although some items will have quite substantial sales, while for others, sales will be minimal. It is axiomatic that an item with sales of more than 5,000 in a month should not be given the same influence in the regression estimator as one with a few transactions. Commodities with very low sales may be at the end of their life cycles or be custom made. Either way, their (quality-adjusted) prices and price changes may be unusual.[35] Such observations with unusual prices should not be allowed to unduly influence

[32] In Chapter 15, Section C.3.2, the issue of adjusting the base- versus the current-period's price is discussed, since there are different data demands.

[33] Other averages may be proposed—for example, the needs of an index representative of the "typical" establishment would be better met by a trimmed mean or median.

[34] Diewert (2002c) also suggests matching items where possible and using hedonic regressions to impute the prices of the missing old and new ones. Different forms of weighting systems, including superlative ones, can be applied to this set of price data in each period for both matched and unmatched data.

[35] Such observations have higher variances of their error terms, leading to imprecise parameter estimates. This would argue for the use of WLS estimators with quantity sold as the weight. This is one of the standard treatments for heteroskedastic errors (see Berndt, 1991)

the index.[36] The estimation of hedonic regression equations by a WLS estimator is preferable. This estimator minimizes the sum of *weighted* squared deviations between the actual prices and the predicted prices from the regression equation, as opposed to OLS estimation, which uses an equal weight for each observation. There is a question as to whether to use quantity (volume) or expenditure weights. The use of quantity weights can be supported by considering the nature of their equivalent "price." Such prices are the average (usually the same) price over a number of transactions. The underlying sampling unit is the individual transaction, so there is a sense that the data may be replicated as being composed of, say, 12 individual observations using an OLS estimator, as opposed to a single observation with a weight of 12 using a WLS estimator. Both would yield the same result. Inefficient estimates arise if the variance of the errors, $V(u_i)$, is not constant—that is, they are heteroskedastic. WLS is equivalent to assuming that the error variances are related to the weights in a multiplicative manner, say, $V(u_i) = \sigma^2 w_i^2$.[37] A priori notions as to whether a hedonic regression model predicts better or worse at different levels of quantities or expenditures may help in identifying which weights are appropriate; however, statistical tests or plots of heteroskedasticity may be more useful.

21.87 The sole use of statistical criteria for deciding on which weighing system to use has rightfully come under some criticism. Diewert (2002c) and Silver (2002) have argued that what matters is whether the estimates are representative of the target index in mind. Conventional target index numbers such as Laspeyres, Paasche, Fisher, and Törnqvist weight price changes by expenditure shares, and the latter two formulas have received support from the axiomatic, stochastic, fixed-base, and economic theoretic approaches, as shown in Chapters 15–18. Thus, value weights are preferred to quantity weights: "The problem with quantity weighting is this: it will tend to give too little weight to cheap models that have low amounts of useful characteristics" (Diewert, 2002c, p. 8). He continues to argue that for a WLS estimator of hedonic time dummy variable indices, expenditure *share* weights should be used, as opposed to the *value* of expenditure, to avoid inflation increasing period 1 value weights, resulting in possible heteroskedastic residuals. Furthermore, for a semilogarithmic hedonic function when models are present in both periods, the average expenditure shares in periods 0 and 1 for *m* items, $\frac{1}{2}(s_{m0} + s_{m1})$, should be used as weights in the WLS estimator. If only matched models exist in the data, then such an estimator may be equivalent to the Törnqvist index. If an observation *m* is available only in one of the periods, its weight should be s_{m0} or s_{m1} accordingly, and the WLS estimator provides a *generalization* of the Törnqvist index.

21.88 Silver (2002) has shown that a WLS estimator using value weights will not necessarily give each observation a weight equal to its relative value. The estimator will give more weight to those observations with high leverage effects and residuals. Observations with values of characteristics with large deviations from their means—say, very old or new models—have relatively high leverage. New and old models are likely to be priced at quite different prices than those predicted from the hedonic regression, even after taking into account their different characteristics. Such prices result, for example, from a pricing strategy designed to skim segments of the market willing to pay a premium for a new model, or from a strategy to charge relatively low prices for an old model to dump it to make way for a new one. In such cases the influence these models have on deriving the estimated coefficients will be over and above that attributable to their value weights. Silver (2002) suggests that leverage effects should be calculated for each observation, and those with high leverage and low weights should be deleted, and the regression re-run. Thus, while quantity or value weights are preferable to no weights (that is, OLS), value weights are more appropriate than quantity ones, and, even so, account should be taken of observations with undue influence.

21.89 Diewert (2002f) has also considered the issue of weighting with respect to the time dummy hedonic indices outlined in Section C.6. The use of WLS by value involves weights being applied to observations in both periods. However, if, for ex-

[36]See Berndt, Ling, and Kyle (2003), Cockburn and Anis (1998), and Silver and Heravi (2002) for examples. Silver and Heravi (2002) show old items have above-average leverage effects and below-average residuals. Not only are they different, but they exert undue influence for their size (number of observations).

[37]Estimating an equation for which each variable is divided by the square root of the weight using OLS is an equivalent procedure.

ample, there is high inflation, then the sales values for a model in the current period will generally be larger than those of the corresponding model in the base period, and the assumption of homoscedastic residuals is unlikely to be met. Diewert (2002f) suggests the use of expenditure *shares* in each period, as opposed to values, as weights for WLS for time dummy hedonic indices. He also suggests that an average of expenditure shares in the periods being compared be used for matched models.

21.90 Data on sales are not always available for weights, but the major selling items can generally be identified. In such cases, it is important to restrict the number of observations of items with relatively low sales, the extent of the restriction depending on the number of observations and the skewness of the sales distribution. In some cases, items with few sales provide the variability necessary for efficient estimates of the regression equation. In other cases, their low sales may be due to factors that make them unrepresentative of the hedonic surface, their residuals being unusually high. An example is low-selling models about to be dumped to make way for new models. Unweighted regressions may thus suffer from a sampling problem—even if the prices are perfectly quality adjusted, the index can be biased because it is unduly influenced by low-selling items with unrepresentative price-characteristic relationships. In the absence of weights, regression diagnostics have a role to play in helping to determine whether the undue variance in some observations belongs to such unusual low-selling items.[38]

Multicollinearity

21.91 There are a priori reasons to expect for some commodities that the variation in the values of one characteristic will not be independent of one or a linear combination of other z characteristics. As a result, parameter estimates will be unbiased, yet imprecise. To illustrate this, a plot of the confidence interval for one parameter estimate against another collinear one is often described as elliptical, since the combinations of possible values they may take can easily drift from, say, high values of β_1 and low β_2 to higher values of β_2 and low of β_1. Since the sample size for the estimates is effectively reduced, relatively small additions to and deletions from the sample may affect the parameter estimates more than would be expected. These are standard statistical issues, and the reader is referred to Maddala (1988) and Kennedy (2003). In a hedonic regression, multicollinearity might be expected because some characteristics may be technologically tied to others. Producers including one characteristic may need to include others for it all to work, while for the consumer side, purchasers buying, for example, an up-market brand may expect a certain bundle of features to come with it. Triplett (2002) argues strongly for the researcher to be aware of the features of the product and consumer market. There are standard, though not completely reliable, indicators of multicollinearity (such as variance inflation factors), but an exploration of its nature is greatly aided by an understanding of the market along with exploration of the effects of including and excluding individual variables on the signs and coefficients and on other diagnostic test statistics (see Maddala, 1988).[39]

21.92 If a subset of the estimated coefficients from a hedonic regression are to be used to quality-adjust a noncomparable replacement price, and if there is multicollinearity *between* variables in this subset *and* other independent variables, then the estimates of the coefficients to be used for the adjustment will be imprecise. The multicollinearity effectively reduces the sample size, and some of the effects of the variables in the subset may be wrongly ascribed to the other independent variables. The extent of this error will be determined by the strength of the multiple-correlation coefficient between all such "independent" variables (the multicollinearity), the standard error or "fit" of the regression, the dispersion of the independent variable concerned, and the sample size. These all affect the precision of the estimates, since they are components in the standard error of the t-statistics.

[38]A less formal procedure is to take the standardized residuals from the regression and plot them against model characteristics that may denote low sales, such as certain brands (makes) or vintage (if not directly incorporated) or some technical feature that makes it unlikely that the item is being bought in quantity. Higher variances may be apparent from the scatter plot. If certain features are expected to have, on average, low sales, but seem to have high variances, leverages, and residuals (see Silver and Heravi, 2002), a case exists for at least downplaying their influence. Bode and van Dalen (2001) use formal statistical criteria to decide between different weighting systems and compare the results of OLS and WLS, finding, as with Ioannidis and Silver (1999), that different results can arise.

[39]Triplett (2002) stresses the point that $\bar{R}^2$ alone is insufficient for this purpose.

Even if multicollinearity is expected to be quite high, large sample sizes and a well-fitting model may reduce the standard errors on the *t*-statistics to acceptable levels. If multicollinearity is expected to be severe, the predicted value for an item's price may be computed using the whole regression and an adjustment made using the predicted value, as explained in Chapter 7, Section E.4, since there is a sense in which it would not matter whether the variation was wrongly attributed to either β_1 or β_2. If dummy variable hedonic *indices* are being calculated (Section B.3 above), the time trend will be collinear with an included variable if a new feature appears in a new month for the vast majority of the items, so that the data are not rich enough to allow the separate effects of the coefficient on the time dummy to be precisely identified. The extent of the imprecision of the coefficient on the time dummy will be determined by the aforementioned factors. A similar argument holds for omitted variable bias.

Omitted-variable bias

21.93 The exclusion of tastes and technology and community characteristics has already been discussed. The concern here is with product characteristics. Consider again the use of a subset of the estimated coefficients from a hedonic regression to quality-adjust a noncomparable replacement price. It is well established that multicollinearity of omitted variables with included variables leads to bias in the estimates of the coefficients of included ones. If omitted variables are *independent* of the included variables, then the estimates of the coefficients on the included variables are unbiased. This is acceptable in this instance; the only caveat is that it may be that the quality adjustment for the replacement item also requires an adjustment for these omitted variables, and this, as noted by Triplett (2002), has to be undertaken using a separate method and data. But what if the omitted variable is multicollinear with a subset of included ones, and these included ones are to be used to quality adjust a noncomparable item? In this case, the coefficient on the subset of the included variables may be wrongly picking up some of the omitted variables' effects. The coefficients will be used to quality-adjust prices for items that differ only with regard to this subset of included variables, and the price comparison will be biased if the characteristics of both included and omitted variables have different price changes. For hedonic *indices* using a dummy time trend, the estimates of quality-adjusted price changes will suffer from a similar bias if omitted variables excluded from the regression are multicollinear with the time change. What are picked up as quality-adjusted price changes over time may, in part, be changes due to the prices of these excluded variables. This requires that the prices on the omitted characteristics follow a different trend. Such effects are most likely when there are gradual improvements in the quality of items, such as the reliability and safety of consumer durables,[40] which are difficult to measure, at least for the sample of items in real time. The quality-adjusted price changes will thus, overstate price changes in such instances.

[40]There are some commodity areas, such as airline comfort, that have been argued to have overall patterns of decreasing quality.

22. Treatment of Seasonal Products

A. Problem of Seasonal Products

22.1 The existence of seasonal products poses some significant challenges for price statisticians. *Seasonal commodities* are products that are either (i) not available in the marketplace during certain seasons of the year or (ii) are available throughout the year but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[1] A commodity that satisfies (i) is termed a *strongly seasonal commodity*, whereas a commodity that satisfies (ii) will be called a *weakly seasonal commodity*. It is strongly seasonal products that create the biggest problems for price statisticians in the context of producing a monthly or quarterly PPI. If a product price is available in only one of the two months (or quarters) being compared, then it is not possible to calculate a relative price for the product, and traditional bilateral index number theory breaks down. In other words, if a product is present in one month but not the next, how can the month-to-month amount of price change for that product be computed?[2] In this chapter, a solution to this problem will be presented that works even if the products produced are entirely different for each month of the year.[3]

[1]This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a, p. 7; 1980b, p. 110; 1980c, p. 68). Diewert (1998b, p. 457) used the terms type 1 and type 2 seasonality.

[2]Zarnowitz (1961, p. 238) was perhaps the first to note the importance of this problem: "But the main problem introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis."

[3]However, the same products must reappear each year for each separate month!

22.2 There are two main sources of seasonal fluctuations in prices and quantities: (i) climate and (ii) custom.[4] In the first category, fluctuations in temperature, precipitation, and hours of daylight cause fluctuations in the demand or supply for many products; for example, think of summer versus winter clothing, the demand for light and heat, vacations, etc. With respect to custom and convention as a cause of seasonal fluctuations, consider the following quotation:

> Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law…. Many of the conventional seasons have considerable effects on economic behaviour. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the first of July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase in bankruptcies in January, and so on. (Wesley C. Mitchell, 1927, p. 237)

22.3 Examples of important seasonal products are the following: many food items; alcoholic beverages; many clothing and footwear items; water, heating oil, electricity; flowers and garden supplies; vehicle purchases, vehicle operation; many entertainment and recreation expenditures; books; insurance expenditures; wedding expenditures; recreational equipment; toys and games; software; air travel; and tourism purchases. For a typical country, seasonal purchases will often amount to one-fifth to one-third of all consumer purchases.[5]

[4]This classification dates back to Mitchell (1927, p. 236) at least: "Two types of seasons produce annually recurring variations in economic activity—those which are due to climates and those which are due to conventions."

[5]Alterman, Diewert, and Feenstra (1999, p. 151) found that over the 40 months between September 1993 and December 1996, somewhere between 23 and 40 percent of U.S. imports and exports exhibited seasonal variations in (continued)

22.4 In the context of producing a monthly or quarterly PPI, it must be recognized that there is no completely satisfactory way of dealing with strongly seasonal products. If a product is present in one month but missing in the next, then none of the index number theories that were considered in Chapters 15–20 can be applied because all of these theories assumed that the dimensionality of the product space was constant for the two periods being compared. However, if seasonal products are present in the market during each season, then, in theory, traditional index number theory can be applied in order to construct month-to-month or quarter-to-quarter price indices. This traditional approach to the treatment of seasonal products will be followed in Sections H, I, and J of this chapter. The reason why this straightforward approach is deferred to the end of the chapter is twofold:

- The approach that restricts the index to products that are present in every period often does not work well in the sense that systematic *biases* can occur; and
- The approach is not fully *representative*; that is, it does not make use of information on products that are not present in every month or quarter.

22.5 In Section B, a modified version of Turvey's (1979) artificial data set is introduced. This data set will be used to numerically evaluate all of the index number formulas that are suggested in this chapter. It will be seen in Section G that large seasonal fluctuations in volumes combined with systematic seasonal changes in price can make month-to-month or quarter-to-quarter price indices behave rather poorly.

22.6 Even though existing index number theory cannot deal satisfactorily with seasonal products in the context of constructing month-to-month indices of producer prices, it can deal satisfactorily with seasonal products if the focus is changed from month-to-month PPIs to PPIs that compare the prices of one month with the prices of the *same* month in a previous year. Thus, in Section C, *year-over-year monthly* PPIs are studied. Turvey's seasonal data set is used to evaluate the performance of these indices, and they are found to perform quite well.

22.7 In Section D, the year-over-year monthly indices defined in Section C are aggregated into an *annual index* that compares all of the monthly prices in a given calendar year with the corresponding monthly prices in a base year. In Section E, this idea of comparing the prices of a current calendar year with the corresponding prices in a base year is extended to annual indices that compare the prices of the last 12 months with the corresponding prices in the 12 months of a base year. The resulting *rolling-year indices* can be regarded as seasonally adjusted price indices. The modified Turvey data set is used to test out these year-over-year indices, and they are found to work very well on this data set.

22.8 The rolling-year indices can provide an accurate gauge of the movement of prices in the current rolling year compared with the base year. However, this measure of price inflation can be regarded as a measure of inflation for a year that is centered around a month that is six months prior to the last month in the current rolling year. As a result, for some policy purposes, this type of index is not as useful as an index that compares the prices of the current month to the previous month, so that more up-to-date information on the movement of prices can be obtained. However, in Section F, it will be shown that under certain conditions, the current month year-over-year monthly index, along with last month's year-over-year monthly index, can successfully *predict* or *forecast* a rolling-year index that is centered around the current month.

22.9 The year-over-year indices defined in Section C and their annual averages studied in Sections D and E offer a theoretically satisfactory method for dealing with *strongly seasonal products*; that is, products that are available only during certain seasons of the year. However, these methods rely on the year-over-year comparison of prices; therefore, these methods cannot be used in the month-to-month or quarter-to-quarter type of index, which is typically the main focus of a producer price program. Thus, there is a need for another type of index, one that may not have strong theoretical foundations but can deal with seasonal products in the context of producing a *month-to-month index*. In Section G, such an index is introduced, and it is implemented using the artificial data set for the products that are available during

quantities, whereas only about 5 percent of U.S. export and import prices exhibited seasonal fluctuations.

Table 22.1. Artificial Seasonal Data Set: Prices

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1970	1	1.14	0	2.48	0	1.30
	2	1.17	0	2.75	0	1.25
	3	1.17	0	5.07	0	1.21
	4	1.40	0	5.00	0	1.22
	5	1.64	0	4.98	5.13	1.28
	6	1.75	3.15	4.78	3.48	1.33
	7	1.83	2.53	3.48	3.27	1.45
	8	1.92	1.76	2.01	0	1.54
	9	1.38	1.73	1.42	0	1.57
	10	1.10	1.94	1.39	0	1.61
	11	1.09	0	1.75	0	1.59
	12	1.10	0	2.02	0	1.41
1971	1	1.25	0	2.15	0	1.45
	2	1.36	0	2.55	0	1.36
	3	1.38	0	4.22	0	1.37
	4	1.57	0	4.36	0	1.44
	5	1.77	0	4.18	5.68	1.51
	6	1.86	3.77	4.08	3.72	1.56
	7	1.94	2.85	2.61	3.78	1.66
	8	2.02	1.98	1.79	0	1.74
	9	1.55	1.80	1.28	0	1.76
	10	1.34	1.95	1.26	0	1.77
	11	1.33	0	1.62	0	1.76
	12	1.30	0	1.81	0	1.50
1972	1	1.43	0	1.89	0	1.56
	2	1.53	0	2.38	0	1.53
	3	1.59	0	3.59	0	1.55
	4	1.73	0	3.90	0	1.62
	5	1.89	0	3.56	6.21	1.70
	6	1.98	4.69	3.51	3.98	1.78
	7	2.07	3.32	2.73	4.30	1.89
	8	2.12	2.29	1.65	0	1.91
	9	1.73	1.90	1.15	0	1.92
	10	1.56	1.97	1.15	0	1.95
	11	1.56	0	1.46	0	1.94
	12	1.49	0	1.73	0	1.64
1973	1	1.68	0	1.62	0	1.69
	2	1.82	0	2.16	0	1.69
	3	1.89	0	3.02	0	1.74
	4	2.00	0	3.45	0	1.91
	5	2.14	0	3.08	7.17	2.03
	6	2.23	6.40	3.07	4.53	2.13
	7	2.35	4.31	2.41	5.19	2.22
	8	2.40	2.98	1.49	0	2.26
	9	2.09	2.21	1.08	0	2.22
	10	2.03	2.18	1.08	0	2.31
	11	2.05	0	1.36	0	2.34
	12	1.90	0	1.57	0	1.97

Table 22.2. Artificial Seasonal Data Set: Quantities

Year *t*	Month *m*	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
1970	1	3,086	0	82	0	10,266
	2	3,765	0	35	0	9,656
	3	4,363	0	98	0	7,940
	4	4,842	0	26	0	5,110
	5	4,439	0	75	700	4,089
	6	5,323	91	82	2,709	3,362
	7	4,165	498	96	1,970	3,396
	8	3,224	6,504	1,490	0	2,406
	9	4,025	4,923	2,937	0	2,486
	10	5,784	865	2,826	0	3,222
	11	6,949	0	1,290	0	6,958
	12	3,924	0	338	0	9,762
1971	1	3,415	0	119	0	10,888
	2	4,127	0	45	0	10,314
	3	4,771	0	14	0	8,797
	4	5,290	0	11	0	5,590
	5	4,986	0	74	806	4,377
	6	5,869	98	112	3,166	3,681
	7	4,671	548	132	2,153	3,748
	8	3,534	6,964	2,216	0	2,649
	9	4,509	5,370	4,229	0	2,726
	10	6,299	932	4,178	0	3,477
	11	7,753	0	1,831	0	8,548
	12	4,285	0	496	0	10,727
1972	1	3,742	0	172	0	11,569
	2	4,518	0	67	0	10,993
	3	5,134	0	22	0	9,621
	4	5,738	0	16	0	6,063
	5	5,498	0	137	931	4,625
	6	6,420	104	171	3,642	3,970
	7	5,157	604	202	2,533	4,078
	8	3,881	7,378	3,269	0	2,883
	9	4,917	5,839	6,111	0	2,957
	10	6,872	1,006	5,964	0	3,759
	11	8,490	0	2,824	0	8,238
	12	5,211	0	731	0	11,827
1973	1	4,051	0	250	0	12,206
	2	4,909	0	102	0	11,698
	3	5,567	0	30	0	10,438
	4	6,253	0	25	0	6,593
	5	6,101	0	220	1,033	4,926
	6	7,023	111	252	4,085	4,307
	7	5,671	653	266	2,877	4,418
	8	4,187	7,856	4,813	0	3,165
	9	5,446	6,291	8,803	0	3,211
	10	7,377	1,073	8,778	0	4,007
	11	9,283	0	4,517	0	8,833
	12	4,955	0	1,073	0	12,558

each month of the year. Unfortunately, due to the seasonality in both prices and quantities in the always available products, this type of index can be systematically biased. This bias is apparent in the modified Turvey data set.

22.10 Since many PPIs are month-to-month indices that use *annual basket quantity weights*, this type of index is studied in Section H. For months when the product is not available in the marketplace, the last available price is carried forward and used in the index. In Section I, an annual quantity basket is again used but instead of carrying forward the prices of seasonally unavailable items, an imputation method is used to fill in the missing prices. The annual basket-type indices defined in Sections H and I are implemented using the artificial data set. Unfortunately, the empirical results are not satisfactory because the indices show tremendous seasonal fluctuations in prices. This volatility makes them unsuitable for users who want up-to-date information on *trends* in general inflation.

22.11 In Section J, the artificial data set is used in order to evaluate another type of month-to-month index that is frequently suggested in the literature on how to deal with seasonal products: the *Bean and Stine Type C* (1924) or *Rothwell* (1958) index. Again, this index does not get rid of the tremendous seasonal fluctuations that are present in the modified Turvey data set.

22.12 Sections H and I show that the annual basket-type indices with carryforward of missing prices (Section H) or imputation of missing prices (Section I) do not get rid of seasonal fluctuations in prices. However, in Section K, it is shown how seasonally adjusted versions of these annual basket indices can be used to successfully *forecast* rolling-year indices that are centered in the current month. In addition, the results in Section K show how these annual basket-type indices can be seasonally adjusted (using information obtained from rolling-year indices from prior periods or by using traditional seasonal adjustment procedures). Hence, these seasonally adjusted annual basket indices could be used as successful indicators of general inflation on a timely basis.

22.13 Section L concludes with several suggestions for dealing with seasonal products.

B. A Seasonal Product Data Set

22.14 It will prove to be useful to illustrate the index number formulas that will be defined in subsequent sections by computing them for an actual data set. Turvey (1979) constructed an artificial data set for five seasonal products (apples, peaches, grapes, strawberries, and oranges) for four years by month, so that there are 5 times 4 times 12 observations, equal to 240 observations in all. At certain times of the year, peaches and strawberries (products 2 and 4) are unavailable, so in Tables 22.1 and 22.2, the prices and quantities for these products are entered as zeros.[6] The data in Tables 22.1 and 22.2 are essentially equal to that constructed by Turvey except that a number of adjustments were made to illustrate various points. The two most important adjustments were as follows:

- The data for product 3 (grapes) were adjusted, so that the annual Laspeyres and Paasche indices (which will be defined in Section D) would differ more than in the original data set;[7] and
- After the aforementioned adjustments were made, each price in the last year of data was escalated by the monthly inflation factor 1.008, so that month-to-month inflation for the last year of data would be at an approximate monthly rate of 1.6 percent per month, compared with about 0.8 percent per month for the first three years of data.[8]

[6]The corresponding prices are not zeros, but they are entered as zeros for convenience in programming the various indices.

[7]After the first year, the price data for grapes was adjusted downward by 30 percent each year and the corresponding volume was adjusted upward by 40 percent each year. In addition, the quantity of oranges (product 5) for November 1971 was changed from 3,548 to 8,548 so that the seasonal pattern of change for this product would be similar to that of other years. For similar reasons, the price of oranges in December 1970 was changed from 1.31 to 1.41 and in January 1971 from 1.35 to 1.45.

[8]Pierre Duguay of the Bank of Canada, while commenting on a preliminary version of this chapter, observed that rolling-year indices would not be able to detect the *magnitude* of systematic changes in the month-to-month inflation rate. The original Turvey data set was roughly consistent with a month-to-month inflation rate of 0.8 percent per month; that is, prices grew roughly at the rate 1.008 each month over the four years of data. Hence this second major (continued)

22.15 Turvey sent his artificial data set to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indices. About 20 countries replied; Turvey summarized the responses as follows:

> It will be seen that the monthly indices display very large differences, for example, a range of 129.12–169.50 in June, while the range of simple annual means is much smaller. It will also be seen that the indices vary as to the peak month or year. (Ralph Turvey, 1979, p. 13)

The (modified) data below will be used to test out various index number formulas in subsequent sections.

C. Year-over-Year Monthly Indices

22.16 It can be seen that the existence of seasonal products that are present in the marketplace in one month but absent the next causes the accuracy of a month-to-month index to fall.[9] A way of dealing with these strongly seasonal products is to change the focus from short-term month-to-month price indices to year-over-year price comparisons for each month of the year. In the latter type of comparison, there is a good chance that seasonal products that appear in February, for example, will also appear in subsequent Februarys, so that the overlap of products will be maximized in these year-over-year monthly indices.

