Wage Formation and Minimum Wage Contracts: Theory and Evidence from Danish Panel Data*

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Abstract

To a large extent, Danish wage contracts are minimum wage contracts, where the wage of an employed worker consists of a negotiated minimum wage and a personal raise. This paper considers bargaining over minimum wages given that the members of the union have different levels of productivity. We show that the total wage of an individual in this case depends on both individual productivity and standard opportunity costs of the union. Further, it is shown that ceteris paribus the minimum wage is lower in this economy than the (total) wage in an economy where all workers have the same productivity. Finally, it is shown that increasing progressivity of the tax system does not necessarily imply a reduction in the minimum wage.

A log-linearized version of the wage equation is estimated for 6 major unions using a panel which is a sub-sample of a representative 5 per cent sample of the Danish population. Estimated parameters of the marginal tax are negative for most male workers whereas this is not the case for women. Even for male workers the coefficients are much smaller than in previous studies based on macro data.

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1. INTRODUCTION

The persistent high levels of registered unemployment in Denmark and other European countries remain one of the main challenges both to economic policy and to economic theory. Labour market reforms, tax reforms and other so-called structural reforms are central policy issues in most European countries in the 1990s. To a large extent the effects of such reforms depend on their effect on the wage determination in the economy. In the light of this fact the present paper derives and estimates wage equations for 6 major union groups in the Danish labour market.

Wage formation depends upon the institutions present in the economy in question. In Denmark two sets of institutions have major effect on wage formation. The two are a general and tax-financed unemployment benefit system and the collective bargaining system that covers most of the private labour market in Denmark.

In the present paper, wage equations are derived in a theoretical model which is an extension of the traditional bargaining model (see e.g. MacDonald & Solow (1981)) designed to yield a better description of the institutions in the Danish labour market. The main extension to the traditional model is that each union organises persons with different levels of productivity, such that some workers are more efficient than others. Wages in the economy reflect these differences in productivity. The wage of an individual worker is given as the minimum wage for the specific union and a personal raise. The union indirectly affects the wage level of the worker by its bargaining power in the negotiations about the minimum wage. This leads to individual wage equations that contain both measures of personal productivity and standard opportunity costs of the union.

An increase in employment tends to reduce the marginal productivity for two reasons. First, production technology of firms exhibits decreasing returns to the index of labour input. Second, increasing employment implies that the firm will have to employ workers with lower productivity than those already employed. This second non-standard effect turns out to be potentially crucial when considering the employment effects of a restructuring of the wage income taxation system, which remains a policy issue in Europe in the 1990s.

Both the European Commission (1994) and OECD (1996) have proposed to fight the unemployment problem through changes in the tax structure.[1] The

[1] In Denmark the Economic Council has put forward a similar proposal, see Economic

idea is to reduce the tax burden of low wage incomes while keeping the level of unemployment benefits after tax constant. The financing of the tax reduction is obtained by an increase in the tax burden among the employed with the highest salaries. Thus, the overall effect on the labour income taxation is an increase in progressivity. The idea is that this shift in the tax structure will increase the incentives for low-skilled unemployed to become employed. Thus, voluntary unemployment may be reduced without reducing the standard of living for the unemployed. Involuntary unemployment may also be reduced since the pre-tax wage for the low-income groups may be reduced. Whether this positive effect is outweighed by the increase in the tax burden and the derived increase in the pre-tax wage for high-income groups is a central policy issue. Those in favour of the "employment tax cut" proposal argue that there will be a net reduction in the involuntary unemployment due to the restructuring. The positive net effect appears because the progressivity of the tax system increases. The argument is that for given levels of the tax burden, an increase in the progressivity (defined as marginal tax rates divided by average tax rates) will reduce the wage level in the economy.

The effects on wage formation of pure increases in progressivity depend on whether the labour market is perfectly competitive or not. In models of perfect labour markets, increased progressivity tends to reduce labour supply and therefore it reduces activity and increases the equilibrium wage rate. On the contrary, standard models of labour market imperfections imply that increased progressivity tends to reduce the wage rate. Consider e.g. the labour market with bargaining over wages. The wage claims of the union are determined such that the sum of the welfare gains to the employed members (due to the increased wages after tax) is equal to the sum of the welfare losses of those members who will become unemployed if the pre-tax wage is raised. The number of members who will become unemployed depends solely on the pre-tax wage. The welfare gain for the employed members depends on the marginal after-tax wages. Therefore, high marginal tax rates tend to reduce the welfare gains of the employed members, and the union tends to profit by having lower wage claims than in the case of low marginal tax rates. Malcomson & Sartor (1987) are the first to find this effect of progressive taxation in case of unionised economies. Similar results are obtained in the case of search theory (Pisauro; 1991) and efficiency wage theory (Hoel; 1990).

In the present paper, we show that in an economy with minimum wage contracts, differences in productivity between workers and some positive bar-

Council (1997).

gaining power of the confederation of employers, the sign of an increase in progressivity of the wage income taxation may be both positive and negative.

The intuition of this result is most easily explained by considering the extreme case, where the bargaining power of the confederation of employers goes to the limit where the employers may dictate the (minimum) wage. Since the real profit of the firm is monotonically decreasing in the (minimum) wage, it is optimal for the confederation of employers to press for reductions in the wage until the threatpoint of the union is reached. In standard wage bargaining, the threatpoint of the union is typically set equal to the opportunity costs of being employed as perceived by the union. This implies that in case of decentralised bargaining the wage is pressed to the point where the wage after tax is equal to the unemployment benefits after tax plus the money value of the utility of leisure. In this standard case, a restructuring of the tax system, which leaves the average tax rate unaffected, does not affect the pre-tax wage. The Nash-Bargaining solution may be considered as a convex combination of this result and the result for the case of the monopoly union where a restructuring of the tax system (leaving the average tax rate constant) leads to a reduction in the pre-tax wage. This implies that the effect of a restructuring which is neutral to the average tax rate will always imply that the wage is reduced. The size of the effect is less the stronger the employers.

In the case of the present paper where the union members differ in productivity and the union maximises the sum of utilities of it members, the threatpoint is still given by the opportunity cost of the employment as perceived by the union. In this case, the employers' confederation will press for wage reductions until the *average* wage rate after tax is equal to the threatpoint of the union. The reason why it is the average wage rate is that the union maximises the sum of utilities. This implies that as long as the average wage rate is higher than the threatpoint, the union will prefer the outcome to the threatpoint. Observe that this outcome implies that some union members receive a lower wage after tax than the opportunity cost of employment perceived by the union. This may be an equilibrium since the opportunity cost of employment as perceived by the individual is lower than the cost perceived by the union. The difference between the two is the unemployment benefits after tax. This is due to the fact that the unemployment, which is a result of a high minimum wage, is involuntary for the individual, who is therefore entitled to unemployment benefits. If, on the other hand, the individual refuses to accept a job-offer with an after tax wage that is lower than the sum of unemployment benefits and the money value of the utility of foregone leisure, then the individual is voluntarily unemployed and therefore not entitled to unemployment benefits.

In fact it is rather common in Denmark that the after tax wage is less than the opportunity cost of employment as perceived by the union. Pedersen & Smith (1995) find that 25 per cent of employed members of unemployment insurance funds gain less than Dkk 500 per month by being employed.

In the paper, we consider the simplest possible progressive tax system, which is a system with a constant marginal tax rate, t, and a unique income tax threshold, D. This implies that a tax restructuring which "leaves the average tax rate constant" is only possible for a unique income level.[2] To fix ideas we assume that this income level is the level of income received by the marginally employed.

Since the threatpoint of the union is equal to the average wage rate, a tax restructuring that increases the marginal tax rate while leaving the average tax rate of the marginally employed constant, will in fact increase the average tax rate of the average employed, because intramarginally employed workers are more productive and therefore receive a higher salary. For a given pre-tax average wage, the after-tax wage rate is therefore reduced. To maintain the outcome of the Nash-bargaining at the threatpoint of the union, the pre-tax average wage must increase, which is only possible through an increase in the minimum wage. Therefore an increase in progressivity of the tax system will increase the minimum wage, if the employers dictate the wage and the average tax rate of the marginally employed is constant.

On the other hand, in case of a monopoly union we get the standard result that increasing progressivity decreases the pre-tax wage rate. Again, considering the Nash-bargaining solution as a convex combination of the two, we may conclude that for a given distribution of productivity there exists a level of bargaining power of the employers, where an increase in progressivity for a constant average tax rate of the marginally employed does not affect the minimum wage.

Given this result, it remains an empirical question whether a pure increase in progressivity reduces or increases the wage rates in the economy. Therefore, it remains an empirical question whether "employment tax cuts" will in fact reduce involuntary unemployment.

Our empirical estimations of wage equations for persons belonging to six major unions in the Danish labour market in the period 1980-90 reveal some general

[2] In the literature mentioned above, it is possible to design general tax systems so that the tax rate is constant for any wage rate in the economy. See section 4 for a discussion of the difference between the tax systems.

tendencies: First of all that variables representing the personal productivity get a higher weight in the wage equation for unions representing high-income groups. Furthermore, the effect of variables measuring personal productivity is larger for men than for women. Similarly, the weight of the unemployment benefits is lower in the wage equations for unions representing high-income groups. Again, there is a difference between the effects in the relation for men and women where the effect of unemployment benefits on the wage is larger for women than for men.

The average tax rate tends to have a significant, increasing effect on the wage for all groups. The effects of marginal tax rate have a significant wage reducing effect for male workers except for members of the building and construction union. For female workers, increases in the marginal tax rate tend to be wage increasing. However, for the large group of unskilled females organised in KAD, the effect is insignificant.

Although the effect of an increase in the marginal tax rates tends to be wage reducing for most groups of male workers, the effect on the pre-tax wage is much less in this study than in previous studies based on macro data (see Hansen *et al.* (1995) for Danish estimates).

The rest of the paper is organised as follows: Section 2 presents a short description of the Danish private labour market and the type of contracts used in the organised part of this market. This reveals that minimum wage contracts are the most widespread type of contracts. Therefore, section 3 sets up a partial equilibrium of a labour market with decentralised bargaining over the minimum wage and competitive markets for intramarginal workers. The effects of changes in the marginal tax rate in this partial equilibrium are discussed in section 4. Section 5 presents the data and the empirical model, while section 6 discusses the empirical results both in relation to the theory presented in section 3 and the previously published empirical results.