22.17 For over a century, it has been recognized that making year-over-year comparisons[10] provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations:

> In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes. (W. Stanley Jevons, 1884, p. 3)

22.18 The economist Flux and the statistician Yule also endorsed the idea of making year-over-year comparisons to minimize the effects of seasonal fluctuations:

> Each month the average price change compared with the corresponding month of the previous year is to be computed. … The determination of the proper seasonal variations of weights, especially in view of the liability of seasons to vary from year to year, is a task from which, I imagine, most of us would be tempted to recoil. (A. W. Flux, 1921, pp. 184–85)

> My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures. (G. Udny Yule, 1921, p. 199)

In more recent times, Zarnowitz also endorsed the use of year-over-year monthly indices:

> There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit "season", and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons. (Victor Zarnowitz, 1961, p. 266)

22.19 In the remainder of this section, it is shown how year-over-year Fisher indices and ap-

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adjustment of the Turvey data was introduced to illustrate Duguay's observation, which is quite correct: the centered rolling-year indices pick up the correct magnitude of the new inflation rate only after a lag of half a year or so. However, they do quickly pick up the direction of change in the inflation rate.

[9]In the limit, if each product appeared in only one month of the year, then a month-to-month index would break down completely.

[10]In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924, p. 31) Type D index.

proximations to them can be constructed.[11] For each month m = 1, 2,...,12, let $S(m)$ denote the set of products that are available for purchase in each year t = 0, 1,...,T. For t = 0, 1,...,T and m = 1, 2,...,12, let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is available in month m of year t for n belongs to $S(m)$. Let $p^{t,m}$ and $q^{t,m}$ denote the month m and year t price and quantity vectors, respectively. Then *the year-over-year monthly Laspeyres, Paasche, and Fisher indices* going from month m of year t to month m of year t + 1 can be defined as follows:

$$(22.1)\ P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t,m}};$$

m = 1, 2,...12;

$$(22.2)\ P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t+1,m}};$$

m = 1,,2,...12;

$$(22.3)\ P_F\left(p^{t,m}, p^{t+1,m}, q^{t,m}, q^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right)}\sqrt{P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right)};$$

m = 1, 2,...,12.

22.20 The above formulas can be rewritten in price relative and monthly revenue share form as follows:

$$(22.4)\ P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right) = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$

m = 1, 2,...12;

$$(22.5)\ P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right) = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

m = 1, 2,...12;

$$(22.6)\ P_F\left(p^{t,m}, p^{t+1,m}, s^{t,m}, s^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right)}\ \sqrt{P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)};$$

m = 1, 2,...,12

$$= \sqrt{\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)} \times \sqrt{\left[\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}};$$

where the monthly revenue share for product $n\in S(m)$ for month m in year t is defined as:

$$(22.7)\ s_n^{t,m} = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(m)} p_i^{t,m} q_i^{t,m}}; \quad ; m = 1,2,...,12;$$

$n\in S(m)$; t = 0,1,...,T;

and $s^{t,m}$ denotes the vector of month m expenditure shares in year t, $[s_n^{t,m}]$ for $n\in S(m)$.

22.21 Current-period revenue shares $s_n^{t,m}$ are not likely to be available. As a consequence, it will be necessary to approximate these shares using the corresponding revenue shares from a base year 0.

22.22 Use the base-period monthly revenue share vectors $s^{0,m}$ in place of the vector of month m and year t expenditure shares $s^{t,m}$ in equation (22.4), and use the base-period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t + 1 revenue shares $s^{t+1,m}$ in equation (22.5). Similarly, replace the share vectors $s^{t,m}$ and $s^{t+1,m}$ in equation (22.6) with the base-period expenditure share vector for month m, $s^{0,m}$. The resulting *approximate year-over-year monthly Laspeyres, Paasche, and Fisher indices* are defined by equations (22.8)–(22.10) below:[12]

[11]Diewert (1996b, pp. 17–19; 1999a, p. 50) noted various separability restrictions on purchaser preferences that would justify these year-over-year monthly indices from the viewpoint of the economic approach to index number theory.

[12]If the monthly revenue shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by equation (22.10) reduces to Fisher's (1922, p. 472) formula 101. Fisher (1922, p. 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992a, p. 143) and Diewert (1995a, p. 29) showed analytically that these two indices approximated each other to the second order. The equally weighted version of equation (22.10) was recom-
(continued)

$$(22.8)\ P_{AL}\left(p^{t,m},p^{t+1,m},s^{0,m}\right) = \sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$

$m = 1, 2,...12;$

$$(22.9)\ P_{AP}\left(p^{t,m},p^{t+1,m},s^{0,m}\right) = \left[\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

$m = 1, 2,...12;$

$$(22.10)\ P_{AF}\left(p^{t,m},p^{t+1,m},s^{0,m},s^{0,m}\right) \equiv \sqrt{P_{AL}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)}\sqrt{P_{P}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)};$$

$m = 1, 2,...,12$

$$= \sqrt{\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right) \times \left[\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}}.$$

22.23 The approximate Fisher year-over-year monthly indices defined by equation (22.10) will provide adequate approximations to their true Fisher counterparts defined by equation (22.6) only if the monthly revenue shares for the base year 0 are not too different from their current-year t and $t + 1$ counterparts. Thus, it will be useful to construct the true Fisher indices on a delayed basis in order to check the adequacy of the approximate Fisher indices defined by equation (22.10).

22.24 The year-over-year monthly approximate Fisher indices defined by equation (22.10) will normally have a certain amount of upward bias, since these indices cannot reflect long-term substitution toward products that are becoming relatively cheaper over time. This reinforces the case for computing true year-over-year monthly Fisher indices defined by equation (22.6) on a delayed basis, so that this substitution bias can be estimated.

22.25 Note that the approximate year-over-year monthly Laspeyres and Paasche indices, P_{AL} and P_{AP} defined by equations (22.8) and (22.9), satisfy the following inequalities:

$$(22.11)\ P_{AL}\left(p^{t,m},p^{t+1,m},s^{0,m}\right) \times P_{AL}\left(p^{t+1,m},p^{t,m},s^{0,m}\right) \geq 1;$$

$m = 1, 2,...,12;$

$$(22.12)\ P_{AP}\left(p^{t,m},p^{t+1,m},s^{0,m}\right) \times P_{AP}\left(p^{t+1,m},p^{t,m},s^{0,m}\right) \leq 1;$$

$m = 1, 2,...,12;$

with strict inequalities if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other.[13] Equation (22.11) says that the approximate year-over-year monthly Laspeyres index *fails the time reversal test* with an upward bias while equation (22.12) says that the approximate year-over-year monthly Paasche index *fails the time reversal test* with a downward bias. As a result, the fixed-weight approximate Laspeyres index P_{AL} has a built-in upward bias while the fixed-weights approximate Paasche index P_{AP} has a built-in downward bias. *Statistical agencies should avoid the use of these formulas*. However, they can be combined, as in the approximate Fisher formula in equation (22.10). The resulting index should be free from any systematic formula bias, although some substitution bias could still exist.

22.26 The year-over-year monthly indices defined in this section are illustrated using the artificial data set tabled in Section B. Although fixed-base indices were not formally defined in this section, these indices have similar formulas to the year-over-year indices that were defined, with the exception that the variable base year t is replaced by the fixed-base year 0. The resulting 12 year-over-year monthly fixed-base Laspeyres, Paasche, and Fisher indices are listed in Tables 22.3 to 22.5.

22.27 Comparing the entries in Tables 22.3 and 22.4, it can be seen that the year-over-year monthly fixed-base Laspeyres and Paasche price indices do not differ substantially for the early months of the year. There are, however, substantial differences between the indices for the last five months of the year by the time the year 1973 is reached. The largest percentage difference between the Laspeyres and Paasche indices is 12.5 percent for month 10 in 1973 (1.4060/1.2496 = 1.125).

mended as an elementary index by Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992a p. 140).

[13]See Hardy, Littlewood, and Polyá (1934, p. 26).

Table 22.3. Year-over-Year Monthly Fixed-Base Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2060	1.2442	1.3062	1.2783	1.2184	1.1734	1.2364	1.1827	1.1049	1.1809	1.2550	1.1960
1973	1.3281	1.4028	1.4968	1.4917	1.4105	1.3461	1.4559	1.4290	1.2636	1.4060	1.5449	1.4505

Table 22.4. Year-over-Year Monthly Fixed-Base Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2023	1.2436	1.3038	1.2773	1.2024	1.1657	1.2307	1.1455	1.0695	1.1274	1.2218	1.1901
1973	1.3190	1.4009	1.4912	1.4882	1.3715	1.3266	1.4433	1.3122	1.1664	1.2496	1.4296	1.4152

Table 22.5. Year-over-Year Monthly Fixed-Base Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2041	1.2439	1.3050	1.2778	1.2104	1.1695	1.2336	1.1640	1.0870	1.1538	1.2383	1.1930
1973	1.3235	1.4019	1.4940	1.4900	1.3909	1.3363	1.4496	1.3694	1.2140	1.3255	1.4861	1.4327

Table 22.6. Year-over-Year Approximate Monthly Fixed-Base Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2025	1.2421	1.3036	1.2757	1.2110	1.1640	1.2267	1.1567	1.0788	1.1309	1.2244	1.1862
1973	1.3165	1.3947	1.4880	1.4858	1.3926	1.3223	1.4297	1.3315	1.1920	1.2604	1.4461	1.4184

Table 22.7. Year-over-Year Approximate Monthly Fixed-Base Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2043	1.2432	1.3049	1.2770	1.2147	1.1687	1.2316	1.1696	1.0918	1.1557	1.2396	1.1911
1973	1.3223	1.3987	1.4924	1.4888	1.4015	1.3341	1.4428	1.3794	1.2273	1.3312	1.4947	1.4344

Table 22.8. Year-over-Year Monthly Chained Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2058	1.2440	1.3058	1.2782	1.2154	1.1720	1.2357	1.1753	1.0975	1.1690	1.2491	1.1943
1973	1.3274	1.4030	1.4951	1.4911	1.4002	1.3410	1.4522	1.3927	1.2347	1.3593	1.5177	1.4432

Table 22.9. Year-over-Year Monthly Chained Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2039	1.2437	1.3047	1.2777	1.2074	1.1682	1.2328	1.1569	1.0798	1.1421	1.2321	1.1908
1973	1.3243	1.4024	1.4934	1.4901	1.3872	1.3346	1.4478	1.3531	1.2018	1.3059	1.4781	1.4305

However, all of the year-over-year monthly series show a nice smooth year-over-year trend.

22.28 Approximate fixed-base year-over-year Laspeyres, Paasche, and Fisher indices can be constructed by replacing current-month revenue shares for the five products with the corresponding base-year monthly revenue shares for the same five products. The resulting approximate Laspeyres indices are equal to the original fixed-base Laspeyres indices, so there is no need to table the approximate Laspeyres indices. However, the approximate year-over-year Paasche and Fisher indices do differ from the fixed-base Paasche and Fisher indices found in Tables 22.4 and 22.5, so these new approximate indices are listed in Tables 22.6 (on preceding page) and 22.7.

22.29 Comparing the entries in Table 22.4 with the corresponding entries in Table 22.6, it can be seen that with few exceptions, the entries correspond fairly well. One of the bigger differences is the 1973 entry for the fixed-base Paasche index for month 9, which is 1.1664, while the corresponding entry for the approximate fixed-base Paasche index is 1.1920, for a 2.2 percent difference (1.1920/1.1664 = 1.022). In general, the approximate fixed-base Paasche indices are a bit bigger than the true fixed-base Paasche indices, as one might expect, because the approximate indices have some substitution bias built in. This is due to the fact that their revenue shares are held fixed at the 1970 levels.

22.30 Turning now to the chained year-over-year monthly indices using the artificial data set, the resultant 12 year-over-year monthly chained Laspeyres, Paasche, and Fisher indices, P_L, P_P, and P_F, where the month-to-month links are defined by equations (22.4)–(22.6), are listed in Tables 22.8 to 22.10.

22.31 Comparing the entries in Tables 22.8 and 22.9, it can be seen that the year-over-year monthly chained Laspeyres and Paasche price indices have smaller differences than the corresponding fixed-base Laspeyres and Paasche price indices in Tables 22.3 and 22.4. This is a typical pattern that was found in Chapter 19: *the use of chained indices tends to reduce the spread between Paasche and Laspeyres indices compared with their fixed-base counterparts*. The largest percentage difference between corresponding entries for the chained Laspeyres and Paasche indices in Tables 22.8 and 22.9 is 4.1 percent for month 10 in 1973 (1.3593/1.3059 = 1.041). Recall that the fixed-base Laspeyres and Paasche indices differed by 12.5 percent for the same month so that *chaining does tend to reduce the spread between these two equally plausible indices.*

22.32 The chained year-over-year Fisher indices listed in Table 22.10 are regarded as the best estimates of year-over-year inflation using the artificial data set.

Table 22.10. Year-over-Year Monthly Chained Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2048	1.2438	1.3052	1.2780	1.2114	1.1701	1.2343	1.1660	1.0886	1.1555	1.2405	1.1926
1973	1.3258	1.4027	1.4942	1.4906	1.3937	1.3378	1.4500	1.3728	1.2181	1.3323	1.4978	1.4368

Table 22.11. Year-over-Year Monthly Approximate Chained Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2056	1.2440	1.3057	1.2778	1.2168	1.1712	1.2346	1.1770	1.0989	1.1692	1.2482	1.1939
1973	1.3255	1.4007	1.4945	1.4902	1.4054	1.3390	1.4491	1.4021	1.2429	1.3611	1.5173	1.4417

Table 22.12. Year-over-Year Monthly Approximate Chained Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2033	1.2424	1.3043	1.2764	1.2130	1.1664	1.2287	1.1638	1.0858	1.1438	1.2328	1.1886
1973	1.3206	1.3971	1.4914	1.4880	1.3993	1.3309	1.4386	1.3674	1.2183	1.3111	1.4839	1.4300

Table 22.13. Year-over-Year Monthly Approximate Chained Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2044	1.2432	1.3050	1.2771	1.2149	1.1688	1.2317	1.1704	1.0923	1.1565	1.2405	1.1912
1973	1.3231	1.3989	1.4929	1.4891	1.4024	1.3349	1.4438	1.3847	1.2305	1.3358	1.5005	1.4358

22.33 The year-over-year chained Laspeyres, Paasche, and Fisher indices listed in Tables 22.8 to 22.10 can be approximated by replacing current-period product revenue shares for each month with the corresponding base-year monthly revenue shares. The resultant 12 year-over-year monthly approximate chained Laspeyres, Paasche, and Fisher indices (P_{AL}, P_{AP}, and P_{AF}), where the monthly links are defined by equations (22.8)–(22.10), are listed in Tables 22.11–22.13. (Tables 22.11 and 22.12 are on the preceding page.)

22.34 The year-over-year chained indices listed in Tables 22.11–22.13 approximate their true chained counterparts listed in Tables 22.8–22.10 closely. For 1973, the largest discrepancies are for the Paasche and Fisher indices for month 9: the chained Paasche is 1.2018, while the corresponding approximate chained Paasche is 1.2183, for a difference of 1.4 percent. The chained Fisher is 1.2181, while the corresponding approximate chained Fisher is 1.2305, for a difference of 1.0 percent. It can be seen that for the modified Turvey data set, the approximate year-over-year monthly Fisher indices listed in Table 22.13 approximate the theoretically preferred (but practically unfeasible) Fisher chained indices listed in Table 22.10 quite satisfactorily. Since the approximate Fisher indices are just as easy to compute as the approximate Laspeyres and Paasche indices, it may be useful to ask statistical agencies to make available to the public these approximate Fisher indices, along with the approximate Laspeyres and Paasche indices.

D. Year-over-Year Annual Indices

22.35 Assuming that each product in each season of the year is a separate annual product is the simplest and theoretically most satisfactory method for dealing with seasonal products when the goal is to construct annual price and quantity indices. This idea can be traced back to Mudgett in the consumer price context and to Stone in the producer price context:

> The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years. (Bruce D. Mudgett, 1955, p. 97)

> The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities. (Richard Stone, 1956, pp. 74–75)

22.36 Using the notation introduced in the previous section, the *Laspeyres, Paasche, and Fisher annual (chain link) indices* comparing the prices of year t with those of year $t + 1$ can be defined as follows:

$$(22.13)\quad P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12}\right) \equiv \frac{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t+1,m}q_n^{t,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t,m}q_n^{t,m}};$$

$$(22.14)\quad P_P(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12}; q^{t+1,1},...,q^{t+1,12})$$

$$\equiv \frac{\sum_{m=1}^{12} \sum_{n \in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{m=1}^{12} \sum_{n \in S(m)} p_n^{t,m} q_n^{t+1,m}};$$

$$(22.15)\quad P_F(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12}; q^{t,1},...,q^{t,12};q^{t+1,1},...,q^{t+1,12})$$
$$\equiv \sqrt{P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12}\right)}$$
$$\times \sqrt{P_P\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t+1,1},...,q^{t+1,12}\right)}.$$

22.37 The above formulas can be rewritten in price relative and monthly revenue share form as follows:

$$(22.16)\quad P_L(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12}; \sigma_1^t s^{t,1},...,\sigma_{12}^t s^{t,12})$$
$$\equiv \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m} \left(p_n^{t+1,m} / p_n^{t,m}\right);$$

$$(22.17)\quad P_P(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1^{t+1} s^{t+1,1}, \sigma_1^{t+1} s^{t+1,1},...,\sigma_{12}^{t+1} s^{t+1,12})$$
$$\equiv \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m} \left(p_n^{t+1,m} / p_n^{t,m}\right)$$
$$\equiv \left[\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^{t+1} s_n^{t+1,m} \left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1}$$
$$= \left[\sum_{m=1}^{12} \sigma_m^{t+1} \sum_{n \in S(m)} s_n^{t+1,m} \left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1}$$
$$= \left[\sum_{m=1}^{12} \sigma_m^{t+1} \left[P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)\right]^{-1}\right]^{-1};$$

$$(22.18)\quad P_F(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12}; \sigma_1^t s^{t,1},...,\sigma_{12}^t s^{t,12};\sigma_1^{t+1} s^{t+1,1},...,\sigma_{12}^{t+1} s^{t+1,12})$$
$$\equiv \sqrt{\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m} \left(p_n^{t+1,m} / p_n^{t,m}\right)}$$
$$\times \sqrt{\left[\sum_{m=1}^{12} n \sum_{\in S(m)} \sigma_m^{t+1} s_n^{t+1,m} \left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1}}$$
$$= \sqrt{\sum_{m=1}^{12} \sigma_m^t \left[P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right)\right]}$$
$$\times \sqrt{\left[\sum_{m=1}^{12} \sigma_m^{t+1} \left[P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)\right]^{-1}\right]^{-1}},$$

where the *revenue share* for month m in year t is defined as

$$(22.19)\quad \sigma_n^t \equiv \frac{\sum_{n \in S(m)} p_n^{t,m} q_n^{t,m}}{\sum_{i=1}^{12} \sum_{j \in S(i)} p_j^{t,i} q_j^{t,i}};\ m = 1,2,...,12;$$

$t = 0,1,...,T$;

and the year-over-year monthly Laspeyres and Paasche (chained-linked) price indices $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$ were defined in the previous section by equations (22.4) and (22.5), respectively. As usual, the annual chain-linked Fisher index P_F defined by equation (22.18), which compares the prices in every month of year t with the corresponding prices in year t + 1, is the geometric mean of the annual chain-linked Laspeyres and Paasche indices, P_L and P_P, defined by equations (22.16) and (22.17). The last equation in equations (22.16), (22.17), and (22.18) shows that these annual indices can be defined as (monthly) share weighted averages of the year-over-year monthly chain-linked Laspeyres and Paasche indices, $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$, defined earlier by equations (22.4) and (22.5). Hence, once the year-over-year monthly indices defined in the previous section have been numerically calculated, it is easy to calculate the corresponding annual indices.

22.38 Fixed-base counterparts to the formulas defined by equations (22.16)–(22.18) can readily be defined: simply replace the data pertaining to period t with the corresponding data pertaining to the base period 0.

22.39 Using the data from the artificial data set in Table 22.1 of Section B, the annual fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 22.14. Viewing Table 22.14, it can be seen that by 1973, the annual fixed-base Laspeyres index exceeds its Paasche counterpart by 4.5 percent. Note that each series increases steadily.

22.40 The annual fixed-base Laspeyres, Paasche, and Fisher indices can be approximated by replacing any current shares with the corresponding base-year shares. The resulting annual approximate fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 22.15. Also listed in the last column of Table 22.15 is the fixed-base geometric Laspeyres annual index, P_{GL}. It is the weighted geometric mean counterpart to the fixed-base Laspeyres index, which is equal to a base-period weighted arithmetic average of the long-term price relative (see Chapter 19). It can be shown that P_{GL} approximates the approximate fixed-base Fisher index P_{AF} to the second order around a point where all of the long-term price relatives are equal to unity.[14] It is evident that the entries for the Laspeyres price indices are exactly the same in Tables 22.14 and 22.15. This is as it should be because the fixed-base Laspeyres price index uses only revenue shares from the base year 1970; consequently, the approximate fixed-base Laspeyres index is equal to the true fixed-base Laspeyres index. Comparing the columns labeled P_P and P_F in Table 22.14 and P_{AP} and P_{AF} in Table 22.15 shows that the approximate Paasche and approximate Fisher indices are quite close to the corresponding annual Paasche and Fisher indices. Thus, for the artificial data set, *the true annual fixed-base Fisher can be closely approximated by the corresponding approximate Fisher index* P_{AF} (or the geometric Laspeyres index P_{GL}), *which can be computed using the same information set that is normally available to statistical agencies*.

Table 22.14. Annual Fixed-Base Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2091	1.1884	1.1987
1973	1.4144	1.3536	1.3837

Table 22.15. Annual Approximate Fixed-Base Laspeyres, Paasche, Fisher, and Geometric Laspeyres Indices

Year	P_{AL}	P_{AP}	P_{AF}	P_{GL}
1970	1.0000	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982	1.0983
1972	1.2091	1.1903	1.1996	1.2003
1973	1.4144	1.3596	1.3867	1.3898

22.41 Using the data from the artificial data set in Table 22.1 of Section B, the annual chained Laspeyres, Paasche, and Fisher indices can readily be calculated using the equations (22.16)–(22.18) for the chain links. The resulting indices are listed in Table 22.16. Viewing Table 22.16, it can be seen that the use of chained indices has substantially narrowed the gap between the Paasche and Laspeyres indices. The difference between the chained annual Laspeyres and Paasche indices in 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas from Table 22.14, the difference between the fixed-base annual Laspeyres and Paasche indices in 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained annual indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices*. Comparing Tables 22.14 and 22.16, it can be seen that for this particular artificial data set, the annual fixed-base Fisher indices P_F are close to their annual chained Fisher counterparts P_{AF}. However, the annual chained Fisher indices should normally be regarded as the more desirable target index to approximate, since this index will normally give better results if prices and revenue shares are changing substantially over time.[15]

22.42 The current-year weights, $s_n^{t,m}$ and $\sigma_m^{\ t}$ and $s_n^{t+1,m}$ and $\sigma_m^{\ t+1}$, which appear in the chain-linked equations (22.16)–(22.18), can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and $\sigma_m^{\ 0}$. This leads to the annual approximate chained

[14]See footnote 12.

[15]"Better" in the sense that the gap between the Laspeyres and Paasche indices will normally be reduced using chained indices under these circumstances. Of course, if there are no substantial trends in prices so that prices are just randomly changing, then it will generally be preferable to use the fixed-base Fisher index.

Table 22.16. Annual Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2052	1.1949	1.2001
1973	1.3994	1.3791	1.3892

Table 22.17. Annual Approximate Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_{AL}	P_{AP}	P_{AF}
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982
1972	1.2051	1.1952	1.2002
1973	1.3995	1.3794	1.3894

Laspeyres, Paasche, and Fisher indices listed in Table 22.17.

22.43 Comparing the entries in Tables 22.16 and 22.17 shows that the approximate chained annual Laspeyres, Paasche, and Fisher indices are extremely close to the corresponding true chained annual Laspeyres, Paasche, and Fisher indices. Therefore, for the artificial data set, the true annual chained Fisher can be closely approximated by the corresponding approximate Fisher index, which can be computed using the same information set that is normally available to statistical agencies.

22.44 The approach to computing annual indices outlined in this section, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indices, should be contrasted with the approach that simply takes the arithmetic mean of the 12 monthly indices. The problem with the latter approach is that months where revenues are below the average (for example, February) are given the same weight in the unweighted annual average as months where revenues are above the average (for example, December).

E. Rolling-Year Annual Indices

22.45 In the previous section, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. However, there is no need to restrict attention to calendar year comparisons; any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the noncalendar year is compared to the January data of the base year, the February data of the noncalendar year is compared to the February data of the base year, and so on.[16] Alterman, Diewert, and Feenstra (1999, p. 70) called the resulting indices *rolling-year* or *moving-year* indices.[17]

22.46 In order to theoretically justify the rolling-year indices from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996b, pp. 32–34; 1999a, pp. 56–61).

22.47 The problems involved in constructing rolling-year indices for the artificial data set that was introduced in Section B are now considered. For both fixed-base and chained rolling-year indices, the first 13 index number calculations are the same. For the year that ends with the data for December of 1970, the index is set equal to 1 for the Laspeyres, Paasche, and Fisher moving-year indices. The base-year data are the 44 nonzero price and quantity observations for the calendar year 1970. When the data for January of 1971 become available, the three nonzero price and quantity entries for January of calendar year 1970 are dropped and replaced with the corresponding entries for January of 1971. The data for the remaining months of the comparison year remain the same; that is, for February through December of the comparison year, the data for the rolling year are set equal to the corresponding entries for February through December of 1970. Thus, the Laspeyres, Paasche, or Fisher rolling-year index value for

[16]Diewert (1983b) suggested this type of comparison and termed the resulting index a *split year* comparison.