2. THE DANISH LABOUR MARKET

The labour market of the private sector in Denmark is dominated by collective bargaining between the individual Employers' Associations organised in the Confederation of Danish Employers Associations (DA) and the unions organised in the Federation of Trade Unions (LO)[3]. DA has rules implying that

[3] Except for a very small Christian Union all unions organising blue collar workers in Denmark are organised in the LO. Unions organising white collar workers negotiate separately

the contracts negotiated by the individual members are to be approved by a majority of the members of the confederation. A similar formal set of rules does not apply in LO.

To a large extent, contracts and working conditions for workers who are not covered by the member organisations of DA and LO mimic the outcome of the bargaining in the organised labour market. In the organised part of the labour market the following types of non-piecework contracts with provisions concerning wages exist: 1) A standard wage-contract, where the wage of a given category of workers is set in the contract. 2) A minimum wage contract where union-employer negotiations determine the minimum wage. The total wage of a given employee is given as the sum of the minimum wage and a personal raise. 3) A so-called minimum payment contract where there is no direct connection between the minimum wage and the total wage of a given employee, except for the fact that the wage has to be at least as high as the minimum wage.

From 1961 to 1989 the coverage of the different types of contracts was almost constant with the standard wage contract covering approximately 50 per cent of the non-piecework employment covered by contracts between the DA and the LO member organisations. Minimum payment contracts and minimum wage contracts covered approximately 40 per cent and 10 per cent respectively. From 1990 there has been a rather dramatic increase in the coverage of non-standard wage-contracts. By 1997 the coverage of the standard wage-contract was reduced to only 16 per cent. 67 per cent of total contracts are either of the minimum wage or the minimum payment type, whereas 17 per cent of the contracts do not have sections on wages (Nicolaisen; 1997).

Unfortunately, our data set only allows us to estimate wage equations for the period from 1980 to 1990 where approximately 50 per cent of the organised labour market were covered by non-standard wage contracts. In figure 1 the distributions of wages in 1981 and 1990 are shown for the unions that are represented in the estimated wage equations at the end of the paper. Observe that wage dispersion tends to increase with the length of the education needed to require the skill in question. Observe also that even if 50 per cent of the organised labour market use a standard wage contract, there does not seem to be a single wage that is dominating in the distributions.

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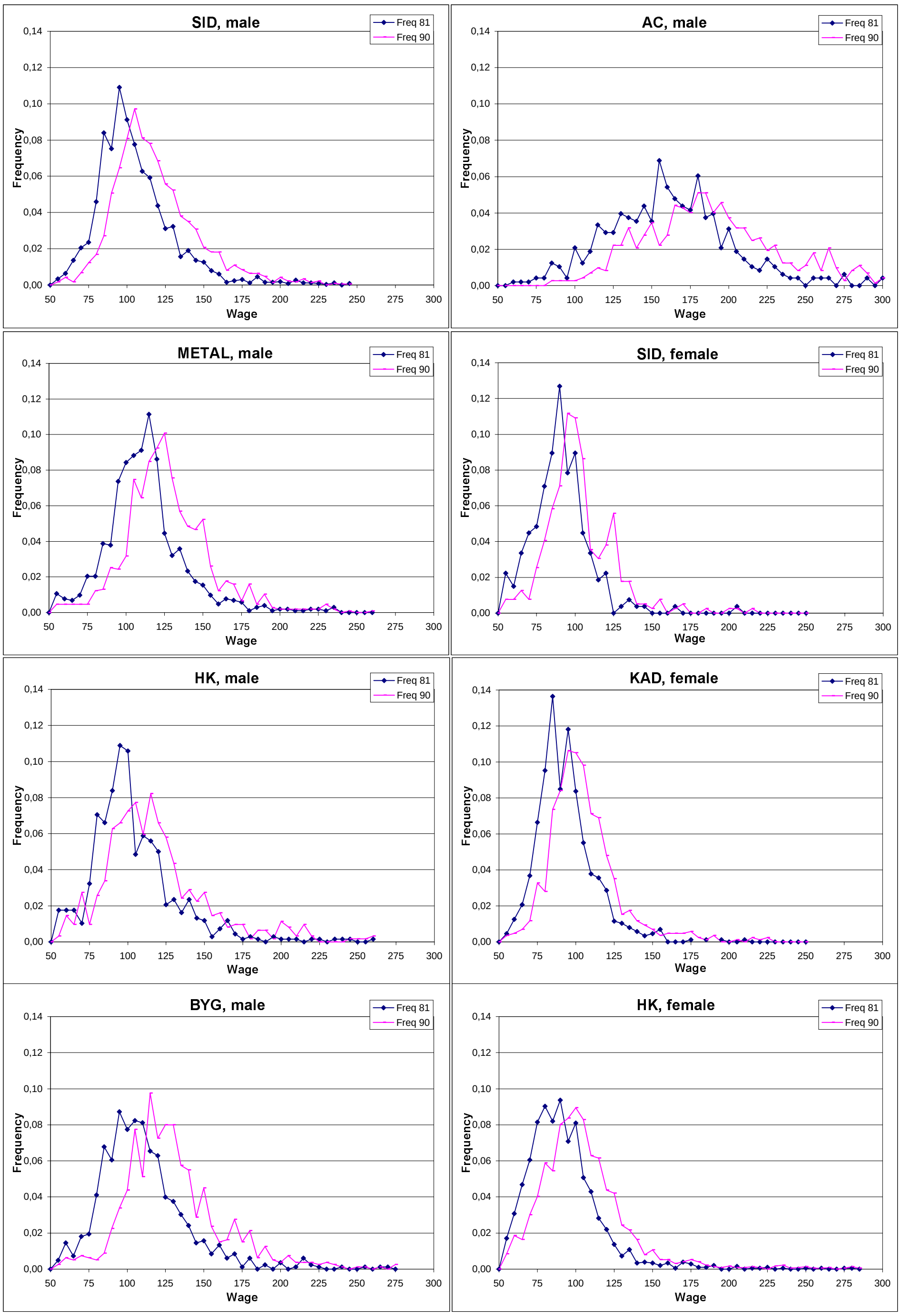
with member organisations of DA.

Figure 1. Frequency of wage rates. Wage rates of 1990 are deflated by consumers price index



3. THE MODEL

In this section we set up a theoretical model of decentralised wage bargaining between unions and employers' associations. Standard theoretical models with imperfect labour markets do not allow for distributions of wages between different workers working within the same collective bargaining area. In bargaining models unions and employers' associations negotiate a unique wage rate for a given part of the labour market. In this paper we want to explain the existence of wage distributions within a given part of the labour market covered by a single collective bargaining agreement.

We assume that the productivity of individual workers differs and that this difference is observable both to the union and the employers. Thus, the firm knows the productivity of a given worker in advance and on this basis the firm decides whether or not to employ the worker in question. We assume that the firms have a "right to manage" so that for a given wage rate, the employment decision is made unilaterally by the firm. Therefore, we start by analysing the behaviour of a representative firm given the distribution of wage rates.

3.1. Behaviour of the representative firm

The production function of the representative firm is assumed to be Cobb Douglas and strictly concave in the index of total labour input. The assumption of decreasing returns to labour input follows from an implicit assumption that the production function exhibits constant returns to scale in capital and labour and the capital stock is assumed to be fixed.

$$y = L^{\alpha}, \quad 0 < \alpha < 1 \tag{3.1}$$

The index of total labour input, L, is given as the sum of labour inputs adjusted for the productivity of the different workers. To simplify the mathematical exposition, we assume that there is infinitely many different productivity levels, so that the total index may be represented by the following integral

$$L = \int_0^1 \rho(i)\ l(i)\ di \tag{3.2}$$

where $\rho(i)$ is a productivity parameter which measures the productivity of a specific category i of workers. As a convention $\rho'(i) < 0$, such that $i = 0$ is the category with the highest productivity. $l(i)$ is the demand for labour of type i.

The maximization problem of the firm, which is assumed to be a price taker in the output market, is to choose the optimal input of each of the categories of workers such that the profit of the firm is maximized.

$$\max_{l(i)} \quad p\left(\int_0^1 \rho\left(i\right)\,l\left(i\right)\,di\right)^{\alpha} - \int_0^1 w\left(i\right)\,l\left(i\right)\,di$$

where p is the output price and $w(i)$ is the wage rate paid to workers in productivity category i. This yields the following condition for optimality for each category of productivity

$$\alpha p\left(\int_0^1 \rho\left(i\right)\,l\left(i\right)\,di\right)^{\alpha-1} \rho\left(i\right) = w\left(i\right), \quad \text{iff } l\left(i\right) > 0 \tag{3.3}$$

This is simply the standard marginal condition under perfect competition that labour is employed until the value of the marginal product is equal to the wage. This holds for productivity types that are employed. Dividing one first order condition with another yields

$$\frac{w\left(i\right)}{\rho\left(i\right)} = \frac{w\left(j\right)}{\rho\left(j\right)} \equiv q, \ \text{ for all } i,j \text{ where } l\left(i\right), l\left(j\right) > 0 \tag{3.4}$$

(3.4) states that since any two different categories of productivity are perfect substitutes then if both types are to be used, the wages corrected for productivity have to be identical across the groups. The productivity-corrected wage is defined as q. The wage of the individual employed category is given by:

$$w\left(i\right) = \rho\left(i\right) q \tag{3.5}$$

Categories not employed by the firm are identified by:

$$w\left(j\right) > \rho\left(j\right) q$$

which simply states that category j is not productive enough to become employed.