[17]Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term *rolling year* seems to be well established in the business literature in the United Kingdom.

January of 1971 compares the prices and quantities of January 1971 with the corresponding prices and quantities of January 1970, and for the remaining months of this first moving year, the prices and quantities of February through December of 1970 are simply compared with the exact same prices and quantities of February through December of 1970. When the data for February of 1971 become available, the three nonzero price and quantity entries for February for the last rolling year (which are equal to the three nonzero price and quantity entries for February of 1970) are dropped and replaced with the corresponding entries for February of 1971. The resulting data become the price and quantity data for the second rolling year. The Laspeyres, Paasche, or Fisher rolling-year index value for February of 1971 compares the prices and quantities of January and February of 1971 with the corresponding prices and quantities of January and February of 1970. For the remaining months of this first moving year, the prices and quantities of March through December of 1971 are compared with the exact same prices and quantities of March through December of 1970. This process of exchanging the price and quantity data of the current month in 1971 with the corresponding data of the same month in the base year 1970 in order to form the price and quantity data for the latest rolling year continues until December of 1971 is reached, when the current rolling year becomes the calendar year 1971. Thus, the Laspeyres, Paasche, and Fisher rolling-year indices for December of 1971 are equal to the corresponding fixed-base (or chained) annual Laspeyres, Paasche, and Fisher indices for 1971 listed in Tables 22.14 or 22.16.

22.48 Once the first 13 entries for the rolling-year indices have been defined as indicated, the remaining fixed-base rolling year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and rearranging them so that the January data in the rolling year is compared with the January data in the base year, the February data in the rolling year is compared with the February data in the base year, and so on. The resulting fixed-base rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in Table 22.18.

22.49 Once the first 13 entries for the fixed-base rolling-year indices have been defined as indicated, the remaining *chained* rolling-year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and comparing them with the corresponding data of the rolling year of the 12 months preceding the current rolling year. The resulting chained rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in the last three columns of Table 22.18. Note that the first 13 entries of the fixed-base Laspeyres, Paasche, and Fisher indices are equal to the corresponding entries for the chained Laspeyres, Paasche, and Fisher indices. Also, the entries for December (month 12) of 1970, 1971, 1972, and 1973 for the fixed-base rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding fixed-base annual Laspeyres, Paasche, and Fisher indices listed in Table 22.14. Similarly, the entries in Table 22.18 for December (month 12) of 1970, 1971, 1972, and 1973 for the chained rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding chained annual Laspeyres, Paasche, and Fisher indices listed in Table 22.16.

22.50 In Table 22.18, the rolling-year indices are smooth and free from seasonal fluctuations. For the fixed-base indices, each entry can be viewed as a *seasonally adjusted annual PPI* that compares the data of the 12 consecutive months that end with the year and month indicated with the corresponding price and quantity data of the 12 months in the base year, 1970. Thus, rolling-year indices offer statistical agencies an *objective* and *reproducible* method of seasonal adjustment that can compete with existing time series methods of seasonal adjustment.[18]

22.51 The use of chained indices substantially narrows the gap between the fixed-base moving-year Paasche and Laspeyres indices as shown in

[18]For discussions on the merits of econometric or time-series methods versus index number methods of seasonal adjustment, see Diewert (1999a, pp. 61–68) and Alterman, Diewert, and Feenstra (1999, pp. 78–110). The basic problem with time-series methods of seasonal adjustment is that the target seasonally adjusted index is difficult to specify in an unambiguous way; that is, there are an infinite number of possible target indices. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Thus, different econometricians will tend to generate different seasonally adjusted series, leading to a lack of reproducibility.

Table 22.18. Rolling-Year Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L (fixed)	P_P (fixed)	P_F (fixed)	P_L (chain)	P_P (chain)	P_F (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0087	1.0085	1.0082	1.0087	1.0085
	2	1.0161	1.0170	1.0165	1.0161	1.0170	1.0165
	3	1.0257	1.0274	1.0265	1.0257	1.0274	1.0265
	4	1.0344	1.0364	1.0354	1.0344	1.0364	1.0354
	5	1.0427	1.0448	1.0438	1.0427	1.0448	1.0438
	6	1.0516	1.0537	1.0527	1.0516	1.0537	1.0527
	7	1.0617	1.0635	1.0626	1.0617	1.0635	1.0626
	8	1.0701	1.0706	1.0704	1.0701	1.0706	1.0704
	9	1.0750	1.0740	1.0745	1.0750	1.0740	1.0745
	10	1.0818	1.0792	1.0805	1.0818	1.0792	1.0805
	11	1.0937	1.0901	1.0919	1.0937	1.0901	1.0919
	12	1.1008	1.0961	1.0984	1.1008	1.0961	1.0984
1972	1	1.1082	1.1035	1.1058	1.1081	1.1040	1.1061
	2	1.1183	1.1137	1.1160	1.1183	1.1147	1.1165
	3	1.1287	1.1246	1.1266	1.1290	1.1260	1.1275
	4	1.1362	1.1324	1.1343	1.1366	1.1342	1.1354
	5	1.1436	1.1393	1.1414	1.1437	1.1415	1.1426
	6	1.1530	1.1481	1.1505	1.1528	1.1505	1.1517
	7	1.1645	1.1595	1.1620	1.1644	1.1622	1.1633
	8	1.1757	1.1670	1.1713	1.1747	1.1709	1.1728
	9	1.1812	1.1680	1.1746	1.1787	1.1730	1.1758
	10	1.1881	1.1712	1.1796	1.1845	1.1771	1.1808
	11	1.1999	1.1805	1.1901	1.1962	1.1869	1.1915
	12	1.2091	1.1884	1.1987	1.2052	1.1949	1.2001
1973	1	1.2184	1.1971	1.2077	1.2143	1.2047	1.2095
	2	1.2300	1.2086	1.2193	1.2263	1.2172	1.2218
	3	1.2425	1.2216	1.2320	1.2393	1.2310	1.2352
	4	1.2549	1.2341	1.2444	1.2520	1.2442	1.2481
	5	1.2687	1.2469	1.2578	1.2656	1.2579	1.2617
	6	1.2870	1.2643	1.2756	1.2835	1.2758	1.2797
	7	1.3070	1.2843	1.2956	1.3038	1.2961	1.3000
	8	1.3336	1.3020	1.3177	1.3273	1.3169	1.3221
	9	1.3492	1.3089	1.3289	1.3395	1.3268	1.3331
	10	1.3663	1.3172	1.3415	1.3537	1.3384	1.3460
	11	1.3932	1.3366	1.3646	1.3793	1.3609	1.3700
	12	1.4144	1.3536	1.3837	1.3994	1.3791	1.3892

Table 22.18. The difference between the rolling-year chained Laspeyres and Paasche indices in December of 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas the difference between the rolling-year fixed-base Laspeyres and Paasche indices in December of 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices*. As in the previous section, the chained Fisher rolling-year index is regarded as the *target seasonally adjusted annual index* when seasonal products are in the scope of the CPI. This type of index is also a suitable index for central banks to

use for inflation targeting purposes.[19] The six series in Table 22.18 are charted in Figure 22.1. The fixed-base Laspeyres index is the highest one, followed by the chained Laspeyres, the two Fisher indices (which are virtually indistinguishable), the chained Paasche, and, finally, the fixed-base Paasche. An increase in the slope of each graph can clearly be seen for the last 8 months, reflecting the increase in the month-to-month inflation rates that was built into the last 12 months of the data set.[20]

22.52 As in the previous section, the current-year weights, $s_n^{t,m}$ and $\sigma_m^{\ t}$ and $s_n^{t+1,m}$ and $\sigma_m^{\ t+1}$, which appear in the chain link equations (22.16)–(22.18) or in the corresponding fixed-base formulas, can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and $\sigma_m^{\ 0}$. This leads to the annual approximate fixed-base and chained rolling-year Laspeyres, Paasche, and Fisher indices listed in Table 22.19.

22.53 Comparing the indices in Tables 22.18 and 22.19, it can be seen that the approximate rolling-year fixed-base and chained Laspeyres, Paasche, and Fisher indices listed in Table 22.19 are close to their true rolling-year counterparts listed in Table 22.18. In particular, the approximate chain rolling-year Fisher index (which can be computed using just base-year expenditure share information along with current information on prices) is close to the preferred target index, the rolling-year chained Fisher index. In December of 1973, these two indices differ by only 0.014 percent (1.3894/1.3892 = 1.00014). The indices in Table 22.19 are charted in Figure 22.2. Figures 22.1 and 22.2 are similar; in particular, the Fisher fixed-base and chained indices are virtually identical in both figures.

[19]See Diewert (2002c) for a discussion of the measurement issues involved in choosing an index for inflation targeting purposes.

[20]The arithmetic average of the 36 month-over-month inflation rates for the rolling-year fixed-base Fisher indices is 1.0091; the average of these rates for the first 24 months is 1.0076; for the last 12 months it is 1.0120; and for the last 2 months it is 1.0156. Thus, the increased month-to-month inflation rates for the last year are not *fully* reflected in the rolling-year indices until a full 12 months have passed. However, the fact that inflation has *increased* for the last 12 months of data compared to the earlier months is picked up almost immediately.

22.54 These tables demonstrate that year-over-year monthly indices and their generalizations to rolling-year indices perform very well using the modified Turvey data set; that is, like is compared to like and the existence of seasonal products does *not* lead to erratic fluctuations in the indices. The only drawback to the use of these indices is that it seems that they cannot give any information on *short-term month-to-month fluctuations in prices. This is most evident if seasonal baskets are completely different for each month, since in this case there is no possibility of comparing prices on a month-to-month basis*. However, in the following section, we learn that a current-period year-over-year monthly index *can* be used to predict a rolling-year index that is centered at the current month.

F. Predicting Rolling-Year Index Using Current-Period Year-over-Year Monthly Index

22.55 In a regime where the long-run trend in prices is smooth, changes in the year-over-year inflation rate for this month compared with last month theoretically could give valuable information about the long-run trend in price inflation. For the modified Turvey data set, this conjecture turns out to be true, as will be seen below.

22.56 The basic idea will be illustrated using the fixed-base Laspeyres rolling-year indices that are listed in Table 22.18 and the year-over-year monthly fixed-base Laspeyres indices listed in Table 22.3. In Table 22.18, the fixed-base Laspeyres rolling-year entry for December of 1971 compares the 12 months of price and quantity data pertaining to 1971 with the corresponding prices and quantities pertaining to 1970. This index number is the first entry in the first column of Table 22.20 and is labeled as P_L. Thus, in the first column of Table 22.20, the fixed-base rolling-year Laspeyres index, P_{LRY} taken from Table 22.18, is tabled starting at December of 1971 and carrying through to December of 1973, 24 observations in all. The first entry of this column shows that the index is a weighted average of year-over-year price relatives over all 12 months in 1970 and 1971. Thus, this index is an average of year-over-year monthly price changes, centered between June and July of the two years whose prices are being compared. As a result, an *approximation* to this annual index

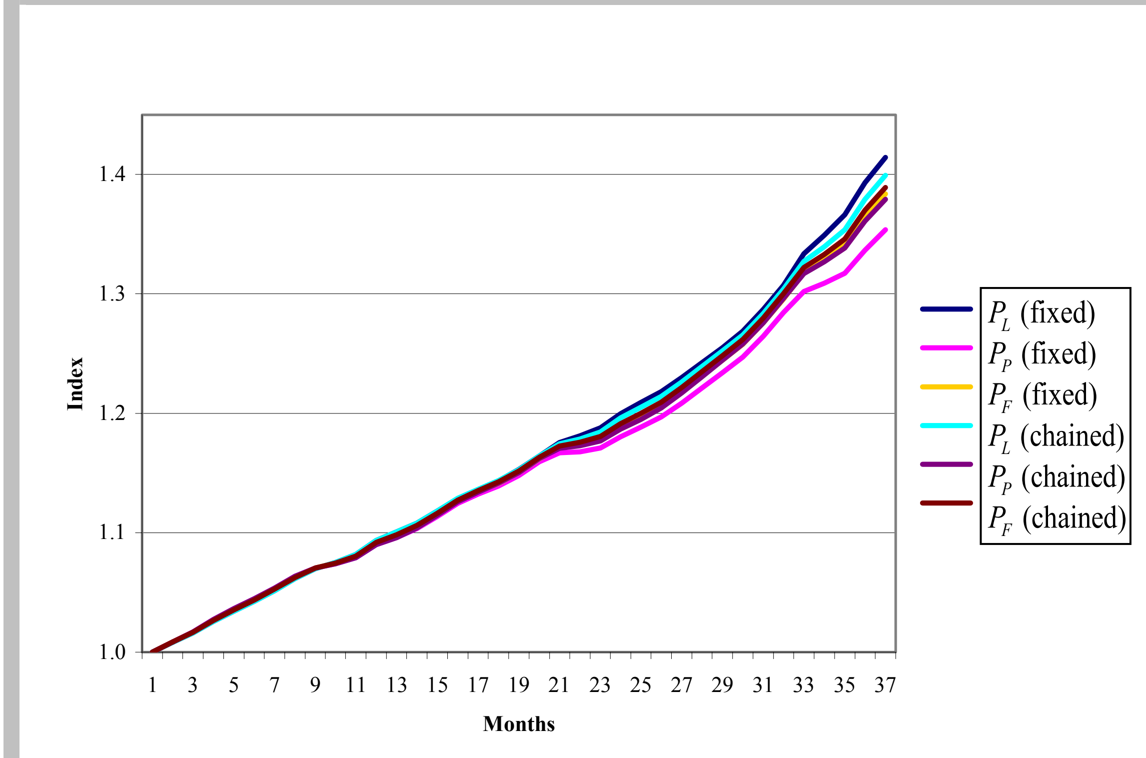
Table 22.19. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_{AL} (fixed)	P_{AP} (fixed)	P_{AF} (fixed)	P_{AL} (chain)	P_{AP} (chain)	P_{AF} (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0074	1.0078	1.0082	1.0074	1.0078
	2	1.0161	1.0146	1.0153	1.0161	1.0146	1.0153
	3	1.0257	1.0233	1.0245	1.0257	1.0233	1.0245
	4	1.0344	1.0312	1.0328	1.0344	1.0312	1.0328
	5	1.0427	1.0390	1.0409	1.0427	1.0390	1.0409
	6	1.0516	1.0478	1.0497	1.0516	1.0478	1.0497
	7	1.0617	1.0574	1.0596	1.0617	1.0574	1.0596
	8	1.0701	1.0656	1.0679	1.0701	1.0656	1.0679
	9	1.0750	1.0702	1.0726	1.0750	1.0702	1.0726
	10	1.0818	1.0764	1.0791	1.0818	1.0764	1.0791
	11	1.0937	1.0881	1.0909	1.0937	1.0881	1.0909
	12	1.1008	1.0956	1.0982	1.1008	1.0956	1.0982
1972	1	1.1082	1.1021	1.1051	1.1083	1.1021	1.1052
	2	1.1183	1.1110	1.1147	1.1182	1.1112	1.1147
	3	1.1287	1.1196	1.1241	1.1281	1.1202	1.1241
	4	1.1362	1.1260	1.1310	1.1354	1.1268	1.1311
	5	1.1436	1.1326	1.1381	1.1427	1.1336	1.1381
	6	1.1530	1.1415	1.1472	1.1520	1.1427	1.1473
	7	1.1645	1.1522	1.1583	1.1632	1.1537	1.1584
	8	1.1757	1.1620	1.1689	1.1739	1.1642	1.1691
	9	1.1812	1.1663	1.1737	1.1791	1.1691	1.1741
	10	1.1881	1.1710	1.1795	1.1851	1.1747	1.1799
	11	1.1999	1.1807	1.1902	1.1959	1.1855	1.1907
	12	1.2091	1.1903	1.1996	1.2051	1.1952	1.2002
1973	1	1.2184	1.1980	1.2082	1.2142	1.2033	1.2087
	2	1.2300	1.2074	1.2187	1.2253	1.2133	1.2193
	3	1.2425	1.2165	1.2295	1.2367	1.2235	1.2301
	4	1.2549	1.2261	1.2404	1.2482	1.2340	1.2411
	5	1.2687	1.2379	1.2532	1.2615	1.2464	1.2540
	6	1.2870	1.2548	1.2708	1.2795	1.2640	1.2717
	7	1.3070	1.2716	1.2892	1.2985	1.2821	1.2903
	8	1.3336	1.2918	1.3125	1.3232	1.3048	1.3139
	9	1.3492	1.3063	1.3276	1.3386	1.3203	1.3294
	10	1.3663	1.3182	1.3421	1.3538	1.3345	1.3441
	11	1.3932	1.3387	1.3657	1.3782	1.3579	1.3680
	12	1.4144	1.3596	1.3867	1.3995	1.3794	1.3894

could be obtained by taking the arithmetic average of the June and July year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 6 and 7 for the year 1971 in Table 22.3, 1.0844 and 1.1103).[21] For the next rolling-year fixed-base Laspeyres index corresponding to the January 1972 entry in Table 22.18, *an approximation to this rolling-year index*, P_{ARY}, could be derived by taking the arithmetic average of the July and August year-over-year monthly

[21]If an average of the year-over-year monthly indices for May, June, July, and August were taken, a better approximation to the annual index could be obtained, and if an average of the year-over-year monthly indices for April, May, (continued) June, July, August, and September were taken, an even better approximation could be obtained to the annual index, and so on.

Figure 22.1. Rolling-Year Fixed-Base and Chained Laspeyres, Paasche, and Fisher Indices



indices pertaining to the years 1970 and 1971 (see the entries for months 7 and 8 for 1971 in Table 22.3, 1.1103 and 1.0783, respectively). These arithmetic averages of the two year-over-year monthly indices that are in the middle of the corresponding rolling-year are listed in the third column of Table 22.20. Table 22.20 shows that column 3, P_{ARY}, does not approximate column 1 particularly well, since the approximate indices in column 3 have some pronounced seasonal fluctuations, whereas the rolling-year indices in column 1, P_{LRY}, are free from seasonal fluctuations.

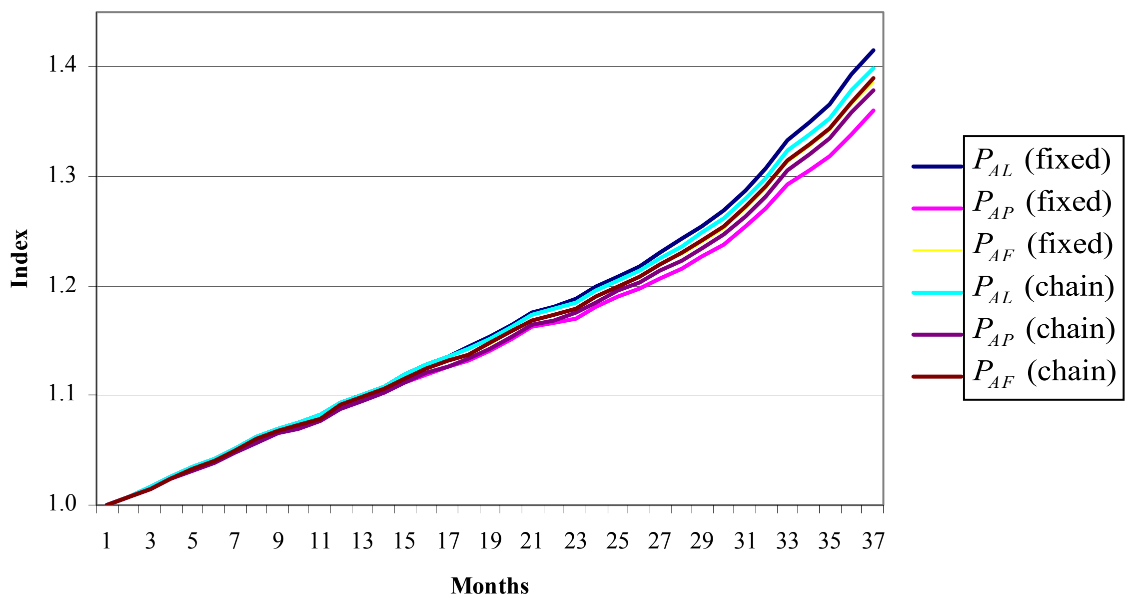
22.57 In the fourth column of Table 22.20, some *seasonal adjustment factors* are listed. For the first 12 observations, the entries in column 4 are simply the ratios of the entries in column 1 divided by the corresponding entries in column 3; that is, for the first 12 observations, the seasonal adjustment factors, *SAF*, are simply the ratio of the rolling-year indices starting at December of 1971 divided by the arithmetic average of the two year-over-year monthly indices that are in the middle of the corresponding rolling year.[22] The initial 12 seasonal adjustment factors are then just repeated for the remaining entries for column 4.

22.58 Once the seasonal adjustment factors have been defined, the approximate rolling-year index P_{ARY} can be multiplied by the corresponding seasonal adjustment factor, *SAF*, to form a *seasonally adjusted approximate rolling-year index*, P_{SAARY}, which is listed in column 2 of Table 22.20.

22.59 Compare columns 1 and 2 in Table 22.20: the rolling-year fixed-base Laspeyres index P_{LRY} and the seasonally adjusted approximate rolling-year index P_{SAARY} are identical for the first 12 observations, which follows by construction since P_{SAARY} equals the approximate rolling-year index

[22]Thus, if *SAF* is *greater than* 1, this means that the two months in the middle of the corresponding rolling year have year-over-year rates of price increase that average out to a number *below* the overall average of the year-over-year rates of price increase for the entire rolling year. The opposite is true if *SAF* is less than 1.

Figure 22.2. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices



P_{ARY} multiplied by the seasonal adjustment factor *SAF*, which in turn is equal to the rolling-year Laspeyres index P_{LRY} divided by P_{ARY}. However, starting at December of 1972, the rolling-year index P_{LRY} differs from the corresponding seasonally adjusted approximate rolling-year index P_{SAARY}. It is apparent that for these last 13 months, P_{SAARY} is surprisingly close to P_{LRY}.[23] P_{LRY}, P_{SAARY}, and P_{ARY} are graphed in Figure 22.3. Due to the acceleration in the monthly inflation rate for the last year of data, it can be seen that the seasonally adjusted approximate rolling-year series, P_{SAARY}, does not pick up this accelerated inflation rate for the first few months of the last year (it lies well below P_{LRY} for February and March of 1973), but in general, it predicts the corresponding centered year quite well.

22.60 The above results for the modified Turvey data set are quite encouraging. If these results can be replicated for other data sets, *statistical agencies will be able to use the latest information on year-over-year monthly inflation to predict reasonably well the (seasonally adjusted) rolling-year inflation rate for a rolling year that is centered around the last two months*. Thus, policymakers and other interested users of the PPI could obtain a reasonably accurate forecast of trend inflation (centered around the current month) some six months in advance of the final estimates.

22.61 The method of seasonal adjustment used in this section is rather crude compared with some of the sophisticated econometric or statistical methods that are available. These more sophisticated methods could be used to improve the forecasts of trend inflation. However, it should be noted that if improved forecasting methods are used, it will be useful to use the rolling-year indices as *targets* for the forecasts rather than using a statistical package that simultaneously seasonally adjusts current data and calculates a trend rate of

[23]The means for the last 13 observations in columns 1 and 2 of Table 22.20 are 1.2980 and 1.2930. A regression of P_L on P_{SAARY} leads to an R^2 of 0.9662 with an estimated variance of the residual of .000214.

Table 22.20. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices

Year	Month	P_{LRY}	P_{SAARY}	P_{ARY}	SAF
1971	12	1.1008	1.1008	1.0973	1.0032
1972	1	1.1082	1.1082	1.0943	1.0127
	2	1.1183	1.1183	1.0638	1.0512
	3	1.1287	1.1287	1.0696	1.0552
	4	1.1362	1.1362	1.1092	1.0243
	5	1.1436	1.1436	1.1066	1.0334
	6	1.1530	1.1530	1.1454	1.0066
	7	1.1645	1.1645	1.2251	0.9505
	8	1.1757	1.1757	1.2752	0.9220
	9	1.1812	1.1812	1.2923	0.9141
	10	1.1881	1.1881	1.2484	0.9517
	11	1.1999	1.1999	1.1959	1.0033
	12	1.2091	1.2087	1.2049	1.0032
1973	1	1.2184	1.2249	1.2096	1.0127
	2	1.2300	1.2024	1.1438	1.0512
	3	1.2425	1.2060	1.1429	1.0552
	4	1.2549	1.2475	1.2179	1.0243
	5	1.2687	1.2664	1.2255	1.0334
	6	1.2870	1.2704	1.2620	1.0066
	7	1.3070	1.2979	1.3655	0.9505
	8	1.3336	1.3367	1.4498	0.9220
	9	1.3492	1.3658	1.4943	0.9141
	10	1.3663	1.3811	1.4511	0.9517
	11	1.3932	1.3827	1.3783	1.0032
	12	1.4144	1.4188	1.4010	1.0127

inflation. What is being suggested here is that the rolling-year concept can be used to make reproducible the estimates of trend inflation that existing statistical methods of seasonal adjustment generate.[24]

22.62 In this section and the previous sections, all of the suggested indices have been based on year-over-year monthly indices and their averages. In the subsequent sections of this chapter, attention will be turned to more traditional price indices that attempt to compare the prices in the current month with the prices in a previous month.