3.2. Characterisation of a partial equilibrium for a given minimum wage

As explained, we assume that the wage contracts in the economy are of the minimum wage type where a labour union and a confederation of employers negotiate a minimum wage. The wages of the employed persons are higher than or equal to this minimum wage. As an initial step we consider the partial equilibrium of a sector in the economy for a given exogenous minimum wage. For persons with a productivity sufficiently high to become employed, given the minimum wage, we assume that a sub-labour market with perfect competition exists so that all persons with sufficiently high productivity are employed and receive the wage that clears the sub-labour market given the level of the minimum wage. Using the description of the firm, the wages under a minimum wage system as defined above may be defined by

$$w(i) = \max\left\{w^{\min}, \rho(i)\, q\right\} \tag{3.6}$$

where $w^{\min}$ is the minimum wage. If $w^{\min} < \rho(i)\, q$ all workers of category i receive the (contingent competitive) wage $\rho(i)\, q$ and are all employed. On the other hand, if $w^{\min} > \rho(i)\, q$ then workers of category i "receive" the minimum wage and are not employed ($l(i) = 0$). The border between these two regimes is given by a category of workers r, called the marginally employed productivity category defined by:

$$w^{\min} \equiv \rho(r)\, q \tag{3.7}$$

We assume there is one representative agent for each category of productivity and that the individual labour supply of this agent is normalized to unity. This could be the case if the length of the working day is fixed by a long-term contract between the union and the confederation of employers, as is the case in Denmark. We ignore the possibility of part-time jobs. This implies that total labour supply in each productivity category is normalized to 1. Due to our convention that $\rho'(i) < 0$, employment will be given by

$$l(i) = \begin{cases} 1 \text{ for } i \leq r \\ 0 \text{ for } i > r \end{cases} \tag{3.8}$$

In partial equilibrium, the first order condition of the firm (3.3) then becomes

$$\alpha \left(\int_0^r \rho(i)\, di \right)^{\alpha - 1} = \frac{q}{p}$$

Substituting the definition of q from (3.7) into the equation, we have that:

$$\alpha \left(\int_0^r \rho(i)\, di \right)^{\alpha - 1} = \frac{w^{\min}}{\rho(r)\, p} \tag{3.9}$$

This equation determines the demand relationship between the minimum wage and the border-category r for a given price level. An increase in $w^{\min}$ implies a decrease in r, such that the number of full-employed categories decreases. Relation (3.9) is the labour demand relationship in traditional wage curve - labour demand representation of the partial equilibrium of the labour market.

3.3. Nash-bargaining

To determine the wage curve we have to set up the Nash-bargaining problem. First consider the behaviour of the union. The union is assumed to be utilitaristic, i.e. to bargain for a minimum wage, $w^{\min}$, such that the sum of the workers' utilities is maximised. The incentive for the union to raise the minimum wage is that this raises the total wage of those becoming employed and this may outweigh the loss due to the unemployment that is the consequence of the minimum wage. The increase in the total wage of the employed appears because the productivity of a given worker depends both on her individual productivity and on the overall level of employment in the economy. The mechanism from total employment to individual productivity is that increased employment reduces the capital labour ratio, which reduces productivity of the individual worker.

We assume that the utility of the workers is homogenous in degree one in consumption and that disutility of work is additively separable.[4] These assumptions imply that the indirect utility function for an employed worker of category i, is given as

$$S\left(w\left(i\right)l\left(i\right);P^{c}\right)=\frac{I_{i}}{P^{c}}-\eta\frac{\gamma}{\gamma+1}l\left(i\right)^{\frac{\gamma+1}{\gamma}}$$

where I_i is the income after tax for an employed worker of category i, P^c is the consumer price index, $\eta > 0$ is a constant representing the weight that the worker attaches to disutility of work, and $\gamma > 1$ is the inverse of the labour supply elasticity. We assume that these preference parameters are identical for all workers and therefore independent of the productivity of the worker in question. A similar expression applies to the unemployed workers with the (obvious) exception that there is no disutility of work.

We assume for simplicity that the tax system is linear and progressive with a constant marginal tax rate t and an income tax threshold, D.

Using (3.5), (3.6) and (3.8), we may write the utility of the union as:

[4] A function of this type is used in e.g. Blanchard & Kiyotaki (1987).

$$V=(1-t)\int_0^r \frac{\rho(i)}{\rho(r)}\frac{w^{\min}}{P^C}di+\frac{1}{P^C}\int_0^r Ddi-\int_0^r \frac{\gamma}{\gamma+1}\eta di+\int_r^1 \frac{b}{P^C}di \quad (3.10)$$

where b is the unemployment benefits after tax

The first two terms on the right-hand side of (3.10) are total real income after tax for the employed members of the union. The third term is the total disutility endured by union members and finally the fourth term is the total real income after tax for unemployed union members.

As we assume that the firm has a "right to manage", we may use the demand relationship (3.9) to substitute for the minimum wage:

$$V=(1-t)\,\alpha\frac{p}{P^C}\left(\int_0^r \rho(i)\ di\right)^{\alpha}+\frac{1}{P^C}Dr-\frac{\gamma}{\gamma+1}\eta r+\frac{b}{P^C}(1-r)$$

The threatpoint of the union, $\bar{V}$, which measures the outcome of the union if no agreement is reached in the negotiations, is defined as the real after tax unemployment benefits for each worker.[5] This implies that the union objective is given by

$$V-\bar{V}=(1-t)\,\alpha\frac{p}{P^C}\left(\int_0^r \rho(i)\,di\right)^{\alpha}+\frac{r}{P^C}\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right) \quad (3.11)$$

Similarly we define the object function of the confederation of employers as the profits deflated by consumer prices. The threatpoint of the employers is set to zero. Inserting the demand for labour into the definition of profits implies that the object function of employers may be written as

$$\frac{\Pi}{P^C}=\frac{(1-\alpha)\,p\,(\int_0^r \rho(i)\ \ di)^{\alpha}}{P^C} \quad (3.12)$$

The Nash-bargaining problem is then given as

$$\arg\max_r\left\{(V-\bar{V})^{\lambda}\left(\frac{\Pi}{P^C}\right)^{(1-\lambda)}\right\}=\arg\max_r\left\{\lambda\log(V-\bar{V})+(1-\lambda)\log\left(\frac{\Pi}{P^C}\right)\right\} \quad (3.13)$$

[5] This is a standard assumption in the literature on wage-bargaining. According to Danish law workers are put in quarantine for a 5-week period before they can receive unemployment benefits after being laid off due to a conflict. However, the quarantine is ignored here.

where $0 \leq \lambda \leq 1$, is the bargaining power of the union.

For given output price, p, the minimum wage, $w^{\min}$, uniquely determines the marginal category of workers who are employed, r. Therefore, by choosing r the negotiators implicitly choose the minimum wage. The minimum wage which is the solution to (3.13) is given as (a derivation is given in appendix)

$$w^{\min}(r) = \frac{1}{(1-t)\alpha}\left((1-\lambda)r\frac{\alpha\rho(r)}{\int_0^r \rho(i)\,di} + \lambda\right)\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right)$$

Defining the average productivity as

$$\hat{\rho}(r) = \frac{\int_0^r \rho(i)\,di}{r}, \quad \hat{\rho}'(r) < 0$$

we may write the minimum wage as

$$w^{\min}(r) = \frac{1}{(1-t)\alpha}\left((1-\lambda)\alpha\frac{\rho(r)}{\hat{\rho}(r)} + \lambda\right)\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right) \qquad (3.14)$$

Relation (3.14) above is the minimum wage curve of the economy.

First, observe that in case of a monopoly union $(\lambda = 1)$, the minimum wage is set in such a way that the after tax value is a mark-up on the cost of becoming employed, which consists of the sum of foregone unemployment benefits after tax and the valuation of the reduction in leisure. This result is similar to the standard result concerning the total wage in economies where a monopoly union sets a standard wage.

Second, consider the case where the federation of employers does have some power in the negotiation and assume that the productivity of all workers is identical so that average and marginal productivity coincides. In this case, the expression for the minimum wage is identical to the result of a standard right to manage Nash-bargaining solution, given the firm has a Cobb-Douglas production technology. In the present case where the productivity of the marginally employed worker is decreasing, we observe that for a given bargaining power of the union (less than 1) and a given evaluation of the alternative to being employed, the minimum wage is lower the lower the productivity of the marginally employed relative to the average productivity. This effect appears because the marginal benefit for the firm of extra employment is lower in this case than in the standard case, where all workers have the same productivity. Therefore, the minimum wage is lower than a standard wage in a similar economy where the workers are identical.

3.4. Partial equilibrium

The partial equilibrium is given at the point where the minimum wage curve defined in relation (3.14) intersects the relation for the minimum wage determined from the demand side given in relation (3.9). This subsection is concerned with the questions of existence and uniqueness of the partial equilibrium.

Productivity of the marginally employed is a decreasing function of employment and this implies that the minimum wage curve becomes a function of the level of employment if the confederation of employers has a positive bargaining power. The sign of partial derivative of the minimum wage curve with respect to the level of employment generally depends upon the curvature of the distribution of productivity. The minimum wage curve is downward sloping if

$$\frac{\rho(r)}{\hat{\rho}(r)} - \frac{r\rho'(r)}{\rho(r)} > 1 \tag{3.15}$$

where $\hat{\rho}(r)$ is the average productivity at the level of employment defined by r. The fact that the minimum wage curve may be decreasing for some levels of employment implies that existence and uniqueness of the partial labour market equilibrium is not guaranteed. The following results apply:

Lemma 3.1. *Assume that the minimum wage curve is given by (3.14) and that the demand for labour is given by (3.9). Sufficient conditions for existence of a partial equilibrium are that the distribution of productivity has the following boundary conditions*

$$\begin{aligned} \lim_{r\to 0} \rho(r) &= \bar{\rho} > 0 \\ \lim_{r\to 1} \rho(r) &= 0 \end{aligned}$$

Proof: See appendix

The crucial assumption ensuring that a partial equilibrium exists is that the productivity of the least productive workers is sufficiently low to ensure that the demand curve for labour in the limit at full employment is below the minimum wage curve (in a standard representation) at this point.

Lemma 3.2. *Assume that the minimum wage curve is given by (3.14) and that the demand for labour is given by (3.9). A sufficient condition for uniqueness of the partial labour market equilibrium is that the inequality (3.15) is* *fulfilled for all $r \in [0;1]$ i.e. the minimum wage curve is monotonically decreasing.*

Proof: See appendix.

The idea of the lemma is to establish a sufficient condition for the slope of the demand-determined minimum wage to become more negative than the slope of the minimum wage curve in a partial equilibrium. This means that the demand-determined minimum wage can only intersect the minimum wage curve from above and therefore the equilibrium is unique given that the condition is fulfilled.

4. CHANGES IN THE MARGINAL TAX RATE

In this section we focus on the effects of a restructuring of the wage income taxation. In the discussion it will be assumed that unemployment benefits after tax are not affected by the changes in the tax system. As unemployment benefits are taxed according to the income tax system, this implies that any effects on the average tax rate of unemployment benefits is offset by changes in the pre-tax benefits.

Following the literature we discuss the effect of tax changes in terms of changes in the marginal tax rate and the average tax rate, respectively. The very simple tax system applied here has only two tax instruments: the constant marginal tax rate and the income tax threshold. This only allows us to fix the average tax rate of a given category of workers when we consider a restructuring of the tax system. To see this, consider the average tax rate of the marginally employed worker (category r). This tax rate, T_r^a, is defined as

$$T_r^a = \frac{tw^{\min} - D}{w^{\min}} \quad \Rightarrow \quad D = \left(t - T_r^a\right) w^{\min}$$

Observe the subscript r on average tax rate. In the following, we want to analyse the effect of an increase in the marginal tax rate for at given average tax rate for the marginal group of workers. Of course, the increase in the income tax threshold which keeps the average tax constant for the marginally employed workers will not be sufficient to keep the average tax rate constant for the intramarginal employed.

The literature on progressive income tax follows Malcomson & Sartor (1987) in defining a general tax function, $\tilde{T}(z)$, where z is a vector of tax instruments. To see the difference between this approach and the present analysis, we consider the typical setting in the literature. First, assume that the vector of wages in the economy is of dimension n so that n different types of workers exist. In this case, the dimension of the vector, z, of tax instruments has to be at least $2n$. Second, assume that wage contracts are of the standard wage type and that each type of labour is an imperfect substitute for other types of labour. For simplicity assume that there is a bargaining for each of the n different wages and that a Nash-equilibrium in wages exists. In this approach there is (in principle) a sufficient number of tax instruments to keep the average tax rate constant for all wages and to define a specific marginal tax rate for each wage rate. Starting in such a Nash-equilibrium, it is possible to analyse the effects of a change in marginal tax rate for a specific type of workers, given that the average tax rate for the type in question is constant, and given that both average and marginal tax rates for all other worker types are constant.

In the present analysis where the union is concerned with the entire distribution of wages, we have chosen to introduce a simple tax system since in our case it seems more relevant to have only a limited number of tax instruments as this is an effective constraint in real world tax systems. When one is concerned with the entire distribution of wages, the shift of the tax function becomes the relevant objective whereas in the case where one is concerned with effects on a specific wage level the formulation used in the literature seems more relevant.

When comparing results, one should be aware of this difference in assumptions concerning the number of tax instruments.

Writing the minimum wage (3.14) as a function of the marginal and average tax rate of the marginally employed workers, yields

$$w^{\min}(r) = \frac{1}{(1-t)\alpha}\left((1-\lambda)\alpha\frac{\rho(r)}{\hat{\rho}(r)} + \lambda\right)\left(b - (t - T_r^a)w^{\min} + \frac{\gamma}{\gamma+1}P^C\eta\right) \quad \Leftrightarrow$$

$$w^{\min}(r) = \frac{\left((1-\lambda)\alpha\frac{\rho(r)}{\hat{\rho}(r)} + \lambda\right)\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t)\alpha + (t - T_r^a)\left((1-\lambda)\alpha\frac{\rho(r)}{\hat{\rho}(r)} + \lambda\right)}$$

The total wage of an employed worker with productivity category i is found using the relations (3.5) and (3.7)

$$w(i) = \rho(i)q = \frac{\rho(i)}{\rho(r)}w^{\min}$$

implying

$$w(i) = \frac{\left((1-\lambda)\,\alpha\frac{\rho(i)}{\hat{\rho}(r)} + \lambda\frac{\rho(i)}{\rho(r)}\right)\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t)\,\alpha + (t - T_r^a)\left((1-\lambda)\,\alpha\frac{\rho(r)}{\hat{\rho}(r)} + \lambda\right)}$$

By definition total unemployment, U, is given by

$$U \equiv \int_0^1 (1 - l(i))\,di = 1 - r$$

such that

$$w(i) = \frac{\left((1-\lambda)\,\alpha\frac{\rho(i)}{\hat{\rho}(1-U)} + \lambda\frac{\rho(i)}{\rho(1-U)}\right)\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t)\,\alpha + \left(t - T_{1-U}^a\right)\left((1-\lambda)\,\alpha\frac{\rho(1-U)}{\hat{\rho}(1-U)} + \lambda\right)} \tag{4.1}$$

The wage of the individual category of productivity depends upon the variables which are typically the result of standard bargaining theory (unemployment benefits, the level of consumer prices, the evaluation of leisure, the level of unemployment within the union in question) and in addition to these, a variable representing the individual level of productivity. Observe that the wage of the workers of productivity type i depends upon the average tax rate of the marginal employed (given our formulation of the tax system). The fact that the wage curve does not only depend upon the taxes paid by the worker herself is a general feature of the minimum wage system in a right to manage setting. This follows from the fact that the firm has to be on its demand curve for labour, which implies that in optimum the wage per produced unit has to be identical for all workers irrespectively of their individual productivity. Therefore, type-specific taxes in general cannot affect the relative distribution of pre-tax wages. The absolute distribution is affected through changes in the number of employed categories of productivity. The fact that we may write the wage as a function of the tax parameters of the marginally employed workers alone is not a general characteristic but follows from the simple formulation of the progressive tax system with a constant marginal tax rate.

A restructuring of the tax system affects the partial equilibrium, which for any productivity category of workers, i, may be found by combining the relation for the wage curve (4.1) and the relation for the demand of type i (3.3). Given that the sufficient condition for uniqueness of the partial equilibrium, (3.15), is fulfilled, then the sign of the effect on the wage rates and the employment may be evaluated simply by evaluating the vertical shift in the wage curve in a standard diagram.

An increase in the marginal tax rate (keeping the average tax rate of the marginally employed constant) may shift the wage curve both up and down, as can be seen from differentiating (4.1) partially with respect to t.

$$\frac{\partial w_i}{\partial t} |r \text{ given} = \frac{w_i \left(\alpha - (1-\lambda)\, \alpha \frac{\rho(1-U)}{\hat{\rho}(1-U)} - \lambda\right)}{(1-t)\, \alpha + \left(t - T^a_{1-U}\right) \left((1-\lambda)\, \alpha \frac{\rho(1-U)}{\hat{\rho}(1-U)} + \lambda\right)} \tag{4.2}$$

Given that the tax system is non-degressive at the level of income received by the marginally employed worker (i.e. $t \geq T^a_{1-U}$), the denominator of (4.2) is positive. In the case of the monopoly union ($\lambda = 1$) the numerator is negative and we get the standard imperfect competition result that an increase in the progressivity of the wage income taxation will reduce the wage rate. Observe that this is true for all productivity types. The result, however, is only comparable to the standard results of the literature for the case of the marginally employed group, since this group is the only one who has a constant average tax rate. All other employed groups will have increased average tax rates.

In the case where all workers have identical productivity we find that an increase in the progressivity of the wage income taxation will reduce the (unique) wage rate for any positive bargaining power of the union where an interior solution to the Nash-bargaining exists. This result is in line with Lockwood & Manning (1993).

Finally, in the case considered here where the productivity of the marginal worker is decreasing in the level of total employment, and the confederation of employers does have some power in the negotiation, we find that the sign of the change in the wage rate of an increase in the marginal tax rate is indeterminate. If (for $0 < \lambda < 1$) the following equation is fulfilled, then an increase in the marginal tax rate will increase the wage rates for all groups

$$\frac{\rho\,(1-U)}{\hat{\rho}\,(1-U)} < \frac{\alpha - \lambda}{\alpha\,(1-\lambda)} \tag{4.3}$$

Observe that the right-hand side is less than 1 for a positive λ. Therefore, the reduction in the marginal productivity is a necessary condition for the non-standard result.

To get an intuition for the non-standard result consider the case where the confederation of employers may dictate the minimum wage, i.e. the limit of the

Nash-bargaining solution for $\lambda \to 0$. In this case the minimum wage becomes

$$\lim_{\lambda \to 0} w^{\min}(r) = \frac{\frac{\rho(r)}{\hat{\rho}(r)}\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t) + (t - T_r^a)\frac{\rho(r)}{\hat{\rho}(r)}} \tag{4.4}$$

Therefore, the wage of productivity category i is given as

$$\lim_{\lambda \to 0} w_i(r) = \frac{\rho(i)}{\rho(r)} \lim_{\lambda \to 0} w^{\min}(r) = \frac{\frac{\rho(i)}{\hat{\rho}(r)}\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t) + (t - T_r^a)\frac{\rho(r)}{\hat{\rho}(r)}}$$

which implies that

$$\int_0^r \lim_{\lambda \to 0} w(i)\,di = \frac{\frac{1}{\hat{\rho}(r)}\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t) + (t - T_r^a)\frac{\rho(r)}{\hat{\rho}(r)}} \int_0^r \rho(i)\,di = \frac{r\left(b + \frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t) + (t - T_r^a)\frac{\rho(r)}{\hat{\rho}(r)}} \tag{4.5}$$

Defining $\hat{T}^a$ as the average tax rate of the average income of employed workers, we may write this tax rate as follows

$$\hat{T}^a = \frac{t\frac{\hat{\rho}(r)}{\rho(r)}w^{\min}(r) - D}{\frac{\hat{\rho}(r)}{\rho(r)}w^{\min}(r)} = \frac{t\frac{\hat{\rho}(r)}{\rho(r)} - (t - T_r^a)}{\frac{\hat{\rho}(r)}{\rho(r)}} = t - (t - T_r^a)\frac{\rho(r)}{\hat{\rho}(r)} \tag{4.6}$$

Inserting this into (4.5) and rearranging yields

$$\int_0^r \left(1 - \hat{T}^a\right) \lim_{\lambda \to 0} w(i) - \frac{\gamma}{\gamma+1}P^C\eta di = rb$$

so that the outcome of the bargaining in this case yields a net utility for the union which is identical to the threatpoint of the union (as one should expect). This implies that the wage of the average employed worker, $\hat{w}$, is equal to the opportunity cost of employment as perceived by the union

$$\lim_{\lambda \to 0} \hat{w} = \frac{b + \frac{\gamma}{\gamma+1}P^C\eta}{1 - \hat{T}^a}$$

The effect of the increase in the marginal tax rate (holding the average tax rate of the marginally employed worker constant) is seen to increase the average tax rate of the average employed (see (4.6)). However this implies for given pre-tax wage that the outcome is worse for the union than the threatpoint. Therefore, the pre-tax wage must increase for a solution to exist. Thus, at the margin where the employers may dictate the wage an increase in progressivity lead to an increase in the minimum wage (since this is necessary for the average wage to increase).