G. Maximum Overlap Month-to-Month Price Indices

22.63 A reasonable method for dealing with seasonal products in the context of picking a target index for a month-to-month PPI is the following:[25]

- Identify products that are produced in both months; and
- For this maximum overlap set of products, calculate one of the three indices recommended in previous chapters; that is, the Fisher, Walsh, or Törnqvist-Theil index.[26]

Thus, the bilateral index number formula is applied only to the subset of products that are present in both periods.[27]

22.64 The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indices), or should the base month be fixed (leading to fixed-base indices)? It seems reasonable to prefer chained indices over fixed-base indices for two reasons:

- The set of seasonal products that overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed-base month (such as January of a base year). The comparisons made using chained indices, therefore, will be more comprehensive and accurate than those made using a fixed-base; and

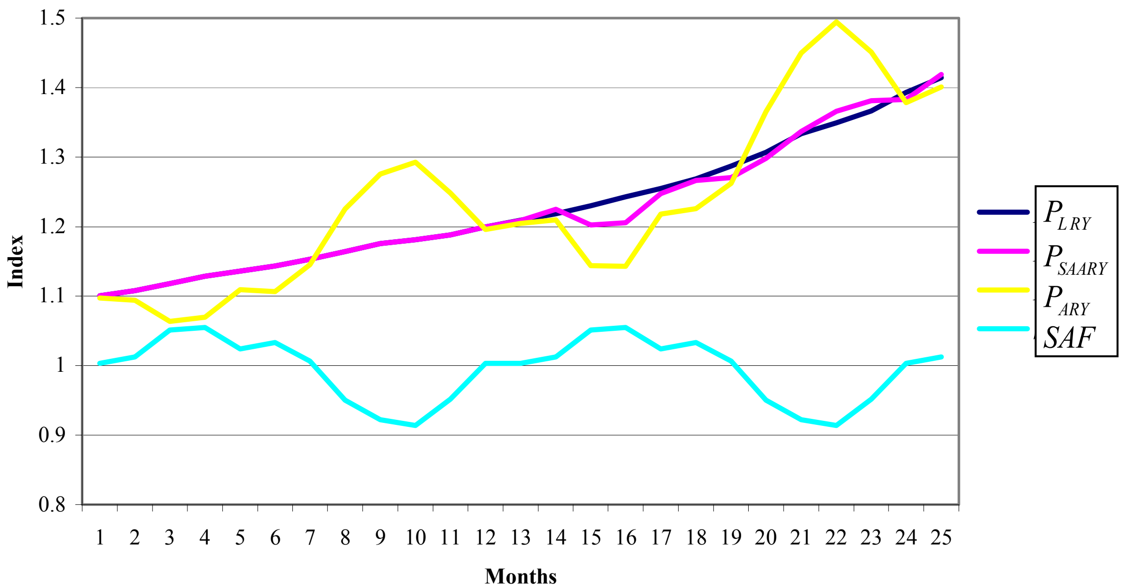
[24]The operator of a statistical seasonal adjustment package has to make somewhat arbitrary decisions on many factors; for example, are the seasonal factors additive or multiplicative? How long should the moving average be and what type? Thus, different operators of the seasonal adjustment package will tend to produce different estimates of the trend and the seasonal factors.

[25]For more on the economic approach and the assumptions on consumer preferences that can justify month-to-month maximum overlap indices, see Diewert (1999a, pp. 51–56).

[26]In order to reduce the number of equations, definitions, and tables, only the Fisher index will be considered in detail in this chapter.

[27]Keynes (1930, p. 95) called this the highest common factor method for making bilateral index number comparisons. This target index drops those strongly seasonal products that are not present in the marketplace during one of the two months being compared. Thus, the index number comparison is not completely comprehensive. Mudgett (1951, p. 46) called the error in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the homogeneity error.

Figure 22.3. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices



- In many economies, on average 2 or 3 percent of price quotes disappear each month due to the introduction of new products and the disappearance of older ones. This rapid sample attrition means that fixed-base indices rapidly become unrepresentative; as a consequence, it seems preferable to use chained indices, which can more closely follow market developments.[28]

22.65 It will be useful to review the notation at this point and define some new notation. Let there be N products that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is in the marketplace[29] in month m of year t (if the product is unavailable, define $p_n^{t,m}$ and $q_n^{t,m}$ to be 0). Let $p^{t,m} \equiv [p_1^{t,m}, p_2^{t,m}, \ldots, p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m}, q_2^{t,m}, \ldots, q_N^{t,m}]$ be the month m and year t price and quantity vectors, respectively. Let $S(t,m)$ be the set of products that is present in month m of year t *and the following month*. Then the maximum overlap Laspeyres, Paasche, and Fisher indices going from month m of year t to the following month can be defined as follows:[30]

$$(22.20)\ P_L\left(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m)\right) = \frac{\sum_{n \in S(t,m)} p_n^{t,m+1} q_n^{t,m}}{\sum_{n \in S(t,m)} p_n^{t,m} q_n^{t,m}};$$

$m = 1, 2, \ldots 11;$

[28]This rapid sample degradation essentially forces some form of chaining at the elementary level in any case.

[29]As was seen in Chapter 20, it is necessary to have a target concept for the individual prices and quantities $p_n^{t,m}$ and $q_n^{t,m}$ at the finest level of aggregation. In most circumstances, these target concepts can be taken to be unit values for prices and total revenues for the quantities purchased.

[30]The equations are slightly different for the indices that go from December to January of the following year. In order to simplify the exposition, these equations are left for the reader.

(22.21) $$P_P\left(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m)\right) = \frac{\sum_{n\in S(t,m)} p_n^{t,m+1} q_n^{t,m+1}}{\sum_{n\in S(t,m)} p_n^{t,m} q_n^{t,m+1}};\ m = 1, 2,...11;$$

(22.22) $$P_F\left(p^{t,m}, p^{t,m+1}, q^{t,m}, q^{t,m+1}, S(t,m)\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m)\right)} \times \sqrt{P_P\left(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m)\right)};$$

$m = 1, 2,...,11.$

Note that P_L, P_P, and P_F depend on the two (complete) price and quantity vectors pertaining to months m and $m + 1$ of year t, $p^{t,m}$,$p^{t,m+1}$,$q^{t,m}$,$q^{t,m+1}$, but they also depend on the set $S(t,m)$, which is the set of products that are present in both months. Thus, the product indices n that are in the summations on the right-hand sides of equations (22.20)–(22.22) include indices n that correspond to products that are present in *both* months, which is the meaning of $n\in S(t,m)$; that is, n belongs to the set $S(t,m)$.

22.66 To rewrite equations (22.20)–(22.22) in revenue share and price relative form, some additional notation is required. Define the revenue shares of product n in month m and $m + 1$ of year t, using the set of products that are present in month m of year t and the subsequent month, as follows:

(22.23) $$s_n^{t,m}(t,m) = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(t,m)} p_i^{t,m} q_i^{t,m}};\ n\in S(t,m);$$

$m = 1, 2,...,11;$

(22.24) $$s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1} q_n^{t,m+1}}{\sum_{i\in S(t,m)} p_i^{t,m+1} q_i^{t,m+1}};\ n\in S(t,m);$$

$m = 1, 2,...,11.$

The notation in equations (22.23) and (22.24) is rather messy because $s_n^{t,m+1}(t,m)$ has to be distinguished from $s_n^{t,m+1}(t,m+1)$. The revenue share $s_n^{t,m+1}(t,m)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month m of year t and the subsequent month, whereas $s_n^{t,m+1}(t,m+1)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month $m + 1$ of year t and the subsequent month. Thus, the set of superscripts, $t,m+1$ in $s_n^{t,m+1}(t,m)$, indicates that the revenue share is calculated using the price and quantity data of month $m + 1$ of year t and (t,m) indicates that the set of admissible products is restricted to the set of products that are present in both month m and the subsequent month.

22.67 Now define vectors of revenue shares. If product n is present in month m of year t and the following month, define $s_n^{t,m}(t,m)$ using equation (22.23); if this is not the case, define $s_n^{t,m}(t,m) = 0$. Similarly, if product n is present in month m of year t and the following month, define $s_n^{t,m+1}(t,m)$ using equation (22.24); if this is not the case, define $s_n^{t,m+1}(t,m) = 0$. Now define the N dimensional vectors:

$$s^{t,m}(t,m) \equiv [s_1^{t,m}(t,m), s_2^{t,m}(t,m),..., s_N^{t,m}(t,m)] \text{ and}$$
$$s^{t,m+1}(t,m) \equiv [s_1^{t,m+1}(t,m), s_2^{t,m+1}(t,m),..., s_N^{t,m+1}(t,m)].$$

Using these share definitions, the month-to-month Laspeyres, Paasche, and Fisher equations (22.20)–(22.22) can also be rewritten in revenue share and price form as follows:

(22.25) $$P_L\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m)\right) \equiv \sum_{n\in S(t,m)} s_n^{t,m}(t,m)\left(p_n^{t,m+1}/p_n^{t,m}\right);$$

$m = 1, 2,...11;$

(22.26) $$P_P\left(p^{t,m}, p^{t,m+1}, s^{t,m+1}(t,m)\right) \equiv \left[\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

$m = 1, 2,...11;$

(22.27) $$P_F\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m), s^{t,m+1}(t,m)\right) \equiv \sqrt{\sum_{n\in S(t,m)} s_n^{t,m}(t,m)\left(p_n^{t,m+1}/p_n^{t,m}\right)} \times \sqrt{\left[\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1}};$$

$m = 1, 2,...,11.$

22.68 It is important to recognize that the revenue shares $s_n^{t,m}(t,m)$ that appear in the maximum overlap month-to-month Laspeyres index defined by equation (22.25) are *not* the revenue shares that could be taken from an establishment production survey for month m of year t; instead, they are the shares that result from revenues on seasonal products that are present in month m of year t but are not present in the following month. Similarly, the revenue shares $s_n^{t,m+1}(t,m)$ that appear in the maximum overlap month-to-month Paasche index defined by equation (22.26) are *not* the expenditure shares that could be taken from an establishment production survey for month $m + 1$ of year t; instead, they are the shares that result from revenues on seasonal products that are present in month $m + 1$ of year t but are not present in the preceding month.[31] The maximum overlap month-to-month Fisher index defined by equation (22.27) is the geometric mean of the Laspeyres and Paasche indices defined by equations (22.25) and (22.26).

22.69 Table 22.21 lists the maximum overlap chained month-to-month Laspeyres, Paasche, and Fisher price indices for the data listed in Section B. These indices are defined by equations (22.25), (22.26), and (22.27).

22.70 The chained maximum overlap Laspeyres, Paasche, and Fisher indices for December of 1973 are 1.0504, 0.1204, and 0.3556, respectively. Comparing these results to the year-over-year results listed in Tables 22.3, 22.4, and 22.5 indicate that the results in Table 22.21 are not at all realistic! These hugely different direct indices compared with the last row of Table 22.21 indicate that *the maximum overlap indices suffer from a significant downward bias for the artificial data set.*

22.71 What are the factors that can explain this downward bias? It is evident that part of the problem has to do with the seasonal pattern of prices for peaches and strawberries (products 2 and 4). These products are not present in the market for each month of the year. For the first month of the year when they become available, they have relatively high prices; in subsequent months, their prices drop substantially. The effects of these initially high prices (compared with the relatively low prices that prevailed in the last month that the

[31]It is important that the revenue shares that are used in an index number formula add up to unity. The use of unadjusted expenditure shares from an establishment survey would lead to a systematic bias in the index number formula.

Table 22.21. Month-to-Month Maximum Overlap Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L	P_P	P_F
1970	1	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777
	3	0.9587	0.9594	0.9590
	4	1.0290	1.0534	1.0411
	5	1.1447	1.1752	1.1598
	6	1.1118	1.0146	1.0621
	7	1.1167	1.0102	1.0621
	8	1.1307	0.7924	0.9465
	9	1.0033	0.6717	0.8209
	10	0.9996	0.6212	0.7880
	11	1.0574	0.6289	0.8155
	12	1.0151	0.5787	0.7665
1971	1	1.0705	0.6075	0.8064
	2	1.0412	0.5938	0.7863
	3	1.0549	0.6005	0.7959
	4	1.1409	0.6564	0.8654
	5	1.2416	0.7150	0.9422
	6	1.1854	0.6006	0.8438
	7	1.2167	0.6049	0.8579
	8	1.2230	0.4838	0.7692
	9	1.0575	0.4055	0.6548
	10	1.0497	0.3837	0.6346
	11	1.1240	0.3905	0.6626
	12	1.0404	0.3471	0.6009
1972	1	1.0976	0.3655	0.6334
	2	1.1027	0.3679	0.6369
	3	1.1291	0.3765	0.6520
	4	1.1974	0.4014	0.6933
	5	1.2818	0.4290	0.7415
	6	1.2182	0.3553	0.6579
	7	1.2838	0.3637	0.6833
	8	1.2531	0.2794	0.5916
	9	1.0445	0.2283	0.4883
	10	1.0335	0.2203	0.4771
	11	1.1087	0.2256	0.5001
	12	1.0321	0.1995	0.4538
1973	1	1.0866	0.2097	0.4774
	2	1.1140	0.2152	0.4897
	3	1.1532	0.2225	0.5065
	4	1.2493	0.2398	0.5474
	5	1.3315	0.2544	0.5821
	6	1.2594	0.2085	0.5124
	7	1.3585	0.2160	0.5416
	8	1.3251	0.1656	0.4684
	9	1.0632	0.1330	0.3760
	10	1.0574	0.1326	0.3744
	11	1.1429	0.1377	0.3967
	12	1.0504	0.1204	0.3556

products were available in the previous year) are not captured by the maximum overlap month-to-month indices, so the resulting indices build up a tremendous downward bias. The downward bias is most pronounced in the Paasche indices, which use the quantities or volumes of the current month. These volumes are relatively large compared to those of the initial month when the products become available, reflecting the effects of lower prices as the quantity made available in the market increases.

22.72 Table 22.22 lists the results using chained Laspeyres, Paasche, and Fisher indices for the artificial data set where the strongly seasonal products 2 and 4 are dropped from each comparison of prices. Thus, the indices in Table 22.22 are the usual chained Laspeyres, Paasche, and Fisher indices restricted to products 1, 3, and 5, which are available in each season. These indices are labeled as $P_L(3)$, $P_P(3)$, and $P_F(3)$.

22.73 The chained Laspeyres, Paasche, and Fisher indices (using only the three year-round products) for January of 1973 are 1.2038, 0.5424, and 0.8081, respectively. From Tables 22.8, 22.9, and 22.10, the chained year-over-year Laspeyres, Paasche, and Fisher indices for January of 1973 are 1.3274, 1.3243, and 1.3258, respectively. Thus, the chained indices using the year-round products, which are listed in Table 22.22, evidently *suffer from substantial downward biases*.

22.74 The data in Tables 22.1 and 22.2 demonstrate that the quantity of grapes (product 3) available in the market varies tremendously over the course of a year, with substantial increases in price for the months when grapes are almost out of season. Thus, the price of grapes decreases substantially as the quantity increases during the last half of each year, but the annual substantial increase in the price of grapes takes place in the first half of the year, when quantities in the market are small. This pattern of seasonal price and quantity changes will cause the overall index to take on a downward bias.[32] To verify that this conjecture is true, see the last three columns of Table 22.22, where chained Laspeyres, Paasche, and Fisher indices are calculated using only products 1 and 5. These indices are labeled $P_L(2)$, $P_P(2)$, and $P_F(2)$, respectively, and for January of 1973, they are equal to 1.0033, 0.9408, and 0.9715, respectively. These estimates based on two year-round products are much closer to the chained year-over-year Laspeyres, Paasche, and Fisher indices for January of 1973, which were 1.3274, 1.3243, and 1.3258, respectively, than the estimates based on the three year-round products. However, it is clear that *the chained Laspeyres, Paasche, and Fisher indices restricted to products 1 and 5 still have substantial downward biases for the artificial data set.* Basically, the problems are caused by the high volumes associated with low or declining prices and the low volumes caused by high or rising prices. These weight effects make the seasonal price declines bigger than the seasonal price increases using month-to-month index number formulas with variable weights.[33]

22.75 In addition to the downward biases that show up in Tables 22.21 and 22.22, all of these month-to-month chained indices show substantial seasonal fluctuations in prices over the course of a year. Therefore, these month-to-month indices are of little use to policymakers who are interested in short-term inflationary trends. *If the purpose of the month-to-month PPI is to indicate changes in general inflation, then statistical agencies should be cautious about including products that show strong seasonal fluctuations in prices in the*

[32]Baldwin (1990) used the Turvey data to illustrate various treatments of seasonal products. He has a good discussion of what causes various month-to-month indices to behave badly. "It is a sad fact that for some seasonal product groups, monthly price changes are not meaningful, whatever the choice of formula" (Andrew Baldwin, 1990, p. 264).

[33]This remark has an application to Chapter 20 on elementary indices where irregular sales during the course of a year could induce a similar downward bias in a month-to-month index that used monthly weights. Another problem with month-to-month chained indices is that purchases and sales of individual products can become irregular as the time period becomes shorter and shorter and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003, p. 125) find an *upward* bias for their chained *weekly* indices for canned tuna compared to a fixed-base index; their bias was caused by variable weight effects due to the timing of advertising expenditures. In general, these drift effects of chained indices can be reduced by lengthening the time period, so that the *trends* in the data become more prominent than the *high-frequency fluctuations*.

Table 22.22. Month-to-Month Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	$P_L(3)$	$P_P(3)$	$P_F(3)$	$P_L(2)$	$P_P(2)$	$P_F(2)$
1970	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777	0.9751	0.9780	0.9765
	3	0.9587	0.9594	0.9590	0.9522	0.9574	0.9548
	4	1.0290	1.0534	1.0411	1.0223	1.0515	1.0368
	5	1.1447	1.1752	1.1598	1.1377	1.1745	1.1559
	6	1.2070	1.2399	1.2233	1.2006	1.2424	1.2214
	7	1.2694	1.3044	1.2868	1.2729	1.3204	1.2964
	8	1.3248	1.1537	1.2363	1.3419	1.3916	1.3665
	9	1.0630	0.9005	0.9784	1.1156	1.1389	1.1272
	10	0.9759	0.8173	0.8931	0.9944	1.0087	1.0015
	11	1.0324	0.8274	0.9242	0.9839	0.9975	0.9907
	12	0.9911	0.7614	0.8687	0.9214	0.9110	0.9162
1971	1	1.0452	0.7993	0.9140	0.9713	0.9562	0.9637
	2	1.0165	0.7813	0.8912	0.9420	0.9336	0.9378
	3	1.0300	0.7900	0.9020	0.9509	0.9429	0.9469
	4	1.1139	0.8636	0.9808	1.0286	1.0309	1.0298
	5	1.2122	0.9407	1.0679	1.1198	1.1260	1.1229
	6	1.2631	0.9809	1.1131	1.1682	1.1763	1.1723
	7	1.3127	1.0170	1.1554	1.2269	1.2369	1.2319
	8	1.3602	0.9380	1.1296	1.2810	1.2913	1.2861
	9	1.1232	0.7532	0.9198	1.1057	1.0988	1.1022
	10	1.0576	0.7045	0.8632	1.0194	1.0097	1.0145
	11	1.1325	0.7171	0.9012	1.0126	1.0032	1.0079
	12	1.0482	0.6373	0.8174	0.9145	0.8841	0.8992
1972	1	1.1059	0.6711	0.8615	0.9652	0.9311	0.9480
	2	1.1111	0.6755	0.8663	0.9664	0.9359	0.9510
	3	1.1377	0.6912	0.8868	0.9863	0.9567	0.9714
	4	1.2064	0.7371	0.9430	1.0459	1.0201	1.0329
	5	1.2915	0.7876	1.0086	1.1202	1.0951	1.1075
	6	1.3507	0.8235	1.0546	1.1732	1.1470	1.1600
	7	1.4091	0.8577	1.0993	1.2334	1.2069	1.2201
	8	1.4181	0.7322	1.0190	1.2562	1.2294	1.2427
	9	1.1868	0.5938	0.8395	1.1204	1.0850	1.1026
	10	1.1450	0.5696	0.8076	1.0614	1.0251	1.0431
	11	1.2283	0.5835	0.8466	1.0592	1.0222	1.0405
	12	1.1435	0.5161	0.7682	0.9480	0.8935	0.9204
1973	1	1.2038	0.5424	0.8081	1.0033	0.9408	0.9715
	2	1.2342	0.5567	0.8289	1.0240	0.9639	0.9935
	3	1.2776	0.5755	0.8574	1.0571	0.9955	1.0259
	4	1.3841	0.6203	0.9266	1.1451	1.0728	1.1084
	5	1.4752	0.6581	0.9853	1.2211	1.1446	1.1822
	6	1.5398	0.6865	1.0281	1.2763	1.1957	1.2354
	7	1.6038	0.7136	1.0698	1.3395	1.2542	1.2962
	8	1.6183	0.6110	0.9944	1.3662	1.2792	1.3220
	9	1.3927	0.5119	0.8443	1.2530	1.1649	1.2081
	10	1.3908	0.5106	0.8427	1.2505	1.1609	1.2049
	11	1.5033	0.5305	0.8930	1.2643	1.1743	1.2184
	12	1.3816	0.4637	0.8004	1.1159	1.0142	1.0638

month-to-month index.[34] If seasonal products are included in a month-to-month index that is meant to indicate general inflation, then a seasonal adjustment procedure should be used to remove these strong seasonal fluctuations. Some simple types of seasonal adjustment procedures will be considered in Section K.

22.76 The rather poor performance of the month-to-month indices listed in the last two tables does not always occur in the context of seasonal products. In the context of calculating import and export price indices using quarterly data for the United States, Alterman, Diewert, and Feenstra (1999) found that maximum overlap month-to-month indices worked reasonably well.[35] However, statistical agencies should check that their month-to-month indices are at least approximately consistent with the corresponding year-over-year indices.

22.77 The various Paasche and Fisher indices computed in this section could be approximated by indices that replaced all current-period revenue shares with the corresponding revenue shares from the base year. These approximate Paasche and Fisher indices will not be reproduced here because they resemble their real counterparts and are themselves subject to tremendous downward bias.

H. Annual Basket Indices with Carryforward of Unavailable Prices

22.78 Recall that the Lowe (1823) index defined in earlier chapters had two reference periods:[36]

- The vector of quantity weights; and
- The base-period prices.

The *Lowe index* for month m was defined by the following equation:

$$(22.28)\quad P_{LO}\left(p^0, p^m, q\right) = \frac{\sum_{n=1}^{N} p_n^m q_n}{\sum_{n=1}^{N} p_n^0 q_n},$$

where $p^0 \equiv [p_1^0,\ldots,p_N^0]$ is the price reference period price vector, $p^m \equiv [p_1^m,\ldots,p_N^m]$ is the current month m price vector, and $q \equiv [q_1,\ldots,q_N]$ is the weight reference year quantity vector. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year will be 1970, and the resulting reference year quantity vector turns out to be:

$$(22.29)\quad q \equiv [q_1,\ldots,q_5] = [53889, 12881, 9198, 5379, 68653].$$

The price reference period for the prices will be December of 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Lowe index with carryforward of missing prices using the modified Turvey data set can be found in column 1 of Table 22.23.

22.79 Baldwin's comments on this type of annual basket (AB) index are worth quoting at length:

> For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256–257) calls it an index of "a hybrid sort." Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is "What would have the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices

[34]However, if the purpose of the index is to compare the prices that producers *actually receive* in two consecutive months, ignoring the possibility that the purchasers may regard a seasonal good as being qualitatively different in the two months, then the production of a month-to-month PPI that has large seasonal fluctuations can be justified.

[35]They checked the validity of their month-to-month indices by cumulating them for four quarters and comparing them to the corresponding year-over-year indices. They found only relatively small differences. However, note that irregular high-frequency fluctuations will tend to be smaller for quarters than for months. For this reason, chained quarterly indices can be expected to perform better than chained monthly or weekly indices.

[36]In the context of seasonal price indices, this type of index corresponds to Bean and Stine's (1924, p. 31) Type A index.

nonetheless retained their own seasonal behaviour?" It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences (Andrew Baldwin, 1990, p. 258).

In spite of Baldwin's somewhat negative comments on the Lowe index, it is the index that is preferred by many statistical agencies, so it is necessary to study its properties in the context of strongly seasonal data.

22.80 Recall that the *Young* (1812) *index* was defined in Chapters 1 and 15 as follows:

$$(22.30)\quad P_Y\left(p^0, p^m, s\right) = \sum_{n=1}^{N} s_n\left(p_n^m / p_n^0\right),$$

where $s \equiv [s_1,\ldots,s_N]$ is the weight reference year vector of revenue shares. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year will be 1970 and the resulting revenue share vector turns out to be

$$(22.31)\quad s \equiv [s_1,\ldots,s_5] = [0.3284, 0.1029, 0.0674, 0.0863, 0.4149].$$

Again, the base period for the prices will be December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Young index with carryforward of missing prices using the modified Turvey data set can be found in column 2 of Table 22.23.

22.81 The *geometric Laspeyres index* was defined in Chapter 19 as follows:

$$(22.32)\quad P_{GL}(p^0, p^m, s) \equiv \prod_{n=1}^{N}(p_n^m / p_n^0)^{s_n}.$$

Thus, the geometric Laspeyres index makes use of the same information as the Young index, except that a geometric average of the price relatives is taken instead of an arithmetic one. Again, the weight reference year is 1970, the price reference-period is December 1970, and the index is illustrated using the modified Turvey data set with carryforward of missing prices. See column 3 of Table 22.23.

22.82 It is interesting to compare the above three indices that use annual baskets to the fixed-base Laspeyres rolling-year indices computed earlier.