As already noted, the effect of an increase in the marginal tax rate (holding the average tax rate of the marginally employed worker constant) in case of a monopoly union is a reduction in the pre-tax wage. Therefore, a bargaining power (less than one) of the union exist for which the effect of an increase in the marginal tax rate does not affect the wages. This bargaining power is given as the λ for which the left- and the right-hand side of (4.3) are equal.

Finally, observe that even in the case where the union does have some bargaining power there may exist union members who receive a wage after tax that is lower than the opportunity cost of employment as perceived by the union. If the wage after tax is also lower than the opportunity cost of employment as perceived by the individual, then these persons would not participate in the labour force. Therefore, we need to make sure that this is not the case.

The difference between the opportunity cost of employment as perceived by the decentralised union and the individual is that the union includes the unemployment benefits after tax in the opportunity costs, since the unemployment caused by a minimum wage is involuntary, and therefore the workers concerned are entitled to unemployment benefits. On the other hand if the individual refuses to accept a job-offer where the wage after tax is lower than the sum of unemployment benefits after tax and the money value of foregone leisure, then the individual is voluntary unemployed and therefore not entitled to unemployment benefits. Following these considerations the participation constraint of the individual with the lowest productivity becomes

$$(1 - T_r^a)\, w^{\min} \geq \frac{\gamma}{\gamma + 1} P^C \eta$$

In the most restrictive case i.e. the limit where the employers' confederation may dictate the wage we may rewrite this constraint to become

$$\frac{\gamma}{\gamma + 1} P^C \eta \leq b \left(\frac{1 - T_{\tilde{r}}^a}{1 - t} \right) \frac{\rho(\tilde{r})}{\hat{\rho}(\tilde{r}) - \rho(\tilde{r})}$$

where $\tilde{r}$ is defined as the level of employment determined by the intersection of the minimum wage curve for $\lambda \to 0$, relation (4.4) and the demand for labour relation (3.9).

Assuming that this inequality is fulfilled implies that all potentially employed will voluntarily participate in the labour force. This may be the case even if the wage after tax is lower than the unemployment benefits after tax.

5. DATA AND EMPIRICAL MODEL

In the empirical estimation performed below we represent the individual productivity variable by standard human capital theory variables such as experience and experience squared and by lagged individual unemployment where the latter both serves as a signal of low productivity and may represent the fact that productivity may depreciate relatively during a period of unemployment.

The treatment of tax data deviates from the theoretical model presented above, since we include personal average tax rate in stead of the average tax rate of the marginally employed within the union. This difference appears because the data divide members of the labour force into members of different unemployment insurance funds. However, members of a specific fund may be paid according to different collective bargaining contracts negotiated by different collective bargaining bodies. Therefore, we have abstained from defining a marginally employed within each unemployment insurance fund. With this qualification the estimated equation is a log-linearized version of (4.1). We estimate this equation for 7 major unions in Denmark using panel data.

The sample used is a panel sample of Danish wage earners covering the 11-year period 1980-90, but since lagged variables are included in the analysis, the estimation period is 1981-90. The sample is a sub-sample of the Danish longitudinal database which is a representative 5 per cent sample of the Danish population. Self-employed persons and assisting wives have been excluded from the sample because reliable wage data are only available for wage earners. Furthermore, the analysis is restricted to workers employed in the private sector since we expect the wage formation in the public sector to be generated by other (political) forces than in the private sector.

We do not have explicit information on the type of wage contract or union membership for each individual. However, we have information on which unemployment insurance fund the individual belongs to. In Denmark the unions administer unemployment insurance funds. Thus, membership of a given UI fund is closely related to being member of a given union. The data includes information on 7 union (UI-fund) groups: unskilled workers (organised in the Unskilled Workers' Union, SID), unskilled female workers (organised in (unskilled) Female Workers' Union, KAD), skilled industrial workers (organised in Metal Workers' Union, METAL), skilled clerical workers (organised in Clerical Workers' Union, HK), building and construction workers (organised in the cartel of unions in the building and construction sector, BAT), and academic workers (organised in the cartel of unions organising academics, AC), and all other unions. The sample is restricted to members of the first six union groups, such that the very heterogeneous group of 'all other unions' is excluded.

The sample is an unbalanced panel, where (mainly young) people enter the sample over time and (mainly older) people leave the sample. Only observations with an observed hourly wage rate are included in the analysis. This means that people who leave the labour force or people who are unemployed for the whole year are excluded from the estimations. This may give rise to a self-selectivity problem, which is often handled by the traditional Heckman selectivity correction. However, we do not correct for the potential selectivity problem in this study.

The hourly wage rate is not observed directly but has to be constructed from annual wage income, divided by working hours, which is calculated from the register on supplementary pension payments (ATP). Annual earnings include overtime pay and to the extent that overtime hours vary over time (or business cycles) variations in the hourly wage rate may also reflect variations in overtime work. The wage rate includes holiday payments but does not include pension payments paid by the employer to a pension scheme. Since the growth of labour market pension schemes did not take place until late in the 1980s, we do not expect that the observed wage rates during the 1980s (contrary to the 1990s) to be influenced much by employers' pension payments.

The marginal and average tax rates (t and T^a) are calculated for each individual on the basis of information on taxable income, and other income information used by the tax authorities when calculating the tax amounts to be paid. Since the information is based on the same administrative registers as the registers used by the tax authorities, we expect the information on marginal and average tax rates to be quite reliable. The wage income of the individual is considered marginal to all other income. Thus, the marginal tax rate is the marginal tax rate on marginal income for the person, after taxes on positive or negative capital income and other sources of income taxes have been paid. The average tax rate is calculated as the average tax rate paid on the individual's observed wage income.

The aggregated unemployment rate is measured by the unemployment rate in the union, to which the person belongs. This variable is based on public statistics covering the whole labour market, including the public sector. The unemployment insurance compensation is measured by the potential hourly compensation rate net of taxes. The potential compensation rate is calculated as 90 per cent of the individual's wage rate up to a maximum, which corresponds to the maximum compensation in the UI scheme for each year. The tax rate on UI payments is calculated as the average tax to be paid by the individual if he or she received no wage income and was unemployed for the whole year.

All variables are measured in nominal terms. Therefore, we include the consumer price index as an explanatory variable in the wage regression. Furthermore, we include different human capital variables in order to capture individual variation in productivity and productivity growth. The traditional variables to include are experience, experience squared, and education. Experience squared is entered into the estimation to allow for a non-linear relationship between wage and experience. One would expect a negative sign of the squared experience coefficient reflecting a decreasing marginal importance of experience in the determination of wages. As we estimate a fixed effect transformation, the time invariant variable for length of education disappears from the regression. Beside these variables the lagged individual unemployment variable is included in order to reflect individual human capital effects, either because previous unemployment implies depreciation of human capital or because it is conceived of as productivity effects to the employer. The variables representing the lagged individual unemployment rate and the aggregated union unemployment rate are measured on a scale from 0 to 1. Since we exclude individuals who are unemployed for the whole year in our analysis because we have no wage information for these individuals, the aggregated unemployment rate in a given union is higher than the average individual unemployment rates in the union.

Sample means for the variables used in the estimations are shown in Table 1 for each union group. The sample means concern the year 1990.

Table 1. Sample means in 1990
(Standard errors in parentheses)

Men	Unskilled (SID)	Metal	Clerical (HK)	Building & Constr.	Academic (AC)
Age	38.209 *(11.714)*	36.609 *(10.848)*	35.160 *(11.838)*	36.310 *(10.299)*	38.280 *(9.167)*
Other income/DKK 100,000	0.083 *(0.162)*	0.089 *(0.190)*	0.080 *(0.202)*	0.121 *(0.168)*	0.220 *(0.370)*
Negative wealth/DKK 100,000	1.895 *(1.940)*	2.164 *(2.028)*	2.209 *(2.253)*	2.375 *(2.113)*	3.963 *(2.907)*
Positive wealth/DKK 100,000	2.553 *(2.557)*	2.994 *(2.680)*	2.733 *(2.808)*	3.168 *(2.726)*	4.761 *(3.471)*
Province (0/1)	0.795 *(0.404)*	0.761 *(0.427)*	0.706 *(0.456)*	0.748 *(0.434)*	0.428 *(0.495)*
Years of education	9.334 *(2.094)*	11.256 *(1.607)*	11.578 *(1.805)*	11.536 *(1.405)*	16.656 *(2.028)*
Married (0/1)	0.480 *(0.500)*	0.478 *(0.500)*	0.422 *(0.495)*	0.496 *(0.500)*	0.548 *(0.498)*
Owner of house (0/1)	0.571 *(0.495)*	0.636 *(0.481)*	0.544 *(0.499)*	0.679 *(0.467)*	0.737 *(0.441)*
Experience	16.691 *(11.237)*	15.879 *(9.480)*	15.181 *(10.976)*	14.392 *(7.879)*	13.047 *(8.812)*
Hourly wage, DKK	123.773 *(32.686)*	128.322 *(28.390)*	119.421 *(41.127)*	132.786 *(34.958)*	206.642 *(59.787)*
Log hourly pot. comp. rate after tax, DKK	4.021 *(0.147)*	4.037 *(0.156)*	4.020 *(0.160)*	4.050 *(0.165)*	4.113 *(0.192)*
Lagged individual unemployment	0.082 *(0.162)*	0.032 *(0.101)*	0.041 *(0.137)*	0.077 *(0.148)*	0.026 *(0.094)*
Union unemployment rate	0.160 *(-)*	0.071 *(-)*	0.091 *(-)*	0.110 *(-)*	0.124 *(-)*
Average tax rate, t^{ave}	0.377 *(0.086)*	0.370 *(0.081)*	0.369 *(0.083)*	0.373 *(0.086)*	0.448 *(0.089)*
Marginal tax rate, t^{m}	0.585 *(0.057)*	0.595 *(0.057)*	0.589 *(0.067)*	0.572 *(0.029)*	0.673 *(0.048)*
Number of individuals in 1990	1558	866	419	655	407

Table 1. (continued)

Women	Unskilled (SID)	Female Union (KAD)	Clerical (HK)
Age	37.909	38.746	36.530
	(11.909)	*(11.886)*	*(11.341)*
Other income/DKK100,000	0.083	0.068	0.066
	(0.188)	*(0.137)*	*(0.185)*
Negative wealth/DKK100,000	1.076	1.081	1.230
	(1.636)	*(1.585)*	*(1.682)*
Positive wealth/DKK100,000	1.142	1.182	1.463
	(1.831)	*(1.894)*	*(1.993)*
Province (0/1)	0.801	0.778	0.627
	(0.400)	*(0.416)*	*(0.484)*
Years of education	9.307	8.879	11.086
	(2.083)	*(1.914)*	*(1.810)*
Married (0/1)	0.494	0.574	0.488
	(0.501)	*(0.495)*	*(0.500)*
Owner of house (0/1)	0.301	0.320	0.362
	(0.460)	*(0.467)*	*(0.481)*
Experience	10.981	11.360	12.164
	(7.084)	*(7.332)*	*(6.913)*
Hourly wage, DKK	103.444	108.396	105.116
	(18.861)	*(25.170)*	*(33.674)*
Log hourly pot. comp. rate after tax, DKK	3.930	3.951	3.933
	(0.122)	*(0.115)*	*(0.145)*
Lagged individual unemployment	0.095	0.087	0.038
	(0.170)	*(0.169)*	*(0.119)*
Union unemployment rate	0.309	0.196	0.126
	(-)	*(-)*	*(-)*
Average tax rate, t^{ave}	0.390	0.364	0.365
	(0.086)	*(0.087)*	*(0.083)*
Marginal tax rate, t^{m}	0.556	0.551	0.558
	(0.037)	*(0.042)*	*(0.054)*
Number of individuals in 1990	176	397	1214

The explanatory variables, marginal and average tax rates, are endogenous since the tax rates depend on the wage level of the individual. Following the theoretical model in the previous section, the same holds for the aggregated union unemployment rate. In order to take care of this problem, these variables are instrumented. The instruments used are: Age, age squared, length of education, other non-wage income, negative wealth, positive wealth, indicators for residence in the province, marriage, ownership of house and year-specific indicators. We regress the calculated average and marginal tax rates on the instruments using OLS (pooled cross-sections). Since the union unemployment rate does not vary across unions, by definition, and since the explanatory power of the variables used as instruments for the tax variables is extremely low, the aggregated union unemployment variable has been instrumented slightly different. Instead of year specific indicators, we have used a trend variable (and second and third order polynomials of the trend variable), and furthermore, the lagged union unemployment rate has been added as an instrument. In order to test for the validity of each of the instruments, we have regressed the calculated residual term from the wage equation on the instruments. If the coefficient of an instrument is significant in this residual regression, the instrument may not be valid. As it is shown in Table 2, most of the instruments pass this test but there seem to be minor problems with the age variables. In a number of other experiments, we have tried different specifications of the model, mainly by varying the price and productivity variables included in the wage regressions and experimenting with the instruments used. If (average) productivity is included in the wage regression, the regressions become extremely unstable with respect to size and sign of the other included variables in the wage equation. As an alternative to the fixed effect estimation we have estimated the model in one-year differences. As expected, this does not change the results notably, and since the fixed effect estimation is exploiting the panel information in the sample more intensively and efficiently, we prefer the fixed effect transformation. The estimated wage equation has the form

$$\log w_{it} = \alpha_i + x_{it}\beta + z_i\gamma + \varepsilon_{it}$$

where x_{it} and z_i are vectors of explanatory time varying and time constant variables, respectively, β and γ are vectors of parameters, and a_i is an individual constant term which captures unobserved time constant heterogeneity. The fixed effect transformation, given by

$$\log w_{it} - \log w_i = (x_{it} - x_i)\,\beta + \varepsilon_{it} - \varepsilon_i$$

where x_i is the individual specific average of x_{it}, eliminates the individual specific intercepts as well as the time constant variables.

Table 2. Test of Instruments. Dependent variable Residual Error Term. (Standard errors in parentheses)

Men	Unskilled (SID)	Metal	Clerical (HK)	Building & Constr.	Academic (AC)
Age	-0.001	-0.008	-0.004	-0.008	0.000
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.002)*
Age squared/100	0.000	0.009	0.004	0.008	-0.000
	(0.001)	*(0.001)*	*(0.002)*	*(0.002)*	*(0.002)*
Other income/DKK100,000	0.027	-0.001	-0.002	0.004	0.006
	(0.009)	*(0.001)*	*(0.012)*	*(0.012)*	*(0.006)*
Negative wealth/DKK100,000	0.001	-0.001	-0.001	0.000	0.001
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
Positive wealth/DKK100,000	-0.000	0.001	0.004	0.002	0.001
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
Province (0/1)	0.001	0.001	-0.001	0.001	0.002
	(0.003)	*(0.003)*	*(0.004)*	*(0.004)*	*(0.004)*
Years of education	-0.000	-0.000	-0.000	0.000	0.000
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
Married (0/1)	-0.003	-0.005	-0.004	-0.001	0.006
	(0.003)	*(0.003)*	*(0.005)*	*(0.004)*	*(0.005)*
Owner of house (0/1)	-0.007	-0.004	-0.016	-0.012	0.006
	(0.004)	*(0.004)*	*(0.006)*	*(0.006)*	*(0.008)*
R^2	0.002	0.017	0.010	0.011	0.005

Women	Unskilled (SID)	Female Union (KAD)	Clerical (HK)
Age	0.004	-0.002	-0.001
	(0.003)	*(0.002)*	*(0.001)*
Age squared/100	-0.005	0.001	0.000
	(0.003)	*(0.002)*	*(0.001)*
Other income/DKK100,000	-0.033	0.068	0.014
	(0.024)	*(0.019)*	*(0.010)*
Negative wealth/DKK100,000	0.002	0.010	0.001
	(0.004)	*(0.003)*	*(0.002)*
Positive wealth/DKK100,000	0.004	-0.003	-0.001
	(0.003)	*(0.003)*	*(0.001)*
Province (0/1)	0.012	-0.006	-0.006
	(0.008)	*(0.005)*	*(0.003)*
Year of education	-0.000	-0.002	0.001
	(0.002)	*(0.001)*	*(0.001)*
Married (0/1)	-0.012	0.005	0.011
	(0.008)	*(0.005)*	*(0.003)*
Owner of house (0/1)	-0.021	-0.022	-0.003
	(0.012)	*(0.009)*	*(0.005)*
R^2	0.008	0.009	0.002

6. RESULTS

The results from the fixed effects estimations of wage functions for separate gender and union categories are shown in Table 3. In other empirical studies of human capital functions, see e.g. Rosholm and Smith (1996), it is usually found that the coefficients to the experience and experience squared variables are positive and negative, respectively, and numerically larger for men than for women. The results in Table 3 are in line with the results found in other empirical studies. The coefficient of the experience squared variable is significantly negative for men, but insignificant for women. However, the career profile may be captured relatively poorly because we do not have information on firm-specific tenure, promotion etc. Therefore, it might be suspected that career effects are also partly captured by the consumer price index variable, log CPI, if variation in this variable is positively correlated with individual wage increases. A priori the coefficient to the CPI variable should be less than one[6], but for the academic union, the coefficient is significantly larger than one. This may indicate that individual wage increases over the career profile during the observation period have been captured by the macro variable for the consumer prices. The CPI coefficient is significantly smaller than one for female unions.

Lagged individual unemployment generally has a negative influence on the wage rate. The general impression from Table 3 is that the union unemployment rate is dominating the individual unemployment rate in the male unions, but still individual unemployment seems to be a strong negative signal or to have negative human capital effects for academic and clerical male workers. For women the negative effects of individual and union unemployment are mainly pronounced for the very heterogeneous group of clerical workers, while the unemployment variables are insignificant for the female union (KAD).

In all union groups, male as well as female, the log hourly compensation rate of the unemployment insurance scheme has a highly significant and positive effect. The effect is considerably larger for the female union groups. This result may indicate that the unemployment compensation rate, which in Denmark is 90 per cent of the wage rate for low-wage groups (but considerably lower for high-wage groups due to the relatively low flat level) works like a floor on the wage distribution. Since a larger fraction of the women in the three union groups included in this study are situated in the lower part of the wage

[6] It should be less than one due to the fact that unemployment benefits after tax are measured in nominal terms and therefore include some of the price effect.

distribution, this may be an explanation of the large positive coefficient of the compensation rate.

In all gender and union groups, the coefficient of the average tax variable $(1 - T^a)$ is significantly negative. Thus, an increase of the average tax rate tends to increase the wage. However, the result is a rather uncontroversial finding since this is in agreement with most theoretical models based on either union behaviour, search theory, efficiency wage setting or competitive labour markets.

The results in Table 3 show that the empirical results concerning the effect of the marginal tax rate are not unambiguous. In all male union groups except building and construction, the coefficient of $(1 - t)$ is significantly positive in line with the standard theoretical result in models of imperfect labour markets. In the three female groups, the coefficient is insignificant. Finally, the coefficient is significantly negative for male workers of the Building and Construction Union, indicating either a competitive labour market or an imperfect labour market where the heterogeneity of the union members implies that the traditional sign of the effect is reversed as explained in the previous section. Thus, the standard result of an imperfect labour market is mainly confirmed for the male unions, while the female unions are either dominated by heterogeneity effect in the productivity or a more competitive labour market, so that the traditional sign of an increase in the marginal tax rate in an imperfect labour market is reversed. It is not obvious that the union power should differ between men and women in the unions organising both gender, e.g. Unskilled Workers' Union (SiD) and Clerical Workers' Union (HK). On the contrary it is more plausible that the results in Table 3 are driven by differences in the distributions of productivity between union members. This explanation probably also applies to the workers in the Building and Construction Cartel.