Table 22.23. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carry-forward Prices

Year	Month	P_{LO}	P_Y	P_{GL}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0554	1.0609	1.0595	1.0091
	2	1.0711	1.0806	1.0730	1.0179
	3	1.1500	1.1452	1.1187	1.0242
	4	1.2251	1.2273	1.1942	1.0298
	5	1.3489	1.3652	1.3249	1.0388
	6	1.4428	1.4487	1.4068	1.0478
	7	1.3789	1.4058	1.3819	1.0547
	8	1.3378	1.3797	1.3409	1.0631
	9	1.1952	1.2187	1.1956	1.0729
	10	1.1543	1.1662	1.1507	1.0814
	11	1.1639	1.1723	1.1648	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.1370	1.1523	1.1465	1.1065
	2	1.1731	1.1897	1.1810	1.1174
	3	1.2455	1.2539	1.2363	1.1254
	4	1.3155	1.3266	1.3018	1.1313
	5	1.4262	1.4508	1.4183	1.1402
	6	1.5790	1.5860	1.5446	1.1502
	7	1.5297	1.5550	1.5349	1.1591
	8	1.4416	1.4851	1.4456	1.1690
	9	1.3038	1.3342	1.2974	1.1806
	10	1.2752	1.2960	1.2668	1.1924
	11	1.2852	1.3034	1.2846	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.2427	1.2710	1.2518	1.2386
	2	1.3003	1.3308	1.3103	1.2608
	3	1.3699	1.3951	1.3735	1.2809
	4	1.4691	1.4924	1.4675	1.2966
	5	1.5972	1.6329	1.5962	1.3176
	6	1.8480	1.8541	1.7904	1.3406
	7	1.7706	1.8010	1.7711	0.0000
	8	1.6779	1.7265	1.6745	0.0000
	9	1.5253	1.5676	1.5072	0.0000
	10	1.5371	1.5746	1.5155	0.0000
	11	1.5634	1.5987	1.5525	0.0000
	12	1.4181	1.4521	1.4236	0.0000

However, the rolling-year index that ends in the current month is centered five and a half months backward. Thus, the above three annual basket-type indices will be compared with an arithmetic average of two rolling-year indices that have their last month five and six months forward. This latter centered rolling-year index is labeled P_{CRY} and is listed in the last column of Table 22.23.[37] Note that zeros are entered for the last six rows of this column, since the data set does not extend six months into 1974. As a result, the centered rolling-year indices cannot be calculated for these last six months.

22.83 It can be seen that the Lowe, Young, and geometric Laspeyres indices have a considerable amount of seasonality in them and do not at all approximate their rolling-year counterparts listed in the last column of Table 22.23.[38] Therefore, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[39] The four series, P_{LO}, P_Y, P_{GL}, and P_{CRY}, listed in Table 22.23 are also plotted in Figure 22.4. The Young price index is generally the highest, followed by the Lowe index, and then the geometric Laspeyres. The centered rolling-year Laspeyres counterpart index, P_{CRY}, is generally below the other three indices (and does not have the strong seasonal movements of the other three series), but it moves in a roughly parallel fashion to the other three indices.[40] Note that the seasonal movements of P_{LO}, P_Y, and P_{GL} are quite regular. This regularity will be exploited in Section K in order to use these month-to-month indices to predict their rolling-year counterparts.

22.84 Part of the problem may be the fact that the prices of strongly seasonal goods have been carried forward for the months when the products are not available. This will tend to add to the amount of seasonal movements in the indices, particularly when there is high general inflation. For this reason, the Lowe, Young, and geometric Laspeyres indices will be recomputed in the following section, using an imputation method for the missing prices rather than simply carrying forward the last available price.

I. Annual Basket Indices with Imputation of Unavailable Prices

22.85 Instead of simply carrying forward the last available price of a seasonal product that is not sold during a particular month, it is possible to use an *imputation method* to fill in the missing prices. Alternative imputation methods are discussed by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001), but the basic idea is to take the last available price and *impute* prices for the missing periods that trend with another index. This other index could be an index of available prices for the general category of product or higher-level components of the PPI. For the purposes of this section, the imputation index is taken to be a price index that grows at the multiplicative rate of 1.008, since the fixed-base rolling-year Laspeyres indices for the modified Turvey data set grow at approximately 0.8 percent per month.[41] Using this imputation method to fill in the missing prices, the Lowe, Young, and geometric Laspeyres indices defined in the previous section can be recomputed. The resulting indices are listed in Table 22.24, along with the centered rolling-year index P_{CRY} for comparison purposes.

22.86 As could be expected, the Lowe, Young, and geometric Laspeyres indices that used imputed prices are on average a bit *higher* than their counterparts that used carryforward prices, but the variability of the imputed indices is generally a bit

[37]This series was normalized to equal 1 in December 1970 so that it would be comparable to the other month-to-month indices.

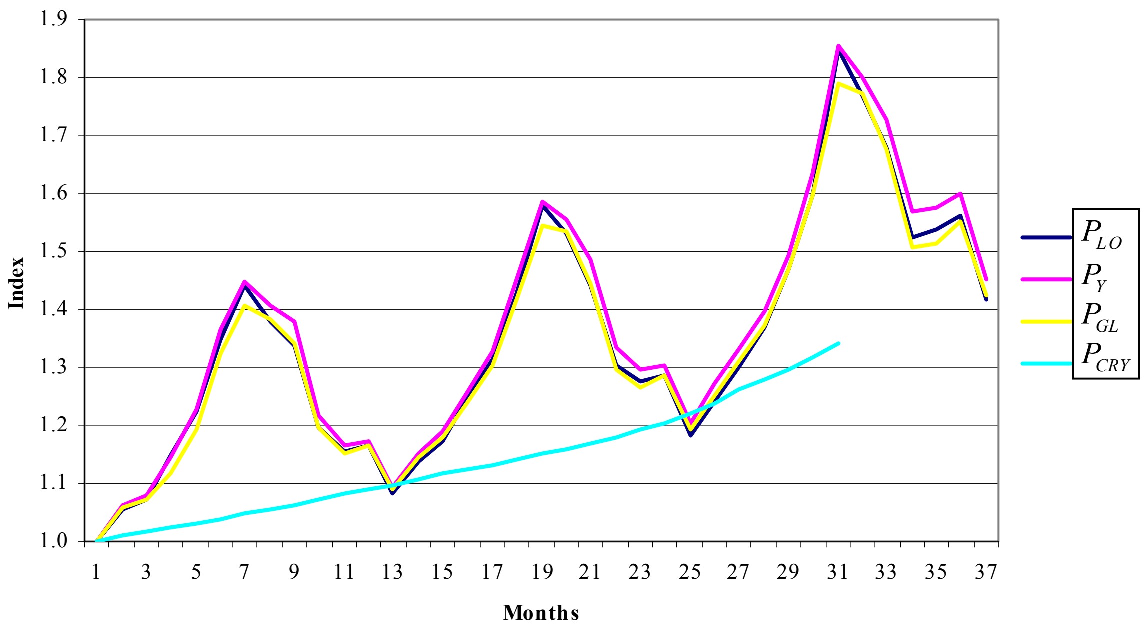
[38]The sample means of the four indices are 1.2935 (Lowe), 1.3110 (Young), 1.2877 (geometric Laspeyres) and 1.1282 (rolling-year). The geometric Laspeyres indices will always be equal to or less than their Young counterparts, since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

[39]In Section K, the Lowe, Young, and Geometric Laspeyres indices will be seasonally adjusted.

[40]In Figure 22.4, P_{CRY} stops at the June 1973 value for the index, which is the last month that the centered index can be constructed from the available data.

[41]For the last year of data, the imputation index is escalated by an additional monthly growth rate of 1.008.

Figure 22.4. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carry-forward Prices



lower.[42] The series that are listed in Table 22.24 are also plotted in Figure 22.5. It is apparent that the Lowe, Young, and geometric Laspeyres indices that use imputed prices still have a huge amount of seasonality in them and do not closely approximate their rolling-year counterparts listed in the last column of Table 22.24.[43] Consequently, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices using imputed prices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[44] As these indices stand, they are not suitable as measures of general inflation going from month to month.

J. Bean and Stine Type C or Rothwell Indices

22.87 The final month-to-month index[45] that will be considered in this chapter is the *Bean and Stine Type C* (1924, p. 31) or *Rothwell* (1958, p. 72) index.[46] This index makes use of *seasonal baskets* in the base year, denoted as the vectors $q^{0,m}$ for the months $m = 1, 2,\ldots,12$. The index also makes use

[42]For the Lowe indices, the mean for the first 31 observations increases (with imputed prices) from 1.3009 to 1.3047, but the standard deviation decreases from 0.18356 to 0.18319; for the Young indices, the mean for the first 31 observations increases from 1.3186 to 1.3224, but the standard deviation decreases from 0.18781 to 0.18730; and for the geometric Laspeyres indices, the mean for the first 31 observations increases from 1.2949 to 1.2994, and the standard deviation also increases slightly from 0.17582 to 0.17599. The imputed indices are preferred to the carryforward indices on general methodological grounds: in high inflation environments, the carryforward indices will be subject to sudden jumps when previously unavailable products become available.

[43]Note also that Figures 22.4 and 22.5 are similar.

[44]In Section K, the Lowe, Young, and geometric Laspeyres indices using imputed prices will be seasonally adjusted.

[45]For other suggested month-to-month indices in the seasonal context, see Balk (1980a, 1980b, 1980c, 1981).

[46]This is the index favored by Baldwin (1990, p. 271) and many other price statisticians in the context of seasonal products.

Table 22.24. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices

Year	Month	P_{LOI}	P_{YI}	P_{GLI}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0568	1.0624	1.0611	1.0091
	2	1.0742	1.0836	1.0762	1.0179
	3	1.1545	1.1498	1.1238	1.0242
	4	1.2312	1.2334	1.2014	1.0298
	5	1.3524	1.3682	1.3295	1.0388
	6	1.4405	1.4464	1.4047	1.0478
	7	1.3768	1.4038	1.3798	1.0547
	8	1.3364	1.3789	1.3398	1.0631
	9	1.1949	1.2187	1.1955	1.0729
	10	1.1548	1.1670	1.1514	1.0814
	11	1.1661	1.1747	1.1672	1.0885
	12	1.0863	1.0972	1.0939	1.0965
1972	1	1.1426	1.1580	1.1523	1.1065
	2	1.1803	1.1971	1.1888	1.1174
	3	1.2544	1.2630	1.2463	1.1254
	4	1.3260	1.3374	1.3143	1.1313
	5	1.4306	1.4545	1.4244	1.1402
	6	1.5765	1.5831	1.5423	1.1502
	7	1.5273	1.5527	1.5326	1.1591
	8	1.4402	1.4841	1.4444	1.1690
	9	1.3034	1.3343	1.2972	1.1806
	10	1.2758	1.2970	1.2675	1.1924
	11	1.2875	1.3062	1.2873	1.2049
	12	1.1888	1.2078	1.1981	1.2203
1973	1	1.2506	1.2791	1.2601	1.2386
	2	1.3119	1.3426	1.3230	1.2608
	3	1.3852	1.4106	1.3909	1.2809
	4	1.4881	1.5115	1.4907	1.2966
	5	1.6064	1.6410	1.6095	1.3176
	6	1.8451	1.8505	1.7877	1.3406
	7	1.7679	1.7981	1.7684	0.0000
	8	1.6773	1.7263	1.6743	0.0000
	9	1.5271	1.5700	1.5090	0.0000
	10	1.5410	1.5792	1.5195	0.0000
	11	1.5715	1.6075	1.5613	0.0000
	12	1.4307	1.4651	1.4359	0.0000

of a vector of *base-year unit-value prices*, $p^0 \equiv [p_1^0,\ldots,p_5^0]$, where the nth price in this vector is defined as

$$(22.33)\quad p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}}\,;\ n = 1,\ldots,5.$$

The *Rothwell price index for month* m *in year* t can now be defined as follows:

$$(22.34)\quad P_R\left(p^0, p^{t,m}, q^{0.m}\right) \equiv \frac{\sum_{n=1}^{5} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{5} p_n^0 q_n^{0.m}}\,;$$

$m = 1,\ldots,12.$

Thus, as the month changes, the quantity weights for the index change. The month-to-month movements in this index, therefore, are a mixture of price and quantity changes.[47]

22.88 Using the modified Turvey data set, the base year is chosen to be 1970 as usual, and the index is started off at December of 1970. The Rothwell index P_R is compared to the Lowe index with carryforward of missing prices P_{LO} in Table 22.25. To make the series a bit more comparable, the *normalized Rothwell index* P_{NR} is also listed in Table 22.25; this index is simply equal to the original Rothwell index divided by its first observation.

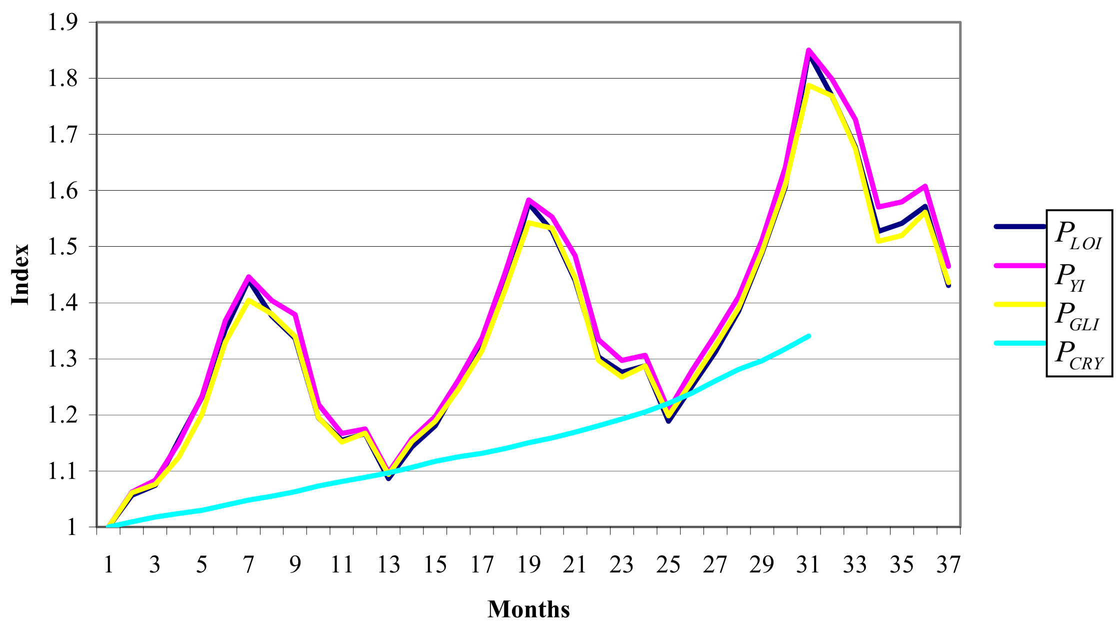
22.89 Viewing Figure 22.6, which plots the Lowe index with the carryforward of the last price and the normalized Rothwell index, it is clear that the Rothwell index has smaller seasonal movements than the Lowe index and is less volatile in general.[48] However, it is evident that there still are large seasonal movements in the Rothwell index, and it may not be a suitable index, for measuring general inflation without some sort of seasonal adjustment.

22.90 In the following section, the annual basket-type indices (with and without imputation) defined earlier in Sections H and I will be season-

[47]Rothwell (1958, p. 72) showed that the month-to-month movements in the index have the form of an expenditure ratio divided by a quantity index.

[48]For all 37 observations in Table 22.25, the Lowe index has a mean of 1.3465 and a standard deviation of 0.20313, while the normalized Rothwell has a mean of 1.2677 and a standard deviation of 0.18271.

Figure 22.5. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices



ally adjusted using essentially the same method that was used in Section F and compared with a standard seasonal adjustment using X-11.

K. Forecasting Rolling-Year Indices Using Month-to-Month Annual Basket Indices

22.91 Recall that Table 22.23 in Section H presented the Lowe, Young, geometric Laspeyres (using carryforward prices), and centered rolling-year indices for the 37 observations running from December 1970 to December 1973 (P_{LO}, P_Y, P_{GL}, and P_{CRY}, respectively). For each of the first three series, define a seasonal adjustment factor, SAF, as the centered rolling-year index P_{CRY} divided by P_{LO}, P_Y, and P_{GL}, respectively, for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for observations 13–24 and then repeat them for the remaining observations. These operations will create three SAF series for all 37 observations (label them SAF_{LO}, SAF_Y, and SAF_{GL}, respectively), but only the first 12 observations in the P_{LO}, P_Y, P_{GL}, and P_{CRY} series are used to create the three SAF series. Finally, define *seasonally adjusted Lowe, Young, and geometric Laspeyres indices* by multiplying each unadjusted index by the appropriate seasonal adjustment factor:

$$(22.35)\quad P_{LOSA} \equiv P_{LO}\, SAF_{LO}\,;\; P_{YSA} \equiv P_Y\, SAF_Y\,;\; P_{GLSA} \equiv P_{GL}\, SAF_{GL}\,.$$

Figure 22.6. Lowe and Normalized Rothwell Indices

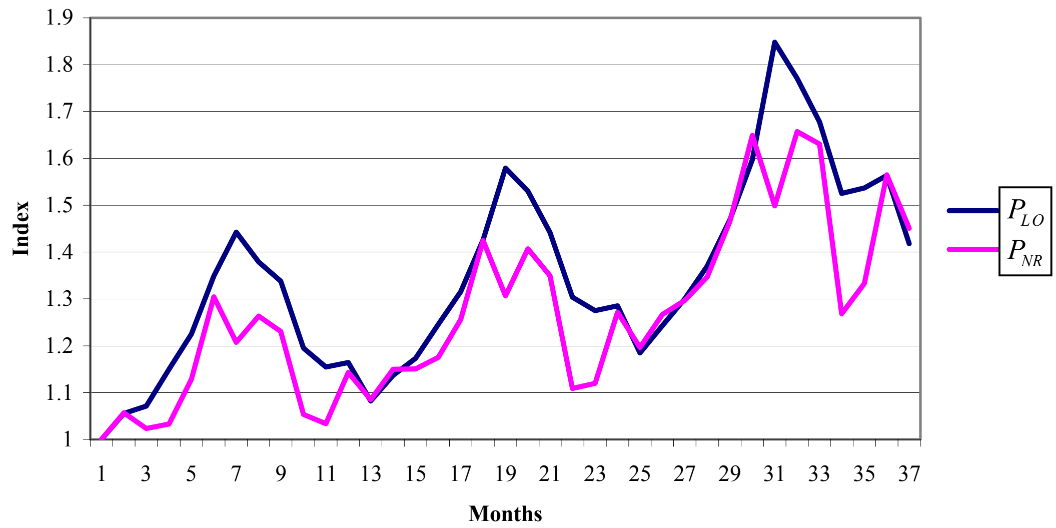


Figure 22.7a. Seasonally Adjusted Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices

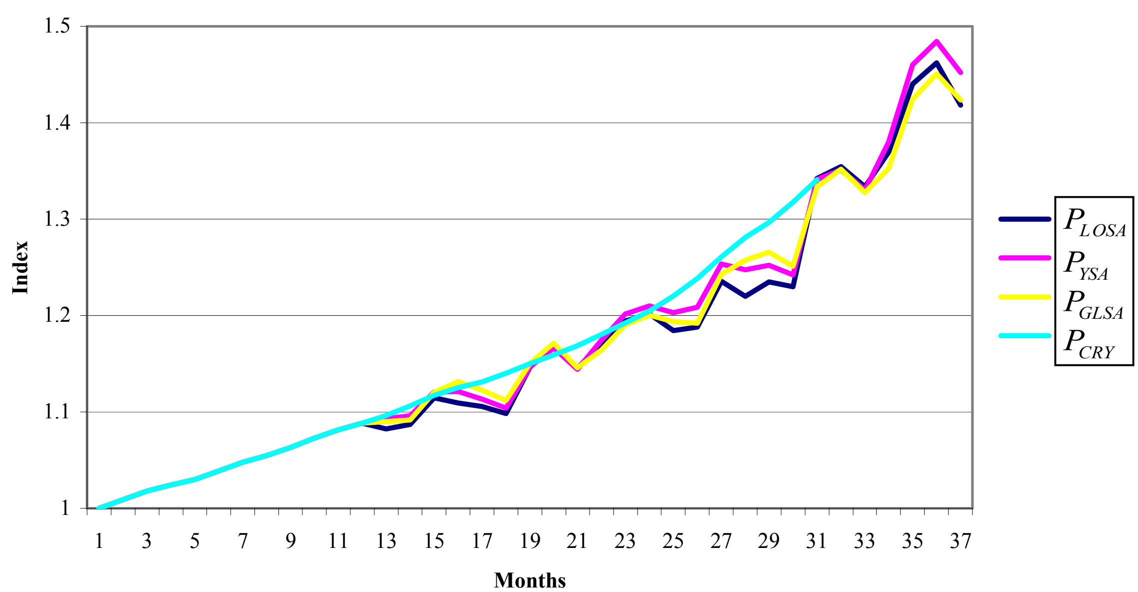


Table 22.25. Lowe with Carryforward Prices, Normalized Rothwell, and Rothwell Indices

Year	Month	P_{LO}	P_{NR}	P_R
1970	12	1.0000	1.0000	0.9750
1971	1	1.0554	1.0571	1.0306
	2	1.0711	1.0234	0.9978
	3	1.1500	1.0326	1.0068
	4	1.2251	1.1288	1.1006
	5	1.3489	1.3046	1.2720
	6	1.4428	1.2073	1.1771
	7	1.3789	1.2635	1.2319
	8	1.3378	1.2305	1.1997
	9	1.1952	1.0531	1.0268
	10	1.1543	1.0335	1.0077
	11	1.1639	1.1432	1.1146
	12	1.0824	1.0849	1.0577
1972	1	1.1370	1.1500	1.1212
	2	1.1731	1.1504	1.1216
	3	1.2455	1.1752	1.1459
	4	1.3155	1.2561	1.2247
	5	1.4262	1.4245	1.3889
	6	1.5790	1.3064	1.2737
	7	1.5297	1.4071	1.3719
	8	1.4416	1.3495	1.3158
	9	1.3038	1.1090	1.0813
	10	1.2752	1.1197	1.0917
	11	1.2852	1.2714	1.2396
	12	1.1844	1.1960	1.1661
1973	1	1.2427	1.2664	1.2348
	2	1.3003	1.2971	1.2647
	3	1.3699	1.3467	1.3130
	4	1.4691	1.4658	1.4292
	5	1.5972	1.6491	1.6078
	6	1.8480	1.4987	1.4612
	7	1.7706	1.6569	1.6155
	8	1.6779	1.6306	1.5898
	9	1.5253	1.2683	1.2366
	10	1.5371	1.3331	1.2998
	11	1.5634	1.5652	1.5261
	12	1.4181	1.4505	1.4143

These three seasonally adjusted annual basket-type indices are listed in Table 22.26, along with the target index, the centered rolling-year index, P_{CRY}. In addition, one could seasonally adjust the original Lowe, Young, and geometric Laspeyres indices using a standard seasonal adjustment procedure such as X-11. Table 22.26 also contains Lowe, Young, and geometric Laspeyres series that have been seasonally adjusted using the X-11 multiplicative model with default settings.[49] The series have been normalized to set December 1970 = 1.0. They are labeled P_{LOx11}, P_{Yx11}, and P_{GLx11}, respectively.

22.92 The first four series in Table 22.26 coincide for their first 12 observations, which follows from the way the seasonally adjusted series were defined. Also, the last six observations are missing for the centered rolling-year series, P_{CRY}, because data for the first six months of 1974 would be required to calculate all of these index values. Note that from December 1971 to December 1973, the three seasonally adjusted annual basket-type indices (P_{LOSA}, P_{YSA}, and P_{GLSA}) can be used to *predict* the corresponding centered rolling-year entries; see Figure 22.7a for plots of these predictions. What is remarkable in Table 22.26 and Figure 22.7a is that *the predicted values of these seasonally adjusted series are fairly close to the corresponding target index values*.[50] This result is somewhat unexpected since the annual basket indices use price information for only two consecutive months, whereas the corresponding centered rolling-year index uses price information for some 25 months![51] It should also be noted that the seasonally adjusted geometric Laspeyres index is generally the best predictor of the corresponding rolling-year index for this data set. In viewing Figure 22.7a, for the first few

[49]Many statistical offices have access to moving average seasonal adjustment programs such as the X-11 system developed by the U.S. Census Bureau and Statistics Canada. The seasonal adjustment performed here ran the data through the multiplicative version of X-11.

[50]For observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8816 is obtained; for the seasonally adjusted Young index, an R^2 of 0.9212 is derived; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9423 is derived. These fits are not as good as the fit obtained in Section F above where the seasonally adjusted approximate rolling-year index was used to predict the fixed-base Laspeyres rolling-year index. This R^2 was 0.9662; recall the discussion around Table 22.20.

[51]However, for seasonal data sets that are not as regular as the modified Turvey data set, the predictive power of the seasonally adjusted annual basket-type indices may be considerably less; that is, if there are abrupt changes in the seasonal pattern of prices, one could not expect these month-to-month indices to accurately predict a rolling-year index.