To conclude, the empirical results found in this study seem to give a more ambiguous picture compared to earlier Danish empirical research based on macro data presented in Hansen *et al.* (1995) and Lockwood *et al.* (1995) concerning support to the view that progressive taxes may be wage moderating. This result only seems to hold in some of the Danish unions and mainly for the male labour market. Even in these cases, the effects seem to be much more limited than in the macro studies.

Table 3. Fixed effect estimation of wage function. Dependent variable log hourly wage rate (standard errors in parentheses)

Men	Unskilled (SID)	Metal	Clerical (HK)	Building & Constr.	Academic (AC)
Experience	0.016	0.018	0.049	-0.000	0.046
	(0.002)	*(0.003)*	*(0.004)*	*(0.004)*	*(0.004)*
Experience squared/100	-0.027	-0.036	-0.076	-0.036	-0.100
	(0.003)	*(0.004)*	*(0.005)*	*(0.005)*	*(0.006)*
LogCPI	0.752	0.952	0.929	1.100	1.397
	(0.044)	*(0.063)*	*(0.087)*	*(0.070)*	*(0.091)*
Log Hourly potential UI compensation, DKK	0.461	0.307	0.352	0.376	0.212
	(0.018)	*(0.022)*	*(0.027)*	*(0.026)*	*(0.025)*
Lagged individual unemployment [0,1]	-0.007	-0.036	-0.150	0.024	-0.159
	(0.011)	*(0.017)*	*(0.028)*	*(0.018)*	*(0.032)*
Union unemployment [0,1]	-0.403	-0.632	-1.451	-3.065	-0.387
	(0.110)	*(0.190)*	*(0.286)*	*(0.290)*	*(0.099)*
$\text{Log}(1-t^{ave})$	-0.435	-0.359	-0.197	-0.164	-0.270
	(0.032)	*(0.044)*	*(0.056)*	*(0.042)*	*(0.040)*
$\text{Log}(1-t^{m})$	0.185	0.426	0.388	-0.173	0.498
	(0.083)	*(0.106)*	*(0.108)*	*(0.078)*	*(0.078)*
R^2	0.58	0.67	0.70	0.63	0.79
Number of observations	14335	7697	4085	6233	2987
Explanatory power, Instrument regression					
R^2, $\text{Log}(1-t^{ave})$ regression	0.42	0.55	0.51	0.51	0.53
R^2, $\text{Log}(1-t^{m})$ regression	0.14	0.19	0.31	0.24	0.31
R^2, Union unemployment regression	0.87	0.73	0.83	0.76	0.93

Table 3. (continued)

Women	Unskilled (SID)	Female Union (KAD)	Clerical (HK)
Experience	0.005 *(0.010)*	-0.001 *(0.005)*	0.002 *(0.003)*
Experience squared/100	0.010 *(0.017)*	0.011 *(0.009)*	-0.008 *(0.006)*
LogCPI	0.397 *(0.147)*	0.794 *(0.084)*	0.304 *(0.047)*
Log Hourly potential UI compensation, DKK	0.618 *(0.061)*	0.476 *(0.037)*	0.935 *(0.019)*
Lagged individual unemployment [0,1]	-0.099 *(0.038)*	-0.007 *(0.025)*	-0.085 *(0.018)*
Union unemployment [0,1]	-0.281 *(0.573)*	0.819 *(0.433)*	-0.467 *(0.179)*
$\text{Log}(1-t^{ave})$	-0.864 *(0.114)*	-0.651 *(0.068)*	-0.947 *(0.042)*
$\text{Log}(1-t^{m})$	-0.366 *(0.189)*	0.201 *(0.163)*	-0.138 *(0.073)*
R^2	0.61	0.57	0.64
Number of observations	1109	3419	10339
Explanatory power, Instrument regression			
R^2, $\text{Log}(1-t^{ave})$ regression	0.29	0.25	0.35
R^2, $\text{Log}(1-t^{m})$ regression	0.18	0.11	0.23
R^2, Union unemployment regression	0.73	0.64	0.75

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Appendix 1: Derivation of the Nash-Bargaining Solution

The maximisation problem is given as

$$\arg\max_r \left(V-\bar{V}\right)^{\lambda}\left(\frac{\Pi}{P^C}\right)^{(1-\lambda)} = \arg\max_r \left(\lambda \log\left(V-\bar{V}\right) + (1-\lambda)\log\left(\frac{\Pi}{P^C}\right)\right)$$

where the two object functions $\left(V-\bar{V}\right)$ and $\left(\frac{\Pi}{P^C}\right)$ are defined in the relations (3.11) and (3.12).

Differentiation of (3.11) with respect to the marginally employed category of productivity yields

$$\begin{aligned}
&\frac{\partial}{\partial r}\left((1-t)\,\alpha\frac{p}{P^C}\left(\int_0^r \rho(i)\,di\right)^{\alpha} + \frac{r}{P^C}\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)\right) \\
= \quad & (1-t)\,\alpha^2\frac{p}{P^C}\left(\int_0^r \rho(i)\,di\right)^{\alpha-1}\rho(r) + \frac{1}{P^C}\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)
\end{aligned}$$

Therefore, differentiation the logarithm of (3.11) yields

$$\frac{(1-t)\,\alpha^2\frac{p}{P^C}\left(\int_0^r \rho(i)\,di\right)^{\alpha-1}\rho(r) + \frac{1}{P^C}\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t)\,\alpha\frac{p}{P^C}\left(\int_0^r \rho(i)\,di\right)^{\alpha} + \frac{r}{P^C}\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)} =$$

$$\frac{(1-t)\,\alpha^2 p\left(\int_0^r \rho(i)\,di\right)^{\alpha-1}\rho(r) + \left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)}{(1-t)\,\alpha p\left(\int_0^r \rho(i)\,di\right)^{\alpha} + r\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)}$$

A similar procedure for the object function of the employers (3.12) implies that

$$\frac{\partial}{\partial r}\frac{(1-\alpha)\,p\left(\int_0^r \rho(i)\;di\right)^{\alpha}}{P^C} = \alpha(1-\alpha)\,\frac{p}{P^C}\left(\int_0^r \rho(i)\;di\right)^{\alpha-1}\rho(r)$$

such that differentiation of the logarithm of (3.12) yields

$$\frac{\alpha(1-\alpha)\,\frac{p}{P^C}\left(\int_0^r \rho(i)\;di\right)^{\alpha-1}\rho(r)}{\frac{(1-\alpha)p\left(\int_0^r \rho(i)\;di\right)^{\alpha}}{P^C}} = \frac{\alpha\rho(r)}{\int_0^r \rho(i)\;di}$$

The first order condition to the Nash-bargaining problem may be written as

$$\lambda\frac{(1-t)\,\alpha^2 p\left(\int_0^r \rho(i)\,di\right)^{\alpha-1}\rho(r) + \left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)}{\left((1-t)\,\alpha p\left(\int_0^r \rho(i)\,di\right)^{\alpha} + r\left(D-b-\frac{\gamma}{\gamma+1}P^C\eta\right)\right)} = -(1-\lambda)\left(\frac{\alpha\rho(r)}{\int_0^r \rho(i)\;di}\right)$$

$\Leftrightarrow$

$$\begin{aligned}
& \lambda (1-t)\, \alpha^2 p \left(\int_0^r \rho(i)\, di\right)^{\alpha-1} \rho(r) - \lambda \left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \\
= \;& -(1-\lambda) \left(\frac{\alpha \rho(r)}{\int_0^r \rho(i)\;\, di}\right) \left((1-t)\, \alpha p \left(\int_0^r \rho(i)\, di\right)^{\alpha} \right. \\
& \left. - r\left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right)\right)
\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}
& \lambda (1-t)\, \rho(r)\, \alpha^2 p \left(\int_0^r \rho(i)\, di\right)^{\alpha-1} - \lambda \left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \\
= \;& -(1-\lambda) \left(\alpha \rho(r)\right) \left((1-t)\, \alpha p \left(\int_0^r \rho(i)\, di\right)^{\alpha-1} \right. \\
& \left. - r\left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \left(\int_0^r \rho(i)\;\, di\right)^{-1}\right)
\end{aligned}$$

Inserting the demand relationship (3.9) repeated below for convenience yields

$$\alpha \left(\int_0^r \rho(i)\;\, di\right)^{\alpha-1} = \frac{w^{\min}}{\rho(r)\, p}$$

$$\begin{aligned}
& \lambda (1-t)\, \rho(r)\, \alpha \frac{w^{\min}}{\rho(r)} - \lambda \left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \\
= \;& -(1-\lambda)\, \alpha \rho(r) \left((1-t) \frac{w^{\min}}{\rho(r)} - r\left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \left(\int_0^r \rho(i)\;\, di\right)^{-1}\right)
\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}
& \lambda (1-t)\, \alpha \frac{w^{\min}}{\rho(r)} - \lambda \left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) + (1-\lambda)\, \rho(r)\, (1-t)\, \alpha \frac{w^{\min}}{\rho(r)} \\
= \;& -(1-\lambda)\, \alpha \rho(r) \left(-r\left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \left(\int_0^r \rho(i)\;\, di\right)^{-1}\right)
\end{aligned}$$

$$\begin{aligned}
& \rho(r)\, (1-t)\, \alpha \frac{w^{\min}}{\rho(r)} = \\
& (1-\lambda)\, \alpha \rho(r) \left(r\left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right) \left(\int_0^r \rho(i)\;\, di\right)^{-1}\right) \\
& + \lambda \left(b - D + \frac{\gamma}{\gamma+1} P^C \eta\right)
\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}
& (1-t)\,\alpha w^{\min} \\
= \quad & (1-\lambda)\,\alpha\rho\,(r)\left(r\left(b-D+\frac{\gamma}{\gamma+1}P^{C}\eta\right)\left(\int_0^r \rho\,(i)\;\; di\right)^{-1}\right) \\
& +\lambda\left(b-D+\frac{\gamma}{\gamma+1}P^{C}\eta\right)
\end{aligned}$$

The final expression is written as

$$w^{\min}\,(r) = \frac{1}{(1-t)\,\alpha}\left((1-\lambda)\,r\frac{\alpha\rho\,(r)}{\int_0^r \rho\,(i)\;di} + \lambda\right)\left(b-D+\frac{\gamma}{\gamma+1}P^{C}\eta\right)$$

Appendix 2: Proof of existence of a partial equilibrium

The idea is to show that demand relationship (3.9) implies a higher minimum wage for the employment going to zero than the minimum wage curve (3.14), similarly for the employment going towards full employment the demand relationship implies a lower minimum wage than the minimum wage curve. By continuity this implies existence of a partial equilibrium.