Table 22.26. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Carryforward Prices and Centered Rolling-Year Index

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0077	1.0088	1.0088
	2	1.0179	1.0179	1.0179	1.0179	1.0009	1.0044	0.9986
	3	1.0242	1.0242	1.0242	1.0242	1.0208	1.0205	1.0029
	4	1.0298	1.0298	1.0298	1.0298	1.0314	1.0364	1.0157
	5	1.0388	1.0388	1.0388	1.0388	1.0604	1.0666	1.0490
	6	1.0478	1.0478	1.0478	1.0478	1.0302	1.0402	1.0258
	7	1.0547	1.0547	1.0547	1.0547	1.0237	1.0409	1.0213
	8	1.0631	1.0631	1.0631	1.0631	1.0572	1.0758	1.0561
	9	1.0729	1.0729	1.0729	1.0729	1.0558	1.0665	1.0626
	10	1.0814	1.0814	1.0814	1.0814	1.0500	1.0598	1.0573
	11	1.0885	1.0885	1.0885	1.0885	1.0598	1.0714	1.0666
	12	1.0824	1.0932	1.0900	1.0965	1.0828	1.0931	1.0901
1972	1	1.0871	1.0960	1.0919	1.1065	1.0856	1.0957	1.0916
	2	1.1148	1.1207	1.1204	1.1174	1.0963	1.1059	1.0992
	3	1.1093	1.1214	1.1318	1.1254	1.1056	1.1173	1.1083
	4	1.1057	1.1132	1.1226	1.1313	1.1076	1.1203	1.1072
	5	1.0983	1.1039	1.1120	1.1402	1.1211	1.1334	1.1229
	6	1.1467	1.1471	1.1505	1.1502	1.1276	1.1387	1.1264
	7	1.1701	1.1667	1.1715	1.1591	1.1361	1.1514	1.1343
	8	1.1456	1.1443	1.1461	1.1690	1.1393	1.1580	1.1385
	9	1.1703	1.1746	1.1642	1.1806	1.1517	1.1676	1.1531
	10	1.1946	1.2017	1.1905	1.1924	1.1599	1.1777	1.1640
	11	1.2019	1.2102	1.2005	1.2049	1.1703	1.1912	1.1762
	12	1.1844	1.2032	1.1938	1.2203	1.1848	1.2031	1.1938
1973	1	1.1882	1.2089	1.1922	1.2386	1.1940	1.2163	1.1998
	2	1.2357	1.2536	1.2431	1.2608	1.2260	1.2480	1.2314
	3	1.2201	1.2477	1.2575	1.2809	1.2296	1.2569	1.2469
	4	1.2349	1.2523	1.2656	1.2966	1.2529	1.2764	1.2678
	5	1.2299	1.2425	1.2514	1.3176	1.2628	1.2820	1.2743
	6	1.3421	1.3410	1.3335	1.3406	1.3175	1.3285	1.3035
	7	1.3543	1.3512	1.3518	0.0000	1.3123	1.3313	1.3069
	8	1.3334	1.3302	1.3276	0.0000	1.3254	1.3460	1.3186
	9	1.3692	1.3800	1.3524	0.0000	1.3489	1.3739	1.3411
	10	1.4400	1.4601	1.4242	0.0000	1.4016	1.4351	1.3962
	11	1.4621	1.4844	1.4508	0.0000	1.4308	1.4691	1.4296
	12	1.4181	1.4521	1.4236	0.0000	1.4332	1.4668	1.4374

months of 1973, the three month-to-month indices underestimate the centered rolling-year inflation rate, but by the middle of 1973, the month-to-month indices are right on target.[52]

22.93 The last three series in Table 22.26 reflect the seasonal adjustment of the Lowe, Young, and geometric Laspeyres using the X-11 program. The seasonally adjusted series (P_{LOx11}, P_{Yx11}, and P_{GLx11})

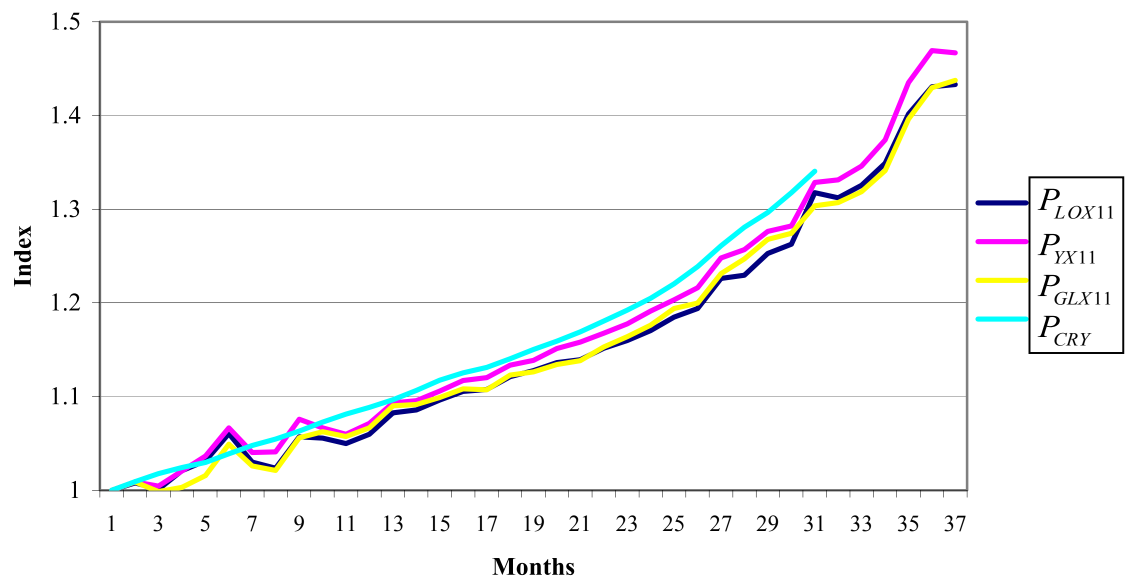
[52]Recall that the last six months of P_{CRY} are missing; six months of data for 1974 would be required to evaluate these (continued) centered rolling-year index values, and these data are not available.

Table 22.27. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices, Seasonally Adjusted Rothwell, and Centered Rolling-Year Indices

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0091	1.0125	1.0131	1.0133
	2	1.0179	1.0179	1.0179	1.0179	1.0179	1.0083	1.0109	1.0057
	3	1.0242	1.0242	1.0242	1.0242	1.0242	1.0300	1.0288	1.0121
	4	1.0298	1.0298	1.0298	1.0298	1.0298	1.0418	1.0460	1.0267
	5	1.0388	1.0388	1.0388	1.0388	1.0388	1.0680	1.0753	1.0574
	6	1.0478	1.0478	1.0478	1.0478	1.0478	1.0367	1.0485	1.0362
	7	1.0547	1.0547	1.0547	1.0547	1.0547	1.0300	1.0450	1.0251
	8	1.0631	1.0631	1.0631	1.0631	1.0631	1.0637	1.0807	1.0615
	9	1.0729	1.0729	1.0729	1.0729	1.0729	1.0607	1.0713	1.0685
	10	1.0814	1.0814	1.0814	1.0814	1.0814	1.0536	1.0634	1.0615
	11	1.0885	1.0885	1.0885	1.0885	1.0885	1.0631	1.0741	1.0704
	12	1.0863	1.0972	1.0939	1.0849	1.0965	1.0867	1.0973	1.0940
1972	1	1.0909	1.0999	1.0958	1.0978	1.1065	1.0948	1.1043	1.1004
	2	1.1185	1.1245	1.1244	1.1442	1.1174	1.1079	1.1168	1.1109
	3	1.1129	1.1250	1.1359	1.1657	1.1254	1.1191	1.1300	1.1224
	4	1.1091	1.1167	1.1266	1.1460	1.1313	1.1220	1.1341	1.1233
	5	1.0988	1.1043	1.1129	1.1342	1.1402	1.1298	1.1431	1.1328
	6	1.1467	1.1469	1.1505	1.1339	1.1502	1.1345	1.1476	1.1377
	7	1.1701	1.1666	1.1715	1.1746	1.1591	1.1427	1.1559	1.1386
	8	1.1457	1.1442	1.1461	1.1659	1.1690	1.1464	1.1632	1.1444
	9	1.1703	1.1746	1.1642	1.1298	1.1806	1.1570	1.1729	1.1594
	10	1.1947	1.2019	1.1905	1.1715	1.1924	1.1639	1.1818	1.1685
	11	1.2019	1.2103	1.2005	1.2106	1.2049	1.1737	1.1943	1.1805
	12	1.1888	1.2078	1.1981	1.1960	1.2203	1.1892	1.2079	1.1983
1973	1	1.1941	1.2149	1.1983	1.2089	1.2386	1.1906	1.2118	1.1954
	2	1.2431	1.2611	1.2513	1.2901	1.2608	1.2205	1.2415	1.2244
	3	1.2289	1.2565	1.2677	1.3358	1.2809	1.2221	1.2483	1.2370
	4	1.2447	1.2621	1.2778	1.3373	1.2966	1.2431	1.2656	1.2542
	5	1.2338	1.2459	1.2576	1.3131	1.3176	1.2613	1.2833	1.2694
	6	1.3421	1.3406	1.3335	1.3007	1.3406	1.3298	1.3440	1.3208
	7	1.3543	1.3510	1.3518	1.3831	0.0000	1.3246	1.3407	1.3158
	8	1.3343	1.3309	1.3285	1.4087	0.0000	1.3355	1.3531	1.3266
	9	1.3712	1.3821	1.3543	1.2921	0.0000	1.3539	1.3780	1.3470
	10	1.4430	1.4634	1.4271	1.3949	0.0000	1.4023	1.4346	1.3971
	11	1.4669	1.4895	1.4560	1.4903	0.0000	1.4252	1.4617	1.4237
	12	1.4307	1.4651	1.4359	1.4505	0.0000	1.4205	1.4540	1.4250

are normalized to December 1970, so that they may easily be compared with the centered rolling-year index, P_{CRY}. Again, these seasonally adjusted series compare rather well with the trend of P_{CRY} and appear to predict the corresponding target values. Figure 22.7b shows a graph of these series, and the X-11 seasonal adjustment appears to provide a somewhat smoother series than those for the first three series in Table 22.26. This occurs because the X-11 program estimates seasonal factors over the whole data series but requires a minimum of three years of monthly data. The seasonal factors (*SAF*) for the first three series are based on the 12 estimated monthly factors for 1971 that are simply re-

Figure 22.7b. Lowe, Young, Geometric Laspeyres, and Centered Rolling Indices Using X-11 Seasonal Adjustment



peated for subsequent years.[53] Although the trends of X-11 series and the target index (P_{CRY}) are similar, the X-11 series are consistently lower than the target series due to the normalization of the X-11 series. December is a month that has a larger seasonal component in the X-11 adjustment than that for the series using the rolling average. Normalizing the X-11 adjusted series for December results in the first few months of the series showing relatively little growth.

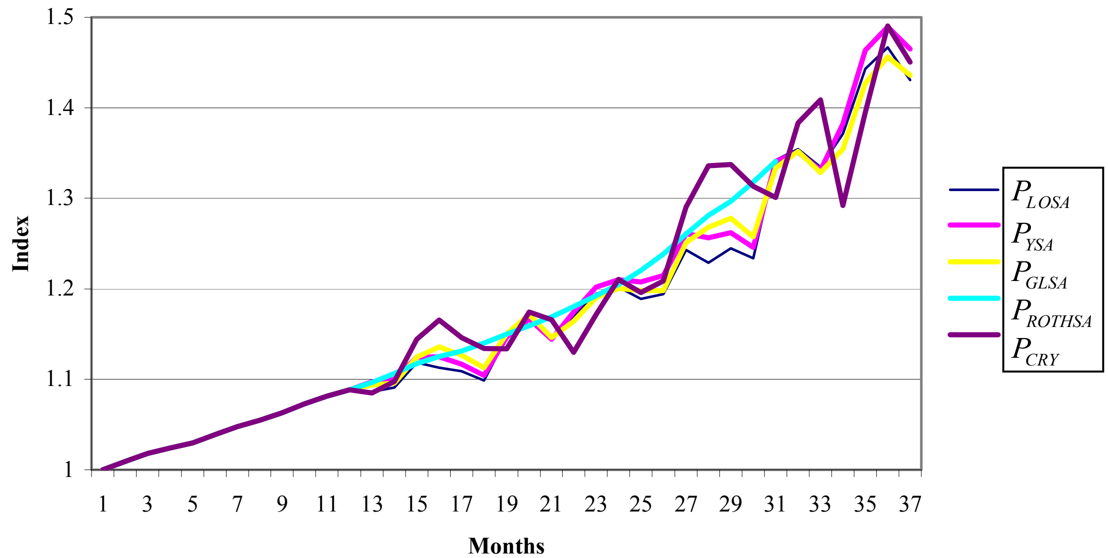
22.94 The manipulations above can be repeated, replacing the *carryforward* annual basket indices with their *imputed* counterparts; that is, using the information in Table 22.24 (instead of Table 22.23) and Table 22.27 replacing Table 22.26. A seasonally adjusted version of the Rothwell index presented in the previous section may also be found in Table 22.27.[54] The eight series in Table 22.27 are also graphed in Figures 22.8a and 22.8b.

22.95 Again, the seasonally adjusted annual basket-type indices listed in the first three data columns of Table 22.27 (using imputations for the missing prices) are reasonably close to the corresponding centered rolling-year index listed in the

[53]Again, for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the X-11 seasonally adjusted Lowe index, an R^2 of 0.9873 is derived; for the X-11 seasonally adjusted Young index, an R^2 of 0.9947 is derived; and for the X-11 seasonally adjusted geometric Laspeyres index, an R^2 of 0.9952 is derived. These fits are better than those obtained above and in Section F. However, the X-11 seasonal adjustment procedure uses the entire data set to do the adjusting, whereas the index number methods of seasonal adjustment used only the first 12 months of data.

[54]The same seasonal adjustment technique that was defined by equation (22.35) was used.

Figure 22.8a. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices; Seasonally Adjusted Rothwell and Centered Rolling-Year Indices

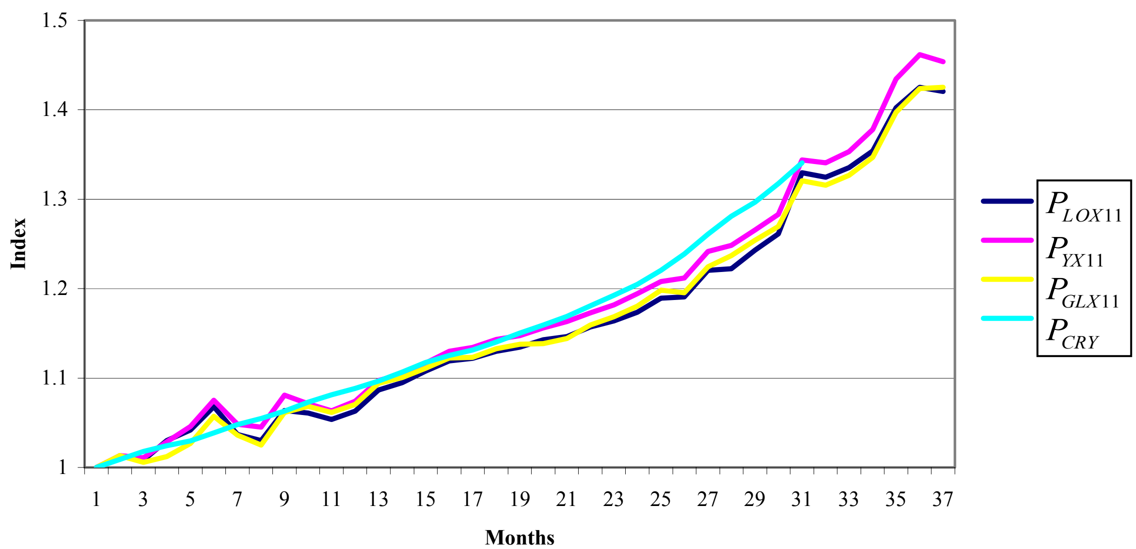


fifth data column of Table 22.27.[55] The seasonally adjusted geometric Laspeyres index is the closest to the centered rolling-year index, and the seasonally adjusted Rothwell index is the furthest away. The three seasonally adjusted month-to-month indices that use annual weights—P_{LOSA}, P_{YSA}, and P_{GLSA}—dip below the corresponding centered rolling-year index, P_{CRY}, for the first few months of 1973 when the rate of month-to-month inflation suddenly increases. But by the middle of 1973, all four indices are fairly close to each other. The seasonally adjusted Rothwell does not do a very good job of approximating P_{CRY} for this particular data set, although this could be a function of the rather simple method of seasonal adjustment that was used. The series adjusted using X-11 again are smoother than the other series and show very similar trends to the target index.

22.96 In comparing the results in Tables 22.26 and 22.7, one can see that it did not make a great deal of difference for the modified Turvey data set whether missing prices are carried forward or imputed; the seasonal adjustment factors picked up the lumpiness in the unadjusted indices that happens when the carryforward method is used. However, the three month-to-month indices that used annual weights and imputed prices did predict the corresponding centered rolling-year indices somewhat better than the three indices that used carry forward prices. Therefore, the use of imputed prices over carryforward prices is recommended.

[55]Again, for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8994 is derived; for the seasonally adjusted Young index, an R^2 of 0.9294 is derived; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9495 is derived. For the seasonally adjusted Rothwell index, an R^2 of 0.8704 is derived, which is lower than the other three fits. For the X-11 seasonally adjusted series, the R^2 values are 0.9644 for the Lowe, 0.9801 for the Young, and 0.9829 for the geometric Laspeyres. All of the Lowe, Young, and geometric Laspeyres indices, using imputed prices, have higher R^2 values than those obtained using carryforward prices.

Figure 22.8b. Lowe, Young, and Geometric Laspeyres Indices Using X-11 Seasonal Adjustment with Imputed Prices and Centered Rolling-Year Indices



22.97 The conclusions that emerge from this section are rather encouraging for statistical agencies that wish to use an annual basket-type index as their flagship index.[56] It appears that for product groups that have strong seasonality, an annual basket-type index can be seasonally adjusted,[57] and the resultant seasonally adjusted index value can be used as a price relative for the group at higher stages of aggregation. The preferred type of annual basket-type index appears to be the geometric Laspeyres index, rather than the Lowe index, but the differences between the two were not large for this data set.

L. Conclusions

22.98 A number of tentative conclusions can be drawn from the results of the sections in this chapter:

- The inclusion of seasonal products in maximum overlap month-to-month indices will frequently lead to substantial biases. Therefore, unless the maximum overlap month-to-month indices using seasonal products cumulated for a year are close to their year-over-year counterparts, the seasonal products should be excluded from the month-to-month index or the seasonal adjustment procedures suggested in Section K should be used;
- Year-over-year monthly indices can always be constructed even if there are strongly seasonal products.[58] Many users will be interested in

[56]Using the results of previous chapters, the use of the annual basket Young index is not encouraged because of its failure of the time reversal test and the resultant upward bias.

[57]It is not necessary to use rolling-year indices in the seasonal adjustment process, but the use of rolling-year indices is recommended because they will increase the objectivity and reproducibility of the seasonally adjusted indices.

[58]There can be problems with the year-over-year indices if shifting holidays or abnormal weather changes normal seasonal patterns. In general, choosing a longer time period will mitigate these types of problems; that is, quarterly sea-
(continued)

these indices; moreover, these indices are the building blocks for annual indices and for rolling-year indices. As a result, statistical agencies should compute these indices. They can be labeled analytic series in order to prevent user confusion with the primary month-to-month PPI;

- Rolling-year indices should also be made available as analytic series. These indices will give the most reliable indicator of annual inflation at a monthly frequency. This type of index can be regarded as a seasonally adjusted PPI. It is the most natural index to use as a central bank inflation target. It has the disadvantage of measuring year-over-year inflation with a lag of six months; thus, it cannot be used as a short-run indicator of month-to-month inflation. However, the techniques suggested in Sections F and K could be used so that timely forecasts of these rolling-year indices can be made using current-price information;
- Annual basket indices can also be successfully used in the context of seasonal commodities. However, many users of the PPI will want to use seasonally adjusted versions of these annual basket-type indices. The seasonal adjustment can be done using the index number methods explained in Section K or traditional statistical agency seasonal adjustment procedures;[59]
- From an a priori point of view, when making a price comparison between any two periods, the Paasche and Laspeyres indices are of equal importance. Under normal circumstances, the spread between the Laspeyres and Paasche indices will be reduced by using chained indices rather than fixed-base indices. As a result, when constructing year-over-year monthly or annual indices, choose the chained Fisher index (or the chained Törnqvist-Theil index, which closely approximates the chained Fisher) as the target index that a statistical agency should aim to approximate. However, when constructing month-to-month indices, chained indices should always be compared with their year-over-year counterparts to check for chain drift. If substantial drift is found, the chained month-to-month indices must be replaced with fixed-base indices or seasonally adjusted annual basket-type indices;[60]
- If current-period revenue shares are not all that different from base-year revenue shares, approximate chained Fisher indices will normally provide a close practical approximation to the chained Fisher target indices. Approximate Laspeyres, Paasche, and Fisher indices use base-period expenditure shares whenever they occur in the index number formula in place of current-period (or lagged current-period) revenue shares. Approximate Laspeyres, Paasche, and Fisher indices can be computed by statistical agencies using their normal information sets; and
- The geometric Laspeyres index is an alternative to the approximate Fisher index that uses the same information. It will normally be close to the approximate Fisher index.

It is evident that more research needs to be done on the problems associated with the index number treatment of seasonal products. A consensus on what is best practice in this area has not yet formed.

sonal patterns will be more stable than monthly patterns, which in turn will be more stable than weekly patterns.

[59]However, there is a problem with using traditional X-11-type seasonal adjustment procedures for adjusting the PPI because final seasonal adjustment factors are generally not available until an additional two or three years' data have been collected. If the PPI cannot be revised, this may preclude using X-11-type seasonal adjustment procedures. Note that the index number method of seasonal adjustment explained in this chapter does not suffer from this problem. It does, however, require the use of seasonal factors derived from a single year of data, so that the year used should reflect a normal seasonal pattern. If the seasonal patterns are irregular, it may be necessary to use the average of two or more years of past adjustment factors. If the seasonal patterns are regular but slowly changing, then it may be preferable to update the index number seasonal adjustment factors on a regular basis.

[60]Alternatively, some sort of multilateral index number formula could be used; for example, see Caves, Christensen, and Diewert (1982a) or Feenstra and Shapiro (2003).

Glossary

Accrual recording

The recording of the value of a purchase or other transaction at the time the obligation to pay is incurred, as distinct from the time payment is made.

Additivity

At current prices, the value of an aggregate is equal to the sum of its components. At constant prices, additivity requires this identity to be preserved for the extrapolated values of the aggregate and its components, when their values in some reference period are extrapolated to some other period using a set of interdependent volume index numbers, or, alternatively, when the values of an aggregate and its components in some period are deflated using a set of interdependent price index numbers based on some other period.

Aggregate

A set of transactions relating to a defined flow of goods and services, such as the total output produced by resident establishments in a given period, or the total purchases of intermediate inputs made by resident establishments in a given period. The term "aggregate" also is used to mean the value of the specified set of transactions.

Aggregation

The process of combining, or adding, different sets of transactions to obtain larger sets of transactions. The larger set is described as having a higher *level* of aggregation than the sets from which it is composed. The term "aggregation" also is used to mean the process of adding the values of lower-level aggregates to obtain higher-level aggregates. It also is used to mean the process by which price indices for lower-level aggregates are averaged, or otherwise combined, to obtain price indices for higher-level aggregates.

Axiomatic approach

The approach to index number theory that determines the choice of index number formula on the basis of its mathematical properties. A list of "tests" is drawn up that require an index to possess certain properties, and the choice of index is made on the basis of the number of tests satisfied. Not all tests may be considered equally important, and the failure to satisfy certain key tests may be considered sufficient grounds for rejecting an index. An important feature of the axiomatic approach is that prices and quantities are considered as separate variables, and no account is taken of possible links between them. Also known as the "test approach."

Base period

The base period generally is understood to be the period with which other periods are compared and whose values provide the weights for a price index. However, the concept of the "base period" is not a precise one and may be used to mean rather different things. Three types of base periods may be distinguished:

(i) the *price reference period*, that is, the period whose prices appear in the denominators of the price relatives used to calculate the index, *or*
(ii) the *weight reference period*, that is, the period, usually a year, whose values serve as weights for the index. However, when hybrid expenditure weights are used in which the quantities of one period are valued at the prices of some other period, there is no unique weight reference period, *or*
(iii) the *index reference period*, that is, the period for which the index is set equal to 100.

The three reference periods may coincide but frequently do not.

Base-weighted index

See "Laspeyres price index."

Basic price

The amount received by the producer from the purchaser for a unit of good or service produced as output. It *includes* subsidies on products and other taxes on production. It *excludes* taxes on products, other subsidies on production, suppliers' retail and wholesale margins, and separately invoiced transport and insurance charges. Basic prices are the prices most relevant for decision making by suppliers.

Basket

The term commonly used for the list of goods and services, together with their relative values of output or input, for which a sample of prices is collected for the purpose of compiling the PPI.

Bias

A systematic error in an index. Bias can arise for a number of reasons, including the design of the sample selected, the price measurement procedures followed, or the index number formula employed.

Book price

See "list price."

Bouncing

The fluctuation or oscillation of prices up and down in a persistent pattern.

Carli price index

An elementary price index defined as the simple, or unweighted, arithmetic average of the current- to base-period price relatives. The Carli index for current period t and price reference period 0 is defined as $P_C \equiv \frac{1}{n}\sum\left(\frac{p^t}{p^0}\right)$.

Carryforward

The situation in which a missing price for a product in the current period is imputed as being equal to the last price observed for that product.

Chain index

An index number series for a given aggregate spanning a long sequence of periods obtained by linking index numbers spanning shorter sequences of periods, each with their own weights. The linking may be made as frequently as the weights change and the data permit, or at specified intervals, such as every 5 or 10 periods. In the limit, the weights may be changed each period, each link in the chain consisting of an index comparing each period with the previous period. See also equation (G.6) of the appendix.

Chain linking

Joining two indices that overlap in one period by rescaling one of them to make its value equal to that of the other in the same period, thus combining them into single time series. More complex methods may be used to link indices that overlap by more than one period. Also known as "chaining."

Chain-linking bias

See "drift."

Characteristics

The physical and economic attributes of a product that identify it and enable it to be classified.

Cif price

Cost, insurance, and freight price. The price of a good delivered at the customs frontier of the importing country, or the price of a service delivered to a resident. It *includes* any insurance and freight charges incurred to that point. It *excludes* any import duties or other taxes on imports and trade and transport margins within the importing country.

Circularity

An index number property such that the algebraic product of the price index comparing period i with period j and the price index comparing period k with period j is equal to the price index that compares period k directly with period i. The property is also known as "transitivity." When the axiomatic approach is used, a price index number may be required to satisfy the "circularity test."

COLI

Cost-of-living index. An index that measures the change between two periods in the minimum expenditures that *would* be incurred by a utility-maximizing consumer, whose preferences or tastes remain unchanged, to maintain a given level of utility (or standard of living or welfare). The COLI is not a fixed-basket index, because consumers may be expected to change the quantities they consume in response to changes in relative prices (see "substitution bias"). The expenditures in one or the other period cannot usually be observed. COLIs cannot be directly calculated but may be approximated by superlative indices. A *conditional cost-of-living index* is one that assumes that all of the factors that may influence the consumer's utility or welfare *other than prices* (such as the physical environment) do not change.

Commensurability test

See "invariance to changes in the units of measurement test."

Commodities

See "products."

Commodity reversal test

A test that may be used under the axiomatic approach, which requires that, for a given set of products, the price index should remain unchanged when the ordering of the products is changed.