First, consider the minimum wage curve. Consider the limit for $r \to 0$. Applying L'Hospital's rule we find the follow minimum wage in this limit

$$\lim_{r\to 0} w^{\min}(r) = \frac{1}{(1-t)\,\alpha}\left((1-\lambda)\,\alpha + \lambda\right)\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right)$$

This minimum wage is identical to the standard wage in the case where workers do not differ in productivity.

Next, consider the limit of the minimum wage curve for $r \to 1$, i.e. in full employment In this case the minimum wage rate becomes

$$\lim_{r\to 1} w^{\min}(r) = \frac{1}{(1-t)\,\alpha}\left((1-\lambda)\frac{\alpha\rho(1)}{\int_0^1 \rho(i)\;di} + \lambda\right)\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right) \quad (*)$$

Observe that since $\rho'(i) < 0$ for all i, then

$$\lim_{r\to 0} w^{\min}(r) > \lim_{r\to 1} w^{\min}(r)$$

The relation for the demand for labour which is repeated below.

$$\frac{\rho(r)\,p\alpha}{\left(\int_0^r \rho(i)\,h\,di\right)^{1-\alpha}} = w^{\min}$$

Considering the limit $r \to 0$ and using the condition that $\rho(0) = \bar{\rho}$, one immediately gets

$$\lim_{r\to 0} w^{\min}(r) = \infty$$

The limit for $r \to 1$ is given as

$$\lim_{r\to 1} w^{\min}(r) = \frac{\rho(1)\,p\alpha}{\left(\int_0^1 \rho(i)\;di\right)^{1-\alpha}} \quad (**)$$

To conclude, we find that in the limit for $r \to 0$ the minimum wage determined from the demand side exceeds the minimum wage determined from the minimum wage curve. In the limit for $r \to 1$ we have to compare (*) to (**). For $0 < \lambda < 1$ a sufficient condition for the minimum wage determined from the minimum wage curve to exceed the minimum wage determined from the demand side is that $\rho(1) = 0$, i.e. that the least productive in the economy cannot contribute to production. Given this condition a partial equilibrium exists.

Appendix 3: Proof of uniqueness of a partial equilibrium

To prove uniqueness we show that the slope of the demand determined minimum wage is more negative than the slope of the minimum wage in a partial equilibrium. Therefore the equilibrium is unique. The result holds subject to the following sufficient condition

$$\frac{\rho(r)}{\hat{\rho}(r)} - \frac{r\rho'(r)}{\rho(r)} > 1$$

Consider the minimum wage curve

$$\begin{aligned} w^{\min} &= \frac{1}{(1-t)\alpha}\left((1-\lambda)r\frac{\alpha\rho(r)}{\int_0^r \rho(i)\ di} + \lambda\right)\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right) \\ &= \left((1-\lambda)\frac{\alpha\rho(r)}{\hat{\rho}(r)} + \lambda\right)\hat{S} \end{aligned}$$

where

$$\hat{S} = \frac{1}{(1-t)\alpha}\left(b - D + \frac{\gamma}{\gamma+1}P^C\eta\right)$$

$$\frac{\partial w^{\min}}{dr} = \alpha\frac{(\rho(r) + r\rho'(r))\int_0^r \rho(i)\ di - r\rho(r)\rho(r)}{\left(\int_0^r \rho(i)\ di\right)^2}S, \quad S = (1-\lambda)\hat{S}$$

The demand determined minimum wage is given by

$$w^{\min} = \rho(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-1}$$

$$\begin{aligned} &\frac{\partial\rho(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-1}}{\partial r} \\ =\ &\rho'(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-1} + \rho(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-2}(\alpha-1)\rho(r) \end{aligned}$$

We want to prove that

$$\begin{aligned} &\rho'(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-1} + \rho(r)p\alpha\left(\int_0^r \rho(i)\ di\right)^{\alpha-2}(\alpha-1)\rho(r) \\ <\ &\frac{(\rho(r) + r\rho'(r))\int_0^r \rho(i)\ di - r\rho(r)\rho(r)}{\left(\int_0^r \rho(i)\ di\right)^2}\alpha S \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}&\rho'(r)\left(\int_0^r \rho(i)\,di\right)^{\alpha+1} + \rho(r)\left(\int_0^r \rho(i)\,di\right)^{\alpha}(\alpha-1)\rho(r)\\ < \;&\left((\rho(r)+r\rho'(r))\int_0^r \rho(i)\,di - r\rho(r)\rho(r)\right)\frac{S}{p}\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}&\rho'(r)(r\hat{\rho}(r))^{\alpha+1} + \rho(r)(r\hat{\rho}(r))^{\alpha}(\alpha-1)\rho(r)\\ < \;&((\rho(r)+r\rho'(r))\,r\hat{\rho}(r) - r\rho(r)\rho(r))\frac{S}{p}\end{aligned}$$

$\Leftrightarrow$

$$\left(\frac{|\rho'(r)|\,r}{\rho(r)} + (1-\alpha)\frac{\rho(r)}{\hat{\rho}(r)}\right)\frac{(r\hat{\rho}(r))^{\alpha}}{r} > \left(\frac{\rho(r)}{\hat{\rho}(r)} + \frac{r\,|\rho'(r)|}{\rho(r)} - 1\right)\frac{S}{p}$$

$\Leftrightarrow$

$$\left(\frac{|\rho'(r)|\,r}{\rho(r)} + \frac{\rho(r)}{\hat{\rho}(r)} - \alpha\frac{\rho(r)}{\hat{\rho}(r)}\right)\frac{(r\hat{\rho}(r))^{\alpha}}{r} > \left(\frac{\rho(r)}{\hat{\rho}(r)} + \frac{r\,|\rho'(r)|}{\rho(r)} - 1\right)\frac{S}{p}$$

Define

$$x = \left(\frac{\rho(r)}{\hat{\rho}(r)} + \frac{r\,|\rho'(r)|}{\rho(r)} - 1\right) > 0$$

where the inequality follows from the sufficient condition stated above. Inserting this into the previous expression yields

$$\left(x + 1 - \alpha\frac{\rho(r)}{\hat{\rho}(r)}\right)\frac{(r\hat{\rho}(r))^{\alpha}}{r} > x\frac{S}{p}$$

$\Leftrightarrow$

$$x\frac{(r\hat{\rho}(r))^{\alpha}}{r} + \left(1 - \alpha\frac{\rho(r)}{\hat{\rho}(r)}\right)\frac{(r\hat{\rho}(r))^{\alpha}}{r} > x\frac{S}{p}$$

$\Leftrightarrow$

$$\left(1 - \alpha\frac{\rho(r)}{\hat{\rho}(r)}\right)\frac{(r\hat{\rho}(r))^{\alpha}}{r} > x\left(\frac{S}{p} - \frac{(r\hat{\rho}(r))^{\alpha}}{r}\right)$$

Which is fulfilled if:

$$\frac{S}{p} \leq \frac{(r\hat{\rho}(r))^{\alpha}}{r}$$

Inserting the definition of S yields

$$\frac{1-\lambda}{(1-t)\,\alpha}\left(\frac{b-D}{p} + \frac{\gamma}{\gamma+1}\frac{P^C}{p}\eta\right)r \leq (r\hat{\rho}(r))^{\alpha}$$

Exploiting the fact that we analyse a partial equilibrium, we now introduce the two relations for the minimum wage. The minimum wage curve is repeated as

$$\frac{w^{\min}}{p} = \frac{1}{(1-t)\,\alpha}\left((1-\lambda)\,\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \lambda\right)\left(\frac{b-D}{p} + \frac{\gamma}{\gamma+1}\frac{P^{C}}{p}\eta\right)$$

such that

$$\frac{w^{\min}}{p} = \left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right)\frac{S}{p}$$

Letting this expression equal to the demand determined minimum wage yields

$$\frac{w^{\min}}{p} = \rho\,(r)\,\alpha\left(\int_0^r \rho\,(i)\;di\right)^{\alpha-1} = \left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right)\frac{S}{p}$$

$\Leftrightarrow$

$$\rho\,(r)\,\alpha\,(r\hat{\rho}\,(r))^{\alpha-1} = \left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right)\frac{S}{p}$$

$\Leftrightarrow$

$$\frac{\rho\,(r)}{r\hat{\rho}\,(r)}\alpha\,(r\hat{\rho}\,(r))^{\alpha} = \left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right)\frac{S}{p}$$

$\Leftrightarrow$

$$(r\hat{\rho}\,(r))^{\alpha} = \frac{r\hat{\rho}\,(r)}{\alpha\rho\,(r)}\left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right)\frac{S}{p}$$

Recalling that we need to prove that

$$\frac{S}{p} \leq \frac{(r\hat{\rho}\,(r))^{\alpha}}{r}$$

and inserting the previous relation yields

$$\frac{\hat{\rho}\,(r)}{\alpha\rho\,(r)}\left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right) \geq 1$$

$\Leftrightarrow$

$$\left(\frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)} + \frac{\lambda}{(1-\lambda)}\right) \geq \frac{\alpha\rho\,(r)}{\hat{\rho}\,(r)}$$

$\Leftrightarrow$

$$\frac{\lambda}{(1-\lambda)} \geq 0$$

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