Compensation of employees

The total remuneration, in cash or kind, payable by enterprises to employees in return for work done by the latter during the accounting period.

Component

A subset of the goods and services that make up some defined aggregate.

Consistency in aggregation

An index is said to be consistent in aggregation when the index for some aggregate has the same value whether it is calculated directly in a single operation, without distinguishing its components, or it is calculated in two or more steps by first calculating separate indices, or subindices, for its components, or subcomponents, and then aggregating them, the same formula being used at each step.

Constant prices test

See "identity test."

Constant quantities test

See "fixed-basket test."

Consumption of fixed capital

The reduction in the value of the fixed assets used in production during the accounting period resulting from physical deterioration, normal obsolescence, or normal accidental damage.

Continuity

The property whereby the price index is a continuous function of its price and quantity vectors.

Constant elasticity of substitution index

A family of price indices that allows for substitution between products. Within an elementary aggregate, the Jevons index is a particular case of a constant elasticity of substitution index. Another case is the Lloyd-Moulton index.

Contract escalation

See "indexation of contracts."

Contract price

A general term referring to a written sales instrument that specifies both the price and shipment terms. A contract may include arrangements for a single shipment or multiple shipments. Usually it covers a period of time in excess of one month. Contracts often are unique in that the price-determining characteristics in one contract are not repeated exactly in any other contract.

Coverage

The set of price transactions that the index actually measures. Coverage may be narrower than scope for practical reasons.

CPA

Classification of Products by Activity. The classification of products by originating activity favored by the European Union. Originating activities are those defined by NACE.

CPC

Central Product Classification. An internationally agreed classification of products based on the physical characteristics of goods or on the nature of the services rendered. Each type of good or service distinguished in the CPC is defined in such a way that it is normally produced by only one activity as defined in the International Standard Industrial Classification of All Economic Activities.

CSWD index

Carruthers, Sellwood, Ward, Dalén index. A geometric average of the Carli and the harmonic mean of price relatives index. It is defined as $P_{CSWD} \equiv \sqrt{P_C \times P_{HR}}$.

Current cost accounting

A method of accounting for the use of assets in which the cost of using the assets in production is calculated at the current price of those assets rather than using the historic cost (that is, the price at which the assets were originally purchased).

Current period

In principle, the "current" period should refer to the most recent period for which an index has been computed or is being computed. However, the term is widely used to refer to any period that is compared with the price reference or index reference period. It is also widely used simply to mean the later of the two periods being compared. The exact meaning varies according to the context.

Current weighted index

See "Paasche price index."

Cutoff sampling

A sampling procedure in which a predetermined threshold is established with all units in the universe at or above the threshold being included in the sample and all units below the threshold being excluded. The threshold is usually specified in terms of the size of some known relevant variable. In the case of establishments, size is usually defined in terms of employment or output.

Deflation

The division of the value of some aggregate by a price index—described as a "deflator"—to revalue its quantities at the prices of the price reference period or to revalue the aggregate at the general price level of the price reference period.

Discount

A deduction from the list or advertised price of a good or a service that is available to specific customers under specific conditions. Examples include cash discounts, prompt payment discounts, volume discounts, trade discounts, and advertising discounts.

Divisia approach

A price or quantity index that treats both prices and quantities as continuous functions of time. By differentiation with respect to time, the rate of change in the value of the aggregate in question is partitioned into two components, one of which is the price index and the other the quantity index. In practice, the indices cannot be calculated directly, but it may be possible to approximate them by chain indices in which the links consist of period-to-period indices linking consecutive periods.

Domain

See "scope" and "coverage."

Double deflation

A method whereby gross value added at constant prices is derived by subtracting the value of intermediate inputs at constant prices from the value of output at constant prices. The method is feasible only when the values at constant prices are additive.

Drift

An index is said to "drift" if it does not return to unity when prices in the current period return to their levels in the base period. Chain indices may drift when prices fluctuate over the periods they cover. Also known as "chain-linking bias."

Drobisch price index

A price index defined as the arithmetic average of the Laspeyres price index and the Paasche price index. It is a symmetric index and a pseudo-superlative index: $P_{DR} \equiv \frac{1}{2}\left(P_L + P_P\right)$.

Durable input

An input that can be continuously used over a period longer than the time period being used in the index, which is generally a month or a quarter. In practice, an input that can be used for several years.

Dutot index

A price index defined as the ratio of the unweighted arithmetic average of the prices in the current period to the unweighted arithmetic average of the prices in the base period. It is an elementary index, defined as

$$P_D \equiv \frac{\frac{1}{n}\sum p^t}{\frac{1}{n}\sum p^0}.$$

Economically significant prices

Prices that have a significant influence on the amounts producers are willing to supply and on the amounts purchasers wish to buy.

Economic approach

The approach to index number theory that assumes the observed price and quantity data are generated as solutions to various economic optimization problems. The quantities are assumed to be functions of the prices and not independent variables. Also known as the "microeconomic approach."

Edgeworth price index

See "Marshall-Edgeworth price index."

Editing

See "input editing" and "output editing."

Elementary aggregate

The lowest level of aggregation for which value data are available and used in the calculation of the PPI. Elementary aggregates consist of relatively homogeneous sets of goods or services. Their values are used as weights when averaging the elementary price indices associated with them to obtain indices for higher-level aggregates. They also may serve as strata from which the products selected for pricing are sampled.

Elementary price index

Specifically, an elementary price index is a price index for an elementary aggregate. As such, it is calculated from individual price observations and usually without using weights. More generally, the term is also sometimes used to describe any price index calculated without weights. Three examples of elementary index number formulas are the Carli, the Dutot, and the Jevons.

Enterprise

An institutional unit in its capacity as a producer of goods and services consisting of one or more establishments. An enterprise may be a corporation, a quasi-corporation, a nonprofit institution, or an unincorporated enterprise.

Establishment

An enterprise, or part of an enterprise, situated in a single location and in which a single, nonancillary productive activity is carried out, or in which the principal productive activity accounts for most of the value added. Also referred to as "LKAU" or "local kind of activity unit."

Error

The difference between the observed value of an index and its "true" value. Errors may be random or systematic. Random errors are generally referred to as "errors." Systematic errors are called "biases."

Evolutionary goods

Goods similar to or extensions of existing goods. They are typically produced on the same production line using production inputs and processes that are largely the same as those used to produce existing goods. It is possible, at least in theory, to adjust for any quality differences between an evolutionary good and an existing good.

Factor reversal test

Suppose the roles of the prices and quantities in a price index are reversed to yield a quantity index of exactly the same functional form as the price index. The factor reversal test used under the axiomatic approach requires that the product of this quantity index and the original price index be identical with the proportionate change in the value of the aggregate in question. Also known as the "product test."

Factory gate price

A basic price with the "factory gate" as the pricing point, that is, the price of the product available at the factory, excluding any separately billed transport or delivery charge.

Farm gate price

A basic price with the "farm gate" as the pricing point, that is, the price of the product available at the farm, excluding any separately billed transport or delivery charge.

FEPI

Final expenditure price index. A measure of the changes in prices paid by consumers, businesses, and government for final purchases of goods and services. Intermediate purchases are excluded.

FIOPI

Fixed-input output price index. The theoretical model for an output PPI based on the assumption of fixed technology and inputs. It requires the index to reflect changes in revenue resulting from the sale of the same products—although not necessarily the same mix of products—produced under the same circumstances and sold under the same terms. In other words, changes in the index arise solely from changes in output prices and are not influenced by changes in inputs. Revenue-maximizing behavior is assumed on the part of the producer.

Fisher price index

A price index defined as the geometric average of the Laspeyres price index and the Paasche price index: $P_F \equiv \sqrt{P_L \times P_P}$. It is a symmetric and superlative index.

Fixed-basket or fixed-weight price index

The traditional conceptualization of a price index. The index measures the change in value of a fixed set of quantities—commonly described as a "fixed basket of goods and services"—between two periods. Because the quantities or weights remain fixed, any change in the index is due to price changes only. In principle, there is no restriction on the quantities that make up the basket. They may be those of one of the two periods being compared, they may refer to the quantities in some third period, or they may constitute a hypothetical basket, such as an average of the quantities in the two periods. Moreover, the quantities may refer to a much longer period of time than the periods of the index: for example, quantities produced over a period of a year or more may be used for a monthly or quarterly PPI. A fixed-basket or fixed-weight index is sometimes described as a "pure price index."

Fixed-basket test

A test that may be used under the axiomatic approach whereby if all the quantities remain unchanged (that is, the sets of quantities in both periods are identical), the price index should equal the proportionate change in the value of the aggregate. Also known as the "constant quantities test."

fob price

Free on board price. The price of a good delivered at the customs frontier of the exporting country. It *includes* the freight and insurance charges incurred to that point and any export duties or other taxes on exports levied by the exporting country.

FOIPI

Fixed-output input price index. The theoretical model for an input PPI based on the assumption of fixed technology and outputs. It requires the index to reflect changes in costs resulting from the purchase of the same inputs—although not necessarily the same mix of inputs—purchased under the same terms to produce the same output with the same technology. In other words, changes in the index arise solely from changes in input prices and are not influenced by changes in outputs. Cost-minimizing behavior is assumed on the part of the producer.

Geometric Laspeyres price index

A price index defined as the weighted geometric average of the current- to base-period price relatives using the value shares of the base period as weights. Also known as the "logarithmic Laspeyres price index." It is defined as $P_{JW} \equiv \prod \left(\frac{p^t}{p^0} \right)^{s^0}$, where $s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}$.

Geometric Paasche price index

A price index defined as the weighted geometric average of the current- to base-period price relatives using the value shares of the current period as weights. Also known as the "logarithmic Paasche price index." It is defined as $\prod \left(\frac{p^t}{p^0} \right)^{s^t}$, where $s^t \equiv \frac{p^t q^t}{\sum p^t q^t}$.

Goods

Physical objects for which a demand exists, over which ownership rights can be established, and whose ownership can be transferred from one institutional unit to another by engaging in transactions on the market. They are in demand because they may be used to satisfy the needs or wants of households or the community or used to produce other goods or services.

Gross sector output

The sum of the sales of output of the establishments in the sector, including the sales of output among themselves, to other sectors in the economy and within the sector. See "net sector output."

Gross value added

The value of output less the value of the intermediate inputs used to produce the output. It is a measure of the contribution to GDP made by an individual producer, industry, or sector.

Harmonic mean of price relatives

An elementary index that constitutes the harmonic average counterpart to the Carli index. It is defined as $P_{HR} \equiv \dfrac{1}{\frac{1}{n}\sum\left(\frac{p^0}{p^t}\right)}$.

Harmonic means price index (also "ratio of harmonic means")

An elementary index that constitutes the harmonic average counterpart to the Dutot index. It is defined as $P_{RH} \equiv \dfrac{\sum n/p^0}{\sum n/p^t}$.

Hedonic method

A regression technique in which observed prices of different qualities or models of the same generic good or service are expressed as a function of the characteristics of the goods or services in question. It is based on the hypothesis that products can be treated as bundles of characteristics and that prices can be attached to the characteristics. The characteristics may be nonnumerical attributes represented by dummy variables. The regression coefficients are treated as estimates of the contributions of the characteristics to the overall prices. The estimates may be used to predict the price of a new quality or model whose mix of characteristics is different from that of any product already on the market. The hedonic method can therefore be used to estimate the effects of quality changes on prices.

Hidden economy

Those activities hidden or nonobserved because they are underground, illegal, informal, undertaken by households for their own use, or missed because of deficiencies in the basic statistical data collection program. Also known as the "nonobserved economy."

Higher-level index

A term sometimes used to distinguish an aggregate index from an elementary index.

Identity test

A test that may be used under the axiomatic approach and that requires if the prices remain unchanged between the two periods (that is, the sets of prices are identical), the price index should equal unity. Also known as the "constant prices test."

Imputed price

The value assigned to a missing price.

Indexation of contracts

A procedure whereby a long-term contract for the provision of goods or services includes a periodic adjustment to the prices paid for the goods or services based on the increase or decrease in the level of a nominated price index. The purpose of indexation is to take inflationary risk out of the contract. Also known as "index linking" and "contract escalation."

Index item

An elementary or lower-level index with a fixed weight within the upper-level index structure.

Index number problem

How to combine the relative changes in the prices and quantities of various products into (i) a single measure of the relative change of the overall price level and (ii) a single measure of the relative change of the overall quantity level. Or, conversely, how a value ratio pertaining to two periods of time can be decomposed in a component that measures the overall change in prices between the two periods—that is, the price index—and a component that measures the overall change in quantities between the two periods—that is, the quantity index.

Index reference period

The period for which the value of the index is set at 100. See also "base period."

Industry

A general term to describe a group of establishments engaged on the same, or similar, kinds of production activity. Also a specific term used to describe establishments engaged in mining and quarrying, manufacturing, electricity, gas and water (Sections C, D, and E of ISIC, Rev. 3).

Input editing

The process of analyzing the prices reported by an individual respondent and querying price changes that are above a specified level or are inconsistent across product lines. Important objectives of the process are to ensure that actual transaction prices are reported and to detect any changes in the specifications.

Input PPI

A measure of the change in the prices of goods and services bought as intermediate inputs by domestic producers. Covers both domestically produced intermediate inputs and imported intermediate inputs. Valuation is at purchasers' prices.

Institutional unit

A national accounts concept defined as an economic entity that is capable, in its own right, of owning assets, incurring liabilities, and engaging in economic activities and transactions with other entities. Enterprises are institutional units. Other kinds of units include households and governments.

Intermediate basket

A basket derived as the average of the baskets of two time periods, usually the base and current periods. The average can be arithmetic, as in the Marshall-Edgeworth price index, or geometric, as in the Walsh price index.

Intermediate consumption

The value of goods and services used or consumed as intermediate inputs by a process of production.

Intermediate inputs

Goods and services, other than fixed assets, used as inputs into the production process of an establishment that are produced elsewhere in the economy or are imported. They may be either transformed or used up by the production process. Land, labor, and capital are primary inputs and are not included among intermediate inputs. Also called "intermediate products."

Intra-company transfer price

The value assigned on a per unit or per shipment basis to goods transferred from one establishment of an enterprise to another. It may or may not be economically significant. However, it is not a market price since ownership of the good does not change hands. See "transfer price."

Invariance to changes in the units of measurement test

A test that may be used under the axiomatic approach and that requires the price index to not change when the physical, or quantity, units to which the prices of the goods or services refer are changed: for example, when the price of a beverage is quoted per liter rather than per pint. Also known as the "commensurability test."

Invariance to proportional change in current- or base-quantities test

A test that may be used under the axiomatic approach and that requires the price index to remain unchanged when the base-period quantities, or the current-period quantities, are multiplied by a positive scalar.

Inverse proportionality in base-year prices test

A test that may be used under the axiomatic approach and that requires if all base-period prices are multiplied by the positive scalar λ, the new price index is $1/\lambda$ times the old price index.

ISIC

International Standard Industrial Classification of All Economic Activities. An internationally agreed classification that allows enterprises and establishments to be classified according to economic activity based on the class of goods produced or services rendered.

Item

A product selected for pricing. A transaction whose price is collected.

Item or product rotation

The deliberate replacement of a sampled item, or product, for which prices are collected, by another product before the replaced product has disappeared from the market or individual establishment. It is designed to keep the sample of products up to date and reduce the need for forced replacements caused by the disappearance of products.

Item substitution

The replacement of a sampled product, or item, by a new product.

Jevons price index

A price index defined as the unweighted geometric average of the current- to base-period price relatives. It is an elementary index and defined as $P_J \equiv \prod \left(\frac{p^t}{p^0} \right)^{1/n}$.

Laspeyres price index

A price index defined as a fixed-weight, or fixed-basket, index that uses the basket of goods and services of the base period. The base period serves as both the weight reference period and the price reference period. It is identical with a weighted arithmetic average of the current- to base-period price relatives using the value shares of the base period as weights. Also called a "base-weighted index." It is defined as $P_L \equiv \frac{\sum p^t q^0}{\sum p^0 q^0} = \sum s^0 \left(\frac{p^t}{p^0} \right)$, where $s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}$.

List price

The price of a product as quoted in the producer's price list, catalogue, Internet site, and the like. The gross price exclusive of all discounts, surcharges, rebates, and the like that apply to an actual transaction. Also known as "book price."

LKAU

Local kind of activity unit. See "establishment."

Lower-level index

See "elementary price index."

Lowe price index

A basket-type family of price indices that compares the prices of period t with those of an earlier period 0, using a certain specified quantity basket q_n. $P_{LO} = \frac{\sum p^t q_n}{\sum p^0 q_n}$. The family of Lowe indices includes, for example, the Laspeyres index ($q_n = q^0$) and the Paasche index ($q_n = q^t$). See Equation (G.1) in the appendix. In practice, statistical offices frequently use a Lowe price index with a quantity basket of period b, where b denotes some period before 0, and hybrid value shares valued at prices in period 0, the price reference period. The share-weighted Lowe index is $P_{LO} = \sum s^{b0} \left(\frac{p^t}{p^0} \right) \left(\frac{p^t}{p^0} \right)$, where $s^{b0} \equiv \frac{p^0 q^b}{\sum p^0 q^b}$.

Lloyd-Moulton price index

A particular case of a constant elasticity of substitution price index. In its weighted form, the Lloyd-Moulton formula is $P_{LM} \equiv \left[\sum s^0 \left(\frac{p^t}{p^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$, where $s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}$; in its unweighted form, $P_{LM} \equiv \left[\sum \frac{1}{n} \left(\frac{p^t}{p^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$.

Market price

The amount of money a willing buyer pays to acquire a good or service from a willing seller. The actual price for a transaction agreed on by the transactors. The net price inclusive of all discounts, surcharges, and rebates applied to the transaction. From the seller's point of view the market price is the basic price; from the buyer's point of view the market price is the purchaser's price. Also referred to as "transaction price."

Marshall-Edgeworth price index

A price index defined as the weighted arithmetic average of the current- to base-period price relatives and that uses the quantities of an intermediate basket as weights. The quantities of the intermediate basket are arithmetic averages of the quantities of the base and current periods. It is a symmetric index and a pseudo-superlative index. It is defined as $P_{ME} \equiv \frac{\sum p^t \frac{1}{2}(q^t + q^0)}{\sum p^0 \frac{1}{2}(q^t + q^0)}$.

Matched-products or -models method

The pricing of identical products or models in consecutive periods to ensure that the measured price change cannot be affected by changes in quality. In other words, pricing to constant quantity. Price changes for perfectly matched products may be described as “pure” price changes. See also “specification pricing.”

Mean value test for prices

A test that may be used under the axiomatic approach and that requires the price index to lie between the minimum price relative and the maximum price relative.

Mean value test for quantities

A test that may be used under the axiomatic approach and that requires the implicit quantity index to lie between the minimum and maximum rates of growth of the individual quantities.

Microeconomic approach

See “economic approach.”

Midperiod price index

A price index that uses either the quantity or value weights from an intermediate period that lies between the base period and the current period when the number of periods between them is odd, or the average of the quantity or value weights for two consecutive intermediate periods that lie between the base period and the current period when the number of periods between them is even.

“Modified,” “short-term change,” or “two-stage” Laspeyres price index

These often-used descriptions of the Laspeyres index have at least three meanings:

As a short-run modified Laspeyres index. This is an index with weight reference period b and price changes between periods 0 and t, where the latter are decomposed into price changes between period 0 and $t-1$ and period $t-1$ and t:

$$P_{MLAS} \equiv \sum s^b \left(\frac{p^{t-1}}{p^0}\right)\left(\frac{p^t}{p^{t-1}}\right), \text{ where } s^b \equiv \frac{p^b q^b}{\sum p^b q^b}.$$

The decomposition helps in dealing with changes in the sampled products. In the absence of changes in the sample, P_{MLAS} reduces to a Young index between t and 0 with weight reference period b:

$$P_{MLAS} \equiv \sum s^b \left(\frac{p^t}{p^0}\right) = P_Y .$$

As a price-updated version of a Young index. It is a fixed-weight index in which the quantities are those of the weight reference period b, but the price reference period is a later period 0 preceding the current period t. The implicit expenditure weights are obtained by revaluing the quantities of the weight reference period b at the prices of the price reference period 0, a procedure described as “price updating.” This modified Laspeyres index between periods 0 and t can be interpreted as a weighted average of the price relatives between 0 and t, using the price-updated expenditure weights. Its definition is $\sum\left(\frac{s^b(p^0/p^b)}{\sum s^b(p^0/p^b)}\right)\left(\frac{p^t}{p^0}\right)$ and corresponds to a Lowe price index between periods 0 and t with weight reference period b. See also “price updating” and “Lowe price index.”

As a two-stage Laspeyres index. The two-stage procedure decomposes a Laspeyres price index between b and t into a Laspeyres price index between b and 0 and a Lowe price index between *0* and t:

$\frac{\sum p^t q^b}{\sum p^b q^b} = \frac{\sum p^0 q^b}{\sum p^b q^b}\frac{\sum p^t q^b}{\sum p^0 q^b}$. See also equation (G.3) in the appendix.

Monotonicity in prices

The property whereby if any current-period price increases or any base-period price decreases, the price index increases.

Monotonicity in quantities

The property whereby if any current-period quantity increases or any base-period quantity decreases, the implicit quantity index that corresponds to the price index increases.

Multifactor productivity

Relates a measure of output to a measure of combined primary inputs. Rates of change of multifactor productivity are typically measured residually, because that change in output cannot be accounted for by the change in combined inputs.

NACE

The acronym for the *General Industrial Classification of Economic Activities within the European Communi-*

ties. The classification is basically a more detailed version of ISIC appropriate to European circumstances.

Net sector output

The sum of the sales of output of the establishments in the sector to other sectors of the economy. Gross sector output for the sector less the sales of the sector's output within the sector.

Net value added

Gross value added less the value of consumption of fixed capital.

New-good problem

Difficulty in comparing prices between two periods when a product enters the basket only in period 2, so that a price for the product does not exist in period 1.

New goods

See "evolutionary goods" and "revolutionary goods."

Nominal prices

Prices charged by providers of general government services such as health and education, and prices that are heavily subsidized through government funding or regulated by government policy. Such prices are not economically significant and therefore do not provide signals of market-driven inflation.

Nonmarket transactions

Transactions covering goods or services that their producers supply to others free or at prices that are not economically significant. Examples of nonmarket transactions include own-account production by establishments for the enterprises of which they form a part, own-account production by unincorporated enterprises owned by households (such as the output of owner-occupiers and subsistence farmers), and services supplied to the community as a whole by establishments owned by general government (such as defense and public order and safety).

Nonprobability sampling

The nonrandom selection of a sample of producers and products based on expert knowledge or judgment. Also known as "nonrandom sampling," "purposive sampling," and "judgmental sampling."

Nonresponse bias

The bias that arises when those who do not respond have different price experiences than those who do respond.

Observation

The price collected or reported for a sampled product or item.

Observation point

Usually a product variety in an establishment. A tightly specified item in a specific establishment.

Order price

The price quoted at the time the order is placed by the purchaser.

Other subsidies on production

The subsidies that resident enterprises may receive as a consequence of engaging in production. For example, subsidies on payroll or workforce or subsidies to reduce pollution. They do not include subsidies on products.

Other taxes on production

The taxes that resident enterprises may pay as a consequence of engaging in production. They consist mainly of current taxes on the labor or capital employed in the enterprise, such as payroll taxes or current taxes on vehicles or buildings. They do not include taxes on products.

Outlier

A term generally used to describe any extreme value in a set of survey data. In a PPI context, it is used for an extreme value that requires further investigation or that has been verified as being correct.

Output

The goods or services produced within an establishment that become available for use outside that establishment, plus any goods and services produced for own final use.

Output editing

The process of comparing the price levels and price movements of similar products among different respondents and querying any outliers.

Output PPI

A measure of the change in the prices of goods and services sold as output by domestic producers. Covers both output sold on the domestic market and output sold as exports. Valuation is at basic prices.

Paasche and Laspeyres bounding test

A test that may be used under the axiomatic approach and that requires the price index to lie between the Laspeyres price index and the Paasche price index.

Paasche price index

A price index defined as a fixed-weight, or fixed-basket, index that uses the basket of goods and services of the current period. The current period serves as the weight reference period and the base period as the price reference period. It is identical with a weighted harmonic average of the current- to base-period price relatives using the value shares of the current period as weights. Also called a "current weighted index." It is defined as

$$P_P \equiv \frac{\sum p^t q^t}{\sum p^0 q^t} = \left[\sum s^t \left(\frac{p^t}{p^0} \right)^{-1} \right]^{-1}, \text{ where } s^t \equiv \frac{p^t q^t}{\sum p^t q^t}.$$

Palgrave price index

A price index defined as the weighted arithmetic average of the current- to base-period price relatives using the current-period value shares as weights.

$$P_{Pal} \equiv \sum s^t \left(\frac{p^t}{p^0} \right).$$

Point-in-time prices

Transaction prices prevailing on a particular day of the month.

Positivity

The property whereby the price index and its constituent vectors of prices and quantities are positive.

PPI

Producer price index. A measure of the change in the prices of goods and services either as they leave their place of production or as they enter the production process. A measure of the change in the prices received by domestic producers for their outputs or of the change in the prices paid by domestic producers for their intermediate inputs. See "output PPI" and "input PPI."

PPS

Probability proportional to size. A sampling procedure whereby each unit in the universe has a probability of selection proportional to the size of some known relevant variable. In the case of establishments, size is usually defined in terms of employment or output.

Price index

A measure reflecting the average of the proportionate changes in the prices of the specified set of goods and services between two periods of time. Usually a price index is assigned a value of 100 in some selected base period, and the values of the index for other periods are intended to indicate the average percentage change in prices compared with the base period.

Price reference period

The prices of a period with which prices the prices in the current-period are compared. The period whose prices appear in the denominators of the price relatives. See also "base period."

Price relative

The ratio of the price of an individual product in one period to the price of that same product in some other period.

Price reversal test

A test that may be used under the axiomatic approach and that requires the quantity index to remain unchanged after the price vectors for the two periods being compared are interchanged.

Price updating

A procedure whereby the quantities in the weight reference period are revalued at the prices of a later period that serves as the price reference period, typically the period preceding the current period. In other words, revaluing the weights to ensure that they are effectively based on the underlying quantities or volumes of the price reference period. The revaluing is achieved by multiplying the expenditure on each product in the weight reference period by the cumulative price change for that product between the weight reference period and the price reference period. Also known as "value updating."

Pricing point

The point in the production or distribution process to which the price refers. For an output PPI, the pricing point is generally where the product leaves its place of production—farm gate or factory gate price. For an input PPI, the pricing point is generally where the product enters the production process; that is, when it is delivered to the customer—purchaser's price.

Pricing to constant quality

See "specification pricing."

Probability sampling

The random selection of a sample of producers and products from a universe of industrial activity in which each producer and product has a known nonzero probability of selection. It ensures that the items to be priced are selected in an impartial and objective fashion, and permits the measurement of the quality of survey results through estimates of the variance or sampling error. Also known as "random sampling."

Producer's index

An index constructed from price data supplied by producers.

Producer's price

The amount received by the producer from the purchaser for a unit of good or service produced as output. It *excludes* any VAT (or similar deductible tax on products) invoiced to the purchaser. It also *excludes* supplier's retail and wholesale margins and separately invoiced transport and insurance charges. (A producer's price for a product is the basic price *plus* any nondeductible tax on products paid by the producer *less* any subsidies on products received by the producer.)

Production

An activity that transforms or combines material inputs into other material outputs—as in agricultural, mining, manufacturing, or construction activities—or transports materials from one location to another. Production also includes storage activities—which, in effect, transport materials in the same location from one time period to another—and the creation of services of all types.

Product line

A group or class of products relatively homogeneous in use and in price behavior.

Product line specification

A statement of the characteristics of the range of products included in a product line. Its purpose is to provide the frame within which individual products may be selected as part of the sample for pricing. It also may describe the products included in a subindex.

Products

Goods and services that are the result of production. They are exchanged and used for various purposes: as inputs in the production of other goods and services, as final consumption, or for investment. Also referred to as "commodities."

Product specification

A detailed list of the characteristics that identify an individual sampled product. Its purpose is to ensure that a consistent price is collected from period to period relating to a consistent product with the same terms of sale in each period. Hence, the characteristics listed cover both the product (name, serial number, description, etc.) and the transaction (class of customer, size of shipment, discounts, payment terms, delivery details, etc.).

Product test

See "factor reversal test."

Proportionality in current prices test

A test that may be used under the axiomatic approach and that requires that if all current-period prices are multiplied by the positive scalar λ, the new price index is λ times the old price index.

Pseudo-superlative index

An index that approximates any superlative index to the second order around an equal price and quantity point.

Purchaser's index

An index constructed from price data supplied by purchasers.

Purchaser's price

The amount paid by the purchaser to take delivery of a unit of a good or service at the time and place required by the purchaser. It *excludes* any VAT (or similar deductible tax on products) that the purchaser can deduct from his or her own VAT liability invoiced to customers. It *includes* suppliers' retail and wholesale margins, separately invoiced transport and insurance charges, and any VAT (or similar deductible tax on products) that the purchaser cannot deduct from his or her own VAT liability. (A purchaser's price for a product is the producer's price *plus* suppliers' retail and wholesale margins, separately invoiced transport and insurance charges, and nondeductible taxes on products payable by the purchaser.) Purchasers' prices are the prices most relevant for decision making by buyers.

"Pure" price change

The change in the price of a good or service whose characteristics do not change over time. When some characteristics do change, that is, a change in quality occurs, the "pure" price change is the price change remaining after eliminating the contribution of the change in quality to the observed price change.

"Pure" price index

A price index based on pricing a constant representative basket of products at the prices of the base period and at the prices of the current period. Because the products and their weights remain constant, any change in the index is due to price changes only. An index that measures "pure" price change. Also called "unequivocal price index."

Quality adjustment

The process—or the result of the process—of estimating what the market price of a replacement product would be if it had the characteristics of the product it replaces and with whose price its price is to be compared. The process requires estimating the market value of any differences in the price-determining characteristics of the two products and adjusting—by addition, subtraction, or multiplication by a coefficient—the observed price of the replacement product. The adjustment is made so that the price comparison between the two products reflects "pure" price change only.

Quantity index

A measure reflecting the average of the proportionate changes in the quantities of a specified set of goods and services between two periods of time. Usually a quantity index is assigned a value of 100 in some selected base period, and the values of the index for other periods are intended to indicate the average percentage change in quantities compared with the base period. See "volume index."

Quantity relative

The ratio of the quantity of a specific product in one period to the quantity of the same product in another period.

Quantity reversal test

A test that may be used under the axiomatic approach and that requires the price index to remain unchanged after the quantity vectors for the two periods being compared are interchanged.

Quantity weights

Weights defined in terms of physical quantities, such as the number or total weight of goods or the number of services. Quantity weights are feasible only at the detailed product level because meaningful aggregation of product weights requires them to be commensurate. See "value weights."

Ratio of harmonic means price index

See "Hamonic means price index."

Rebasing

There is some ambiguity in the concept of the base year. Rebasing may mean

- Changing the weights in an index,
- Changing the price reference period of an index number series, or
- Changing the index reference period of an index number series.

The weights, the price reference period, and the index reference period may be changed at the same time, but not necessarily so.

Rebate

A discount paid to the customer after the transaction has occurred.

Replacement product

The product chosen to replace a sampled product either because it has disappeared completely from the market or because its market share has fallen either in a specific establishment or in the market as a whole.

Representative item

A product selected for pricing within an elementary aggregate because purchases of the product account for a significant proportion of the total purchases of all products within the aggregate or because the price change for the product is close to the average for all products within the aggregate.

Revenue

The value of output sold. The value of invoiced sales of goods or services supplied to third parties during the reference period. Used interchangeably with "sales" and "turnover."

Revolutionary goods

Goods that are significantly different from existing goods. They are generally produced on entirely new production lines using production inputs and processes considerably newer than those used to produce existing goods. It is virtually impossible, both in theory and in practice, to adjust for any quality differences between a revolutionary good and any existing good.

Reweighting

Introducing a new set of weights into the index.

RMSE

Root mean square error. A measure of total error defined as the square root of the sum of the variance and the square of the bias.

Sample augmentation

Maintaining and adding to the sample of establishments in the survey panel to ensure it continues to be representative of the population of establishments. A fixed sample of establishments tends to become depleted as establishments cease producing or stop responding. Recruiting new establishments also facilitates the inclusion of new products in the PPI.

Sample rotation

Limiting the length of time that establishments stay on the survey panel by dropping a proportion of them after a certain period of time and replacing them with a new sample of establishments. Generally done only with the smaller respondents, for whom it is felt that responding to surveys imposes a significant burden. Rotation is designed to keep the sample up to date. It also helps to alleviate the problems caused by sample depletion.

Sampling error

A measure of the chance of a difference between the results obtained from the units sampled and the results that would have been obtained from a complete enumeration of all units in the universe.

Sampling frame

The list of the units in the universe from which a sample of units is to be selected. It provides for each unit the details required to pick the sample, such as the unit's location, size, and activities.

Sauerbeck price index

A price index defined as the weighted arithmetic average of the current- to previous-period price relatives using the values of the base period as weights. The price reference period is the previous period (that is, the period immediately before the current period), and the weight reference period is some other fixed period before the previous period. A time-series index is derived by chaining, which, because the weight reference period remains fixed, can result in a serious upward drift in the index when price changes are large and erratic.

Scope

The domain of price transactions that the PPI aims to measure. The conceptual boundaries of the PPI in terms of the products, transactions, geographical areas, and producers to which it refers.

Seasonal products

Products that are either not available on the market during certain seasons or periods of the year or are available throughout the year but with regular fluctuations in their quantities and prices that are linked to the season or time of the year.

Sector

A general term used to describe a group of establishments engaged in similar kinds of economic activity. A sector can be a subgroup of an economic activity—as in "coal mining sector"—or a group of economic activities—as in "service sector"—or a cross-section of a group of economic activities—as in "informal sector." Also a specific term used in the *1993 SNA* to denote one of the five mutually exclusive institutional sectors that group institutional units on the basis of their principal functions, behavior, and objectives, namely: nonfinancial corporations, financial corporations, general government, nonprofit institutions serving households, and households.

Services

Outputs produced to order that cannot be traded separately from their production. Ownership rights cannot be established over services, and by the time their production is completed, they must have been provided to the consumers. However, as an exception to this rule, there is a group of industries, generally classified as service industries, some of whose outputs have characteristics of goods. These are the industries concerned with the provision, storage, communication, and dissemination of information, advice, and entertainment in the broadest sense of those terms. The products of these industries, where ownership rights can be established, may be classified either as goods or services depending on the medium by which these outputs are supplied.

Shipment price

The price at the time the order is delivered to the purchaser.

SNA

System of National Accounts. A coherent, consistent, and integrated set of macroeconomic accounts, balance sheets, and tables based on a set of internationally agreed concepts, definitions, classifications, and accounting rules.

Specification pricing

The pricing methodology whereby a manageable sample of precisely specified products is selected, in consultation with each reporting establishment, for repeat pricing. Products are fully defined in terms of all characteristics that influence their transaction prices. The objective is to price to constant quality to produce an index showing "pure" price change.

Splicing

The introduction of a replacement item and attributing any price change between the replacement item in the period it is introduced and the replaced item in the period before the introduction to the change in quality.

Spot market price

A generic term referring to any short-term sales agreement. Generally it refers to single-shipment orders with delivery expected in less than one month. Goods sold on this basis are usually off the shelf and not subject to customization. Spot market prices are subject to discounting and directly reflect current market conditions.

Stage of processing

The classification of goods and services according to their position in the chain of production. However, unlike the classification by stage of production, a product is allocated to only one stage, even though it may occur in several stages. Goods and services are classified either as primary products, intermediate products, or finished products.

Stage of production

The classification of goods and services according to their position in the chain of production but allowing for the multifunction nature of products. Unlike the classification by stage of processing, a product is allocated to each stage to which it contributes and not assigned solely to one stage. Goods and services are classified as primary products, intermediate products, or finished products.

Stochastic approach

The approach to index number theory that treats each price relative as an estimate of a common price change. Hence, the expected value of the common price change can be derived by the appropriate averaging of a random sample of price relatives drawn from a defined universe.

Subsidies on products

The subsidies on goods or services produced as the outputs of resident enterprises that become payable as the result of the production of those goods or services. They are payable per unit of good or service produced.

Subsidized prices

Prices that differ from market prices in that some significant portion of variable or fixed costs are covered by a revenue source other than the selling price.

Substitution bias

The bias that arises when the index number formula used for an output PPI systematically understates average price increases because it does not take into account that producers seeking to maximize revenue from a given technology and inputs may shift production to items with above-average relative price increases. Or, the bias that arises when the index number formula used for an input PPI systematically overstates average price increases because it does not take into account that producers seeking to minimize costs with a given technology and output may shift purchases of inputs to items with below-average relative price increases.

Superlative index

Indices that are “exact” for a “flexible aggregator.” A flexible aggregator is a second-order approximation to an arbitrary cost, production, utility, or distance function. Exactness implies that a particular index number can be derived directly from a specific flexible aggregator. The Fisher price index, the Törnqvist price index, and the Walsh price index are superlative indices. Superlative indices are generally symmetric.

Surcharge

An addition to the list price of a good or a service. Generally of short duration reflecting unusual cost pressures affecting the producer. For example, a fuel surcharge for transport operators.

Symmetric index

An index that treats the two periods being compared symmetrically by giving equal weight, or importance, to the price and value data in both periods. The price and value data for both periods enter into the index number formula in a symmetric or balanced way.

Taxes on products

The taxes on goods or services produced as the outputs of resident enterprises that become payable as the result of the production of those goods or services. They are payable per unit of good or service produced.

Test approach

See “axiomatic approach.”

Time reversal test

A test that may be used under the axiomatic approach and that requires if the prices and quantities in the two periods being compared are interchanged, the resulting price index is the reciprocal of the original price index. When an index satisfies this test, the same result is obtained whether the direction of change is measured forward in time from the first to the second period or backward from the second to the first period.

Törnqvist price index

A price index defined as the weighted geometric average of the current- to base period-price relatives in which the weights are the simple unweighted arithmetic averages of the value shares in the two periods. It is a symmetric index and a superlative index. Also known as the “Törnqvist-Theil price index.”

It is defined as $\ln P_T \equiv \sum ½\left(s^0 + s^t\right)\ln\left(\frac{p^t}{p^0}\right)$,

where $s^j \equiv \frac{p^j q^j}{\sum p^j q^j}$; $j = t, 0$. Also written as

$$P_T \equiv \left(\frac{p^t}{p^0}\right)^{\frac{\left(s^0+s^t\right)}{2}}.$$

Total factor productivity

See “multifactor productivity.”

Transaction

The buying and selling of a product on terms mutually agreed by the buyer and seller.

Transaction price

See “market price.”

Transfer price

A price adopted for bookkeeping purposes used to value transactions between affiliated enterprises integrated under the same management at artificially high or low levels to effect an unspecified income payment or capital transfer between those enterprises. See "intra-company transfer price."

Transitivity

See "circularity."

"True" index

A theoretically defined index that lies between the Laspeyres price index and the Paasche price index. For a theoretical output price index, the Laspeyres output price index is the lower bound and the Paasche output price index is the upper bound. For a theoretical input price index, the situation is reversed: the Paasche output price index is the lower bound and the Laspeyres output price index is the upper bound. See "FIOPI" and "FOIPI."

Turnover

See "revenue."

Unequivocal price index

See "'pure' price index."

Unique product

A product that is manufactured only once to the specification of an individual customer.

Unit-value index

A "price" index that measures the change in the average value of units. These may not be homogeneous, and the unit-value index therefore may be affected by changes in the mix of items as well as by changes in their prices.

Unit-value "mix" problem

The change in the value of a unit-value index, thereby implying a "price change," that arises from a change in the relative quantities of the items covered without any change in their prices.

Universe

The population of producers and products to be sampled.

Upper-level index

An index constructed from elementary or lower-level indices. Weights are used to combine them.

Value

At the level of a single homogeneous good or service, value is equal to the price per unit of quantity multiplied by the number of quantity units of that good or service. Unlike price, value is independent of the choice of quantity unit. Values are expressed in terms of a common unit of currency and are commensurate and additive across different products. Quantities, on the other hand, are not commensurate and additive across different products even when measured in the same kind of physical units.

Value added

Gross value added is the value of output less the value of intermediate consumption; it is a measure of the contribution to GDP made by an individual producer, industry, or sector; gross value added is the source from which the primary incomes of the *SNA* are generated.
Net value added is the value of output less the values of both intermediate consumption and consumption of fixed capital.

Value-added PPI

The weighted average of an output PPI and an input PPI.

Value updating

See "price updating."

Value weights

The measures of the relative importance of products in the index. The weight reference period values or shares of the various components of output (or input) covered by the index. Being commensurate and additive across different products, value weights can be used at aggregation levels above the detailed product level. See "quantity weights."

VAT

Value-added tax. A wide-ranging tax usually designed to cover most or all goods and services. It is collected in stages by enterprises obliged to pay the government only the difference between the VAT on their sales and the VAT on their purchases for intermediate consumption or capital formation. VAT is not usually charged on exports.

Virtual corporation

A partnership among several enterprises sharing complementary expertise created expressly to produce a product with a short perspective life span, with the production of the product being controlled through a computerized network. The corporation is disbanded with the conclusion of the product's life span.

Volume index

The weighted average of the proportionate changes in the quantities of a specified set of goods and services between two periods of time. The quantities compared must be homogeneous, while the changes for the different goods and services must be weighted by their economic importance as measured by their values in one or the other, or both, periods.

Walsh price index

A price index defined as the weighted arithmetic average of the current- to base-period price relatives that uses the quantities of an intermediate basket as weights. The quantities of the intermediate basket are geometric averages of the quantities of the base and current periods. It is a symmetric index and a superlative index and is represented as $P_W \equiv \dfrac{\sum p^t \left(q^t q^0\right)^{\frac{1}{2}}}{\sum p^0 \left(q^t q^0\right)^{\frac{1}{2}}}$.

Weights

A set of numbers between zero and one that sum to unity and are used when calculating averages. Value shares sum to unity by definition and are used to weight price relatives, or elementary price indices, when these are averaged to obtain price indices or higher-level indices. Although quantities are frequently described as weights, they cannot serve as weights for the prices of different types of products whose quantities are not commensurate and use different units of quantity that are not additive. The term "quantity weights" generally is used loosely to refer to the quantities that make up the basket of goods and services covered by an index and included in the value weights. See "quantity weights" and "value weights."

Weight reference period

The period whose value shares serve as weights for a set of price relatives or elementary price indices. It does not have to have the same duration as the periods for which the index is calculated and, in the case of a PPI, is typically longer—a year or more, rather than a month or quarter. Nor does it have to be a single period as in the case of symmetric indices such as the Marshall-Edgeworth, the Walsh, and the Törnqvist price indices.

Wholesale price index

A measure that reflects changes in the prices paid for goods at various stages of distribution up to the point of retail. It can include prices of raw materials for intermediate and final consumption, prices of intermediate or unfinished goods, and prices of finished goods. The goods are usually valued at purchasers' prices. For historical reasons some countries call their PPI a "wholesale price index" even though the index no longer measures changes in wholesale prices.

Young index

Specifically, a weighted average of price ratios between the current year t and the price reference year 0, where the weights are value shares (s_n) that sum to 1.

The Young index is thus defined as $P_{YO} \equiv \sum s_n \left(\dfrac{p^t}{p^0}\right)$.

Special cases include the Laspeyres index when $s_n = s^0 = \dfrac{p^0 q^0}{\sum p^0 q^0}$ and the Paasche index when s_n are hybrid weights using period t quantities valued at period 0 prices, that is, $s_n = s^{0t} \equiv \dfrac{p^0 q^t}{\sum p^0 q^t}$.

Appendix G.1: Some Basic Index Number Formulas and Terminology

1. A *basket(-type)* price index (called here a *Lowe* price index after the index number pioneer who first proposed this general type of index) is an index of the form[1]

(G.1) $\dfrac{\sum p_n^t q_n}{\sum p_n^0 q_n}$,

which compares the prices of period t with those of (an earlier) period 0, using a certain specified quantity basket. The family of Lowe indices includes some well-known indices as special cases:

[1]The sums are understood to be running over all items n.

- When $q_n = q_n^0$, we get the Laspeyres index;
- When $q_n = q_n^t$, we get the Paasche index;
- When $q_n = (q_n^0 + q_n^1)/2$, we get the Marshall-Edgeworth index; and
- When $q_n = (q_n^0 q_n^t)^{1/2}$, we get the Walsh index.

In practice, statistical offices frequently work with a Lowe index in which $q_n = q_n^b$, where *b* denotes some period before 0.

2. A useful feature of a basket price index for period *t* relative to period 0 is that it can be decomposed, or factored, into the product of two, or more, indices of the same type: for instance, as the product of an index for period *t* – 1 relative to period 0 and an index for period *t* relative to period *t* – 1. Formally,

$$\text{(G.2)}\quad \frac{\sum p_n^t q_n}{\sum p_n^0 q_n} = \frac{\sum p_n^{t-1} q_n}{\sum p_n^0 q_n} \frac{\sum p_n^t q_n}{\sum p_n^{t-1} q_n}.$$

The index on the right side of equation (G.2) is described as a two-stage index. It is identical to the single-basket index that compares period *t* directly with period 0, provided the same set of prices is available and used in all three periods 0, *t* – 1, and *t*.

In particular, when $q_n = q_n^0$, equation (G.2) turns into

$$\text{(G.3)}\quad \frac{\sum p_n^t q_n^0}{\sum p_n^0 q_n^0} = \frac{\sum p_n^{t-1} q_n^0}{\sum p_n^0 q_n^0} \frac{\sum p_n^t q_n^0}{\sum p_n^{t-1} q_n^0}.$$

The left side of equation (G.3) is a direct Laspeyres index. Note that only the first of the indices that make up the two-stage Laspeyres index on the right side is itself a Laspeyres index, the second being a Lowe index for period *t* relative to period *t* – 1 that uses the quantity basket of period 0 (not *t* – 1). Some statistical offices describe the two-stage Laspeyres index given in equation (G.3) as a "modified Laspeyres" index.

3. In a time-series context, say, when *t* runs from 1 to *T*, the series

$$\text{(G.4)}\quad \frac{\sum p_n^1 q_n}{\sum p_n^0 q_n}, \frac{\sum p_n^2 q_n}{\sum p_n^0 q_n}, \ldots, \frac{\sum p_n^T q_n}{\sum p_n^0 q_n}$$

is termed a series of *fixed*-basket price indexes. In particular, when $q_n = q_n^0$, we get a series of Laspeyres indexes.

4. At period *T*, one could change to a new quantity basket *q'* and calculate from this period onward

$$\text{(G.5)}\quad \frac{\sum p_n^{T+1} q_n'}{\sum p_n^T q_n'}, \frac{\sum p_n^{T+2} q_n'}{\sum p_n^T q_n'}, \frac{\sum p_n^{T+3} q_n'}{\sum p_n^T q_n'}, \ldots .$$

To relate the prices of periods *T* + 1, *T* + 2, *T* + 3, …. to those of period 0, *chain linking* can be used to transform (G.5) into a series of the form

$$\text{(G.6)}\quad \frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+1} q_n'}{\sum p_n^T q_n'}, \frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+2} q_n'}{\sum p_n^T q_n'}, \frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+3} q_n'}{\sum p_n^T q_n'}, \ldots .$$

This could be termed a series of *chain-linked* fixed-basket price indexes. In particular, when $q_n = q_n^0$ and $q_n' = q_n^T$, we get a series of chain-linked Laspeyres indexes. Since the basket was changed at period *T*, the adjective "fixed'" applies literally over only a certain number of time intervals. The basket was fixed from period 1 to period *T* and is fixed again from period *T* + 1 onward. When the time intervals during which the basket is kept fixed are of the same length, such as one, two, or five years, this feature can be made explicit by describing the index as an annual, biannual, or five-yearly chain-linked fixed-basket price index.

5. A *weighted arithmetic*[2]*-average (-type)* price index (called here a *Young* price index after another index number pioneer) is an index of the form

$$\text{(G.7)}\quad \sum w_n (p_n^t / p_n^0),$$

which compares the prices of period *t* with those of period 0, using a certain set of weights adding up to 1. Note that any basket price index in the form of equation (G.1) can be expressed in the form of equation (G.7), since

$$\text{(G.8)}\quad \frac{\sum p_n^t q_n}{\sum p_n^0 q_n} = \sum \frac{p_n^0 q_n}{\sum p_n^0 q_n} \frac{p_n^t}{p_n^0}.$$

In particular, when

(i) $$\text{(G.9)}\quad w_n = s_n^0 \equiv p_n^0 q_n^0 / \sum p_n^0 q_n^0,$$

[2]To distinguish from geometric or other kinds of average.

that is, period 0 value shares, equation (G.7), turns into the Laspeyres index. When

(ii) (G.10) $w_n = p_n^0 q_n^t / \sum p_n^0 q_n^t$,

that is, hybrid period (0,*t*) value shares, we get the Paasche index. One could also think of setting

(iii) (G.11) $w_n = s_n^b \equiv p_n^b q_n^b / \sum p_n^b q_n^b$,

that is, period *b* value shares. In practice, however, instead of working with equation (G.11), one frequently works with

(iv) (G.12) $w_n = s_n^b (p_n^0 / p_n^b) / \sum s_n^b (p_n^0 / p_n^b)$
$= p_n^0 q_n^b / \sum p_n^0 q_n^b$;

that is, *price-updated* period *b* value shares.

Note that hybrid value shares, such as equation (G.10) or (G.12), typically are not constructed by multiplying sums of prices of one period with quantities of another period. They must be constructed using price relatives and actual expenditure shares as in the first part of equation (G.12).

6. In a time-series context, when *t* runs from 1 to *T*, the series

(G.13) $\sum w_n (p_n^1 / p_n^0)$, $\sum w_n (p_n^2 / p_n^0)$,
….., $\sum w_n (p_n^T / p_n^0)$

is termed a series of *fixed* weighted arithmetic-average price indexes. In particular, when the weights are equal to the period 0 expenditure shares, we get a series of Laspeyres indexes, and when the weights are equal to the price-updated period *b* expenditure shares, we get a series of Lowe indices in which the quantities in the basket are those of period *b*.

7. In period *T* one could change to a new set of weights *w'* and calculate from this period onward

(G.14) $\sum w_n' (p_n^{T+1} / p_n^T)$, $\sum w_n' (p_n^{T+2} / p_n^T)$,
$\sum w_n' (p_n^{T+3} / p_n^T)$, …. ,

or, using chain linking to relate the prices of periods $T + 1$, $T + 2$, $T + 3$, …. to those of period 0,

(G.15) $\sum w_n (p_n^T / p_n^0) \sum w_n' (p_n^{T+1} / p_n^T)$,
$\sum w_n (p_n^T / p_n^0) \sum w_n' (p_n^{T+2} / p_n^T)$, …. .

This could be termed a series of *chain-linked* fixed-weight arithmetic-average price indexes. In particular, when $w_n = s_n^0$ and $w_n' = s_n^T$, we get a series of chain-linked Laspeyres indices. When $w_n = s_n^b (p_n^0 / p_n^b) / \sum s_n^b (p_n^0 / p_n^b)$ and $w_n' = s_n^{b'} (p_n^T / p_n^{b'}) / \sum s_n^{b'} (p_n^T / p_n^{b'})$ for some later period *b'*, we get a series of chain-linked Lowe indices.

8. Again, since the weights were changed at period *T*, the adjective “fixed” applies literally over only certain time intervals. The weights were fixed from period 1 to period *T* and are again fixed from period *T* + 1 onward. When the time intervals during which the weights are kept fixed are of the same length, this feature can be made explicit by adding a temporal adjective such as annual, biannual, or five-yearly.

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